Nucleon axial form factors from two-flavour Lattice QCD

In Collaboration with:
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Outline

- Structure of the axial matrix elements of nucleon
- Lattice calculation of axial matrix elements
- Extraction of Form factors
  - Plateau method
  - Summation method
  - Excited state ansatz
- Results on \( G_A(Q^2), \langle r_A^2 \rangle \)
- Results on \( G_P(Q^2) \)
- Conclusions
Axial structure of the Nucleon

From Lorenz invariance, CPT and Isospin symmetry,

\[ \langle N, p' | A^\mu (x) | N, p \rangle = e^{iq \cdot x} \bar{u}(p') \left( \gamma^\mu \gamma^5 G_A(q^2) + \gamma^5 \frac{q^\mu}{2m} G_P(q^2) \right) u(p) \]

Axial structure strongly influenced by chiral symmetry breaking,

\[ g_A/g_V = 1.2701(25) \quad \text{Ref: PDG 2012} \]

\[ Q^2 \sim 1 \text{ GeV}^2 \]

\[ G_A \approx \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \quad \langle r_A^2 \rangle = -\frac{6}{G_A(0)} \frac{dG_A(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]
Axial structure of the Nucleon

Combined Electroproduction data

$\nu$ scattering
$M_A = 1.026 \pm 0.021$ GeV
$\langle r_A^2 \rangle = 0.443 \pm 0.028$ fm

$e - \pi^+$ scattering
$M_A = 1.069 \pm 0.016$ GeV
$\langle r_A^2 \rangle = 0.408 \pm 0.020$ fm

$\Delta M_A = 0.043 \pm 0.026$ GeV

Nucleon matrix elements on the Lattice

- Standard Nucleon operator with covariant gaussian smearing at source and sink.
- Sequential inversion with fixed sink and momentum injected at the operation insertion.
- Calculation performed at several $t_s$, quark masses and volumes

\[ C_3 \sim \frac{e^{-E_q t}}{2E_q} \frac{e^{-m(ts-t)}}{2m} \text{Tr}[\Gamma_{\alpha\beta}\langle N, -\vec{p}, s|A^\mu|N, 0, s'\rangle_{\alpha\beta}] \]
# CLS Ensembles

\( N_f = 2 \) non-perturbatively \( \mathcal{O}(a) \) improved gauge configurations

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[Coordinated Lattice Simulations](https://twiki.cern.ch/twiki/bin/view/CLS/WebHome)
Issues in Form factor Extraction

- Matrix elements extracted from conveniently defined ratio as,

\[
R_{A_{\mu}} = \frac{C_{3,A_{\mu}}(q^2, t, t_s)}{C_2(q^2, t, t_s)} \sqrt{\frac{C_2(q^2, t, t_s - t)}{C_2(0, t, t_s - t)}} \frac{C_2(0, t)}{C_2(0, t_s)} \frac{C_2(q, t)}{C_2(q, t_s)}
\]

\[
R_{A_3} = \frac{i}{\sqrt{2E_q(m + E_q)}} \left( (m + E_q)G_A(q^2) - \frac{G_P(q^2)}{2m}q_3^2 \right)
\]

- As \( m_\pi \rightarrow m_{\pi phys} \), systematic effects need to addressed.
  - Chiral extrapolation to physical quark mass
  - Finite Volume and lattice spacing effects
  - Excited state contributions (as shown in G. von Hippel’s Talk)
  - At finite \( Q^2 \), significant impact on form factors.
Methods of Form factor Extraction

❖ Plateau method

❖ Fit the largest $t_s$ to a constant plateau.

❖ Summed operator insertions - Summation method

$$S(t_s) = \sum_{t=0}^{t_s} R(\bar{q}, t, t_s) \rightarrow C(\Delta, \Delta') + t_s (G + \mathcal{O}(e^{-\Delta t_s}) + \mathcal{O}(e^{-\Delta' t_s}))$$

❖ Explicit excited state ansatz,

$$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-\Delta t_s}$$

$$G_{X,eff}(Q^2; t, t_s) = G_X(Q^2) + C_1 e^{-\Delta t} + C_2 e^{-\Delta'(t_s-t)}$$

with assumptions for gap and individual and simultaneous $t_s$ fits.
Axial charge

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0$ GeV$^2$

Summation method

$g_A$ vs $t$

Axial charge

Excited State fits

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm

$g_A = 1.146 \pm 0.045$, $\Delta = 0.248 \pm 0.146$

Fit Ansatz: $g_A - c_1 e^{-\Delta t_s/2}$

$am_\pi = 0.0830(2)$

$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-\Delta t_s}$

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm

$g_A = 1.179 \pm 0.013$

Fit Ansatz: $g_A - c_1 e^{-2m_\pi(t_s/2)}$

$am_\pi = 0.0830(2)$

$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-2m_\pi t_s}$
Axial charge

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0$ GeV$^2$

$g_{A,\text{eff}}(t_s) = g_A + C_1 e^{-\Delta t_s}$
Axial charge

$g_{A,\text{eff}}(t_s) = g_A + C_1 e^{-2m_\pi t_s}$

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0$ GeV$^2$
Axial form factor

$G_A(Q^2)$

$N6, \, \beta = 5.5, \, m_\pi = 340 \, \text{MeV}, \, L = 2.4 \, \text{fm}, \, Q^2 = 0.25 \, \text{GeV}^2$

$t_s = 13$
$t_s = 16$
$t_s = 19$
$t_s = 22$
Axial form factor

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0.25$ GeV$^2$

$G_A(Q^2)$ vs $t$

$G_A(Q^2) = 5\, m = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0.25$ GeV$^2$

Summation Method
Axial form factor

\[ G_{\text{A}}(Q^2) = G_{\text{X}}(Q^2) + C_1 e^{-\Delta t} + C_2 e^{-\Delta'(t_s-t)} \]

N6, \( \beta = 5.5, \ m_\pi = 340 \text{ MeV}, \ L = 2.4 \text{ fm}, \ Q^2 = 0.25 \text{ GeV}^2 \)

3 parameter fit
Axial form factor

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0.7$ GeV$^2$
Axial form factor

Summation method results

Effect of removal of lowest $t_s$ on extraction of $G_A(Q^2)$

Consistent results albeit with higher uncertainties
Axial form factor

Summation method results

Effect of addition of $t_s = 25$ on extraction of $G_A(Q^2)$
Axial form factor

Excited state fit results

Effect of removal of lowest $t_s$ on extraction of $G_A(Q^2)$

$$G_{X, eff}(Q^2; t, t_s) = G_X(Q^2) + C_1 e^{-\Delta t} + C_2 e^{-\Delta'(t_s-t)}$$
Axial form factor

Excited state fit results

Effect of addition of $t_s = 25$ on extraction of $G_A(Q^2)$

Consistent results albeit with pronounced effects
Axial Form factor

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm

$G_A(Q^2)$ vs. $Q^2$ [GeV$^2$]

- Experiment $\nu$ scattering
- Plateau
- Summation
- Excited-state fit
Axial Form factor

$O7, \beta = 5.5, m_\pi = 270 \text{ MeV}, L = 3.2 \text{ fm}$
Axial radius of nucleon

Results for Plateau Method

Results for Summation Method
Induced pseudoscalar form factor

\[ \langle N, p' | A^\mu(x) | N, p \rangle = e^{iq \cdot x} \bar{u}(p') \left( \gamma^\mu \gamma^5 G_A(q^2) + \gamma^5 \frac{q^\mu}{2m} G_P(q^2) \right) u(p) \]
Induced pseudoscalar form factor

\[
\langle N, p' | A^\mu (x) | N, p \rangle = e^{iq \cdot x} \bar{u}(p') \left( \gamma^\mu \gamma^5 G_A (q^2) + \gamma^5 \frac{q^\mu}{2m} G_P (q^2) \right) u(p)
\]

Chiral symmetry breaking and PCAC constrain structure to pion pole

\[
G_P (Q^2) = G_A (Q^2) \frac{2M_N}{Q^2 + m^2_\pi}
\]

Experimentally measured in muon capture by proton

\[
g_P \equiv \frac{m_\mu}{2M_N} G_P (Q^2 = -0.88m^2_\mu)
\]
Induced pseudoscalar form factor

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0.25$ GeV$^2$

$t_s = 13$
$t_s = 16$
$t_s = 19$
$t_s = 22$
Induced pseudoscalar form factor

$G_P(Q^2)$

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 2.4$ fm, $Q^2 = 0.25$ GeV$^2$
Induced pseudoscalar form factor

\[ G_P(Q^2) \]

N6, \( \beta = 5.5, \ m_\pi = 340 \text{ MeV}, \ L = 2.4 \text{ fm}, \ Q^2 = 0.25 \text{ GeV}^2 \]
Induced pseudoscalar form factor

N6, $\beta = 5.5$, $m_\pi = 340$ MeV, $L = 3.2$ fm

\[ G_P(Q^2) \]

\[ Q^2 \text{ [GeV}^2] \]
Induced pseudoscalar form factor

$O7, \beta = 5.5, m_\pi = 270 \text{ MeV}, L = 3.2 \text{ fm}$

\[ G_P(Q^2) \]

\[ Q^2 \text{ [GeV}^2] \]

 Phenomenology
 Plateau
 Summation
 Excited-state fit
Conclusions

❖ For the case of $g_A$, summation method can identify the ground state of the matrix element where the plateau can be misleading.

❖ Preliminary results for axial form factor show varying excited state contributions at different momentum transfers.
  ❖ Preliminary results show excited state contributions to $\langle r_A^2 \rangle$ hidden by statistics.

❖ Large excited state contributions to $G_P(Q^2)$ are observed and it remains under investigation.
Thank You
Backup Slides
Individual excited state fits

N6, $\beta=5.5$, $m_{\pi}=340\text{MeV}$, $L=2.4\text{fm}$, $Q^2=0.7\text{GeV}^2$
Individual excited state fits

$N6, \beta=5.5, m_\pi=340\text{MeV}, L=2.4\text{fm}, Q^2=0.25\text{GeV}^2$