Deconfinement transition
as a black hole formation
by the condensation of QCD string

Masanori Hanada
花田 政範
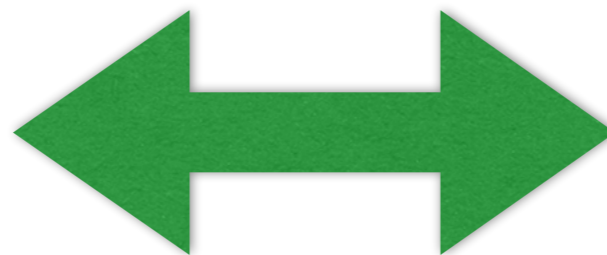Hana　Da　　Masa　Nori

Kyoto U. & Stanford U.

(I skip work in progress about real-time evolution because
it turned out to be impossible to explain everything in 20 minutes)

Maldacena's conjecture: deconfining phase = black hole
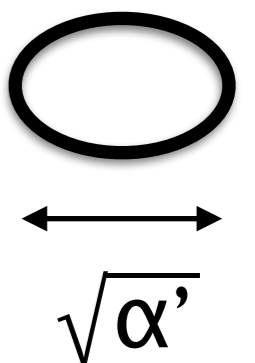
SYM   STRING

$1/\lambda$ α'/R_{BH}^2

$g_{YM}^2 \sim 1/N$ g_s

$\lambda = \infty, N = \infty$ corresponds to supergravity.

assumed to be correct without proof,
and applied to QGP

Is it correct?

Is it correct only at large-N, strong coupling?

(supergravity, or Einstein gravity)

Or correct including $1/\lambda$ and $1/N$ corrections?
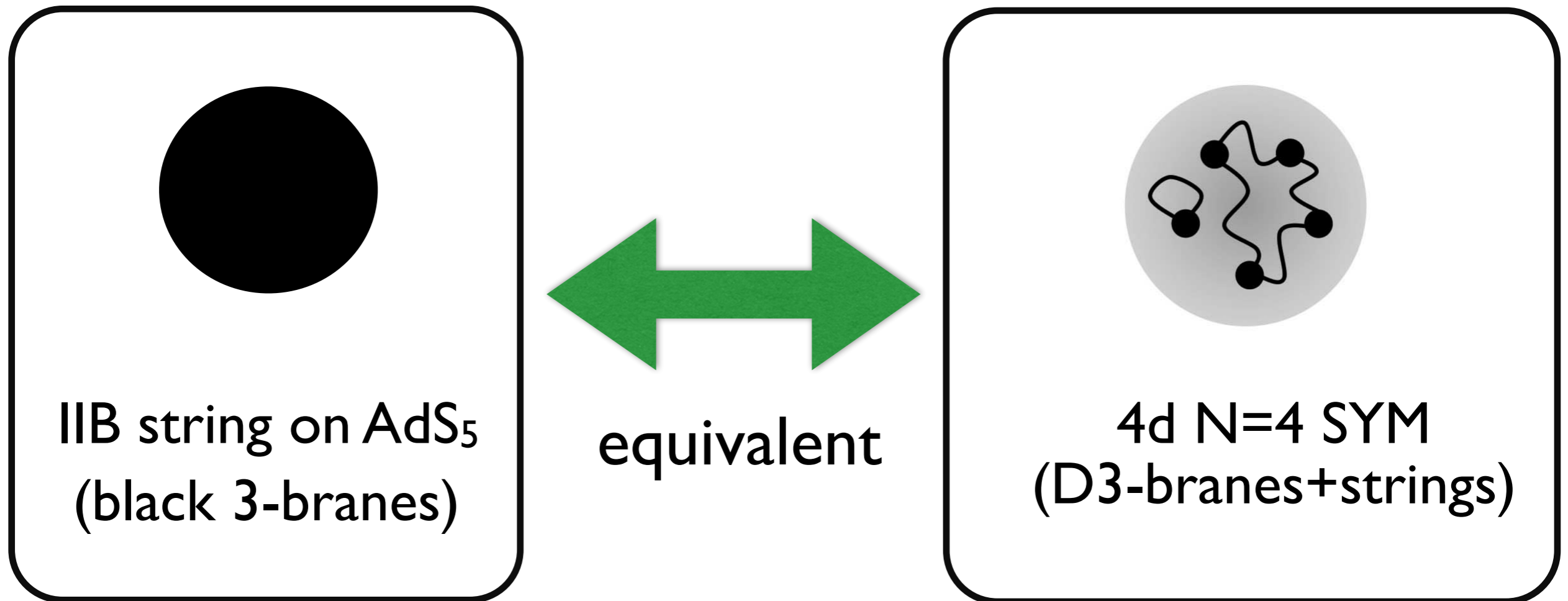
(superstring theory)

If correct, why? Can we understand it intuitively?

I want to answer to these questions, because

(1) I want to understand quantum gravity.
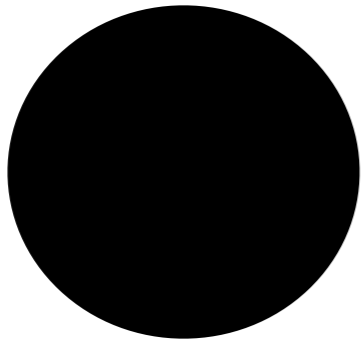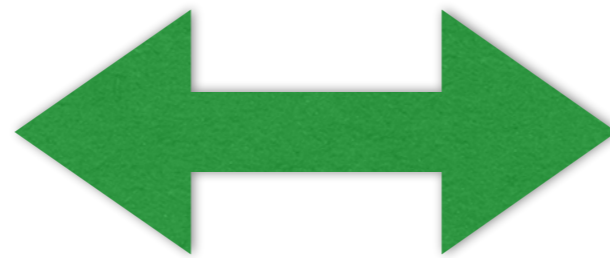
(2) I want to understand QGP.

AdS/CFT correspondence

IIB string on AdS$_5$
(black 3-branes)

equivalent

4d N=4 SYM
(D3-branes+strings)

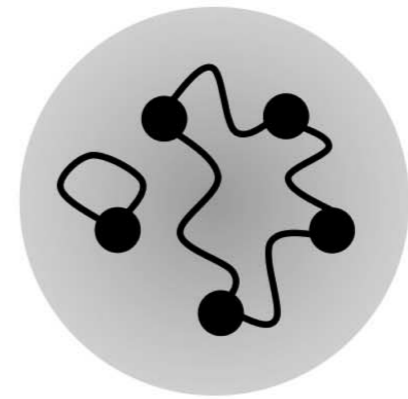(Maldacena1997)

Black hole = bunch of D0-branes

(+ strings between them)

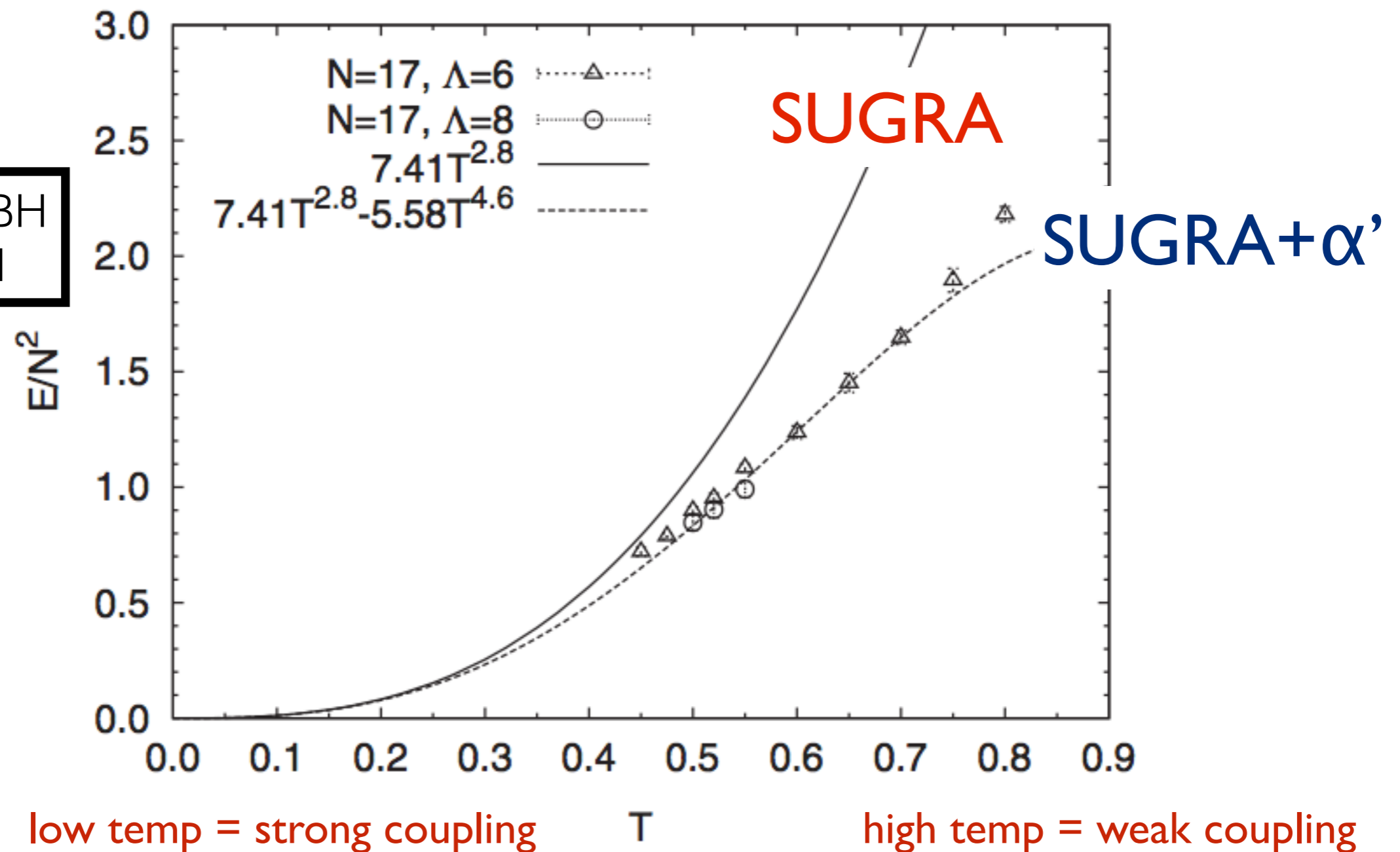IIA string around black 0-brane (near horizon) ⟷ equivalent ⟷ (0+1)-d maximal SYM (D0-branes+strings)

(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

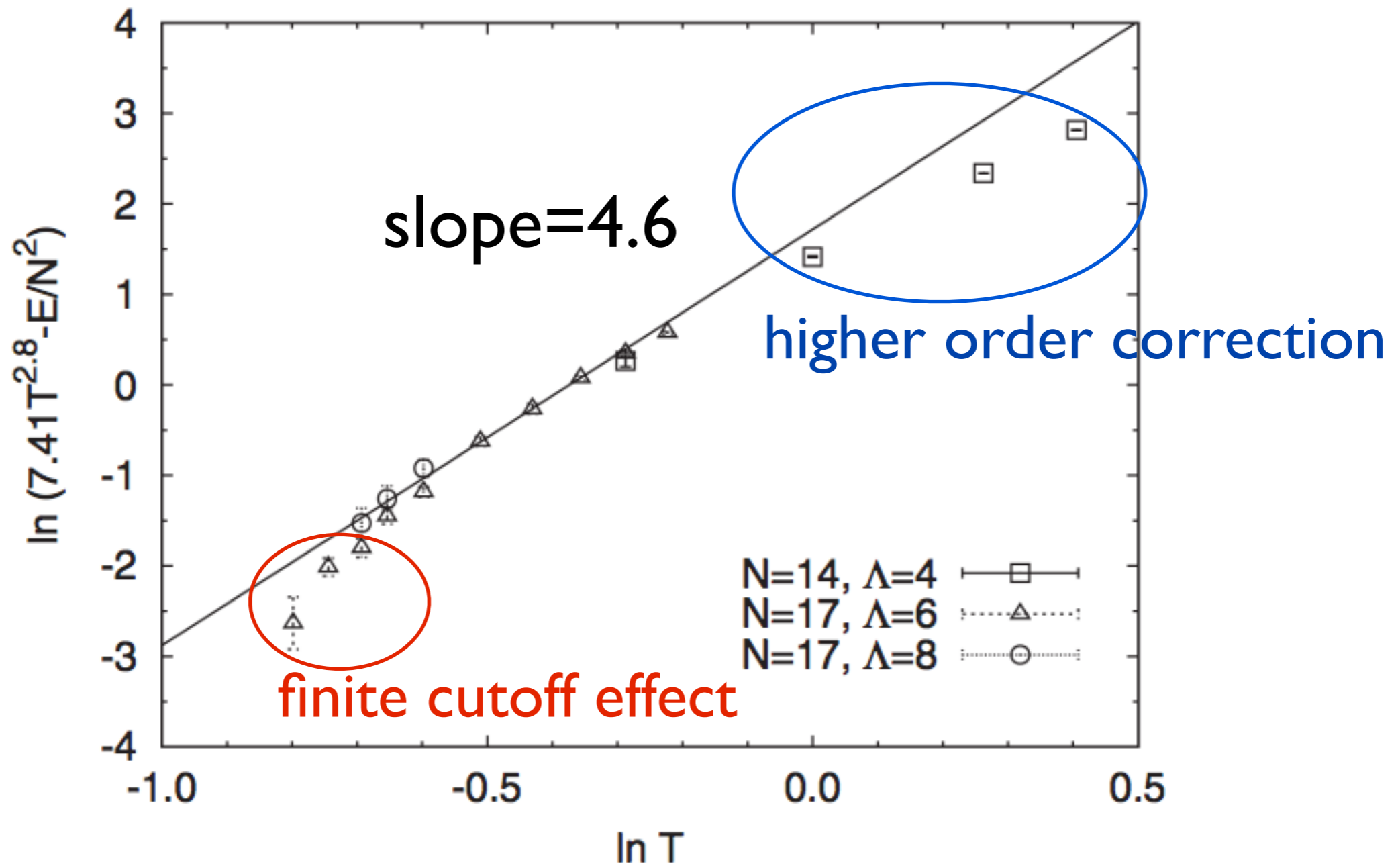Quantitative test is possible by studying SYM numerically.

low temp = strong coupling   T   high temp = weak coupling

M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

($\lambda^{-1/3}T$: dimensionless effective temperature)

Maldacena conjecture is correct at finite coupling & temperature!

M.H.-Hyakutake-Nishimura-Takeuchi, PRL 2009

1/N correction

Dual gravity prediction (Y. Hyakutake, PTEP 2013)

$$E/N^2 = 7.41T^{2.8} - 5.58T^{4.6} + ...$$
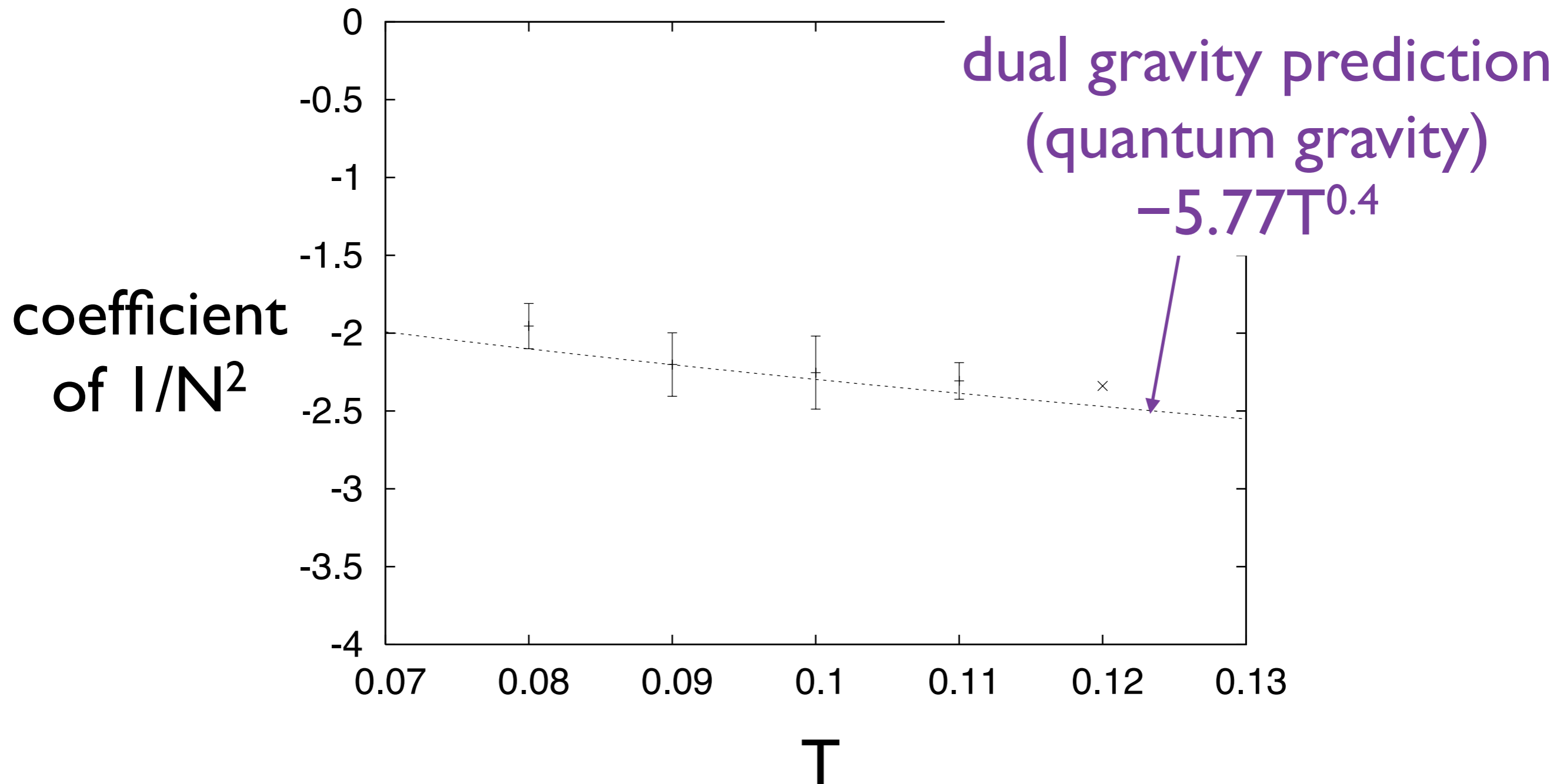
$$+ (1/N^2)(-5.77T^{0.4} + aT^{2.2} + ...)$$

$$+ (1/N^4)(bT^{-2.6} + cT^{-2.0} + ...)$$

$$+ .....$$

QUANTUM string effect

Can it be reproduced from YM?

Maldacena conjecture is correct at finite-N !

coefficient of $1/N^2$

dual gravity prediction (quantum gravity) $-5.77T^{0.4}$

M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Maldacena's conjecture is correct
at finite temperature,
including 1/λ and 1/N corrections,
at least to the next-to-leading order.

So you can use it for learning about QGP at finite-N!

&

You can apply your knowledge about QGP to solve SYM
plasma, which tells us about quantum gravity!
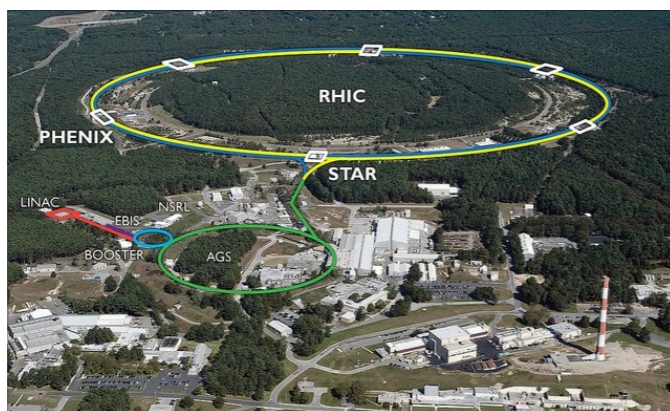
heavy-ion colliders are
machines for quantum gravity!

But why does it hold? We want to understand it intuitively, so that we can understand physics behind it.

It should give us new perspective for both QGP and BH.

microscopic descriptions of the black hole (black brane)

(1) D-branes + open strings

Polchinski, …

(2) condensation of closed strings

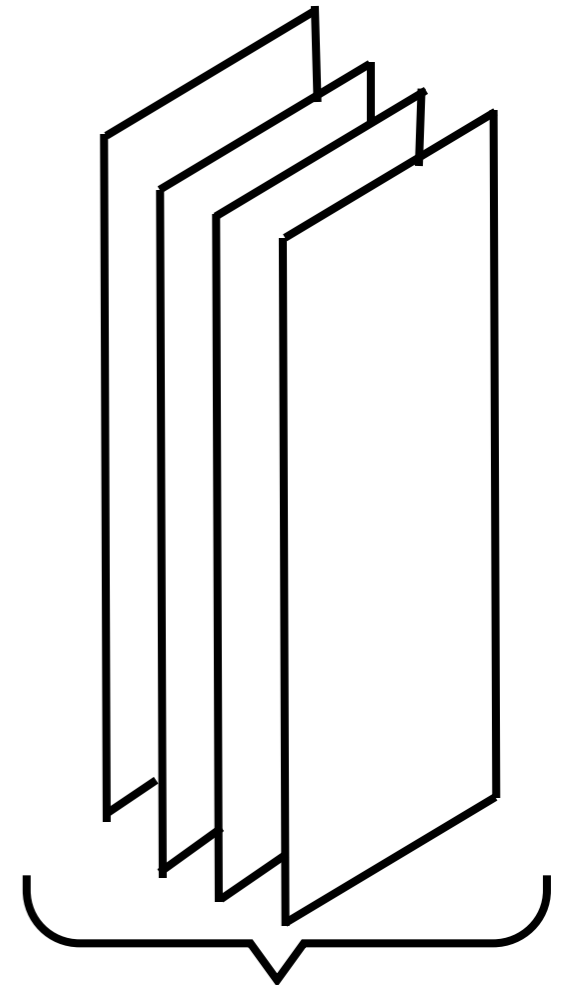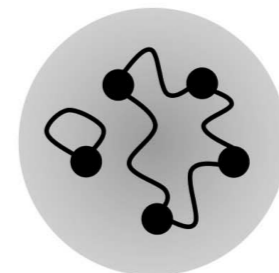Susskind, Horowitz-Polchinski, …

BH = D-branes + open strings
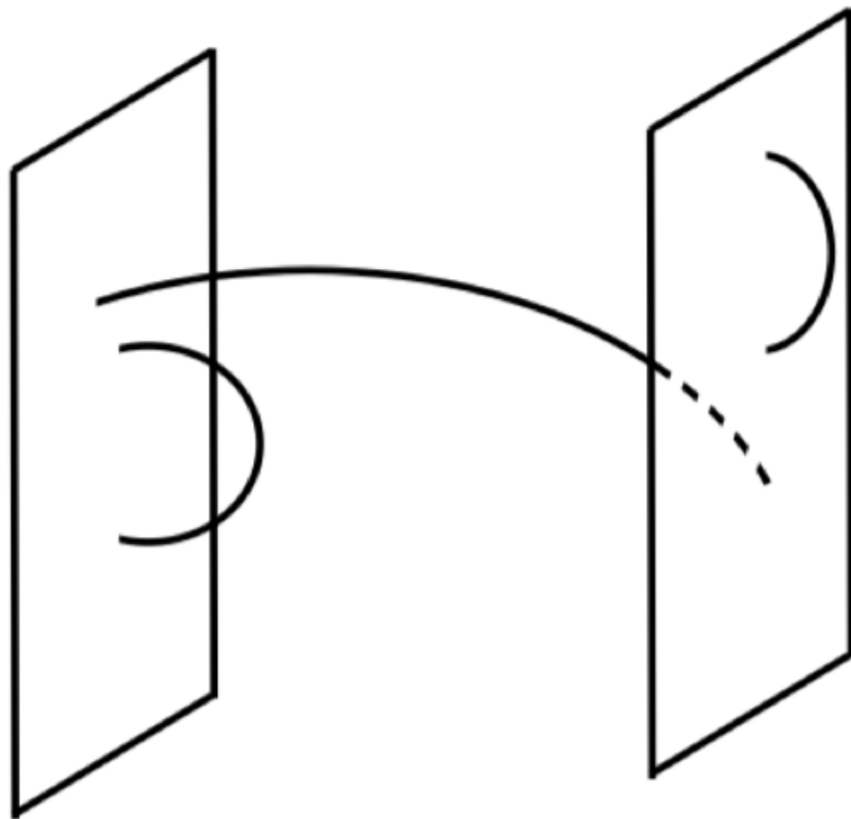
U(N) YM

U(2) YM

(i,j)-component of matrices
= string between i-th and j-th D-branes

N Dp-branes

large N ➞ heavy ➞ BH

Black hole from closed string

(e.g. Susskind 1993)

long, winding string with length L

energy = tension × L

entropy $\sim$ L

when L >> 1, huge energy and entropy are
packed in a small region → _black hole_

How are they related?

long, winding strings = black brane + open strings

open strings
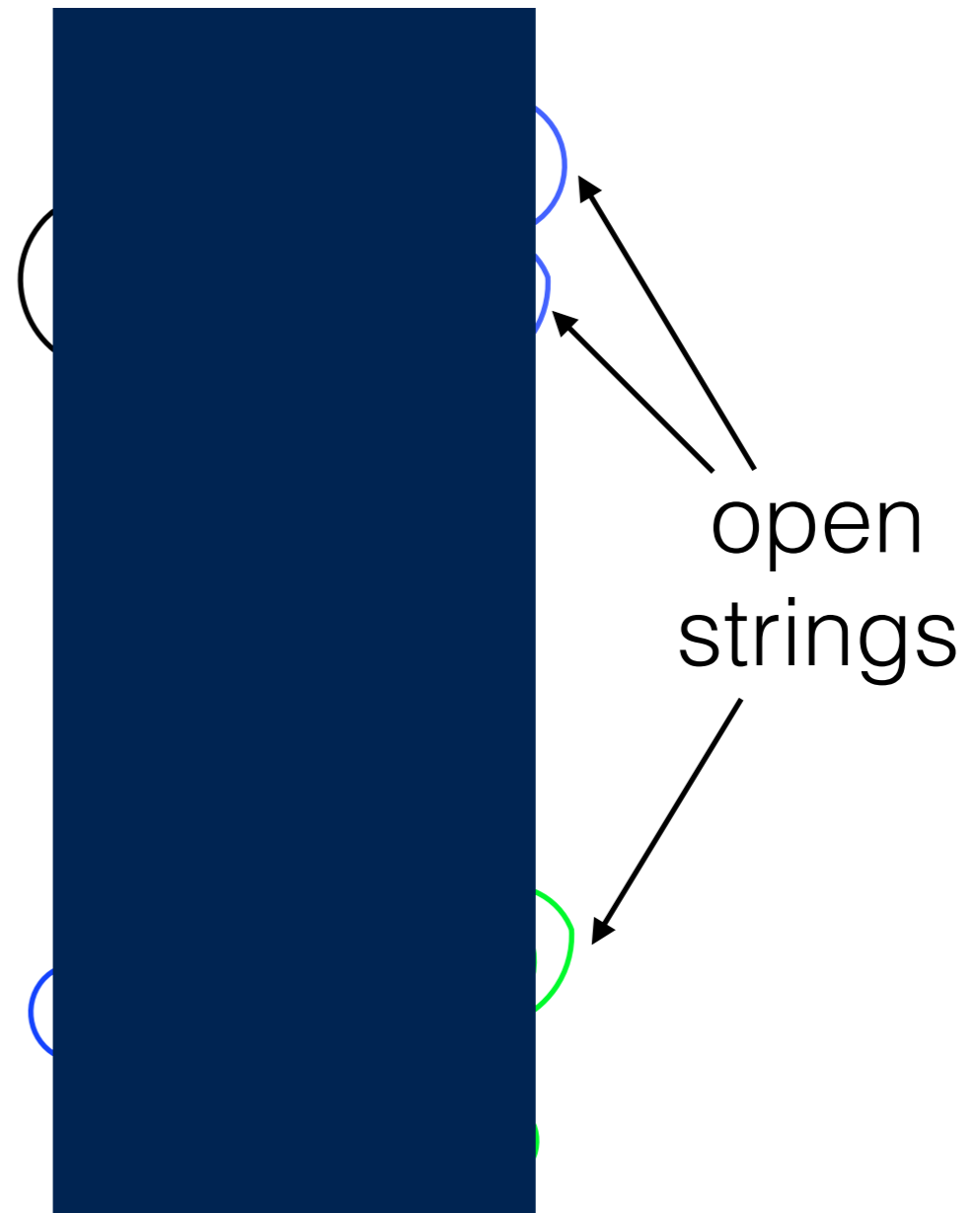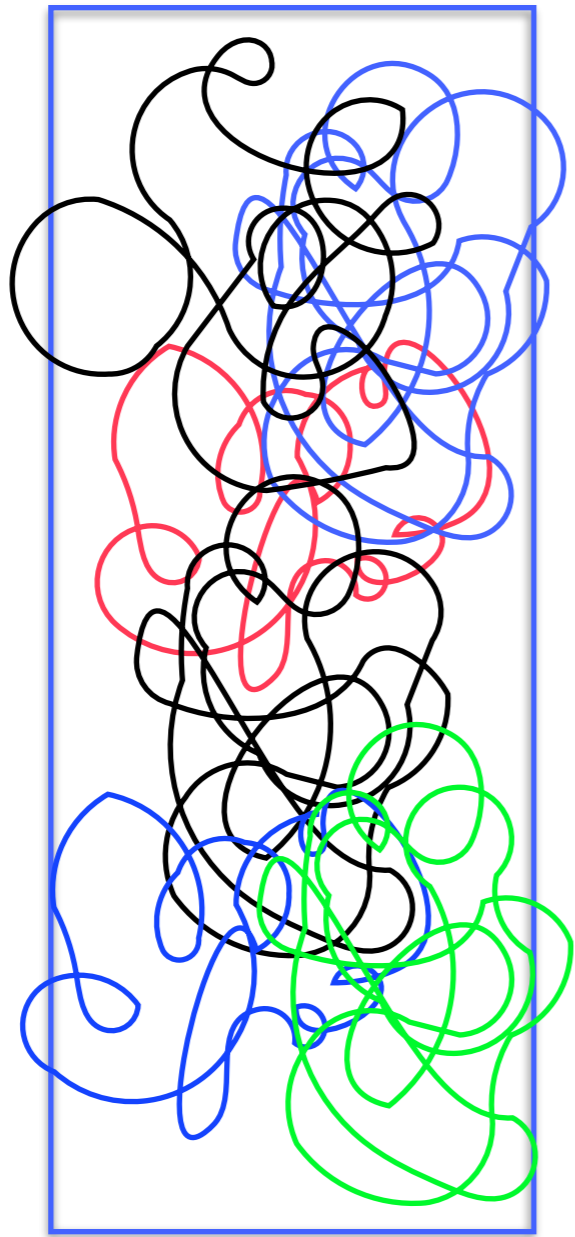
The meaning of N (# of D-branes) becomes clear later.

Gauge theory description

confining phase: 't Hooft, 1974

deconfining phase: M.H.-Maltz-Susskind, 2014

Lattice gauge theory description at strong coupling

Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

$$H = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E^{\alpha}_{\mu,\vec{x}}\right)^2 + \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}})\right)$$

$$[E^{\alpha}_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu\nu}\delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}}$$

$$\sum_{\alpha=1}^{N^2} \tau^{\alpha}_{ij} \tau^{\alpha}_{kl} = \frac{\delta_{il}\delta_{jk}}{N^2}$$

Hilbert space is expressed by <u>Wilson loops.</u>

(closed string)

strong coupling limit

$$H = \frac{N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} (E_{\mu,\vec{x}}^{\alpha})^2$$

(λ=1 for simplicity)

$$\frac{L}{2}$$

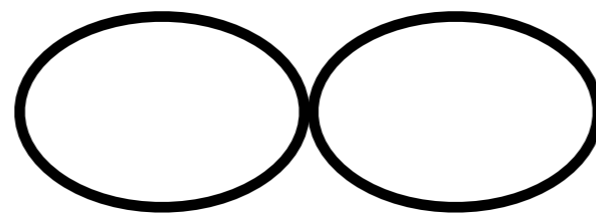L = length of string

2 strings

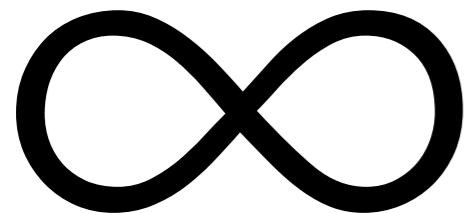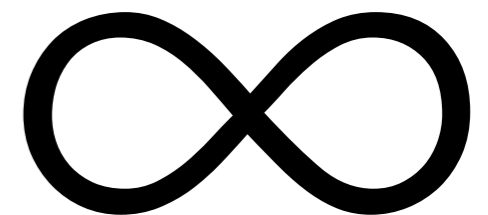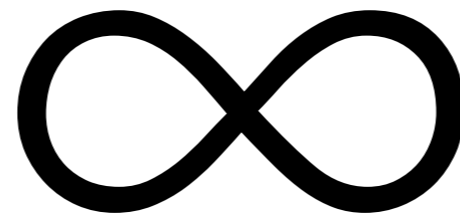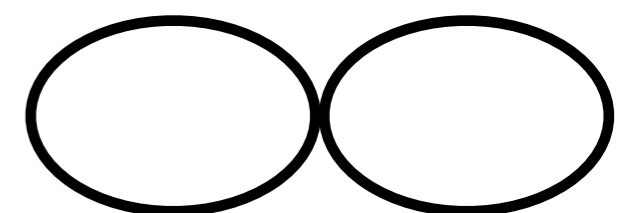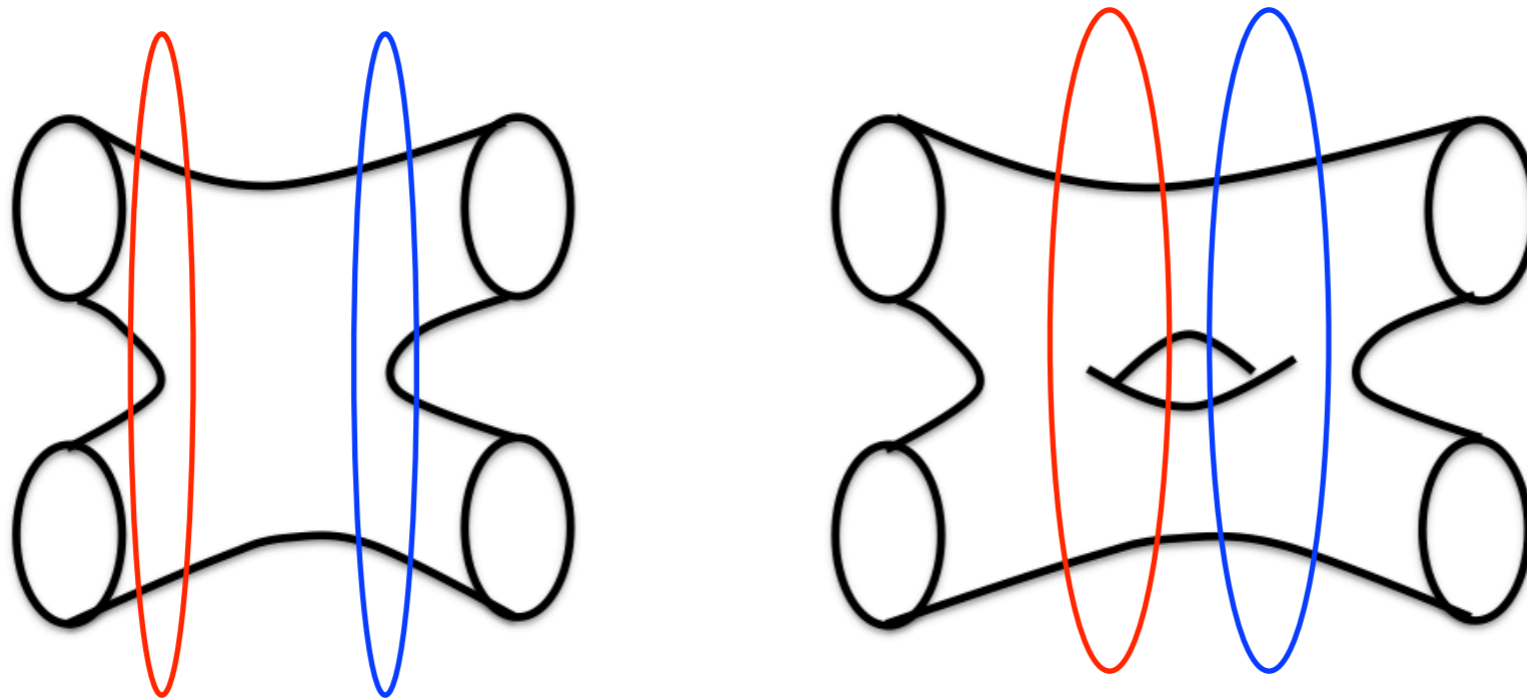$$\frac{L}{2} \qquad + \frac{1}{N}$$

1 string

$$\frac{L}{2} \qquad + \frac{1}{N}$$

splitting ~ 1/N

joining ~ 1/N

1/N² for each loop of closed strings

"large-N limit is the theory of free string"

Strings out of YM: <u>deconfining</u> phase

M.H.-Maltz-Susskind, 2014

related previous work: Patel; Kalaydzhyan-Shuryak; …

Hilbert space is always the same. Why don't we express the deconfining phase by using Wilson loops?

- interaction (joining/splitting) is 1/N-suppressed
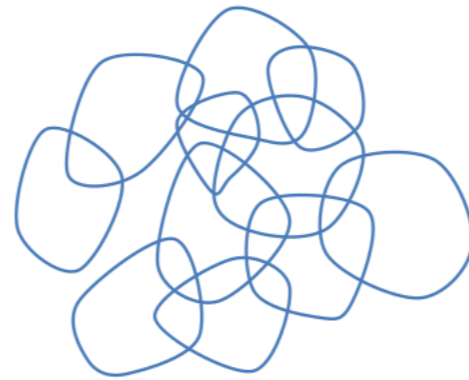
    "large-N limit is the theory of free string"

- It is true when L is $O(N^0)$. ($\rightarrow$ confining phase)

- In deconfinement phase, total length of the strings is $O(N^2)$ $\rightarrow$ number of intersections is $O(N^2)$ $\rightarrow$ interaction is **not** negligible

    large-N limit is still very dynamical!

confining phase
= gas of short strings
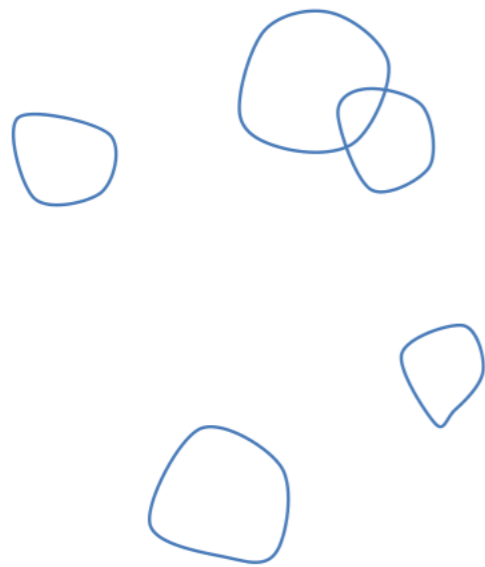
long and winding string,
which is interpreted as BH,
appears

as the density of strings increase,
interaction between strings
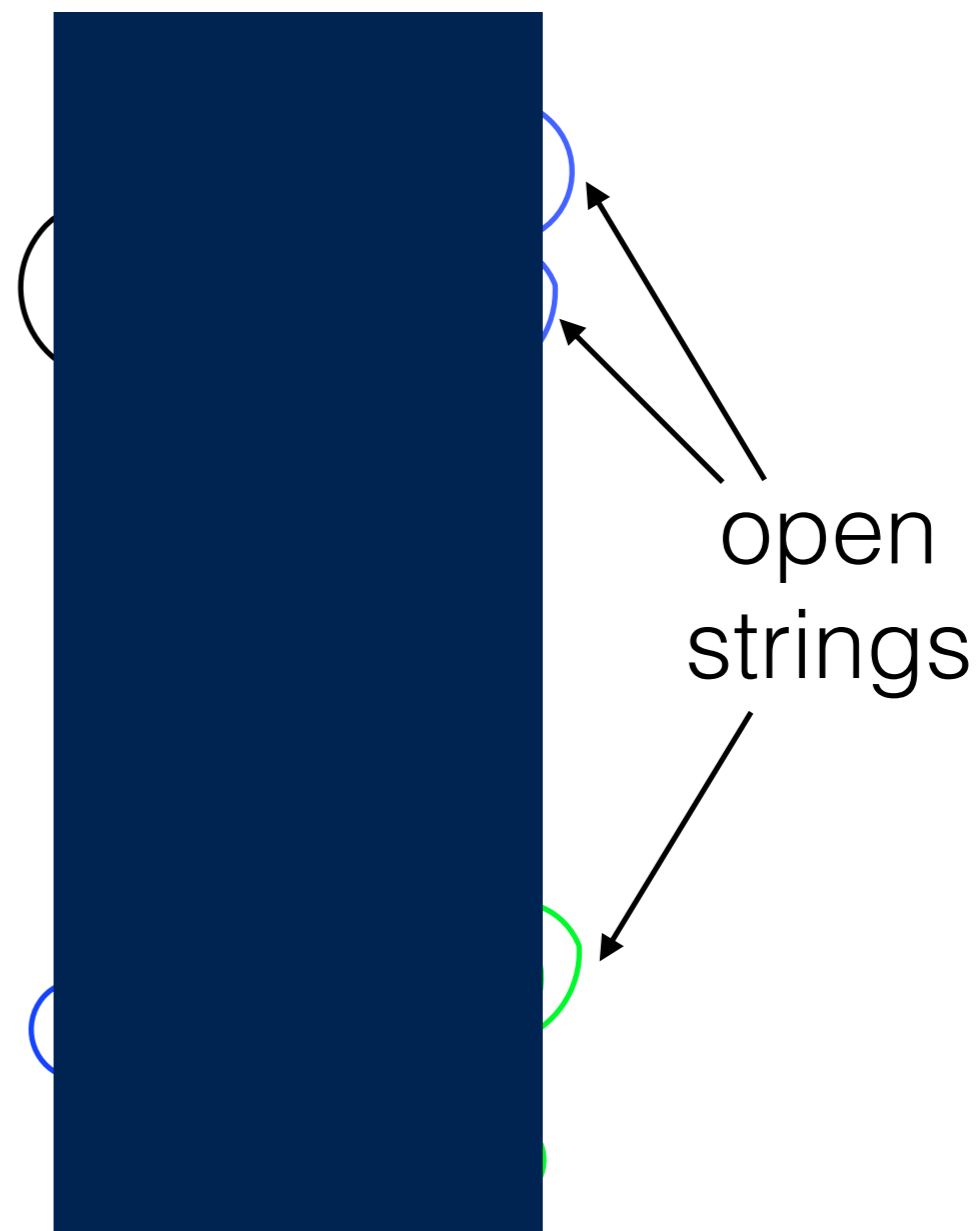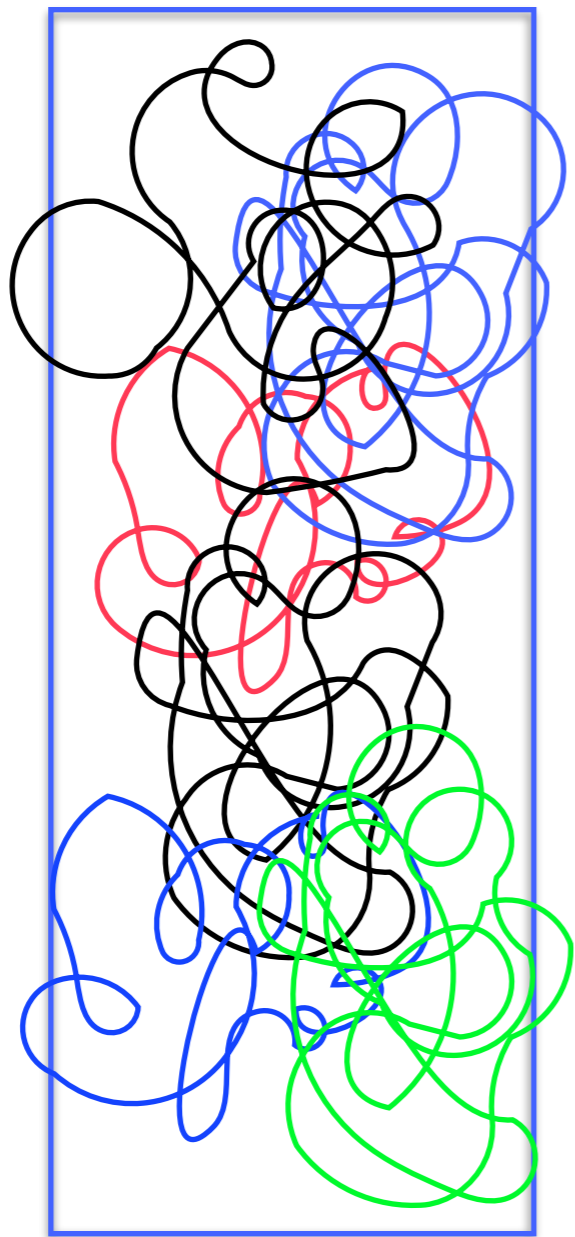becomes important,and…

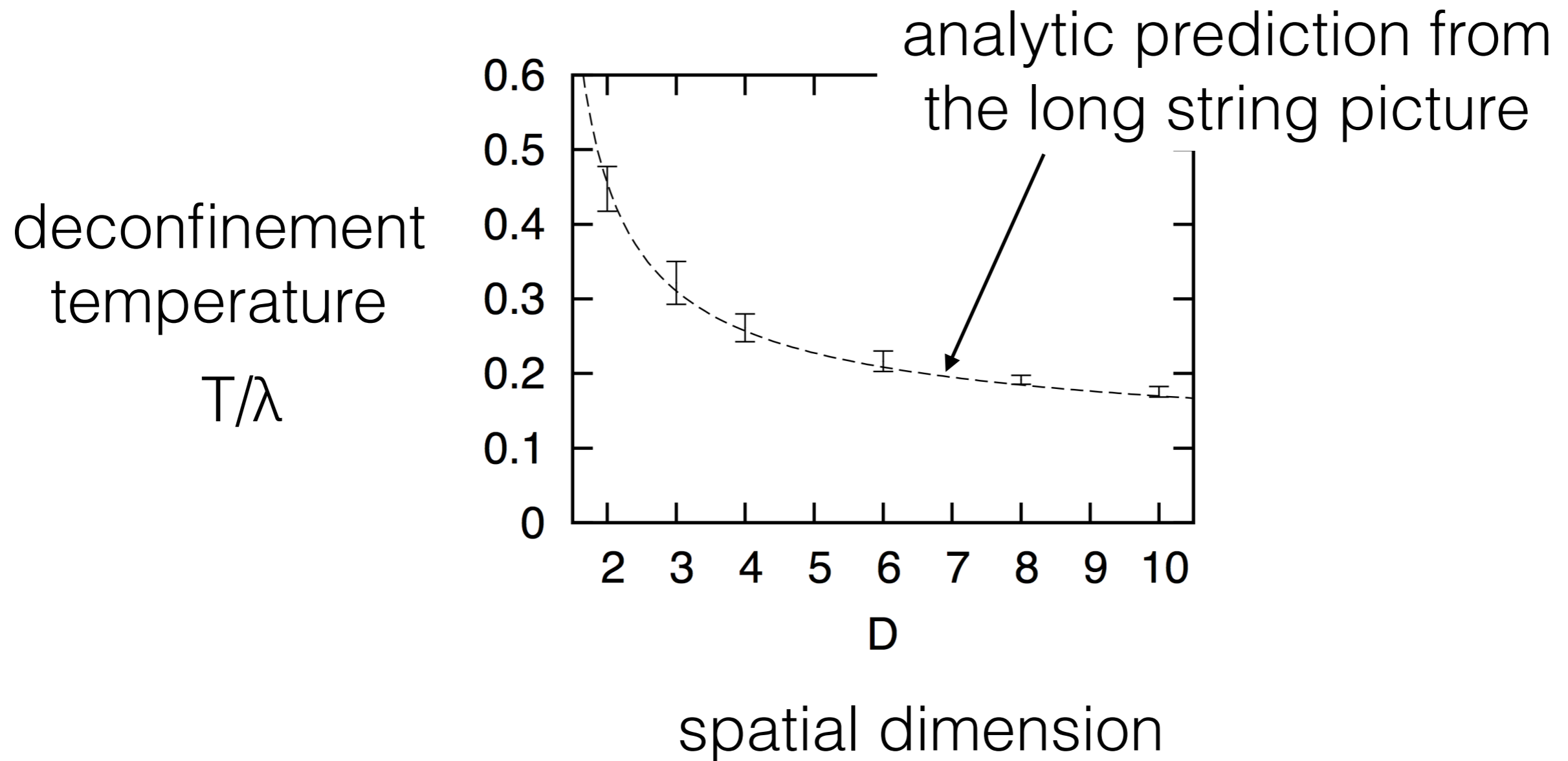# (de)confinement of probe charges

confine
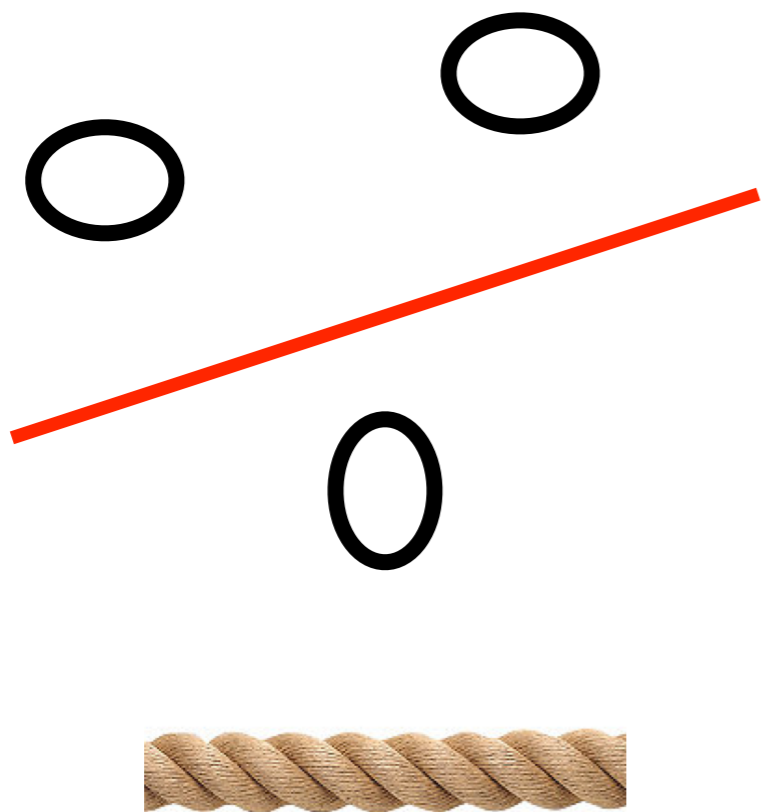
deconfine
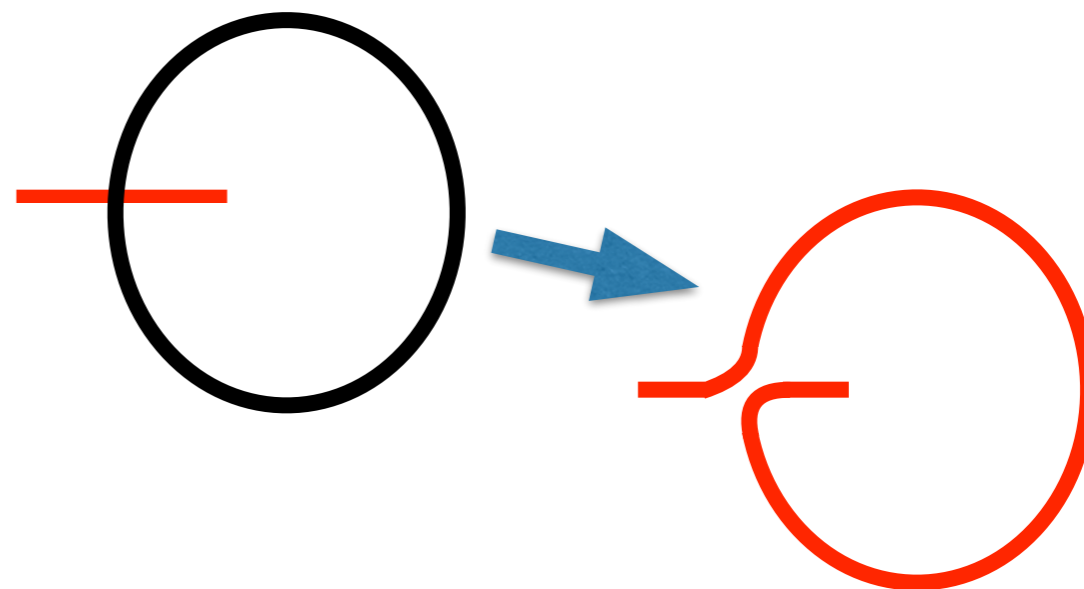
long, winding *QCD*-strings = black brane + open *QCD*-strings

open strings

open strings = Wilson lines, which have N color d.o.f at endpoints
→ black brane is made from N Dp-branes

D-dim square lattice at strong coupling

analytic prediction from
the long string picture

deconfinement
temperature

T/λ

0.6
0.5
0.4
0.3
0.2
0.1
0

2 3 4 5 6 7 8 9 10

D

spatial dimension

conclusion

Maldacena's conjecture is correct
at finite temperature,
including 1/λ and 1/N corrections,
at least to the next-leading order.

so, lattice/nuclear theorists can study
quantum gravity, by studying field theory.
You can do something string theorists cannot do.
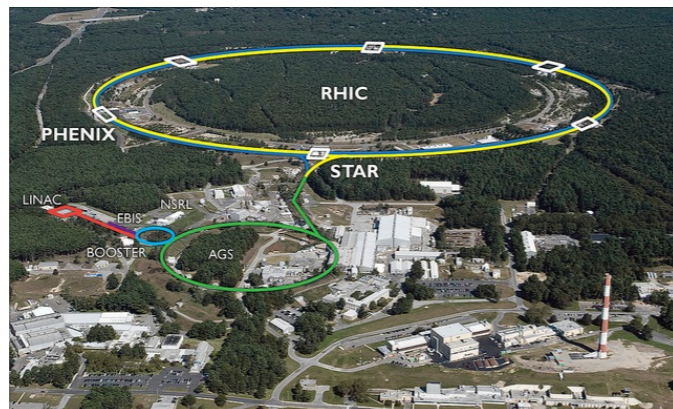
heavy-ion colliders are
machines for quantum gravity!

conclusion

deconfinement phase $=$ $=$
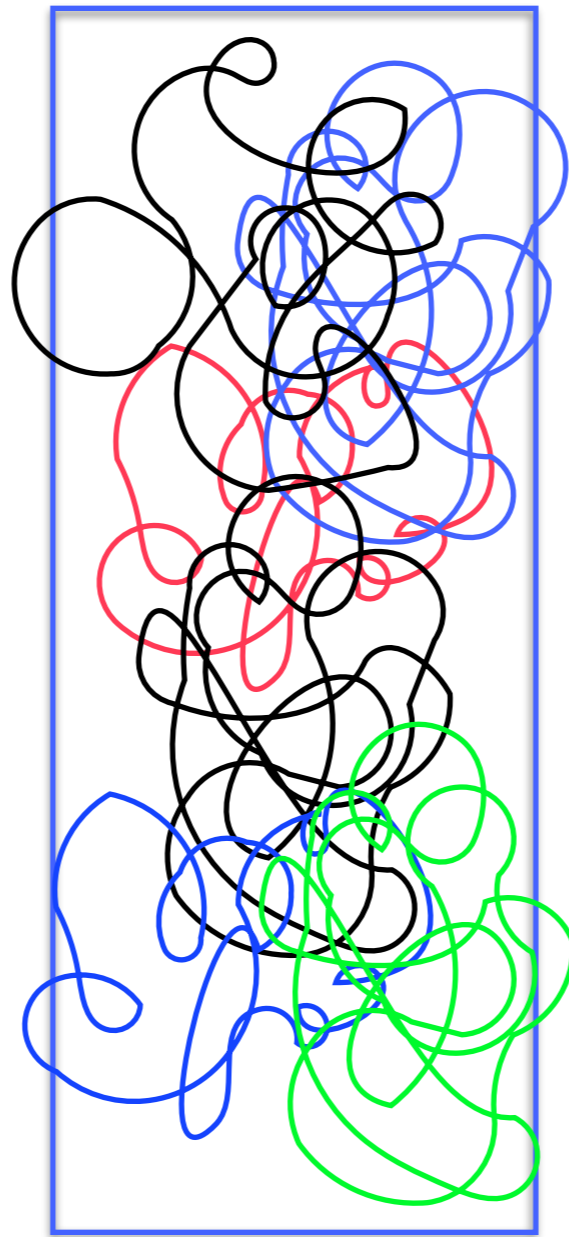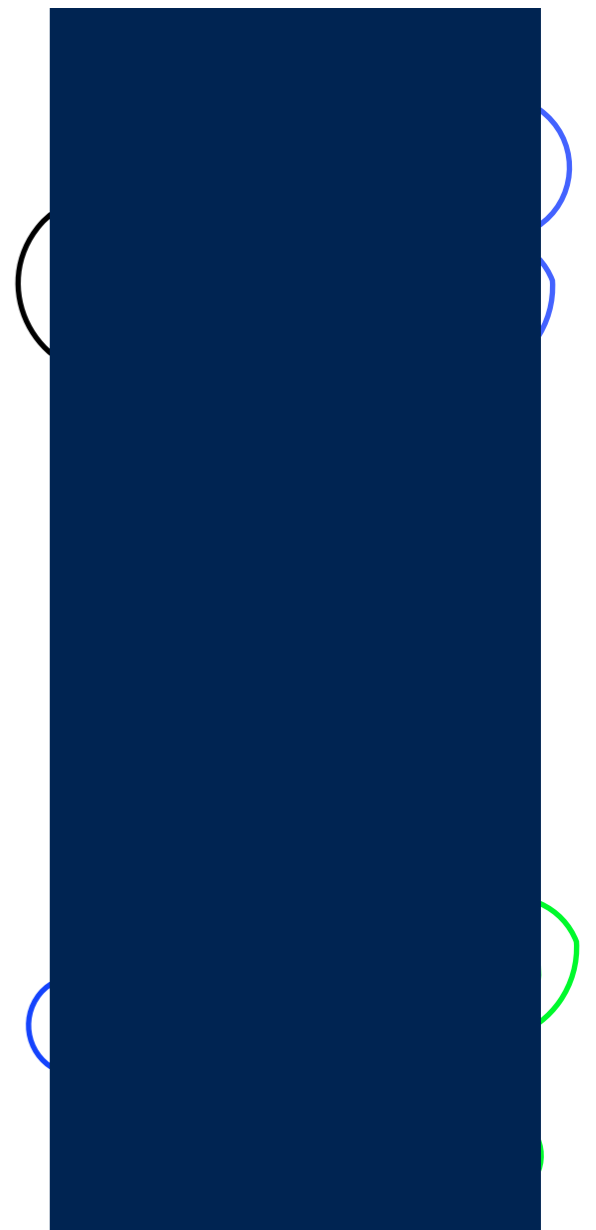
Strong coupling limit contains the essence.
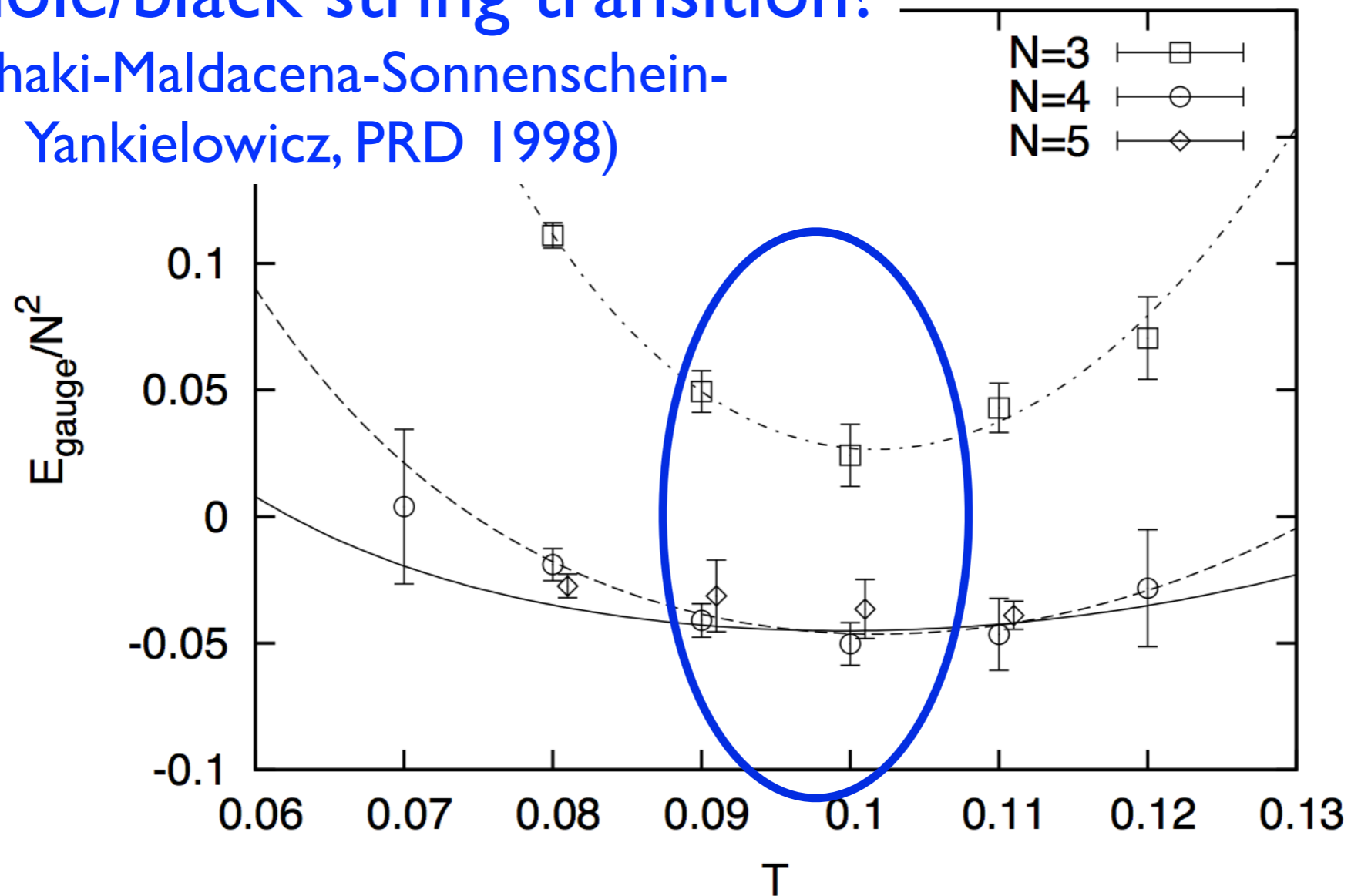Stringy picture should be useful for learning about QGP.

backup
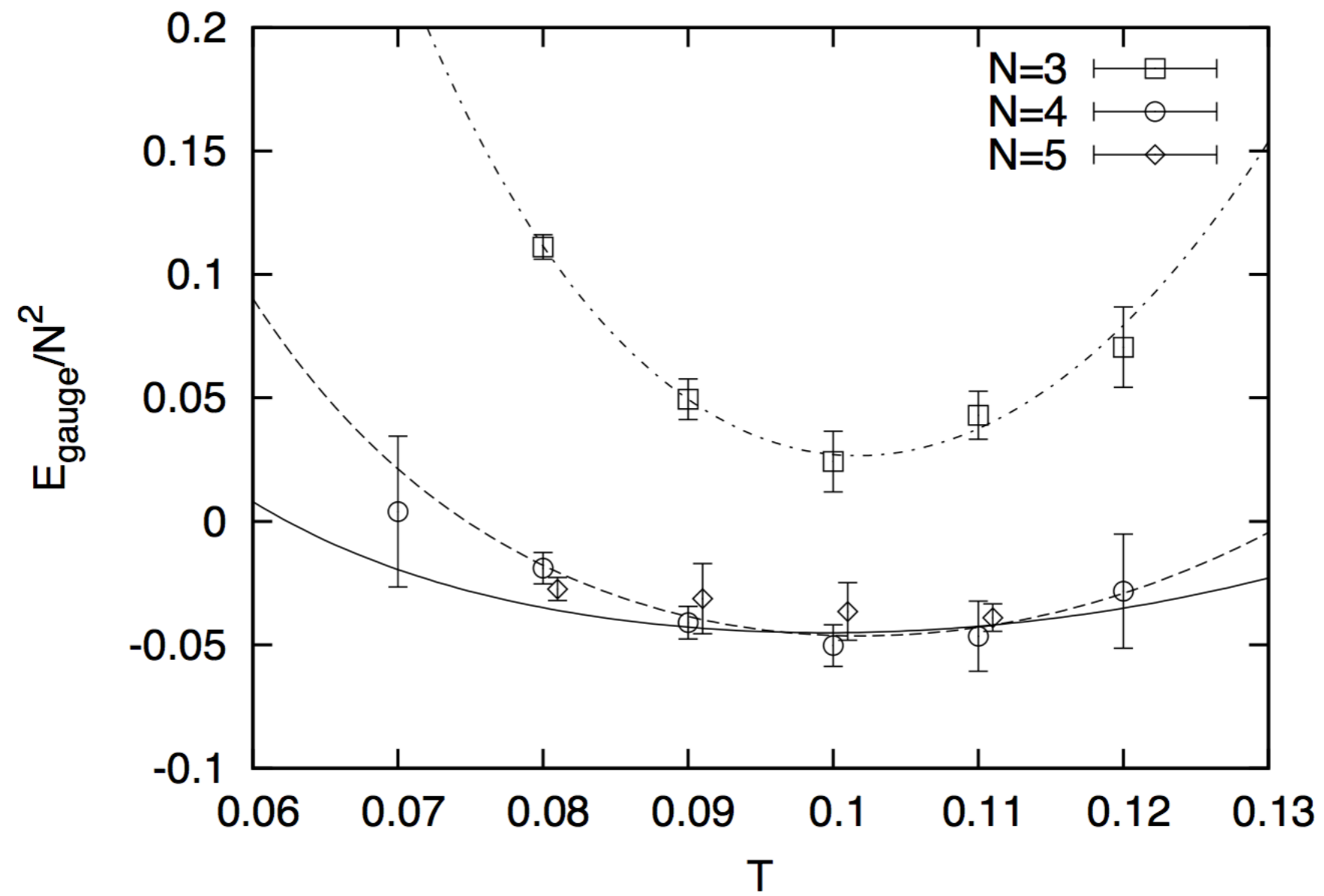
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

negative specific heat
→ the same as Schwarzschild BH
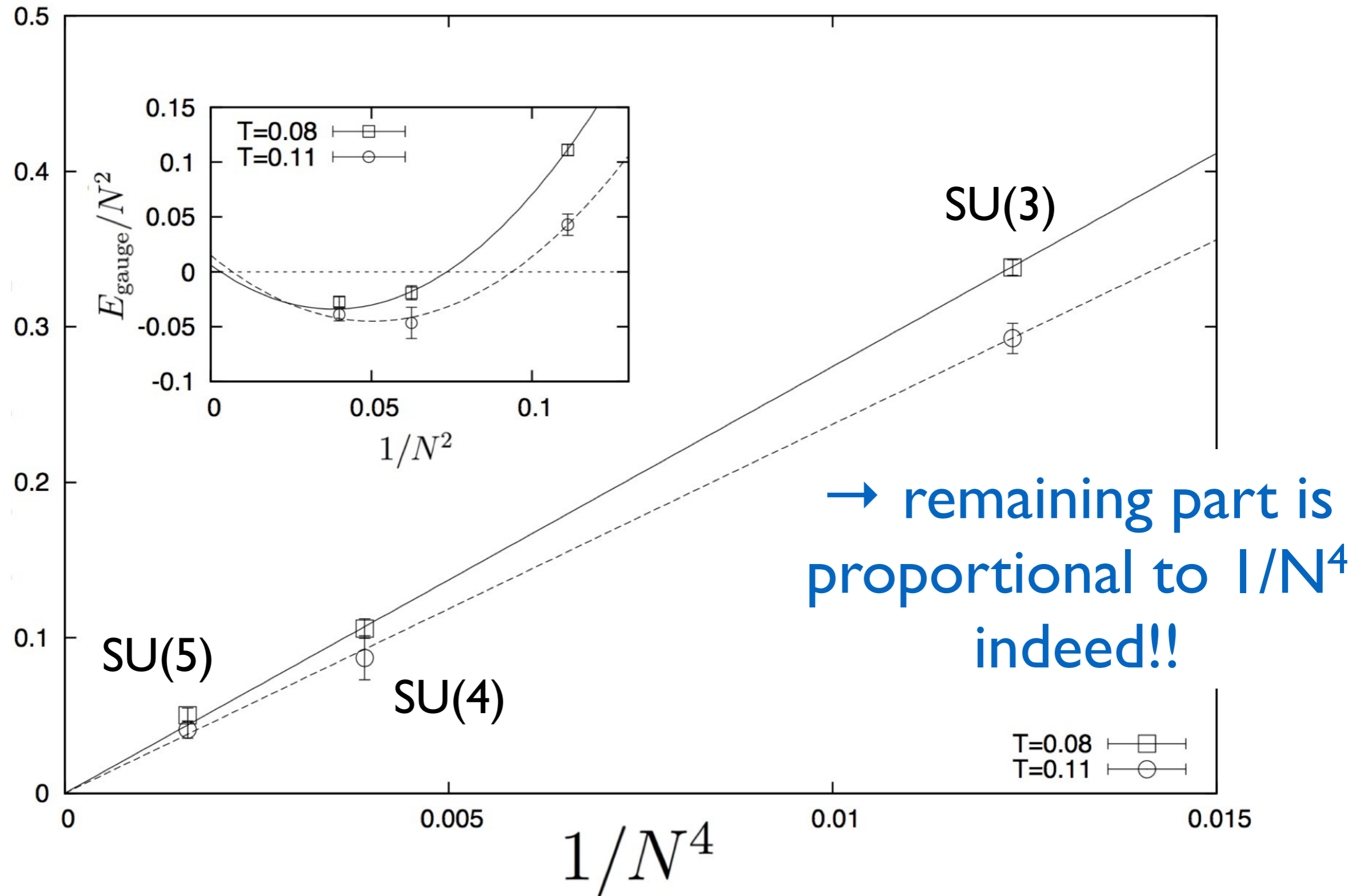
M.H.-Hyakutake-Ishiki-Nishimura, Science 2014
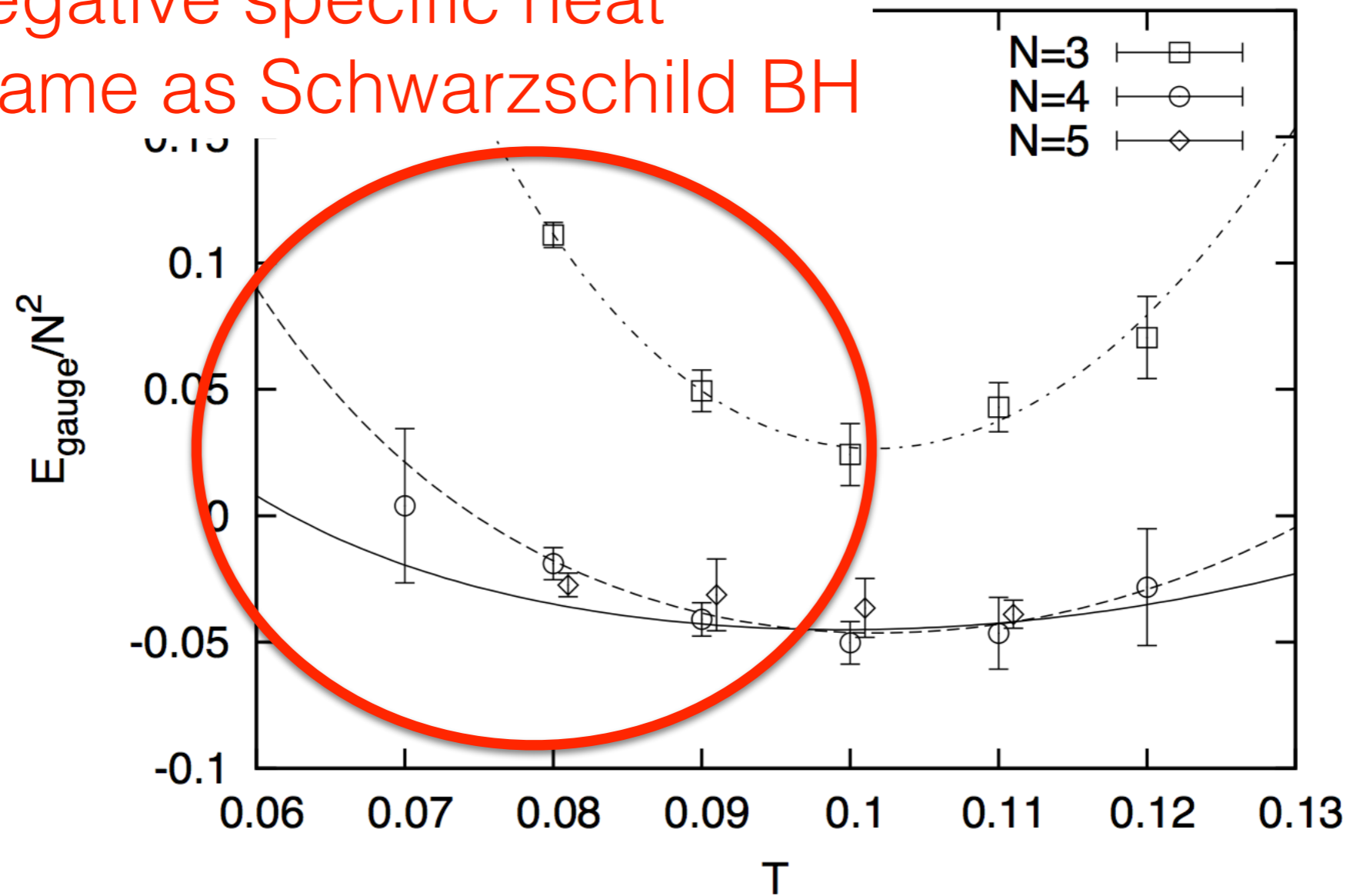
Black hole/black string transition?

(Itzhaki-Maldacena-Sonnenschein-
Yankielowicz, PRD 1998)

M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

$E/N^2 - (7.41T^{2.8} - 5.77T^{0.4}/N^2)$ vs. $1/N^4$

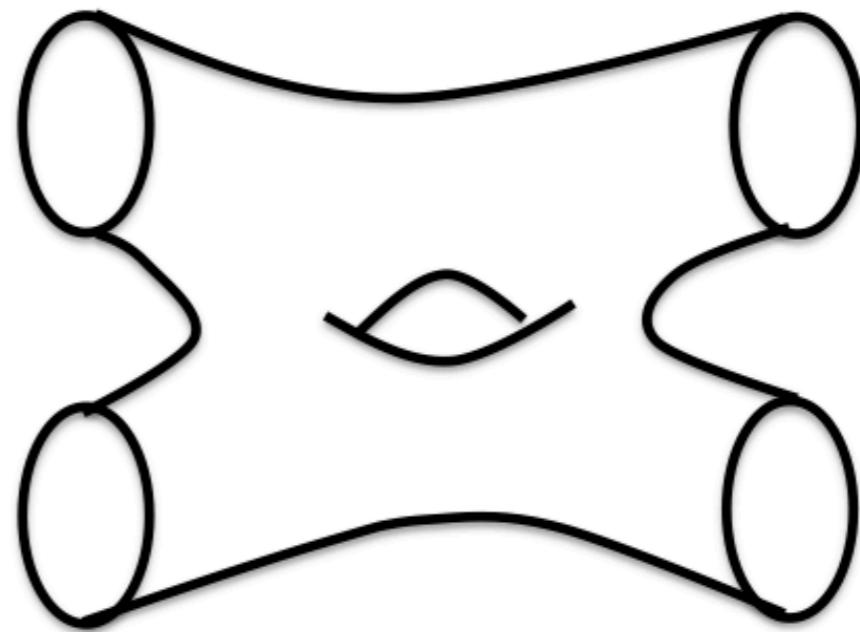→ remaining part is proportional to $1/N^4$ indeed!!

M.H.-Hyakutake-Ishiki-Nishimura, Science 2014

Strings out of YM:

't Hooft's argument for the <u>confining</u> phase
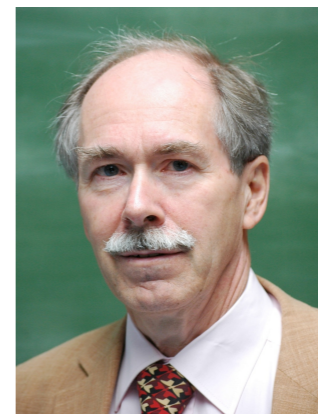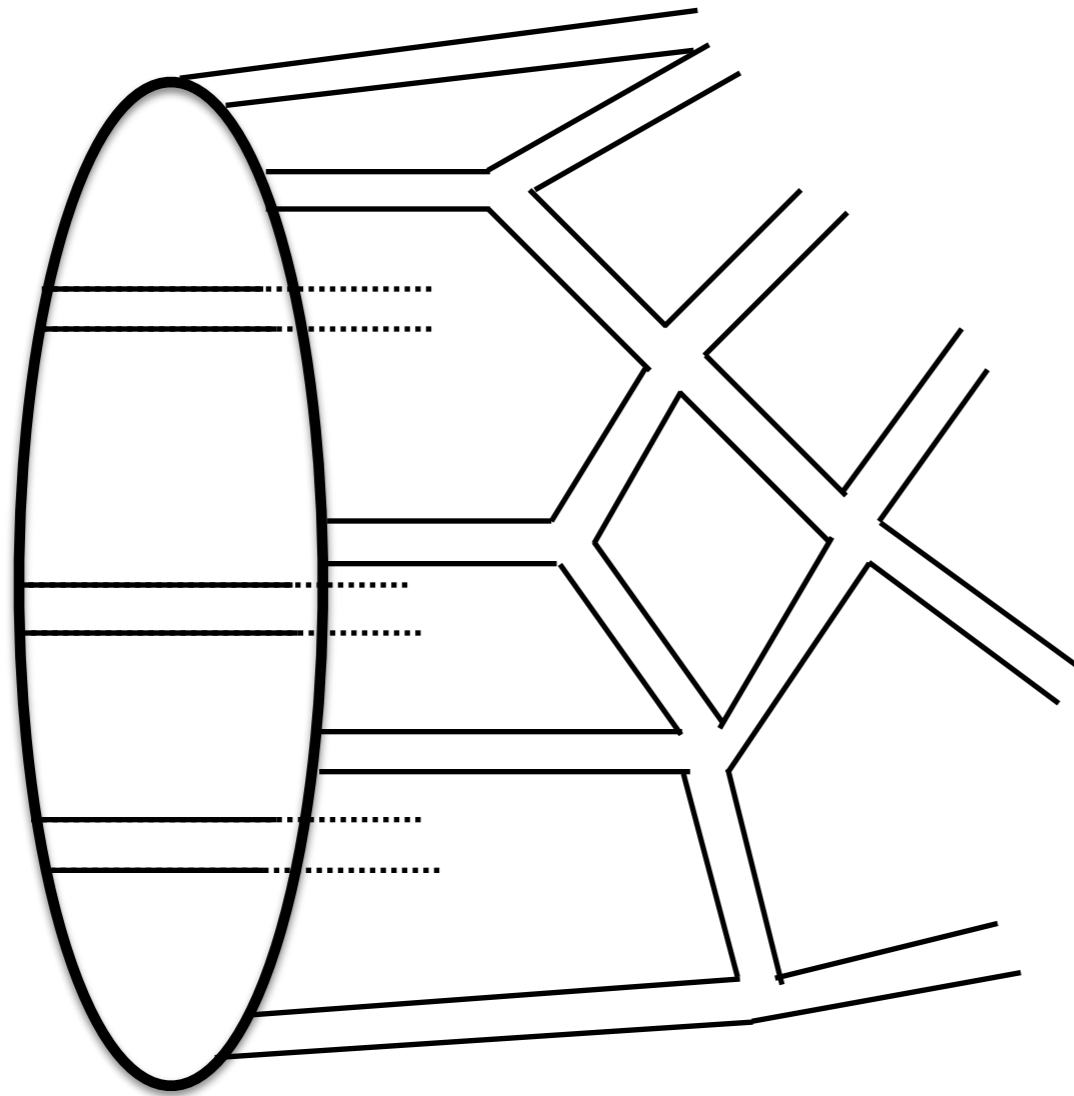
scattering of strings
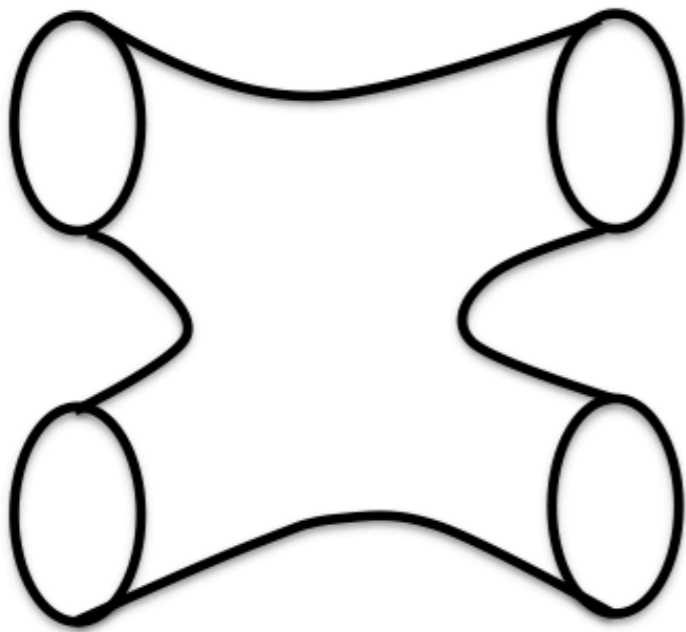
tree

one-loop
$\sim g_s{}^2$

g closed string loops → genus g surface
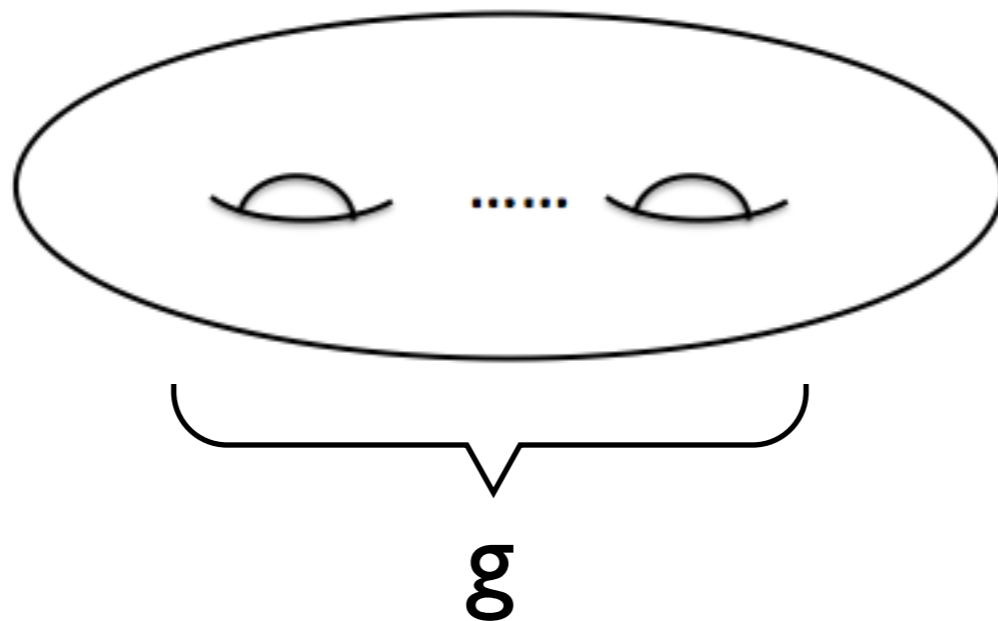
$\sim g_s{}^{2g}$

g

One takes into account the quantum effect order by order, by increasing g one by one.
→ *perturbative* formulation

Main idea

Feynman diagram
= "fishnet" made of gluons
= string worldsheet

How can they be related
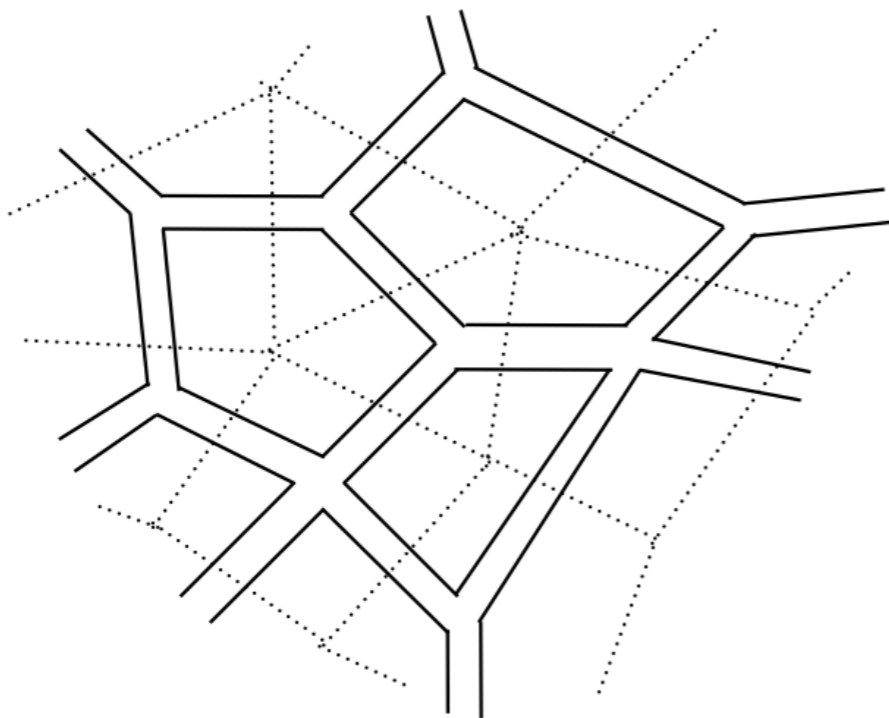without ambiguity?

Wilson loop = creation operator of closed string

Main idea

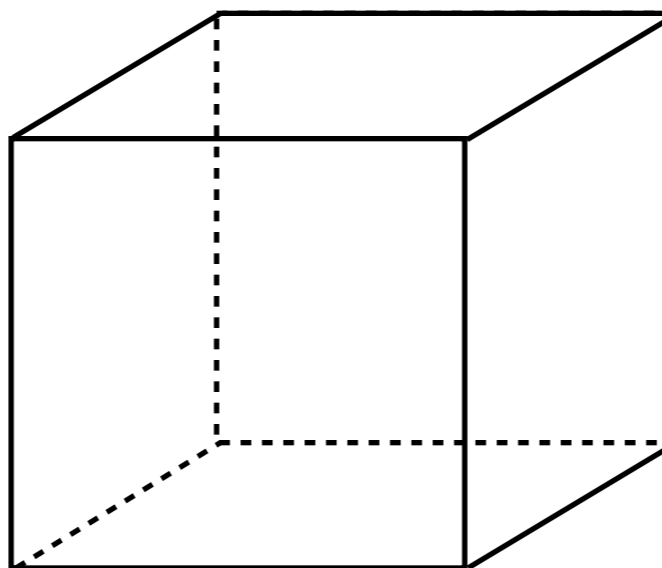Feynman diagram
||
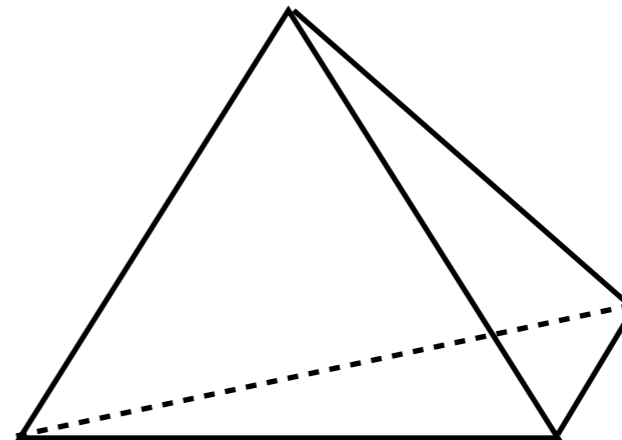triangulation/quadrangulation
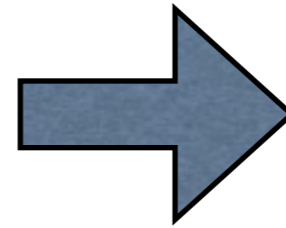of string worldsheet

"fish net"

1/N expansion
||
genus expansion
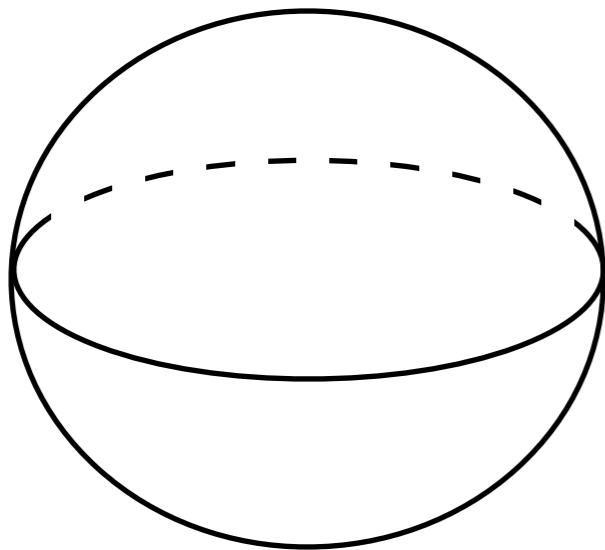
two-sphere (g=0)

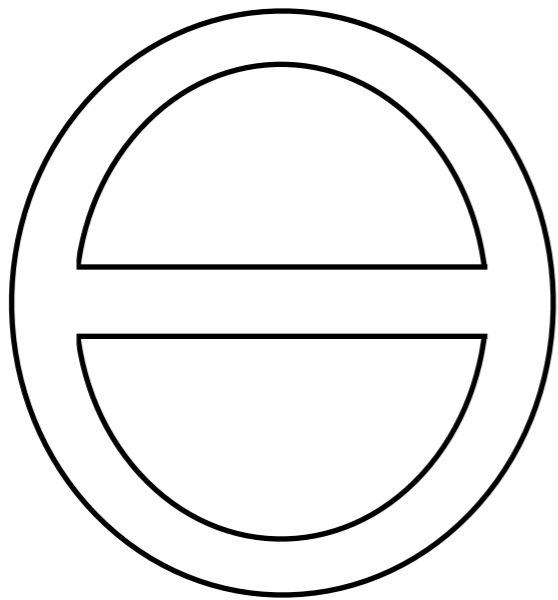$$S = \frac{N}{4\lambda} \int d^4 x \, Tr F_{\mu\nu}^2$$
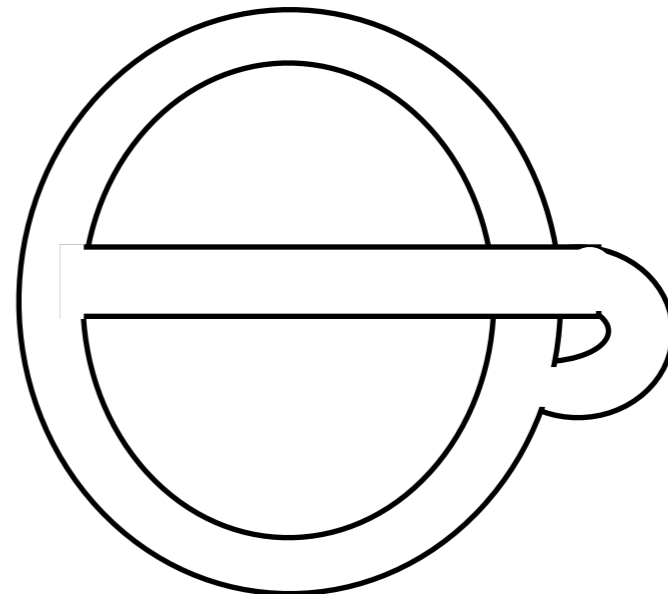
(U(N) gauge group)

vertex $\sim$ N
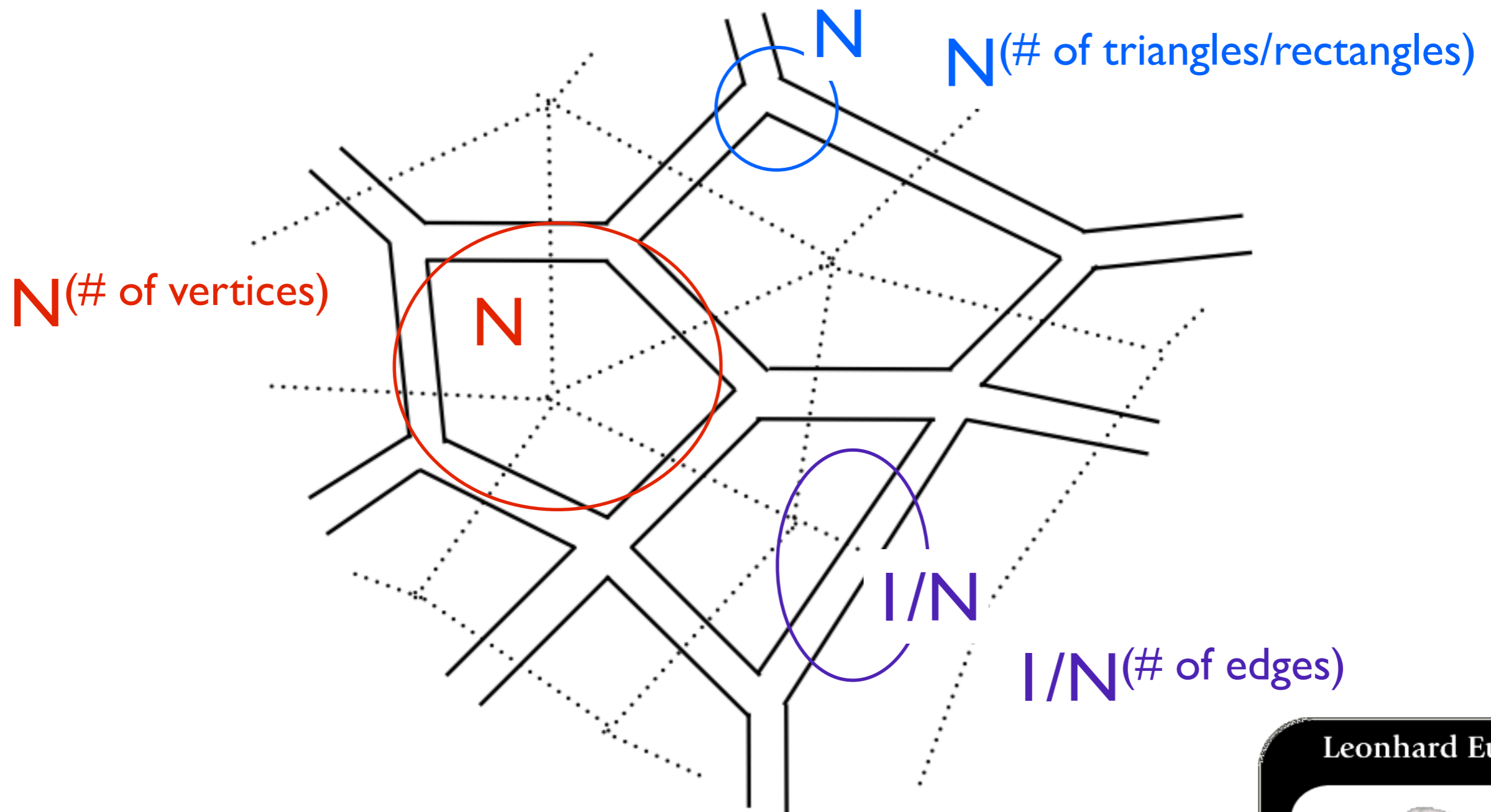
index loop $\sim$ N

propagator $\sim$ 1/N

planar diagram

$N^2 \times N^{-3} \times N^3 = N^2$

nonplanar diagram
(genus one)

$N^2 \times N^{-3} \times N^1 = N^0$

N(# of triangles/rectangles)

N(# of vertices)

1/N(# of edges)

N(# of vertices)
× 1/N(# of edges)
× N(# of triangles/rectangles)
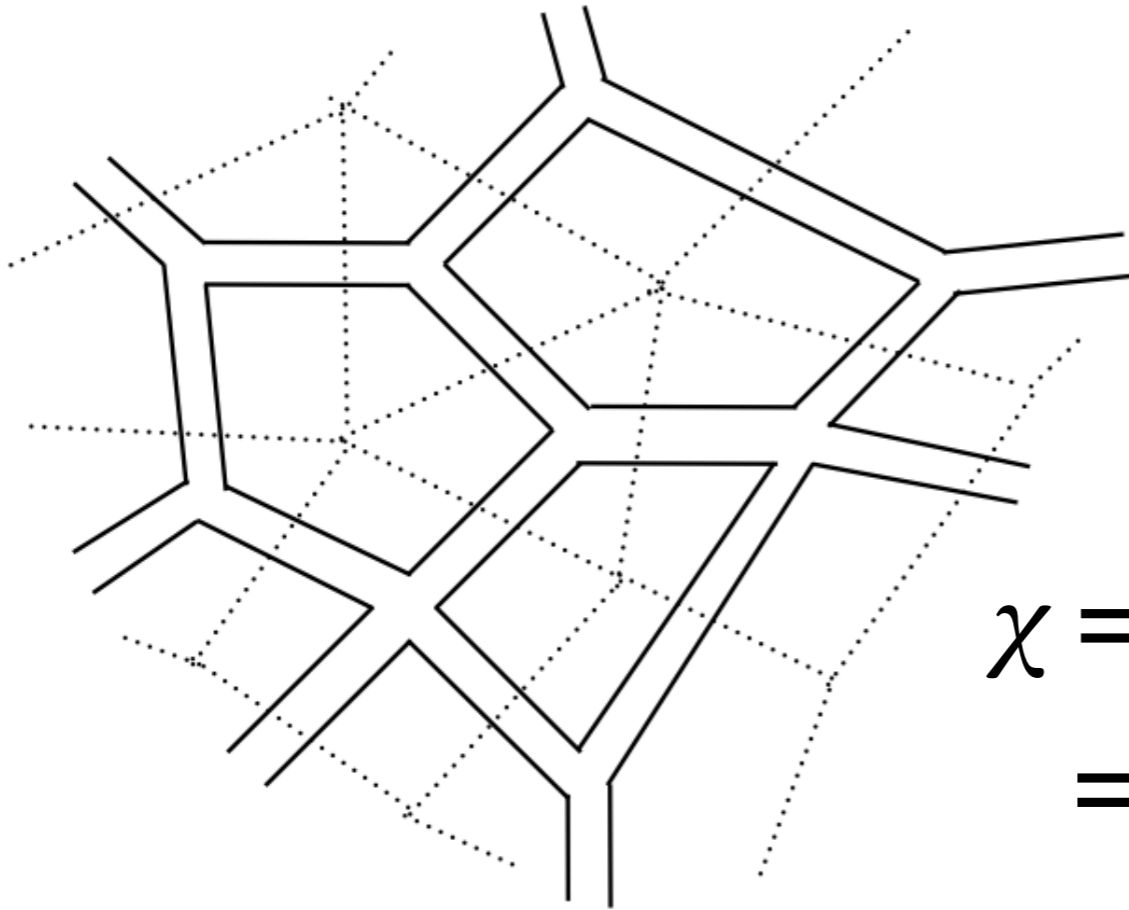
$= N^{\chi}$

Leonhard Euler

1707 - 1783
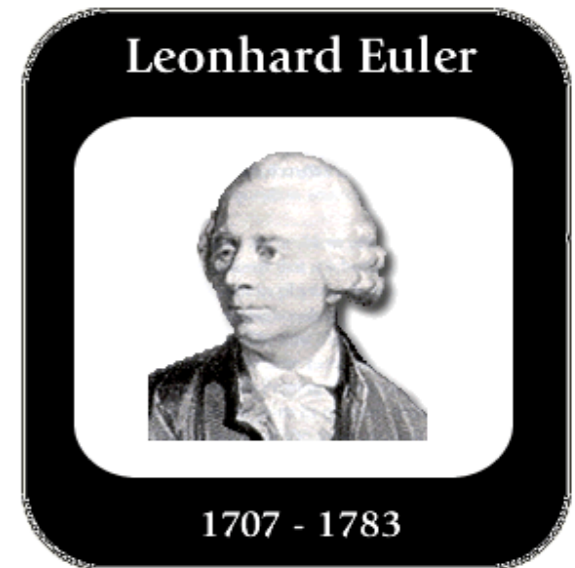
vertex $\sim$ N $\sim$ triangle/rectangle

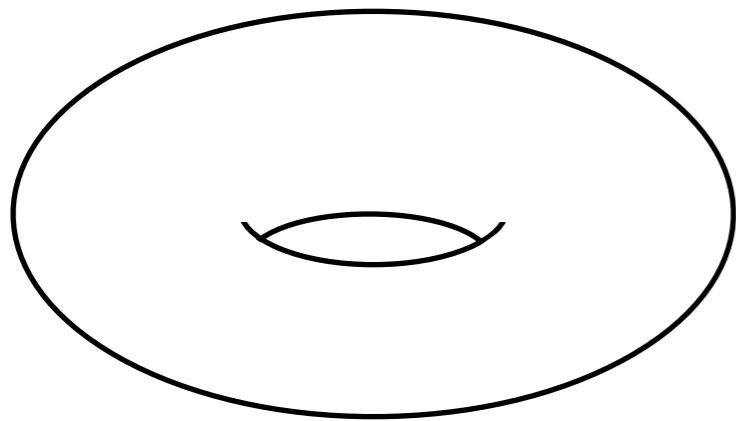index loop $\sim$ N $\sim$ vertex

propagator $\sim$ 1/N $\sim$ edges
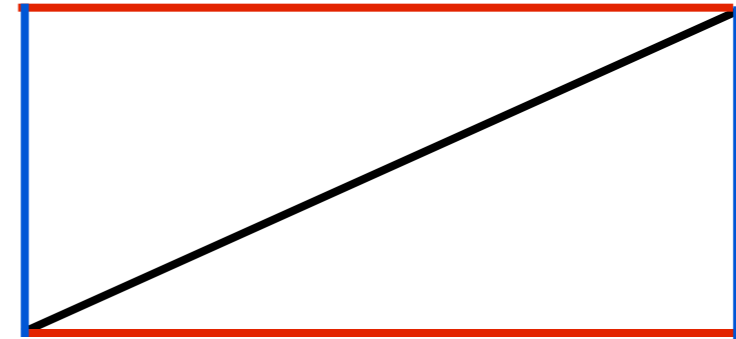
$\sim$N$^{\chi}$

Leonhard Euler

1707 - 1783

χ = Euler number

= (# vertices) − (# propagators)
+ (# index loops)

= (# triagnles/quadrangles)
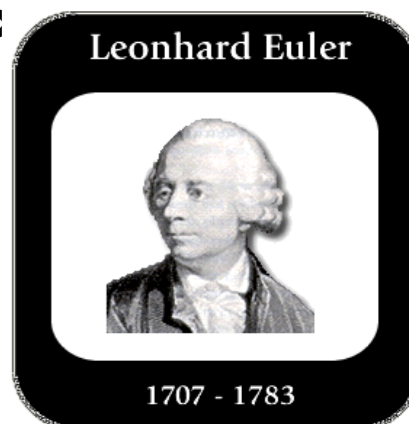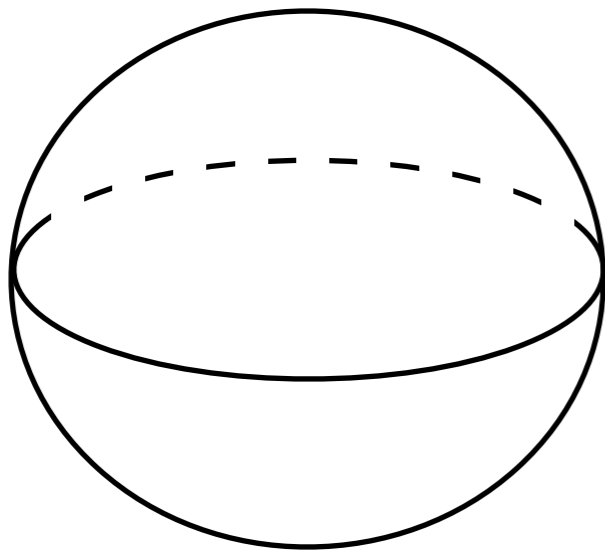− (# edges)+ (# vertices)

torus

triangulation of torus

Euler number

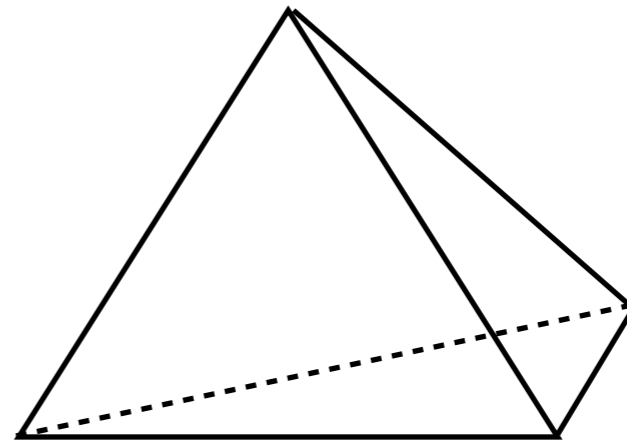$\chi = (\#\text{triangles})-(\#\text{edges})+(\#\text{vertices})=2-3+1=0$

more generally,

$\chi = (\#\text{triangles})-(\#\text{edges})+(\#\text{vertices})=2-2g$
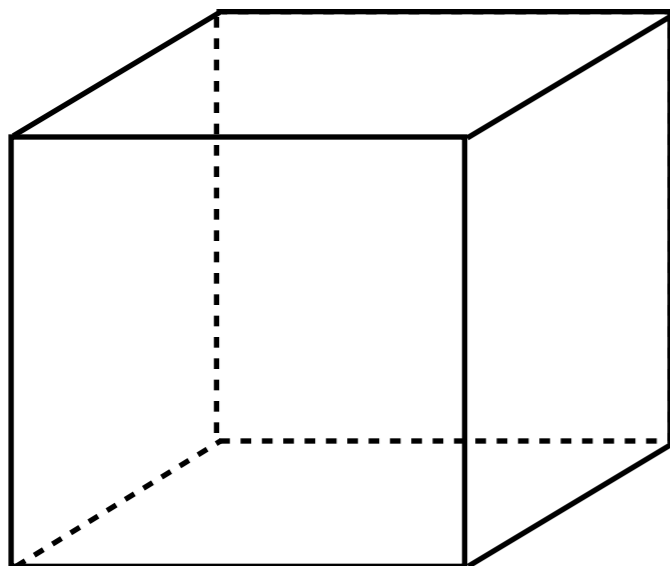
where   g = (#genus)

Leonhard Euler

1707 - 1783

two-sphere (g=0)

4 triangles
6 edges
4 vertices

$$4-6+4 = 2 = 2-2g$$
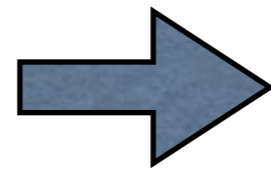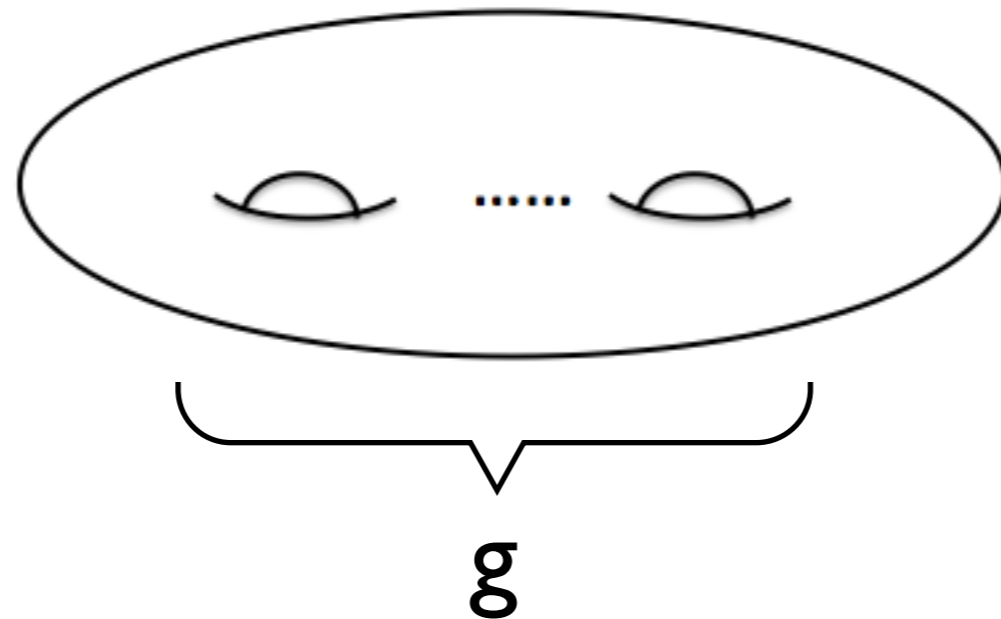
6 squares
12 edges
8 vetices

$$6-12+8 = 2 = 2-2g$$

genus-g diagram = diagram which can be drawn
on genus-g surface

➡️ g closed string loops

$(1/N)^{2g-2} = g_s^{2g-2}$

$1/N = g_s$

g

Yang-Mills gives *nonperturbative* formulation of string theory.

large-N limit is free string theory.

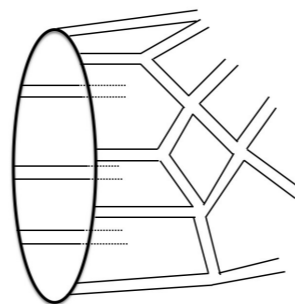# Lattice gauge theory description at strong coupling

Understand it by using the Hamiltonian formulation of lattice gauge theory (Kogut-Susskind, 1974)

$$H = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} \left(E_{\mu,\vec{x}}^{\alpha}\right)^2 + \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu<\nu} \left(N - \mathrm{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U_{\mu,\vec{x}+\hat{\nu}}^{\dagger} U_{\nu,\vec{x}}^{\dagger})\right)$$

$$[E_{\mu,\vec{x}}^{\alpha}, U_{\nu,\vec{y}}] = \delta_{\mu\nu} \delta_{\vec{x}\vec{y}} \cdot \tau^{\alpha} U_{\nu,\vec{y}}$$

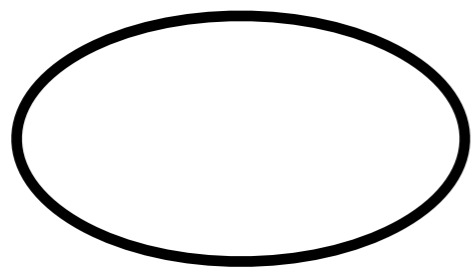$$\sum_{\alpha=1}^{N^2} \tau_{ij}^{\alpha} \tau_{kl}^{\alpha} = \frac{\delta_{il}\delta_{jk}}{N^2}$$

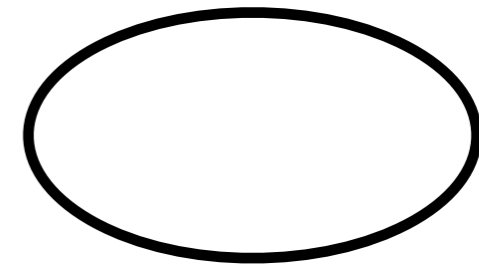Hilbert space is expressed by <u>Wilson loops.</u>

(closed string)

strong coupling limit

$$H = \frac{N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha=1}^{N^2} (E^{\alpha}_{\mu,\vec{x}})^2$$
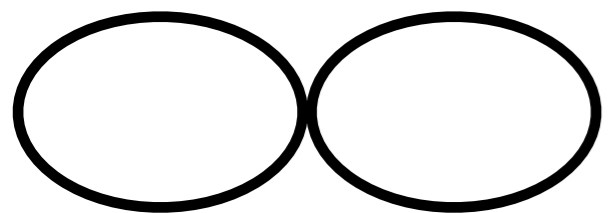
(λ=1 for simplicity)

$$\longrightarrow \frac{L}{2}$$
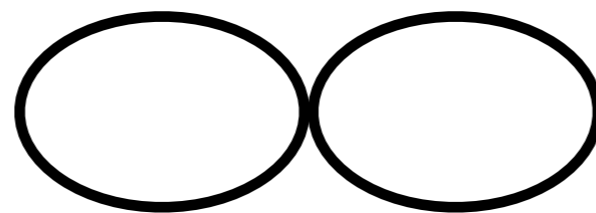
L = length of string

2 strings

$$\longrightarrow \frac{L}{2} \cdots + \frac{1}{N} \infty$$

1 string

$$\longrightarrow \frac{L}{2} \infty + \frac{1}{N} \cdots$$

$$W_C = \mathrm{Tr}(U_{\mu,\vec{x}} M_C) \qquad W_{C'} = \mathrm{Tr}(U_{\mu,\vec{x}} M_{C'})$$
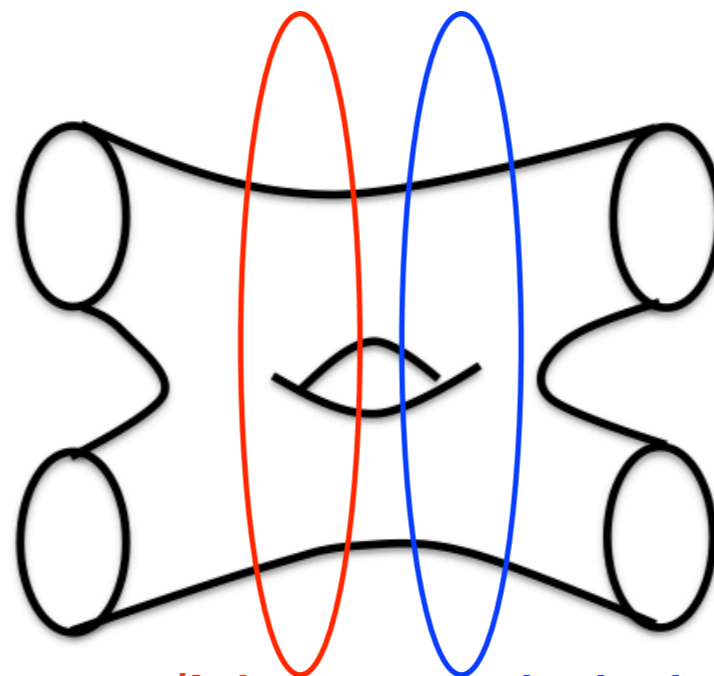
$$H|W_C, W_{C'}\rangle$$

$$= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle$$

$$+ \lambda N \sum_\alpha \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_C) \cdot \mathrm{Tr}(\tau^\alpha U_{\mu,\vec{x}} M_{C'})|0\rangle$$

$$= \frac{\lambda(L+L')}{2}|W_C, W_{C'}\rangle + \frac{\lambda}{N}\mathrm{Tr}(U_{\mu,\vec{x}} M_C U_{\mu,\vec{x}} M_{C'})|0\rangle$$
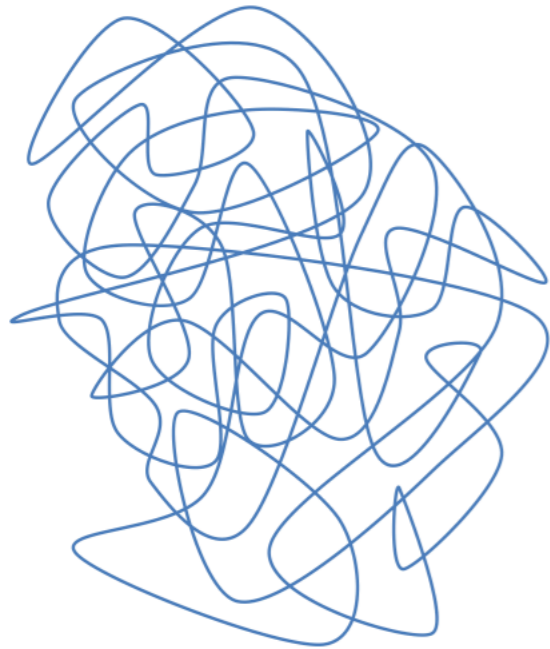

splitting $\sim$ 1/N   joining $\sim$ 1/N

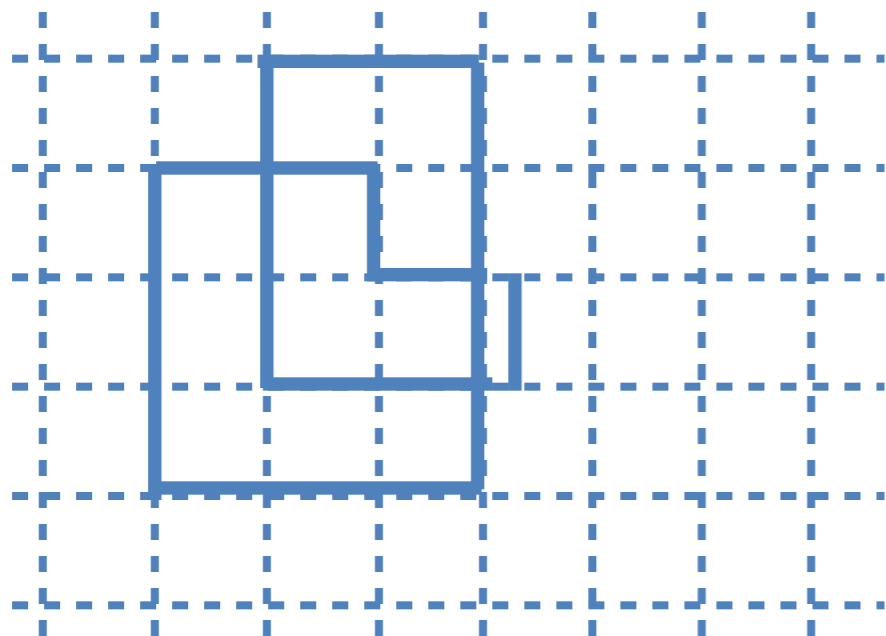# D-dim square lattice at strong coupling

deconfining phase = long string

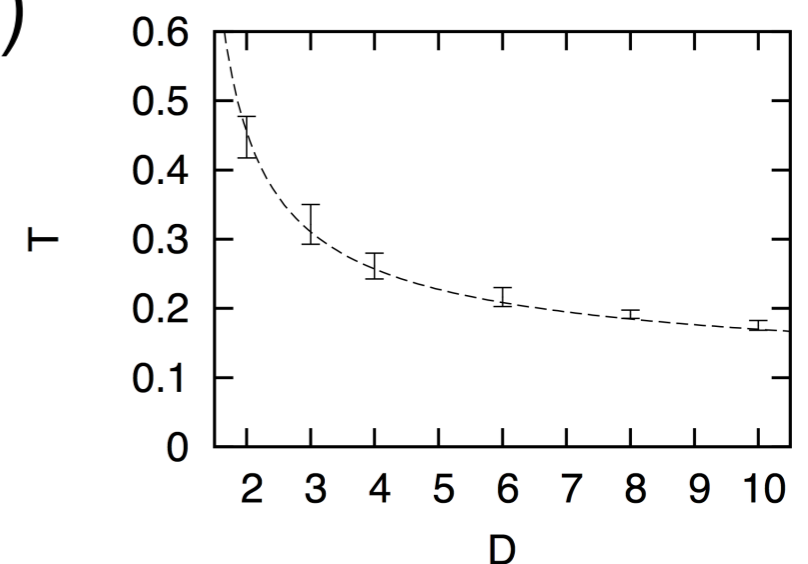huge mass and entropy are packed
in a small region → BH

$$E = L(T)/2, \quad S = L(T) \times \log(2D-1)$$

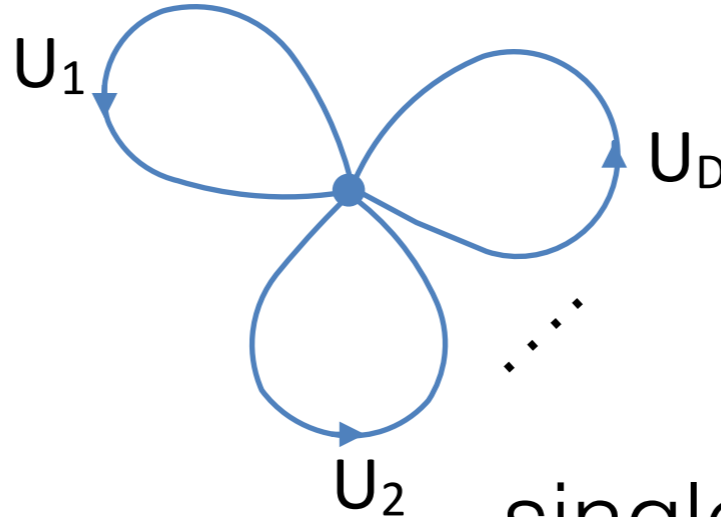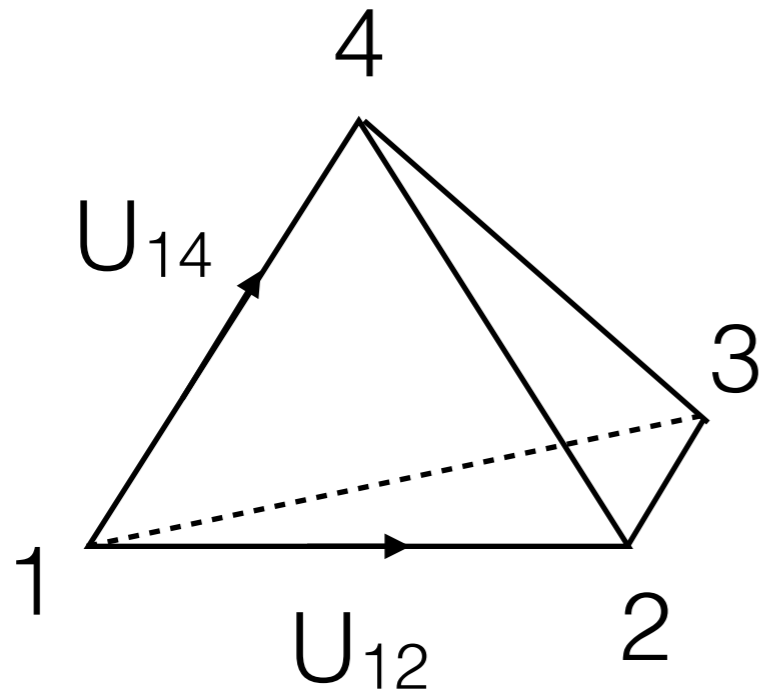$$F = E - TS = L(T) \times (1/2 - T \times \log(2D-1))$$

$$L \sim N^2$$

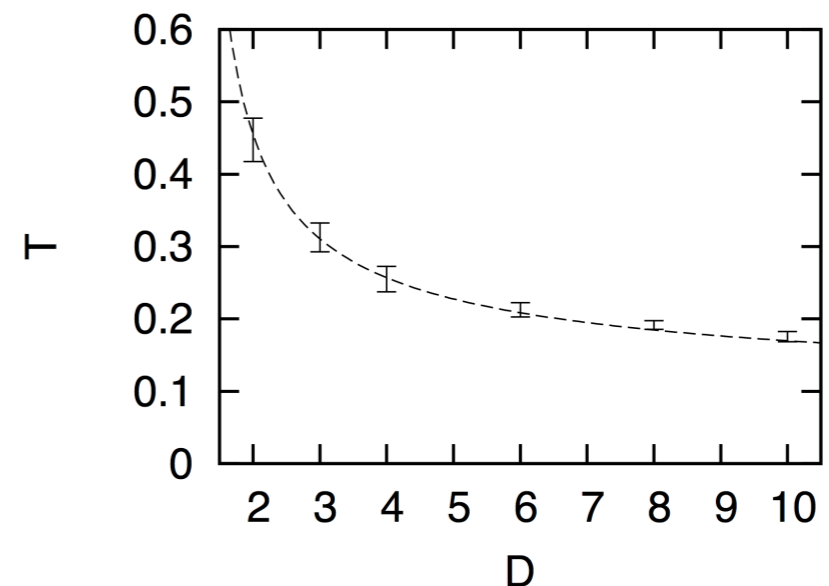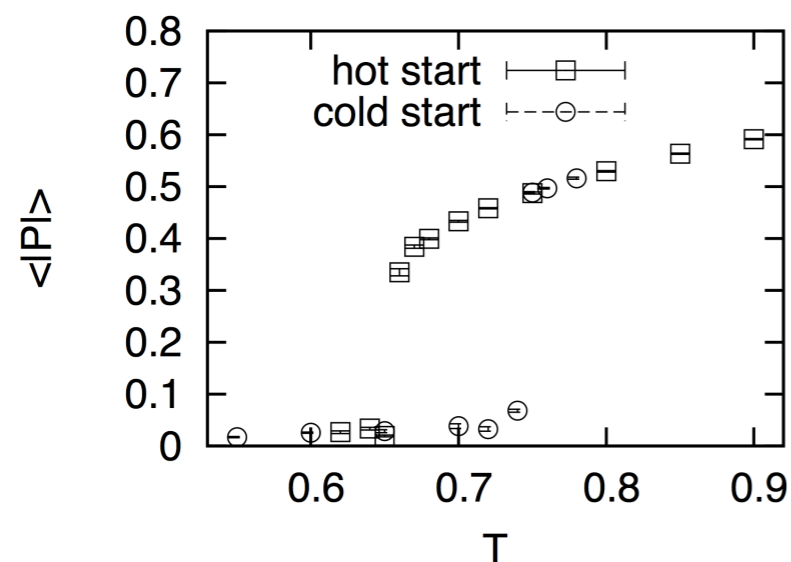$$T_c = \frac{1}{2\log(2D-1)}$$

matrix models at strong coupling

$$T_C = \frac{1}{2\log(2D-1)}$$

single-site with D-links
(Eguchi-Kawai model)

tetrahedron $\quad T_C = \frac{1}{2\log 2} = 0.72\ldots$

(Equivalent to large-volume lattice
via Eguchi-Kawai equivalence)

Why L $\sim$ N²?

- Tr(UU'U''…..)

  length $\gtrsim$ N² ➡️ factorizes to shorter traces

  N² is the upper bound.
  Beyond there, the counting changes;
  <u>not much gain for the entropy.</u>