A Feynman-Hellmann approach to nonperturbative renormalization of lattice operators

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CSSM/QCDSF/UKQCD collaborations

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- Connection between "lattice world" and "real world": renormalization constants Z
- Must know them as accurate as possible
- Nonperturbative approach: widely used scheme is $\mathrm{RI}' - \mathrm{MOM}$

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Z_{\mathcal{O}}^{-1}(p) &= Z_q^{-1}(p)\,\frac{1}{12}\mathrm{tr}\left(\Gamma_{\mathcal{O}}(p)\Gamma^{-1}_{\mathrm{Born},\,\mathcal{O}}(p)\right) \\
Z_q(p) &= \frac{\mathrm{tr}(-i\sum_\lambda \gamma_\lambda \sin(ap_\lambda) aS^{-1}(p))}{12\sum_\lambda \sin^2(ap_\lambda)}
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- Simulations with dynamical fermions: vertex function $\Gamma_{\mathcal{O}}(p)$ can contain disconnected contributions
- Three-point functions and disconnected contributions: technically very demanding
- Alternative approach: Feynman-Hellmann (FH) method which needs two-point functions only - at the expense of modified actions
- We present first results for the local operators $\mathcal{O} = A_3, S$
- Setting: $32^3 \times 64$ lattice, $\beta = 5.5$, $N_f = 3$, $a = 0.074(2)\,\mathrm{fm}$, 8 momentum tuples, 9 configurations/tuple
- Axial vector operator: $\kappa = 0.12090$
- Scalar operator: $\kappa = 0.12099, 0.12095, 0.12092$
- Action: SLiNC fermions with tree-level improved Symanzik gluons

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- Modified action ($\kappa_u = \kappa_d = \kappa_s = \kappa_{\rm sea}$)

$$\mathcal{S}_{\rm mod}(\lambda_{\rm sea}) = \mathcal{S}_G(U) + \sum_q \overline{\psi}_q\, M(\kappa_{\rm sea})\, \psi_q - \lambda_{\rm sea} \sum_q \overline{\psi}_q\, \mathcal{O}\, \psi_q$$

- Modified propagator $S_{ij}^{\rm mod}$ from the fermion matrix (after integration over the fermion fields)

$$S_{ij}^{\rm mod}(\lambda_{\rm sea}, \lambda_{\rm val}) =$$

$$\frac{\int DU\, (M(\kappa_{\rm val}) - \lambda_{\rm val}\, \mathcal{O})_{ij}^{-1}\, \det\, (M(\kappa_{\rm sea}) - \lambda_{\rm sea}\, \mathcal{O})^{N_f}\, \exp\, [-\mathcal{S}_G(U)]}{\int DU\, \det\, (M(\kappa_{\rm sea}) - \lambda_{\rm sea}\, \mathcal{O})^{N_f}\, \exp\, [-\mathcal{S}_G(U)]}$$

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- Expectation values $\langle \ldots \rangle$ are taken for $\lambda_{\mathrm{sea}} = 0$
- $\frac{\partial}{\partial \lambda_{\mathrm{val}}} \rightarrow$ connected contributions
- $\frac{\partial}{\partial \lambda_{\mathrm{sea}}} \rightarrow$ disconnected contributions
- Obtain three-point function (e.g., singlet case)

$$\frac{\partial}{\partial \lambda} S^{\mathrm{mod}}(\lambda, \lambda)\Big|_{\lambda=0} = \langle M^{-1}\, \mathcal{O}\, M^{-1} \rangle + N_f \{\ldots\} = G_{\mathcal{O}}^{\mathrm{conn.+disc.}}$$

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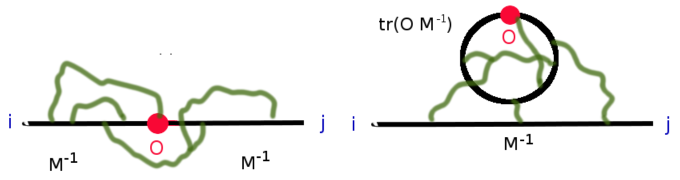

Figure: Fermion-line connected (left) and disconnected (right) contributions.

- $\frac{\partial}{\partial \lambda_{\rm val}} \to$ connected contributions

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- At least two values of parameter λ, detailed investigations in *[CSSM/QCDSF/UKQCD-collaboration, arXiv:1405.3019, 2014, cf. also talk of J. Zanotti]*
- With a reasonable choice of the λ values we compute

$$G_{\mathcal{O}}(p) \approx \frac{1}{\Delta\lambda}\left[S^{\mathrm{mod}}(\lambda_2; p) - S^{\mathrm{mod}}(\lambda_1; p)\right], \Delta\lambda = \lambda_2 - \lambda_1$$

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Axial vector A_3

- Amputated Born Green function: $\Gamma_{\mathrm{Born},A_3} = \imath\gamma_5\gamma_3$
- Values for λ:
  non-singlet case: $\lambda_{\mathrm{val}} = (0, 0.0125)$, $\lambda_{\mathrm{sea}} = 0$
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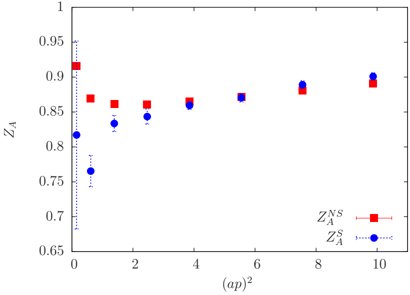

Figure: The non-singlet (NS) and singlet (S) renormalization factors Z_A. Fit range for RGI: $(2 < (ap)^2 < 10)$

Check: derivative

- Realiability of the numerical approximation of $\frac{\partial}{\partial \lambda} S^{\mathrm{mod}}\Big|_{\lambda=0}$
- For the singlet case we have the additional point $\lambda_{\mathrm{val}} = \lambda_{\mathrm{sea}} = 0$
- If $1/12\,\mathrm{tr}(S_0^{-1} S^{\mathrm{mod}}(\lambda_i, \lambda_i; p) S_0^{-1} \Gamma_{\mathrm{Born}}^{-1})$ $(i = 1, 2, 3)$ on a straight line (negligible $O(\lambda^2)$)

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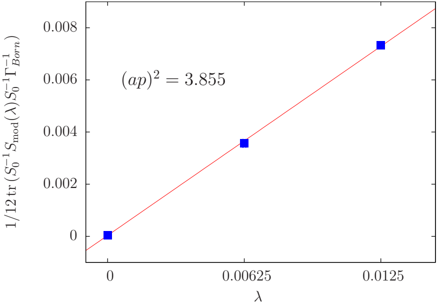

Figure: $1/12\,\mathrm{tr}(S_0^{-1} S^{\mathrm{mod}}(\lambda_i, \lambda_i; p) S_0^{-1} \Gamma_{\mathrm{Born}}^{-1})$ for three different λ values together with a linear fit at $(ap)^2 = 3.855$.

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- Non-singlet case: comparison with standard three-point approach
- Comparison with new results of *[Cyprus/CSSM/QCDSF/UKQCD, 2014]*

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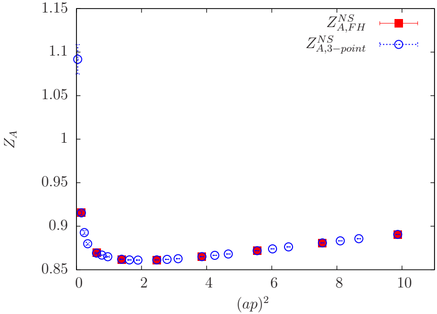

Figure: Comparison of the non-singlet Z factors calculated with the FH method ($Z^{\mathrm{NS}}_{\mathrm{A,FH}}$) and via the three-point function ($Z^{\mathrm{NS}}_{\mathrm{A,3-point}}$).

RGI

- In order to transform to RGI use intermediate scheme (MOM)
- $\gamma_A(non - singlet) = 0\,, \gamma_A(singlet) \neq 0\ \rightarrow$ momentum dependence for singlet case
- After performing the transformation the remaining $(ap)^2$ dependence is parametrized as

$$Z^{\mathrm{RGI}}_{\mathrm{data}} = Z^{\mathrm{RGI}} + c_1 (ap)^2 + c_2 \left((ap)^2\right)^2$$

- Fit range: $2 < (ap)^2 < 10$
- Results:
  $Z^{\mathrm{RGI}}_{\mathrm{A,NS}} = 0.847(2)$
  $Z^{\mathrm{RGI,3-point}}_{\mathrm{A,NS}} = 0.849(8)$ *[Cyprus/CSSM/QCDSF/UKQCD, 2014]*
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  $Z^{\mathrm{RGI}}_{\mathrm{A,NS}} = 0.847(2)$
  $Z^{\mathrm{RGI,3-point}}_{\mathrm{A,NS}} = 0.849(8)$ *[Cyprus/CSSM/QCDSF/UKQCD, 2014]*
  $Z^{\mathrm{RGI}}_{\mathrm{A,S}} = 0.861(9)$

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- $\gamma_A(non - singlet) = 0\,, \gamma_A(singlet) \neq 0 \ \rightarrow$ momentum dependence for singlet case
- After performing the transformation the remaining $(ap)^2$ dependence is parametrized as

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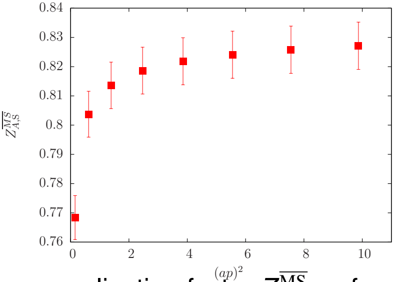

Figure: The renormalization factor $Z_{\mathrm{A,S}}^{\overline{\mathrm{MS}}}$ as function of $(ap)^2$

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- $Z_{\mathrm{A,S}}^{\overline{\mathrm{MS}}} = 0.802(8)$ at $p^2 = 4\,\mathrm{GeV}^2$

Ratio NS/S

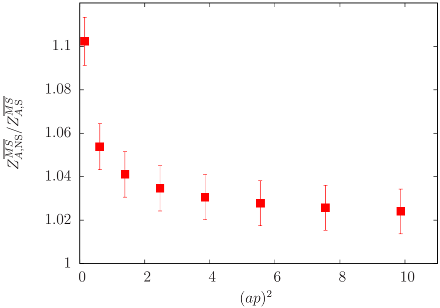

Figure: The ratio $Z^{\overline{\mathrm{MS}}}_{\mathrm{A,NS}}/Z^{\overline{\mathrm{MS}}}_{\mathrm{A,S}}$.

The ratio is close to 1 - supported by LPT: the difference between the non-singlet and singlet Z factors starts at two-loop only and is very small *[cf. talk of H. Panagopoulos]*

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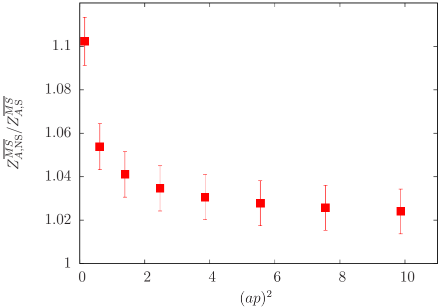

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Scalar operator S

- Scalar operator coupling $\lambda\,\bar{\psi}\,1\,\psi \leftrightarrow m\,\bar{\psi}\,\psi$
- Mass term serves as coupling term of the scalar operator
- Partially quenched quarks $\rightarrow$ non-singlet case
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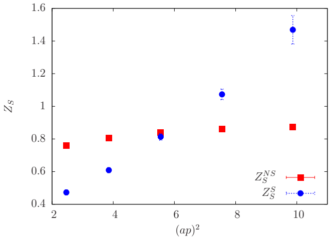

Figure: The non-singlet and singlet renormalization factors Z_S in the momentum range of interest.

- Non-singlet case checked with three point function approach *[Cyprus/CSSM/QCDSF/UKQCD, 2014]*

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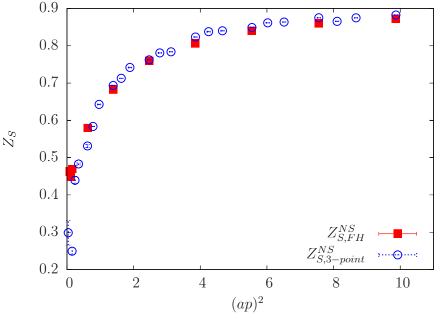

Figure: Comparison of the non-singlet Z_S computed with the Feynman-Hellman method and from the three-point function.

RGI and $\overline{\mathrm{MS}}$ results

- RGI results:
  $Z_{\mathrm{S,NS}}^{\mathrm{RGI}} = 0.549(5)$
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  $Z_{\mathrm{S,S}}^{\mathrm{RGI}} = 0.246(19)$
- $\overline{\mathrm{MS}}$ results at $p^2 = 4\,\mathrm{GeV}^2$:
  $Z_{\mathrm{S,NS}}^{\overline{\mathrm{MS}}} = 0.740(7)$
  $Z_{\mathrm{S,NS}}^{\overline{\mathrm{MS}},3-point} = 0.736(53)$ *[Cyprus/CSSM/QCDSF/UKQCD, 2014]*
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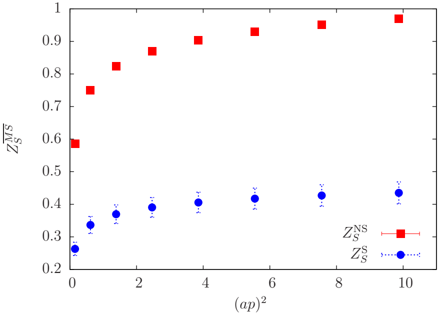

Figure: Non-singlet and singlet renormalization factors for the scalar operator in the $\overline{\mathrm{MS}}$ scheme.

The ratio is 2.23(18) for RGI and $\overline{\mathrm{MS}}$

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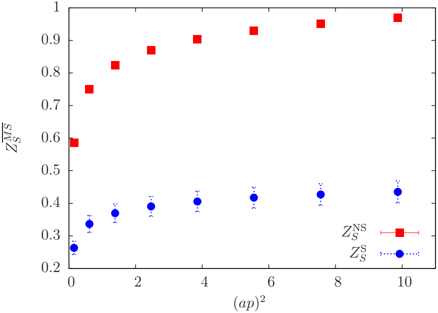

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Summary

- Presented Feynman-Hellman method for nonperturbative calculation of renormalization factors
- FH makes use of two-point functions $\rightarrow$ better signals
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- Straightforward inclusion of disconnected contributions $\rightarrow$ singlet operators
- First results for axial vector and scalar operators
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- Future steps: improved operators, improved statistics, additional operators

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