QCD in magnetic fields: from the butterfly to the phase diagram

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Marios Costa, Zoltán Fodor, Sándor Katz, Tamás Kovács,
Stefan Krieg, Haris Panagopoulos, Andreas Schäfer, Kálmán Szabó

and all of my other collaborators and friends

Motivation

for *significant contributions to our understanding of QCD matter in strong magnetic fields and to QCD thermodynamics.*

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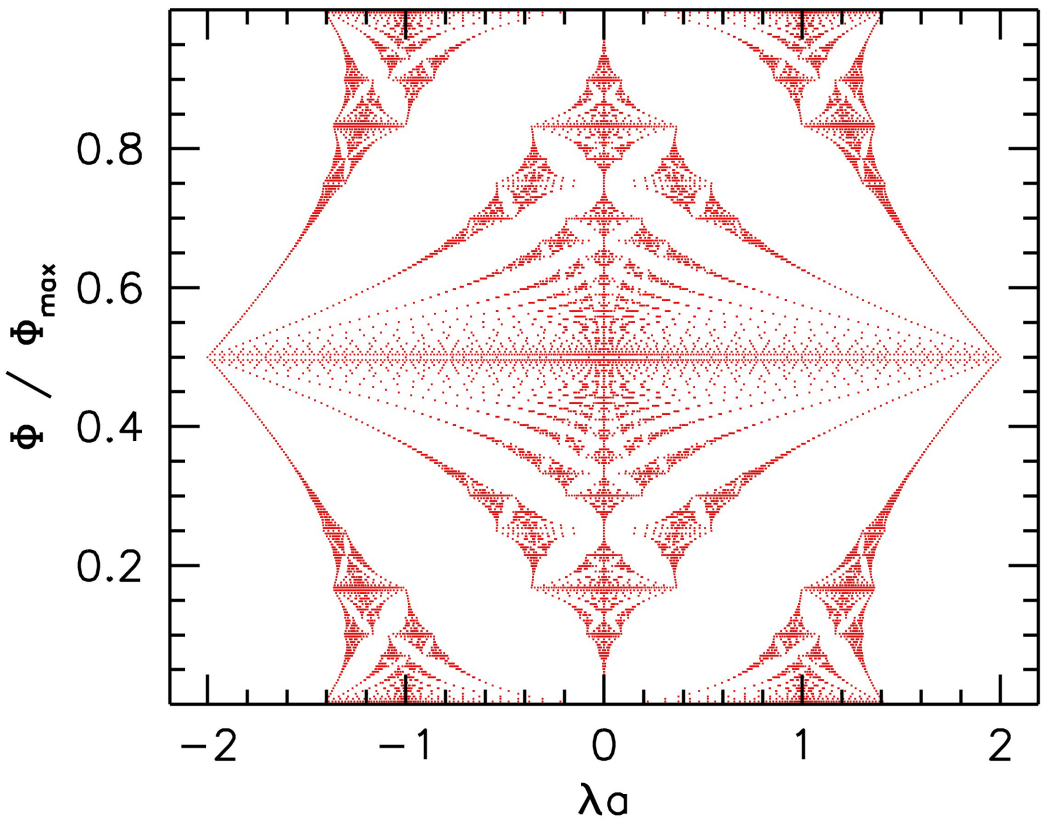

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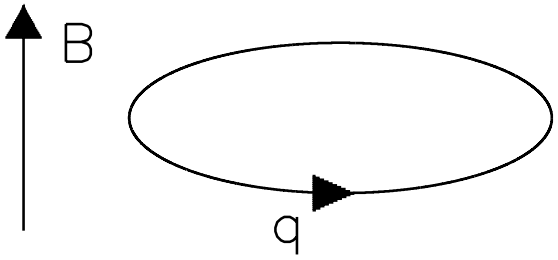
- magnetic response: connects various fields in physics

From the butterfly to the phase diagram

1: the butterfly

Landau versus Bloch

- free quark (electron)
- exposed to magnetic field in continuum space: Landau orbits



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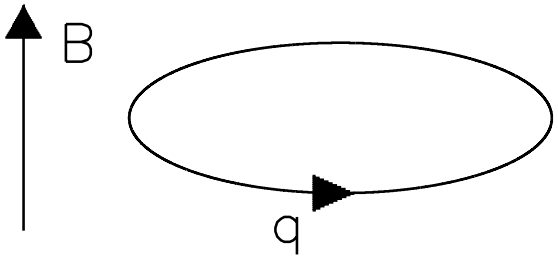

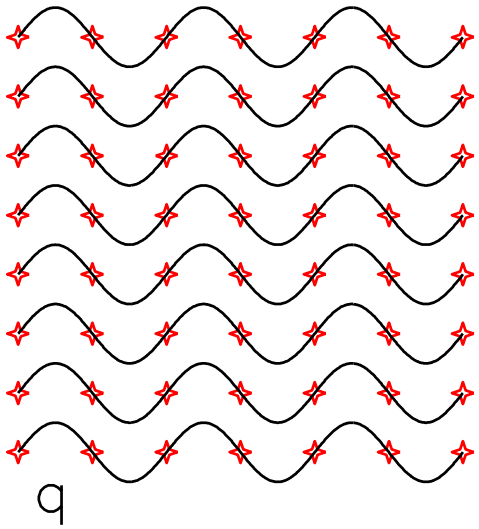

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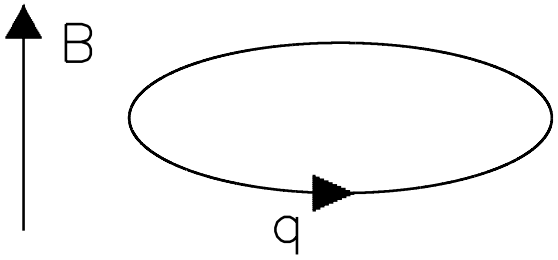
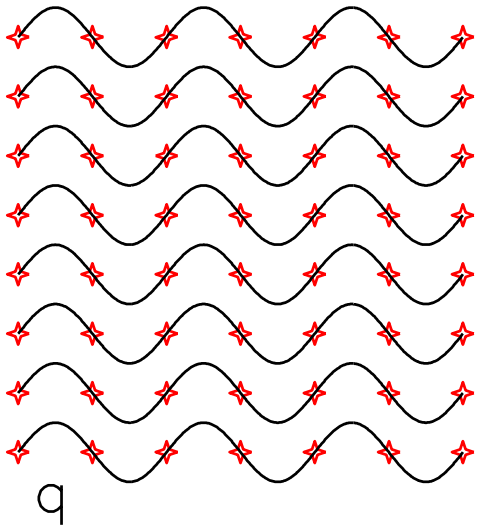
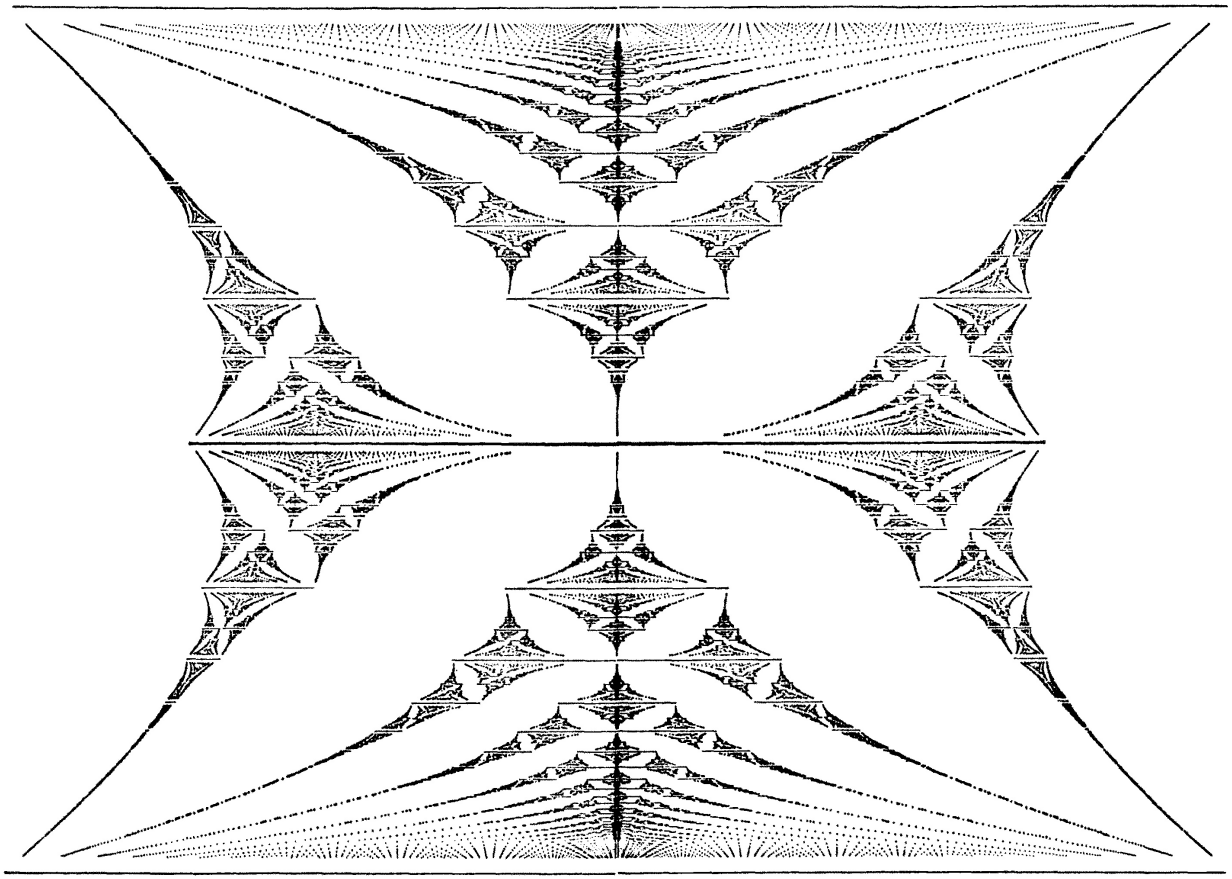




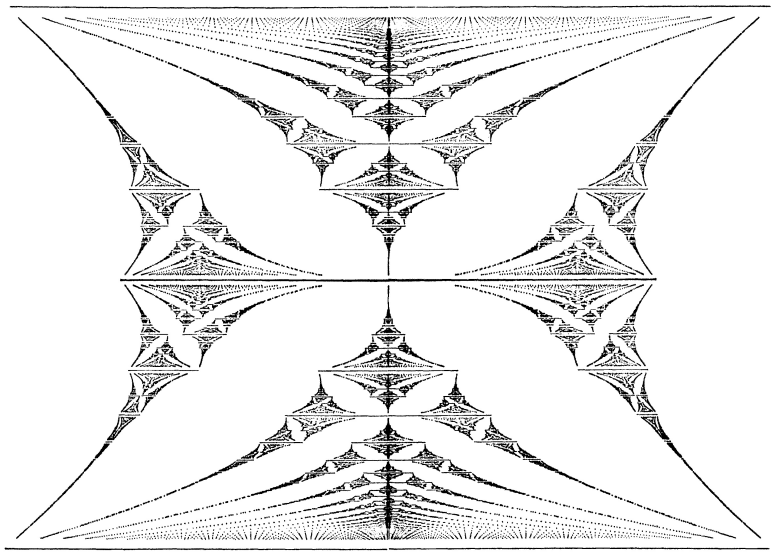
- what happens if the two are combined?

Hofstadter's butterfly [Hofstadter '76]



- Bloch electrons immersed in a magnetic field
  energy levels versus magnetic flux $\Phi = a^2 qB/2\pi$

Hofstadter's butterfly [Hofstadter '76]



- true fractal structure (if the lattice is infinite)
- energies accumulate into bands if flux $\Phi \in \mathbb{Q}$ ($2\pi/a^2$ and qB are commensurable)
- energies isomorphic to the Cantor set if $\Phi \notin \mathbb{Q}$ ($2\pi/a^2$ and qB are incommensurable)

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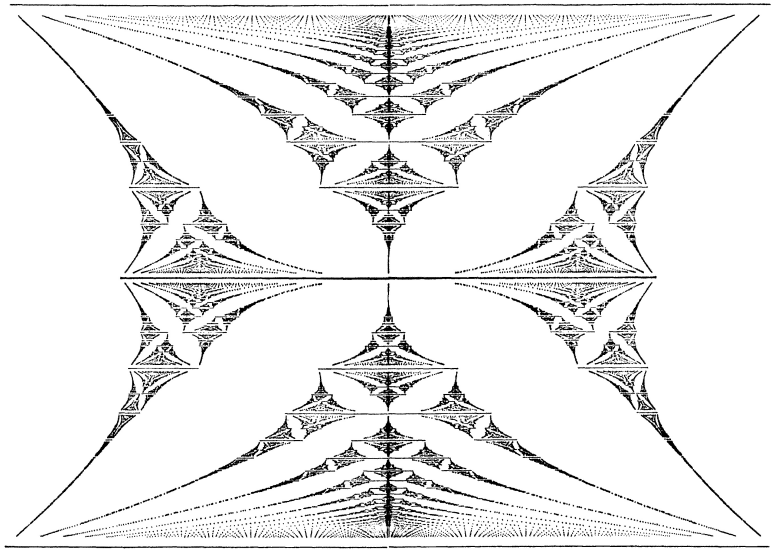

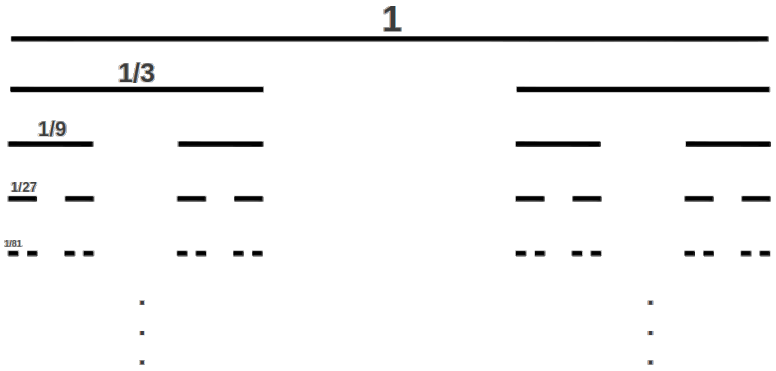

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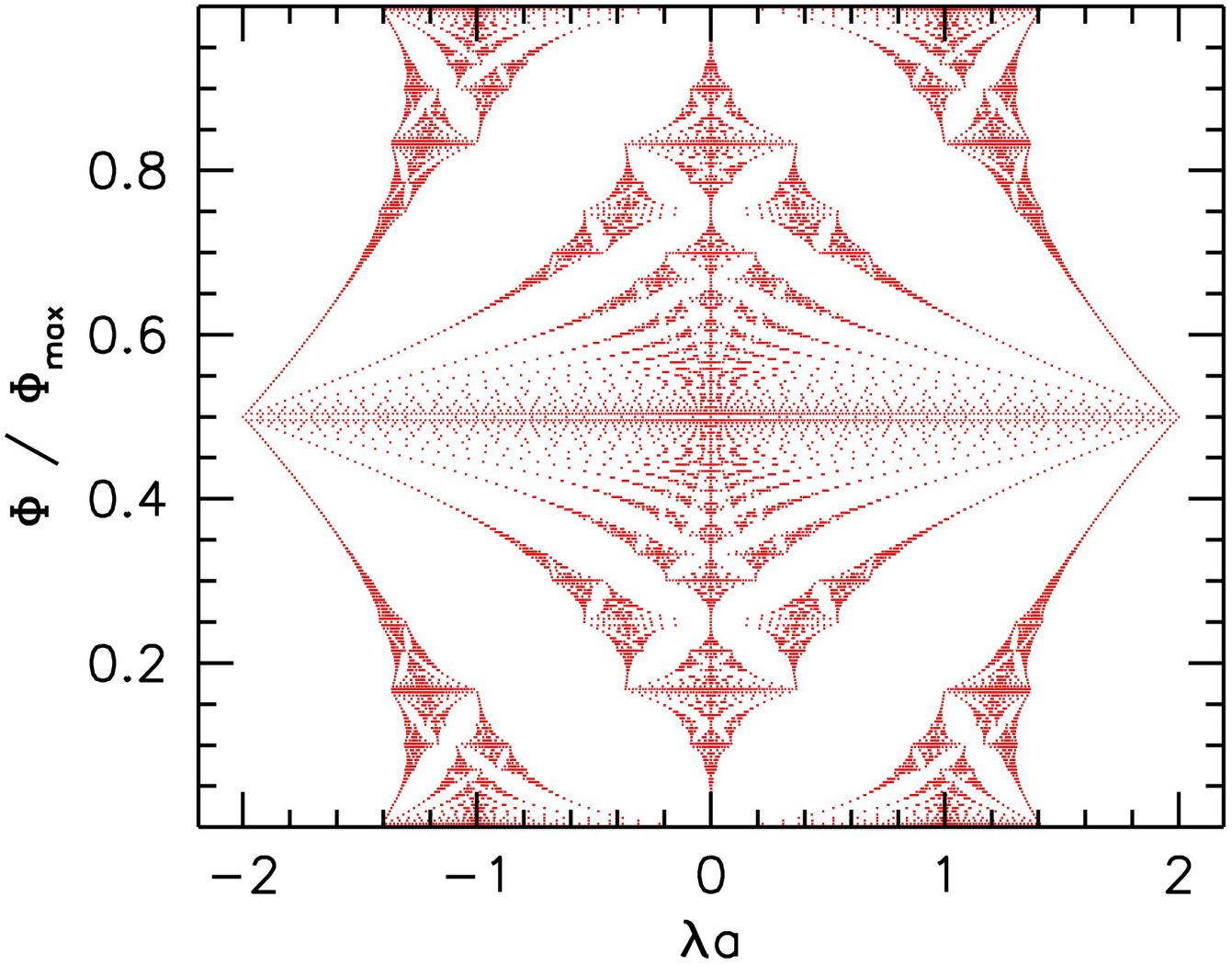
Hofstadter's butterfly: free quarks on the lattice



- Dirac equation for quarks in a magnetic field in 2D
  [GE, unpublished]

Hofstadter's butterfly: free quarks on the lattice

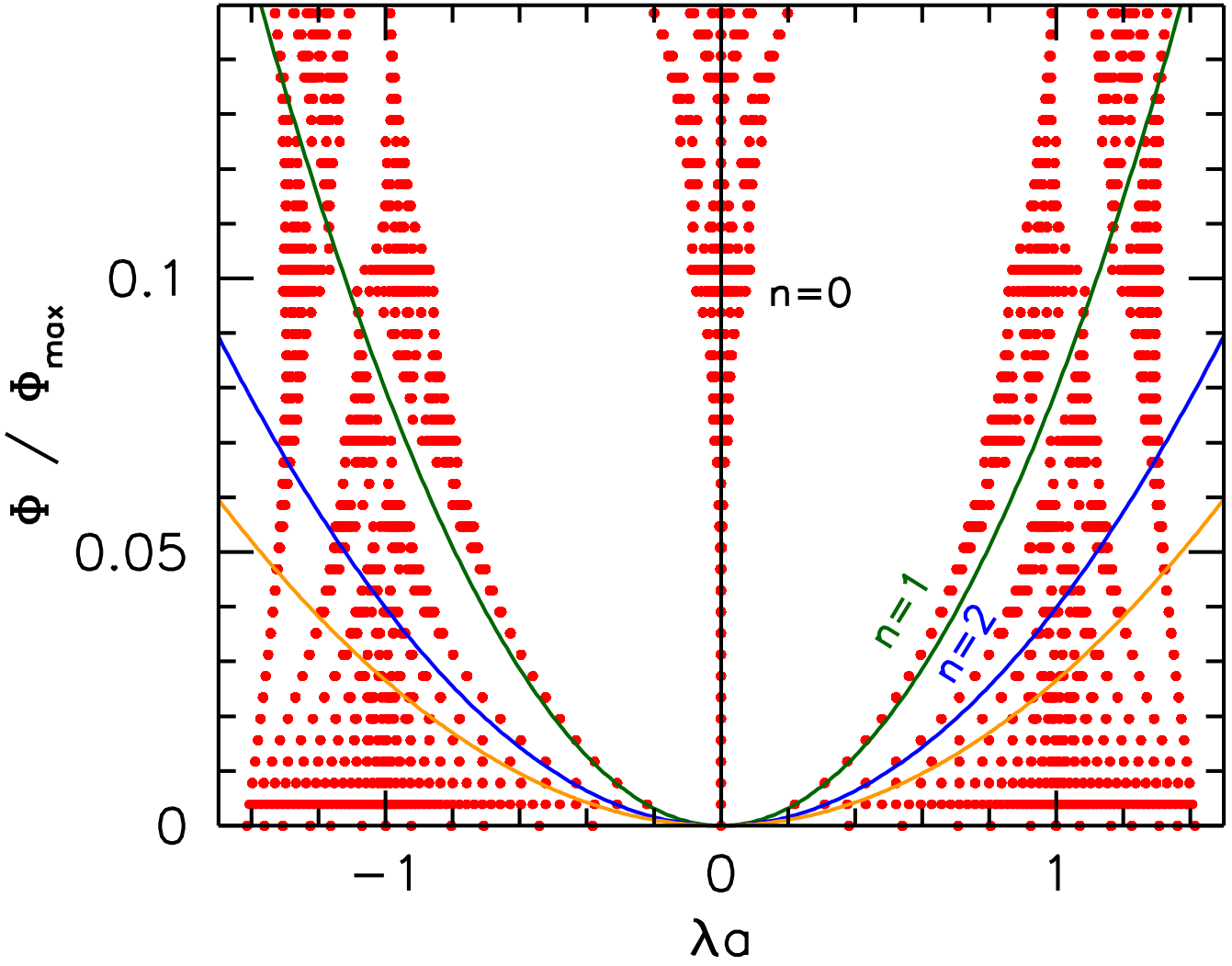
- crystal in solid state physics

  versus

  regulator in quantum field theory
- $\Phi = a^2 \cdot qB/2\pi \xrightarrow{a \to 0} 0$

  $\Rightarrow$ butterfly disappears in the continuum limit
- but, low-Φ behavior contains continuum physics

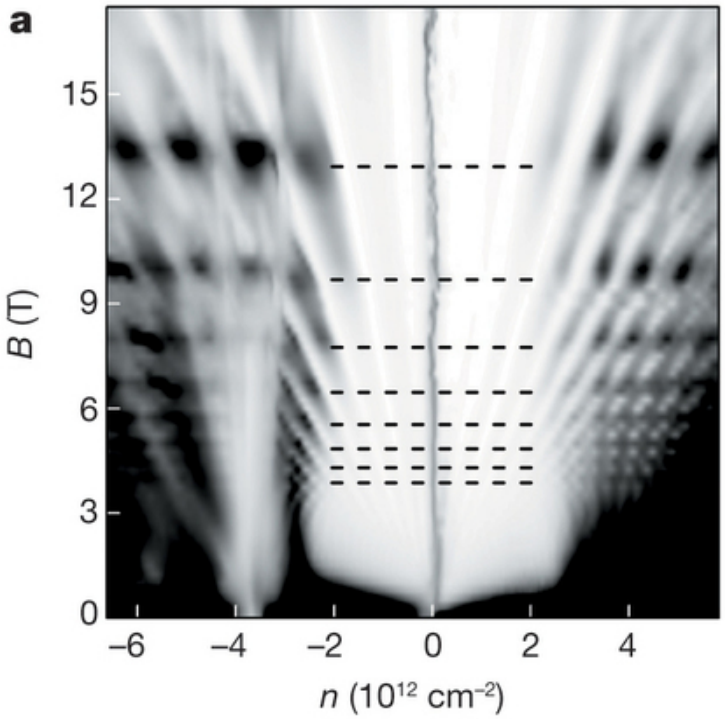
Hofstadter's butterfly: free quarks on the lattice



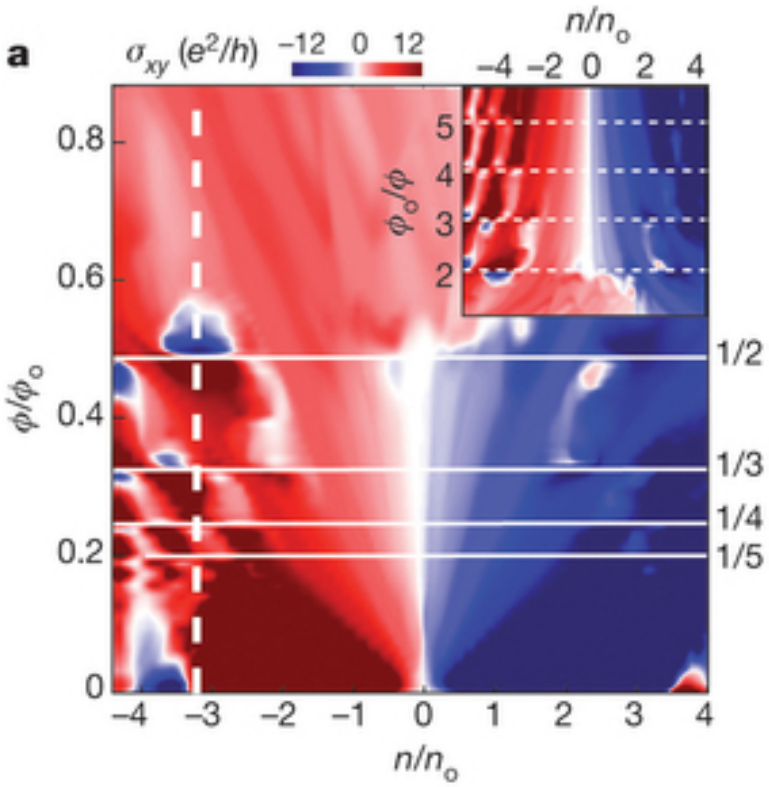
- Landau levels at low $B \to$ dissolve into bands [GE, unpublished]

Hofstadter's butterfly: experiments

- "catching the butterfly"



[Ponomarenko et al '13]



[Dean et al '13]

2: the condensate

Hofstadter's butterfly: impact on QCD [GE, unpublished]

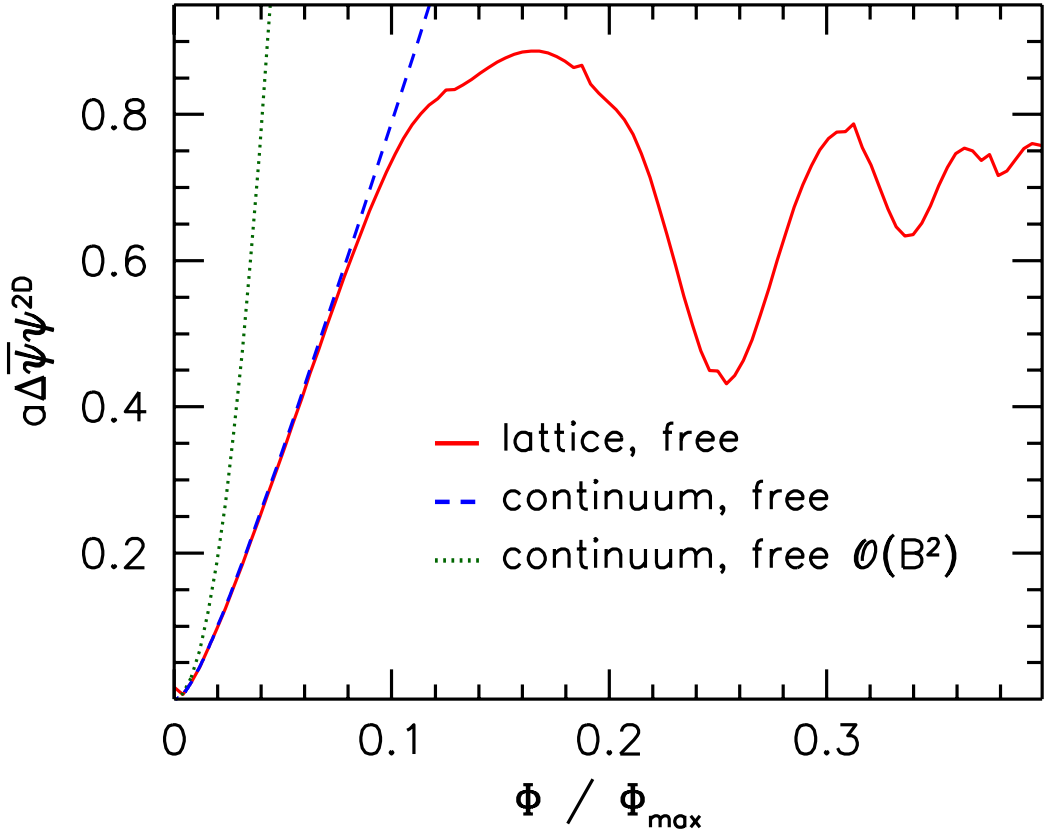
- the butterfly disappears in the continuum limit, but its wings around $a^2 qB = 2\pi\Phi \approx 0$ contain physical information
- in contrast to electron energies, Dirac eigenvalues cannot be measured
- physical observable composed of the eigenvalues: condensate of quarks with mass m

$$\bar{\psi}\psi^{2\mathrm{D}} = \sum_{\lambda} \frac{m}{\lambda^2 + m^2}$$

- nonzero quark mass washes out the fractal structure up to $qB \propto m^2$

  $\rightarrow$ animation

Hofstadter's butterfly and the condensate



- in the continuum: $\bar{\psi}\psi^{2\mathrm{D}} \propto \beta_1 \cdot B^2 + \mathcal{O}(B^4)$
  due to electric charge renormalization [GE 1301.1307]
  $\Rightarrow$ tendency dictated by $\beta_1 > 0$ (no asympt. freedom in QED)

A threefold correspondence for free quarks

wings of Hofstadter's butterfly at low magnetic fields

$\simeq$

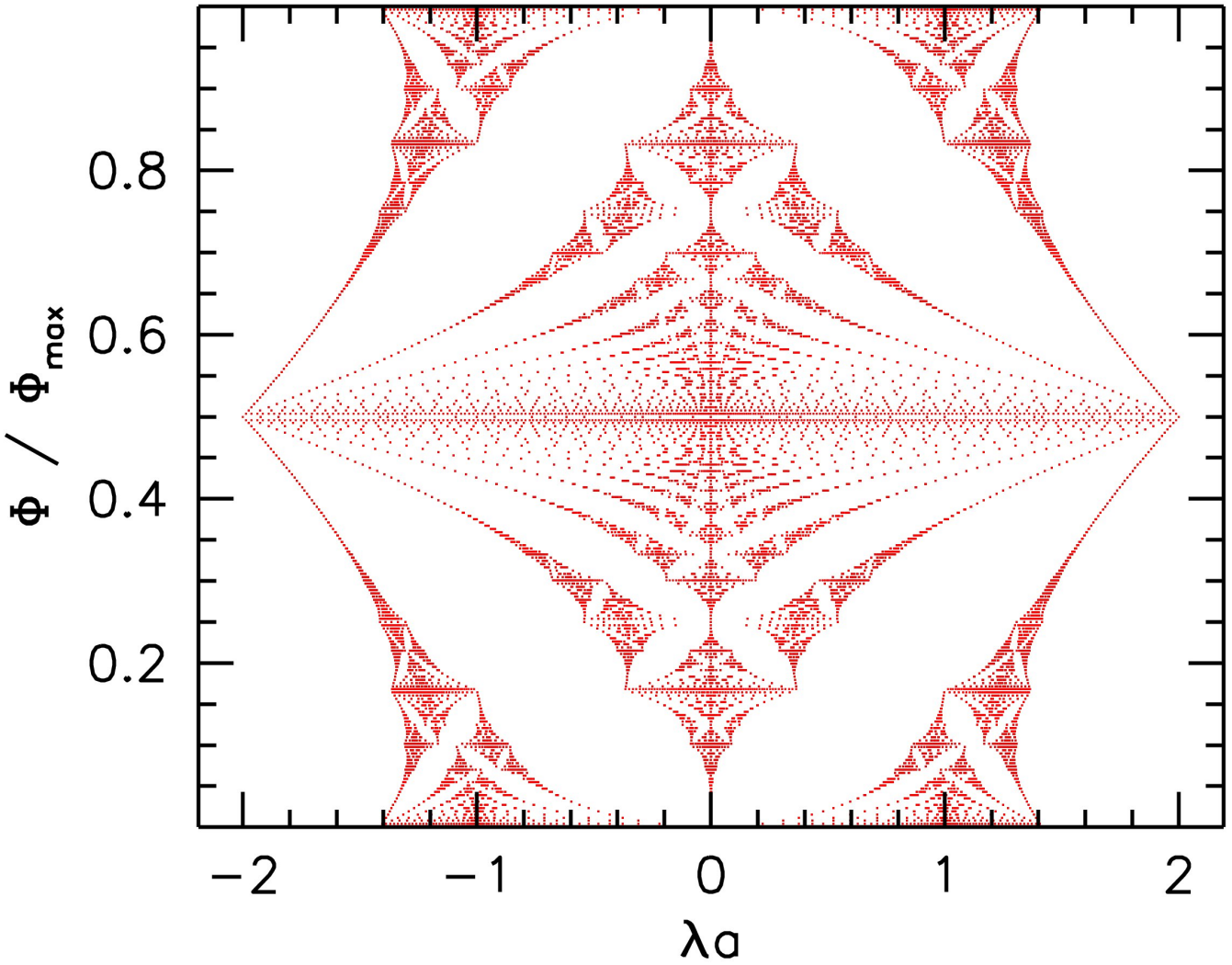
no asymptotic freedom in QED

$\simeq$

enhancement of quark condensate by magnetic fields

3: the QCD phase diagram

Hofstadter's butterfly: impact on QCD [GE, unpublished]



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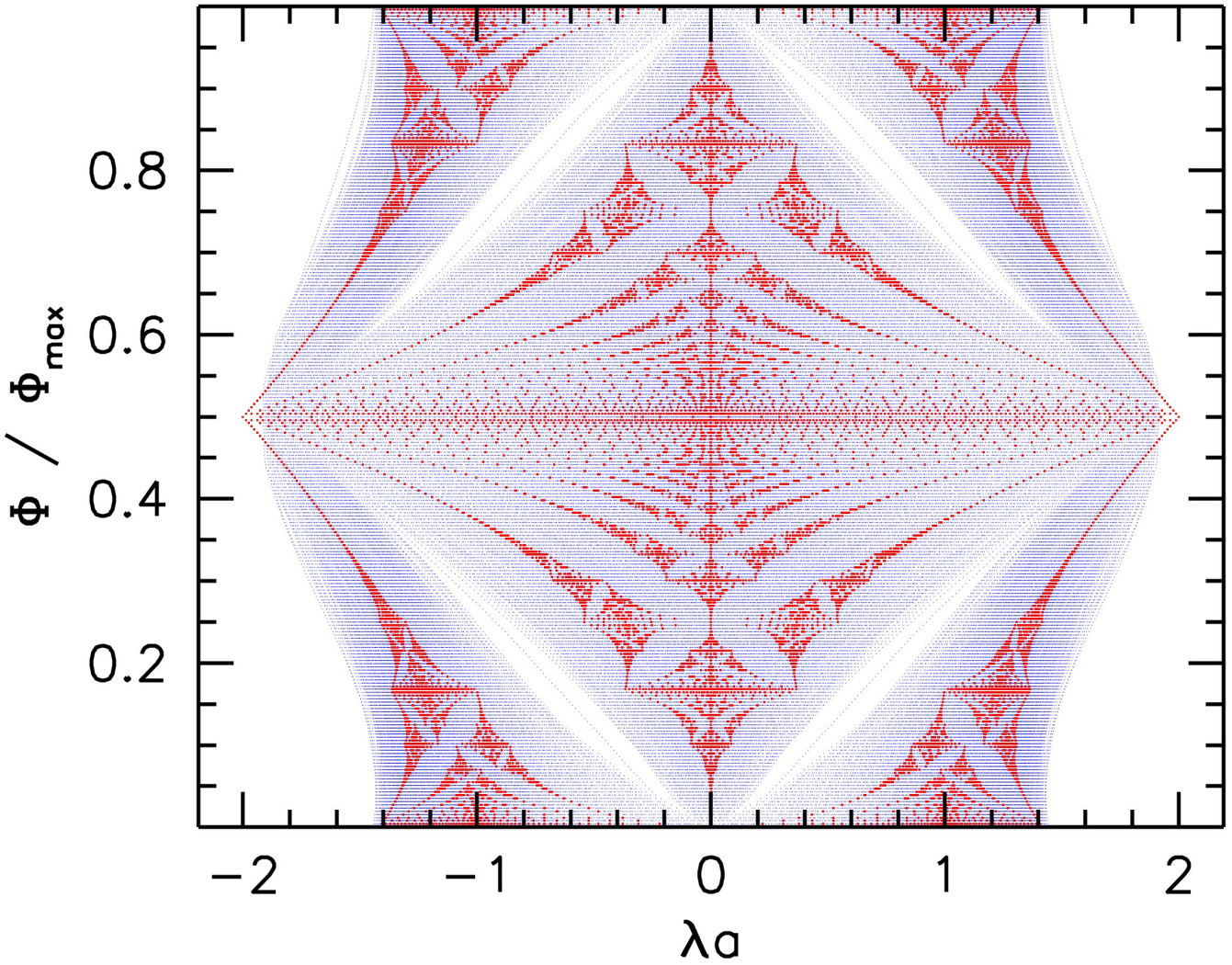
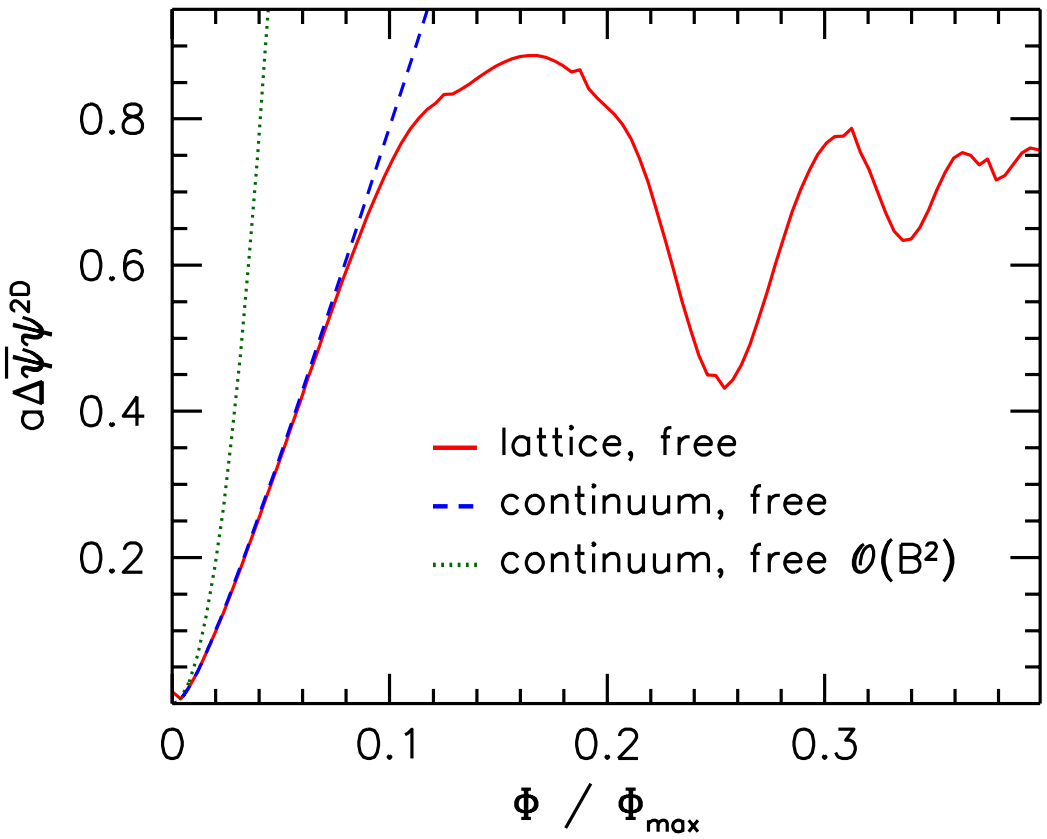


- Dirac equation for quarks in a magnetic field in 2D with QCD interactions switched on

Hofstadter's butterfly and the condensate



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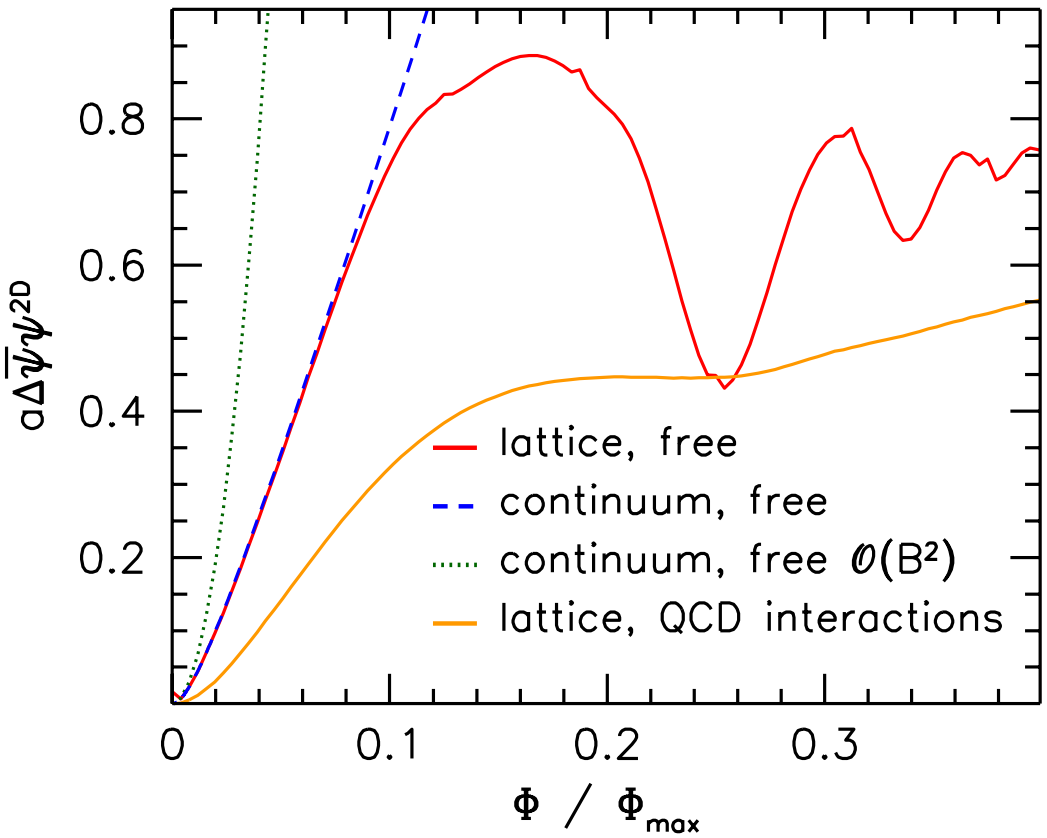

- QCD interactions wash out the fractal structure, but qualitative tendency remains

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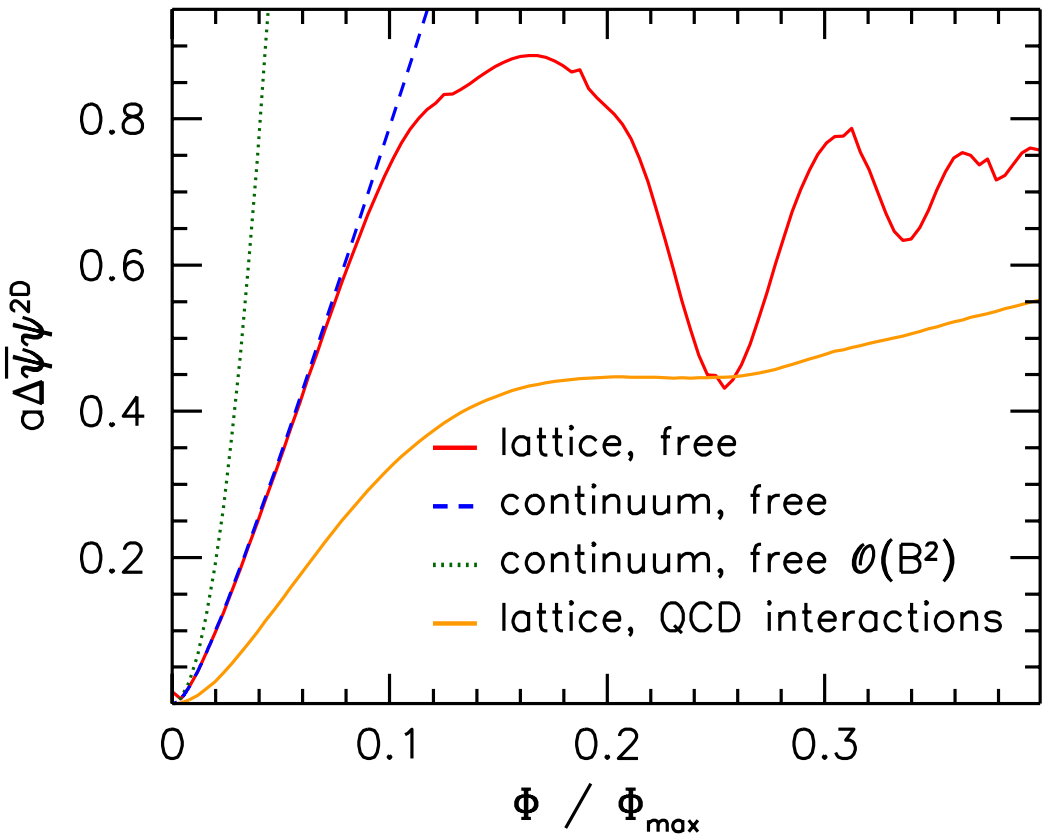
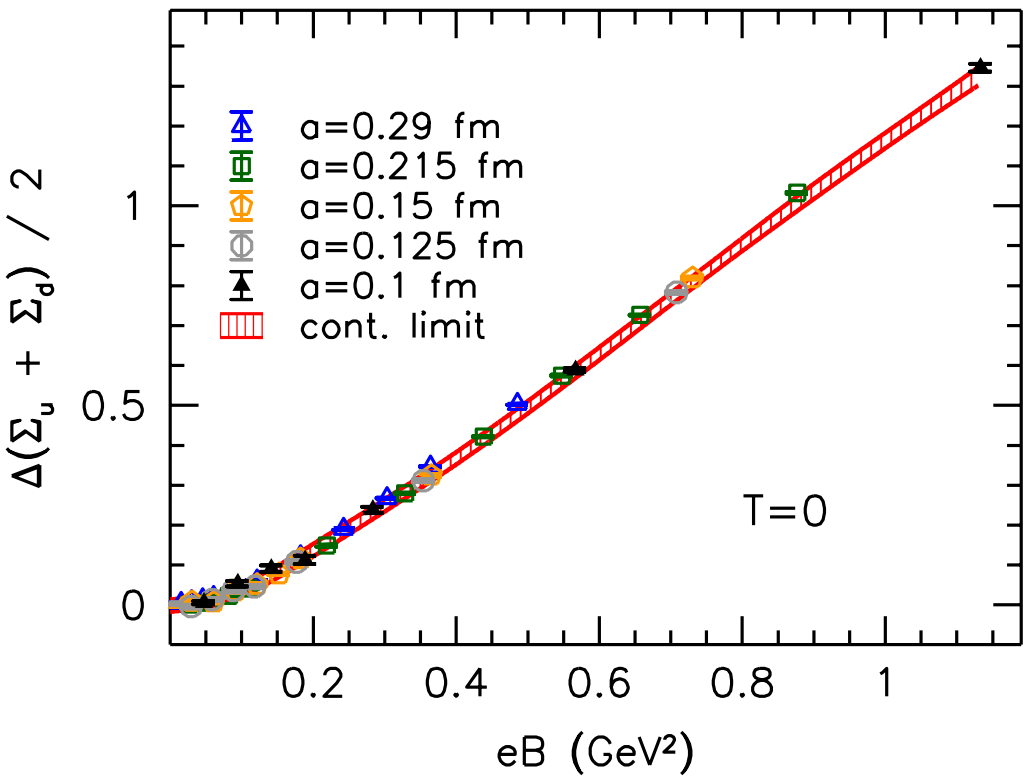


- QCD interactions wash out the fractal structure, but qualitative tendency remains
- subtleties: charge renormalization, interacting case, massless limit etc. [Bali, Bruckmann, GE, Katz, Schäfer 1406.0269]

Quark condensate in 4D QCD at $T = 0$

- $\bar{\psi}\psi$ grows with B even in the interacting case in 4D
  [Bali, Bruckmann, GE, Fodor, Katz, Schäfer 1206.4205]



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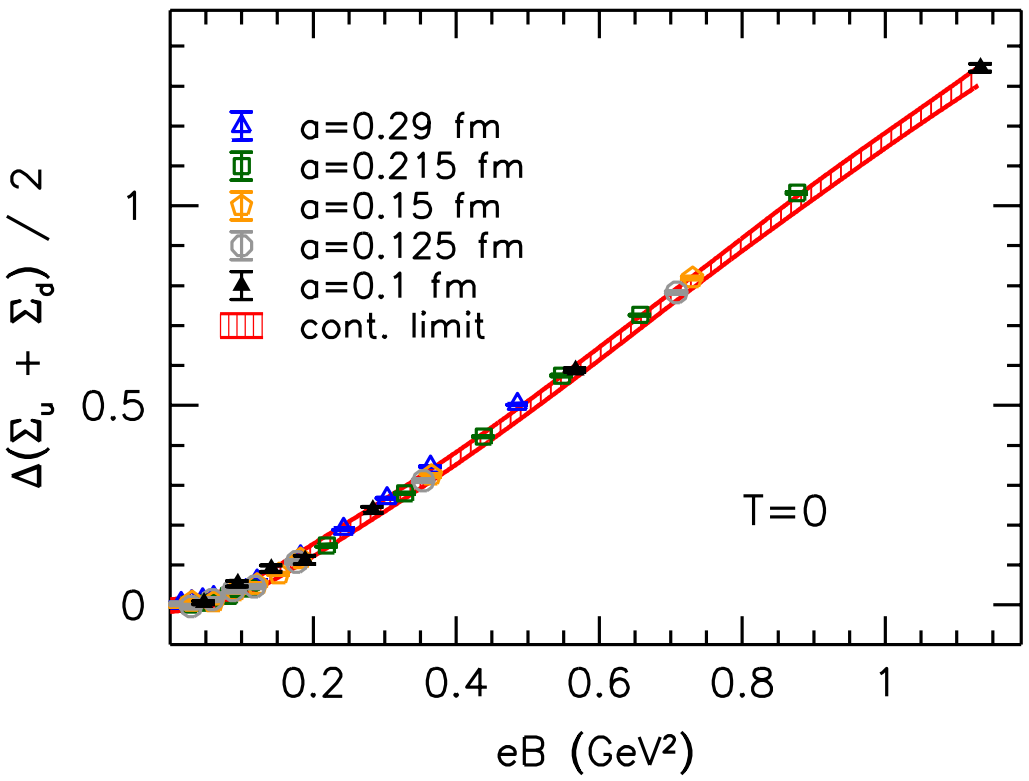

- has been long known as 'magnetic catalysis', due to dimensional reduction for $B \to \infty$ [Gusynin et al hep-ph/9509320]

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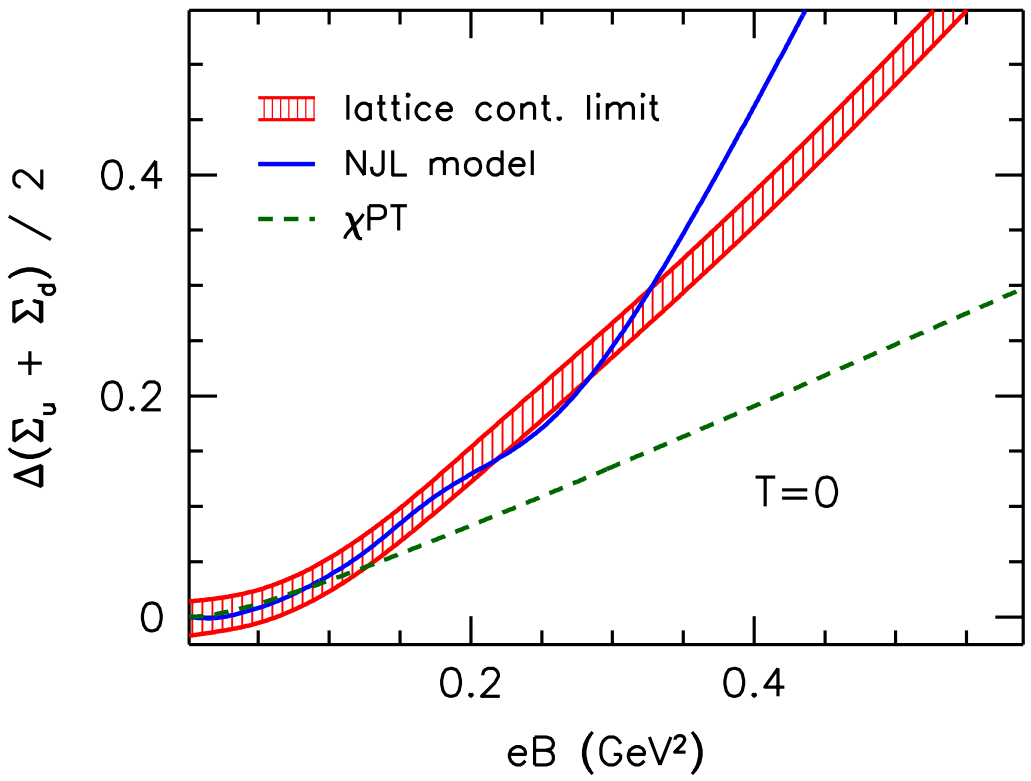
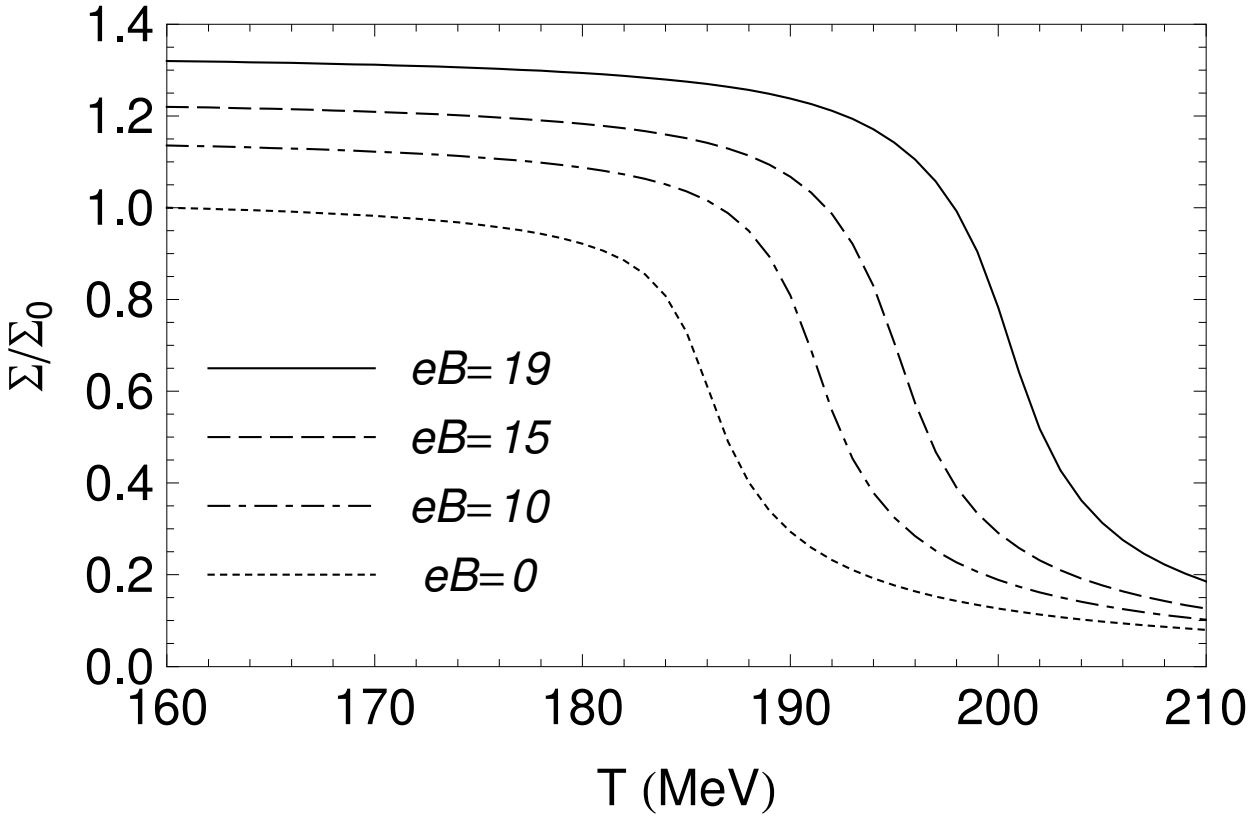


- captured by all low-energy models, e.g. χPT, NJL, ...

Quark condensate in 4D QCD at $T > 0$: models

- low-energy models predict magnetic catalysis to persist even at high temperatures (e.g. PNJL model [Gatto et al 1012.1291])



- result: $T_c(B)$ increases

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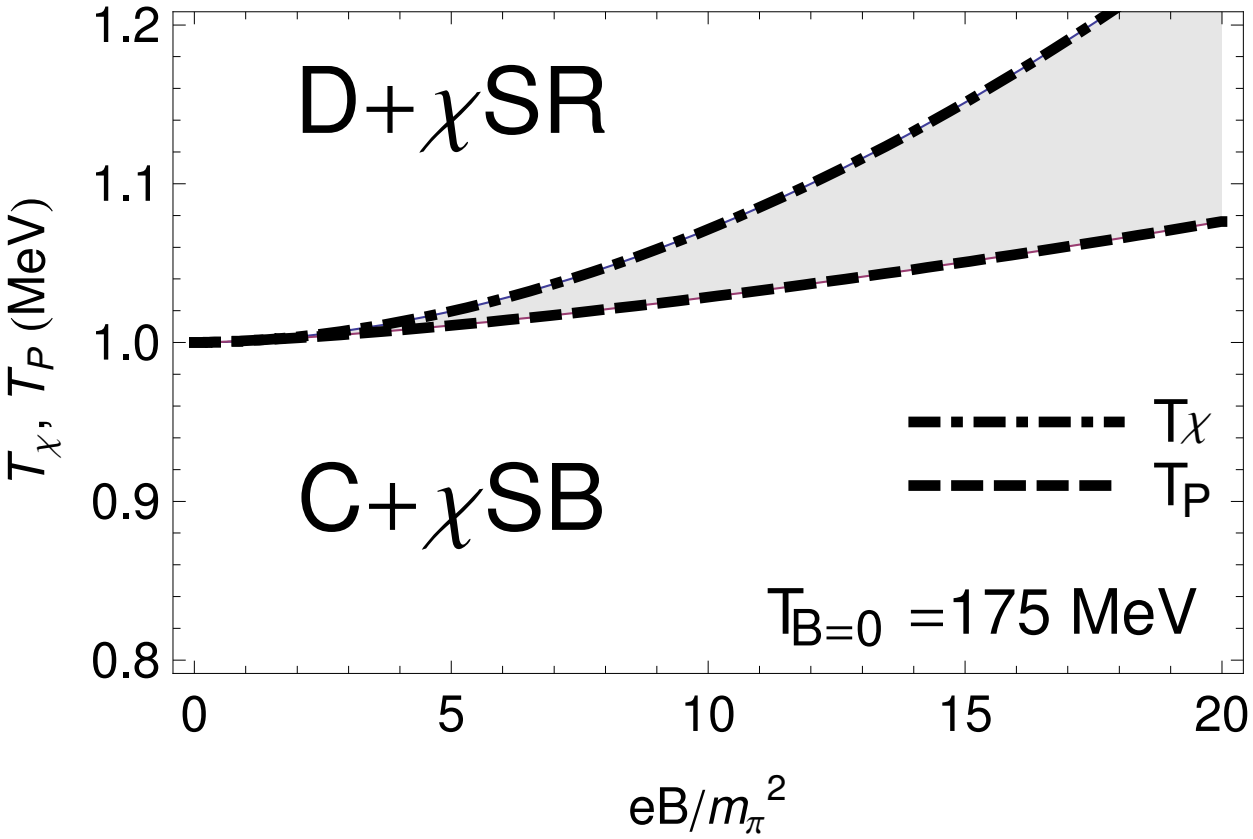
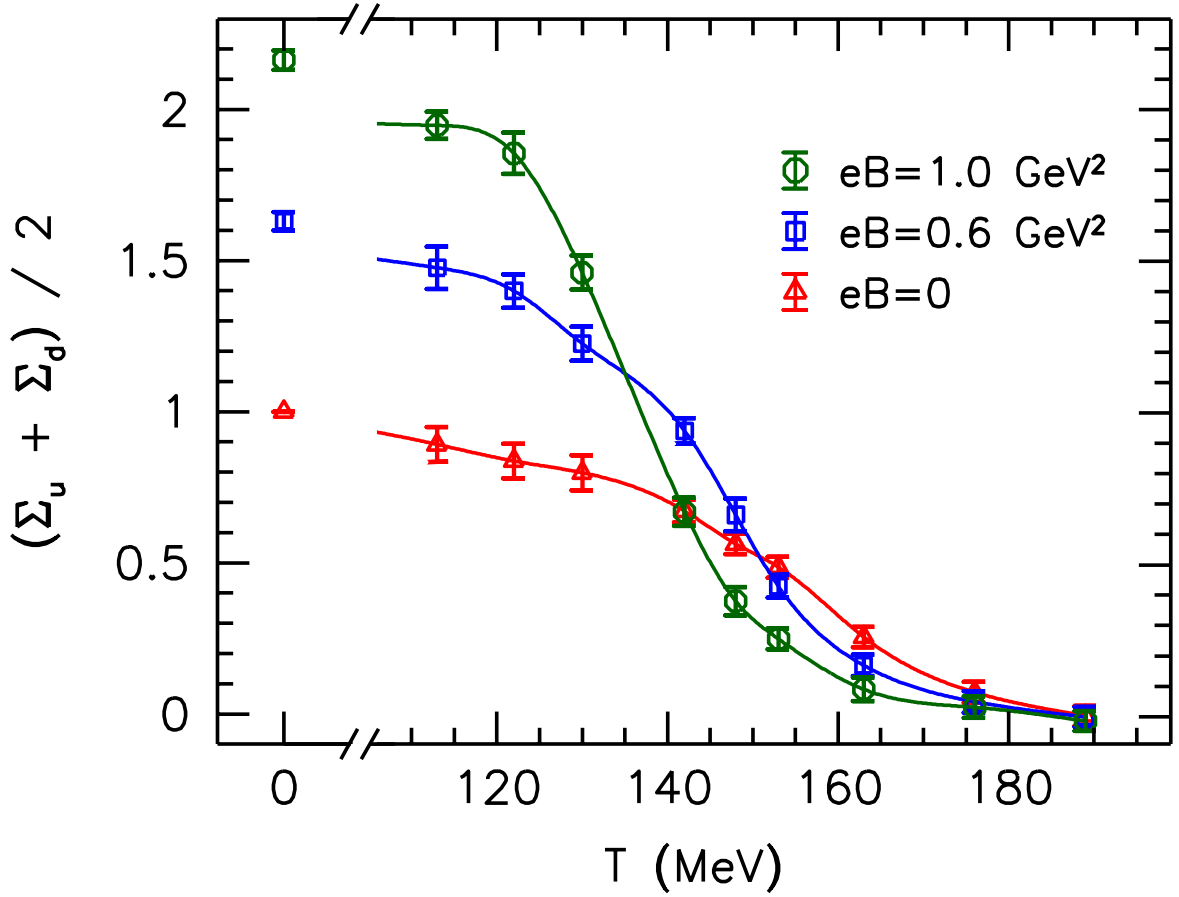


- result: $T_c(B)$ increases

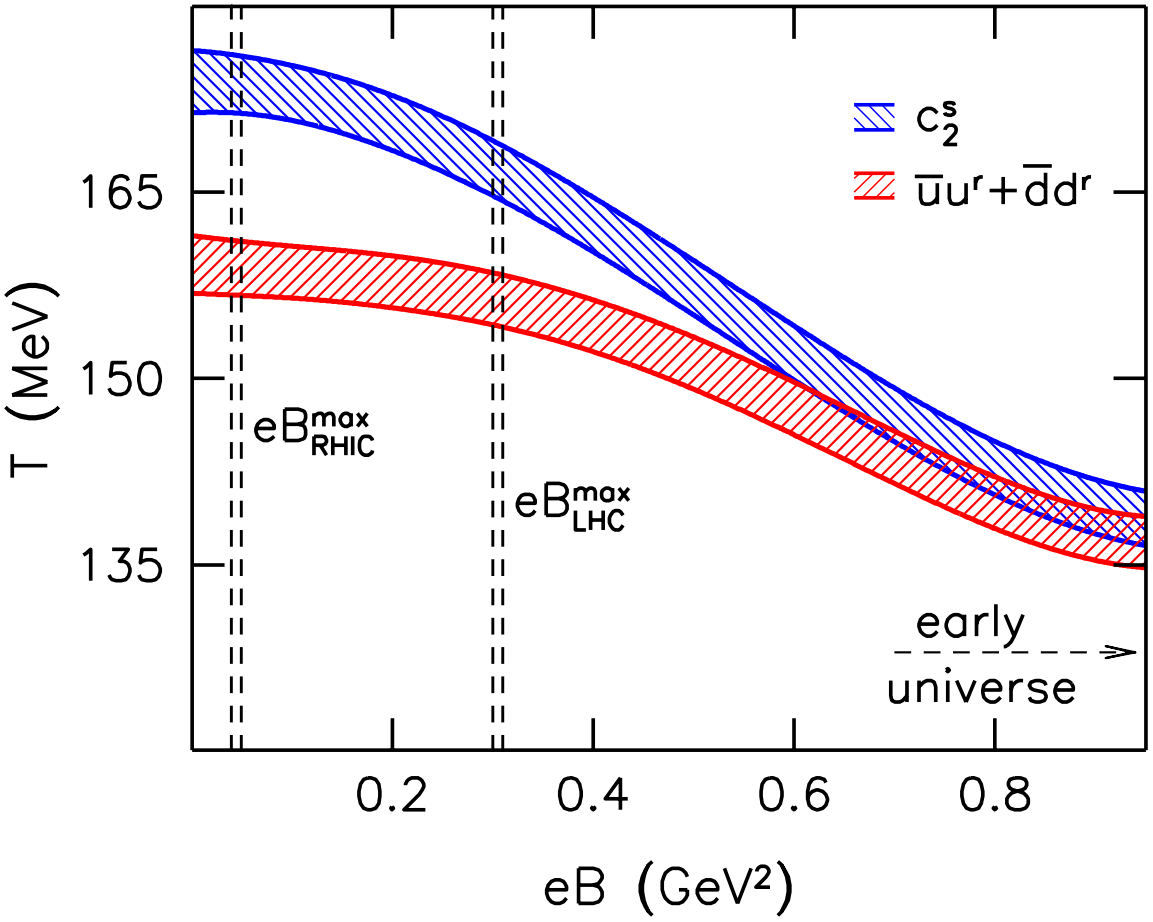
Quark condensate in 4D QCD at $T > 0$: lattice

- going to finite temperatures changes the response qualitatively dubbed 'inverse magnetic catalysis' around T_c
  [Bruckmann, GE, Kovács 1303.3972]

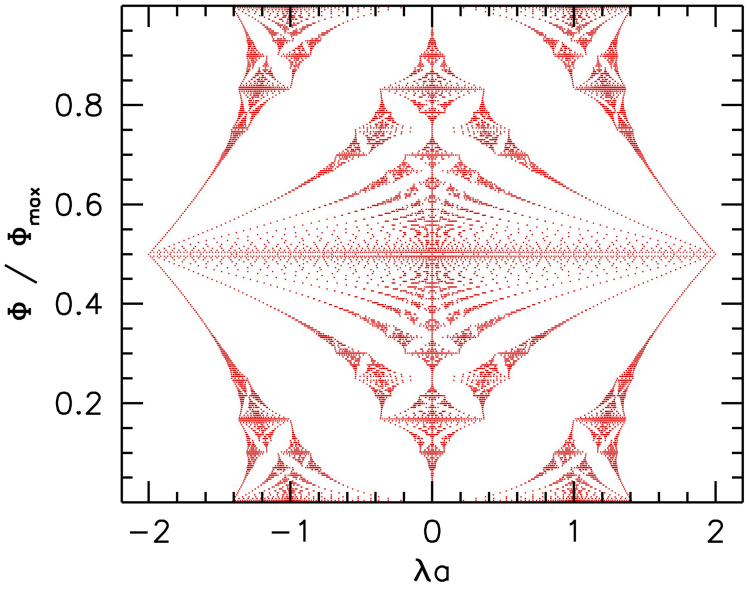


Phase diagram

- impact on the QCD phase diagram: $T_c(B)$ decreases
  [Bali, Bruckmann, GE, Fodor, Katz, Krieg, Schäfer, Szabó 1111.4956]

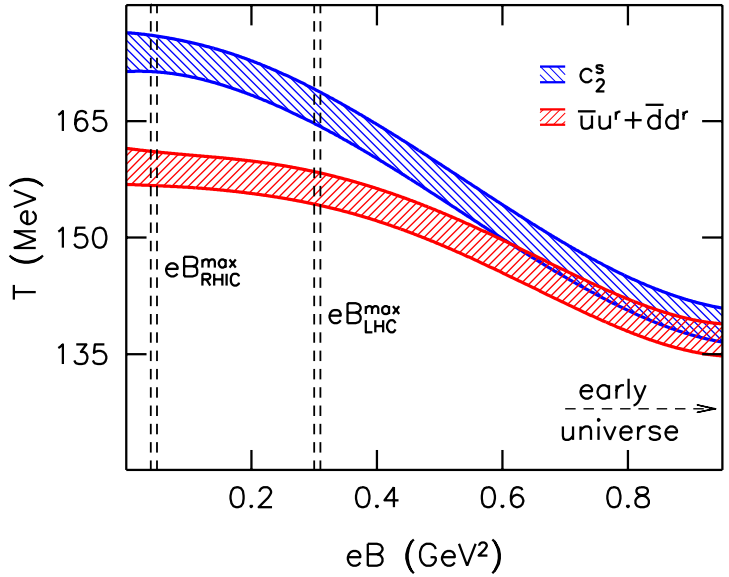


Summary I.



- three intertwined phenomena at $T = 0$
- wings of Hofstadter's butterfly at small B
  $\rightarrow$ solid state physics
- no asymptotic freedom in QED ($\beta_1 > 0$)
  $\rightarrow$ perturbative QFT
- magnetic catalysis of the QCD condensate
  $\rightarrow$ nonperturbative QFT, χSB

Summary II.

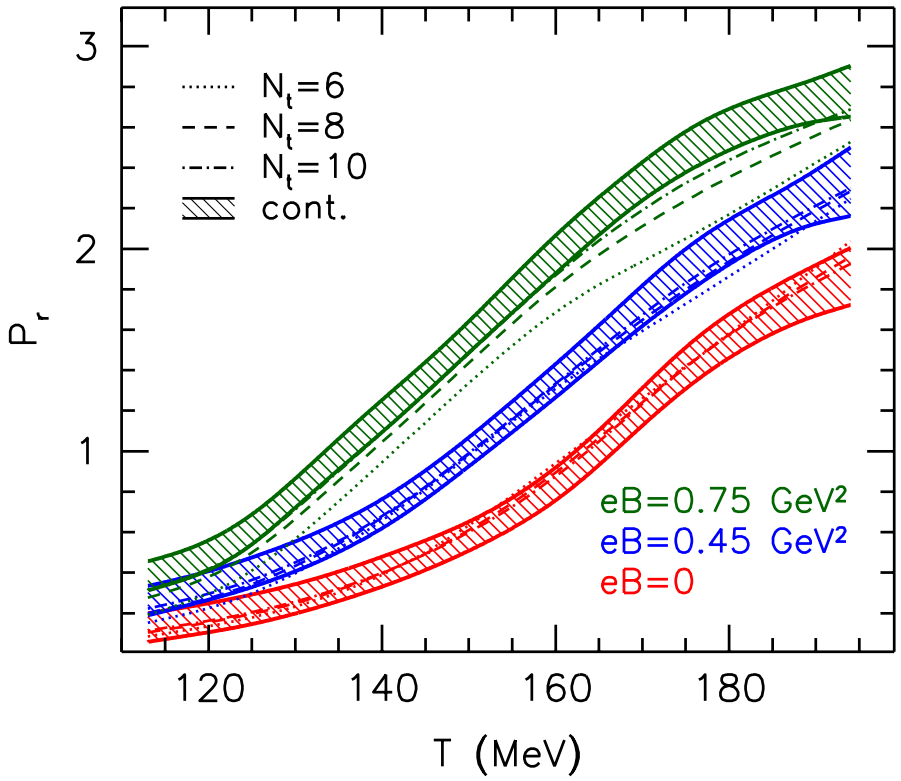


- inverse magnetic catalysis appears around $T \sim T_c$
- impact on phase diagram
- input to improve low-energy models
- relevant for phenomenology: non-central heavy-ion collisions and early universe

- more on magnetic fields: [D'Elia Sat 08:30]
  [Braguta Thu 14:35, Buividovich Fri 15:15, Kochetkov Fri 17:30, Larina poster]
  [Levkova Fri 18:10, Mariti Fri 17:50, Negro Wed 10:00, Valgushev Thu 16:15]

Inverse catalysis

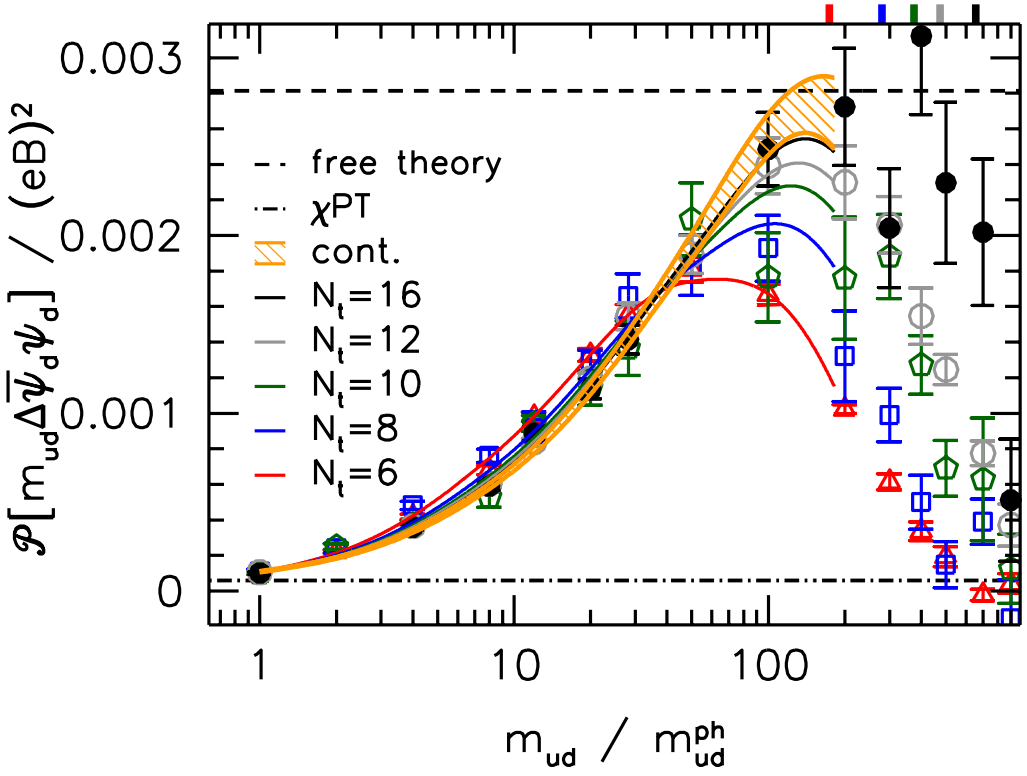
- B changes the typical gauge configurations
- most important dof: Polyakov loop



- Polyakov loop 'draws' condensate with itself
- effect only visible for light quarks [Bruckmann, GE, Kovács 1303.3972] (otherwise indirect effect of B on gluons is weak)

Magnetic catalysis – mass dependence

- $\bar{\psi}\psi = c \cdot (qB)^2$
- free quarks (interacting, $m \to \infty$): $c = N_c \cdot \beta_1^{\mathrm{spinor}}$
- free pions (interacting, $m \to 0$): $c = \beta_1^{\mathrm{scalar}}/4$



- intermediate masses: smooth dependence

[Bali, Bruckmann, GE, Katz, Schäfer 1406.0269]