The static three-quark potential from the Polyakov loop correlation function

Yoshiaki Koma (Numazu College of Technology)

Miho Koma (Nihon University)

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Contents

Introduction

- **Heavy quarkonium:** $Q\bar{Q}$

  $\Rightarrow$ **potential NRQCD** [cf. Brambilla etal.(RMP77)]

$$H = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} + \underbrace{V^{(0)}(x_1, x_2)}_{\text{well known}} + \underbrace{O(\frac{1}{m})}_{\text{known}}$$

[Koma etal.(PRL97)ff]

  $\Rightarrow$ **quarkonium can be studied "quantum mechanically"**

- **Heavy baryon:** QQQ

$$H = \frac{\vec{p}_1^{\,2}}{2m_1} + \frac{\vec{p}_2^{\,2}}{2m_2} + \frac{\vec{p}_3^{\,2}}{2m_3} + \underbrace{V^{(0)}(x_1, x_2, x_3)}_{\text{known*}} + \underbrace{O(\frac{1}{m})}_{\text{unknown}}$$

  $\Rightarrow$ **precise functional form of the three-quark potential ?**

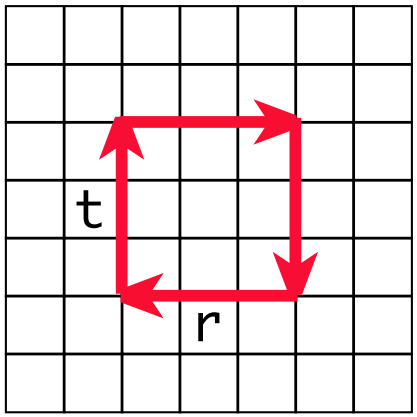
  *[cf. Tahakashi etal.(PRD65), Alexandrou etal.(PRD65)]

How to compute the potential

- **Inter-quark potential can be extracted from the Wilson loop or Polyakov loop corrlation function (PLCF)**

Wilson loop

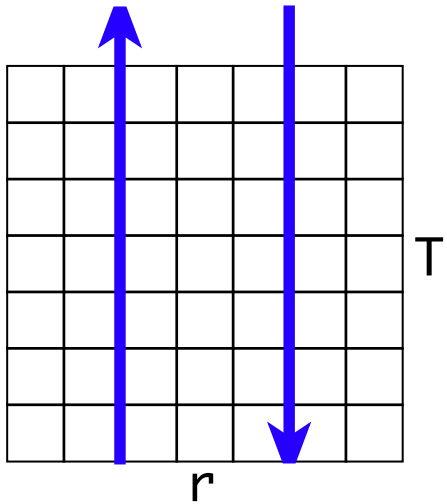
$$V^{(0)}(r) = -\lim_{t\to\infty}\frac{1}{t}\ln\langle W(r,t)\rangle$$



W(r,t)

PLCF

$$V^{(0)}(r) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle P(0)P(r)^*\rangle$$



P(0)P(r)*

(popular at finite temperature)

Wilson loop vs. PLCF

- $W(r,t) = \mathbb{L}(r,0)^*_{\alpha\gamma}\{\mathbb{T}(0)\mathbb{T}(a)\cdots\mathbb{T}(t-a)\}_{\alpha\beta\gamma\delta}\mathbb{L}(r,t)_{\beta\delta}$

$$\Rightarrow \langle W(r,t)\rangle = \sum_{n=0}^{\infty} w_n(r,t)e^{-E_n(r)t} \quad (w_n(r,t) = \langle 0|\mathbb{L}(r,0)^*|n(r)\rangle\langle n(r)|\mathbb{L}(r,t)|0\rangle)$$

$$\Rightarrow -\frac{1}{t}\ln\langle W(r,t)\rangle = \underbrace{E_0(r)}_{=V^{(0)}} - \underbrace{\frac{1}{t}\ln w_0(r,t) + O(\frac{1}{t}e^{-(E_1-E_0)t})}_{\text{unwanted contributions}}$$

- $P(0)P(r)^* = \{\mathbb{T}(0)\mathbb{T}(a)\cdots\mathbb{T}(T-a)\}_{\alpha\alpha\gamma\gamma}$

$$\Rightarrow \langle P(0)P(r)^*\rangle = \sum_{n=0}^{\infty} w_n e^{-E_n(r)T} \qquad (w_0 = 1)$$

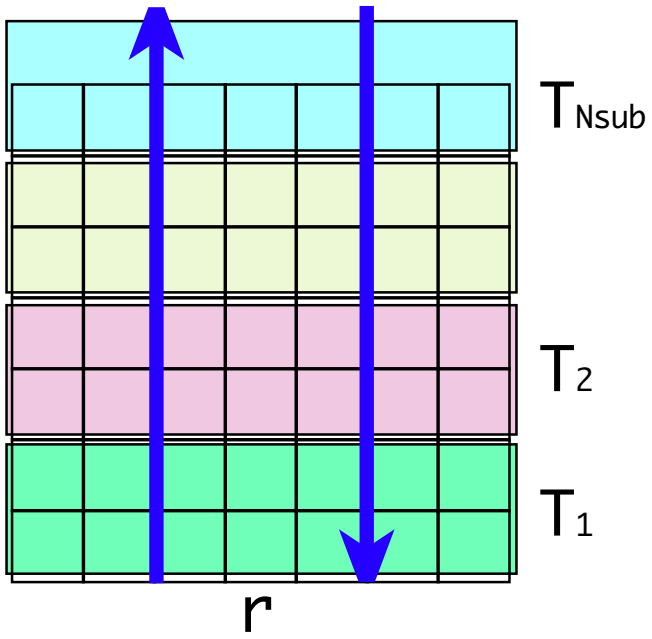
$$\Rightarrow -\frac{1}{T}\ln\langle P(0)P(r)^*\rangle = \underbrace{E_0(r)}_{=V^{(0)}} + \underbrace{O(\frac{1}{T}e^{-(E_1-E_0)T})}_{\text{can be neglected}}$$

PLCF at zero temperature

- **Difficult to obtain accurate PLCFs for larger r and T, since the expectation values are exponentially smaller**

  $\Rightarrow$ **use the multilevel algorithm**

  [Lüscher&Weisz, JHEP0109(2001)010, JHEP0207(2002)049]



$$P(0)P(r)^* = [\mathbb{T}_1][\mathbb{T}_2]\cdots[\mathbb{T}_{N_{\rm sub}}]$$

construct the PLCF form the average of the sublattice correlator $\mathbb{T}(r)$

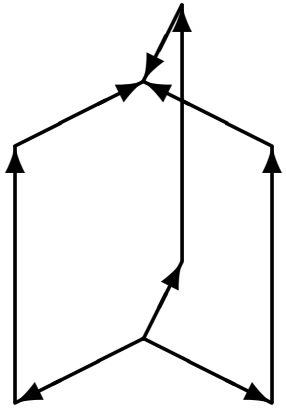
$\Rightarrow$ **exponential noize reduction**

(e.g.) $N_{\rm sub} = 4$

$$O(10^{-20}) \leftarrow O(10^{-5})\cdot O(10^{-5})\cdot O(10^{-5})\cdot O(10^{-5})$$

Three-quark potential

- **Wilson loop** [Takahashi etal.,(PRD65), Alexandrou etal.(PRD65)]

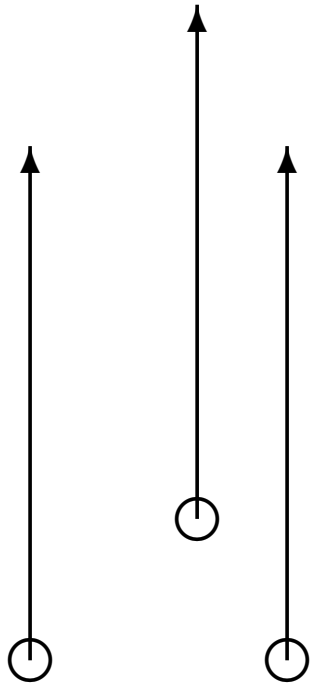


$\Rightarrow$ **junction in source**

(may cause systematic effects)

- **PLCF**

NO junction $\Leftarrow$



$$P(x_1)P(x_2)P(x_3)$$

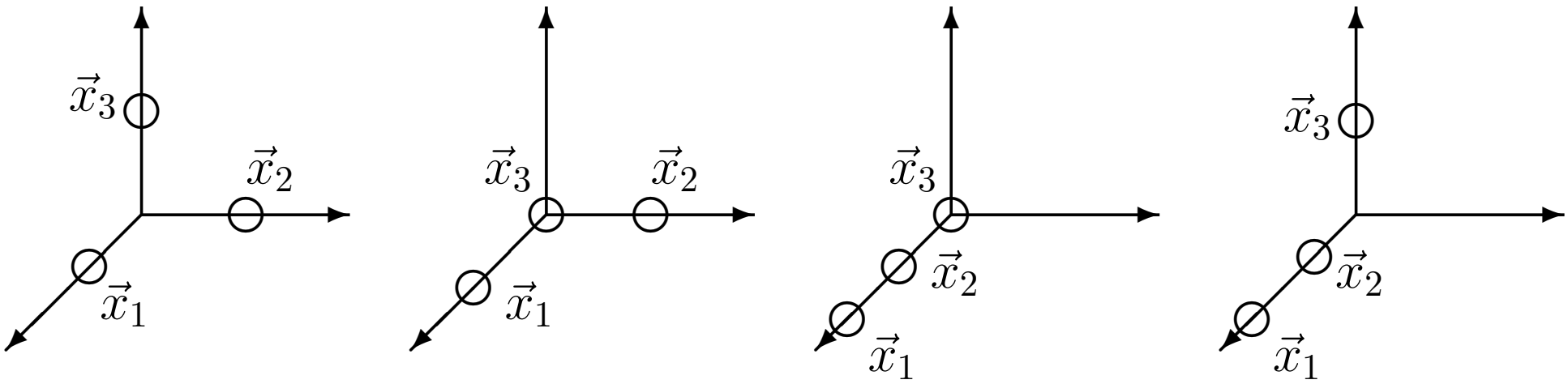
Results

- **Simulation details**
  - **SU(3) Wilson gauge action**

β	L^3T	a [fm]	$N_{\rm sub}$	$N_{\rm iupd}$	$N_{\rm conf}$
5.85	24^4	0.123	8	500000	8
6.00	24^4	0.093	6	500000	$1\sim8$
6.30	24^4	0.059	4	400000	4

[NEC-SX8@RCNP, Osaka Univ]

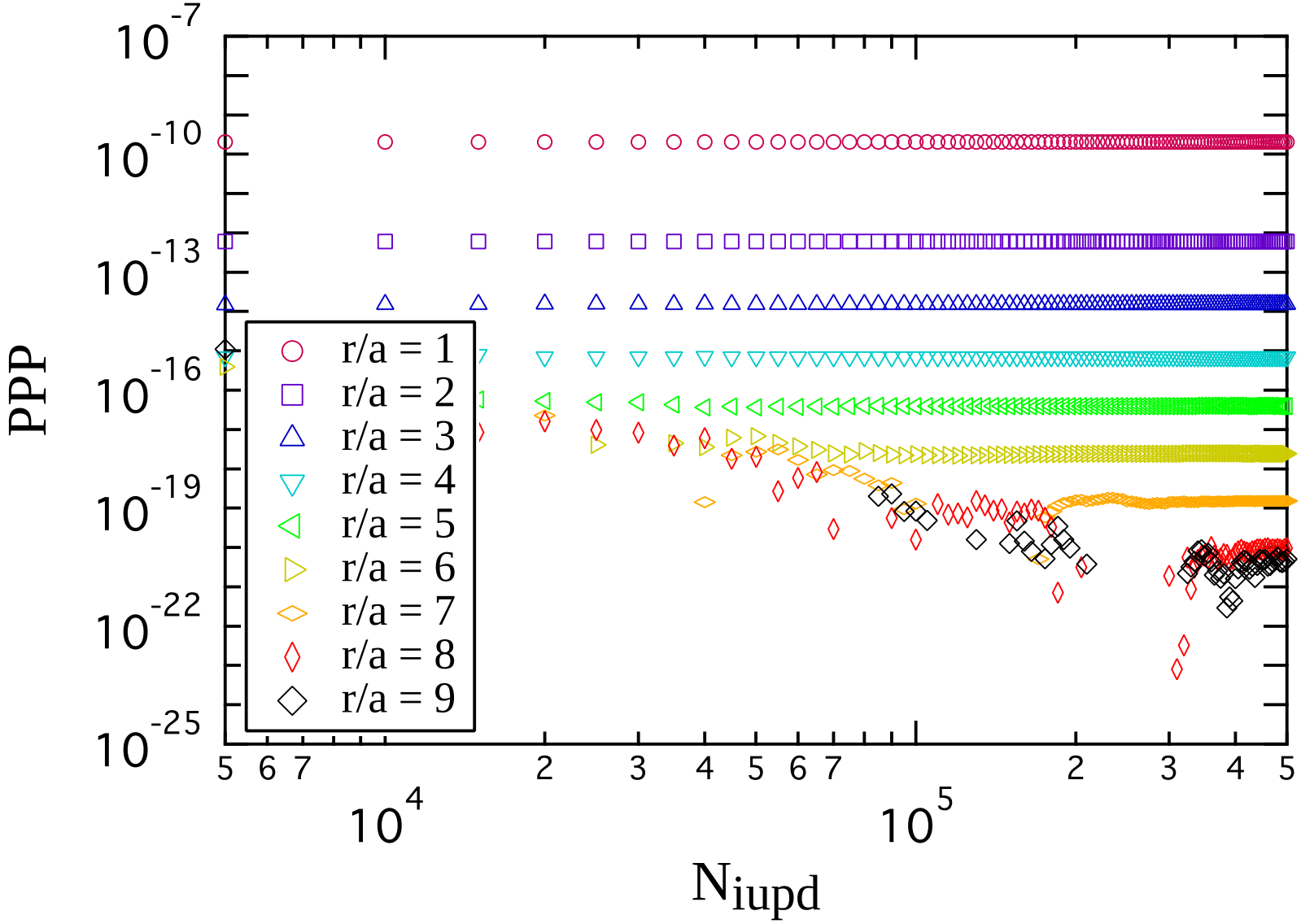
  - **quark locations**



(regular, isosceles) (right) (line) (obtuse)

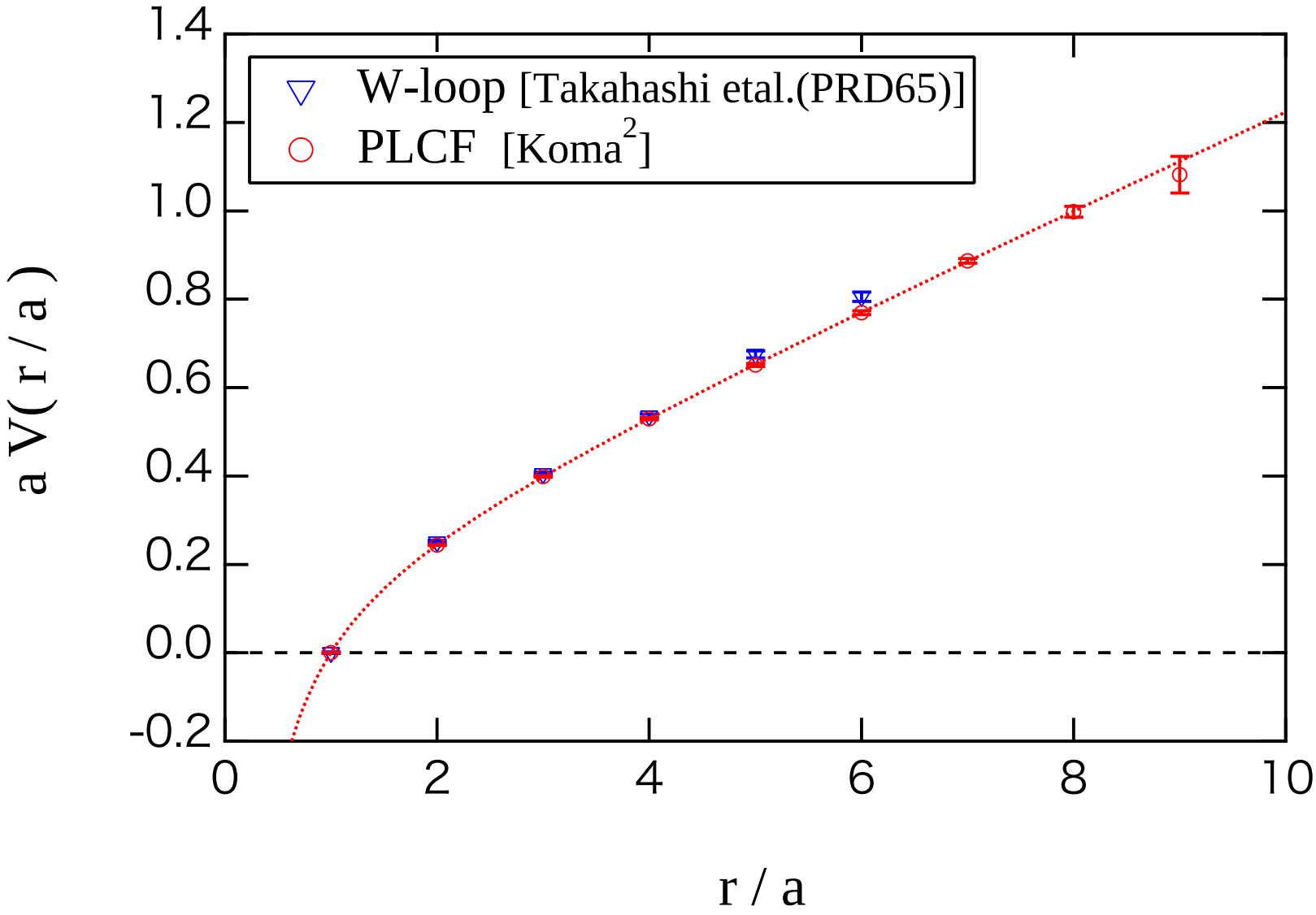
Three-quark PLCF (regular)

- IUPD history of PLCF for $\vec{x}_i = r\vec{e}_i$ ($\beta = 6.0$, 24^4 lattice, $N_{\rm sub} = 6$)



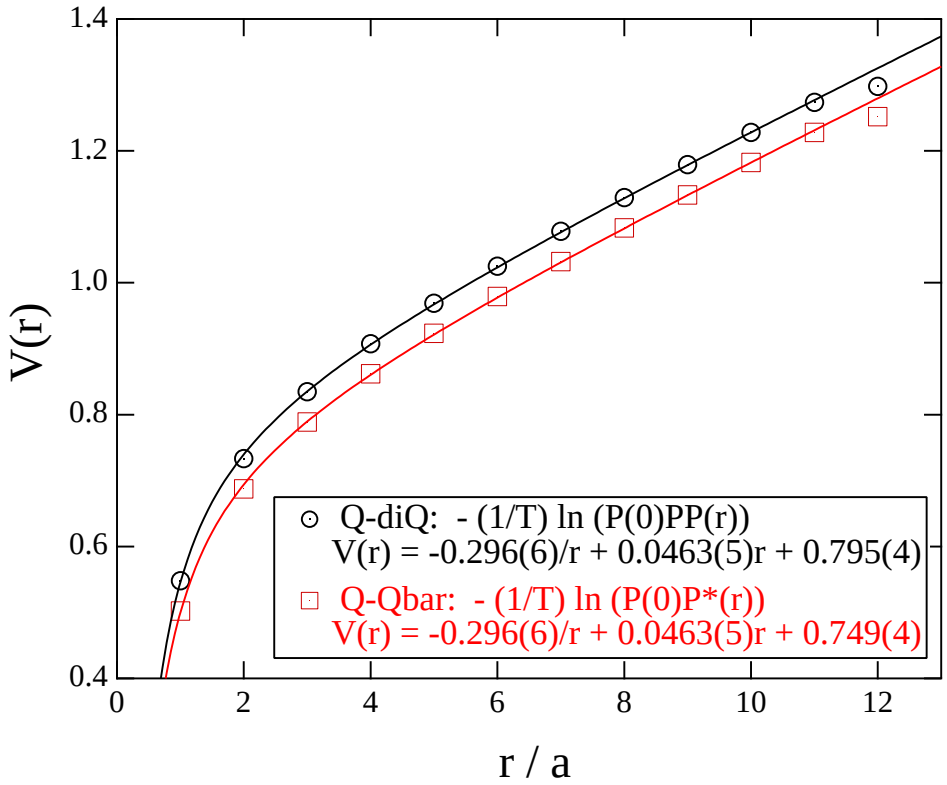
Three-quark potential (regular)

- $\vec{x}_i = r\vec{e}_i$ ($\beta = 6.0$, 24^4 lattice, $N_{\rm sub} = 6$, $N_{\rm iupd} = 500000$, $N_{\rm conf} = 8$)



Three-quark potential (Q-QQ system)

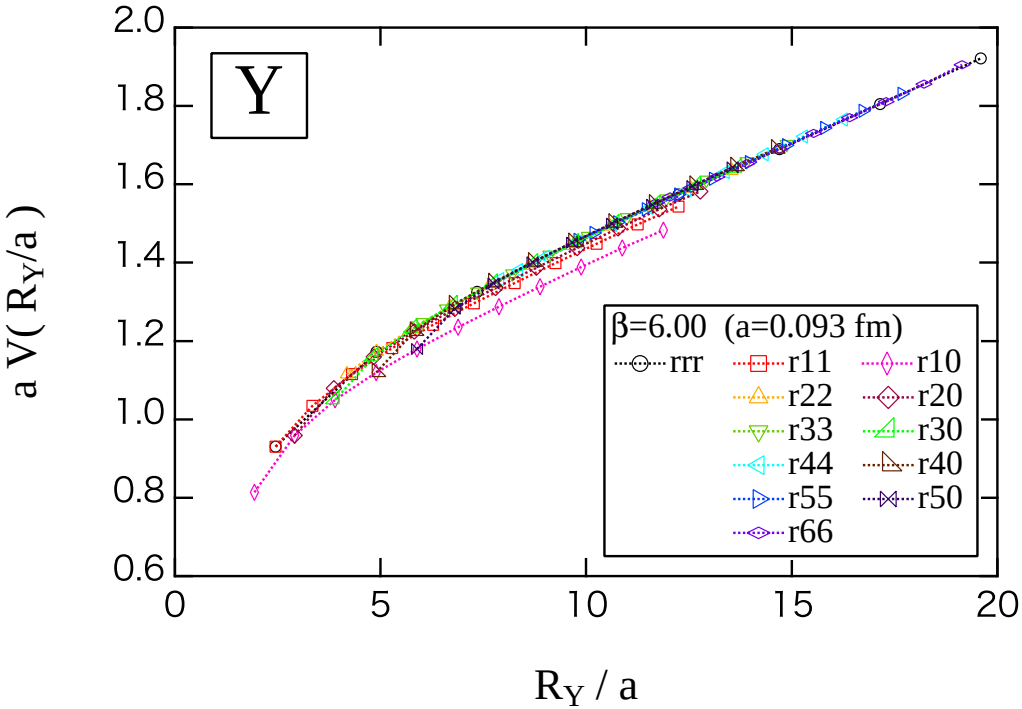
- $\vec{x}_1 = r\vec{e}_1,\ \vec{x}_2 = \vec{x}_3 = \vec{0}$ ($\beta = 6.0$, 24^4 lattice, $N_{\rm sub} = 6$)

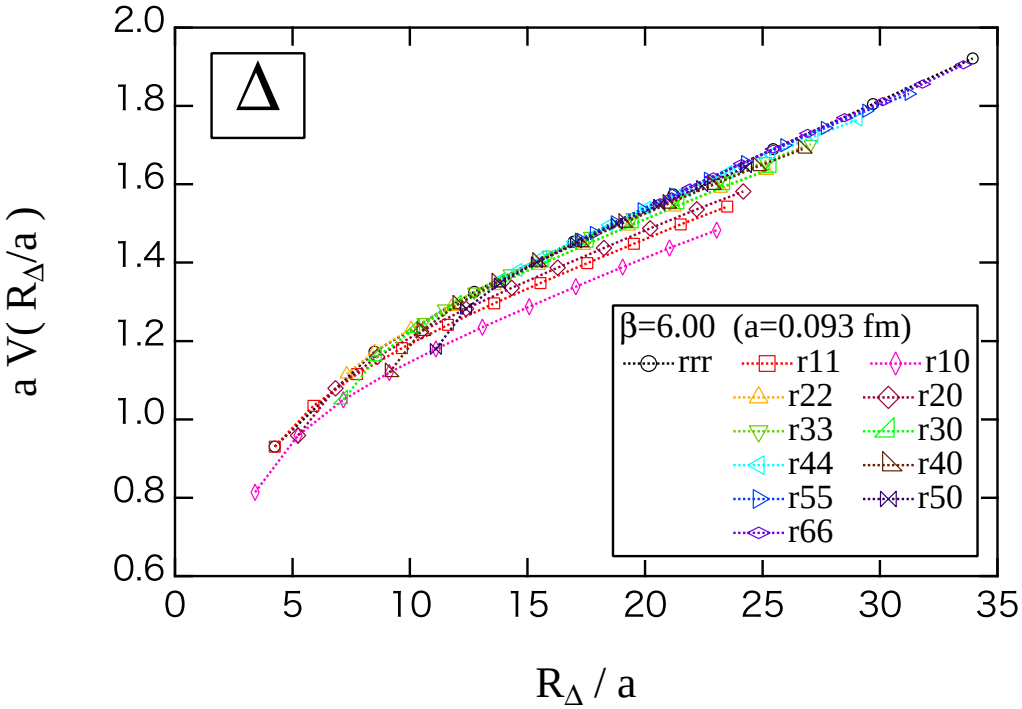


- Q-$\bar{Q}$ and Q–QQ potential are the same except constant shift
  $3 \otimes (\bar{3} \oplus 6) \Rightarrow 3 \otimes \bar{3}$ confirmed [cf. Bissey,Signal&Leinweber, PRD80]

Three-quark potential (regular, isosceles, right)

- **Various 3-quark locations with Y and Δ parametrization ($\beta = 6.0$, 24^4 lattice, $N_{\rm sub} = 6$)**





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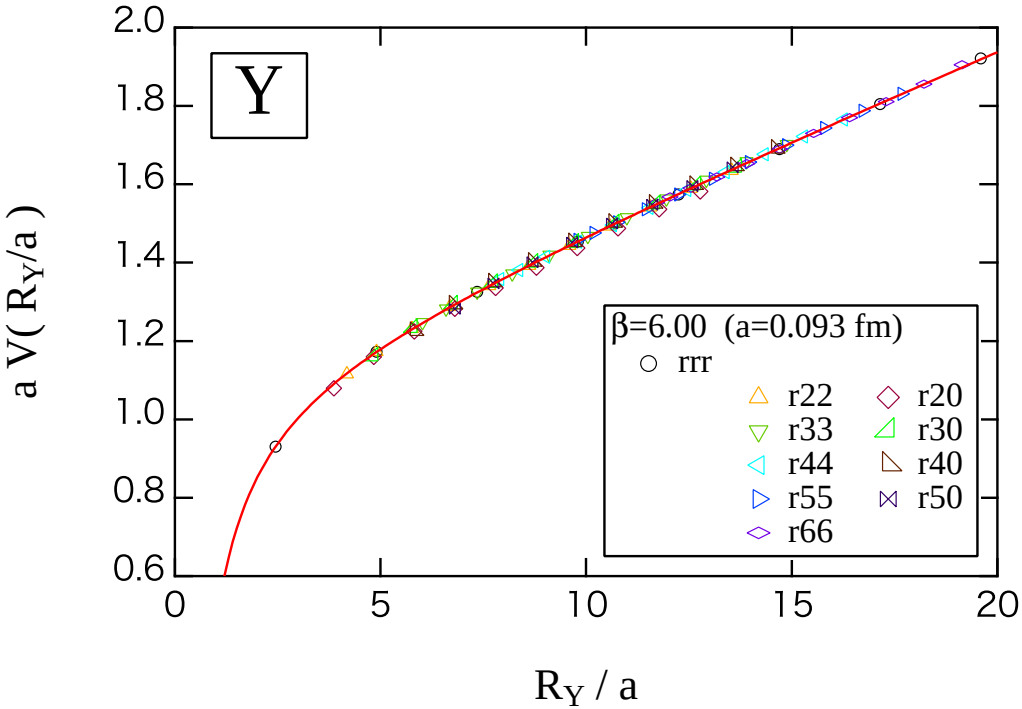
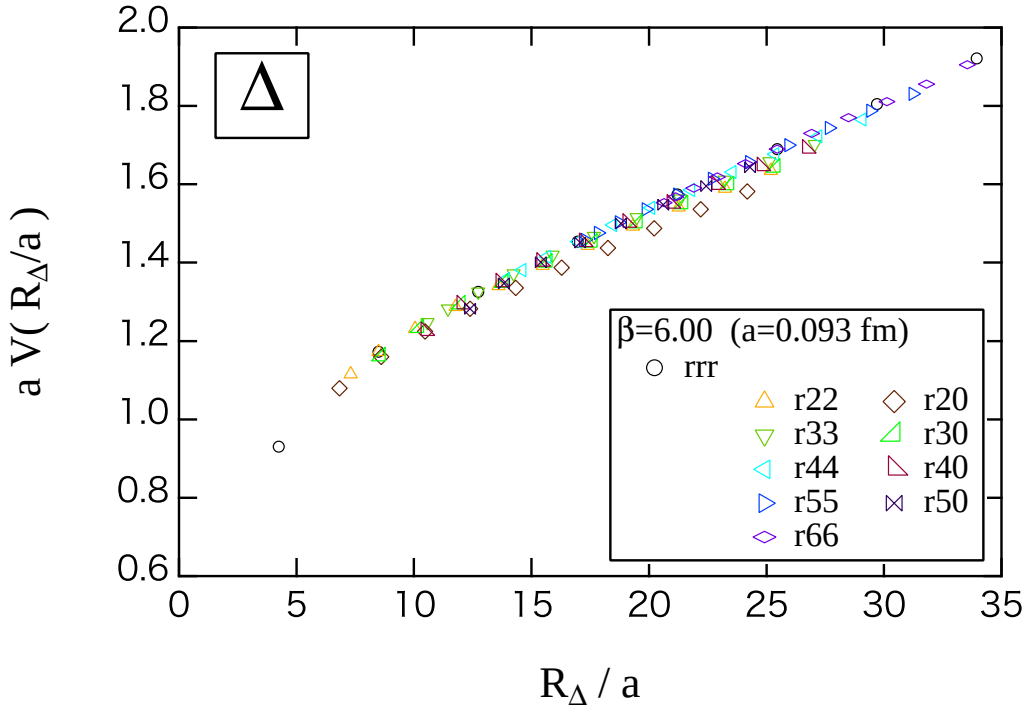
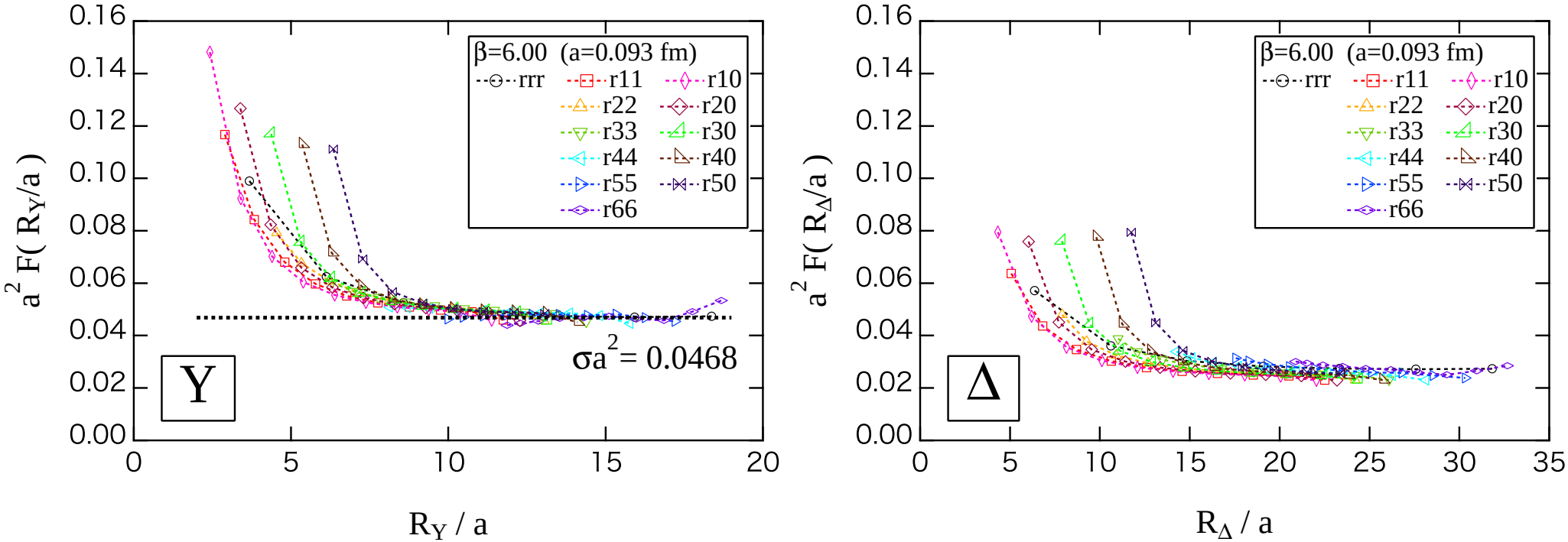




- **The potential with Y parametrization falls into ONE CURVE whenever three quarks are separated with each other**

Three-quark force (regular, isosceles, right)

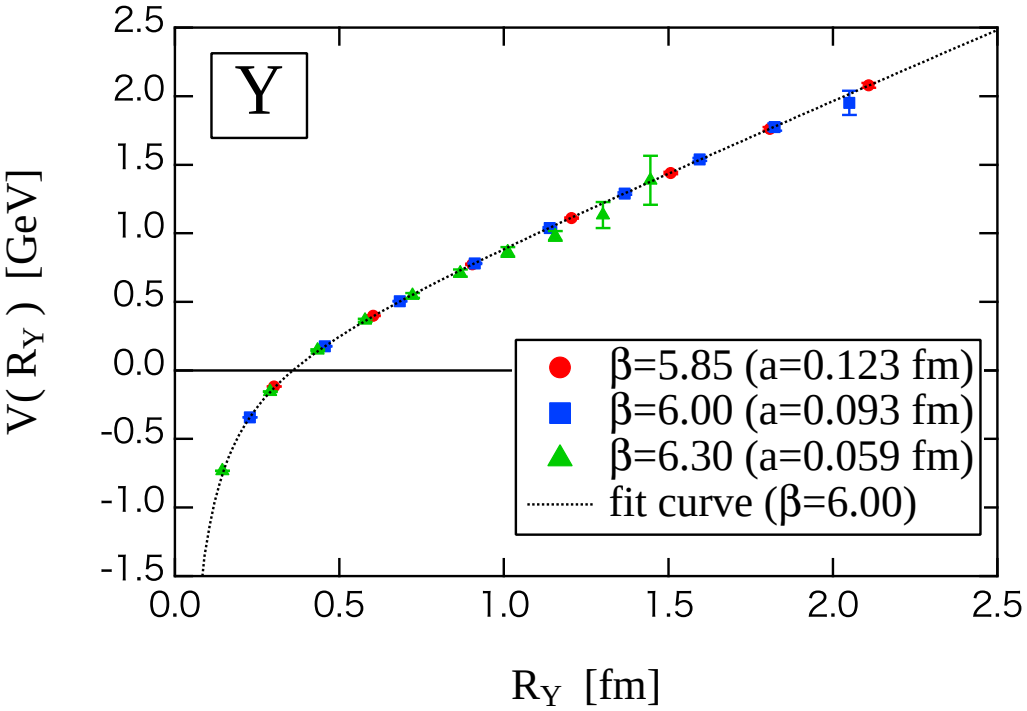
- **Various 3-quark locations with Y and Δ parametrization ($\beta = 6.0$, 24^4 lattice, $N_{\rm sub} = 6$)**

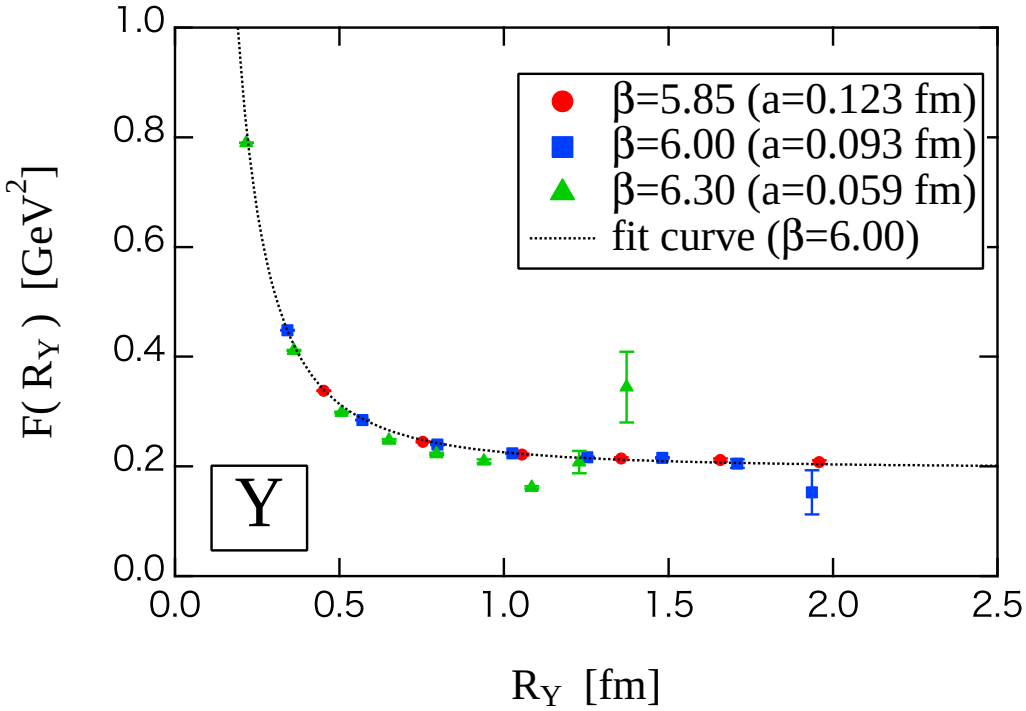


- **"String tension" from Y parametrization is consistent with that of the $Q\bar{Q}$ potential**

Three-quark potential & force (scaling test)

- **Regular triangle with Y parametrization ($\beta = 5.85, 6.00, 6.30$, 24^4 lattice)**





- **The potential and force show good scaling behaviors**

Summary

- **We have investigated the static three-quark potential in SU(3) lattice gauge theory at zero temperature with the Polyakov loop correlation function (PLCF) by employing the multilevel algorithm**

  $\Rightarrow$ **PLCF allows to investigate the static potential of various 3-quark locations in detail with less systematic effects**

  $\Rightarrow$ **The potential with Y-parametrization falls into one curve whenever three quarks are separated with each other**

  $\Rightarrow$ **"String tension" from Y-parametrization is consistent with that of the $Q\bar{Q}$ potential**

  $\Rightarrow$ **The potential and force show good scaling behaviors with Y-parametrization**

- **The present method is promising for computing relativistic corrections for the three-quark system**