A dynamical study of the chirally rotated Schrödinger functional in QCD

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Problem:

- Symanzik's effective theory + chiral symmetry of continuum QCD $\Rightarrow$ **automatic O(a) improvement** for massless Wilson-fermions in finite volume with p.b.c.'s. (Frezzotti, Rossi '04; Shindler '05; Sint '05; Aoki, Bär '06)
- (standard) SF b.c.'s are **incompatible** with this argument!

  Consider: $\psi \to \gamma_5\,\psi,\ \ \overline{\psi} \to -\overline{\psi}\,\gamma_5$, using $P_\pm \equiv \frac{1}{2}(1+\gamma_0)$,

$$\begin{aligned} P_+\psi(x)|_{x_0=0} &= 0, \\ \overline{\psi}(x)P_-|_{x_0=0} &= 0, \end{aligned} \quad \xrightarrow{\gamma_5} \quad \begin{aligned} P_-\psi(x)|_{x_0=0} &= 0, \\ \overline{\psi}(x)P_+|_{x_0=0} &= 0. \end{aligned}$$

Solution:

- Find alternative SF b.c.'s and/or symmetry transformations of continuum QCD. (Frezzotti, Rossi '05; Sint '05)
- Generalize the ideas of tmQCD to the SF is a natural solution $\Rightarrow$ **chirally rotated SF (χSF)**. (Sint '05)

The chirally rotated Schrödinger functional

A chiral rotation to the Schrödinger functional (Sint '05)

- Given the isospin doublets ψ and $\overline{\psi}$ satisfying standard SF b.c.'s, we consider the **chiral rotation**,

$$\psi \equiv R\chi \equiv e^{i\frac{\pi}{2}\gamma_5\frac{\tau^3}{2}}\chi, \quad \overline{\psi} \equiv \overline{\chi}R \equiv \overline{\chi}e^{i\frac{\pi}{2}\gamma_5\frac{\tau^3}{2}}.$$

- The fields χ and $\overline{\chi}$ satisfy the **chirally rotated SF** b.c.'s,

$$\begin{aligned} \widetilde{Q}_+\chi(x)|_{x_0=0} &= 0, \\ \overline{\chi}(x)\widetilde{Q}_+|_{x_0=0} &= 0, \end{aligned} \qquad \widetilde{Q}_\pm \equiv \frac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3).$$

- χSF b.c.'s are **invariant** w.r.t. the continuum QCD symmetry,

$$\chi \to \gamma_5\tau^1\chi, \quad \overline{\chi} \to -\overline{\chi}\gamma_5\tau^1.$$

$\Rightarrow$ **automatic O(a) improvement can be rescued!**

- Since R is a non-anomalous field transformation,

$$\boxed{\langle O[\psi,\overline{\psi}]\rangle_{P_\pm} = \langle O[R\chi,\overline{\chi}R]\rangle_{\widetilde{Q}_\pm}.}$$

On the lattice, we expect these relations to hold among **renormalized** correlation functions (up to cutoff effects)!

The chirally rotated Schrödinger functional

Renormalization and O(a) improvement (Sint '11)

- For Wilson-fermions, the correct χSF b.c.'s are realized by **fine-tuning** a dim. 3 boundary counterterm (e.g. at $x_0 = 0$),

$$\overline{\chi} Q_- \chi \xrightarrow{R} -\overline{\psi} i\gamma_5 \tau^3 P_- \psi,$$

$\Rightarrow$ **breaks** parity and flavor symmetry: its coefficient, $z_f(g_0)$, can be fixed by imposing parity **restoration**.

- **Automatic (bulk) O(a) improvement:**
  - $\Rightarrow$ **NO** bulk O(a) effects for $\gamma_5\tau^1$-even obs. $(O \xrightarrow{\gamma_5\tau^1} +O)$,
  - $\Rightarrow$ bulk O(a) effects are located in $\gamma_5\tau^1$-odd obs. $(O \xrightarrow{\gamma_5\tau^1} -O)$.

- Full O(a) improvement needs the tuning of a **couple** of O(a) boundary counterterms. PT is generally good! (P. Vilaseca's talk)

- The setup has been studied in the **quenched** approximation
  - ✓ Automatic O(a) improvement, and universality relations.
  - ✓ Tuning of $m_0 \to m_{\rm crit}$ and z_f quite independent once the action is improved. (Leder, Sint '10; Gonzalez Lopez, Jansen, Renner, Shindler '12)

Renormalization in the χSF

Correlation functions we need . . .

(Leder, Sint '10)

Boundary-to-bulk:

$$g_X^{f_1 f_2}(x_0) = -\frac{1}{2}\langle \overline{\chi}_{f_1}(x)\Gamma_X \chi_{f_2}(x)\, \mathcal{O}_5^{f_2 f_1}\rangle,$$

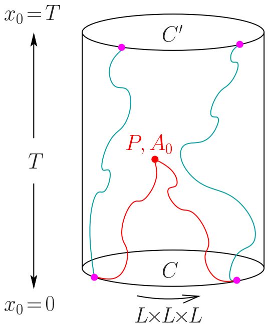
$$l_Y^{f_1 f_2}(x_0) = -\frac{1}{6}\sum_k \langle \overline{\chi}_{f_1}(x)\Gamma_{Y_k} \chi_{f_2}(x)\, \mathcal{O}_k^{f_2 f_1}\rangle,$$

$$\Gamma_X = A_0, V_0, S, P,$$
$$\Gamma_{Y_k} = A_k, V_k, T_{k0}, \widetilde{T}_{0k}.$$

Boundary-to-boundary:

$$g_1^{f_1 f_2} = -\frac{1}{2}\langle \mathcal{O}_5^{f_1 f_2}\, \mathcal{O}_5'^{f_2 f_1}\rangle$$

$$l_1^{f_1 f_2} = -\frac{1}{6}\sum_k \langle \mathcal{O}_k^{f_1 f_2}\, \mathcal{O}_k'^{f_2 f_1}\rangle$$



$f_1, f_2 = u, d, u'd'$

$\mathcal{O}_5^{f_1 f_2}, \mathcal{O}_k^{f_1 f_2} \equiv$ bilinears of non-Dirichlet boundary quark-field components.

Renormalization in the χSF

Renormalization conditions from universality relations, an example: Z_A (Leder, Sint '10)

Universality relations:

We consider $\gamma_5\tau^1$-even correlation functions ($\widetilde{V} \equiv$ conserved current),

$$f_A^R = g_A^{R\,uu'} = -ig_V^{R\,ud}, \qquad \Rightarrow \qquad Z_A\, g_A^{uu'} = -ig_{\widetilde{V}}^{ud} + \mathrm{O}(a^2),$$

$$k_V^R = l_V^{R\,uu'} = -il_A^{R\,ud}, \qquad \Rightarrow \qquad Z_A\, l_A^{ud} = \;\; il_{\widetilde{V}}^{uu'} + \mathrm{O}(a^2).$$

Renormalization conditions:

We can turn the universality relations around and define ren. conditions,

$$Z_A^g \equiv \left.\frac{-ig_{\widetilde{V}}^{ud}(x_0)}{g_A^{uu'}(x_0)}\right|_{x_0=\frac{T}{2}}, \qquad Z_A^l \equiv \left.\frac{il_{\widetilde{V}}^{uu'}(x_0)}{l_A^{ud}(x_0)}\right|_{x_0=\frac{T}{2}}.$$

Z's so obtained are **O(a) improved**:

- **NO** need for operator improvement i.e. c_A or $c_{\widetilde{V}}$.
- O(a) boundary effects **cancel** out in the ratios.

Lattice setup

... and a bit about the code

Setup:

- $N_\mathrm{f} = 2$ non-perturbatively O(a) improved Wilson-fermions.
- We consider a LCP defined by $C = C' = 0$, $T = L$, $L \approx 0.6$ fm $\Rightarrow a \approx [0.0755, 0.0658, 0.0486]$ fm, $L/a = 8, 12, 16$.

Tuning conditions:

$$\mathbf{m}_\mathrm{crit} : m_\mathrm{PCAC} = \left.\frac{\partial_0 g_A^{ud}(x_0)}{2 g_P^{ud}(x_0)}\right|_{x_0=\frac{T}{2}} \stackrel{!}{=} 0; \qquad \mathbf{z_f} : O^{\gamma_5\tau^1\text{-odd}} \stackrel{!}{=} 0.$$

Code:

- The code is based on the openQCD package. (Lüscher, Schaefer '12)
- Simulations of χSF + SF O(a) improved Wilson-fermions.
- Several algorithmic features inherited: multilevel Hasenbusch twisted-mass- + EO-preconditioning, twisted-mass reweighing, ...

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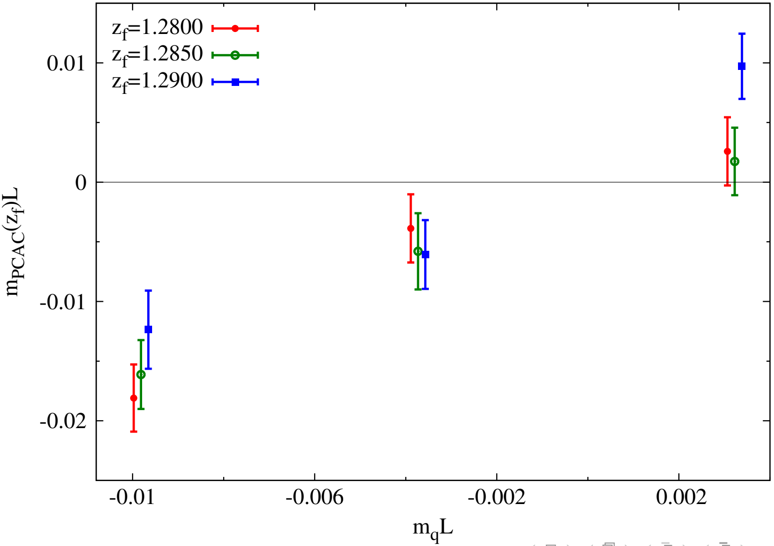
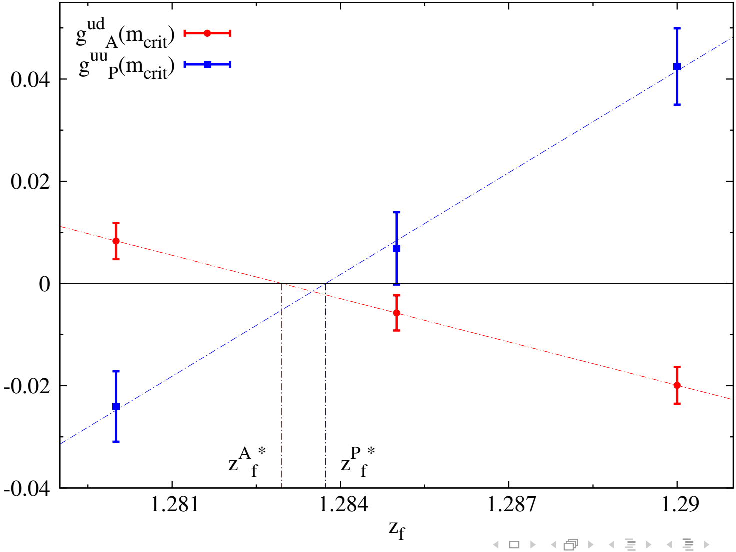
Determination of $m_{\rm crit}$ and z_f

$m_{\rm PCAC}$ as a function of $m_q = m_0 - m_{\rm crit}$, for $a \approx 0.076$ fm, $L/a = 8$, and different z_f's



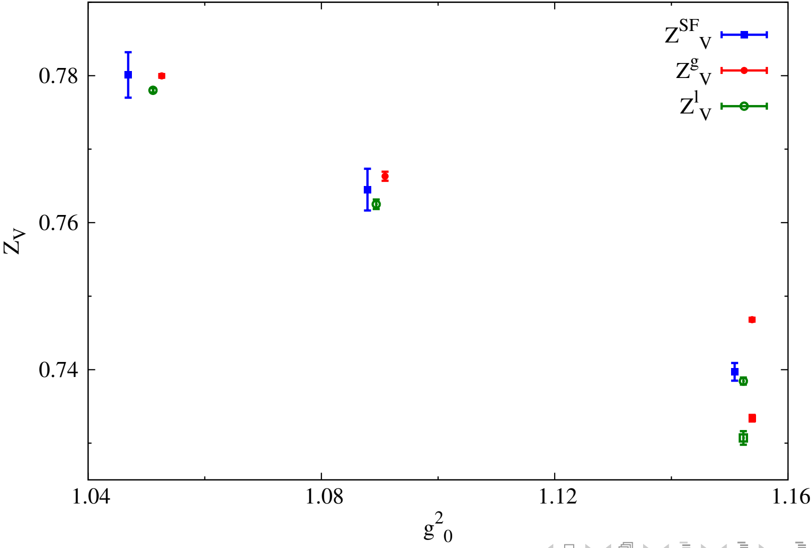
Determination of $m_{\rm crit}$ and z_f

$\gamma_5\tau^1$-odd functions g_A^{ud} and $g_P^{uu'}$ as a function of z_f, for $a \approx 0.076$ fm, $L/a = 8$



Renormalization constants

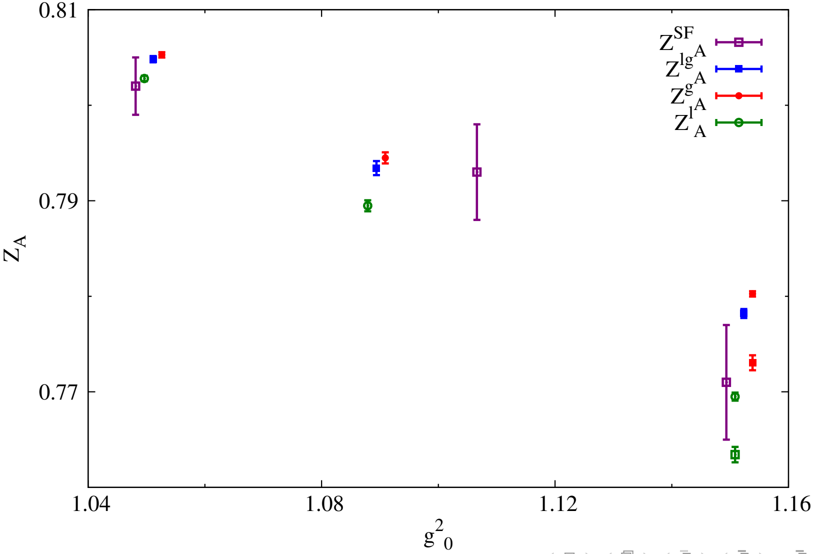
Comparison among different definitions of Z_V as a function of g_0^2 (Della Morte *et. al.* '05)



Renormalization constants

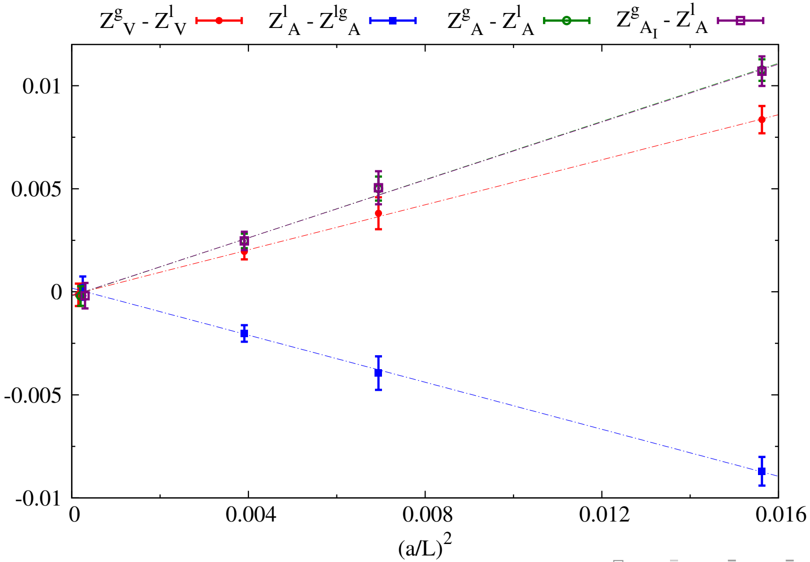
Comparison among different definitions of Z_A as a function of g_0^2

(Della Morte *et. al.* '05; Della Morte, Sommer, Takeda '08; Fritzsch *et. al.* '12)



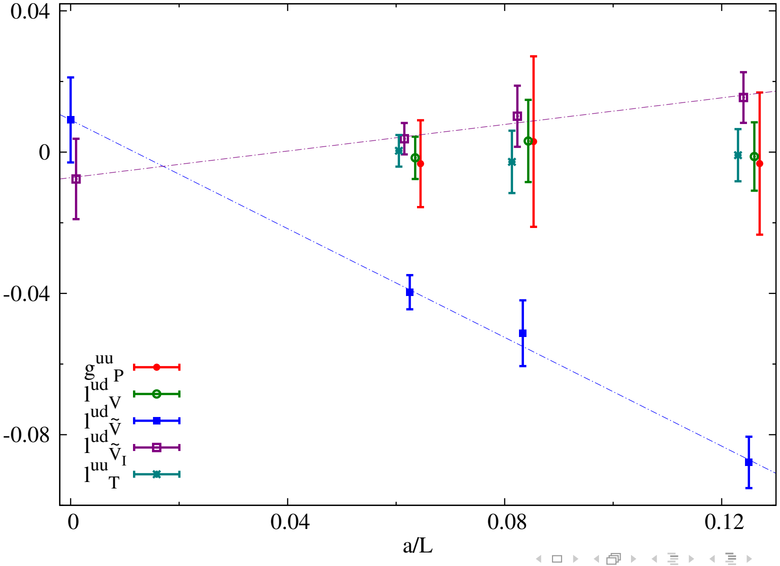
Test of universality I

Continuum limit extrapolations for $Z_{V,A}$ differences



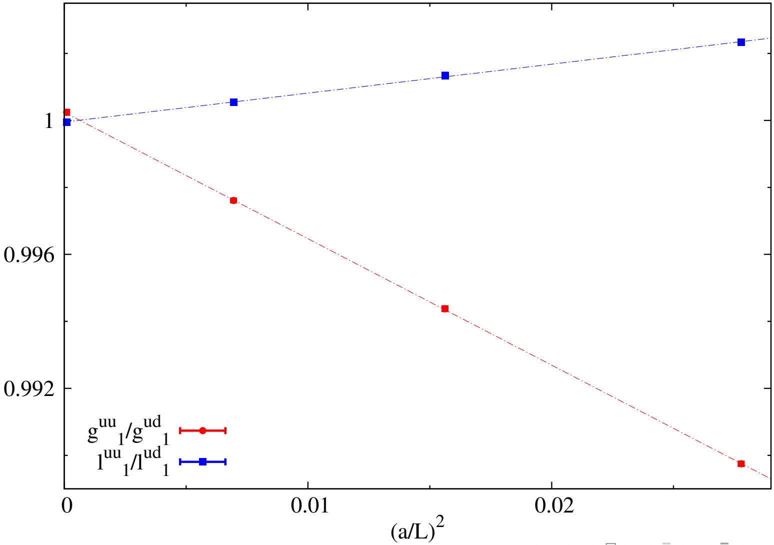
Test of automatic O(a) improvement

Continuum limit extrapolations of $\gamma_5\tau^1$-odd correlation functions



Flavor symmetry restoration

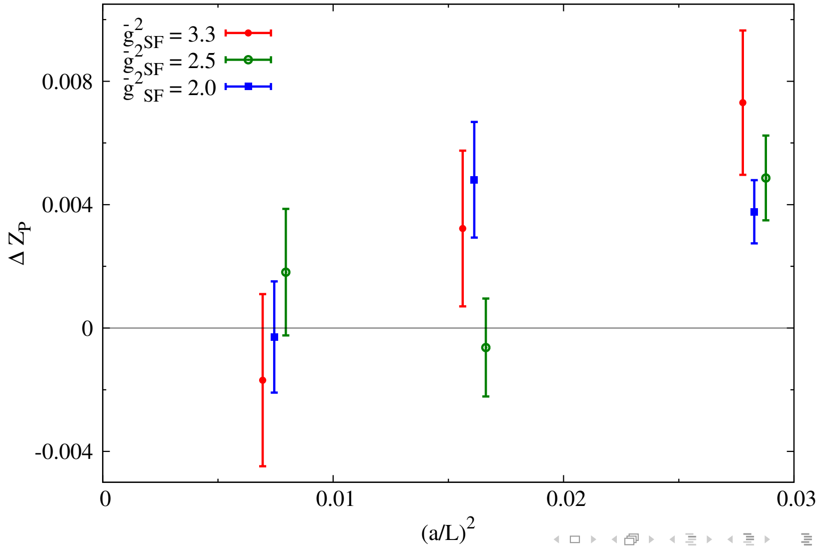
Continuum limit extrapolations of ratios of correlation functions for $\bar{g}^2_{\mathrm{SF}}(L) \approx 2.0$



Test of universality II

Continuum limit extrapolations for $\Delta Z_P = 1 - \frac{Z_P^{\chi\mathrm{SF}}}{Z_P^{\mathrm{SF}}}$ for different LCP's

(Della Morte *et. al.* '05)



Outlook & Conclusions

Conclusions:

- Automatic O(a) improvement seems at work.
- Very flexible setup to devise renormalization conditions that allows good control over cutoff effects, and good precision.
- The additional tuning of z_f does not present any particular difficulty, and does not add too much work.

Outlook:

- This is a natural setup for renormalization problems in tmLQCD at maximal twist.
- Determination of renormalization constants for $N_\mathrm{f} = 2 + 1$ can be obtained from a mixed setup: 2 χSF + 1 SF fermions.
- Renormalization of more complicated operators, in particular 4-quark and twist-2 operators; very difficult w/o automatic O(a) improvement!


Thanks