The QCD Equation of State

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Hadronic matter deconfines above $T \sim 155$ MeV.

Quarks and gluons are the relevant degrees of freedom.

Theory still non-perturbative because of infrared sector.

Equation of State needed for hydrodynamic description.

Important for

- heavy-ion collisions,
- cooling of the early universe.

Study EOS in the region 140 MeV to 400 MeV.

Spans ‘transition’ region around $154(9)$ MeV.
Simulate 2+1 flavor QCD using HISQ/tree action.

- Vary $\beta = 10/g^2$ keeping $M_{s\bar{s}} \equiv 695$ MeV.
- $\beta$ varied in the interval 5.9–7.825.
- $N_\tau = 6, 8, 10, 12$; $N_\sigma/N_\tau = 4$.
- LCP: $m_l = m_s/20$ ($M_\pi \approx 160$ MeV).
- RHMC with mass preconditioning.
- Scale set by $r_1 = 0.3106(14)(8)(4)$ fm.
- We determine
  - $r_0 = 0.4688(41)$ fm cf. 0.48(1)(1) fm
  - $w_0 = 0.1749(14)$ fm cf. 0.1755(18)(4) fm.
Lattice details

Basic formulae

$Z(\beta, N_\sigma, N_\tau) = \int \prod_{x,\mu} dU_{x,\mu} e^{-\left(\beta S_G(U) - S_F(U)\right)}$

$T \frac{d}{dT} \left( \frac{p}{T^4} \right) = \frac{\varepsilon - 3p}{T^4} = \frac{\Theta_{G}^{\mu\mu}(T)}{T^4} + \frac{\Theta_{F}^{\mu\mu}(T)}{T^4}$,

$\Theta_{G}^{\mu\mu}(T) = R_\beta [\langle s_G \rangle_0 - \langle s_G \rangle_\tau] N_\tau^4$,

$\Theta_{F}^{\mu\mu}(T) = -R_\beta R_m \left[ 2m_l \left( \langle \bar{\psi}\psi \rangle_{l,0} - \langle \bar{\psi}\psi \rangle_{l,\tau} \right) + m_s \left( \langle \bar{\psi}\psi \rangle_{s,0} - \langle \bar{\psi}\psi \rangle_{s,\tau} \right) \right] N_\tau^4$. 
Lattice details
Nonperturbative beta function

$$R_\beta(\beta) = \frac{r_1}{a} \left( \frac{d(r_1/a)}{d\beta} \right)^{-1}$$
Basic formulae
Nonperturbative beta function
Quark mass along LCP
Scale setting
Trace anomaly
Comparison to HRG

Lattice details
Quark mass along LCP

\[ R_m(\beta) = \frac{1}{m_s(\beta)} \frac{d m_s(\beta)}{d \beta} \]
Lattice details

Scale setting

Scale set using $f_\eta$, $f_K$, $M_\phi$ and $w_0$ agree within errors.
Lattice details

Trace anomaly

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Lattice details

Comparison to HRG

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Various fit strategies agree. Final result from a simultaneous fit
- interpolating in $T$,
- extrapolating in $1/N_T^2$.

Use a basis of cubic splines:
- Smooth:
  Value, first and second derivatives continuous in $T$.
- Smooth extrapolation:
  Coefficient of each spline linear/quadratic function of $1/N_T^2$.
- Smooth matching to Hadron Resonance Gas:
  $T = 130 \text{ MeV}, \ T - \text{derivative} = \text{HRG}$. 

$N_T = \infty \Rightarrow \text{Value and } T\text{-derivative} = \text{HRG}$. 

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Continuum Extrapolation

B-Splines

With $n$ ‘knots’, the B-spline basis has $n + 4$ splines.

HRG condition at 130 MeV reduces this to $n + 2$ splines.

Implemented using statistical package R.
Continuum Extrapolation

Degrees of Freedom

This total number of free parameters is

\[ n + 2 + n + 4 + n \]

continuum T dep each power of \( \frac{1}{N_T^2} \) knot positions

For each choice of \( n \) minimize uncorrelated \( \chi^2 \).
Choose \( n \) by minimizing \( \chi^2 + 2 \text{d.o.f.} \).
Errors in fit estimated by drawing bootstrap samples using
- estimated error on each measurement,
- a conservative 10% error on HRG value and slope.
Add 2% error on \( T \) (scale uncertainty) at the end.

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Continuum Extrapolation

Final fit (Black band) using $N_\tau = 8, 10, 12$ data and 2 knots.

Data at $N_\tau = 6$ not captured by lowest order in $1/N_\tau^2$. 
HRG agrees with continuum extrapolation below $\sim 150$ MeV.
Results
Pressure, Energy, Entropy

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Results
Comparison to Stout (Wuppertal-Budapest) results

Good agreement with stout results (Grey bands).
Systematic difference at high temperatures.
Results

Speed of Sound

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Simplified parameterization of data for phenomenology.

\[ \frac{C_V}{T^3} \quad \text{free} \]

\[ \text{cont.exp.} \]

\[ \text{combined fit to } p, \epsilon \]

HRG

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Conclusions

Comparison with previous studies

- Good agreement with stout (Wuppertal-Budapest) results.
  - Possible systematic deviation above 350 MeV.
  - Probably unimportant for heavy ion phenomenology.

- Disagreement with previous asqtad and p4 understood
  - Large cutoff effects in p4 and asqtad.
  - Data at $N_\tau = 12$ and continuum extrapolation.