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Beta function of three-dimensional QED

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$\mathrm{U}(1)$ gauge theory, $d = 3$, $N_f = 2$ (4 Dirac components), $m = 0$:

$$S = \int d^3x \left(\frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i \not{D} \psi_i \right)$$

THE ISSUE: Confinement vs. conformality

The problem: $3d \Longrightarrow$ IR problems, volume sensitivity (`Hands, Kogut, Scorzato, Strouthos`)

THE PHYSICS of QED3

(Appelquist *et al.*)

Nothing non-Abelian here! Just

1. Logarithmic Coulomb potential $+$ transverse photons
2. Dynamical screening by massless charges

Which one wins? Hypothesis:

- Small N_f: Confinement, mass generation $m \sim e^2$
- Large N_f: Screening ... meaning what?

Running coupling $e^2(q)$:

$$\frac{de^2}{d\log q} = N_f b_1 e^4/q + \cdots, \qquad b_1 > 0 \quad \text{(screening!)}$$

Dimensionless $g^2(q) = e^2/q$

$$\frac{dg^2}{d\log q} = -g^2 + N_f b_1 g^4 + \cdots$$

- Small N_f: 1st term drives g^2 large $\Longrightarrow$ $\langle\bar\psi\psi\rangle$ condensate, mass for fermions $\Longrightarrow$ decoupling, $\log r$ confinement
- Large N_f: IR flow to fixed point at $g^2 = (N_f b_1)^{-1}$ $\Longrightarrow$ Conformal physics, no length scale

Familiar questions: just like technicolor candidate theories

(TRIGGER WARNING: Technicolor survivors)

CALCULATING THE β FUNCTION: the Schrödinger Functional

Continuum SF definition of $g(L)$: (Lüscher *et al.*, ALPHA collaboration)

- Cubical Euclidean box, volume L^3, massless limit
- Fix the gauge field at $t = 0, L$: $A_x = A_y = \pm\phi/L$
  $\Rightarrow$ background field $E_x = E_y = -2\phi/L^2$. Note L is the only scale.
- Consider $\Gamma \equiv -\log Z$, compare to classical action of bkgd field:

$$\Gamma = \frac{1}{e^2(L)} \int d^3x \, \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{e^2(L)} \frac{\#}{L}$$

$\Longrightarrow$ running coupling $g^2(L) = e^2(L)L$
(actually calculate $d\Gamma/d\phi =$ some Green function $= K(\phi)/g^2(L)$)

One loop:

$$\frac{1}{g^2(L)} = \frac{1}{g^2(\mu)\mu L} + N_f b_1 + \cdots$$

Beta function for $u \equiv 1/g^2$

$$\tilde{\beta}(1/g^2) \equiv \frac{d(1/g^2)}{d\log L} = -\frac{1}{g^2} + N_f b_1 + O(g^2)$$

. . . a straight line, crosses zero at $u = N_f b_1$.

LATTICE CALCULATION

Non-compact U(1) gauge field, Wilson–clover fermions, nHYP smearing

$$S = \frac{\beta}{2} \sum_{\substack{n \\ \mu<\nu}} (\nabla \times A)^2_{n\mu\nu} + \bar{\psi} D \psi$$

Bare couplings $\beta = 1/(e_0^2 a)$, $\kappa = \kappa_c(\beta)$, volume $(L = Na)^3$

$\Longrightarrow$ inverse coupling $u(L)$

Compare $L \to sL$ at fixed a

$\Longrightarrow$ Discrete beta function

$$R(u, s) \equiv \frac{u(sL) - u(L)}{\log s}$$

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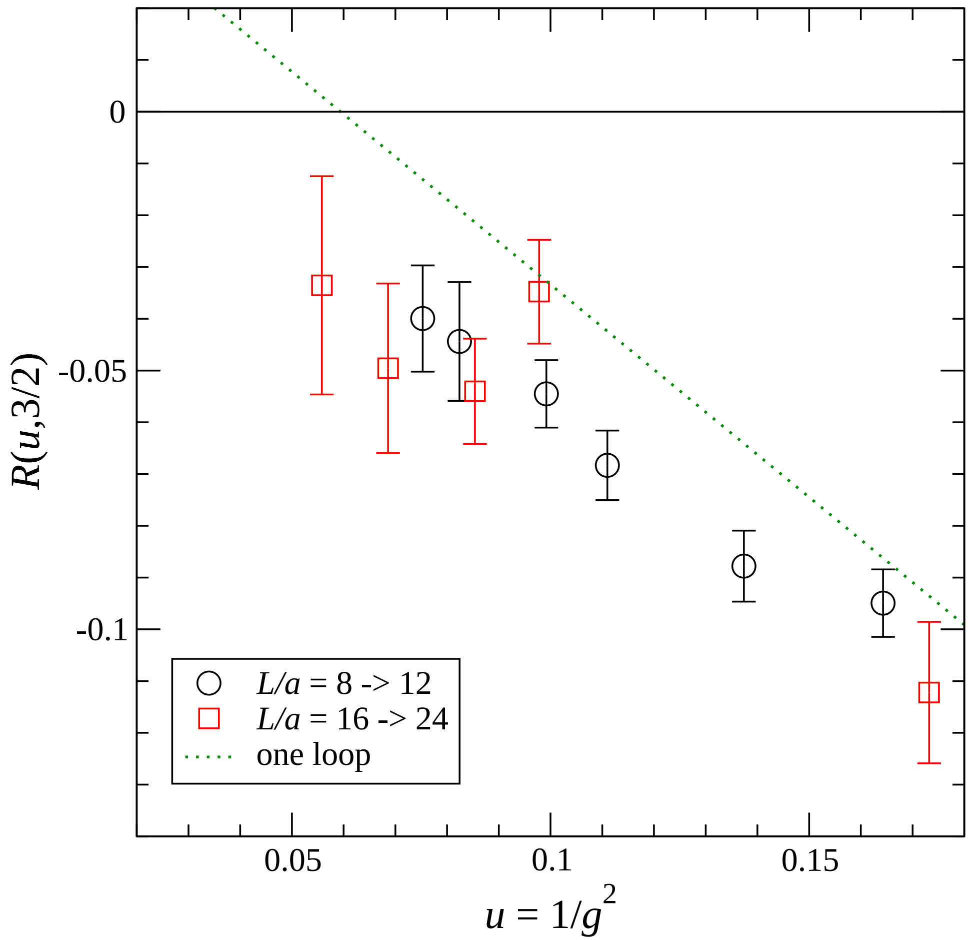


In strong coupling, $R(u)$ avoids fixed point — but lattice spacing is still fixed . . .

BETA FUNCTION: SLOPE ANALYSIS

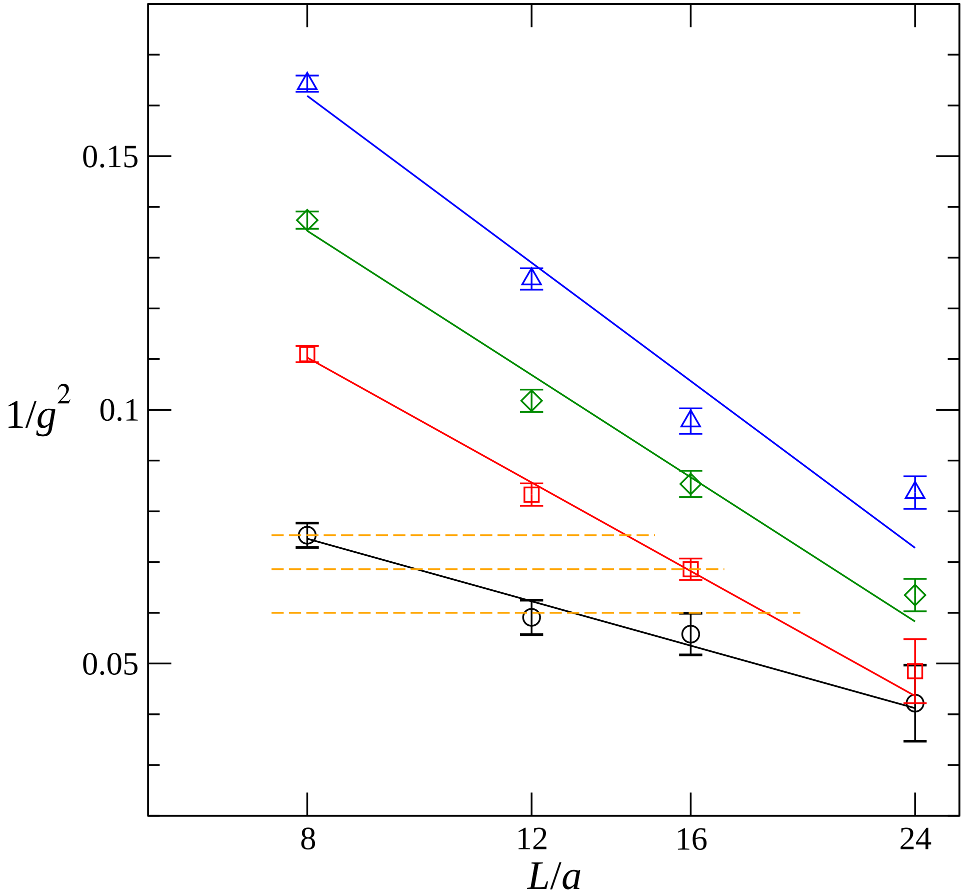
Look for levelling off of beta fn:

Plot $1/g^2$ vs. $\log L$ at fixed bare coupling β.
Slope $\Longrightarrow$ beta fn if $\sim$constant.

Success at 2 strongest couplings.

Fix $1/g^2$, get 2 slopes at
2 β's $\Longleftrightarrow$ 2 lattice spacings

$\Longrightarrow$ extrapolate $a/L \to 0$



Top to bottom: $\beta = 1.0, 0.8, 0.6, 0.4$

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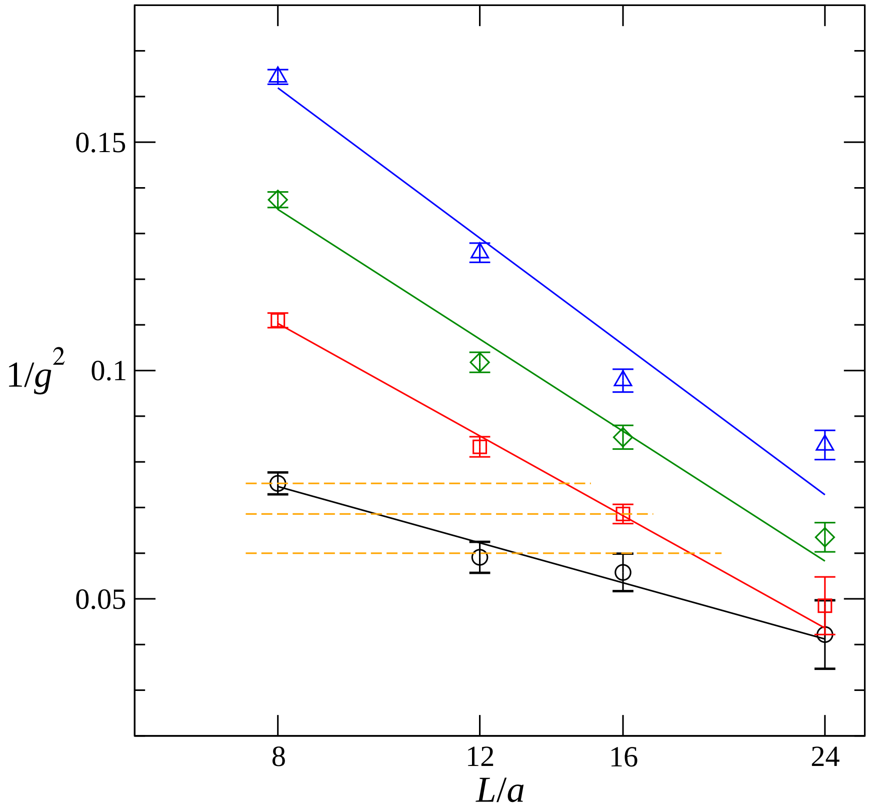

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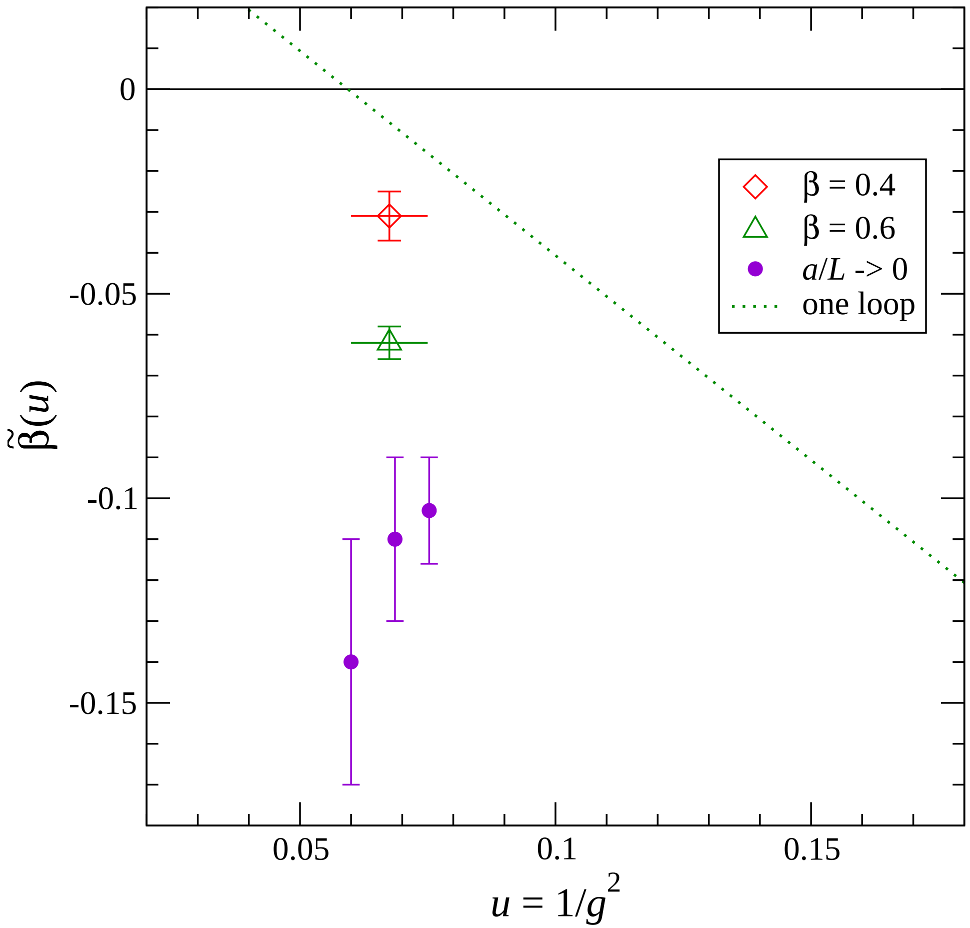
Extrapolated slopes



CONCLUSION: Beta function avoids zero $\Longrightarrow$ $N_f = 2$ QED3 confines.