The QCD Equation of State

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Introduction

EOS of quark-gluon plasma

- Hadronic matter deconfines above $T \sim 155$ MeV.
- Quarks and gluons are the relevant degrees of freedom.
- Theory still non-perturbative because of infrared sector.
- Equation of State needed for hydrodynamic description.
- Important for
  - heavy-ion collisions,
  - cooling of the early universe.
- Study EOS in the region 140 MeV to 400 MeV.
- Spans ‘transition’ region around 154(9) MeV.

Introduction
Current calculation

- Simulate 2+1 flavor QCD using HISQ/tree action.
- Vary $\beta = 10/g^2$ keeping $M_{s\bar{s}} \equiv 695$ MeV.
- β varied in the interval 5.9–7.825.
- $N_\tau =$ 6, 8, 10, 12; $N_\sigma/N_\tau = 4$.
- LCP: $m_l = m_s/20$ ($M_\pi \approx 160$ MeV).
- RHMC with mass preconditioning.
- Scale set by $r_1 = 0.3106(14)(8)(4)$ fm.
- We determine
  - $r_0 = 0.4688(41)$ fm cf. 0.48(1)(1) fm
  - $w_0 = 0.1749(14)$ fm cf. 0.1755(18)(4) fm.

Lattice details

Basic formulae

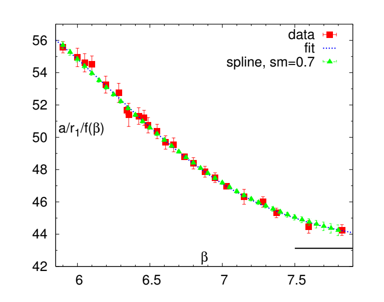
$$Z(\beta, N_\sigma, N_\tau) = \int \prod_{x,\mu} dU_{x,\mu} e^{-(\beta\, S_G(U) - S_F(U))}$$

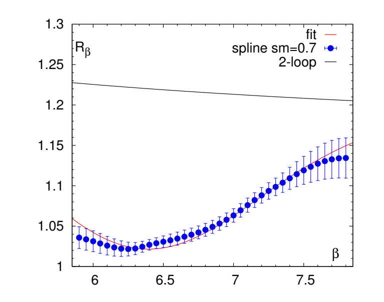
$$\begin{aligned}
T\frac{d}{dT}\left(\frac{p}{T^4}\right) &= \frac{\varepsilon - 3p}{T^4} = \frac{\Theta_G^{\mu\mu}(T)}{T^4} + \frac{\Theta_F^{\mu\mu}(T)}{T^4}\,, \\
\frac{\Theta_G^{\mu\mu}(T)}{T^4} &= R_\beta \left[\langle s_G\rangle_0 - \langle s_G\rangle_\tau\right] N_\tau^4\,, \\
\frac{\Theta_F^{\mu\mu}(T)}{T^4} &= -R_\beta R_m \left[2m_l \left(\langle\bar\psi\psi\rangle_{l,0} - \langle\bar\psi\psi\rangle_{l,\tau}\right)\right. \\
&\qquad \left. + m_s \left(\langle\bar\psi\psi\rangle_{s,0} - \langle\bar\psi\psi\rangle_{s,\tau}\right)\right] N_\tau^4\,.
\end{aligned}$$

Lattice details

Nonperturbative beta function

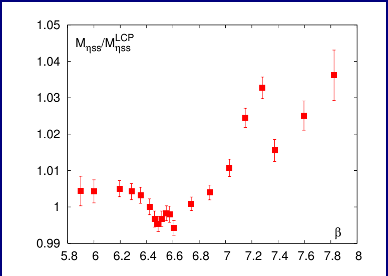
$$R_\beta(\beta) = \frac{r_1}{a} \left(\frac{\mathrm{d}(r_1/a)}{\mathrm{d}\beta} \right)^{-1}$$

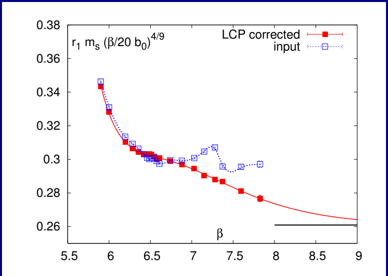




Lattice details
Quark mass along LCP

$$R_m(\beta) = \frac{1}{m_s(\beta)} \frac{\mathrm{d}m_s(\beta)}{\mathrm{d}\beta}$$

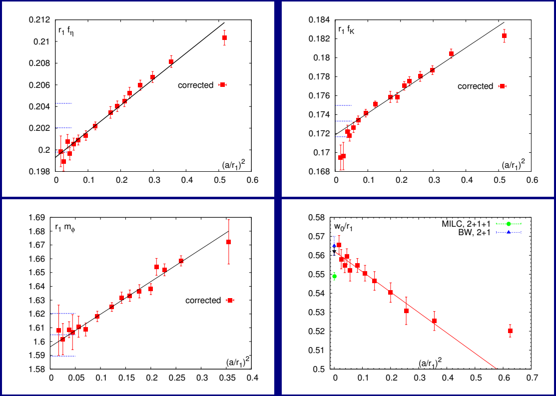




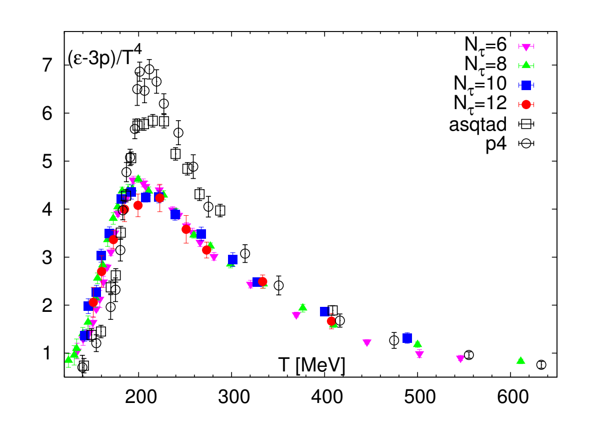
Lattice details

Scale setting

Scale set using
f_η, f_K, M_ϕ and w_0
agree
within errors.

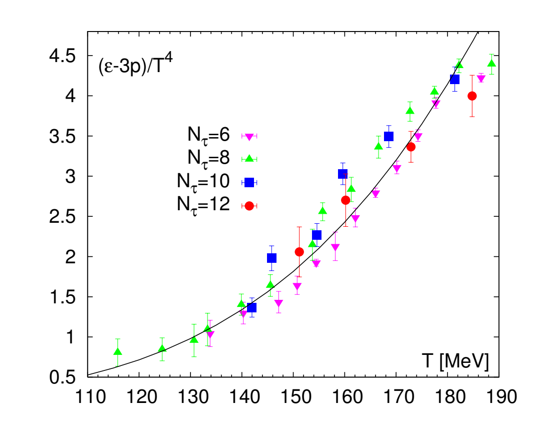


Lattice details
Trace anomaly



Lattice details

Comparison to HRG



Continuum Extrapolation

Strategy

Various fit strategies agree. Final result from a simultaneous fit

- interpolating in T,
- extrapolating in $1/N_\tau^2$.

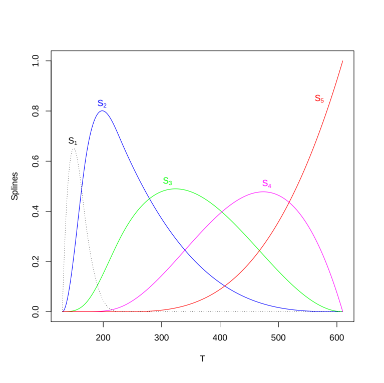
Use a basis of cubic splines:

- Smooth:
  Value, first and second derivatives continuous in T.
- Smooth extrapolation:
  Coefficient of each spline linear/quadratic function of $1/N_\tau^2$.
- Smooth matching to Hadron Resonance Gas:
  $T = 130$ MeV, $N_\tau = \infty \Rightarrow$ Value and T-derivative $=$ HRG.

Continuum Extrapolation
B-Splines

With n 'knots', the B-spline basis has $n+4$ splines.

HRG condition at 130 MeV reduces this to $n+2$ splines.



Implemented using statistical package R.

Continuum Extrapolation

Degrees of Freedom

This total number of free parameters is

$$\underbrace{n+2}_{\text{continuum T dep}} \quad + \quad \underbrace{n+4}_{\text{each power of } 1/N_\tau^2} \quad + \quad \underbrace{n}_{\text{knot postions}}$$

For each choice of n minimize uncorrelated χ^2.
Choose n by minimizing $\chi^2 + 2$d.o.f.
Errors in fit estimated by drawing bootstrap samples using

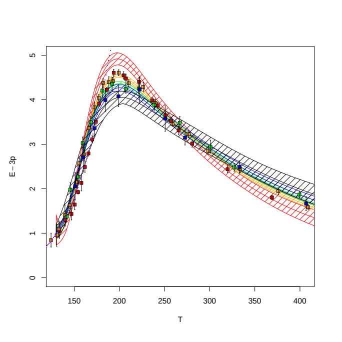
- estimated error on each measurement,
- a conservative 10% error on HRG value and slope.

Add 2% error on T (scale uncertainty) at the end.

Continuum Extrapolation
Fits

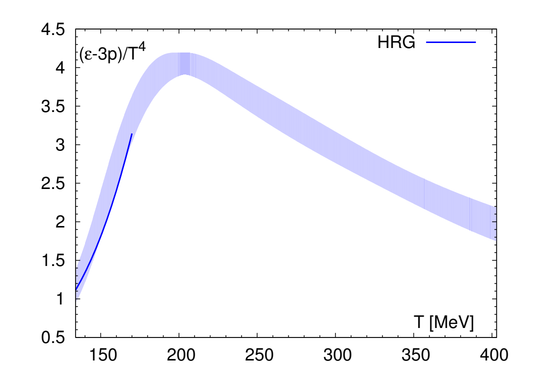
Final fit (Black band) using $N_\tau =$ 8, 10, 12 data and 2 knots.

Data at $N_\tau = 6$ not captured by lowest order in $1/N_\tau^2$.



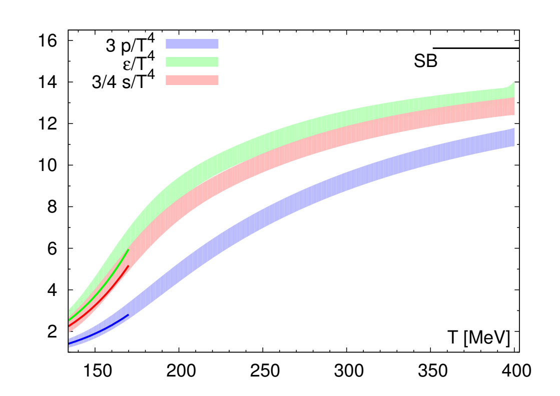
Results
Integration measure

HRG agrees with continuum extrapolation below ~ 150 MeV.



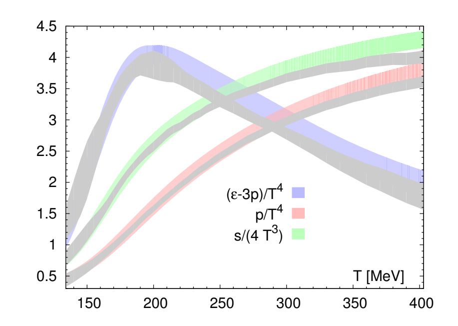
Results

Pressure, Energy, Entropy

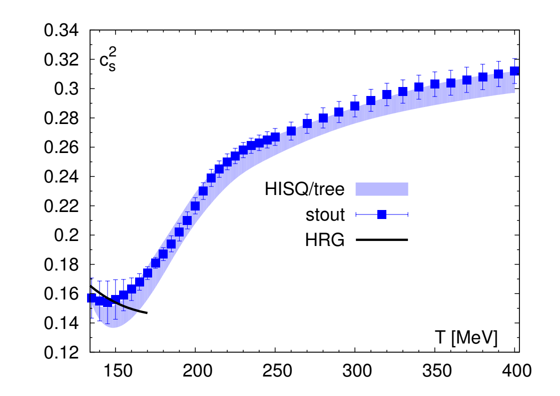


Results
Comparison to Stout (Wuppertal-Budapest) results

Good agreement with stout results (Grey bands).
Systematic difference at high temperatures.



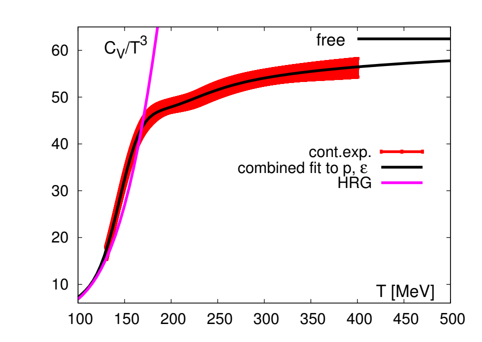
Results
Speed of Sound



Results

Specific Heat

Simplified parameterization of data for phenomenology.



Conclusions

Comparison with previous studies

- Good agreement with stout (Wuppertal-Budapest) results.
  - Possible systematic deviation above 350 MeV.
  - Probably unimportant for heavy ion phenomenology.
- Disagreement with previous asqtad and p4 understood
  - Large cutoff effects in p4 and asqtad.
  - Data at $N_\tau = 12$ and continuum extrapolation.