Polyakov loop correlations at large N

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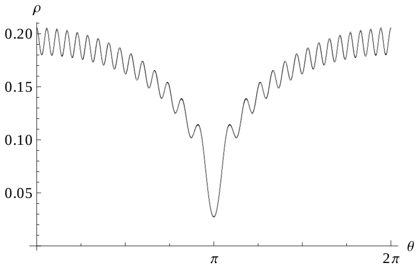
Large N transition in contractible Wilson loops.

- The single eigenvalue distribution of smeared Wilson loops undergoes a "compactification" transition on the unit circle at $N=\infty$.
- Below is an example at $N=29$ of a 6×6 smeared Wilson loop of size 0.6 Fermi



Problem: approximately calculate σ in units of Λ_{QCD} by matching EST (effective string theory) and PT (YM perturbation theory) at large N at the transition point.

EST

Previous work has led me to conclude that:

- Long distance is described by EST, but EST is limited for loops with kinks.
- EST is insensitive to N: it does not simplify as $N \to \infty$.
- To test PT-EST matching we need MC for smooth loops.
- On the lattice this leaves Polyakov as the single loop type option.
- Problem: find a large N transition associated with Polyakov loops. This is not the finite temperature transition – it should occur in the low temperature phase.

My talk addresses this problem, but does not solve it.

Setup

- $U_P(x) = \mathscr{P}\, e^{i \oint_{x_4}^{x_4} A_4(\vec{x},\tau) d\tau}$
- $P_R(\vec{x}) = \frac{1}{d_R} \chi_R(U_P(x))$
- $G_R(r) = \langle P_R(0) P_{\bar{R}}(r) \rangle$
- $W_R(l,r) = \log G_R(r)$, where l is length of compact direction.

Example – a universal EST prediction:

- Define: $\mathscr{F}_R(l) = \lim_{r\to\infty} \partial^2 W_R(l,r)/\partial l \partial r$
- Let $\mathscr{F}_R(l) = \sigma_n \hat{F}_R(l\sqrt{\sigma_n})$, where n is the N-ality.
- In this case EST has a lot to say: $\hat{F}(x) = 1 + c_1/x^2 + c_2/x^4 + c_3/x^6 + \ldots$, with $c_{1,2,3}$ universal calculable numbers, independent of n.

Generalities

- In general, some EST universal predictions exist for $W_R(\infty, r), W_R(l, \infty)$. These predictions do not depend on any microscopic details, get no benefit from making N large, and are oblivious to the representation content of the loop.
- In this respect EST is weaker than strong coupling. However, EST is Lorentz invariant.

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Eigenvalue-eigenvalue correlations in 2D YM

- For Polyakov loops need to consider at the least a two point function of single-eigenvalue densities $\rho^{(1)}(\theta;U)=\frac{2\pi}{N}\sum_{k=1}^{N}\delta_{2\pi}(\theta-\theta_k)$
- Use the same type of modelling as for Wilson loops: 2D YM.
- Need the "cylinder propagator" $Z_N(U_{P_1},U_{P_2}|t)=\sum_R \chi_R(U_{P_1})e^{-\frac{t}{2N}C_2(R)}\chi_{\overline{R}}(U_{P_2})$
- Calculate $\langle\rho_1^{(1)}(\alpha)\rho_2^{(1)}(\beta)\rangle_c=\int dU_{p_1}dU_{p_2}\rho_1^{(1)}(\alpha)\rho_2^{(1)}(\beta)[Z_N(U_{p_1},U_{p_2}|t)-1]$
- This can be done using the character expansion $\rho^{(1)}(\theta;U)=1+\frac{1}{2N}\lim_{\varepsilon\to 0^+}\sum_{p=0}^{N-1}\sum_{q=0}^{\infty}(-1)^p e^{-\varepsilon(p+q+1)}[e^{i(p+q+1)\theta}\chi_{(p,q)}(U)+e^{-i(p+q+1)\theta}\chi_{\overline{(p,q)}}(U)]$
- For simplicity, restrict to odd N.
- Using $C(p,q)=(p+q+1)(N-\frac{p+q+1}{N}+q-p)$, I obtained $\langle\rho_1^{(1)}(\alpha)\rho_2^{(1)}(\beta)\rangle_c=\frac{1}{N^2}\sum_{p=0}^{N-1}\sum_{q=0}^{\infty}(-1)^p e^{-\frac{t}{2N}C(p,q)}\cos[(p+q+1)(\alpha-\beta)]$
- Taking the large N limit gives: $N^2\langle\rho_1^{(1)}(\alpha)\rho_2^{(1)}(\beta)\rangle\sim\Re\sqrt{\frac{N}{t}}ue^{-\frac{t}{2}+\frac{t}{2N^2}}\int\frac{dx}{\sqrt{2\pi}}e^{-\frac{N}{2t}x^2+\frac{1}{2t}x^2}\frac{1+u^N e^{-N(x+\frac{t}{2})+\frac{t}{2}}}{1+ue^{-x-\frac{t}{2}+\frac{t}{2N}}}\frac{1}{1-ue^{-\frac{t}{2}}}$, where $u=\exp[i(\alpha-\beta)]$.

How well does the large N limit work?

Using a saddle point approximation, the result is given by the sum of a rapidly oscillating piece and a non-oscillating piece:

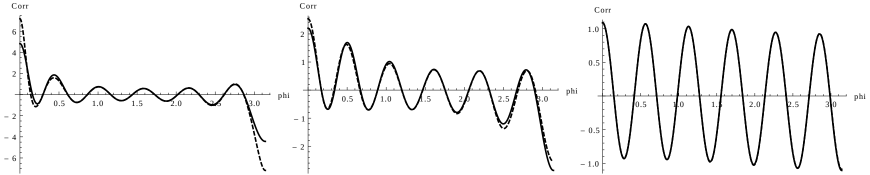
$$\frac{1}{2}\frac{\sinh\frac{t}{2}\cos\varphi + e^{\frac{t}{2}}\left(\sinh\frac{t}{2}\cos N\varphi - \sin\varphi\sin N\varphi\right)}{\sinh^2\frac{t}{2} + \sin^2\varphi},$$

where $\varphi = \alpha - \beta$. Observations:

- This smooth expression differs from the universal form for random hermitian matrix models, likely because of the absence of the potential term of the latter.
- **More important for us is that there is no large N transition separating regimes of small t and large t.**

The approximate large N formula is compared with the exact finite N formula below with the solid line showing the exact result. One sees that approximate large N expression deteriorates when N decreases, when $\varphi \approx k\pi, k \in Z$ and when t is small relative to 1.

$N = 11$, $t = 0.3, 1, 5$



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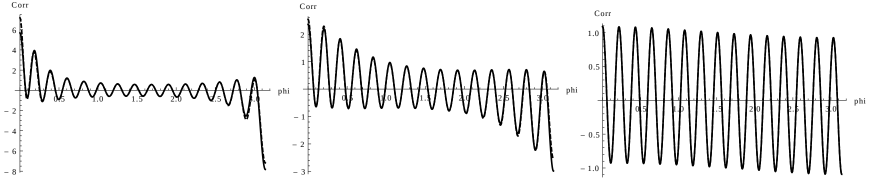

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General structure

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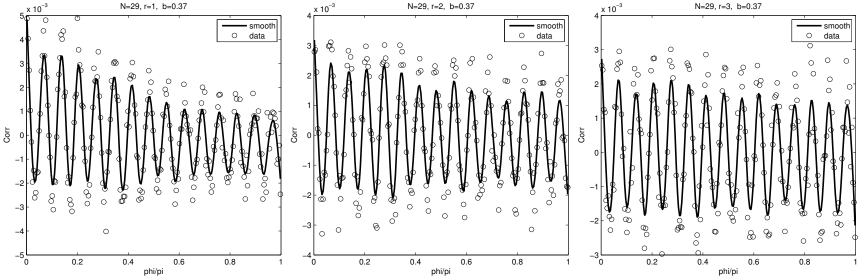
- For finite N, there is no reason for $\rho^{(2)}$ to depend only on the angle difference since the $Z(N)$ symmetry only provides invariance under simultaneous shifts of α and β by $2\pi k/N$.
- Initial simulations were done collecting two dimensional histograms in the α, β plane. Is was found that within practical numerical accuracy collapsing the histograms along constant $\alpha - \beta$ lines did not loose any information.
- This means that we may as well redefine $\rho^{(2)}$:

$$\rho^{(2)}(\alpha-\beta) = \frac{N}{2\pi}\int_{-\pi/N}^{\pi/N} d\theta \langle \rho_1^{(1)}(\alpha+\theta)\rho_2^{(1)}(\beta+\theta)\rangle_c$$

producing a $\rho^{(2)}$ depending only on the angle difference on account of the $Z(N)$ symmetry.

4D examples

In addition to raw data, I show a smoothed curve obtained by a cubic spline smoothing method. The method of smoothing consists of a minimization of a weighted combination of an average of the curve curvature and deviation from the data. The smoothing procedure is quite ad-hoc, and only serves to produce curves to guide the eye.



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Conclusions

- There is no large N phase transition for large enough Polyakov loops as their separation is varied.
- To get a large N transition one would have to shrink the compact direction, while maintaining the system in the now metastable confined phase. This may be possible using quenching techniques and would be of theoretical interest also in considerations of a possible fundamental QCD-string.
- One also needs to extend EST to include the presence of all $n \in Z(N)$-type strings and the associated group theoretical vertex.