Fermion mass generation without a chiral condensate?

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Introduction

- Symmetries can forbid the inclusion of mass terms.
- Chiral symmetries $\Longrightarrow$ massless fermions.
- Can interactions that preserve these symmetries generate fermion masses dynamically ?

Conventionally by Spontaneous Symmetry Breaking signaled through a non-zero chiral condensate

Can we achieve fermion mass generation without a chiral condensate ?

$\downarrow$

We have a lattice model in 3D in which a 4 fermion interaction makes this is possible

Our Lattice model

Staggered fermion action for two flavors ψ^1 and ψ^2 in 3D :

$$S_0 = \sum_{x,y} \left\{ \overline{\psi^1_x} D_{x,y} \psi^1_y + \overline{\psi^2_x} D_{x,y} \psi^2_y \right\}$$

All fields are Grassmann fields.

Notice the similarity to the Dirac Action !

where ,

$$D_{x,y} = \frac{1}{2} \sum_{\widehat{\alpha}} \{ \delta_{x,y+\widehat{\alpha}} - \delta_{x,y-\widehat{\alpha}} \} \eta_{\alpha,x}$$

$$\eta_{1,x} = 1 ; \eta_{2,x} = (-1)^{x_1} ; \eta_{3,x} = (-1)^{x_1 + x_2}$$

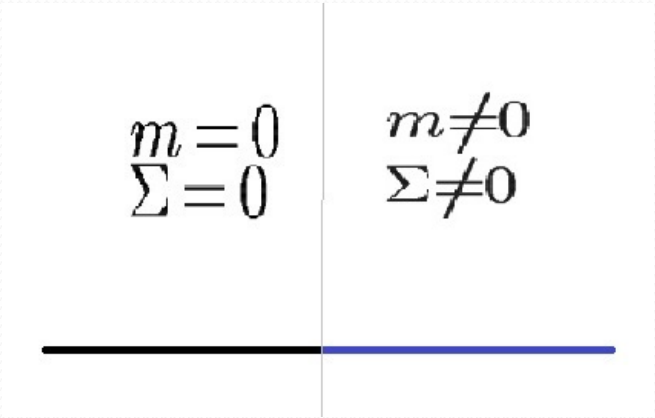
Four-fermion Interaction :

$$S_I = -U \sum_x \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x$$

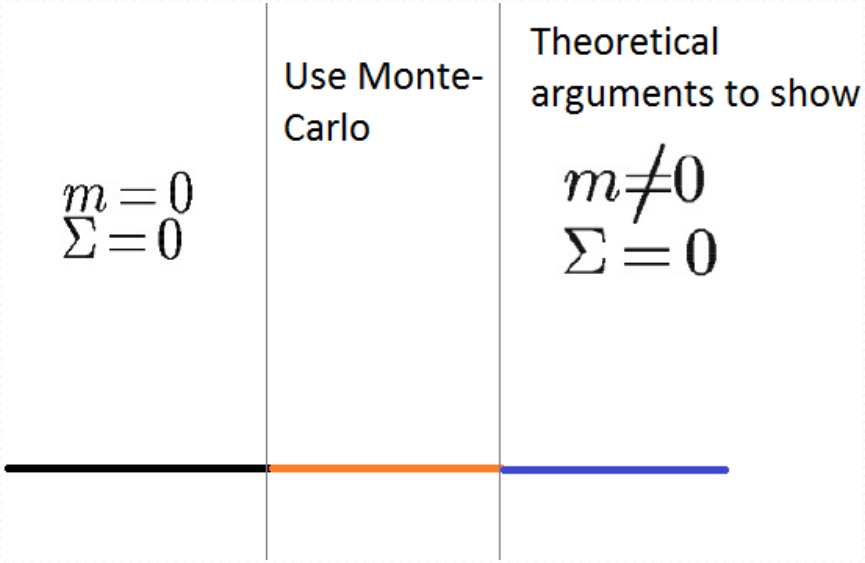
In addition to the usual discrete space-time symmetries*, the action has a continuous SU(4) symmetry.

Maarten F.L. Golterman and Jan Smit. Selfenergy and Flavor Interpretation of Staggered Fermions. Nucl.Phys., B245:61, 1984.

For four-fermion interactions, we expect*



We get



*NPB388, 1992, page 243 Bock, De and Smit
*NPB344, 1990, page 207 Bock et. al.,

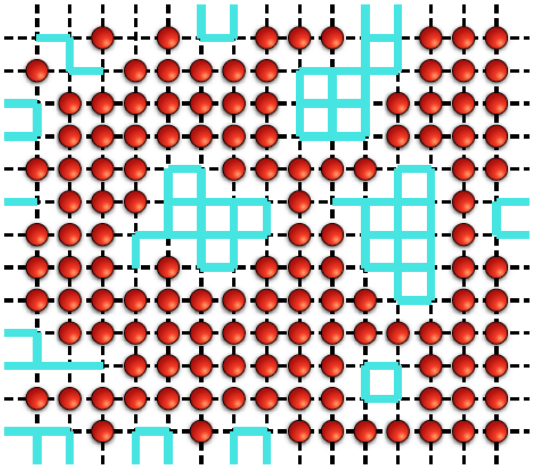
*The Fermion Bag approach

$$Z = \int [d\overline{\psi} d\psi] e^{-S_0} e^{U \sum_x \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x}$$

$$= \int [d\overline{\psi} d\psi] e^{-S_0} \prod_x e^{U \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x}$$

$$= \int [d\overline{\psi} d\psi] e^{-S_0} \prod_x [1 + U \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x]$$

$$= \int [d\overline{\psi} d\psi] e^{-S_0} \prod_x \sum_{m=0,1} \left(U \overline{\psi^1_x} \psi^1_x \overline{\psi^2_x} \psi^2_x \right)^{m_x}$$



Fermion Bag configuration

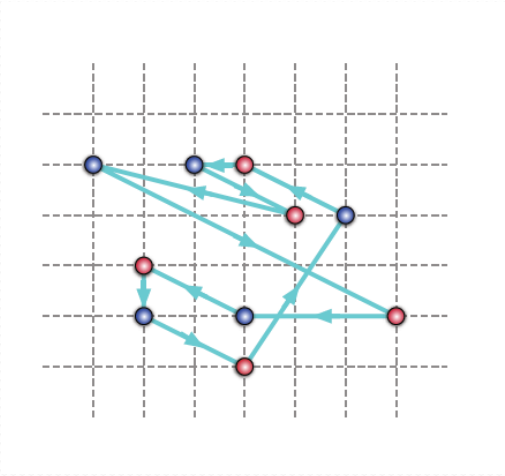
Assigning m_x= 0 or 1 to each lattice site

- m_x= 0 ≡ ***free sites***
- m_x= 1 ≡ ***monomers***

*S Chandrasekharan and Anyi Li. The generalized fermion-bag approach. Procd of Sc., LATTICE2011:058, 2011.
Shailesh Chandrasekharan. The Fermion bag approach to lattice field theories. PRD 82:025007, 2010.

Weak coupling limit

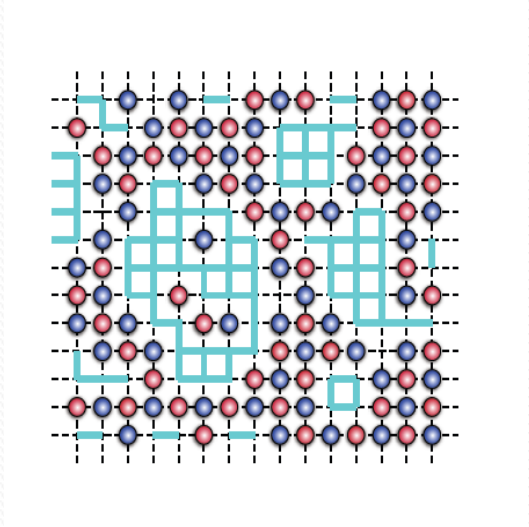
$$Z = Det[D] \sum_{\{m_x\}} U^k Det(D_1^{-1}) Det(D_1^{-1})$$



D^{-1} kxk matrix of propagators.

Strong coupling limit

$$Z = \sum_{\{m_x\}} U^k Det(W_1) Det(W_1)$$



W_1 is a
(V-k) x (V-k) matrix

- Can show that each determinant can be expressed as a square of smaller determinants.

Extracting the condensate Σ

Σ is usually defined as:

$$\Sigma = \lim_{m \to 0} \lim_{V \to \infty} \langle \overline{\psi} \psi \rangle$$

With massless fermions, we can instead compute the bosonic susceptibility :

$$\chi = \frac{1}{2\,V} \sum_{x,y} \left\langle \overline{\psi^1}_x \psi^1{}_x \overline{\psi^1}_y \psi^1{}_y \right\rangle$$

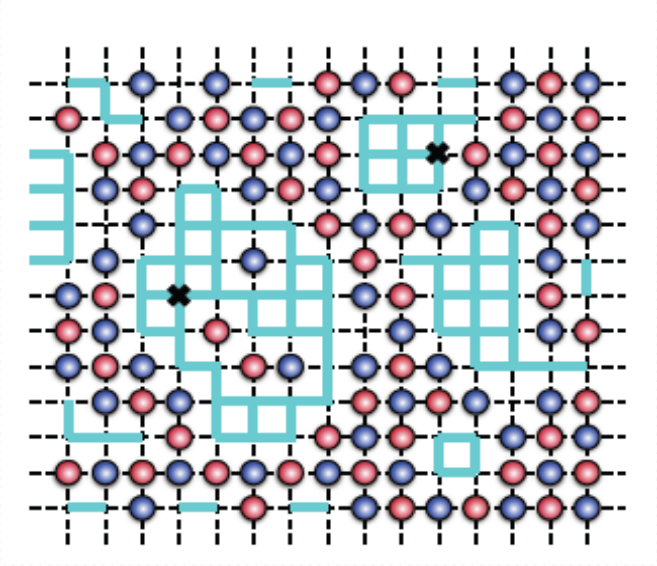
$$= \frac{V}{2} \Sigma^2 + \text{constant}$$

$\chi = \text{const.} \Rightarrow \text{condensate} = 0$

Fermionic correlator

$$\langle \overline{\psi^1_x} \psi^1_y \rangle = \frac{1}{Z} Det(W) W^{-1}{}_{x,y}$$

- W can be written in terms of bags in block diagonal form.
- Can show that the correlator is zero unless x and y belong to the same bag.



W *and* W^{-1} are block diagonal

$$\begin{bmatrix} A & o & o & o \\ o & B & o & o \\ o & o & C & o \\ o & o & o & D \end{bmatrix}$$

Need for a path of free sites connecting o & t for a non-zero correlator G(x,y) !

Bosonic correlator

$$\langle \overline{\psi^1_x} \psi^1_x \overline{\psi^1_y} \psi^1_y \rangle = \frac{1}{Z} Det(W_1)\ \mathrm{Det}(\mathrm{W}_2)$$

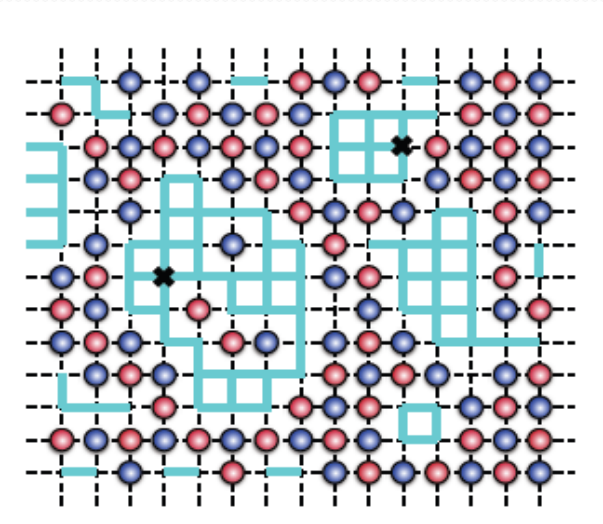
Since, for a non-zero determinant,
Number of even sites = Number of odd sites

Can show that,

$$Det(W_1) \neq 0 \ \Rightarrow Det(W_2) = 0$$

and

$$Det(W_2) \neq 0 \ \Rightarrow Det(W_1) = 0$$



The two flavors ensure :

Need for a path of free sites connecting o & t for a non-zero correlator G(o,t) !

Fermion Mass

- Small U → Irrelevant coupling

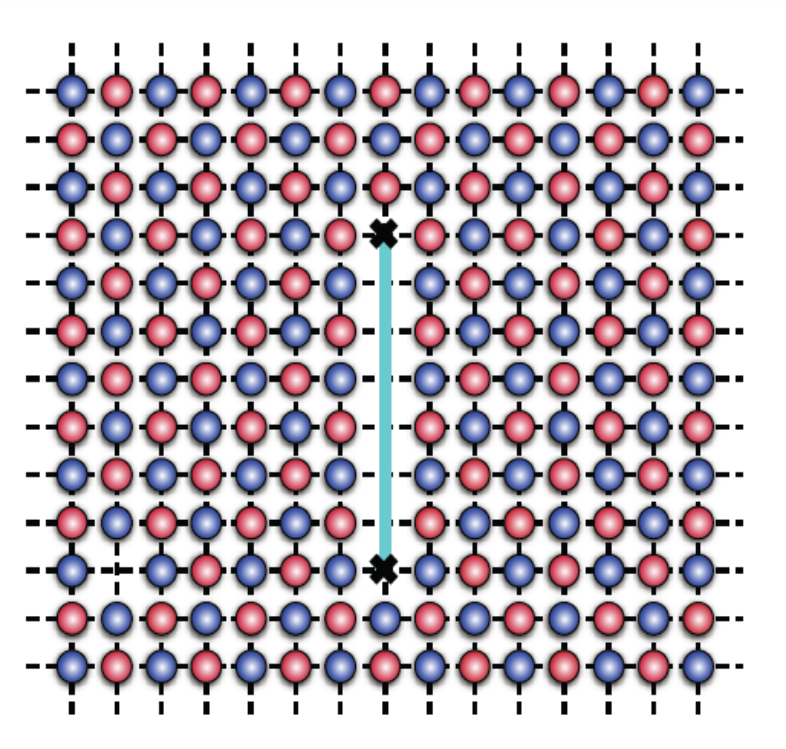
⇒ massless fermions.

- For very large U

$G(t) \sim e^{-(y-x)\ \ln U} \sim e^{-m(y-x)}$

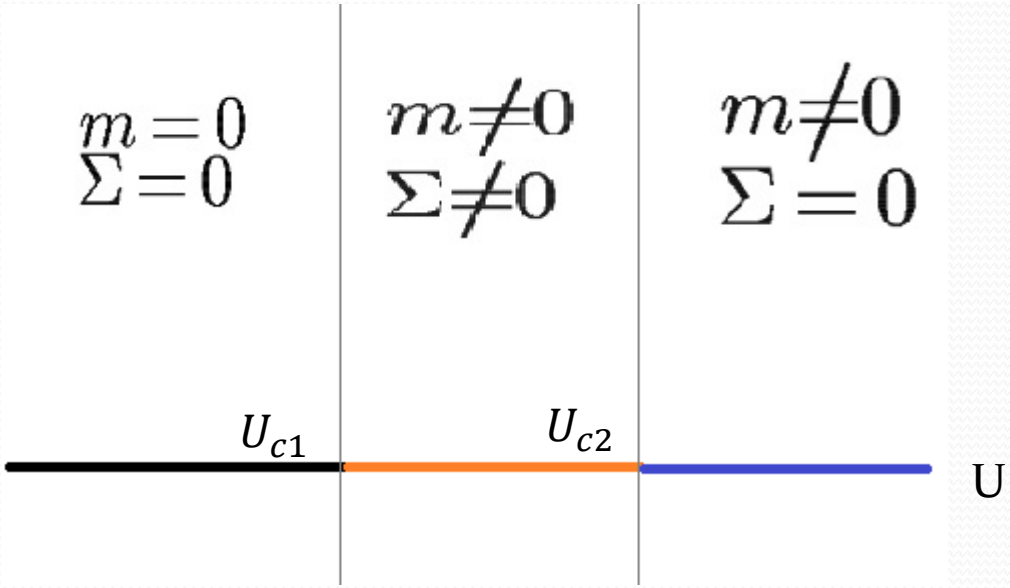
⇒ massive fermions.

- Exponential decay of all correlators indicates a zero condensate at very large U.

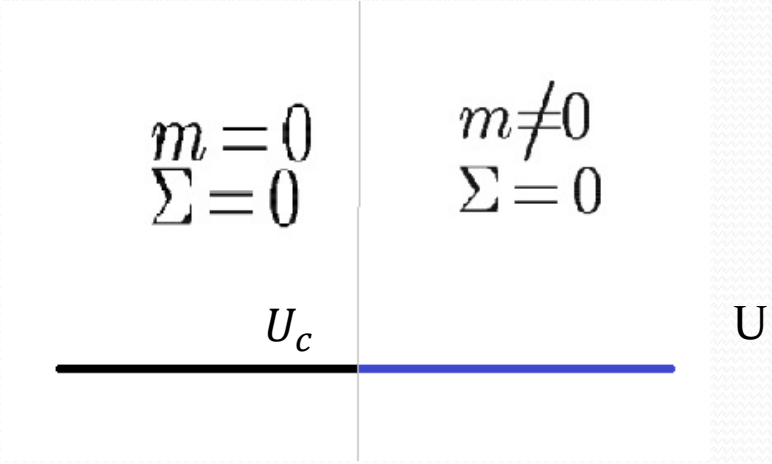


Phase Transition

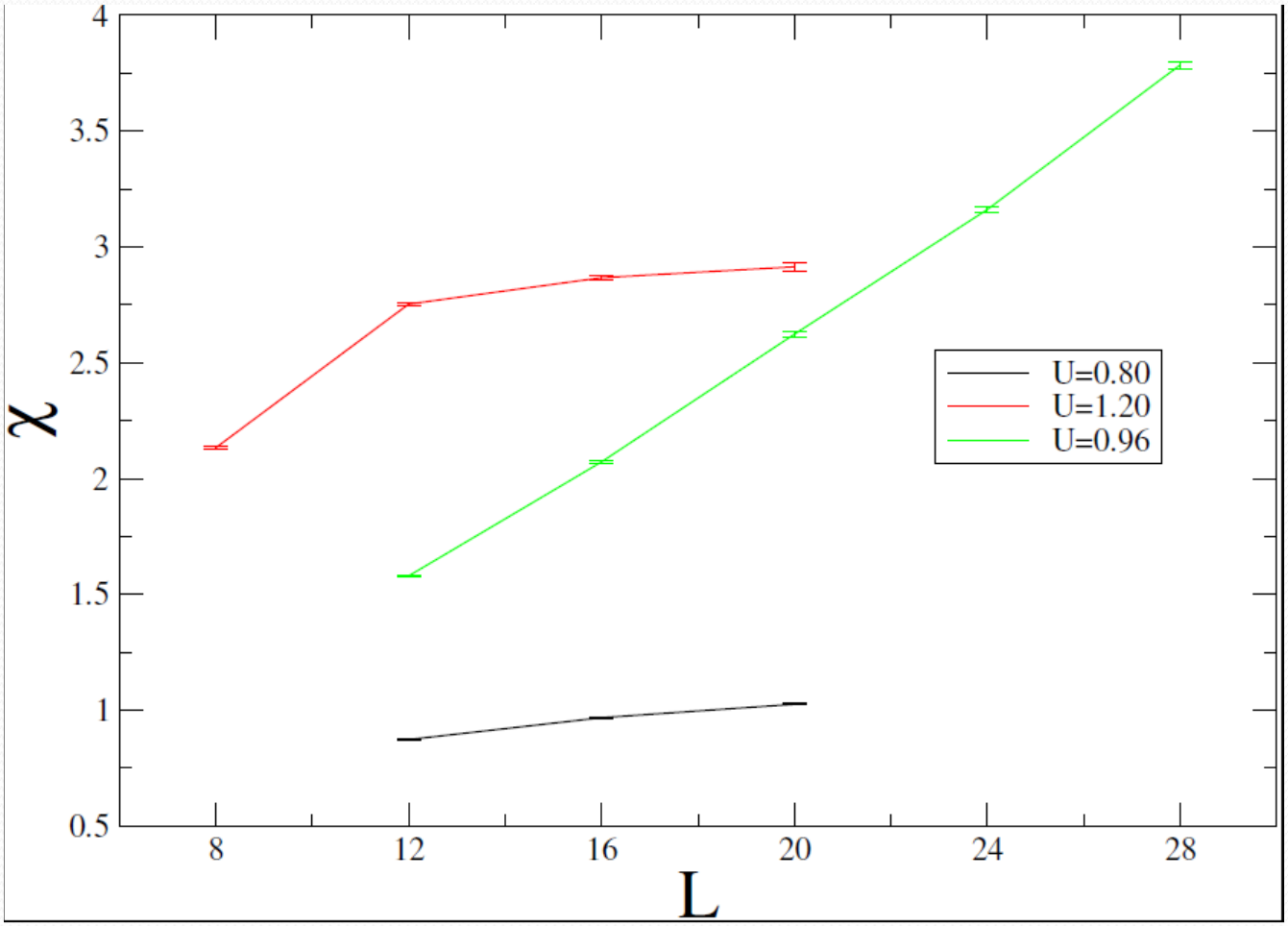
Expectation from traditional understanding



We seem to observe



Preliminary results



Have used Open Science Grid (OSG) for computations

χ = const. ⇒ condensate = 0

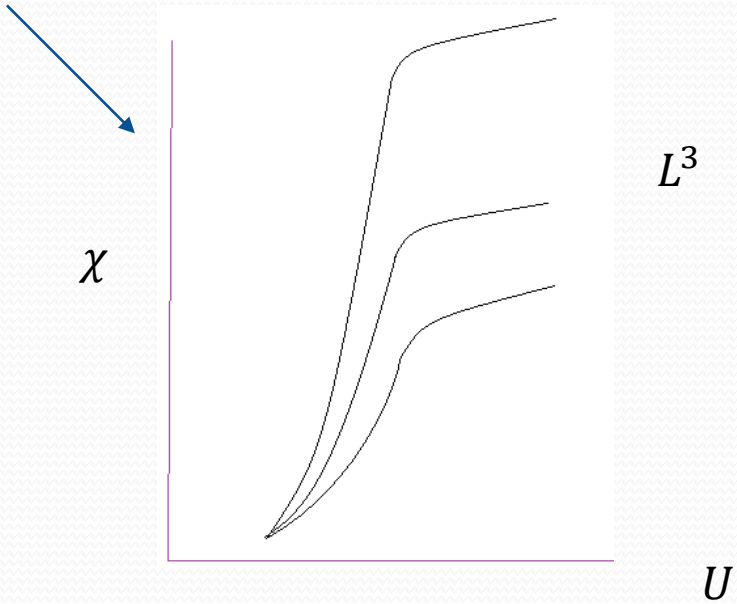
$Near\ \ U_c = 0.96,$
$\chi\ grows\ with\ L$

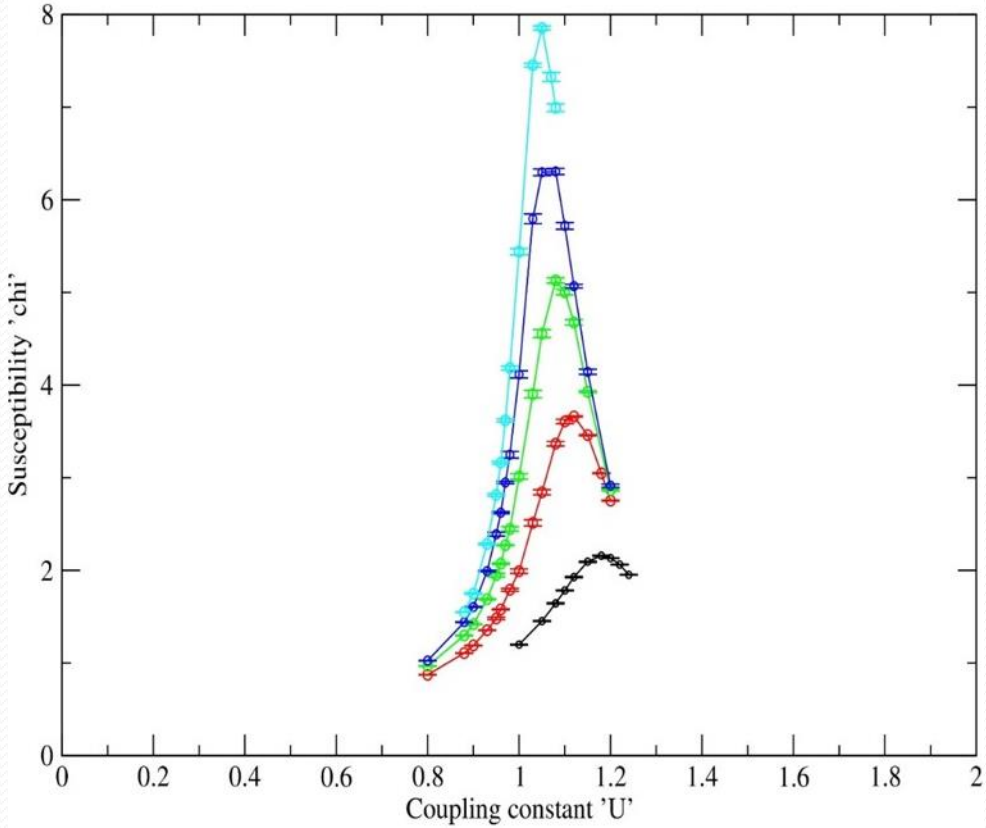
Susceptibility saturates indicating a zero condensate

Results contd.

- Susceptibility reaches a maximum for intermediate U
- Then decreases and then saturates.

Instead of

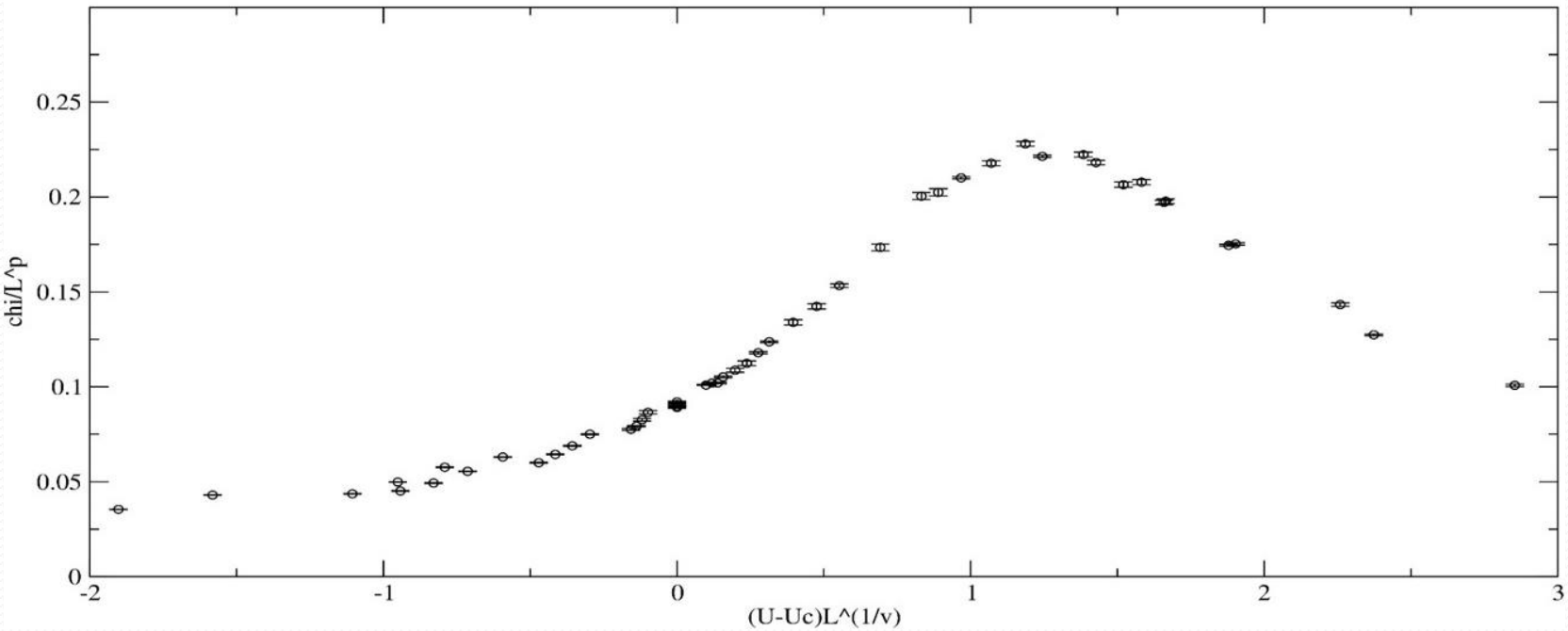




susceptibility vs coupling U for cubical lattices of length 8,12,16,20,24

Evidence that Uc is a 2nd order critical point

For a second order transition we expect : $\chi = L^p \; f\left[(U - U_c) L^{\frac{1}{v}} \right]$



Plot of $\chi \,/\, L^p vs \;\; (U - U_c) L^{\frac{1}{v}}$

Errors too small ?

Preliminary calc. of Critical exponents :

$$\eta = 0.878(1), \qquad v = 1.21(2), U_c = 0.958\ (2)$$

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Caveats

- We do observe some rare large fluctuations in data for large U. Are these statistically significant ?
- Results using two very different Monte-Carlo algorithms seem to give consistent results.
- Considering the unconventional nature of the result, we are trying to develop another algorithm that gives results without fluctuations.

Conclusions

- We have found a lattice model in 3D, in which fermions acquire a mass at large couplings, but without a fermion bilinear condensate.

  Possibly no spontaneous symmetry breaking ?
- The transition from massless to massive phase seems second order. If true, we could have an interesting 3D continuum field theory.
- Similar result in 4D could be exciting for particle physics.

Thank You

Back up slides

S_0 can be written as :

$$\begin{pmatrix} \overline{\psi^1_{x,e}} & \psi^1_{x,e} & \overline{\psi^2_{x,e}} & \psi^2_{x,e} \end{pmatrix} \begin{pmatrix} M & 0 & 0 & 0 \\ 0 & M & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & M \end{pmatrix} \begin{pmatrix} \psi^1_{x,o} \\ \overline{\psi^1_{x,o}} \\ \psi^2_{x,o} \\ \overline{\psi^2_{x,o}} \end{pmatrix}$$

$\Rightarrow S_0$ invariant under SU(4)

At every site, $\overline{\psi^1_x}\psi^1_x\overline{\psi^2_x}\psi^2_x$ S_I invariant under SU(4)

Thus the SU(4) symmetry is preserved by the interaction.