QCD in magnetic fields: from the butterfly to the phase diagram

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and all of my other collaborators and friends
Motivation

for significant contributions to our understanding of QCD matter in strong magnetic fields and to QCD thermodynamics.
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- direct and indirect impact on QCD dynamics at $T = 0$ changes at $T > 0$, implications for the QCD phase diagram
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- direct and indirect impact on QCD dynamics at $T = 0$ changes at $T > 0$, implications for the QCD phase diagram
- magnetic response: connects various fields in physics
From the butterfly to the phase diagram
1: the butterfly
Landau versus Bloch

- free quark (electron)
  - exposed to magnetic field in continuum space: Landau orbits
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  - on a (crystal) lattice: Bloch waves
Landau versus Bloch

- free quark (electron)
  - exposed to magnetic field in continuum space: Landau orbits
  - on a (crystal) lattice: Bloch waves

- what happens if the two are combined?
Hofstadter’s butterfly [Hofstadter ’76]

- Bloch electrons immersed in a magnetic field
  energy levels versus magnetic flux $\Phi = a^2 qB/2\pi$
Hofstadter’s butterfly [Hofstadter ’76]

- true fractal structure (if the lattice is infinite)
- energies accumulate into bands if flux $\Phi \in \mathbb{Q}$
  $(2\pi/a^2$ and $qB$ are commensurable)
- energies isomorphic to the Cantor set if $\Phi \not\in \mathbb{Q}$
  $(2\pi/a^2$ and $qB$ are incommensurable)
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Hofstadter’s butterfly: free quarks on the lattice

- Dirac equation for quarks in a magnetic field in 2D
  [GE, unpublished]
Hofstadter’s butterfly: free quarks on the lattice

• crystal in solid state physics
  versus
  regulator in quantum field theory
• $\Phi = a^2 \cdot qB/2\pi \xrightarrow{a \to 0} 0$
  $\Rightarrow$ butterfly disappears in the continuum limit
• but, low-$\Phi$ behavior contains continuum physics
Hofstadter’s butterfly: free quarks on the lattice

- Landau levels at low $B \to$ dissolve into bands [GE, unpublished]
Hofstadter’s butterfly: experiments

- “catching the butterfly”

[Ponomarenko et al ’13] [Dean et al ’13]
2: the condensate
Hofstadter’s butterfly: impact on QCD [GE, unpublished]

- the butterfly disappears in the continuum limit, but its wings around $a^2 qB = 2\pi \Phi \approx 0$ contain physical information.
- In contrast to electron energies, Dirac eigenvalues cannot be measured.
- Physical observable composed of the eigenvalues: condensate of quarks with mass $m$

$$\bar{\psi} \psi^{2D} = \sum_{\lambda} \frac{m}{\lambda^2 + m^2}$$

- Nonzero quark mass washes out the fractal structure up to $qB \propto m^2$
  → animation
Hofstadter’s butterfly and the condensate

in the continuum: \( \bar{\psi}\psi^{2D} \propto \beta_1 \cdot B^2 + O(B^4) \)
due to electric charge renormalization \([\text{GE 1301.1307}]\)
\(\Rightarrow\) tendency dictated by \(\beta_1 > 0\) (no asympt. freedom in QED)
A threefold correspondence for free quarks

wings of Hofstadter’s butterfly at low magnetic fields

≃

no asymptotic freedom in QED

≃

enhancement of quark condensate by magnetic fields
3: the QCD phase diagram
Hofstadter’s butterfly: impact on QCD [GE, unpublished]

- Dirac equation for quarks in a magnetic field in 2D
• Dirac equation for quarks in a magnetic field in 2D with QCD interactions switched on.
Hofstadter’s butterfly and the condensate

QCD interactions wash out the fractal structure, but qualitative tendency remains. Subtleties: charge renormalization, interacting case, massless limit, etc. (Bali, Bruckmann, GE, Katz, Schäfer 1406.0269)
Hofstadter’s butterfly and the condensate

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Quark condensate in 4D QCD at $T = 0$

- $\bar{\psi} \psi$ grows with $B$ even in the interacting case in 4D

[Bali, Bruckmann, GE, Fodor, Katz, Schäfer 1206.4205]
Quark condensate in 4D QCD at $T = 0$

- $\bar{\psi}\psi$ grows with $B$ even in the interacting case in 4D
  [Bali, Bruckmann, GE, Fodor, Katz, Schäfer 1206.4205]

- has been long known as ‘magnetic catalysis’, due to dimensional reduction for $B \to \infty$ [Gusynin et al hep-ph/9509320]
Quark condensate in 4D QCD at $T = 0$

- $\bar{\psi}\psi$ grows with $B$ even in the interacting case in 4D
  
  [Bali, Bruckmann, GE, Fodor, Katz, Schäfer 1206.4205]

- captured by all low-energy models, e.g. $\chi$PT, NJL, ...
Quark condensate in 4D QCD at $T > 0$: models

- low-energy models predict magnetic catalysis to persist even at high temperatures (e.g. PNJL model [Gatto et al 1012.1291])

![Graph showing the effect of magnetic field on quark condensate]

- result: $T_c(B)$ increases
Quark condensate in 4D QCD at $T > 0$: models

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- result: $T_c(B)$ increases
Quark condensate in 4D QCD at $T > 0$: lattice

- going to finite temperatures changes the response qualitatively dubbed ‘inverse magnetic catalysis’ around $T_c$

[Bruckmann, GE, Kovács 1303.3972]
Phase diagram

- impact on the QCD phase diagram: $T_c(B)$ decreases

[Bali, Bruckmann, GE, Fodor, Katz, Krieg, Schäfer, Szabó 1111.4956]
Summary 1.

- three intertwined phenomena at $T = 0$
  - wings of Hofstadter’s butterfly at small $B$ → solid state physics
  - no asymptotic freedom in QED ($\beta_1 > 0$) → perturbative QFT
  - magnetic catalysis of the QCD condensate → nonperturbative QFT, $\chi_{SB}$
Summary II.

- inverse magnetic catalysis appears around $T \sim T_c$
  - impact on phase diagram
  - input to improve low-energy models
  - relevant for phenomenology: non-central heavy-ion collisions and early universe

- more on magnetic fields: [D’Elia Sat 08:30]
  [Braguta Thu 14:35, Buividovich Fri 15:15, Kochetkov Fri 17:30, Larina poster]
  [Levkova Fri 18:10, Mariti Fri 17:50, Negro Wed 10:00, Valgushev Thu 16:15]
**Inverse catalysis**

- $B$ changes the typical gauge configurations
- most important dof: Polyakov loop

- Polyakov loop ‘draws’ condensate with itself
- effect only visible for light quarks [Bruckmann, GE, Kovács 1303.3972]
  (otherwise indirect effect of $B$ on gluons is weak)
Magnetic catalysis – mass dependence

- \( \bar{\psi} \psi = c \cdot (qB)^2 \)
- free quarks (interacting, \( m \to \infty \)): \( c = N_c \cdot \beta_1^{\text{spinor}} \)
- free pions (interacting, \( m \to 0 \)): \( c = \beta_1^{\text{scalar}} / 4 \)

- intermediate masses: smooth dependence

[Bali, Bruckmann, GE, Katz, Schäfer 1406.0269]