Study of Kinematic Reconstruction Methods for an Electron-Ion Collider (EIC)

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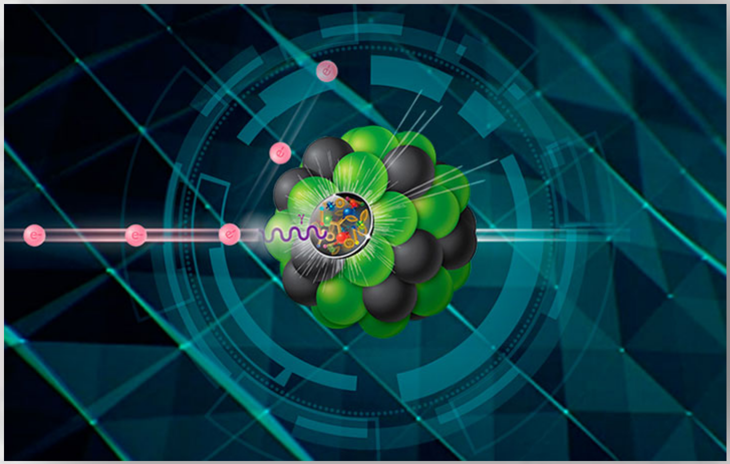

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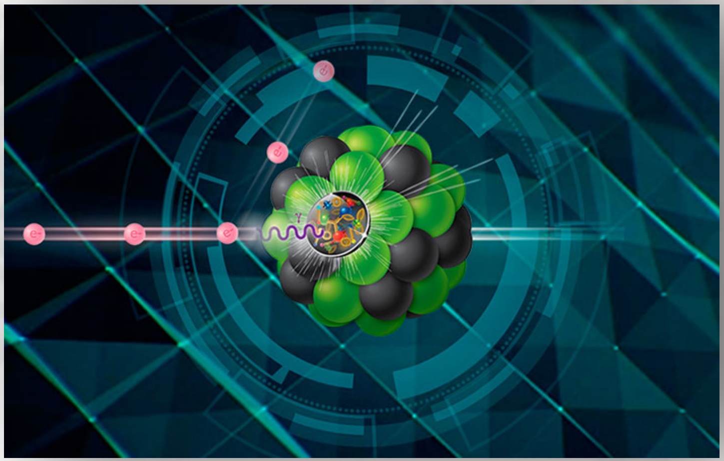


DIS Relativistic Kinematics

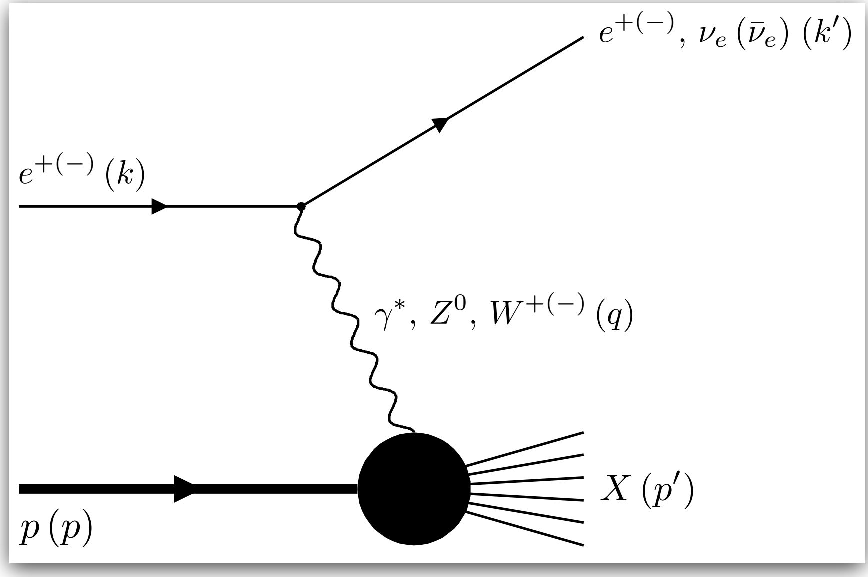
Basics

- The relativistic invariant description of any type of particle collision is an essential element for any modern high-energy collider experiment, in particular involving hadron beams.
- Standard Model electroweak gauge boson exchange process in ep collisions:

$$e(k) + P(p) \to l(k') + X(p')$$

- Relativistic invariant variables: s, t, u, Q^2, x, y and W^2:

$$\begin{aligned}
s &= (k+p)^2 \simeq 4E_e E_P \\
t &= (p-p')^2 \\
u &= (k'-p)^2 \\
Q^2 &= -(k-k')^2 = -(p-p')^2 = -t = -q^2 \\
x &= \frac{Q^2}{2(p\cdot q)} \simeq -\frac{t}{u+s} \qquad 0 \le x \le 1 \\
y &= \frac{p\cdot q}{p\cdot k} \simeq \frac{u+s}{s} \qquad 0 \le y \le 1 \\
W^2 &= (p+q)^2 = (p')^2 = m_p^2 + \frac{Q^2}{x}(1-x) \simeq s+t+u
\end{aligned}$$



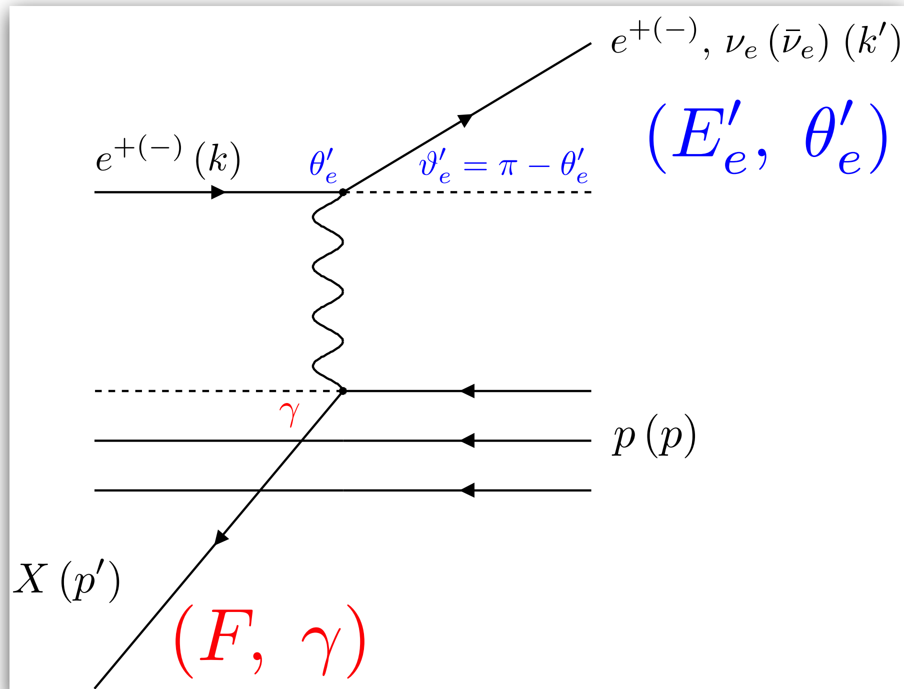
$$Q^2 \simeq s \cdot x \cdot y$$

ep/eA Collider Observables

- Four-vector kinematics
  - Starting from the process discussed in section 1, ep/eA scattering is viewed as the scattering of a lepton (e) with a parton resulting in a scattered lepton (e) (Energy E' and polar angle θ') and a scattered parton (Energy F and polar angle γ) in the final state:

$$k = \begin{pmatrix} E_e \\ 0 \\ 0 \\ -E_e \end{pmatrix}$$

$$p' = \begin{pmatrix} \sum_h E_h \\ \sum_h p_{X,h} \\ \sum_h p_{Y,h} \\ \sum_h p_{Z,h} \end{pmatrix} = \begin{pmatrix} F \\ F \sin\gamma \cos\beta \\ F \sin\gamma \cos\beta \\ F \cos\gamma \end{pmatrix}$$



$$k' = \begin{pmatrix} E'_e \\ E'_e \sin\theta'_e \cos\phi'_e \\ E'_e \sin\theta'_e \sin\phi'_e \\ E'_e \cos\theta'_e \end{pmatrix}$$

$$p = \begin{pmatrix} E_P \\ 0 \\ 0 \\ E_P \end{pmatrix}$$

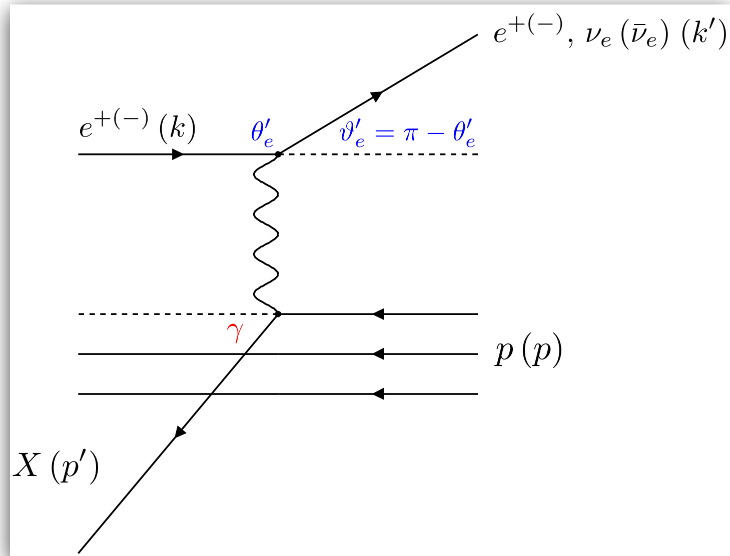
  - The positive direction by convention is taken to be the proton beam direction.
  - Measured quantities: E', θ', F and γ

Formulation of DIS Kinematic Variables

Reconstruction methods 1

- Inclusive DIS processes require two independent variables such y and Q^2. Those can be formulated using any combination of the measured four observable variables: E', θ', F and γ

1. E', θ'
2. E', γ
3. E', F
4. θ', γ
5. θ', F
6. F, γ



- In fixed-target experiments, it is only method 1 which is used to reconstruct DIS kinematic variables. However, in a collider environment the ability to also reconstruct F and γ provides a means to optimize the reconstruction of kinematic variables through other combinations different from E' and θ'.

Formulation of DIS Kinematic Variables

- Reconstruction methods 2
  - Starting from the basic definition of y and Q^2, one arrives at the following expressions:

1

$$Q^2[E'_e,\theta'_e] = 2E_eE'_e(1+\cos\theta'_e)$$
$$y[E'_e,\theta'_e] = 1 - \frac{E'_e}{2E_e}(1-\cos\theta'_e)$$

2

$$Q^2[E'_e,\gamma] = 4E_e^2(y[E'_e,\gamma]-1) + 4E_eE'_e$$
$$y[E'_e,\gamma] = \frac{1}{2}\left(\left(1-\frac{E_e}{2E'_e}\right)(1-\cos\gamma) \pm \frac{E'_e}{E_e}\sqrt{\frac{1}{4}(1-\cos\gamma)^2 + \frac{E_e}{E'_e}\left(1-\frac{E_e}{E'_e}\right)\sin^2\gamma}\right)$$

3

$$Q^2[E'_e,F] = \frac{4E_e^2(E_e-F)}{E'_e+F-2E_e} + 4E_eE'_e$$
$$y[E'_e,F] = \frac{E'_e-E_e}{E'_e+F-2E_e}$$

4

$$Q^2[\theta'_e,\gamma] = 4E_e^2\frac{\sin\gamma(1+\cos\theta'_e)}{\sin\gamma+\sin\theta'_e-\sin(\gamma+\theta'_e)}$$
$$y[\theta'_e,\gamma] = \frac{\sin\theta'_e(1-\cos\gamma)}{\sin\gamma+\sin\theta'_e-\sin(\theta'_e+\gamma)}$$

5

$$Q^2[\theta'_e,F] = 4E_e^2(1-y[F,\theta'_e])\left(\frac{1+\cos\theta'_e}{1-\cos\theta'_e}\right)$$
$$y[\theta'_e,F] = 1 - \frac{1}{4E_e}(1-\cos\theta'_e)\left((2E_e-F) \pm \sqrt{(2E_e-F)^2 - 8E_e\frac{(E_e-F)}{(1-\cos\theta'_e)}}\right)$$

6

$$Q^2[F,\gamma] = \frac{F^2\sin^2\gamma}{1-y[F,\gamma]}$$
$$y[F,\gamma] = \frac{F}{2E_e}(1-\cos\gamma)$$

Topology of ep/eA Kinematics

- x-Q² plane 1
  - Q² as a function of x reveals substantial insight into the topology of ep events by considering their dependence on fixed quantities of E′, θ′, F or γ and by graphically displaying the final state lepton and struck quark both in terms of the magnitude of the energy and direction (Angles with respect to proton direction).
  - Q² as a function of x for fixed E′, θ′, F and γ:

$$Q^2[x,E'_e] = \frac{xs\left(1-\frac{E'_e}{E_e}\right)}{1-\frac{xs}{4E_e^2}}$$

Fixed E'_e

2GeV steps: 2GeV-100GeV

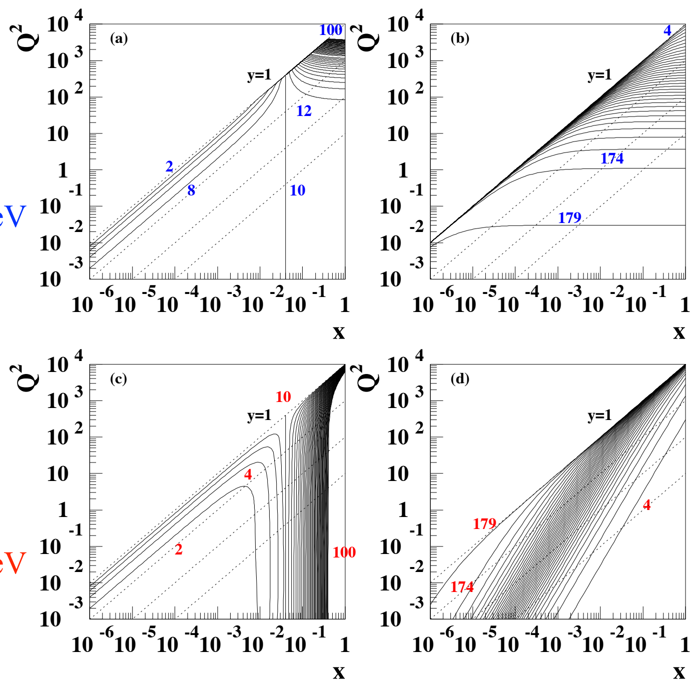
$E_e = 10\,\mathrm{GeV}$

$E_p = 250\,\mathrm{GeV}$

$$Q^2[x,F] = \frac{4E_eF - sx}{\frac{4E_e^2}{sx}-1}$$

Fixed F

2GeV steps: 2GeV-100GeV



Fixed θ'_e

2° steps: 4°-179°

$$Q^2[x,\theta'_e] = \frac{xs}{\frac{xs}{4E_e^2}\tan^2\frac{\theta'_e}{2}+1}$$

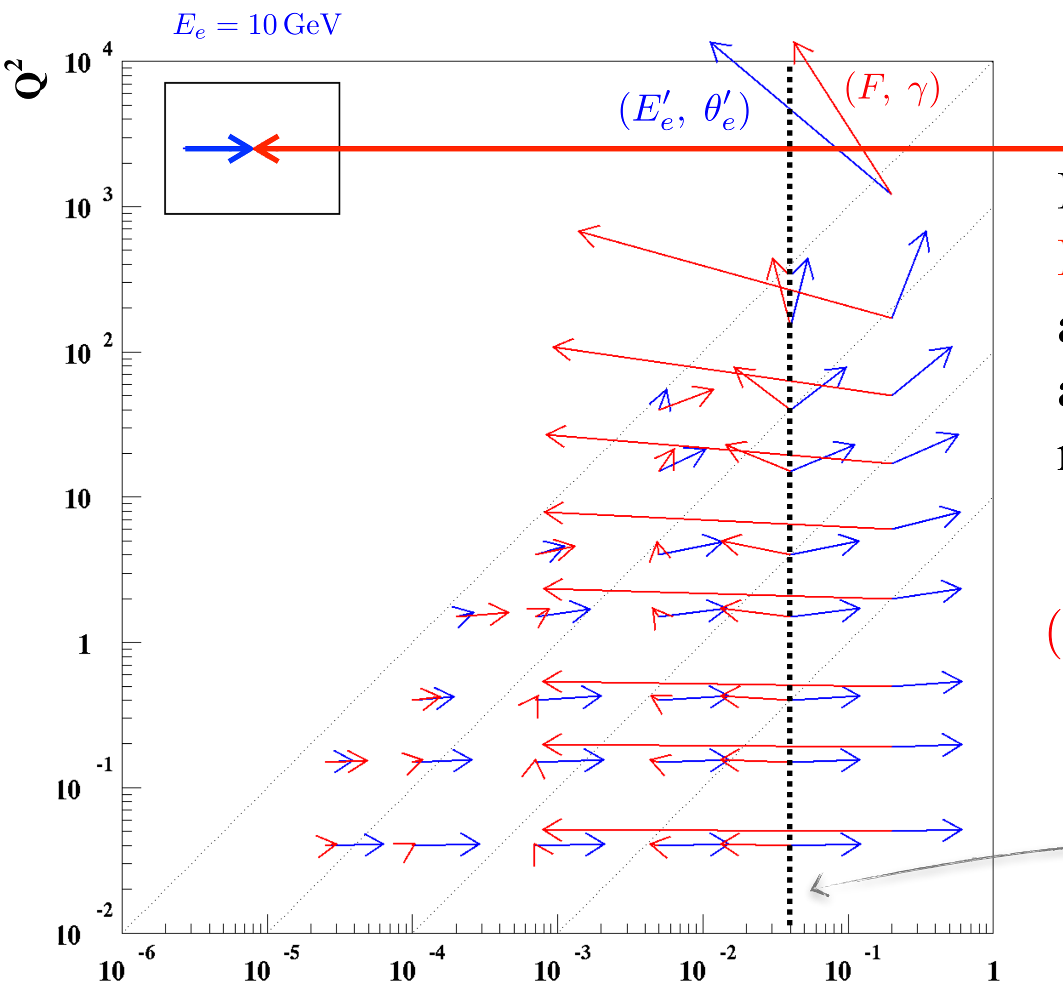
Fixed γ

2° steps: 4°-179°

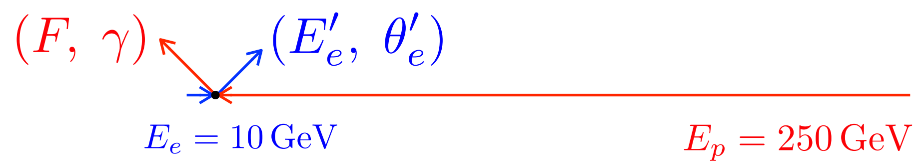
$$Q^2[x,\gamma] = \frac{sx}{\frac{4E_e^2}{sx}\cot^2\frac{\gamma}{2}+1}$$

Topology of ep/eA Kinematics

- x-Q² plane 2



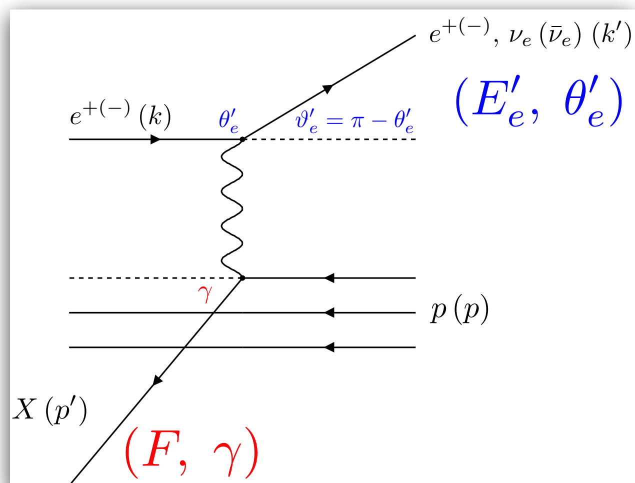
For fixed (x,Q²) points, calculate E', θ', F and γ and plot at each (x,Q²) point arrows of length E' under an angle θ' and arrows of length F under an angle γ, respectively.



Kinematic peak at $x=E_e/E_p=0.04$
where $E'=F=E_e=10$GeV

Resolution of DIS Kinematic Variables

- Error propagation 1
  - The event topology diagrams already show a clear issue for method 1 for large values of x where the resolution in x and y deteriorates. This is one of the main motivations to seek alternative reconstruction methods as evaluated in section 3.
  - Analytical error propagation for all reconstruction methods 1 - 6 have been determined. Study kinematic variable resolution from methods 2, 3, 4 and 5 have been studied and compared to methods 1 (E', θ') and 6 (F, γ).
  - Examples for methods 1 (E', θ') and 6 (F, γ):



Resolution of DIS Kinematic Variables

Error propagation 2

- Examples for methods 1 (E', θ') and 6 (F, γ):

$$\frac{\partial Q^2}{\partial E'_e} = 2E_e(\cos\theta'_e + 1) \qquad \frac{\partial y}{\partial E'_e} = \frac{\cos\theta'_e - 1}{2E_e}$$

$$\frac{\partial Q^2}{\partial \theta'_e} = -2E_e E'_e \sin\theta'_e \qquad \frac{\partial y}{\partial \theta'_e} = -\frac{E'_e \sin\theta'_e}{2E_e}$$

$$(\delta Q^2)^2 = \left(\frac{\partial Q^2}{\partial E'_e}\right)^2 (\delta E'_e)^2 + \left(\frac{\partial Q^2}{\partial \theta'_e}\right)^2 (\delta\theta'_e)^2$$

$$(\delta y)^2 = \left(\frac{\partial y}{\partial E'_e}\right)^2 (\delta E'_e)^2 + \left(\frac{\partial y}{\partial \theta'_e}\right)^2 (\delta\theta'_e)^2$$

$$\frac{\partial Q^2}{\partial F} = \frac{F^2 \sin^2\gamma(1-\cos\gamma)}{2E_e(1-\frac{F(1-\cos\gamma)}{2E_e})^2} + \frac{2F\sin^2\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} \qquad \frac{\partial y}{\partial F} = \frac{1-\cos\gamma}{2E_e}$$

$$\frac{\partial Q^2}{\partial \gamma} = \frac{F^3 \sin^3\gamma}{2E_e(1-\frac{F(1-\cos\gamma)}{2E_e})^2} + \frac{2F^2\sin\gamma\cos\gamma}{1-\frac{F(1-\cos\gamma)}{2E_e}} \qquad \frac{\partial y}{\partial \gamma} = \frac{F\sin\gamma}{2E_e}$$

$$(\delta Q^2)^2 = \left(\frac{\partial Q^2}{\partial F}\right)^2 (\delta F)^2 + \left(\frac{\partial Q^2}{\partial \gamma}\right)^2 (\delta\gamma)^2$$

$$(\delta y)^2 = \left(\frac{\partial y}{\partial F}\right)^2 (\delta F)^2 + \left(\frac{\partial y}{\partial \gamma}\right)^2 (\delta\gamma)^2$$

- Relative resolution: Method 1(E', θ')

$$\left(\frac{\delta Q^2}{Q^2}\right) = \frac{\delta E'_e}{E'_e} \oplus \tan\left(\frac{\theta'_e}{2}\right)\delta\theta'_e$$

Q^2 resolution worsens for large θ', need excellent θ' resolution!

$$\left(\frac{\delta y}{y}\right) = \left(1-\frac{1}{y}\right)\frac{\delta E'_e}{E'_e} \oplus \left(\frac{1}{y}-1\right)\cot\left(\frac{\theta'_e}{2}\right)\delta\theta'_e$$

y resolution worsens for small y, need excellent E' resolution!

- Relative resolution: Method 2 (F, γ)

$$\left(\frac{\delta Q^2}{Q^2}\right) = \left(\frac{2-y}{1-y}\right)\frac{\delta F}{F} \oplus \left[2\cot\gamma + \left(\frac{y}{1-y}\right)\cot\left(\frac{\gamma}{2}\right)\right]\delta\gamma$$

Q^2 resolution worsens for large y, need excellent F resolution!

$$\left(\frac{\delta y}{y}\right) = \frac{\delta F}{F} \oplus \cot\left(\frac{\gamma}{2}\right)\delta\gamma$$

y resolution worsens for small γ, need excellent γ resolution!

Summary

- A full derivation of DIS kinematic variable reconstruction methods has been completed as a function of all combinations of two measured observables E', θ', F and γ
- The well-known deterioration of method 1/6 is clearly seen. All other reconstruction methods have also been analytically evaluated including a full analytical error propagation.
- The full scope of optimizing the kinematic variable reconstruction is still being evaluated based on the completed full analytical derivations beyond methods 1 and 6, such as method 3 and 4: Preparation of error propagation plots vs. kinematics variables.
- Extend to MC simulation with simple smearing to quantify acceptance and purity for y-Q^2 bins!