Beta function of three-dimensional QED

B. Svetitsky
Tel Aviv University

with O. Raviv and Y. Shamir

U(1) gauge theory, $d = 3$, $N_f = 2$ (4 Dirac components), $m = 0$:

$$S = \int d^3 x \left( \frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i \not{D} \psi_i \right)$$

THE ISSUE: Confinement vs. conformality

The problem: $3d \rightarrow$ IR problems, volume sensitivity

(Hands, Kogut, Scorzato, Strouthos)
Nothing non-Abelian here! Just

1. Logarithmic Coulomb potential + transverse photons

2. Dynamical screening by massless charges

Which one wins? Hypothesis:

- Small $N_f$: Confinement, mass generation $m \sim e^2$
- Large $N_f$: Screening . . . meaning what?

Running coupling $e^2(q)$:

$$\frac{de^2}{d\log q} = N_f b_1 e^4 / q + \cdots, \quad b_1 > 0 \quad (\text{screening!})$$

Dimensionless $g^2(q) = e^2 / q$

$$\frac{dg^2}{d\log q} = -g^2 + N_f b_1 g^4 + \cdots$$

- Small $N_f$: 1st term drives $g^2$ large $\implies \langle \bar{\psi} \psi \rangle$ condensate, mass for fermions $\implies$ decoupling, $\log r$ confinement
- Large $N_f$: IR flow to fixed point at $g^2 = (N_f b_1)^{-1} \implies$ Conformal physics, no length scale

Familiar questions: just like technicolor candidate theories  

(Appelquist et al.)
CALCULATING THE $\beta$ FUNCTION: the Schrödinger Functional

Continuum SF definition of $g(L)$:  
(Lüscher et al., ALPHA collaboration)

- Cubical Euclidean box, volume $L^3$, massless limit
- Fix the gauge field at $t = 0, L$: $A_x = A_y = \pm \phi / L$
  \[ \Rightarrow \text{background field } E_x = E_y = -2\phi / L^2. \]
  Note $L$ is the only scale.
- Consider $\Gamma \equiv -\log Z$, compare to classical action of bkgd field:
  \[ \Gamma = \frac{1}{e^2(L)} \int d^3x \frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{e^2(L)} \frac{\#}{L} \]
  \[ \implies \text{running coupling } g^2(L) = e^2(L)L \]
  (actually calculate $d\Gamma / d\phi = $ some Green function = $K(\phi) / g^2(L)$)

One loop:
\[ \frac{1}{g^2(L)} = \frac{1}{g^2(\mu)\mu L} + N_f b_1 + \cdots \]

Beta function for $u \equiv 1 / g^2$
\[ \tilde{\beta}(1/g^2) \equiv \frac{d(1/g^2)}{d \log L} = -\frac{1}{g^2} + N_f b_1 + O(g^2) \]

... a straight line, crosses zero at $u = N_f b_1$. 
LATTICE CALCULATION

Non-compact $\text{U}(1)$ gauge field, Wilson–clover fermions, nHYP smearing

$$S = \frac{\beta}{2} \sum_{\mu < \nu} (\nabla \times A)^2_{\nu \mu} + \bar{\psi}D\psi$$

Bare couplings $\beta = 1/(e_0^2 a), \kappa = \kappa_c(\beta)$,

volume $(L = Na)^3$

$\Rightarrow$ inverse coupling $u(L)$

Compare $L \rightarrow sL$ at fixed $a$

$\Rightarrow$ Discrete beta function

$$R(u, s) \equiv \frac{u(sL) - u(L)}{\log s}$$
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In strong coupling, $R(u)$ avoids fixed point — but lattice spacing is still fixed . . .
BETA FUNCTION: SLOPE ANALYSIS

Look for levelling off of beta fn:

Plot $1/g^2$ vs. $\log L$ at fixed bare coupling $\beta$.
Slope $\implies$ beta fn if $\sim$constant.

Success at 2 strongest couplings.

Fix $1/g^2$, get 2 slopes at
2 $\beta$'s $\iff$ 2 lattice spacings

$\implies$ extrapolate $a/L \to 0$

Top to bottom: $\beta = 1.0, 0.8, 0.6, 0.4$
CONCLUSION: Beta function avoids zero $\Rightarrow N_f = 2$ QED3 confines.