Nucleon axial form factors from two-flavour Lattice QCD

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Outline

- Structure of the axial matrix elements of nucleon
- Lattice calculation of axial matrix elements
- Extraction of Form factors
  - Plateau method
  - Summation method
  - Excited state ansatz
- Results on $G_A(Q^2), \quad \langle r_A^2 \rangle$
- Results on $G_P(Q^2)$
- Conclusions

Axial structure of the Nucleon

From Lorenz invariance, CPT and Isospin symmetry,

$$\langle N, \mathbf{p}' | A^\mu(x) | N, \mathbf{p} \rangle = e^{iq\cdot x} \bar{u}(\mathbf{p}') \left(\gamma^\mu \gamma^5 G_A(q^2) + \gamma^5 \frac{q^\mu}{2m} G_P(q^2) \right) u(\mathbf{p})$$

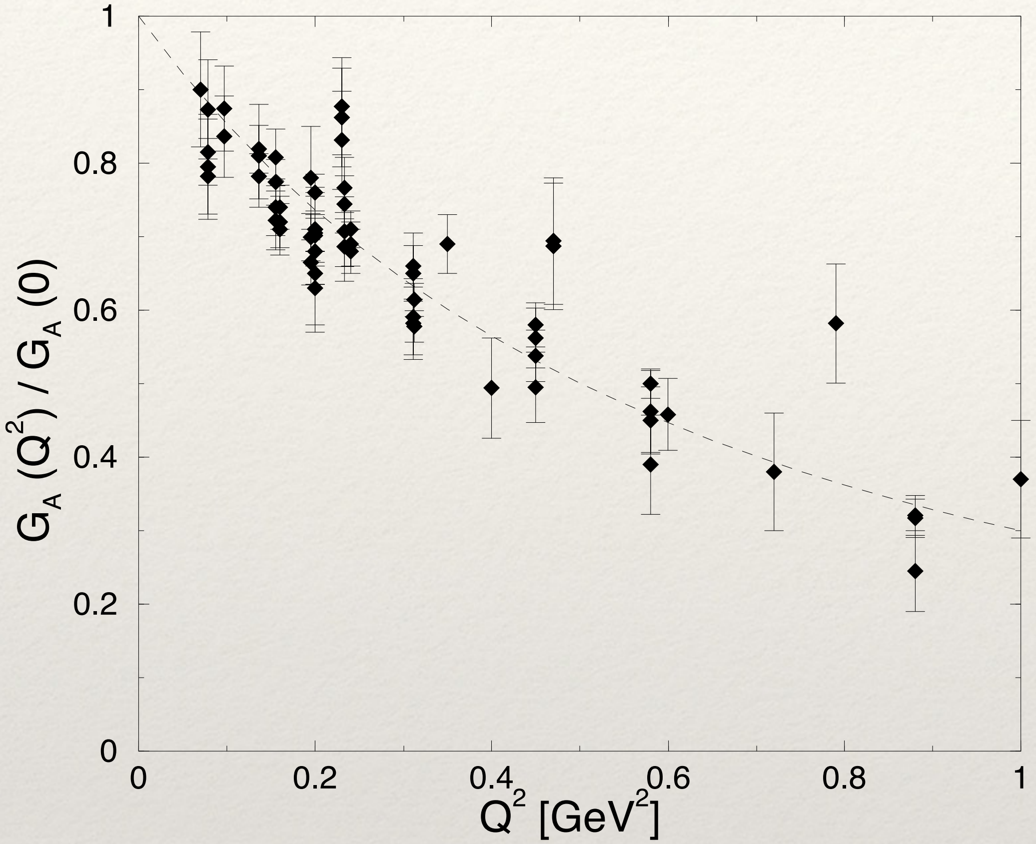
Axial structure strongly influenced by chiral symmetry breaking ,

$$g_A / g_V = 1.2701(25)$$ Ref: PDG 2012

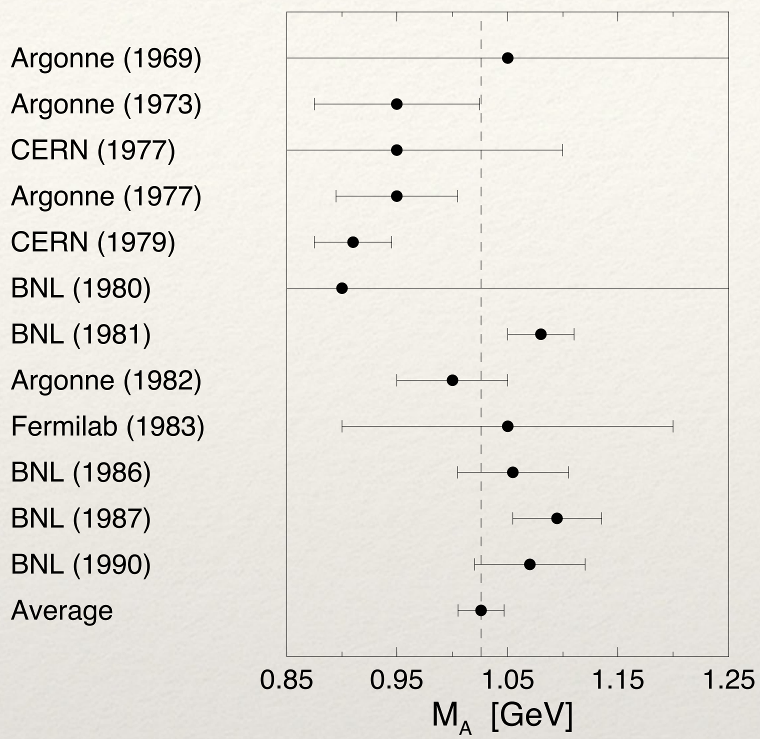
$$Q^2 \sim 1 \text{ GeV}^2$$

$$G_A \approx \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \qquad \langle r_A^2 \rangle = -\frac{6}{G_A(0)} \frac{\mathrm{d}G_A(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

Axial structure of the Nucleon



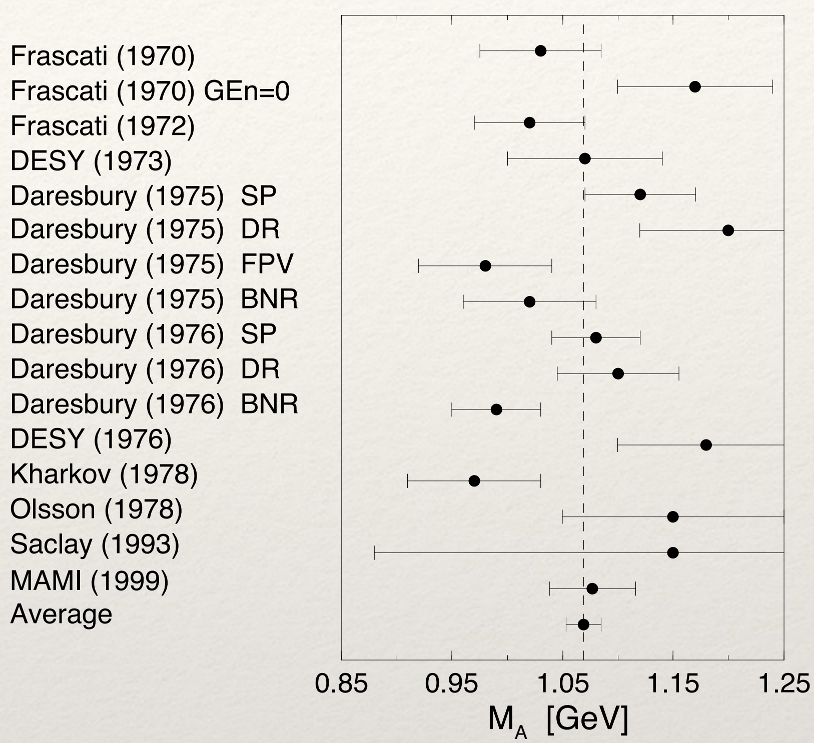
Combined Electroproduction data



ν scattering

$$M_A = 1.026 \pm 0.021 \text{ GeV}$$

$$\langle r_A^2 \rangle = 0.443 \pm 0.028 \text{ fm}$$



$e - \pi^+$ scattering

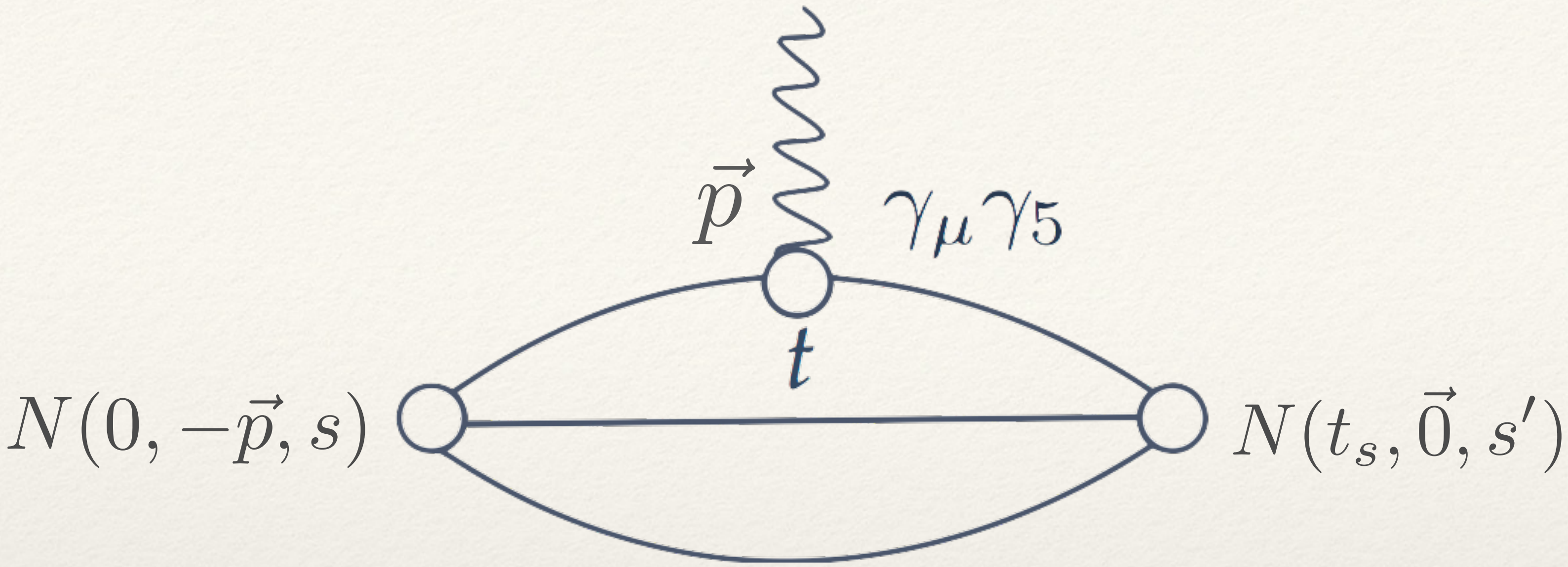
$$M_A = 1.069 \pm 0.016 \text{ GeV}$$

$$\langle r_A^2 \rangle = 0.408 \pm 0.020 \text{ fm}$$

$$\Delta M_A = 0.043 \pm 0.026 \text{ GeV}$$

Bernard, Elouadrhiri, Meißner J.Phys. G28 (2002) R1-R35

Nucleon matrix elements on the Lattice



- Standard Nucleon operator with covariant gaussian smearing at source and sink.

- Sequential inversion with fixed sink and momentum injected at the operation insertion.

$$C_3 \sim \frac{e^{-E_q t}}{2E_q} \frac{e^{-m(ts-t)}}{2m} \mathrm{Tr}[\Gamma_{\alpha\beta} \langle N, -\vec{p}, s | A^\mu | N, \vec{0}, s' | \rangle_{\alpha\beta}]$$

- Calculation performed at several t_s, quark masses and volumes

CLS Ensembles

$N_f = 2$ non-perturbatively $\mathcal{O}(a)$ improved gauge configurations

Run	β	a [fm]	L	m	L [fm]	m	N
A3				473	2.5	6	2128
A4	5.2	0.079	32	363	2.5	4.7	3200
A5				312	2.5	4	4000
B6			*48*	*262*	*3.8*	*5*	*2544*
E5			32	451	2	4.7	4000
F6	5.3	0.063	48	324	3	5	3600
F7				277	3	4.2	3000
G8			*64*	*195*	*4*	*4*	*4176*
N5			48	430	2.4	5.2	1908
N6	5.5	0.05		340	2.4	4	3784
O7			*64*	*270*	*3.2*	*4.4*	*1960*

Coordinated Lattice Simulations [*https://twiki.cern.ch/twiki/bin/view/CLS/WebHome*]

Issues in Form factor Extraction

- Matrix elements extracted from conveniently defined ratio as,

$$R_{A_\mu} = \frac{C_{3,A_\mu}(\vec{q},t,t_s)}{C_2(\vec{q},t,t_s)} \sqrt{\frac{C_2(\vec{q},t,t_s-t)}{C_2(\vec{0},t,t_s-t)} \frac{C_2(\vec{0},t)}{C_2(\vec{q},t)} \frac{C_2(\vec{0},t_s)}{C_2(\vec{q},t_s)}}$$

$$R_{A_3} = \frac{i}{\sqrt{2E_q(m+E_q)}} \left((m+E_q)G_A(q^2) - \frac{G_P(q^2)}{2m} q_3^2 \right)$$

- As $m_\pi \to m_{\pi phys}$, systematic effects need to addressed.
  - Chiral extrapolation to physical quark mass
  - Finite Volume and lattice spacing effects
  - Excited state contributions (as shown in G. von Hippel's Talk)
    - At finite Q^2, significant impact on form factors.

Methods of Form factor Extraction

- Plateau method
    - Fit the largest t_s to a constant plateau.
- Summed operator insertions - Summation method

$$S(t_s) = \sum_{t=0}^{t_s} R(\vec{q}, t, t_s) \to C(\Delta, \Delta') + t_s(G + \mathcal{O}(e^{-\Delta t_s}) + \mathcal{O}(e^{-\Delta' t_s}))$$

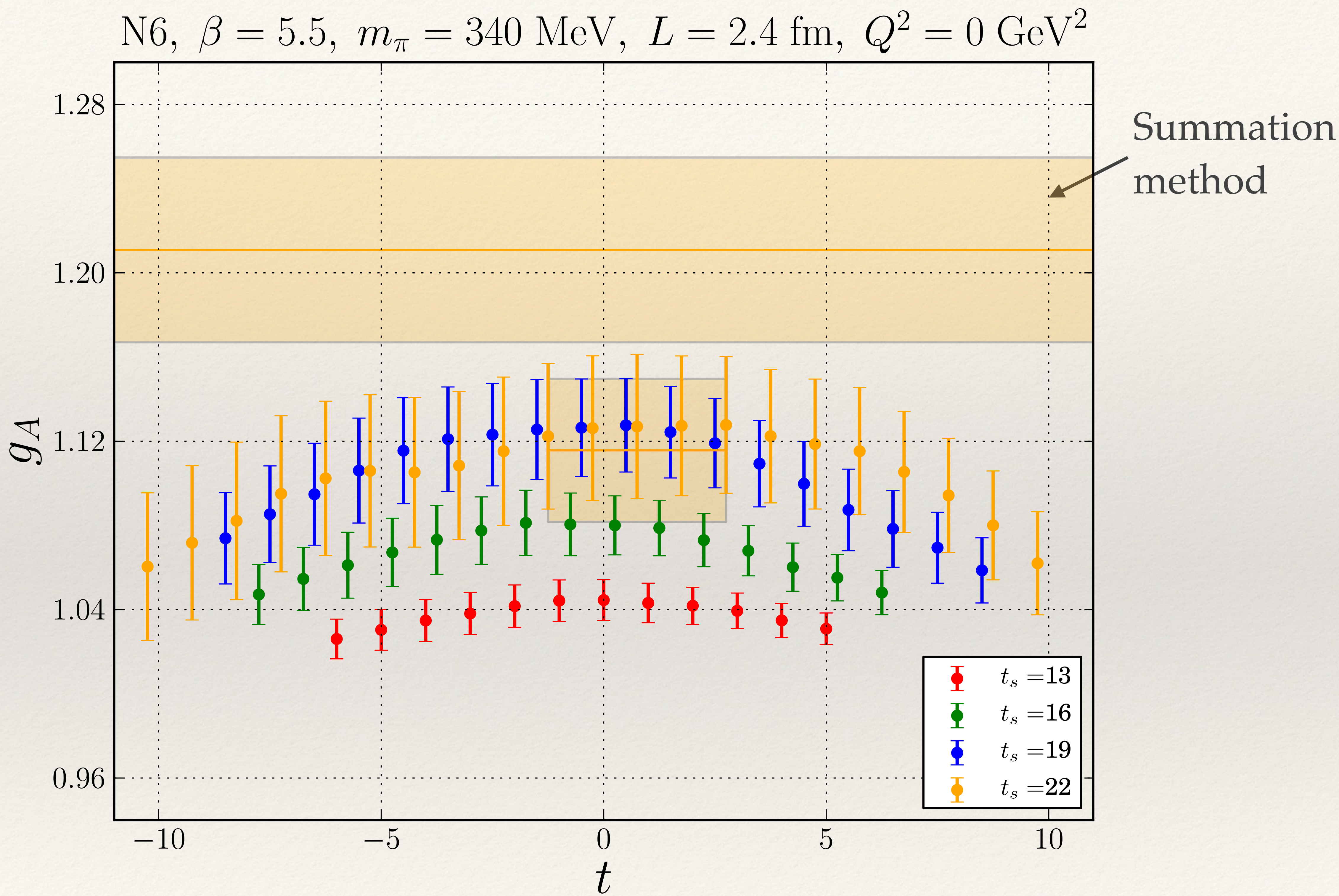
- Explicit excited state ansatz,

$$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-\Delta t_s}$$

$$G_{X,eff}(Q^2; t, t_s) = G_X(Q^2) + C_1 e^{-\Delta t} + C_2 e^{-\Delta'(t_s - t)}$$

with assumptions for gap and individual and simultaneous t_s fits.

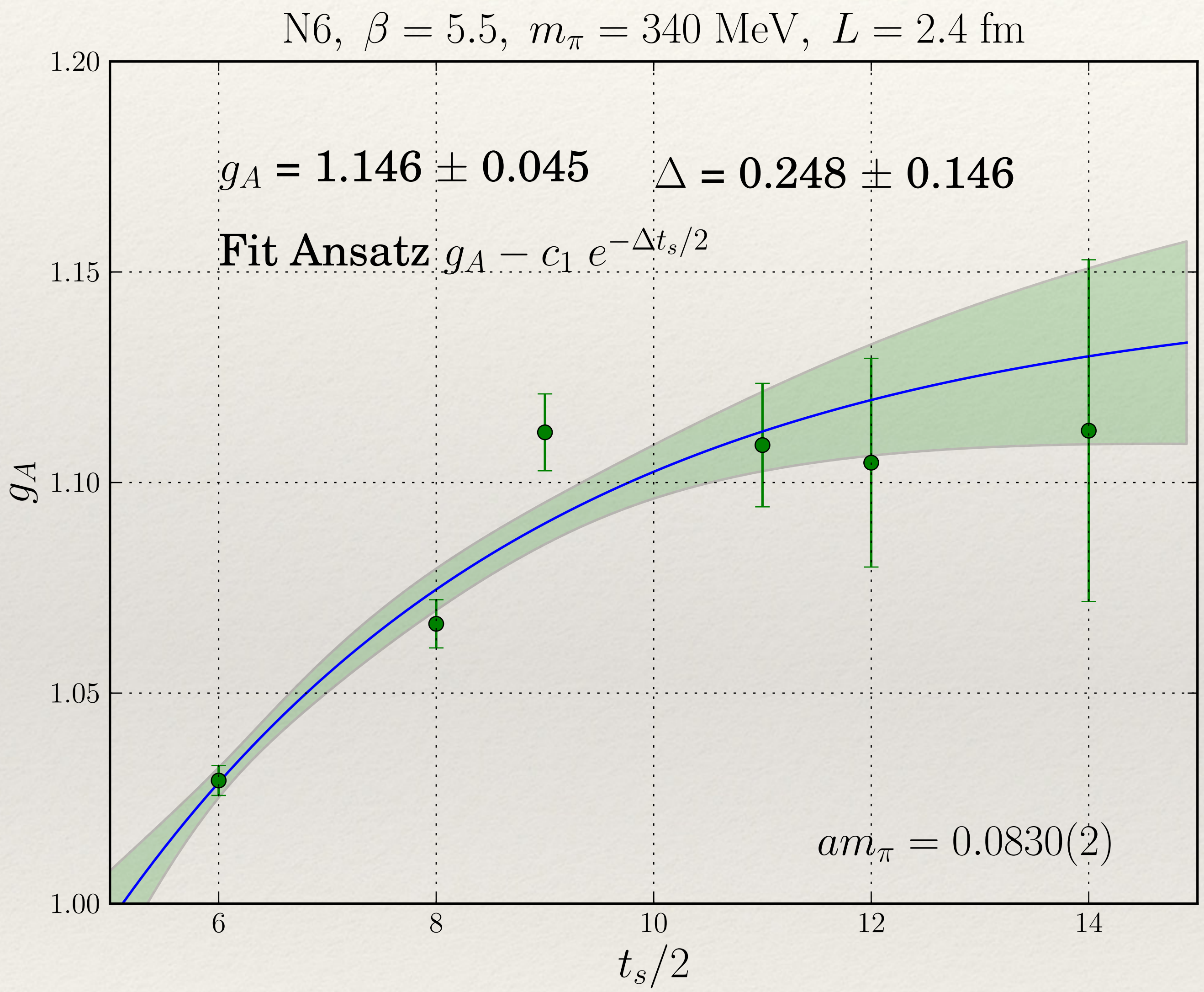
Axial charge

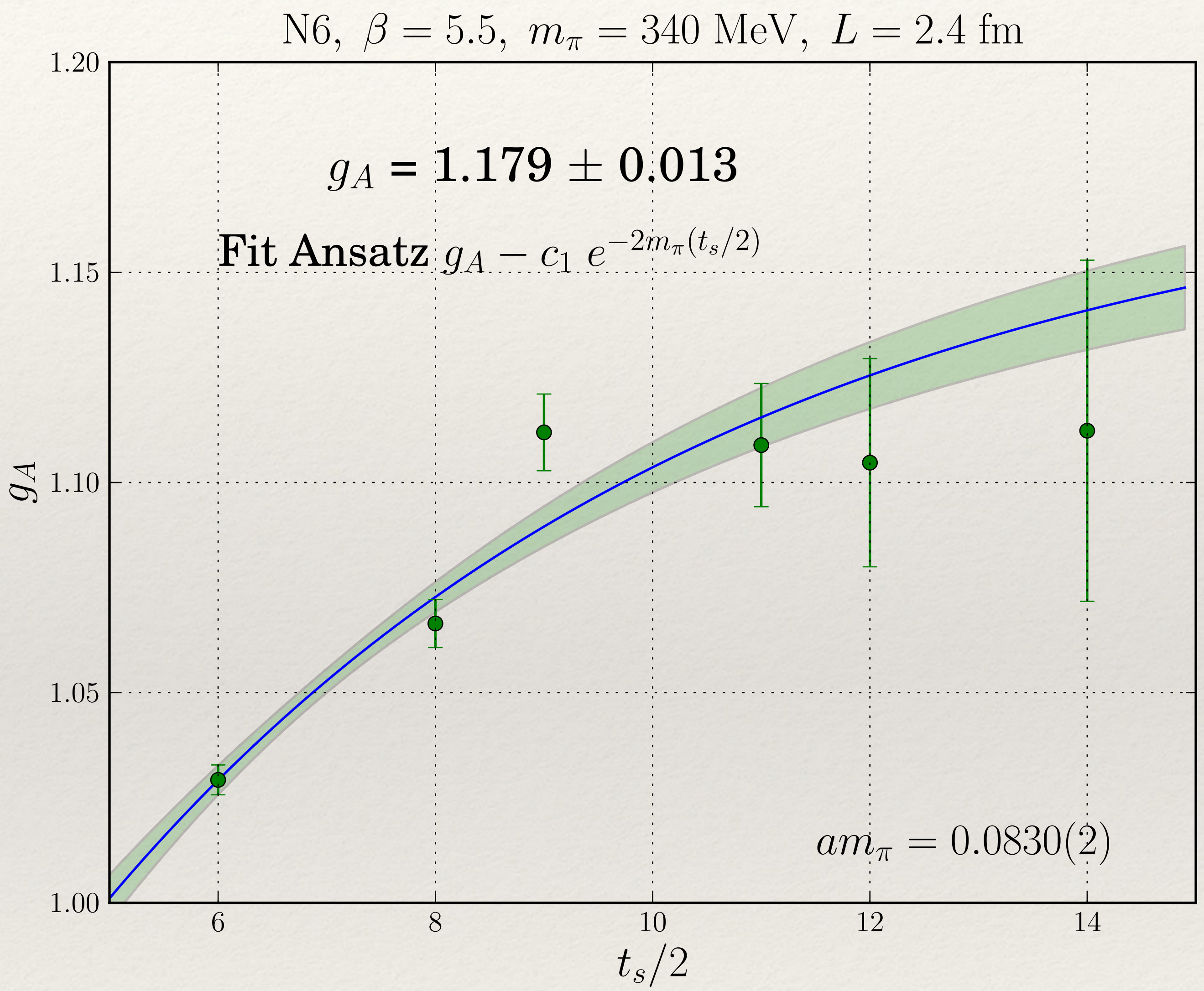


[arXiv:1311.5804], Phys.Rev. D86 (2012) 074502

Axial charge

Excited State fits

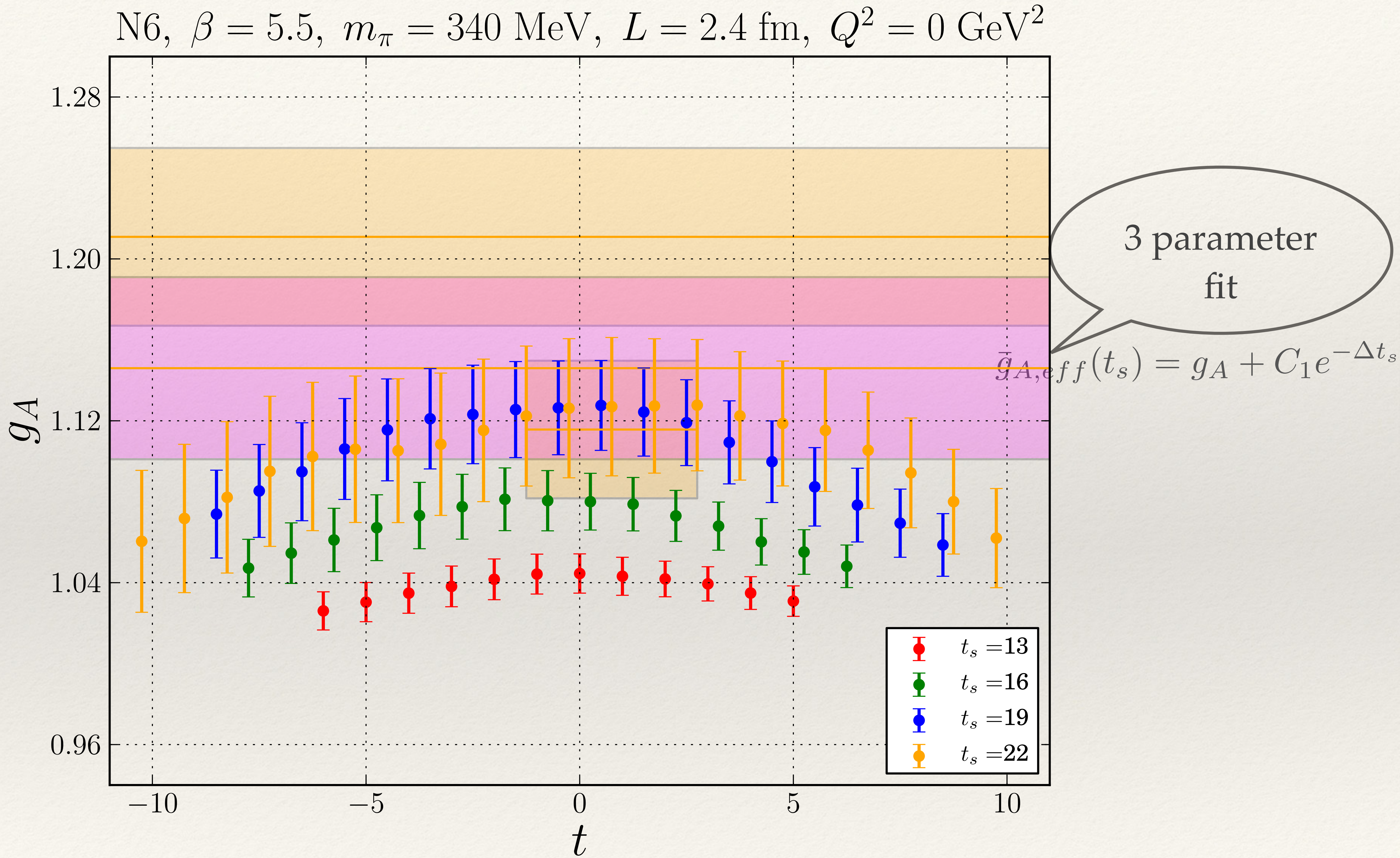




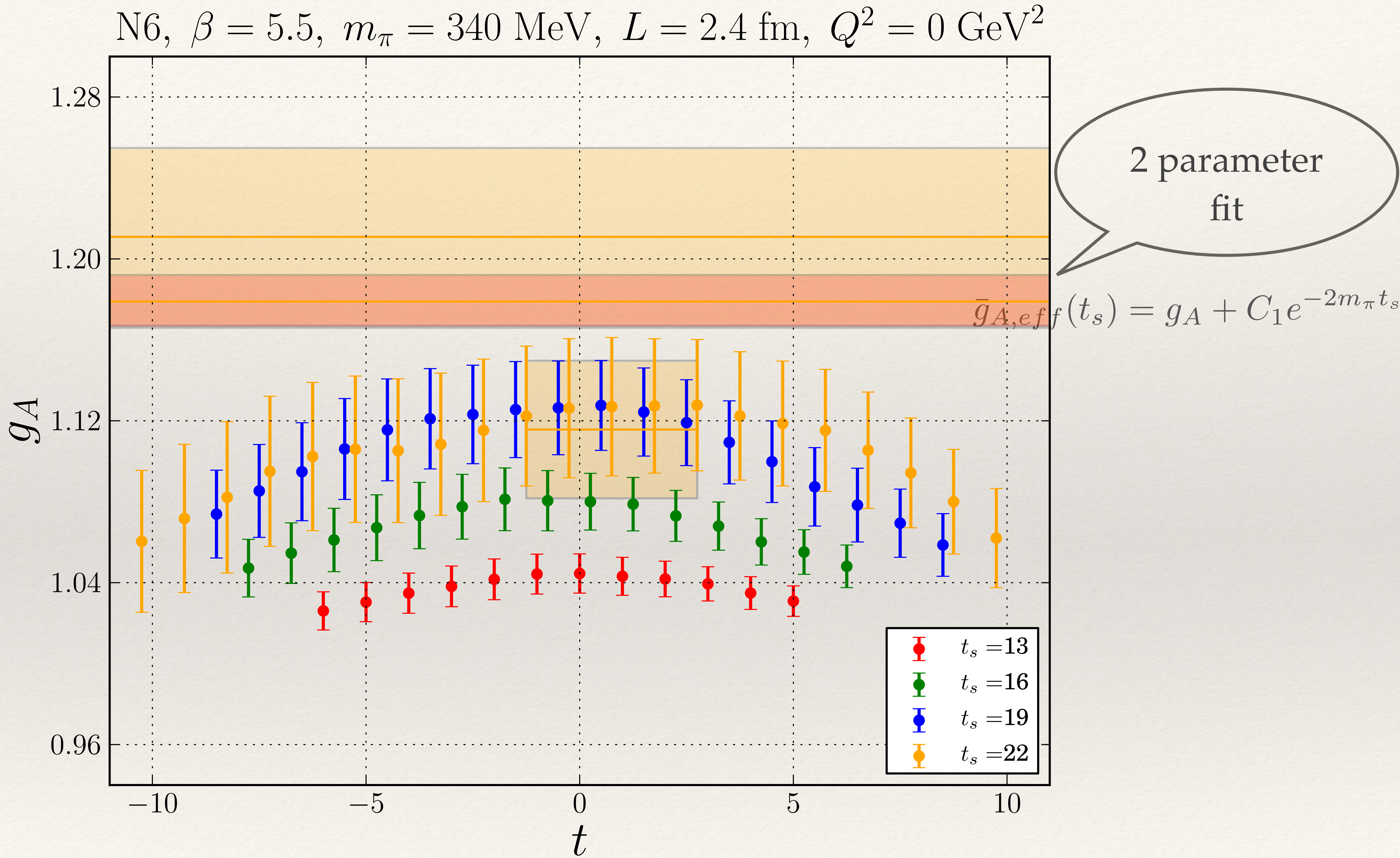
$$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-\Delta t_s}$$

$$\bar{g}_{A,eff}(t_s) = g_A + C_1 e^{-2m_\pi t_s}$$

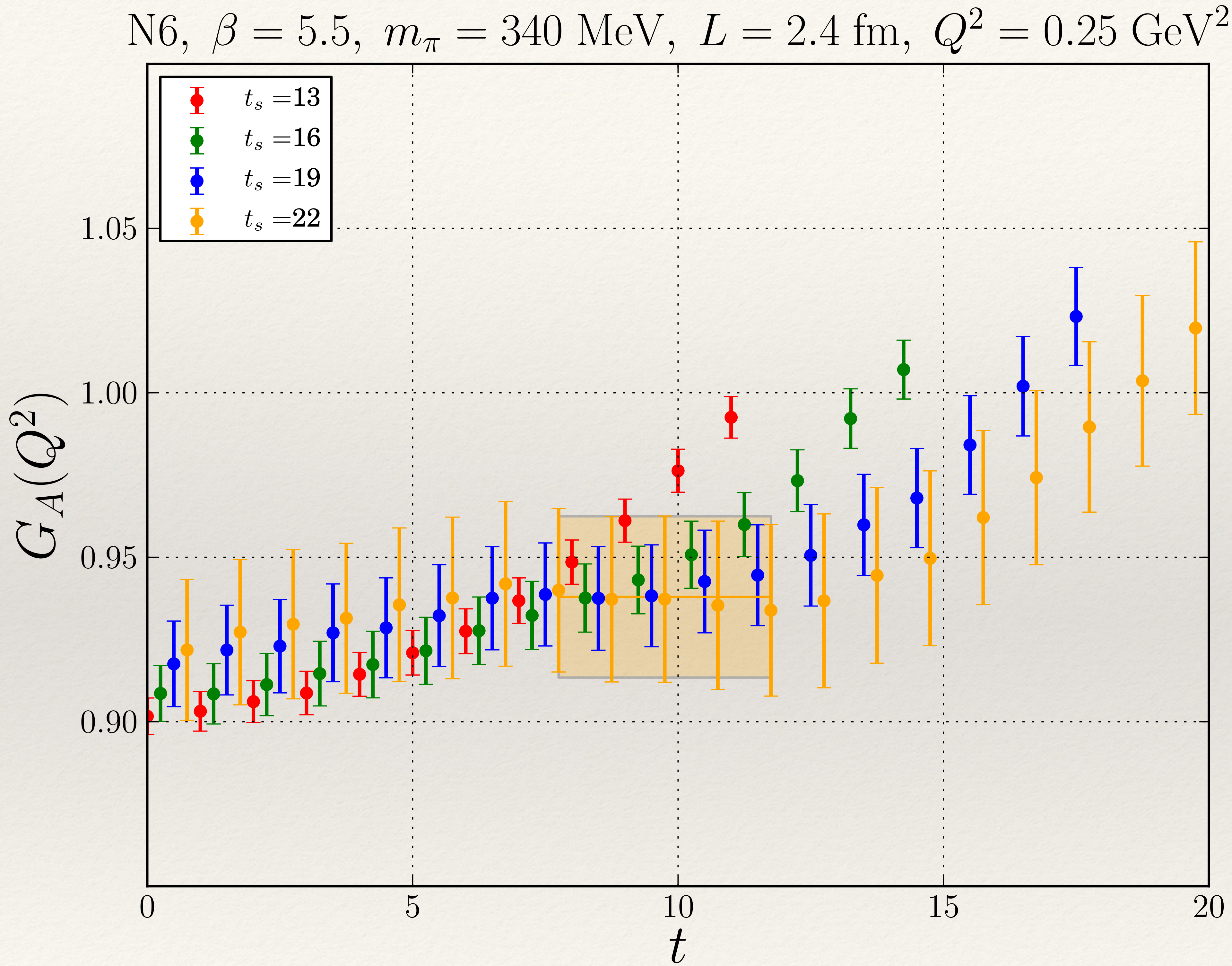
Axial charge



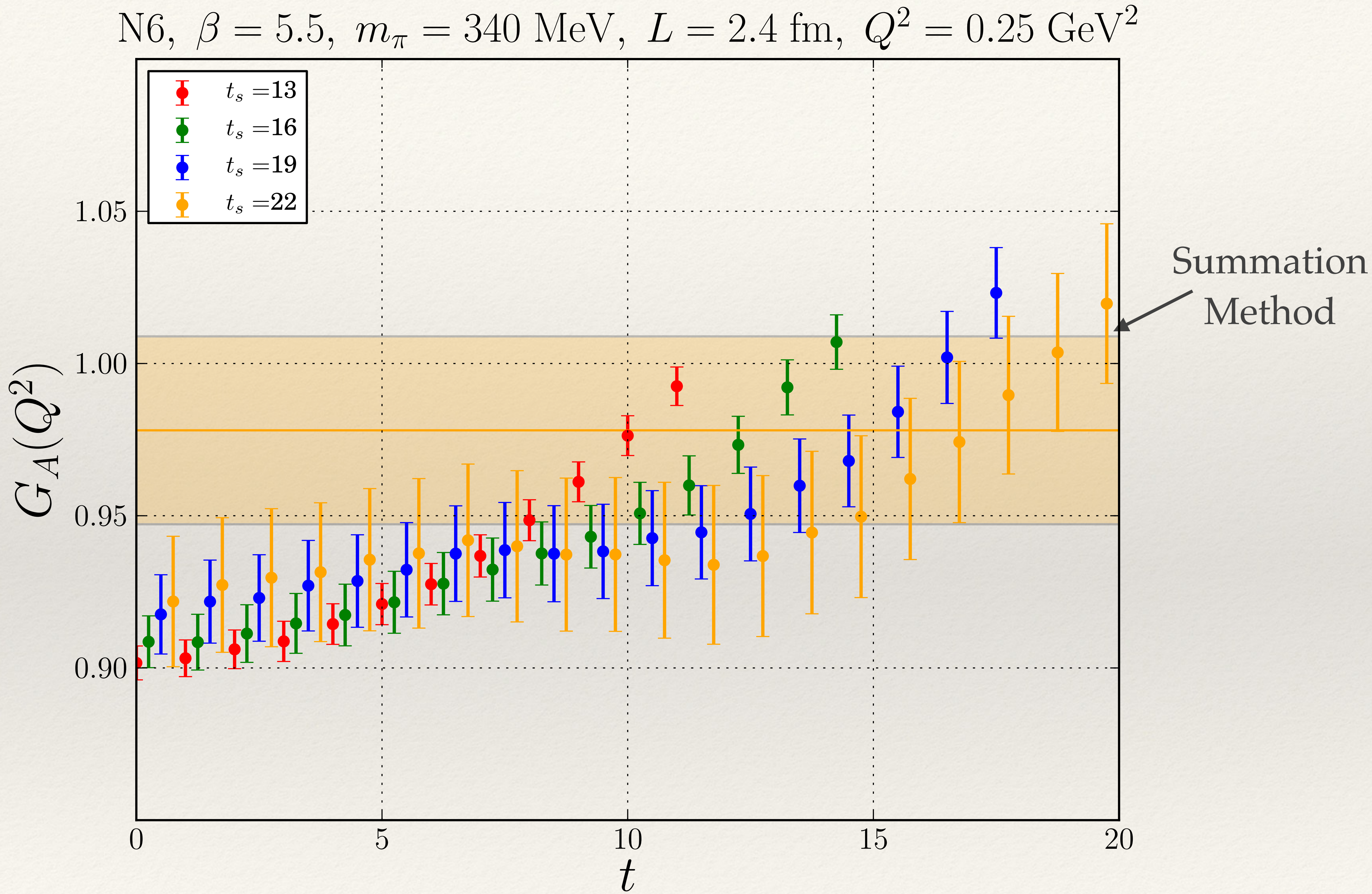
Axial charge



Axial form factor



Axial form factor



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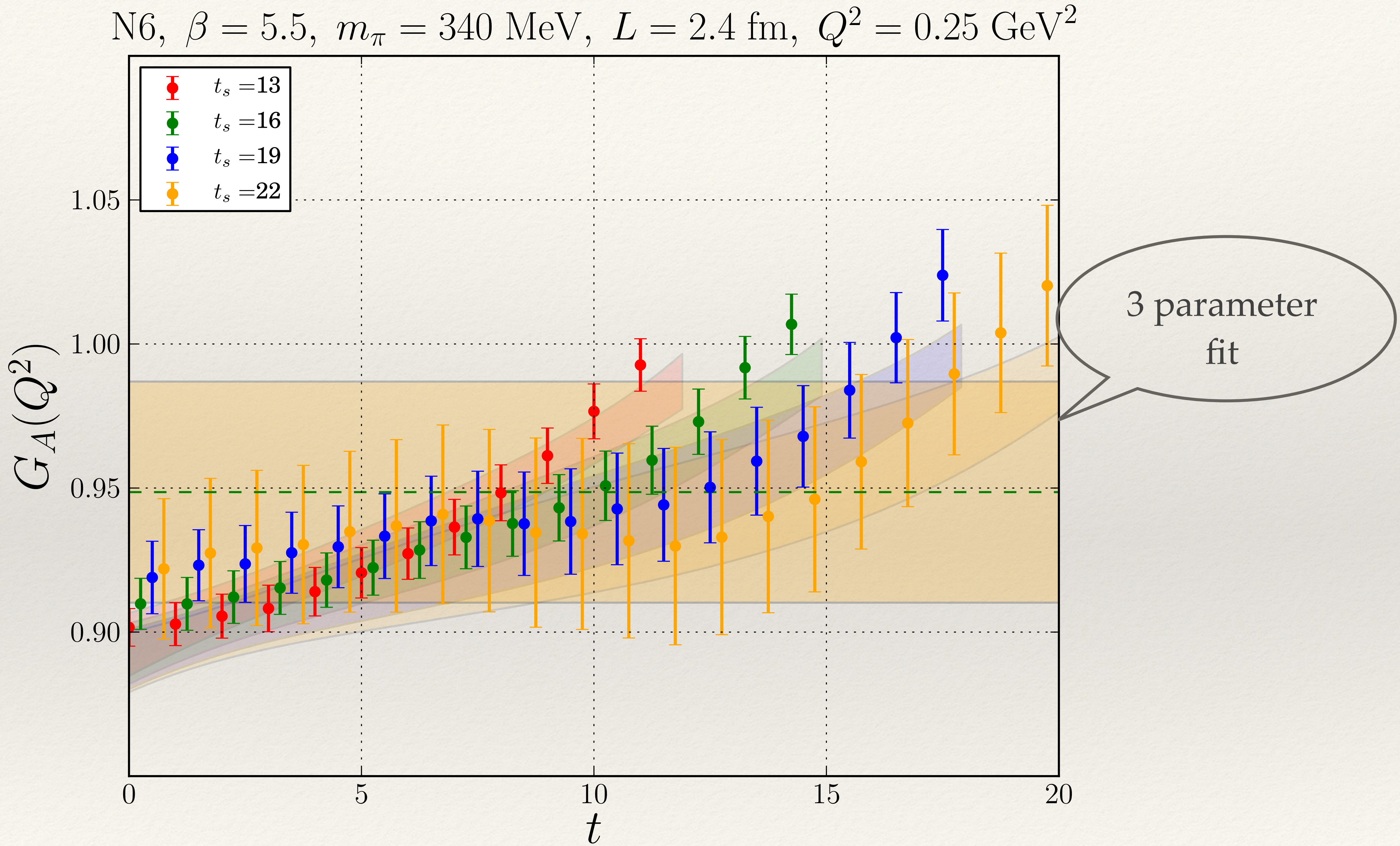
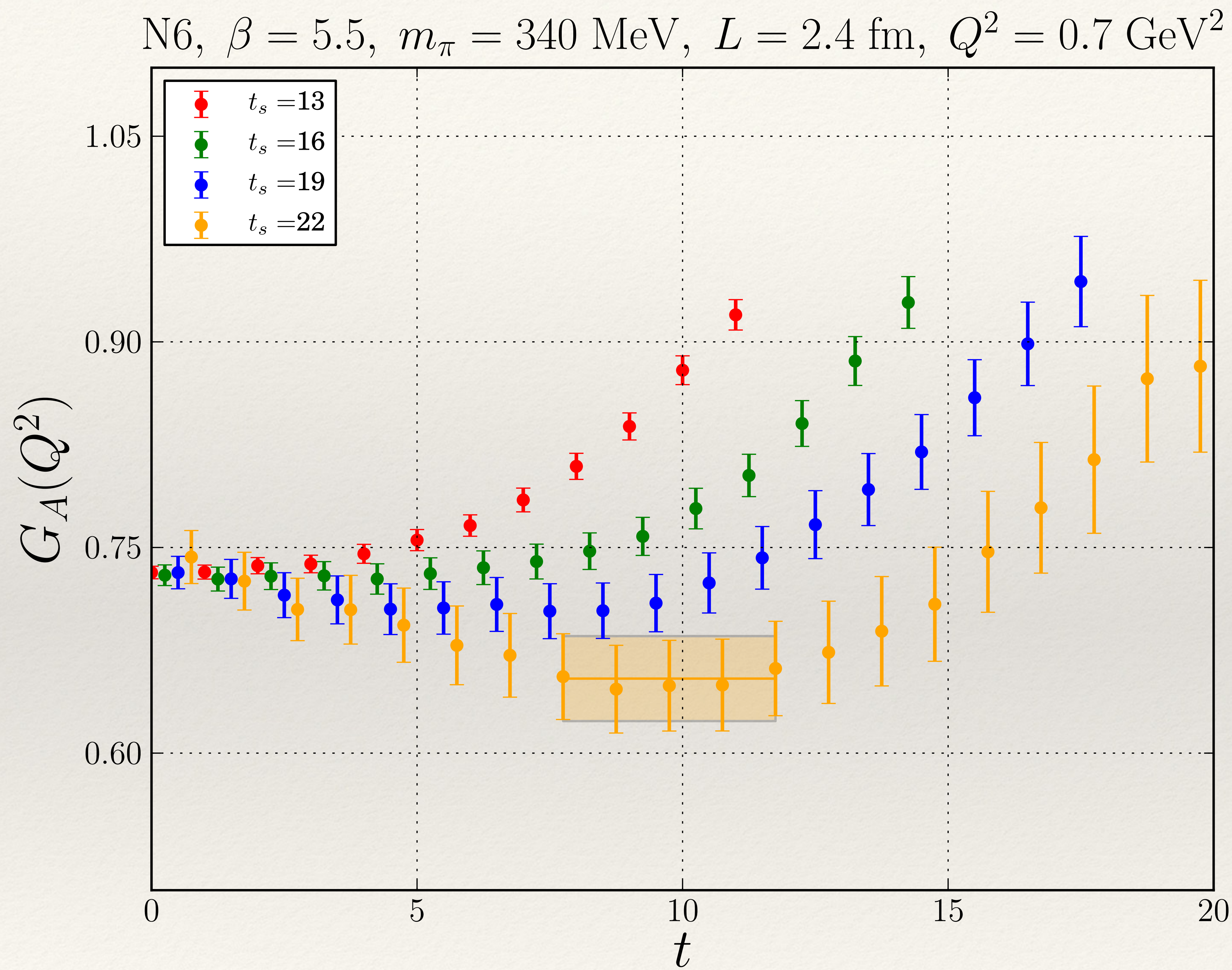


$$G_{X,eff}(Q^2; t, t_s) = G_X(Q^2) + C_1 e^{-\Delta t} + C_2 e^{-\Delta'(t_s - t)}$$

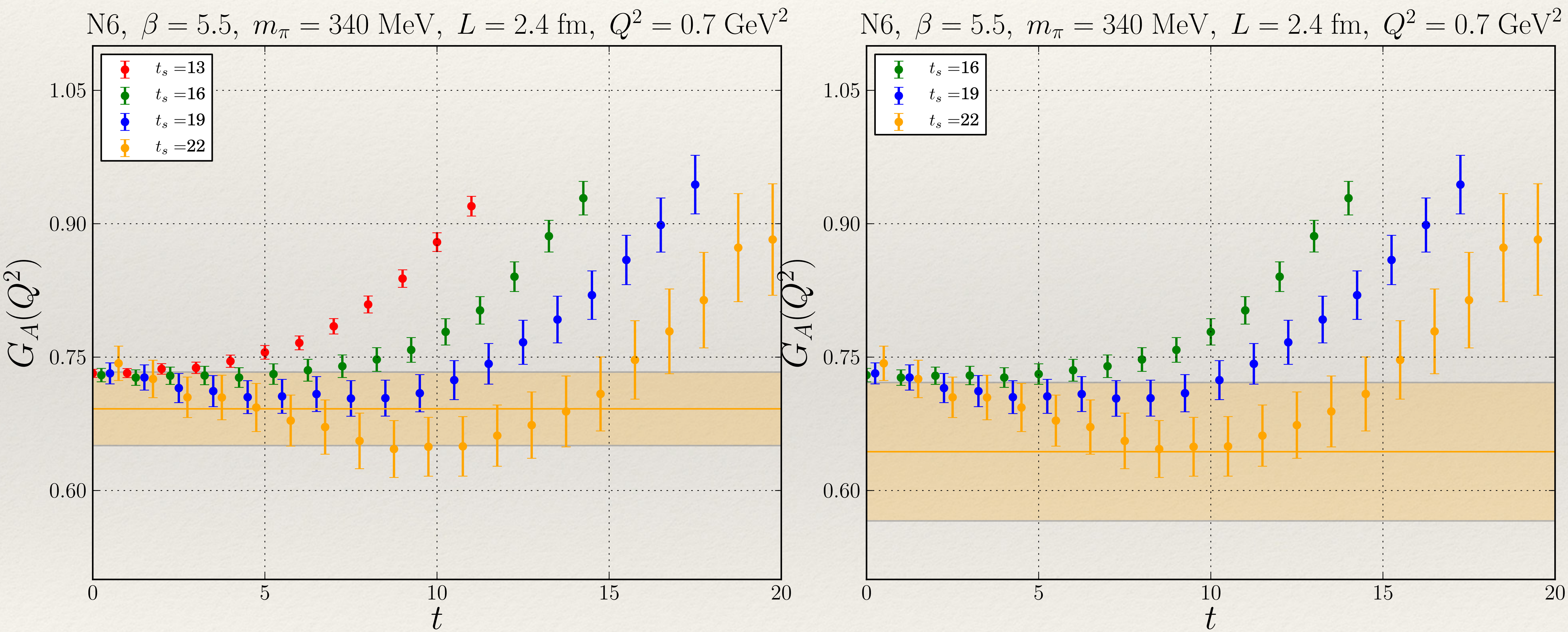
Axial form factor



Axial form factor

Summation method results

Effect of removal of lowest t_s on extraction of $G_A(Q^2)$

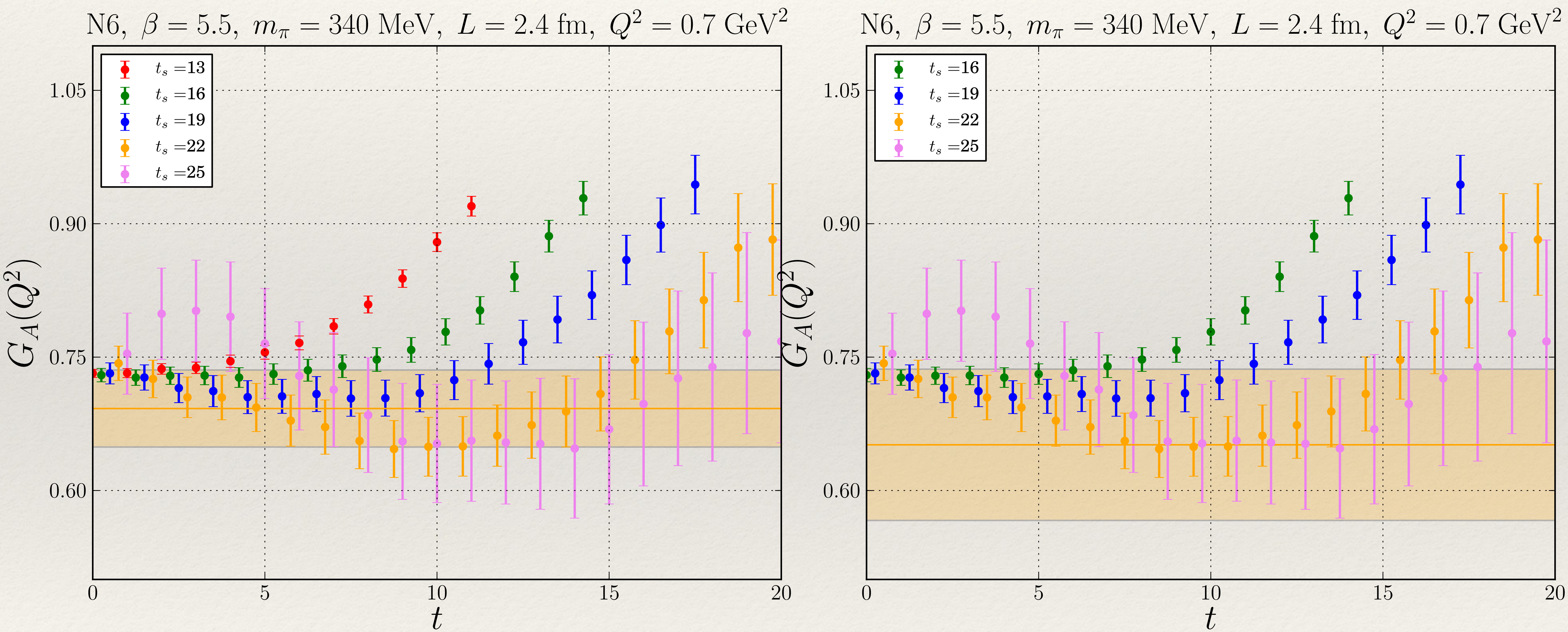


Consistent results albeit with higher uncertainties

Axial form factor

Summation method results

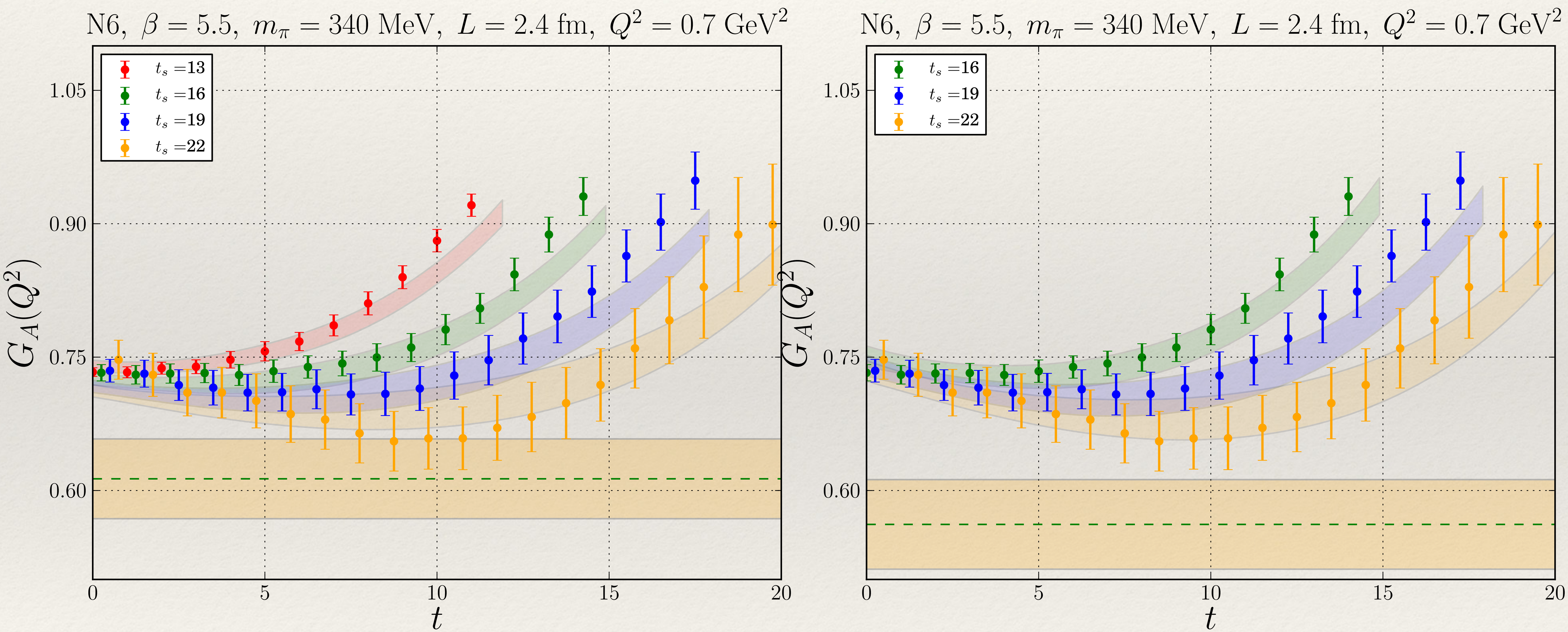
Effect of addition of ts = 25 on extraction of $G_A(Q^2)$



Axial form factor

Excited state fit results

Effect of removal of lowest t_s on extraction of $G_A(Q^2)$



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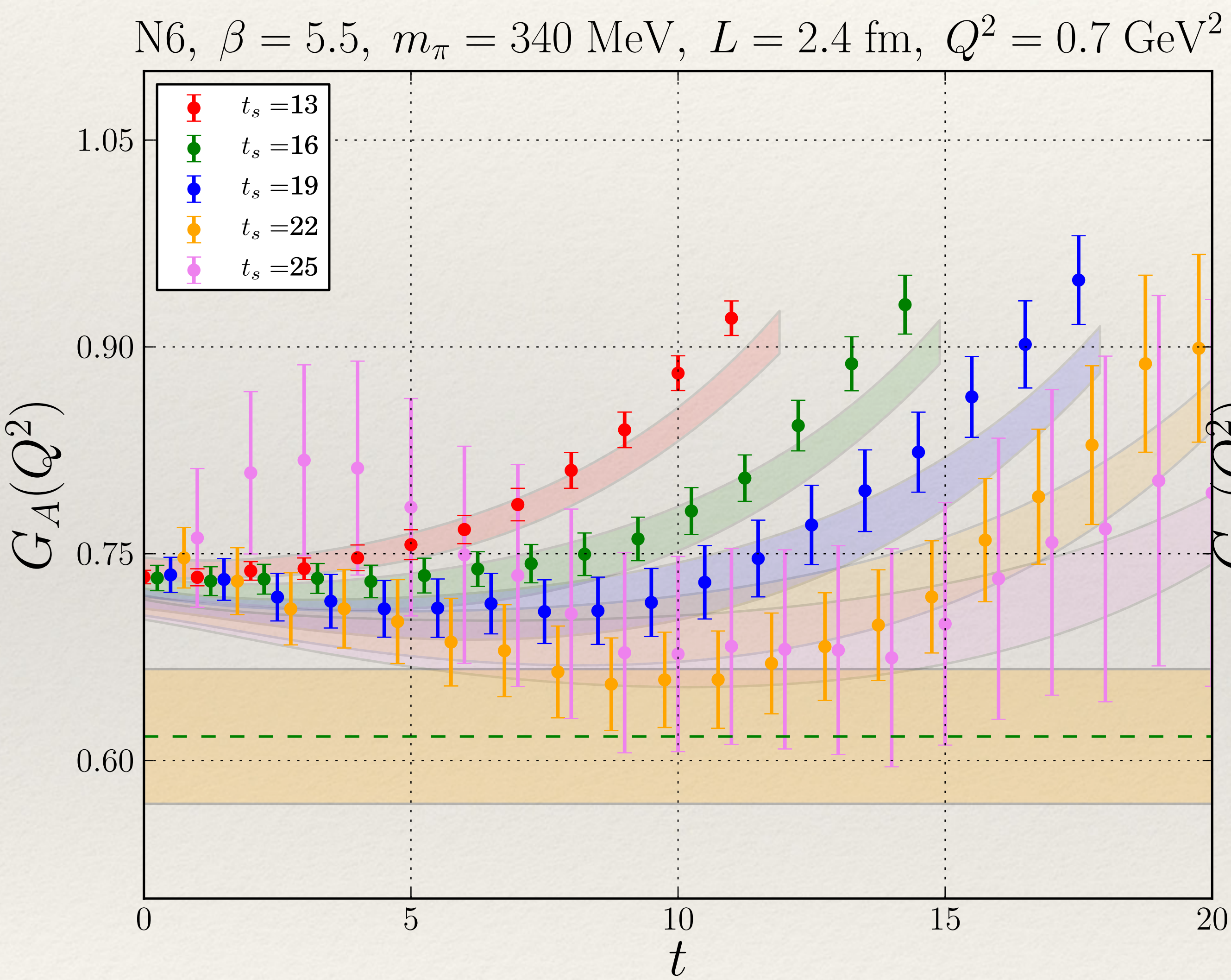
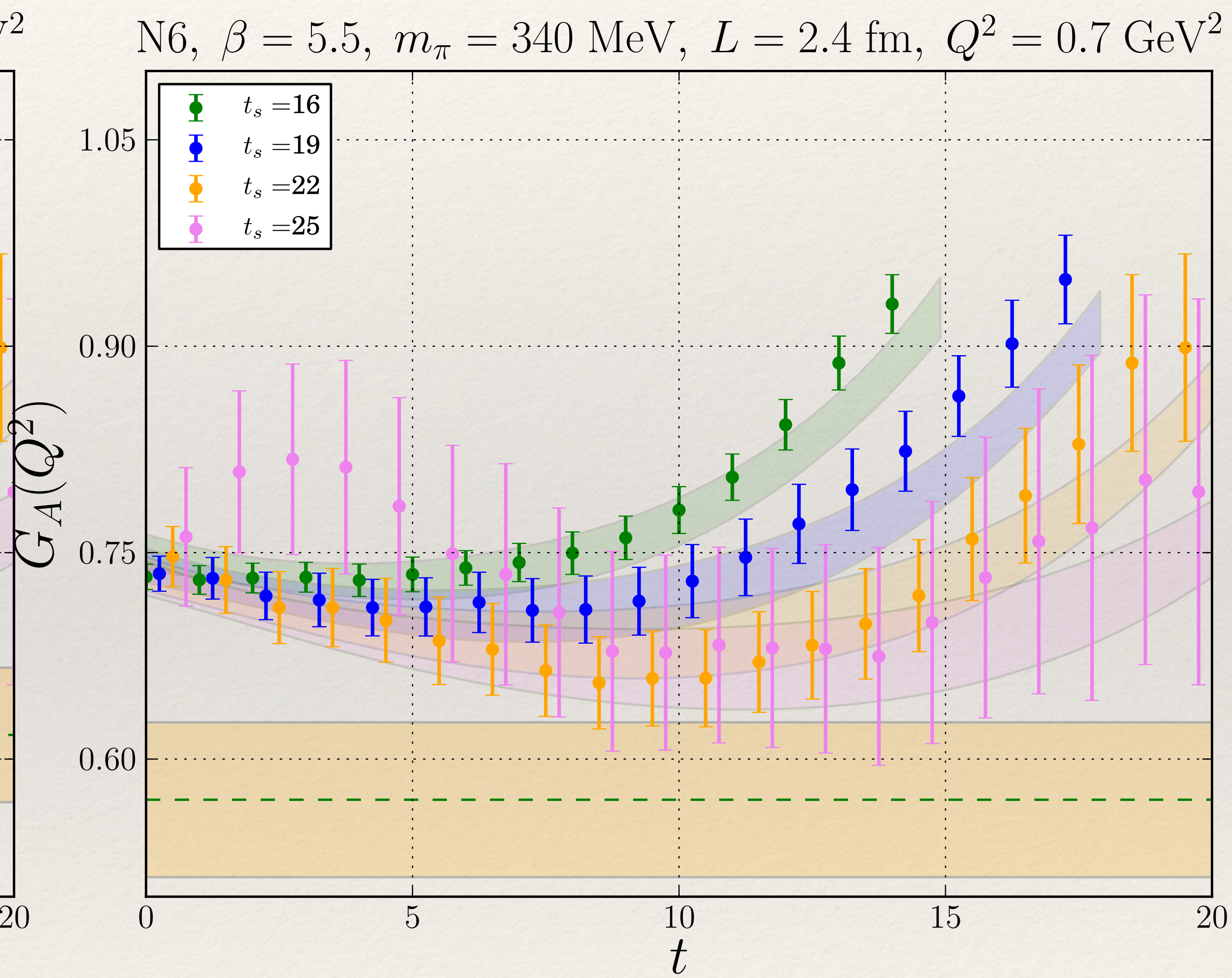
Axial form factor

Excited state fit results

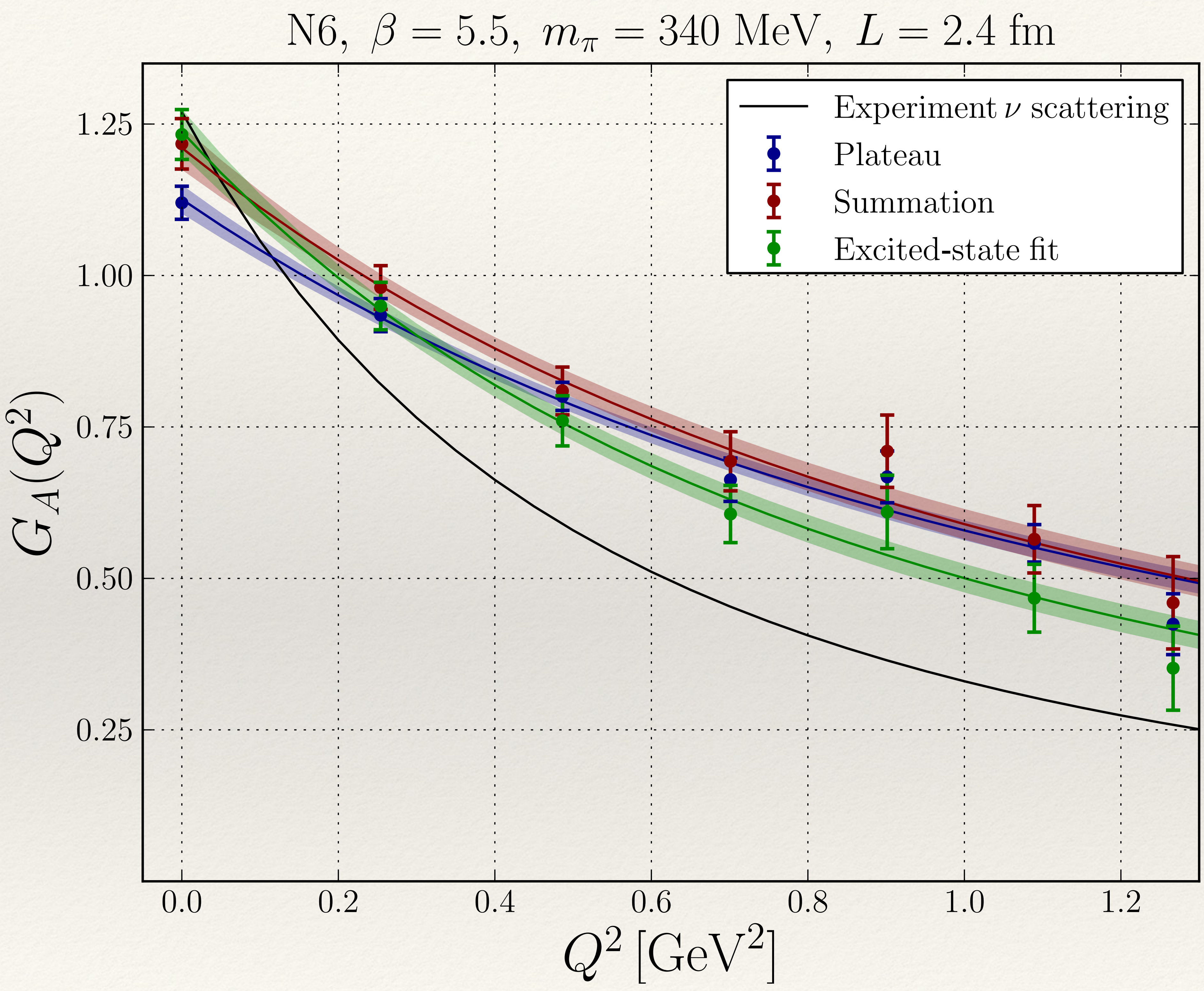
Effect of addition of ts = 25 on extraction of $G_A(Q^2)$



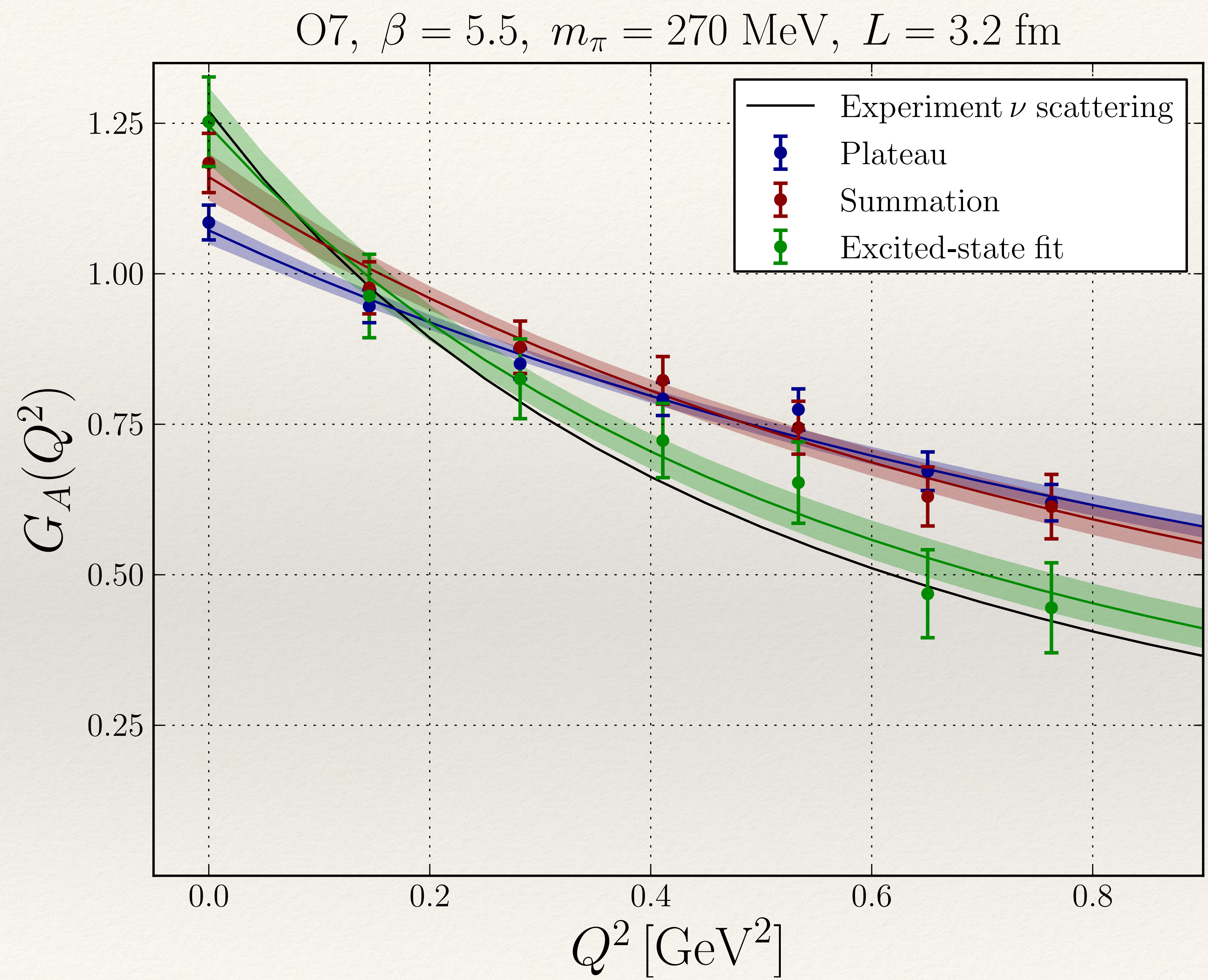


Consistent results albeit with pronounced effects

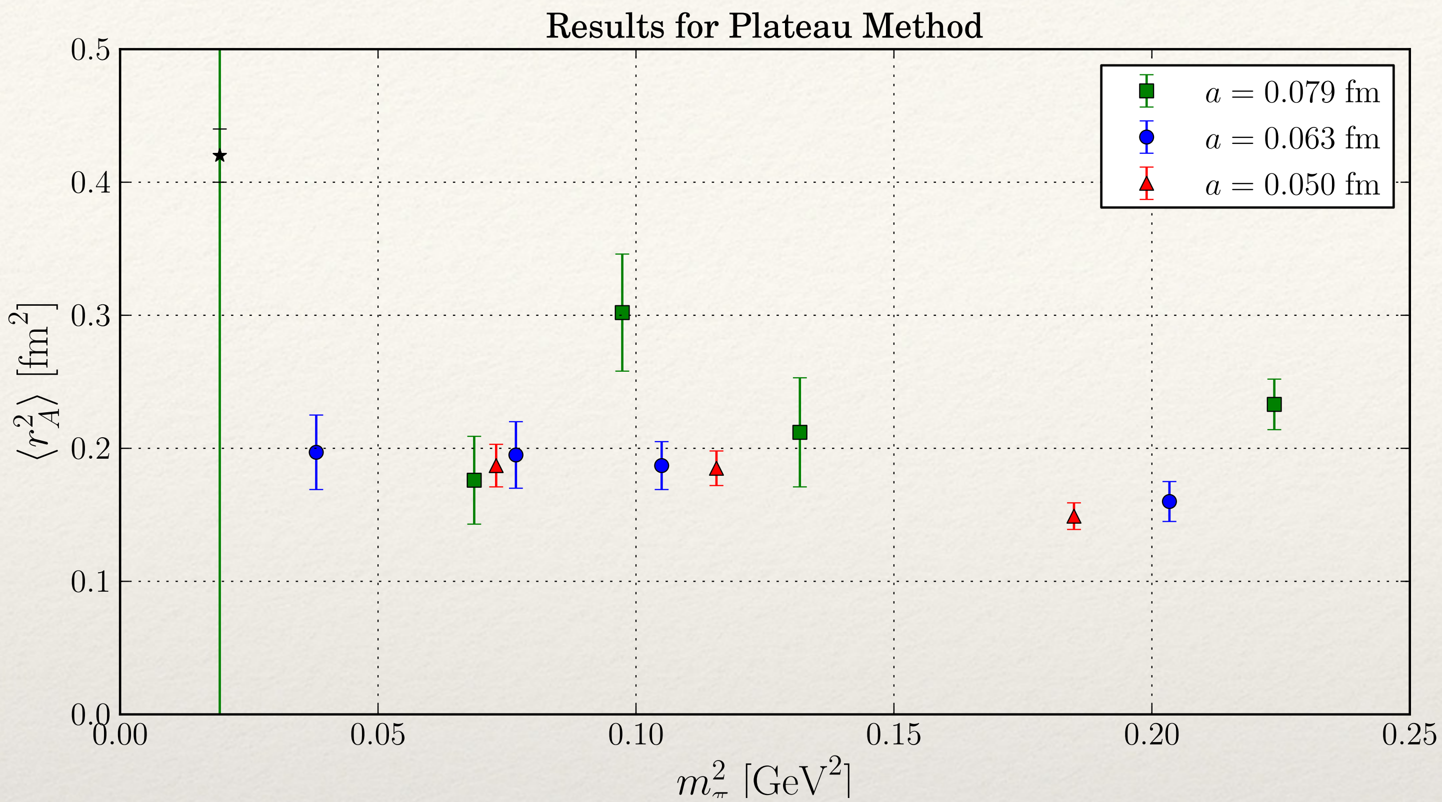
Axial Form factor

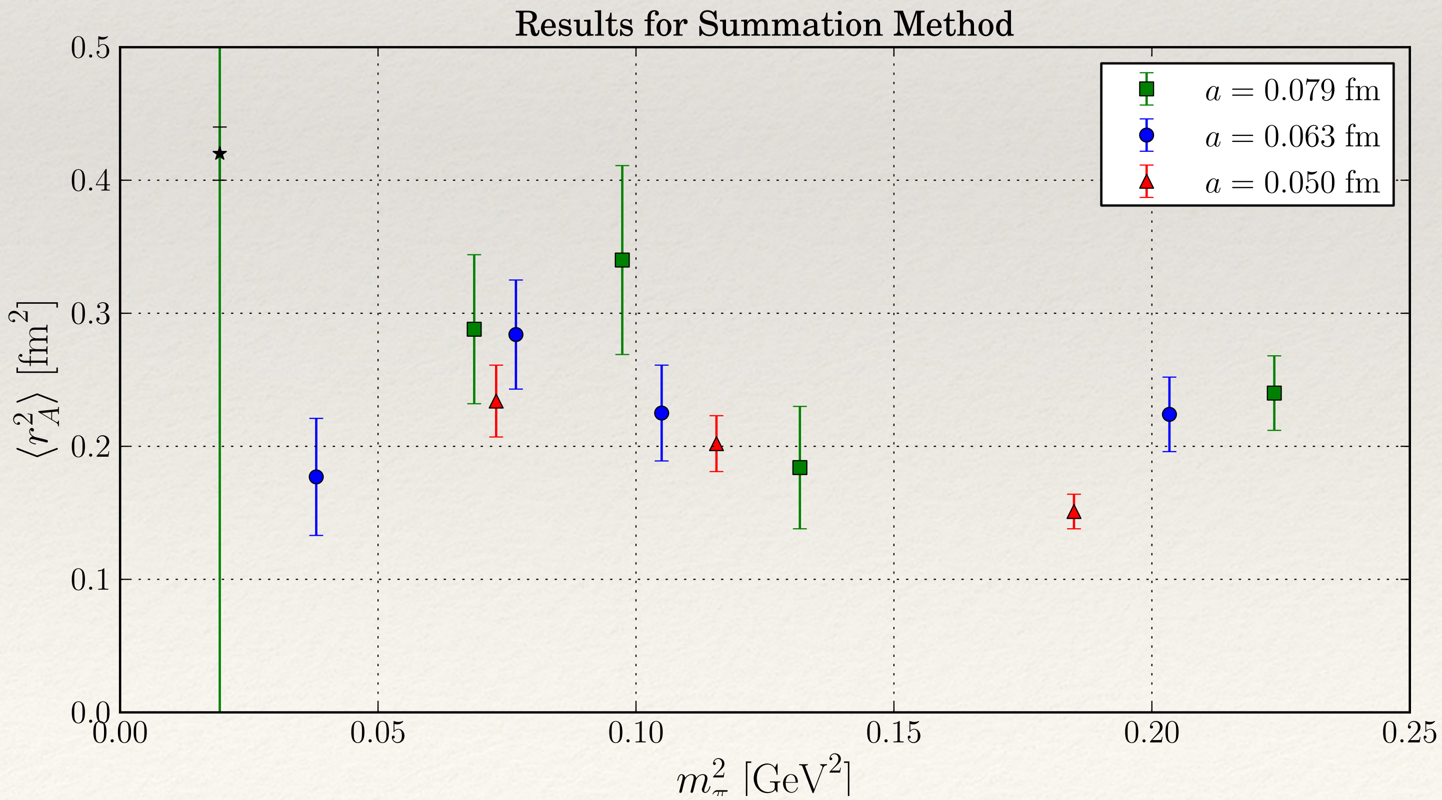


Axial Form factor



Axial radius of nucleon





Induced pseudoscalar form factor

$$\langle N, \mathbf{p}' | A^\mu(x) | N, \mathbf{p} \rangle = e^{iq\cdot x} \bar{u}(\mathbf{p}') \left(\gamma^\mu \gamma^5 G_A(q^2) + \gamma^5 \frac{q^\mu}{2m} \boxed{G_P(q^2)} \right) u(\mathbf{p})$$

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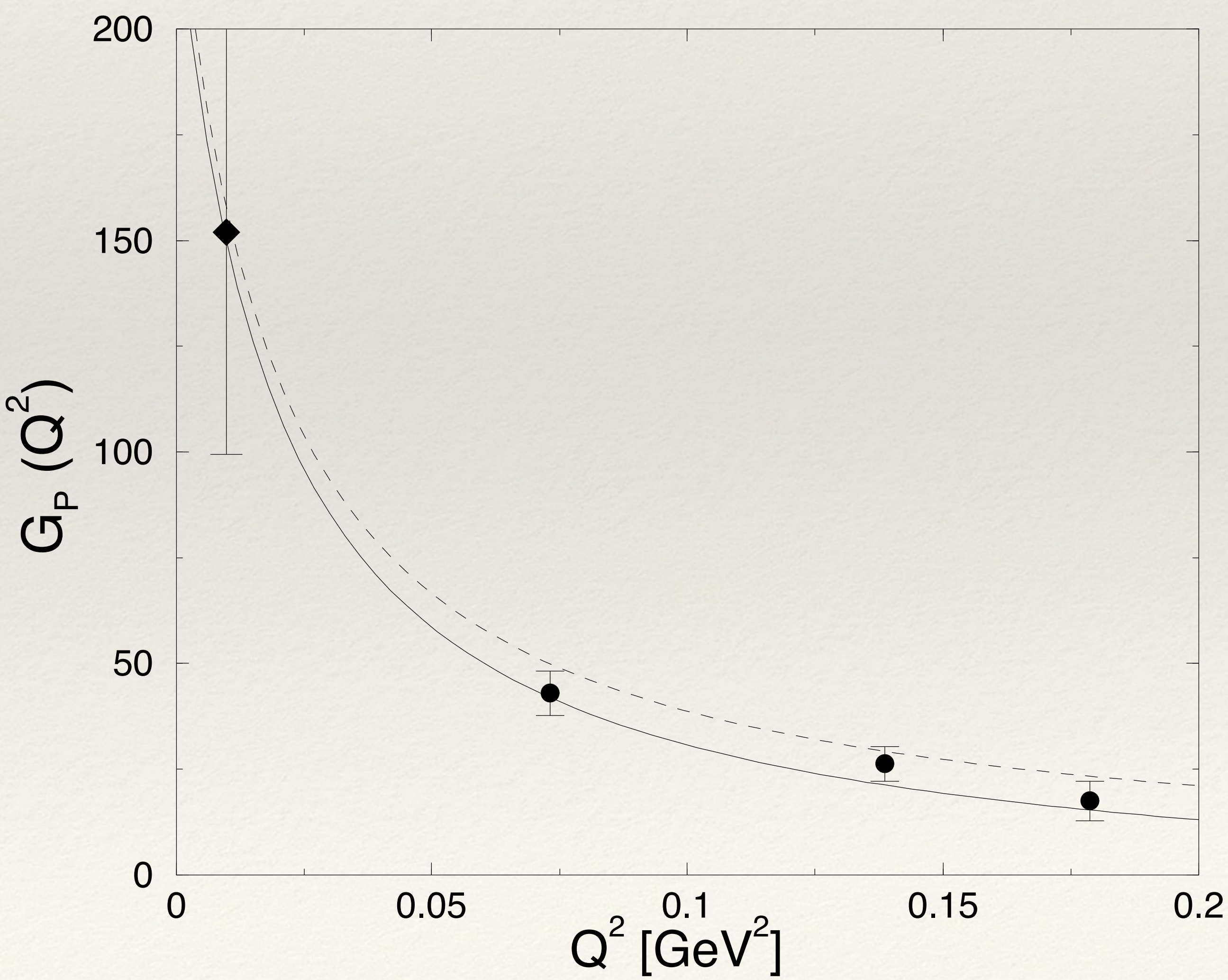
Chiral symmetry breaking and PCAC constrain structure to pion pole

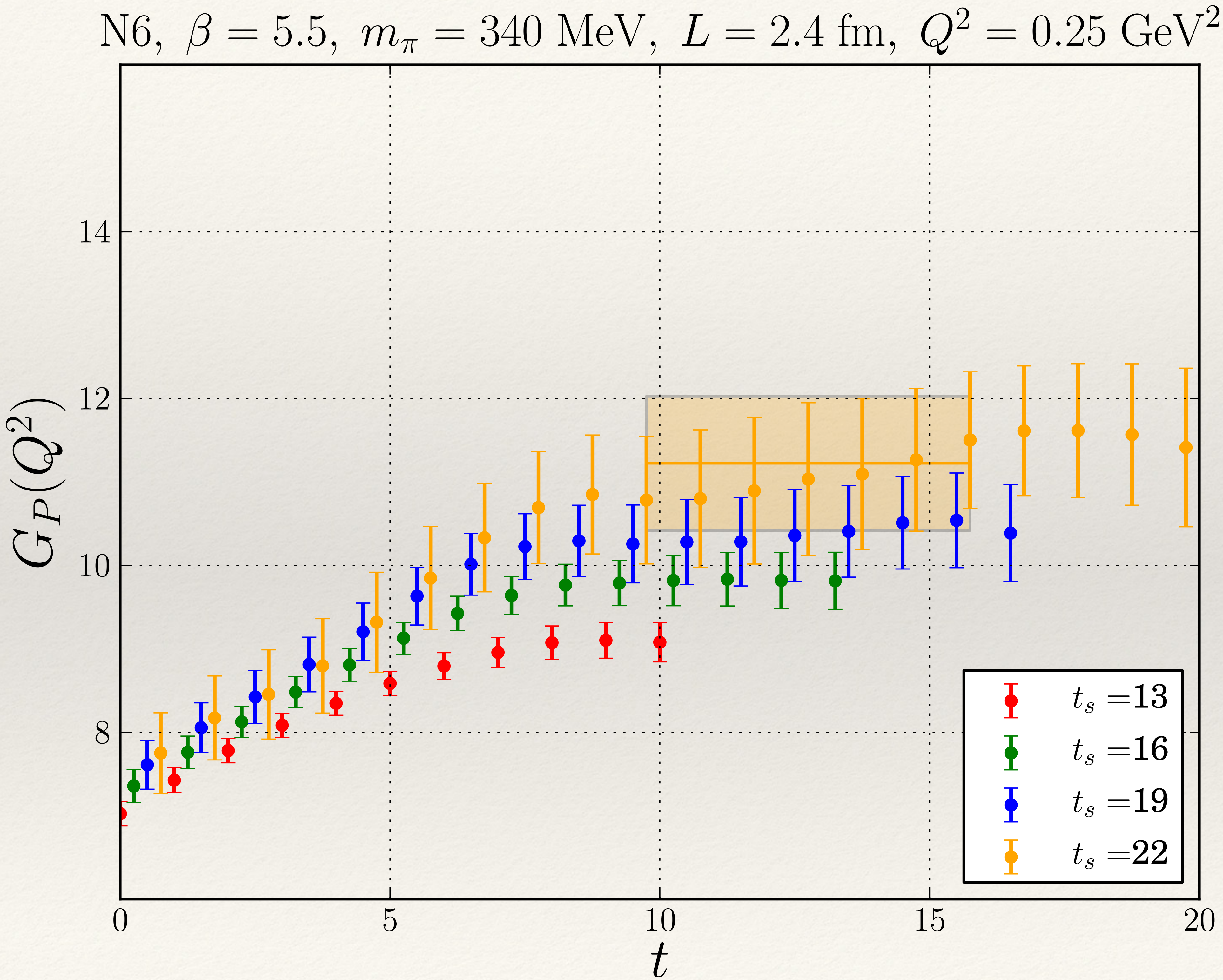
$$G_P(Q^2) = G_A(Q^2)\frac{2M_N}{Q^2 + m_\pi^2}$$

Experimentally measured in muon capture by proton

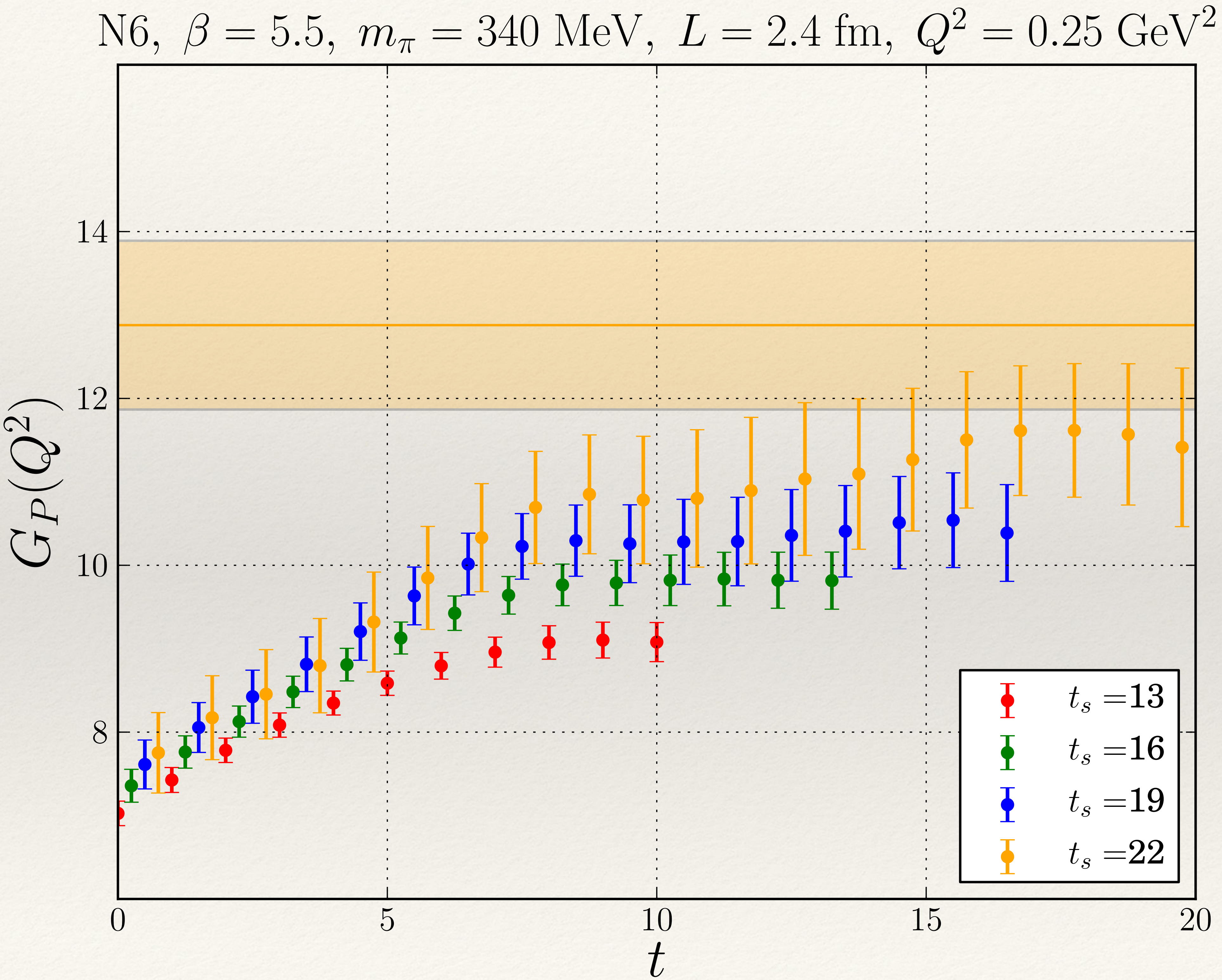
$$g_P \equiv \frac{m_\mu}{2M_N} G_P(Q^2 = -0.88 m_\mu^2)$$



Induced pseudoscalar form factor



Induced pseudoscalar form factor



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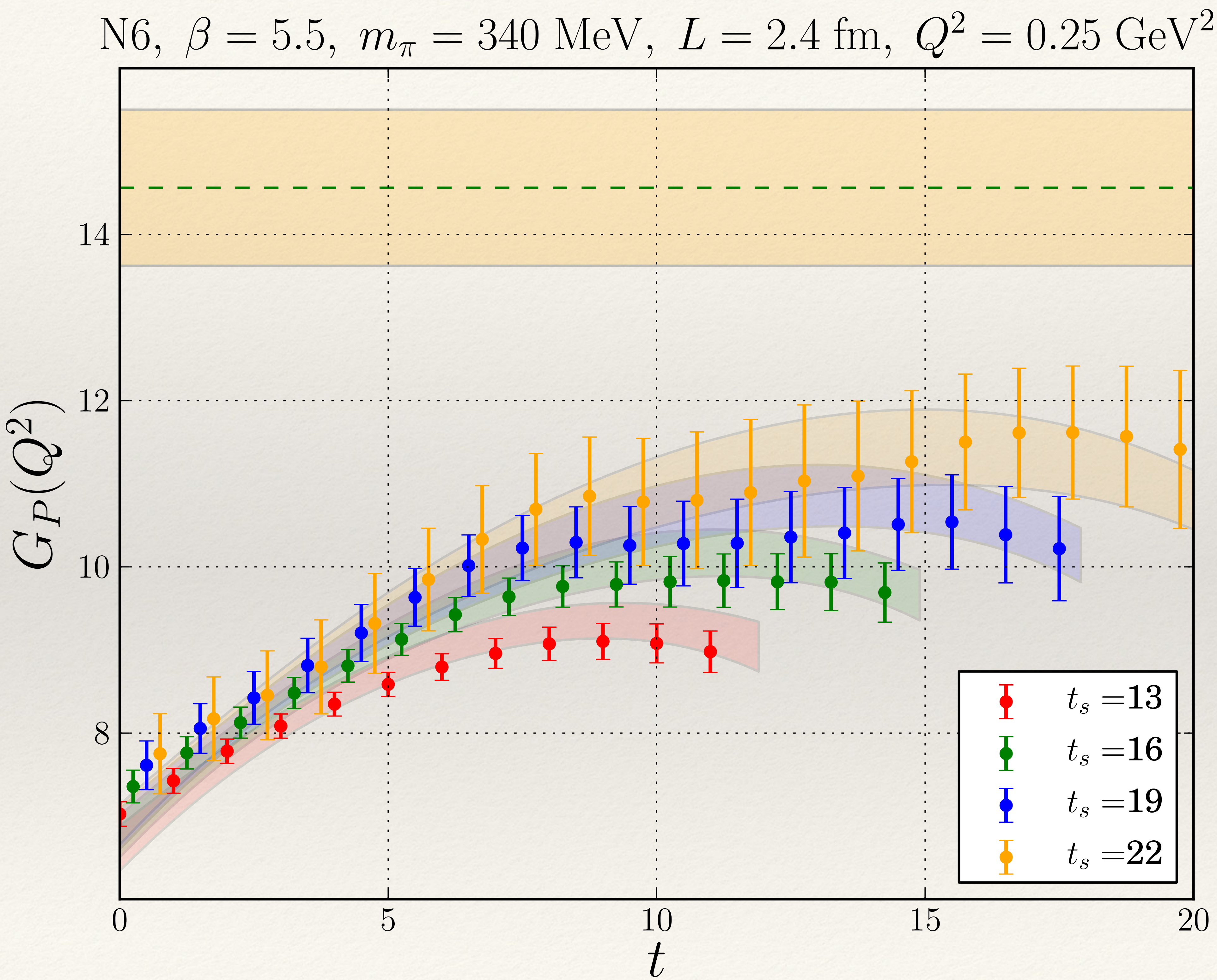

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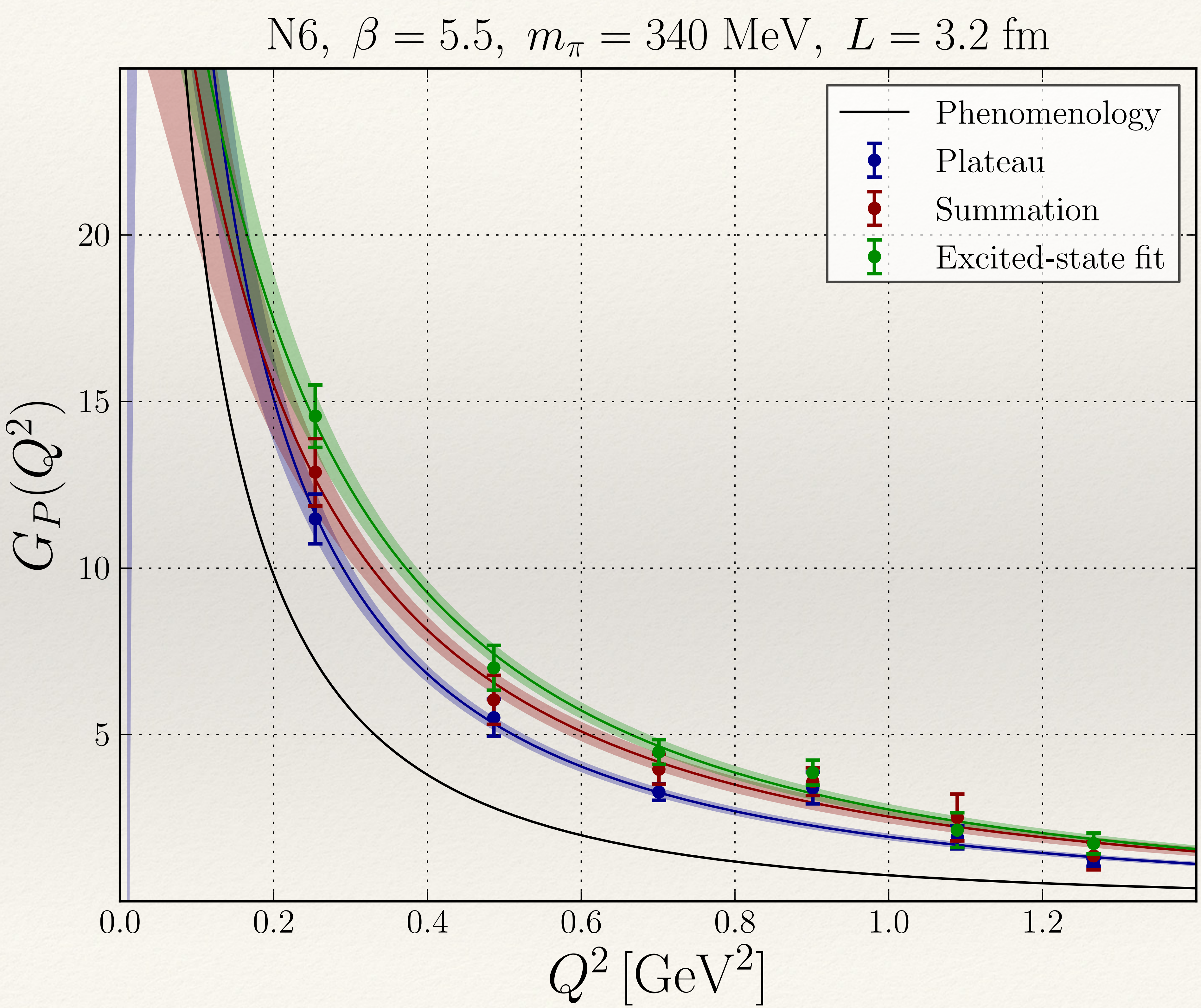
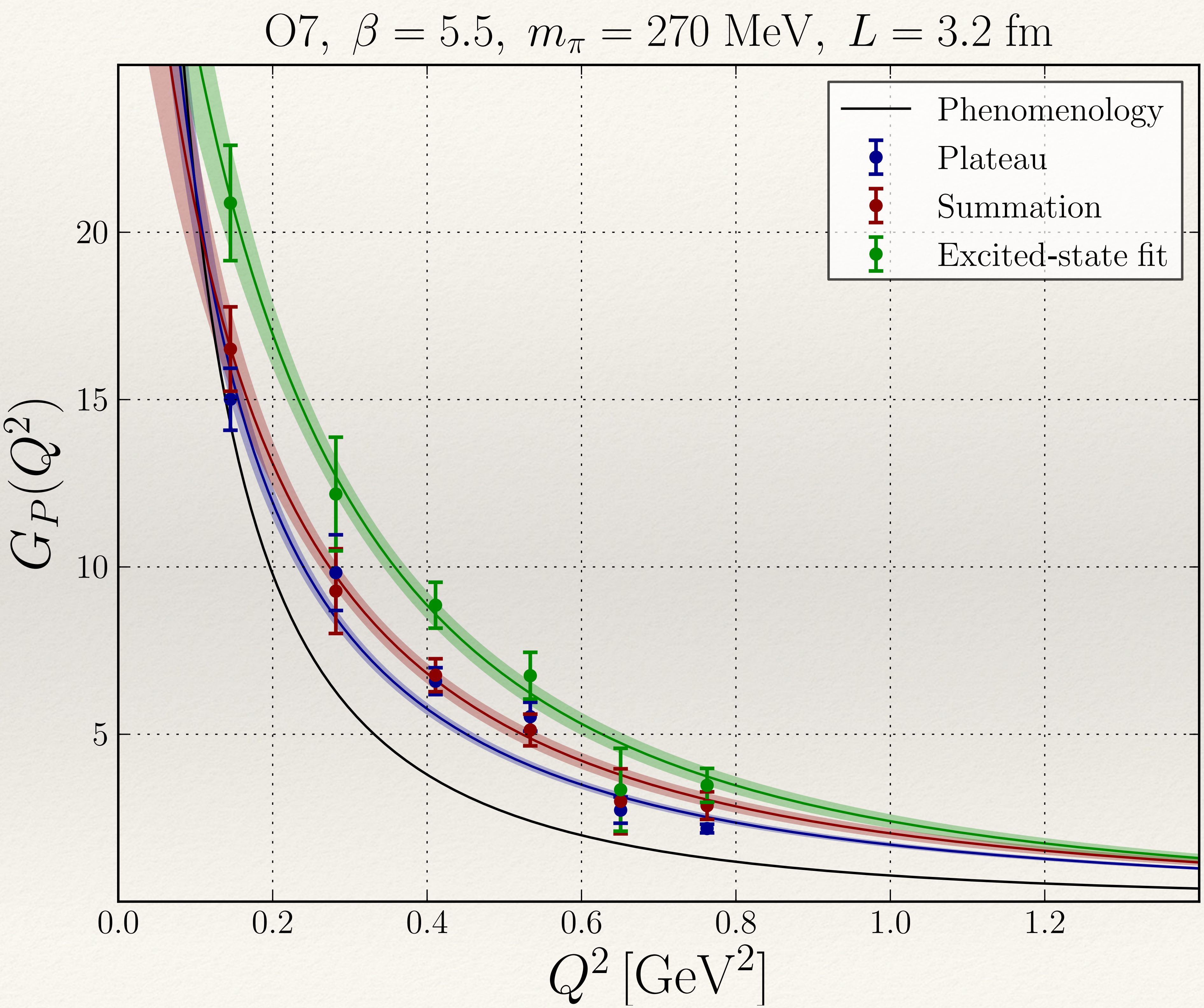


Induced pseudoscalar form factor



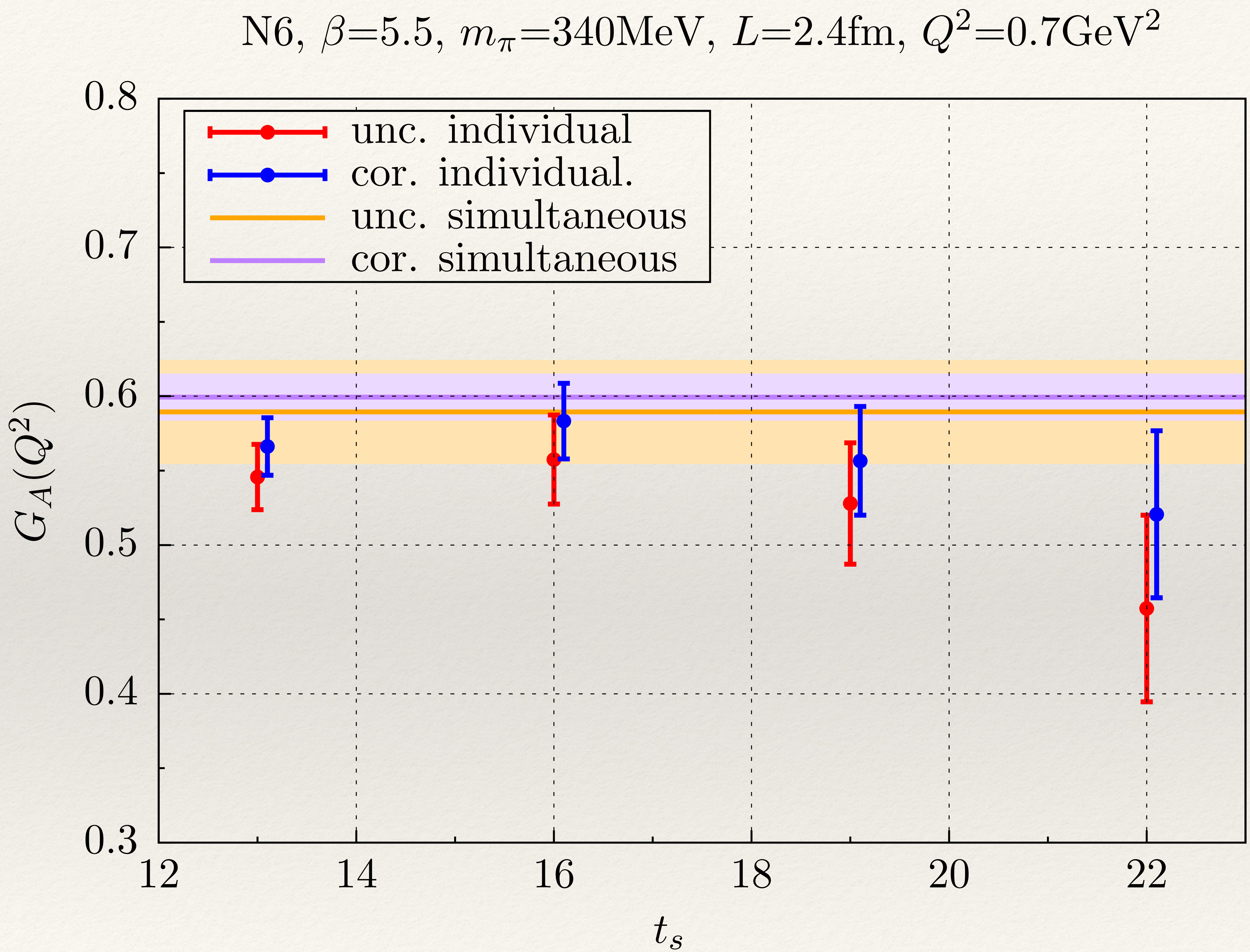
Conclusions

- For the case of g_A, summation method can identify the ground state of the matrix element where the plateau can be misleading.
- Preliminary results for axial form factor show varying excited state contributions at different momentum transfers.
    - Preliminary results show excited state contributions to $\langle r_A^2 \rangle$ hidden by statistics.
- Large excited state contributions to $G_P(Q^2)$ are observed and it remains under investigation.

Thank You

Backup Slides

Individual excited state fits



Individual excited state fits