Glueball masses in 2+1 dimensional $SU(N)$ gauge theories with twisted boundary conditions

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Statement of the problem & notation

- Problem: calculate glueball masses in pure gauge $SU(N)$ theory on spatial two-torus of size L with twisted boundary conditions.
- For numerical investigation: lattice model with Wilson action, periodic b.c. in time:

$$S = Nb \sum_{n \in \mathbb{Z}^3_{(L,L,T)}} \sum_{\mu \neq \nu} (N - z^*_{\mu\nu}(n) P_{\mu\nu}(n)),$$

  where $z_{\mu\nu}(n) = \exp(i\epsilon_{ij}\frac{2\pi k}{N})$ at corner plaquettes in each (1,2)-plane, and 1 everywhere else.
- Inverse 't Hooft coupling: $b = 1/g^2 N$
- Integer $\bar{k}$ defined as: $k\bar{k} = 1 \pmod N$

Motivation

Garcia-Perez, Gonzalez-Arroyo, Okawa, 13, 14:

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- Electric flux energies (calculated from Polyakov loop correlators) in the theory only depend on NL and twist parameters.
- More precisely: on $x = \frac{NL}{4\pi b}$ and $\tilde{\theta} = \frac{2\pi \bar{k}}{N}$
- Shown in perturbation theory to all orders, and also by lattice simulations in wide range of b.
- Can avoid tachyonic instabilities by keeping $k, \bar{k} \propto N$, just as in Twisted Eguchi-Kawai model (Gonzalez-Arroyo, Okawa, 10)
- Can be thought of as a strong form of TEK-like volume independence, also valid for finite N.

What we calculate

- Goal (long term): verify if this result holds also in the zero electric flux (glueball) sector.
- Goal (for this talk): calculate the mass of the lightest 0^{++} glueball as a function of b, for 2 different values of N.
- We take two theories chosen so that LN and $\tilde{\theta}$ are close:
  1. $N = 5$, $L = 14$, $\bar{k} = 2$ ($NL = 70$, $\tilde{\theta} \approx 2.513$)
  2. $N = 17$, $L = 4$, $\bar{k} = 7$ ($NL = 68$, $\tilde{\theta} \approx 2.587$)

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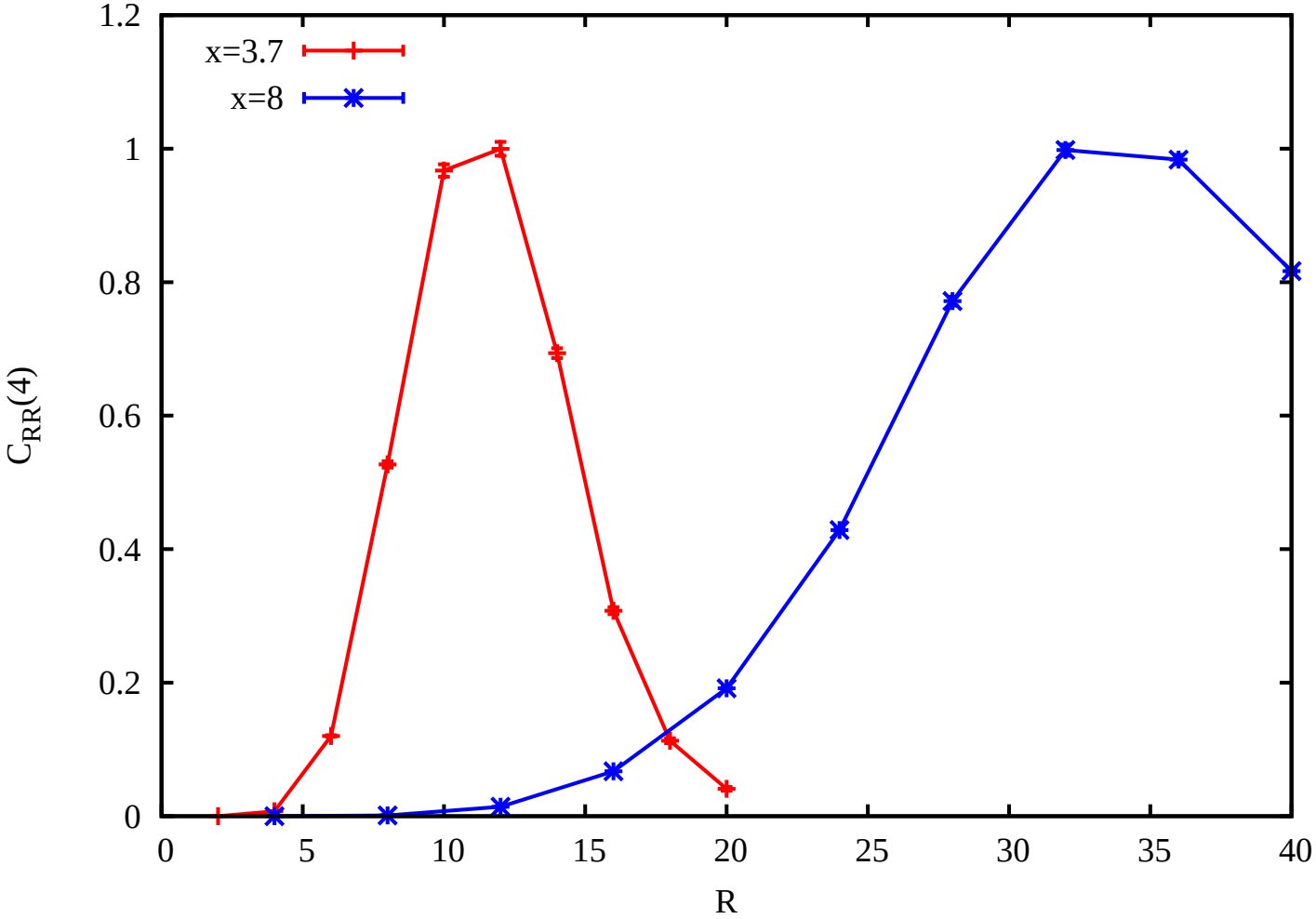
- Knowledge from earlier works:
  - $x \lesssim 0.5\ (b \gtrsim 10)$ - perturbative, small volume region
  - $0.5 \lesssim x \lesssim 4\ (1.5 \lesssim b \lesssim 10)$ - intermediate region
  - $x \gtrsim 4\ (b \lesssim 1.5)$ - large volume region

  Intent: probe glueball mass in all 3 regions.

How we calculate

- Use correlations of rectangular Wilson loops and moduli of multi-winding Polyakov loops $|\mathrm{Tr}\, P^n|^2$
- Use 3 different levels of smearing and large loops, trying to follow the physical size of the glueball (including loops larger than L for small and moderate x)
- Construct: $C_{ij}(t) = \sum_{t'} \langle O_i(t'+t) O_j(t') \rangle - \langle O_i(t'+t) \rangle \langle O_j(t') \rangle$

How we calculate



$C_{RR}(4)$ is the (normalized) correlator of $W(R,R)$ at distance 4 lattice sites, $N = 5, L = 14, \bar{k} = 2$.

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- Construct: $C_{ij}(t) = \sum_{t'} \langle O_i(t'+t) O_j(t') \rangle - \langle O_i(t'+t) \rangle \langle O_j(t') \rangle$
- Do GEVP:

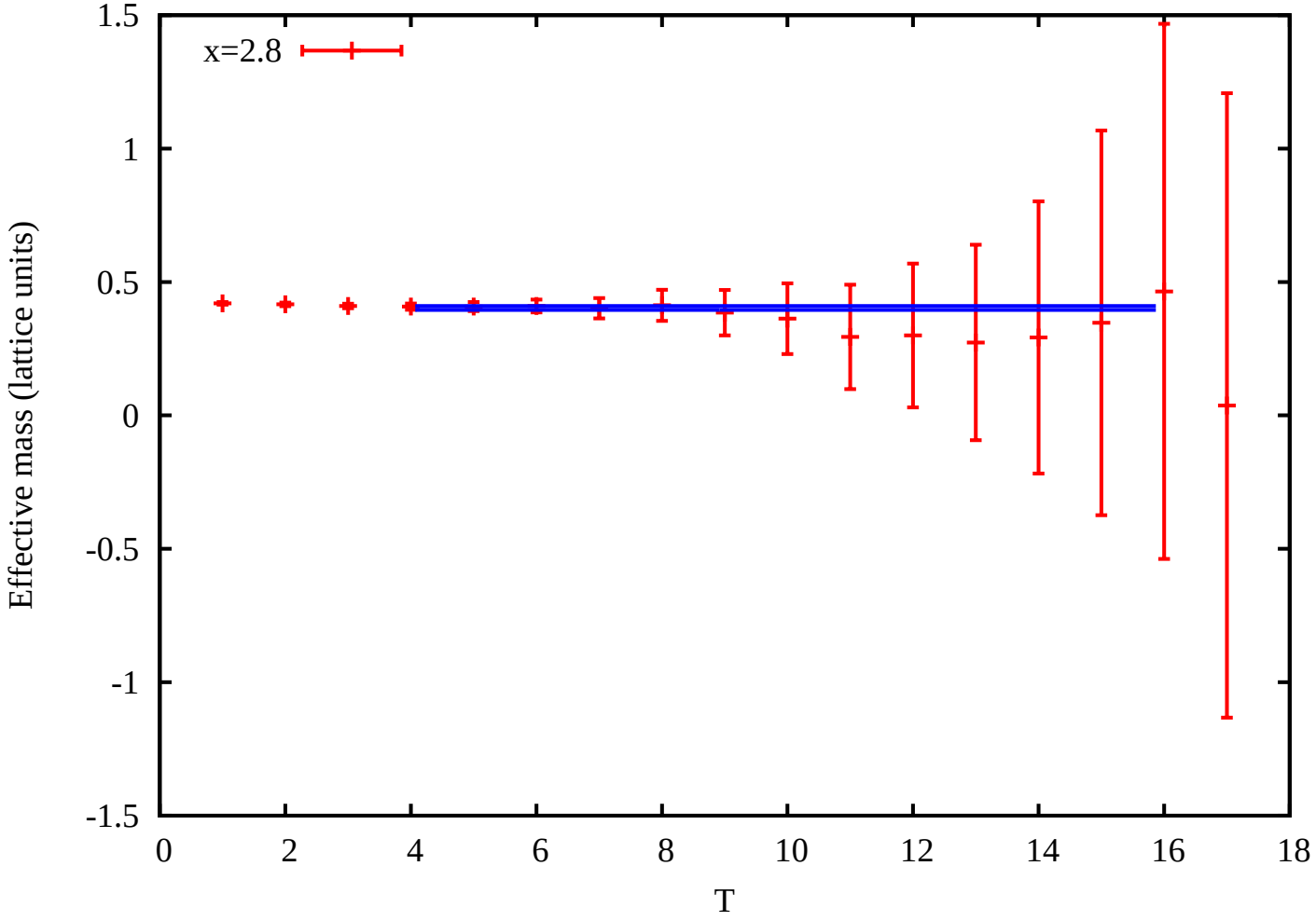
$$C(t_1) v = C(t_0) \lambda v$$

  to find v, use them to change the basis $C(t) \to \tilde{C}(t)\ \forall t$ and fit to diagonal elements of $\tilde{C}(t)$ (after finding the plateau)
- Technicalities: use ≈ 12 operators for $C_{ij}(t)$, estimate if basis allows reliable GEVP by first solving it on *non-symmetrized* $C(t)$, use quad precision for GEVP and basis change

Results: caveat

Beware: results preliminary, all errors only statistical.

Results: example mass plateau



Effective mass plateau for $N = 5, L = 14, \bar{k} = 2, b = 2$ $(x = 2.8)$, $Nmeas = 10^5$

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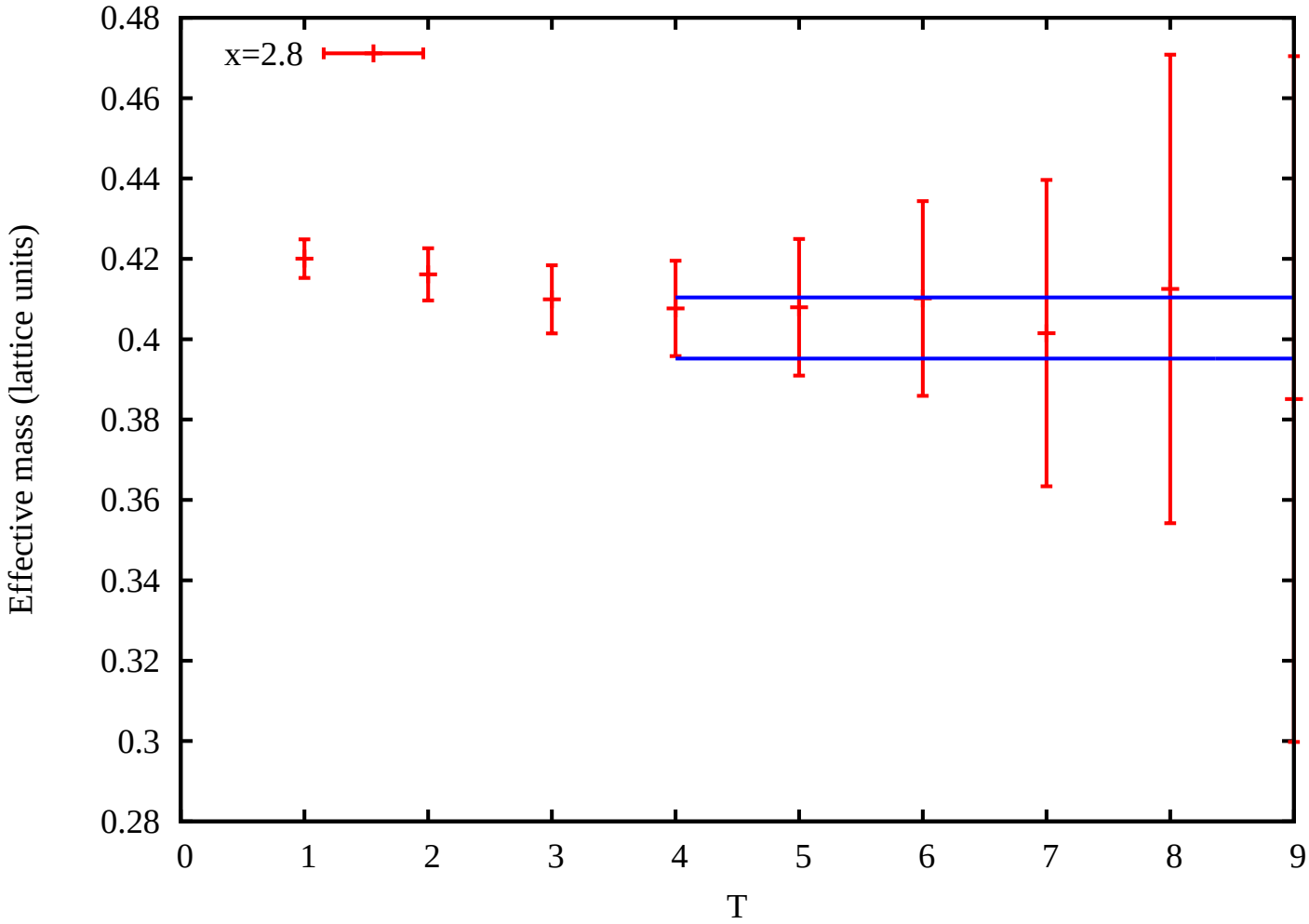

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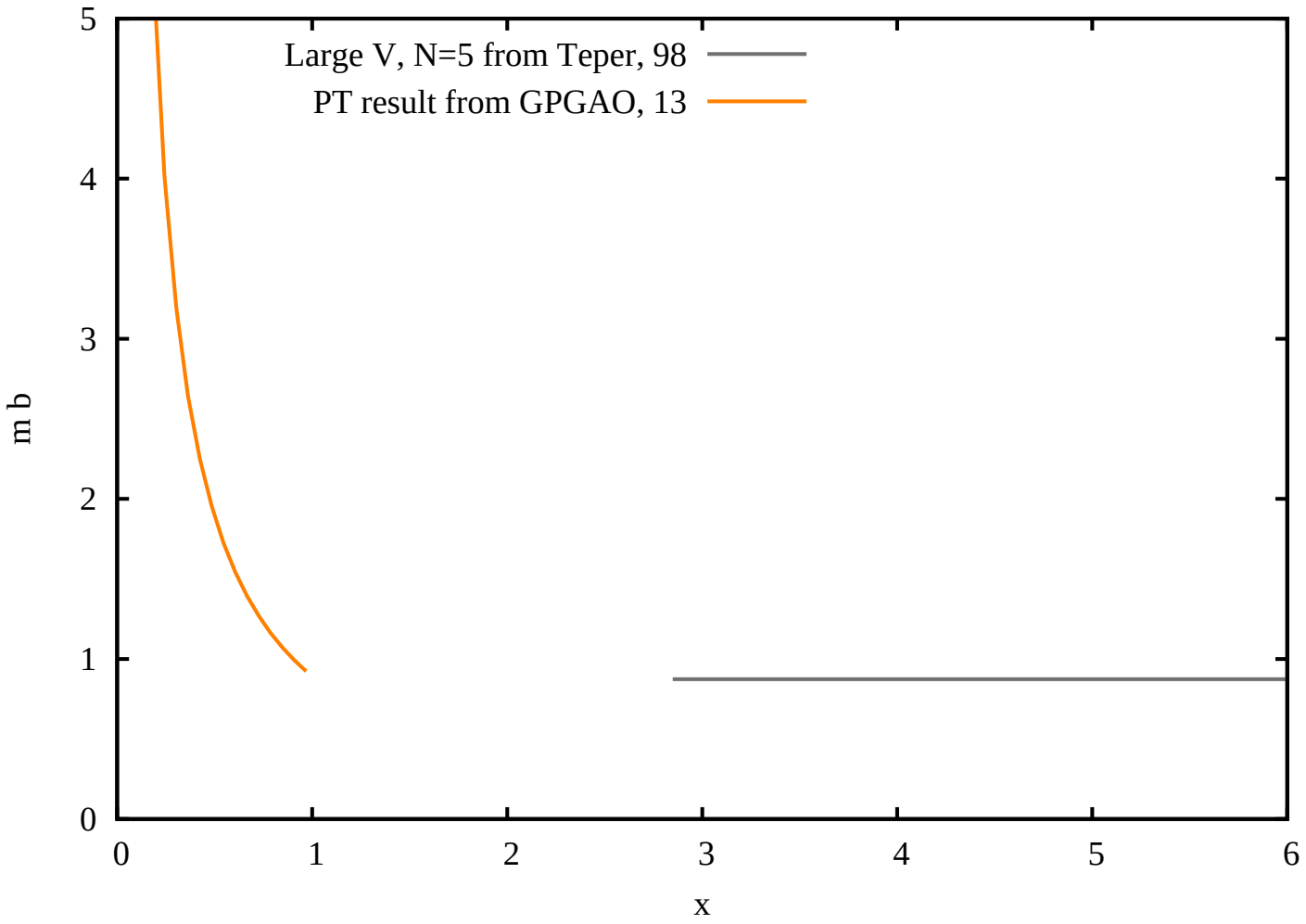

The theoretical expectations

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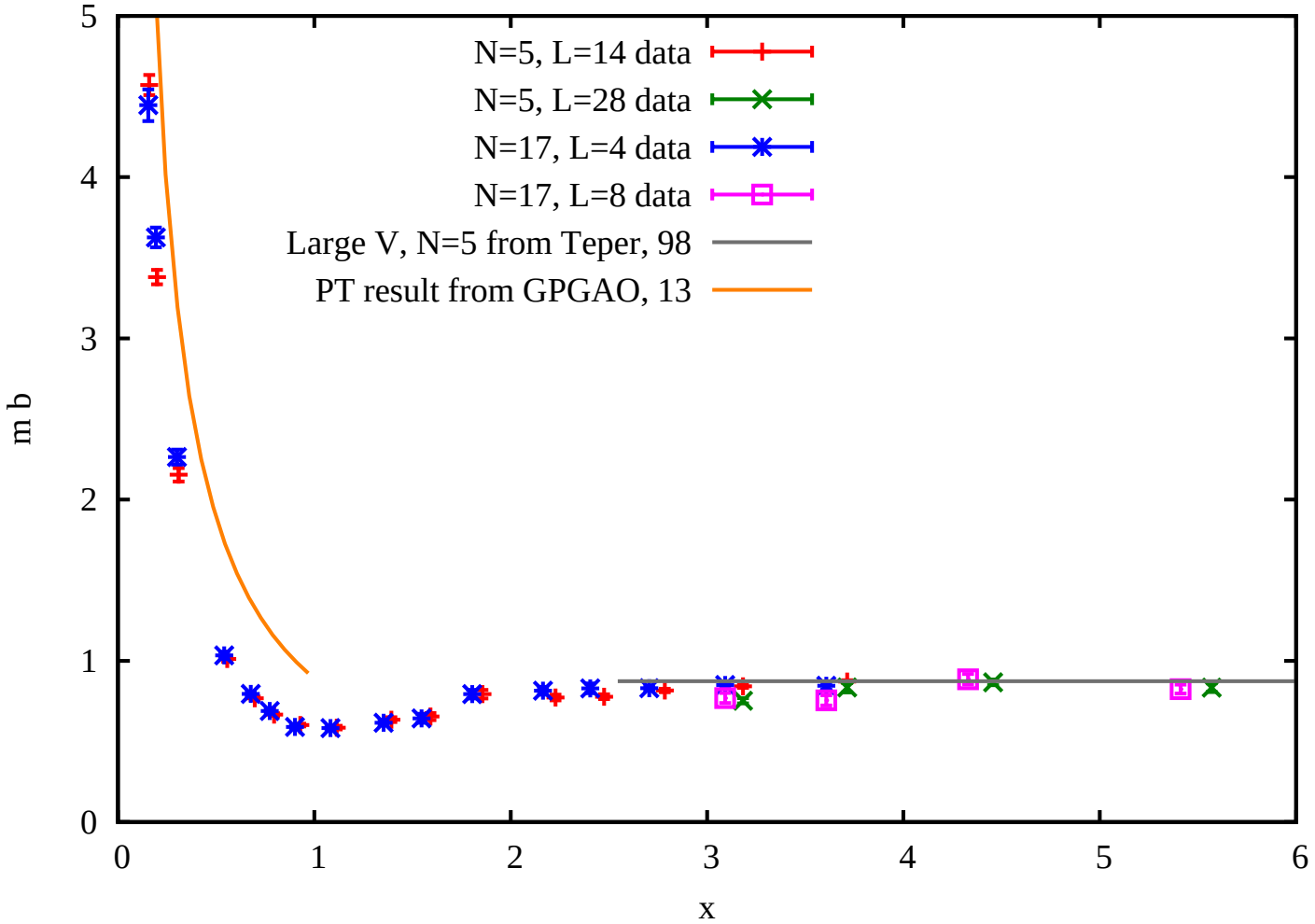

The results for both theories, note the results for doubled L in the large x region.

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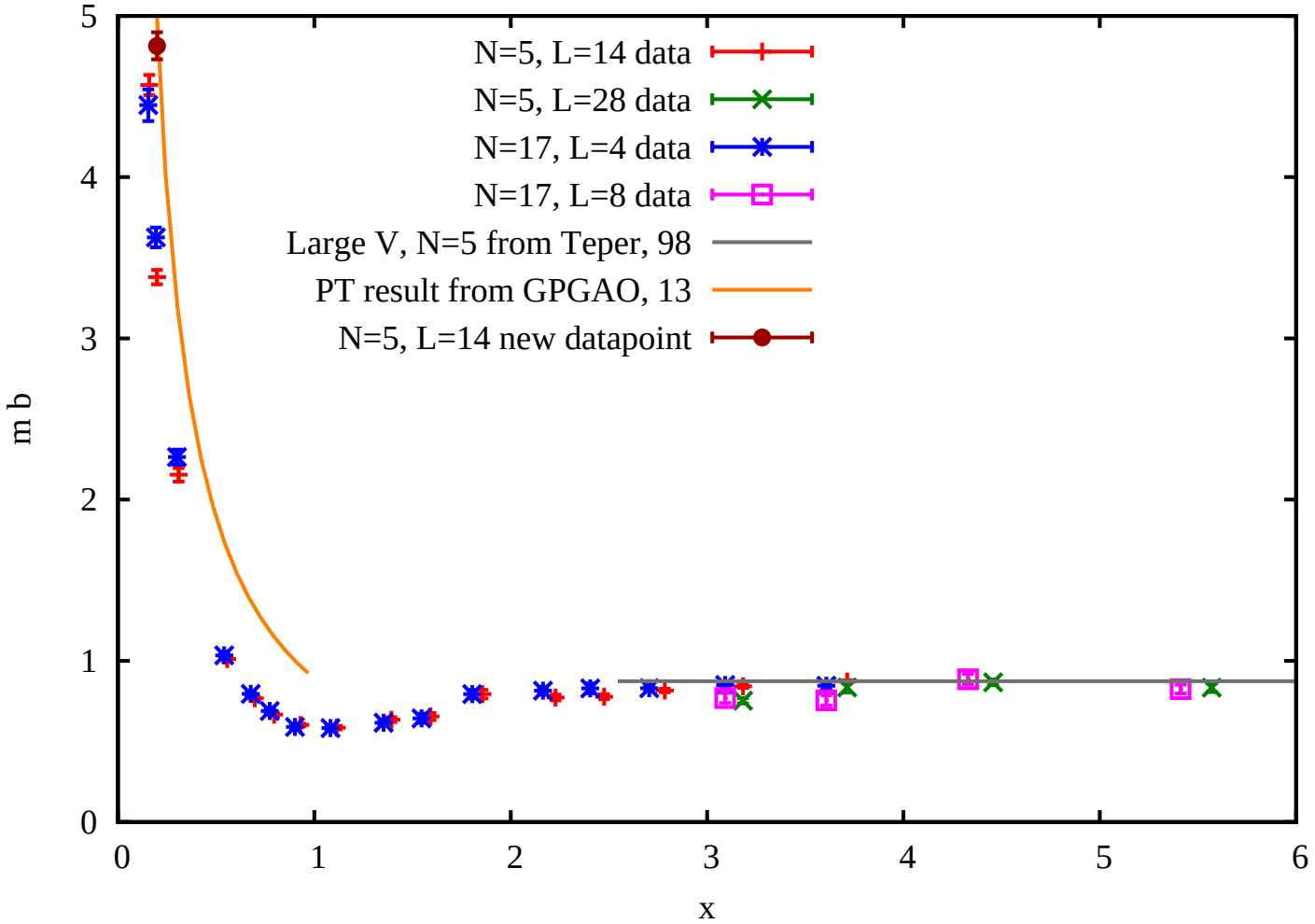

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Conclusions & outlook

- Extracted 0^{++} glueball mass in large range of couplings with constant lattice size L for $N = 5$ and $N = 17$ with matching NL and electric flux $\tilde{\theta}$.
- Large volume region under good control, small volume still needs improved analysis.
- $N = 5$ and $N = 17$ data in good agreement, as expected by the x-scaling hypothesis!

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TODO list:

- Get the systematics right, especially in the small-x region - autocorrelations, replicas, larger T, other operators?
- Add other quantum numbers, especially 2^{++}.
- Investigate other $\bar{k}$ values.

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Thank you for your attention!