Quark mass dependence of finite temperature phase transitions in QCD with many flavors of Wilson fermions

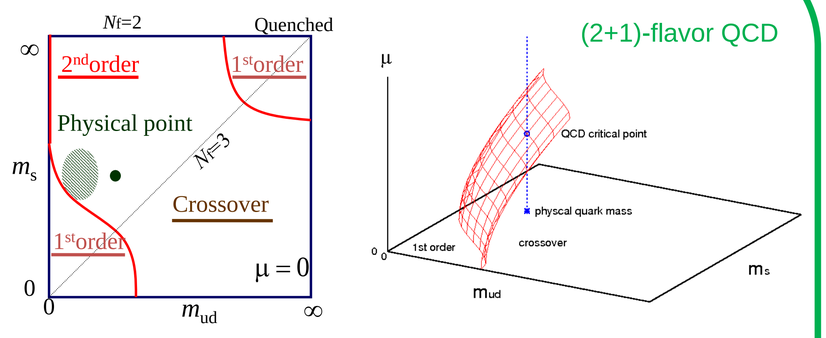
Shinji Ejiri (Niigata Univ.[1]), Norikazu Yamada(KEK[2])

[1]Department of Physics, Niigata University, Niigata 950-2181, Japan

[2]Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801, Japan

Purpose

Quark mass dependence of QCD transitions



The determination of the boundary of 1st order region is important.

2-flavor QCD in the chiral limit

O(4) universality crass or First order transition?

UA(1) symmetry restores?

On the line of physical mass,

the crossover at low density ➡ 1st order transition at high density?

However, the 1st order region is very small, and simulations with very small quark mass are required. ➡ Difficult to study.

$T\neq0$ phase transition in (2+many)-flavor QCD

Technicolor model constructed by many-flavor QCD

Chiral phase transition of QCD

→ Electroweak phase transition at finite temperature

Nambu-Goldstone bosons

3 bosons are absorbed into the gauge bosons. (3 massless bosons)

The other bosons have not observed yet. (The other bosons: heavy)

2 techni-felmions are massless, and the others are heavy.

Electro-weak baryogenesis

Strong first order transition: required.

From the analogy of 2+1-flavor QCD,

1st order at small mass; 2nd order or crossover at large mass.

It is important to determine the endpoint of the first order region in (2+many)-flavor QCD.

Good test ground for the study of (2+1)-flavor QCD

(Ejiri, Yamada, Physical Review Letters 110, 172001 (2013))

The critical line exists in the heavy quark region for many-flavor.

As increasing N_f, the critical mass becomes larger.

Easy to investigate the critical line.

Tricritical scaling: the same as (2+1)-flavor QCD

massive quark mass infinity limit → 2-flavor QCD

Abstract

We investigate the phase transitions of (2+N_f)-flavor QCD, where two light flavors and N_f massive flavors exist, to discuss the electroweak baryogenesis in realistic technicolor scenario. Because an appearance of a first order phase transition at finite temperature is a necessary condition for the baryogenesis, it is important to study the nature of phase transition in the case of massless 2 flavors. Performing simulations of 2-flavor QCD with Wilson fermions and using the reweighting method, we calculate probability distribution functions of the many-flavor QCD. Through the shape of the distribution function, we determine the boundary of the first order region in the parameter space of the light quark mass and heavy quark mass.

It is found that the light quark mass dependence of the critical mass of heavy quarks is very small in the region we investigated. From the light quark mass dependence, it is even possible to extract the nature of the transition of massless 2-flavor QCD. Our current result of small dependence suggests that the critical mass of heavy quark remains finite in the chiral limit of 2-flavors and there exists a second order transition region on the line of the 2-flavor massless limit.

Computational method

Distribution function & the effective potential

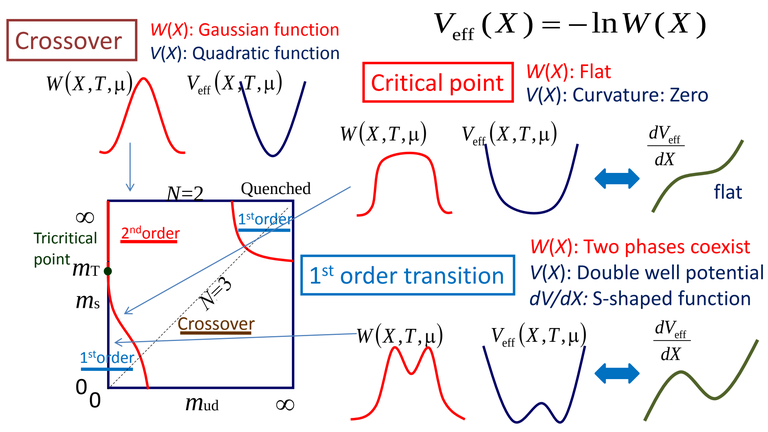
(S. Ejiri, Phys. Rev. D 77, 014508 (2008), S. Ejiri, N. Yamada, Phys. Rev. Lett. 110, 172001 (2013))

$$W(X;m,T,\mu)\equiv\int DU\delta(X-\hat{X})\prod_{f=1}^{N_f}\det M(m_f,\mu_f)e^{-S_g}$$ (Histogram)

Quark determinant Gauge action

X: order parameters, total quark number, average plaquette, etc.

$$V_{\rm eff}(X)=-\ln W(X)$$



Reweighting method for plaquette distribution

$$W(P,\beta,m,\mu)\equiv\int DU\delta(\hat{P}-P)\prod_{f=1}^{N_f}\det M(m_f,\mu_f)e^{6N_{\rm site}\beta\hat{P}}$$

$S_g=-6N_{\rm site}\beta\hat{P}$ $(\beta=6/g^2)$

plaquette P (1x1 Wilson loop for the standard action)

$$R(P,\beta,\beta_0 m,m_0,\mu)\equiv W(P,\beta,m,\mu)/W(P,\beta_0,m_0,0)$$ (Reweight factor)

$$R(P)=\frac{\left\langle\delta(\hat{P}-P)e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}\right\rangle_{(\beta_0,\mu=0)}}{\left\langle\delta(\hat{P}-P)\right\rangle_{(\beta_0,\mu=0)}}\equiv\left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}\right\rangle_{P:\rm fixed}$$

Effective potential:

$$V_{\rm eff}(P,\beta,m,\mu)=-\ln[W(P,\beta,m,\mu)]=V_{\rm eff}(P,\beta_0,m_0,0)-\ln R(P,\beta,\beta_0 m,m_0,\mu)$$

$$\ln R(P)=6N_{\rm site}(\beta-\beta_0)P+\ln\left\langle\prod_f\frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}\right\rangle_{P:\rm fixed}$$

First order transition point: 2 phases coexist

Plaquette distribution function

Performing simulations of 2-flavor QCD,

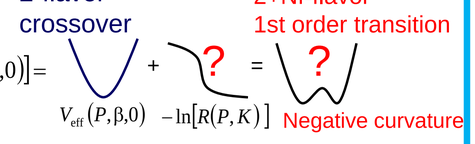
Dynamical effect of N_f-flavors are included by the reweighting.

We assume N_f-flavors are heavy. ($\Omega_R+i\Omega_I$: Polyakov loop)

Hopping parameter (κ) expansion (Wilson quark)

$$N_f\ln\left(\frac{\det M(\kappa,\mu)}{\det M(0,0)}\right)=N_f\left(288N_{\rm site}\kappa^4P+12\cdot2^{N_t}N_s^3\kappa^{N_t}(\cosh(\mu/T)\Omega_R+i\sinh(\mu/T)\Omega_I)+\cdots\right)$$

Effective potential



$$V_{\rm eff}(P,\beta,\kappa)=-\ln[R(P,\kappa)W(P,\beta,0)]=V_{\rm eff}(P,\beta,0)-\ln[R(P,K)]$$

$$\ln R(P)=\ln\left\langle e^{6N_{\rm site}(\beta-\beta_0)\hat{P}}\prod_f\frac{\det M(m_f,\mu_f)}{\det M(m_0,0)}\right\rangle_{P:\rm fixed}\approx\ln\left\langle\exp(6hN_s^3\hat{\Omega}_R)\right\rangle_{P:\rm fixed}+(\text{linier term of }P)$$

(degenerate mass case at μ=0)

$$V_{\rm eff}(P,\beta,h,\mu)=V_{\rm eff}(P,\beta_0,0,0)-\ln\bar{R}(P,h,\mu)+\ (\text{linier term of }P)$$

$$\bar{R}(P)=\left\langle\exp(6N_s^3h\Omega_R)\right\rangle_{P:\rm fixed}$$ (μ=0 case)

Wilson quark $h=2N_f(2\kappa_h)^{N_t}$ Staggered quark $h=N_f/(4(2m_h)^{N_t})$

Slope of the effective potential

Linear term of P is irrelevant to the shape of $\frac{dV_{\rm eff}}{dP}(P,\beta,h)$

β-dependence is only in the linear term.

and gives just constant shift. $\left(\because\frac{dV_{\rm eff}}{dP}(\beta_2)=\frac{dV_{\rm eff}}{dP}(\beta_1)-6N_{\rm site}(\beta_2-\beta_1)\right)$

$$\frac{dV_{\rm eff}}{dP}(P,\beta,h)=\underbrace{\frac{dV_{\rm eff}}{dP}(P,\beta_0,0)}_{\text{2-flavor}}-\frac{d\ln\bar{R}}{dP}(P,h)+(\text{constant shift in }P)$$

If the first derivative of $V_{\rm eff}$ is S-shaped function,

➡ First order transition (double-well potential)

Expectations

N_f-dependence of critical mass

When the critical point h_c exists in the heavy quark region, the critical mass increases as N_f increases.

$$h=2N_f(2\kappa_h)^{N_t}\ \Rightarrow\ \kappa_h^c=\frac{1}{2}\left(\frac{h_c}{2N_f}\right)^{1/N_t}$$

When N_f is large, κ is small. Then, the hopping parameter (κ) expansion is good.

On the hand, when N_f is small, the κ-expansion is bad.

Slope of the critical line at degenerate mass

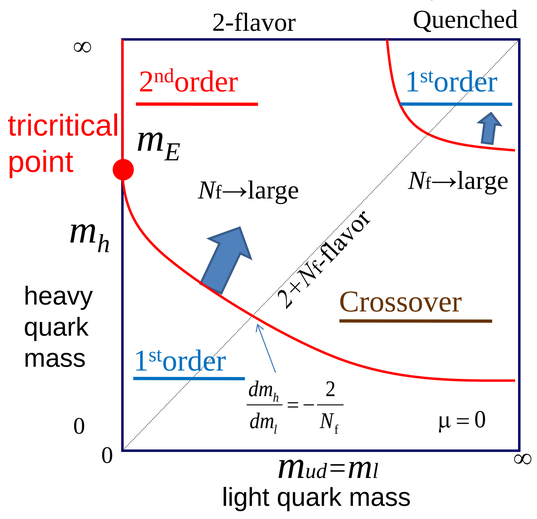
$$\ln\left[\left(\frac{\det M(\kappa_l)}{\det M(\kappa_0)}\right)^2\left(\frac{\det M(\kappa_h)}{\det M(\kappa_0)}\right)^{N_f}\right]\approx\frac{d\ln\det M}{d\kappa}\bigg|_{\kappa_0}\left[2(\kappa_l-\kappa_0)-N_f(\kappa_h-\kappa_0)\right]+\cdots$$

The reweighting factor is one along the line of

$$2(m_l-m_0)-N_f(m_h-m_0)=0\ \Leftrightarrow\ \frac{dm_h}{dm_l}=-\frac{2}{N_f}\ \text{at}\ m_l=m_h=m_0$$

Slope of the critical line: smaller as N_f increases.

Phase structure of (2+N_f)-flavor QCD



As increasing N_f,

the critical mass becomes larger.

When the tricritical point exists on m_l=0,

The tricritical scaling is the same as (2+1)-flavor QCD.

$$m_{ud}^c\sim(m_E-m_h)^{5/2}$$
$$m_{ud}^c\sim\mu^5$$

Tricritical point: m_E

One can test the tricritical scaling.

Existence of the tricritical point can be confirmed.

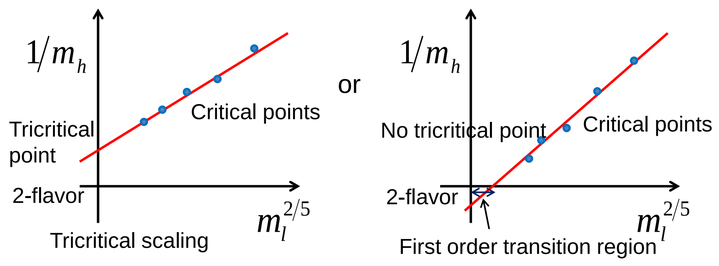
Nature of 2-flavor QCD in the chiral limit

2nd order or 1st order?

Light quark mass (m_l) dependence of the critical line

Trictitical scaling behavior or

First order transition region in 2-flavor QCD ?



Similar study in QCD with an imaginary chemical potential:

(Bonati, D' Elia, de Forcrand, Philipsen, Sanfilippo, arXiv:1311.0473)

Results

Simulations

Iwasaki gauge action +

N_f=2 clover -Wilson fermion action,

κ=0.145, 0.475, 0.150, 0.1505,

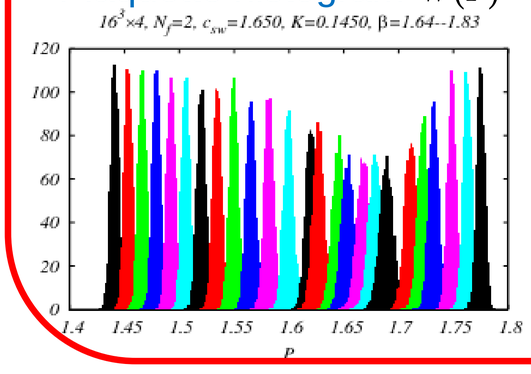
m_π/m_ρ = 0.6647, 0.5761, 0.4677, 0.4575,

16^3x4 lattice.

Dynamical heavy quark effect is added by the reweighting method.

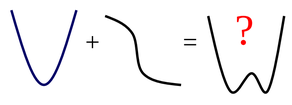
det$M$: Hopping parameter expansion

Plaquette histogram $W(P)$

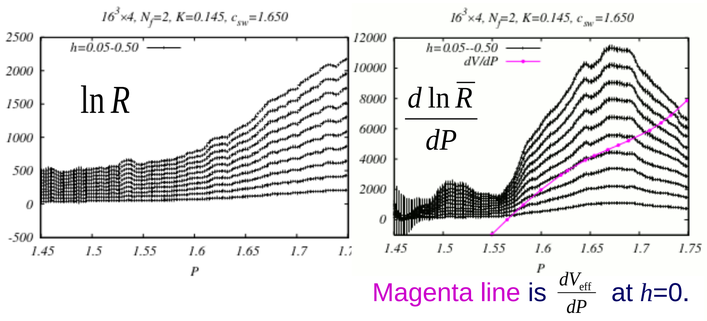


Effective potential $V_{\rm eff}=-\ln W$

$$V_{\rm eff}(P,\beta,0)-\ln\bar{R}(P,h)=V_{\rm eff}(P,\beta,h)$$



$h=2N_f(2\kappa_h)^{N_t}$ or $h=N_f/(4(2m_h)^{N_t})$



Magenta line is $\frac{dV_{\rm eff}}{dP}$ at h=0.

Slope of $V_{\rm eff}$ at h=0

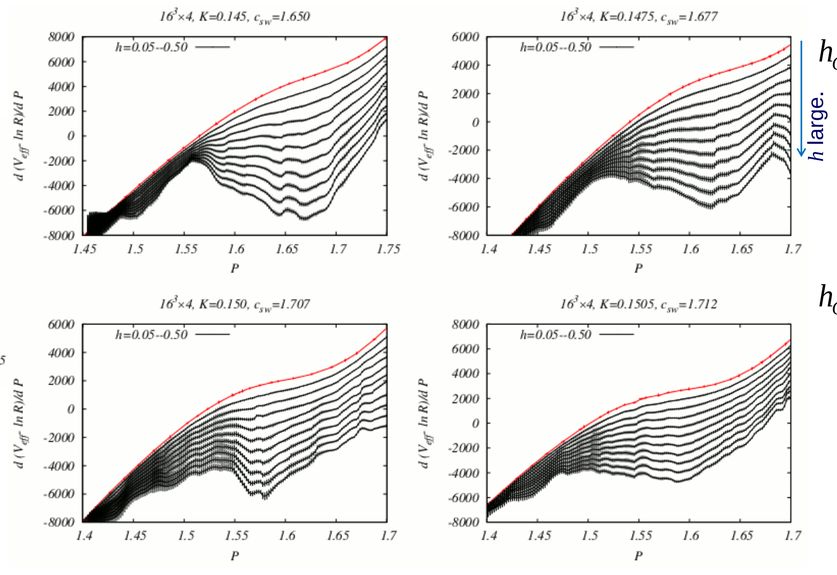
At the peak of the distribution ($\bar{P}$) obtained at β_0, $\frac{dV_{\rm eff}}{dP}(\bar{P},\beta_0)=0$

Hence, the slope at β is given by

$$\frac{dV_{\rm eff}}{dP}(\bar{P},\beta)=\frac{dV_{\rm eff}}{dP}(\bar{P},\beta_0)-6N_{\rm site}(\beta-\beta_0)=-6N_{\rm site}(\beta-\beta_0)$$

Slope of the effective potential at finite h

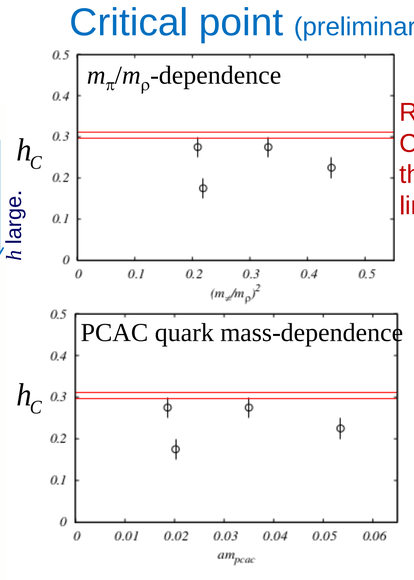
$$\frac{dV_{\rm eff}(P,\beta,h)}{dP}=\frac{dV_{\rm eff}(P,\beta,0)}{dP}-\frac{d\ln\bar{R}(P,h)}{dP}$$



We calculate every Δh=0.05 from h=0 to 0.50.

These become S-shaped functions for large h. ➡1st order transition

Critical point (preliminary)



Red line: Critical in the m_l=∞ limit

the light quark mass dependence of the critical heavy quark mass is very small.

The first order region in 2-flavor QCD is not suggested.