Dynamical QCD+QED simulation with staggered quarks

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Outline

- Introduction to QED effects in lattice QCD studies
- Dynamical QCD+QED simulation in the RHMC algorithm
- Meson spectrum tests on the dynamical QED+QED ensembles
- Future plan

QED effects relevant to lattice QCD studies

QED effects play an important role in many topics studied via lattice QCD:

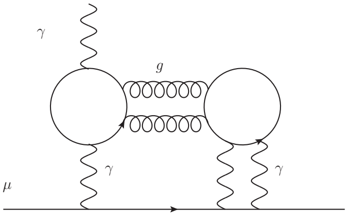
- The EM interaction contributes to isospin breaking in the meson and baryon mass spectrum. ($M_{\pi^+} - M_{\pi^0}$, $M_{K^+} - M_{K^0}$, $M_p - M_n$, etc.)
- Accurate determination of quark masses, e.g., m_u and m_d, requires us to account for electromagnetic effects.
- Hadron polarization from an external EM field
- Muon g-2
- Chiral magnetic effect

Many of these topics have been investigated with quenched-QED lattice calculations. More details can be found in many review talks

QED effects in lattice QCD studies

However, some questions are directly related to the sea quark QED effect.

- Pseudoscalar meson masses have a contribution from sea quark charges: $M^2(QED) = Y_1 \sum q_{\rm sea}^2 + ...$
- In the muon g-2 hadronic light-by-light (HLBL) calculation sea quarks interact with photons



Dynamical QCD+QED simulations directly include QED effects. QCDSF and BMW are also working on the dynamical QCD+QED simulation with non-compact QED. (arXiv:1311.4554, arXiv:1406.4088).

Compact and non-compact QED

Non-compact QED uses gauge potential $A_\mu \in (-\infty, \infty)$ as the variable to represent the U_1 field.

- $S_{\mathrm{QED}} = \frac{1}{4}\sum_{x,\mu,\nu}(\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x))^2$
- In the quenched QED approximation, the QED action can be written as a closed form of A_μ and sampled directly after gauge fixing. $\rightarrow$ All QED fields are generated independently.

Compact QED uses QED link U_μ as variable.

- $S_{\mathrm{QED}} = \beta\sum_{x,\mu,\nu}(1 - \Box_{\mu\nu})$, where the $\Box_{\mu\nu}$ is the U_1 plaquette.
- The U_1 code can be in a similar form of the $SU(3)$ code. Easy to implement.
- The U_1 gauge fixing can be done apart from the RHMC evolution.

Dynamical QCD simulation using RHMC

An observable $\langle \hat{O} \rangle$ is given by:

$$\langle \hat{O} \rangle = \frac{1}{Z(\beta)} \int \prod_{x,\mu} dU_\mu(x) \hat{O} (\det M_F)^\delta \exp\{-S_G\} \tag{1}$$

We generate gauge field U with probability distribution:

$$P_U = \frac{1}{Z(\beta)} [\det M_F(U)]^\delta \exp\{-S_G(U)\} = \frac{1}{Z} \exp\{-S_{\text{eff}}(U)\} \tag{2}$$

$$S_{\text{eff}} = S_G(U) + \delta \mathrm{Tr} \ln M_F(U) \tag{3}$$

We add a conjugate momentum p to S_{eff} to form a effective Hamiltonian

$$H(p, U) = \frac{p^2}{2} + S_{\text{eff}}(U) \tag{4}$$

The evolution of the system is given by Hamilton's equations

$$\begin{cases} \dot{U} &= p \\ \dot{p} &= -\frac{\partial S_{\text{eff}}}{\partial U} \end{cases} \tag{5}$$

Dynamical QCD simulation using RHMC

The force term is calculated by:

$$\frac{\partial S_{\text{eff}}}{\partial U_\mu} = \frac{\partial S_G}{\partial U_\mu} - \delta \mathrm{Tr}\left[\frac{\partial M_F(U)}{\partial U_\mu} M_F^{-1}(U)\right] \tag{6}$$

Use pseudo-fermion ϕ to calculate the fermion determinant:

$$S_{\text{eff}} = S_G(U) + \Phi^+ M_F^{-1} \Phi \tag{7}$$

$$\frac{\partial S_{\text{eff}}}{\partial U_\mu} = \frac{\partial S_G}{\partial U_\mu} - \Phi^+ M_F^{-1}(U) \frac{\partial M_F(U)}{\partial U_\mu} M_F^{-1}(U) \Phi \tag{8}$$

Gauge force and fermion force are used to update the conjugate momentum p.

Dynamical QCD+QED simulation using RHMC

Changes in the dynamical QCD+QED configuration generation code:

- We start from the MILC dynamical QCD configuration generation code.
- Add QED field link (U_μ^{QED}) and its conjugate momentum to lattice site object.
- Add QED contribution to the total action.
  $S = S_G^{\mathrm{QCD}} + S_G^{\mathrm{QED}} + \Phi^+ M_F^{-1}(U)\Phi$.
  The new fermion determinant includes both of the QCD and QED effects.
- Add functions to update QED field(U_μ^{QED}) and its conjugate momentum with RHMC algorithm.
- Change the fermion force in both QED and QCD momenta update function.

Some remarks regarding QCD+QED code

- Fermion can "see" both QCD and QED fields. The total gauge link is:

$$U_\mu(x) = U_\mu^{QCD}(x) U_\mu^{QED}(x) \ . \tag{9}$$

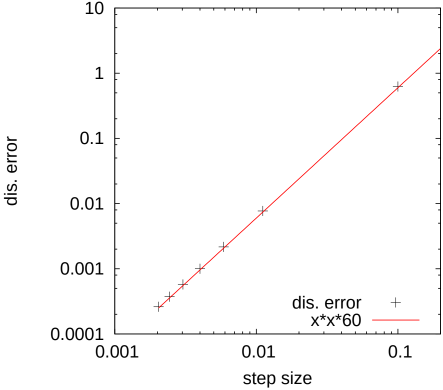
The total link $U_\mu(x)$ is the new link variable we smear in the code. This is also used in many non-compact lattice QCD + quenched QED studies.

- The QCD equations of motion are still the same as before. For QED field, we have

$$\begin{aligned} \dot{U}_\mu^{QED} &= iH_\mu^{QED}(x) U_\mu^{QED}(x) \qquad (10) \\ \dot{H}_\mu^{QED} &= iU_\mu^{QED}(x) \frac{\partial S_{\text{eff}}(U)}{\partial U_\mu^{QED}} = iU_\mu(x) \frac{\partial S_{\text{eff}}(U)}{\partial U_\mu}\Big|_{\text{Trace}} \qquad (11) \end{aligned}$$

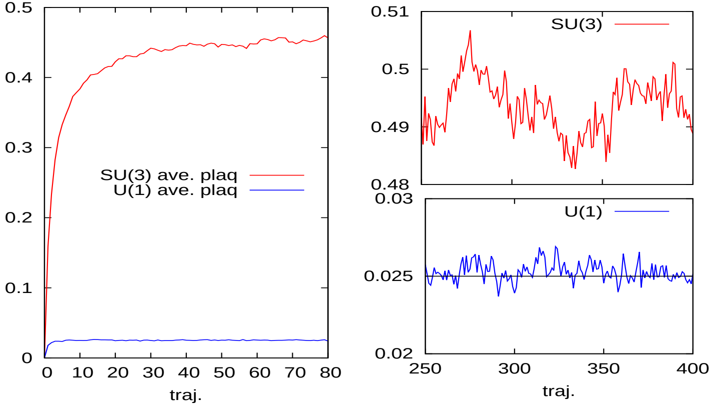
- This algorithm works for different smearing methods(OL, OFN, asqtad...).

RHMC Integrator Test



We run the QCD+QED code with a fixed trajectory length = (number of steps) x (step size). We plot the change of the Hamiltonian vs. step size using the leapfrog algorithm.

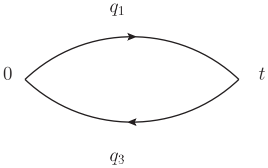
Evolution of the gauge configuration



The evolution of the average $SU(3)$ and $U(1)$ plaquette. The $SU(3)$ average plaquette in the right plot is shifted downwards. The black horizontal line is the theoretical predicted value in the weak coupling limit.

Meson spectrum from dynamical QCD+QED simulation

- Pseudoscalar meson spectrum can be measured accurately.
- The spectrum studies tell us the basic information about the lattice ensembles generated.
- We only consider electric charge neutral meson here(here in QCD, we have color neutral meson), because we have not fixed the $U(1)$ gauge. Meson correlator is from:



- The total EM charge is $q_{13} = q_1 - q_3$. $q_{1,3}$ can be $q_{u,d,s}$. The convention is: charge neutral meson $q_{13} = 0$

Meson spectrum from dynamical QCD+QED simulation

- The ChPT description on the total mass from QCD+QED is (with fixed quark masses,):

$$
\begin{aligned}
m^2 &= m^2(\mathrm{QCD}) + m^2(\mathrm{QED}) && (12)\\
m^2(\mathrm{QED}) &= Aq_{\mathrm{val}}^2 + Bq_{val}q_{\mathrm{sea}} + Cq_{\mathrm{sea}}^2 && (13)\\
\delta m^2 &= m^2(e_{\mathrm{val}} \neq 0) - m^2(e_{\mathrm{val}} = 0) && (14)\\
&\propto e^2 \propto \alpha_{\mathrm{EM}} && (15)
\end{aligned}
$$

meson spectrum on QCD+QED ensembles

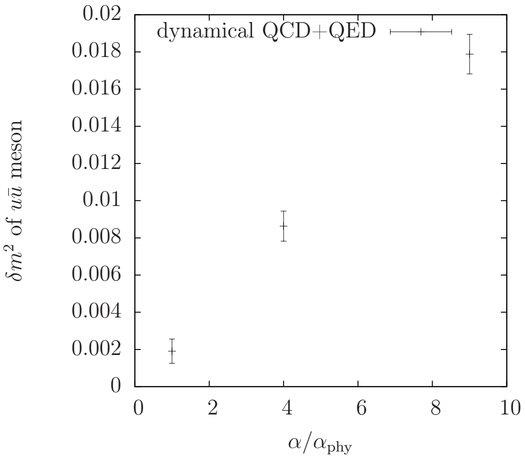


Figure : The δm^2 measured on $12^3 \times 32$, β_{QCD}=5.5 lattices. No smearing is used on the staggered fermion.

Plan of the future work

- We will continue small test runs.
- Tune the input parameters *beta*, charge, masses, etc.
- Physics projects on the new QCD+QED ensembles, LEC fit, mass splittings, decay constants, g-2 etc.
- Port to GPU platform.