Models of Walking Technicolor on the Lattice

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Introduction

We study models for the Higgs sector in which the Higgs is composite.

In particular, we study Technicolor models – QCD-like theories with massless fermions, where the Goldstone pion-like excitations play the role of the Higgs field, giving mass to the $W^{\pm}$ and Z.

Of particular interest are walking-Technicolor models, where there is a range of mass scales over which the running coupling evolves very slowly. Such models can avoid the phenomenological problems with naive Technicolor.

QCD with 2 colour-sextet quarks is a candidate walking-Technicolor model.

Need to distinguish whether this theory walks or is conformal.

Attractive because it has just the right number of Goldstone bosons (3) to give mass to the $W^{\pm}$ and Z.

Other groups are studying this model: Lattice Higgs Collaboration and DeGrand, Shamir & Svetitsky.

We study this theory at finite temperature to see if the coupling at the chiral transition evolves as predicted by asymptotic freedom for a finite-temperature transition.

QCD with 3 colour-sextet quarks, which is believed to be conformal, is studied for comparison.

We simulate these theories, latticized with unimproved staggered fermions, using the RHMC method.

Does QCD with 2 colour-sextet quarks have a light Higgs with standard-model properties? What other light particles are in its spectrum? Can any of its particles be dark-matter candidates? What about its S (,T and U) parameter(s)?

We have also considered $SU(2)$ Yang-Mills with 3 Majorana/Weyl fermions. However, it is unclear how to embed the weak gauge group into this theory to give masses to the Ws and Z.

QCD with 2 colour-sextet quarks at finite temperature

We simulate QCD with 2 color-sextet quarks at finite temperature by simulating on an $N_s^3 \times N_t$ lattice with $N_s >> N_t$. Since $T = 1/N_t a$, increasing N_t with T fixed decreases a. Assuming the chiral phase transition is a finite-temperature transition, yields a convenient T, T_χ. Measuring g or $\beta = 6/g^2$ at T_χ gives a running coupling at a sequence of as which approach zero as $N_t \rightarrow \infty$.

$$N_t = 12$$

Much of the past year has been devoted to increasing the statistics for our simulations on $24^3 \times 12$ lattices at quark masses $m = 0.0025$ and $m = 0.005$, close to the chiral transition.

For our largest mass $m = 0.01$ we have extended our simulations at low βs to determine the position of the deconfinement transition.

Because the β dependence of the chiral condensate is so smooth for the masses we use, we determine the position β_χ of this transition from the peaks in the (disconnected) chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = V\left[\langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2\right]$$

extrapolated to $m = 0$. V is the space-time volume.

For $m = 0.01$ in the range $5.7 \leq \beta \leq 5.9$, near the deconfinement transition, we run for 50,000 trajectories for each β with βs spaced by 0.02. In the range $6.6 \leq \beta \leq 6.9$, near the chiral transition we run for 25,000 trajectories per β with βs spaced by 0.02. Elsewhere in the range $5.7 \leq \beta \leq 7.2$ we run for 10,000 trajectories for βs spaced by 0.1.

For $m = 0.005$ in the range $6.6 < \beta \leq 6.9$, we run for 50,000 trajectories per β at βs spaced by 0.02. At $\beta = 6.6$ we run for 100,000 trajectories. Elsewhere in the range $6.4 \leq \beta \leq 7.2$, we run 10,000 trajectories per β for βs spaced by 0.1.

For $m = 0.0025$ in the range $6.7 \leq \beta \leq 6.9$, we will run for 100,000 trajectories per β with βs spaced by 0.02. In the range $6.6 \leq \beta < 6.7$, we will run for 50,000 trajectories per

β. These runs are nearing completion. Elsewhere in the range $6.5 \leq \beta \leq 7.2$ we run 10,000 trajectories per β at βs spaced by 0.1.

Figure 1 shows the chiral condensates measured in these simulations. Note that while these suggest that this condensate will vanish in the chiral limit for large enough β values, they do not allow a precise determination of β_χ.

Figure 2 shows the chiral susceptibilities from these runs. The peak of the $m = 0.0025$ susceptibility yields an estimate of β_χ, namely $\beta_\chi = 6.77(1)$.

Combining this with our $N_t = 8$ results yields:

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) = 0.08(2) \ ,$$

significantly smaller than the 2-loop perturbative prediction:

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.12 \ .$$

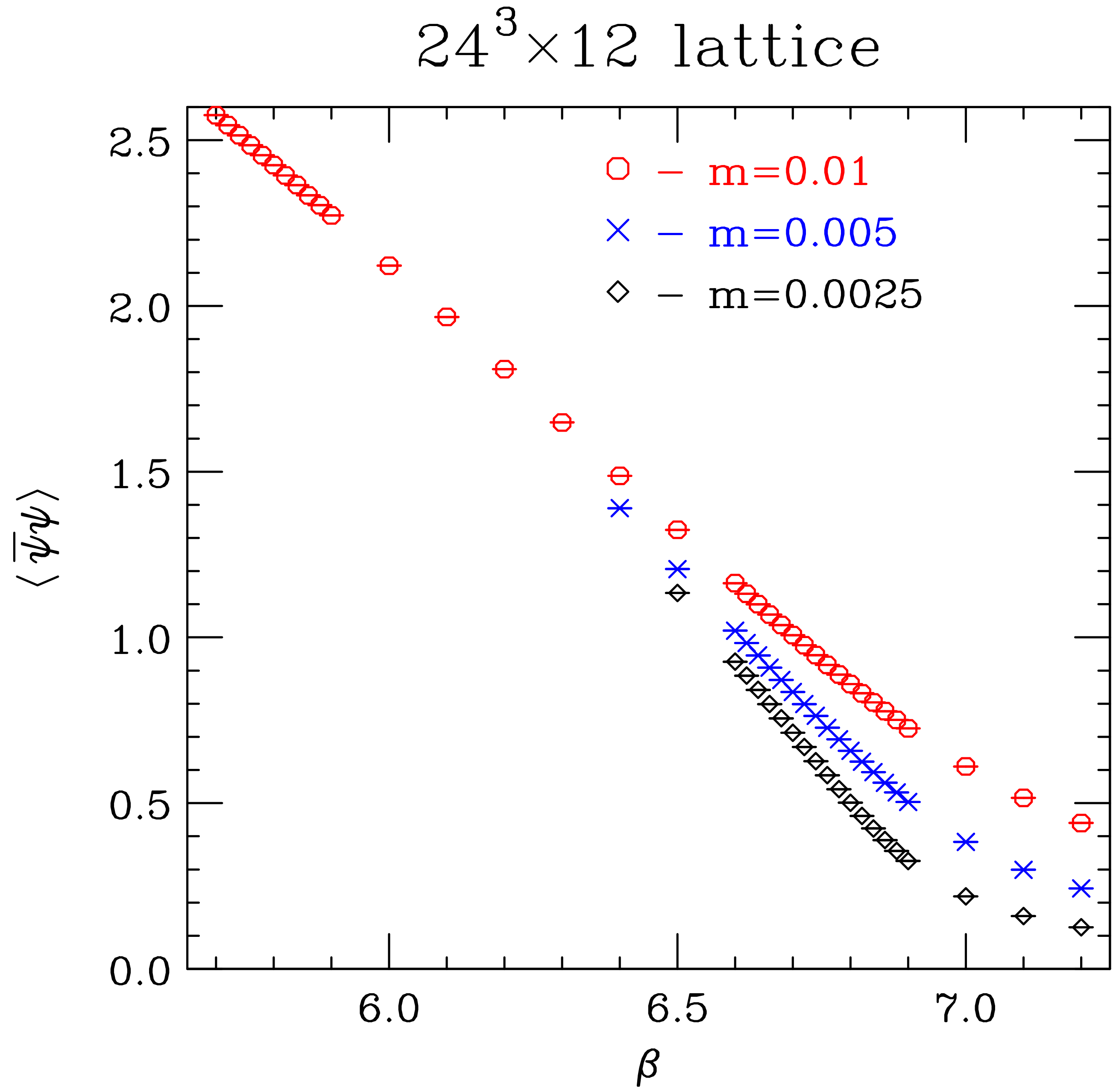

Figure 1: Chiral condensates on a $24^3 \times 12$ lattice.

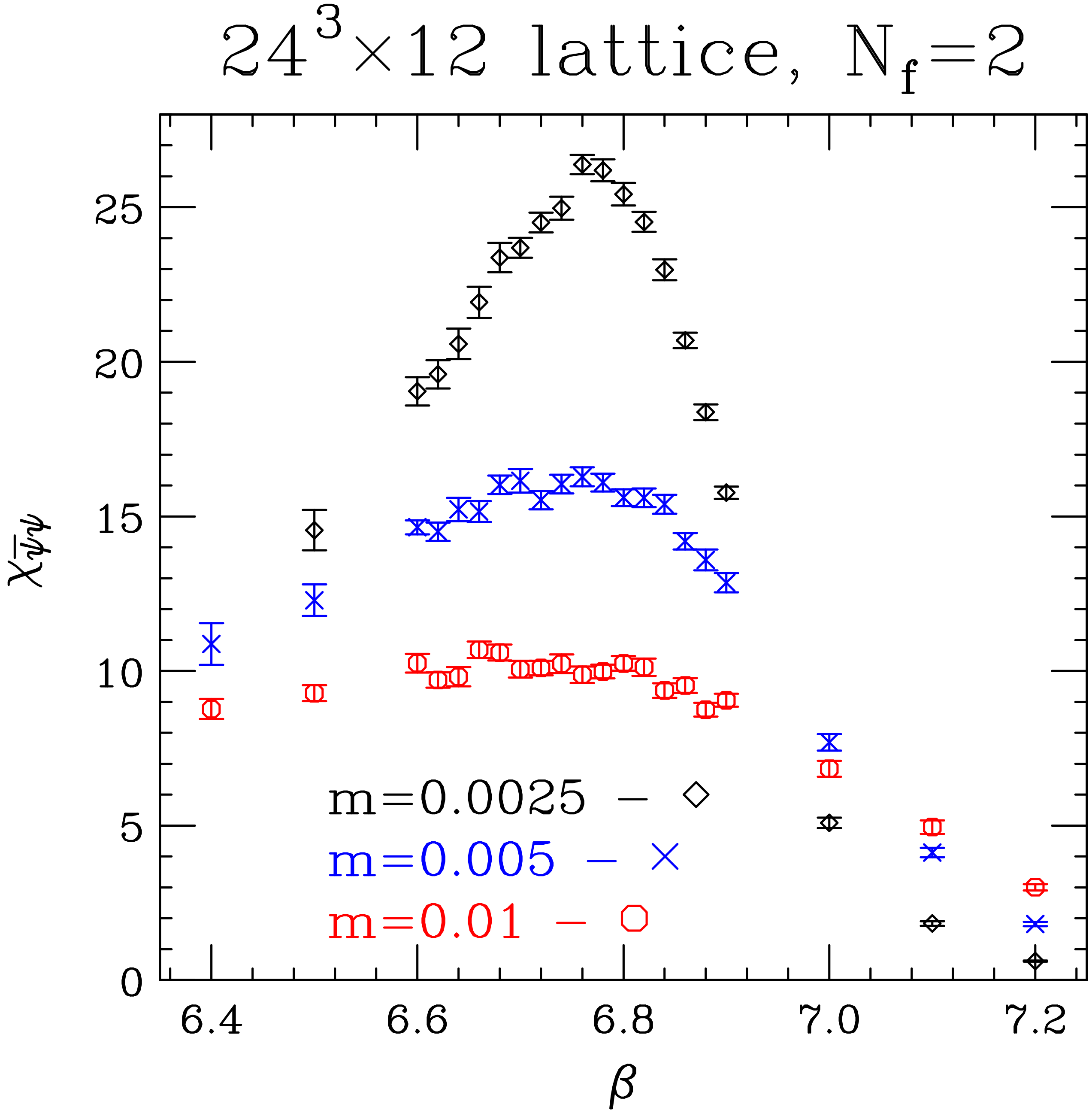

Figure 2: Chiral susceptibilities on a $24^3 \times 12$ lattice.

Figure 3 shows the Wilson Lines from these simulations.

Figure 4 shows histograms of the magnitudes of Wilson Lines for $m = 0.01$ near to the deconfinement transition. From this we deduce that $\beta_d = 5.81(1)$ for $m = 0.01$. This should be close to the value for $m = 0$.

N_t	β_d	β_χ
4	5.40(1)	6.3(1)
6	5.54(1)	6.60(2)
8	5.65(1)	6.69(1)
12	5.81(1)	6.77(1)

Table 1: $N_f = 2$ deconfinement and chiral transitions for $N_t = 4, 6, 8, 12$.

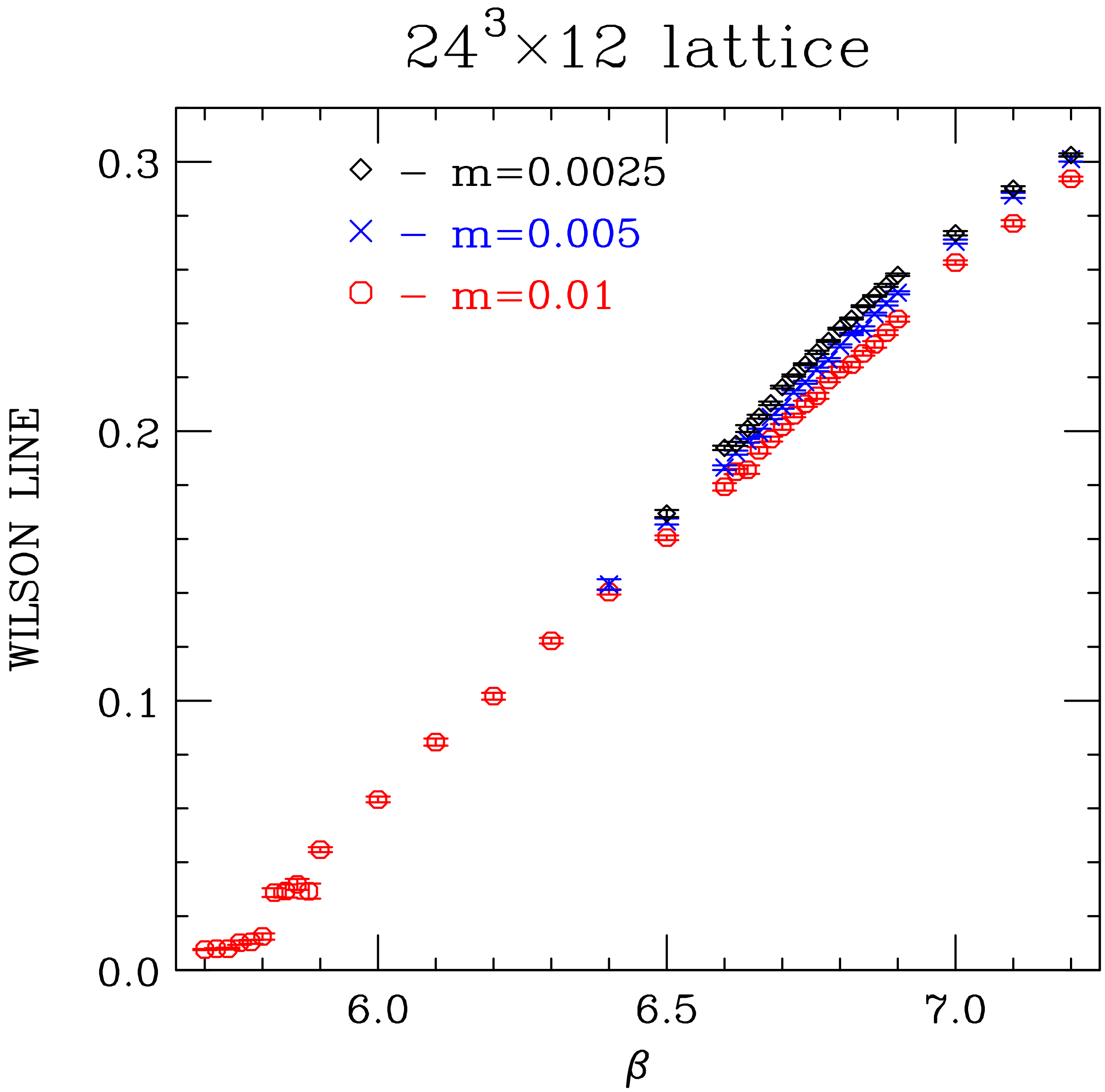

Figure 3: Wilson Lines (Polyakov Loops) on a $24^3 \times 12$ lattice: States with real Wilson Lines only.

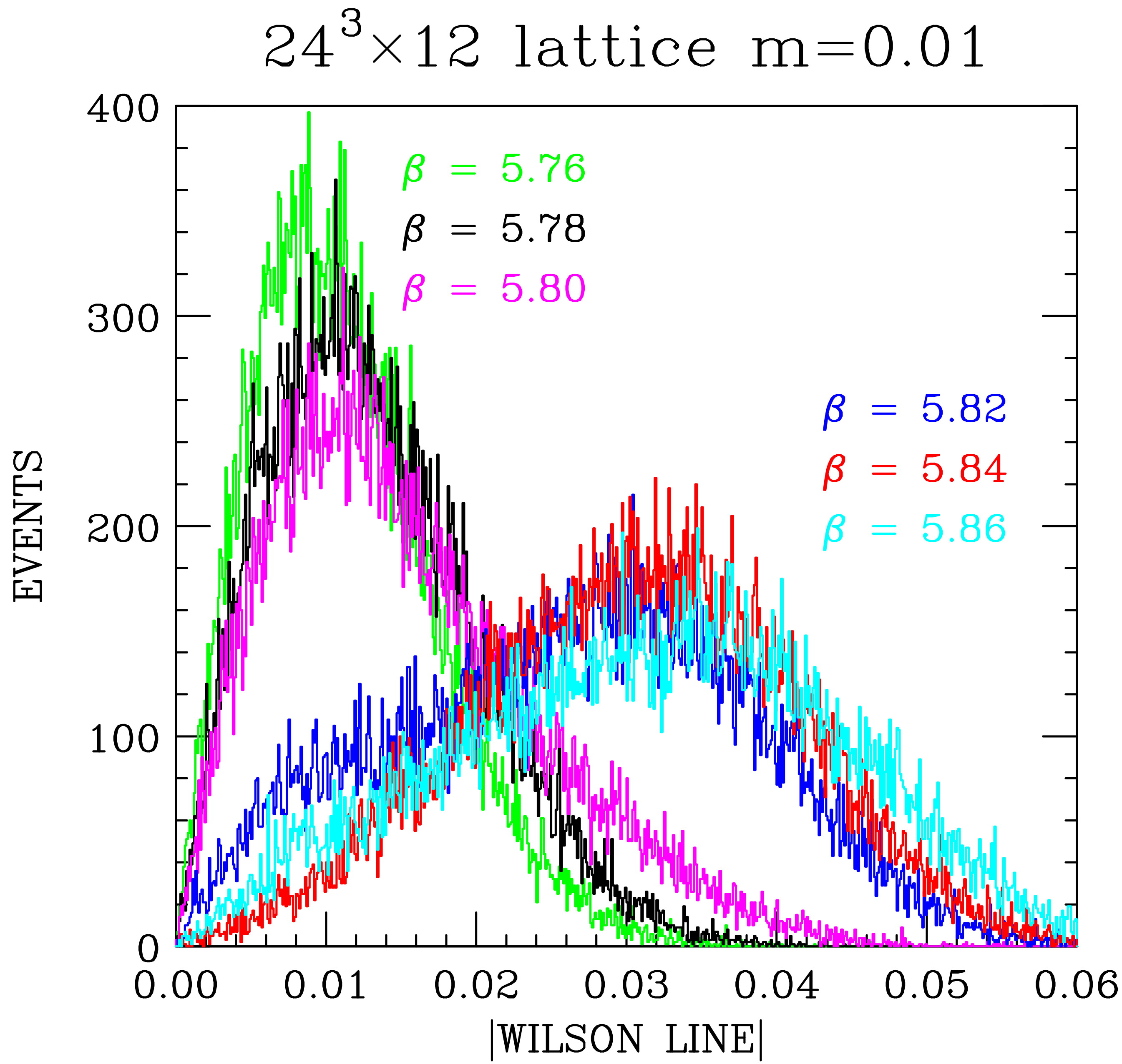

Figure 4: Histograms of magnitudes of Wilson Lines for βs close to the deconfinement transition for $m = 0.01$.

QCD with 2 colour-sextet quarks at zero temperature

Planned simulations and measurements

Start with simulations on a $36^3 \times 72$ lattice at $\beta = 6.1$(?), at several ms. [Deconfinement transition for $\beta_d(N_t = 36) \sim 6.25$–$6.4$.]

Unfortunately, $\beta = 6.1$ is still too small to access the continuum limit. However, we hope that we will be able to get results which are qualitatively correct.

Repeat on $48^3 \times 96$ lattice.

Measure f_π and spectrum of local mesons (connected).

Measure non-local pion spectrum. How large is taste breaking?

Measure glueball spectrum. Are glueballs light?

Measure meson spectrum including disconnected terms. Is there a light η/η'?

Is there a light 0^{++} state with sufficient fermion content to be the Higgs? Is its mass $\approx \frac{1}{2} f_\pi$?

Does this Higgs have the right couplings to $W^{\pm}$, Z and γ?

Measure S parameter contributions.

Determine scaling behaviour of the chiral condensate to extract γ_m.

Determine mass dependence of meson (and glueball) masses, and scaling behaviour.

Measure the $Q\overline{Q}$ potential.

Examine the β dependence of the masses.

QCD with 3 colour-sextet quarks at finite temperature

We simulate lattice QCD with 3 colour-sextet quarks at finite temperature for comparison with the 2-flavour case.

This theory is believed to be conformal with an infrared fixed point.

The chiral transition should be a bulk transition fixed at a finite constant β_χ for N_t sufficiently large.

We have simulated this theory at $N_t = 4$, 6 and 8, and are now starting $N_t = 12$ simulations.

For $N_t = 6$ we simulate on a $12^3 \times 6$ lattice at $m = 0.02$, $m = 0.01$ and $m = 0.005$.

Close to the chiral transition, $6.2 \leq \beta \leq 6.4$ at the lowest quark mass ($m = 0.005$) we simulate at βs separated by 0.02, with 100,000 trajectories per β.

We estimate the position of the chiral transition as the peak in the chiral susceptibility for $m = 0.005$.

For $N_t = 8$ we simulate on a $16^3 \times 8$ lattice at $m = 0.01$ and

$m = 0.005$.

Close to the chiral transition, $6.28 \leq \beta \leq 6.5$ at the lowest mass ($m = 0.005$) we simulate at βs separated by 0.02. with 100,000 trajectories per β.

We estimate the position of the chiral transition as the peak in the chiral susceptibility for $m = 0.005$.

The results for the positions of the chiral and deconfinement transitions for $N_t = 4$, 6 and 8 are given in table 2.

N_t	β_d	β_χ
4	$5.275(10)$	$6.0(1)$
6	$5.375(10)$	$6.278(2)$
8	$5.45(10)$	$6.37(1)$

Table 2: $N_f = 3$ deconfinement and chiral transitions for $N_t = 4, 6, 8$. In each case we have attempted an extrapolation to the chiral limit.

Since

$$\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) = 0.09(1)$$

we have yet to see evidence of a bulk transition.

We are therefore starting $N_t = 12$ simulations on a $24^3 \times 12$ lattice.

Figure 5 shows the $m = 0.005$ chiral susceptibilities for $N_t = 6$, $N_t = 8$ and preliminary results for $N_t = 12$.

Figure 6 shows the chiral condensates, both unsubtracted and subtracted for our $16^3 \times 8$ simulations. The subtracted condensates use the definition of the Lattice Higgs Collaboration:

$$\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle - \left(m_V \frac{\partial}{\partial m_V}\langle\bar{\psi}\psi\rangle\right)_{m_V=m} .$$

Note, although it is clearer that the subtracted condensate will vanish in the continuum limit for β sufficiently large than is the case for the unsubtracted condensate, it still does not yield an accurate estimate of β_χ.

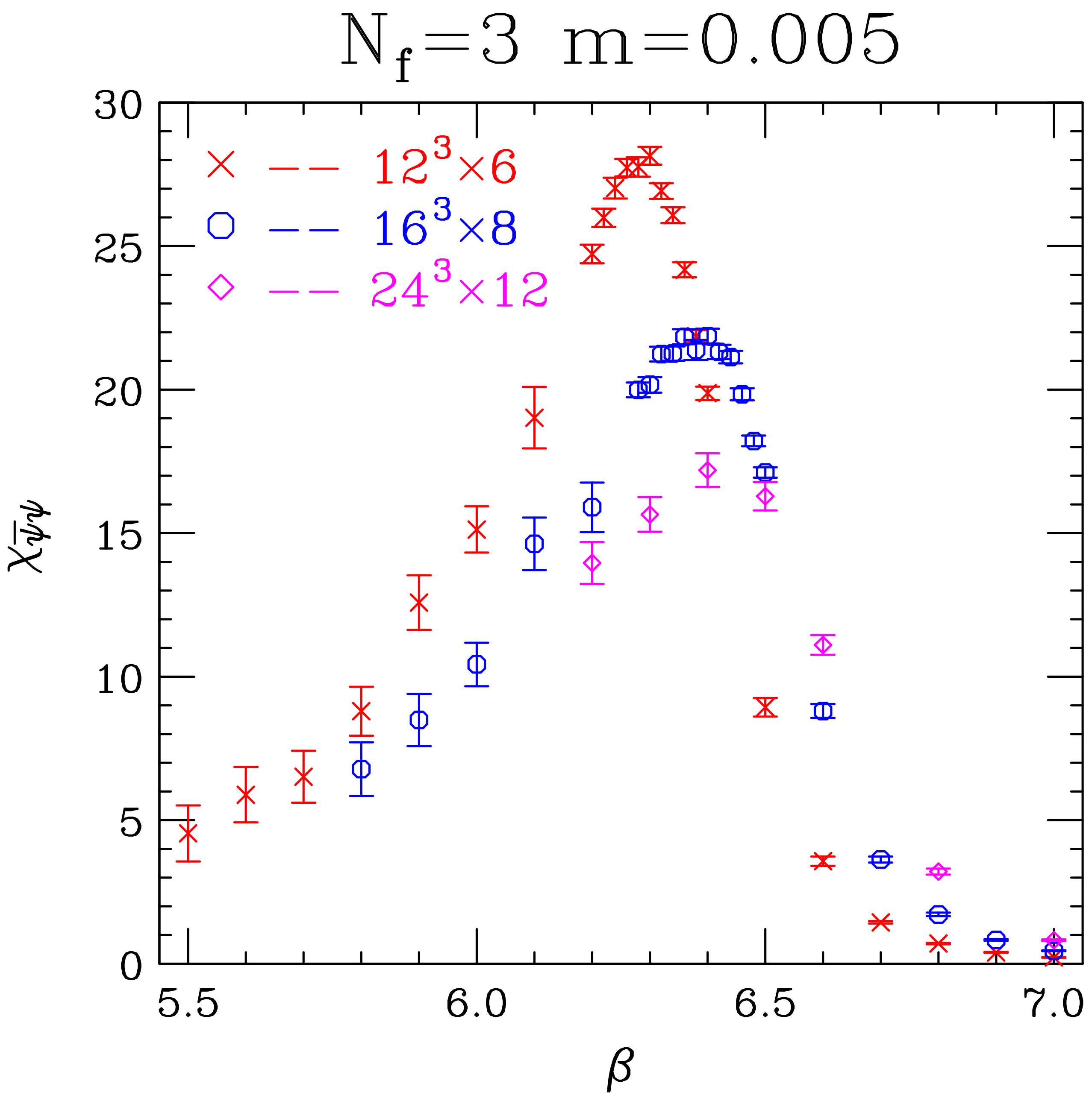

Figure 5: Chiral susceptibilities for $N_f = 3$, $m = 0.005$ on $12^3 \times 6$, $16^3 \times 8$ and $24^3 \times 12$ lattices.

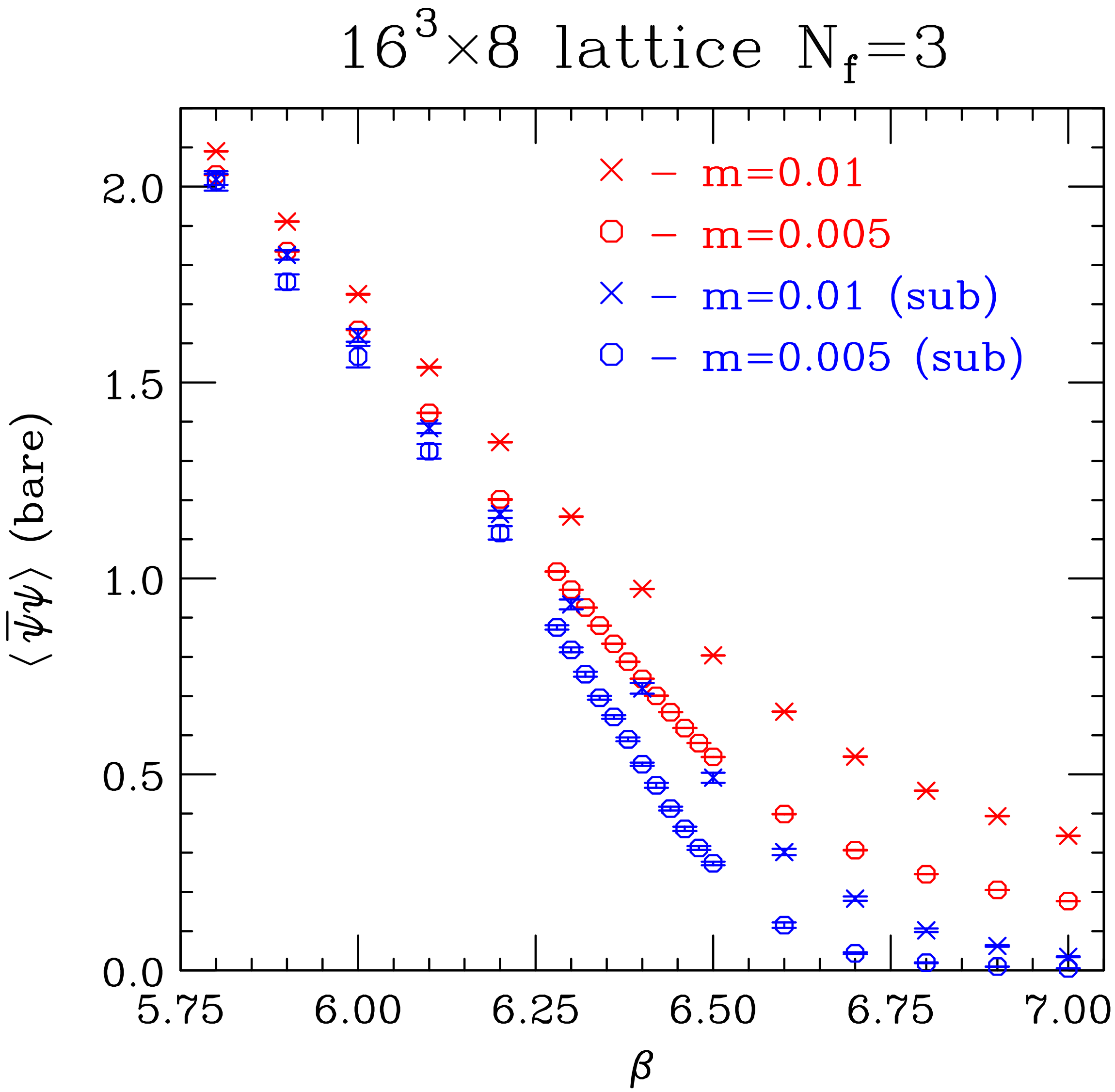

Figure 6: Chiral condensates on a $16^3 \times 8$ lattice for $m = 0.005$ and $m = 0.01$. The red graphs are unsubtracted, lattice regulated condensates. The blue graphs have been subtracted using the method of the Lattice Higgs Collaboration.

QCD_2 with 3 Majorana/Weyl colour-adjoint quarks

The symmetries of this theory are easiest to see in terms of 2-component (Weyl) fermions.

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\psi^\dagger i\sigma^\mu \overleftrightarrow{D}_\mu \psi + \frac{m}{2}\left[\psi^T i\sigma_2\psi - \psi^\dagger i\sigma_2\psi^*\right]$$

where ψ is a 3-vector in colour_2 space and in flavour space.

If $m = 0$, the chiral flavour symmetry is $SU(3)$.

The Majorana mass term reduces this flavour symmetry to the real elements of $SU(3)$, i.e. to $SO(3)$.

Thus when $m = 0$ and the chiral symmetry breaks spontaneously, the chiral condensate is $\langle\psi^T i\sigma_2\psi - \psi^\dagger i\sigma_2\psi^*\rangle$. and the spontaneous symmetry breaking pattern is

$$SU(3) \rightarrow SO(3)$$

The unbroken generators of $SU(3)$ are the 3 imaginary generators. These form a spin-1 representation under the unbroken $SO(3)$.

The 5 broken generators are the 5 real generators. They, as well as the 5 corresponding Goldstone bosons, form a spin-2 representation of $SO(3)$.

The problem occurs when one tries to embed the weak $SU(2) \times U(1)$ group in such a way as to give masses to $W^{\pm}$ and Z.

This is easiest to see if we consider the case where the Weinberg angle is zero. Then we need to embed $SU(2)$ in such a way that all 3 components are broken spontaneously. Thus we would need to make a set of $SU(2)$ generators from the 5 real $SU(3)$ generators. However, the $SU(2)$ algebra requires that at least one of its generators is complex, so this is impossible.

The only Weinberg angle which would work is $\pi/2$ where the photon is pure $SU(2)$ and the Z is pure $U(1)$.

Discussion and Conclusions

- We simulate lattice QCD with 2 colour-sextet quarks at finite temperature to distinguish whether it is QCD-like and walks, or if it is a conformal field theory.
- We run on lattices with $N_t = 4, 6, 8, 12$. β_χ increases by $0.08(2)$ between $N_t = 8$ and $N_t = 12$. While this increase favours the walking scenario, this increase is significantly smaller than the 2-loop prediction of ≈ 0.12.
- Is this because 2-loop perturbation theory is inadequate for this lattice action and β? Are there sizable finite volume corrections? Will the theory finally prove to be conformal?
- If walking, this theory is a promising walking-technicolor theory. We have outlined a program for checking its zero-temperature properties. Does it have a light Higgs? Does it satisfy the precision electroweak constraints? Does it have a Dark Matter candidate? What about its particle spectrum?..........
- We simulate QCD with 3 colour-sextet quarks which should be

conformal. The increase in β_χ between $N_t = 6$ and $N_t = 8$ is still appreciable ($0.09(1)$), so we don't yet have evidence for β_χ approaching a finite constant as $N_t \to \infty$.

- We are now simulating at $N_t = 12$. This shows some promise.
- QCD_2 with 3 Majorana/Weyl quarks does not appear to be a Technicolor candidate.

These simulations were performed on Hopper, Edison and Carver at NERSC, Kraken at NICS, Stampede at TACC, and Fusion and Blues at LCRC, Argonne.