Angularity event shapes in DIS at the NNLL accuracy

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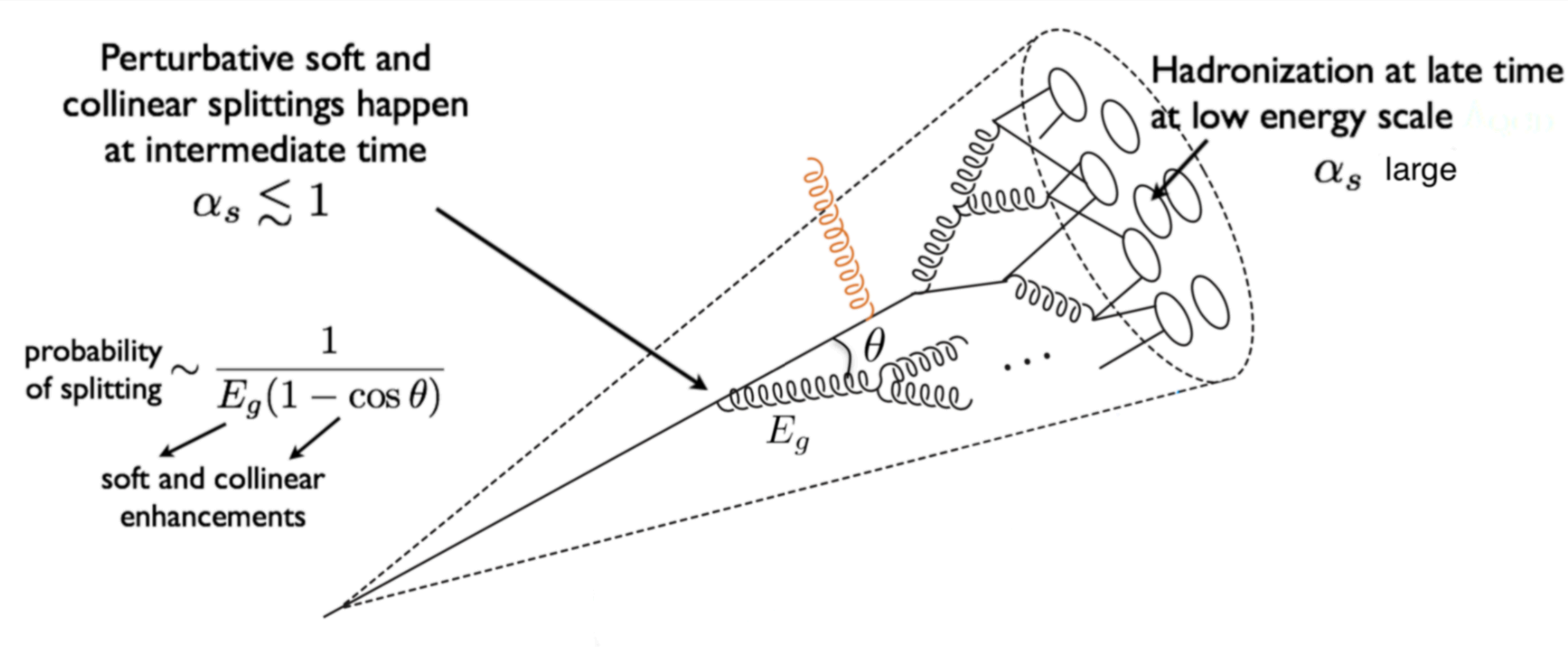
Online

Contents

Jet?

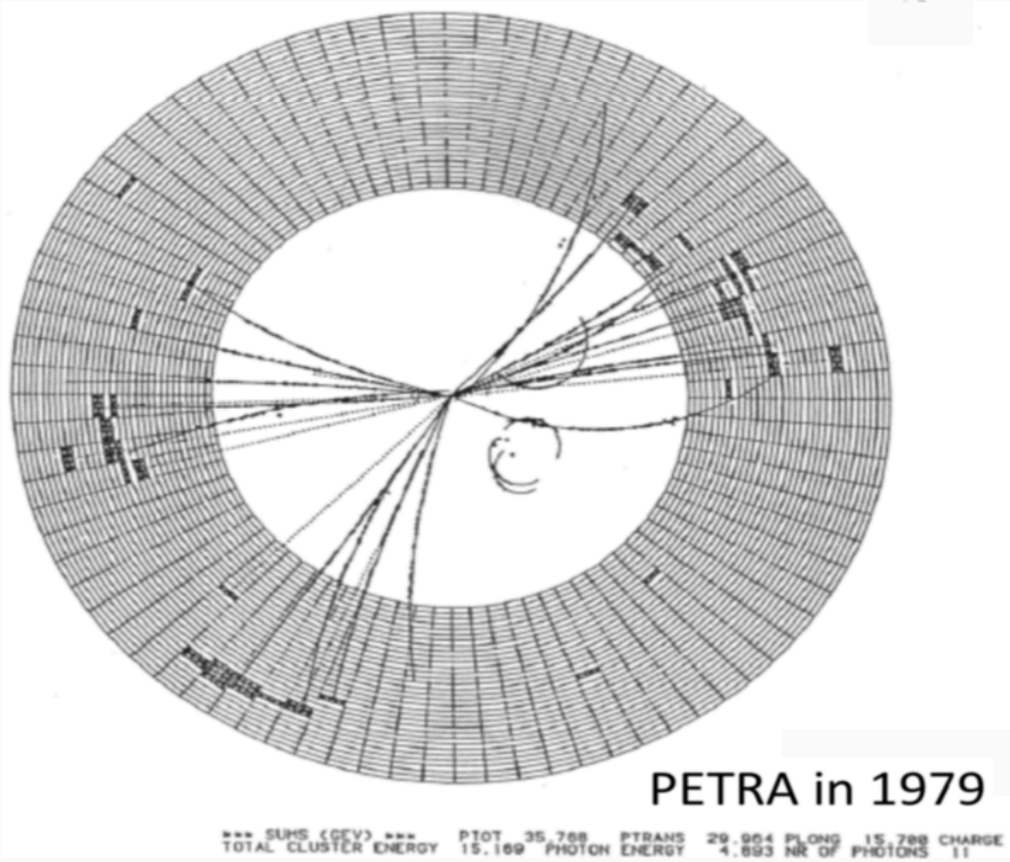
- In high energy scattering, the most common final states are collimated branches of strongly interacting particles, called jet.

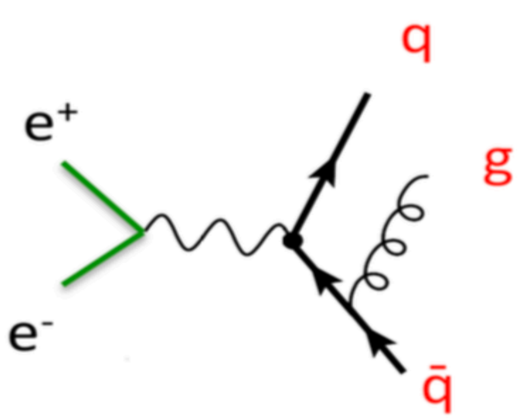


- Jet contains rich information to probe strong dynamics.

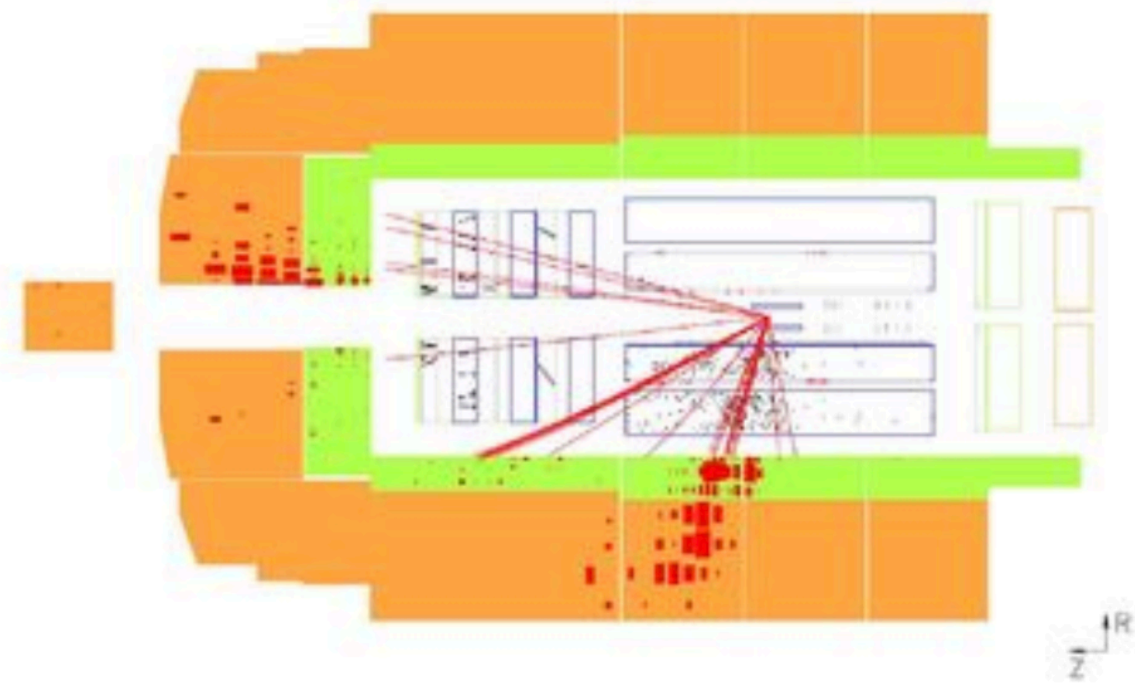
Jet production in collider

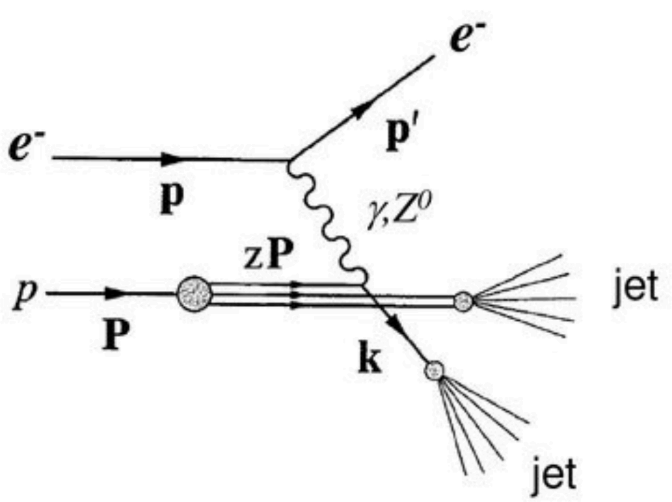
3-jet event: discovery of gluon





2-jet event: H1 detector at HERA





Jet Observables are called event shapes

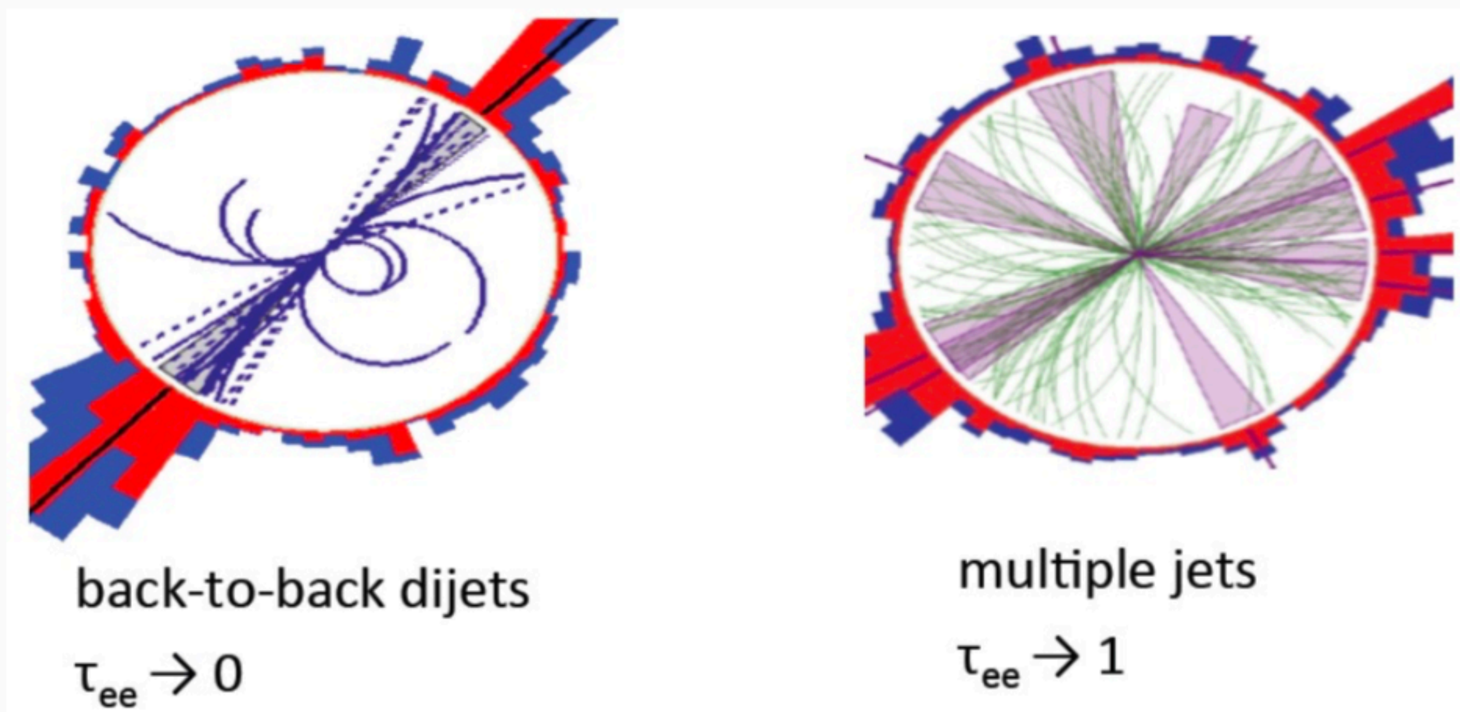
e.g., Thrust, Jet Broadening, Angularity…

Thrust

One of the most precisely measured observables!

$$\tau = \frac{2}{Q} \sum_{i \in \mathcal{X}} \left|\mathbf{p}_{\perp}^{i}\right| e^{-|\eta_i|}$$

Thrust event shape characterised the geometry of the collision!



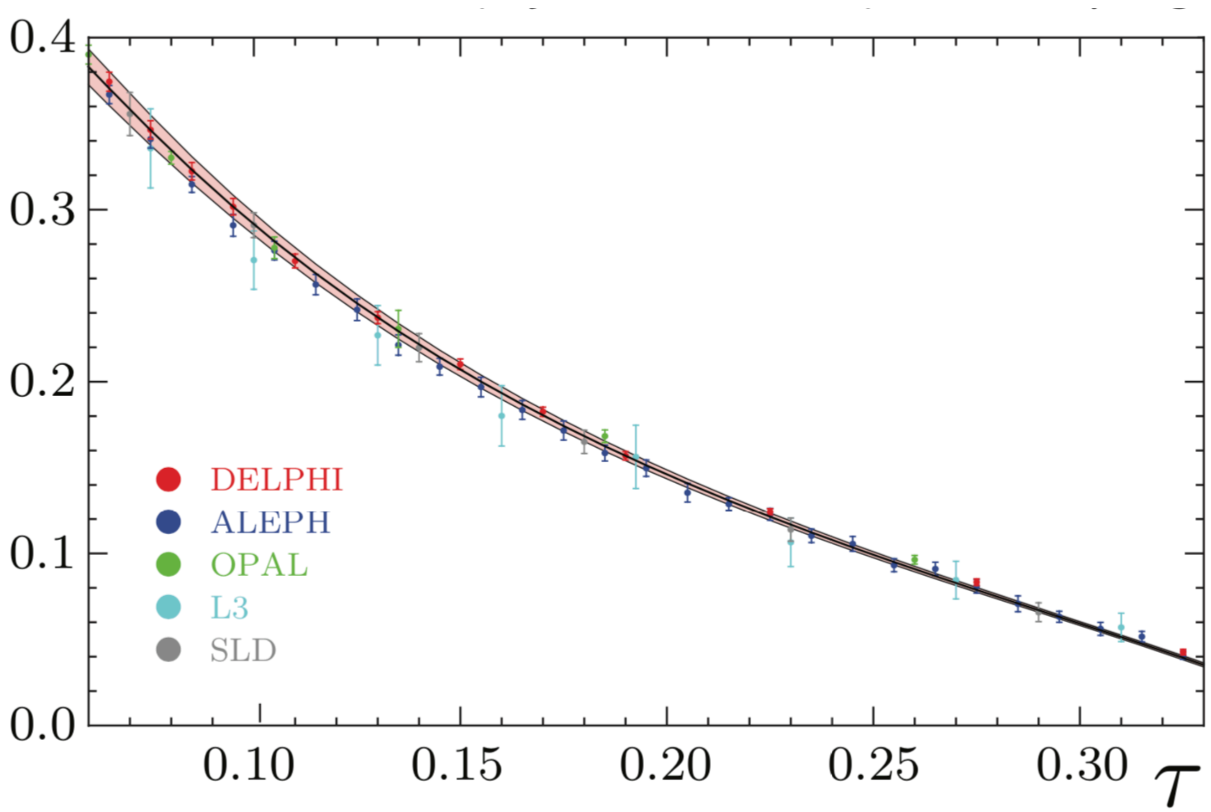
LEP: [P. Achard *et al.*, JHEP 1110, 143 (2011)]

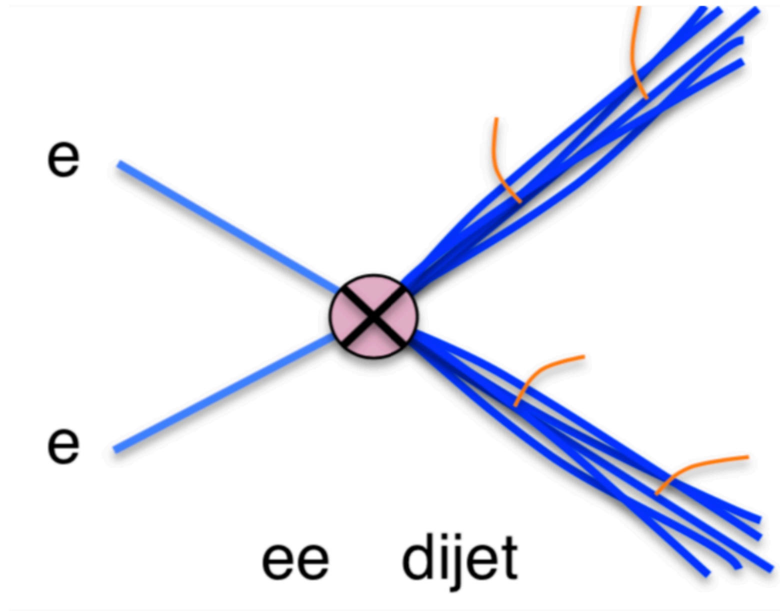
HERA by the ZEUS and H1 collaborations :

[1]C. Adloff *et al.* [H1 Collaboration], Phys. Lett. B 406, 256 (1997)
[2] C. Adloff *et al.* [H1 Collaboration], Eur. Phys. J. C 14, 255 (2000)
[3] A. Aktas *et al.* [H1 Collaboration], Eur. Phys. J. C 46, 343 (2006)
[4] J. Breitweg *et al.* [ZEUS Collaboration], Phys. Lett. B 421, 368 (1998)
[5] S. Chekanov *et al.* [ZEUS Collaboration], Eur. Phys. J. C 27, 531 (2003)
[6] S. Chekanov *et al.* [ZEUS Collaboration], Nucl. Phys. B 767, 1 (2007)

Precision jet Physics in e+e-

LEP data for thrust (dijets)



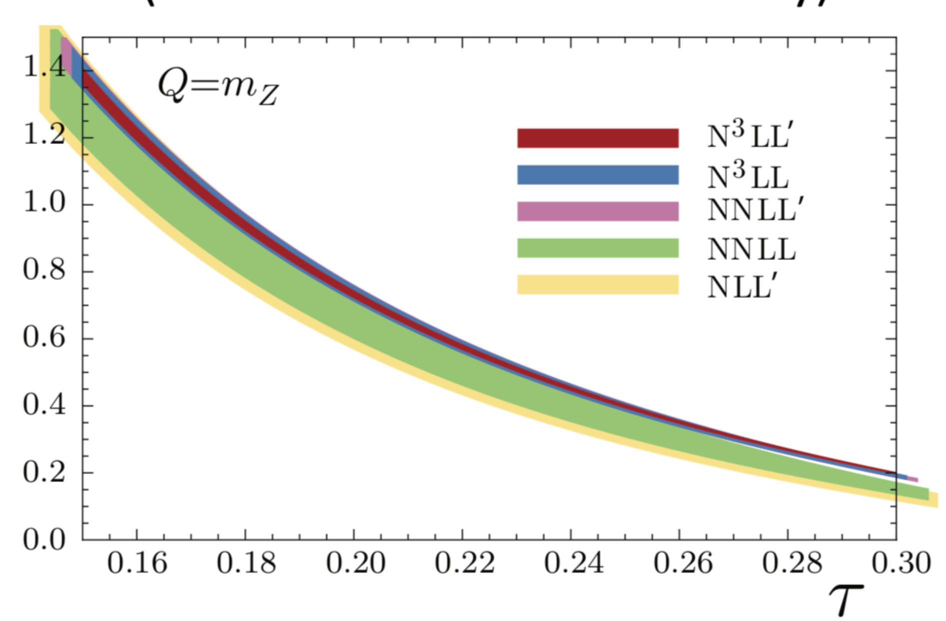


determination of strong coupling at 1% precision

$$\alpha_s(m_Z) = 0.1135 \pm 0.0011$$

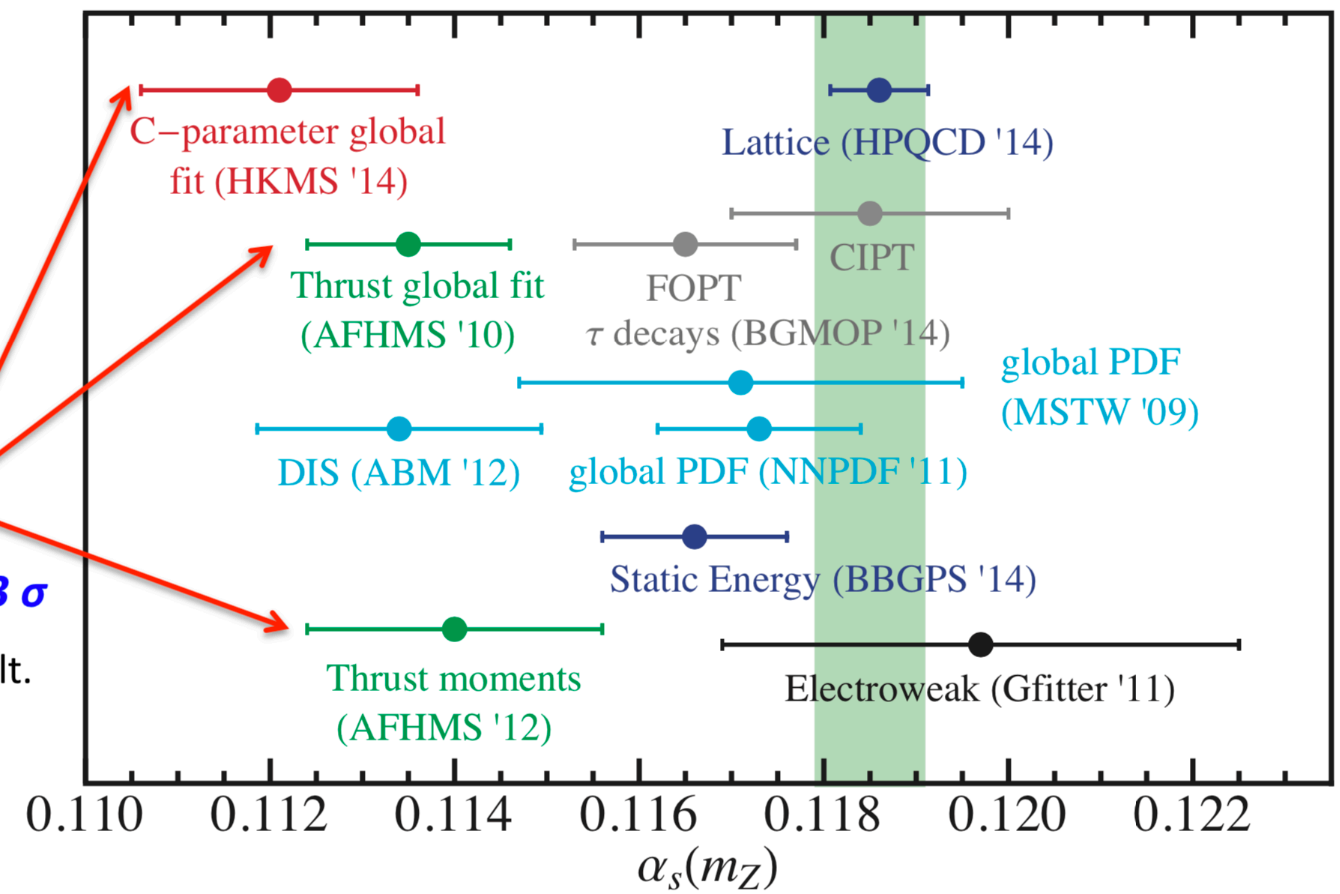
Prediction by recent theoretical advance

(soft collinear effective theory)



—more I. Stewart **Phys.Rev. D83 (2011) 074021**

Puzzle in strong coupling constant!!



$e^+e^- \rightarrow$ dijet

discrepancy > 3 σ

from Lattice result.

- Tension between e+e− → 2-jet result and Lattice result!
- Need a new test from an independent experiment!

DIS: e P -> 2-jet

Angularity!

Precision jet Physics in DIS

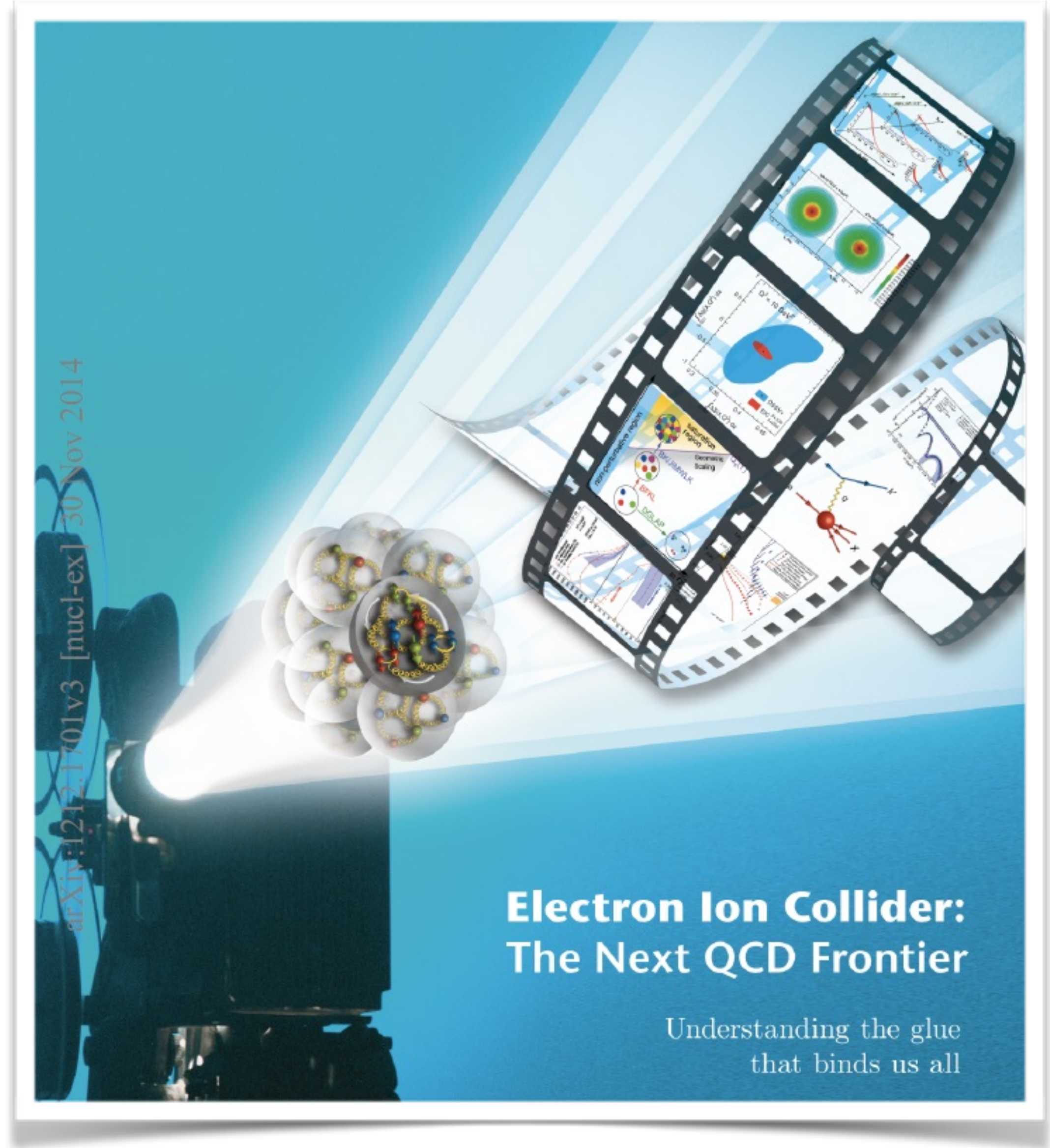
Shedding light on the discrepancy in strong coupling

DIS: e P -> 2-jet

Angularity!

Precision prediction to Future EIC

One of early milestones!



Precision jet Physics in DIS

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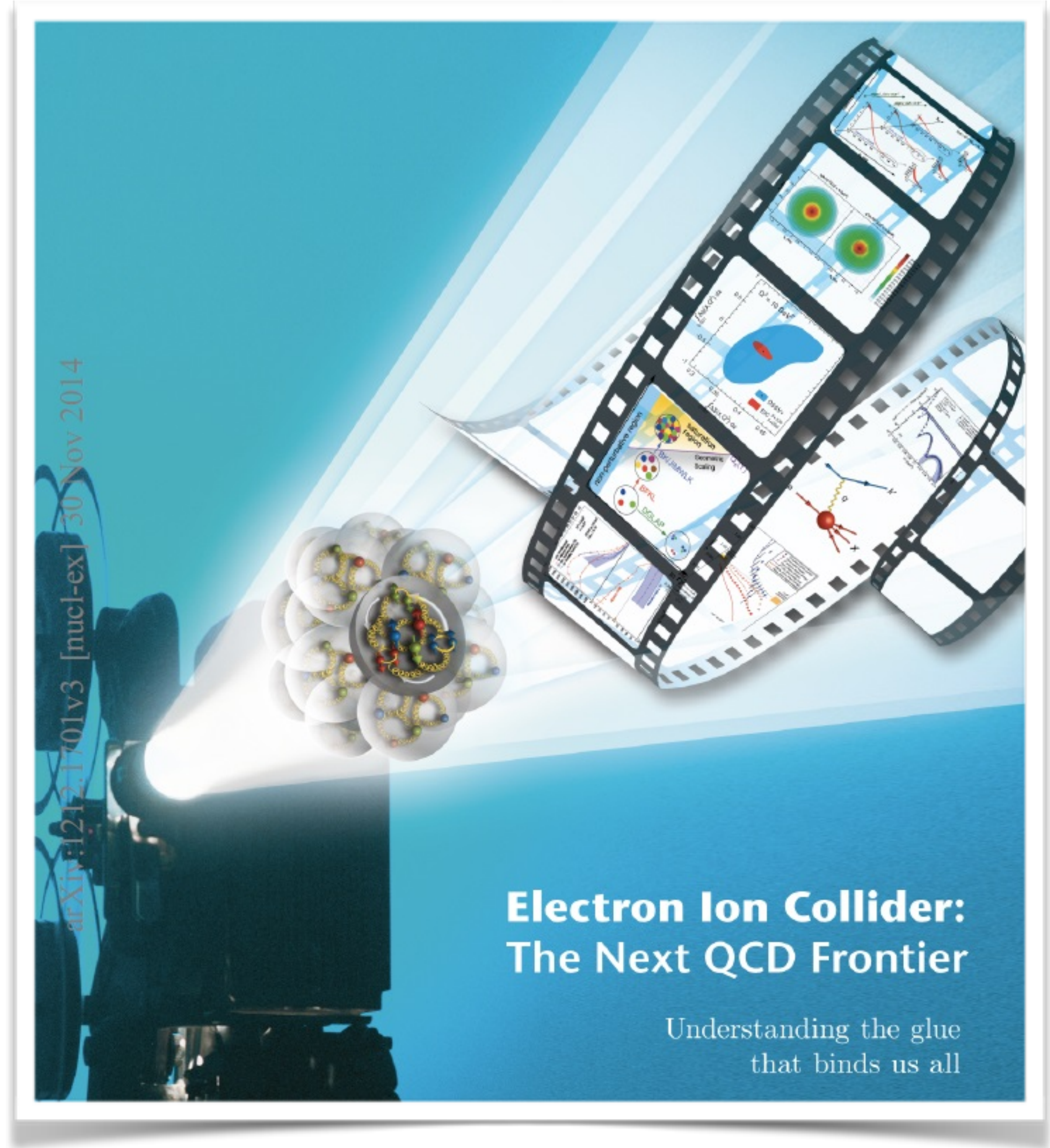
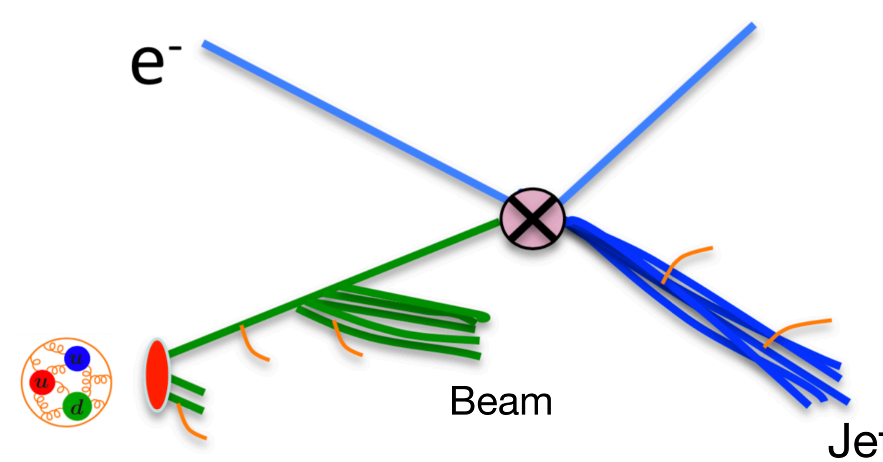


Jet: form **Final state radiation(FSR)** to investigate *short distance phenomenon, QCD dynamics*

Beam: from **Initial state Radiation (ISR)** encodes *internal structure of hadrons*— PDF, GPD,TMD, Spin structure.



We do…

Investigation of **angularity event shape in DIS** to push the frontier of the precision jet physics in the future Electron-Ion Collider (EIC)

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Investigation of **angularity event shape in DIS** to push the frontier of the precision jet physics in the future Electron-Ion Collider (EIC)

Tool

Soft-collinear effective theory (SCET)

a systematic way to achieve high precision in high-energy scattering

- Soft-collinear effective theory (SCET) is a systematic expansion of QCD in a small parameter λ which characterizes the scale of collinear and soft radiation from energetic massless partons.

Angularity in DIS

$$\tau_a = \frac{2}{Q} \sum_{i \in \mathcal{X}} \left| \mathbf{p}_\perp^i \right| e^{-|\eta_i|(1-a)}$$

A more general event shape!

Depends on a continuous parameter

provide access from thrust to jet broadening in continuous manner

—C. F. Berger, T. Kucs and G. F. Sterman' 2003

Angularity in DIS

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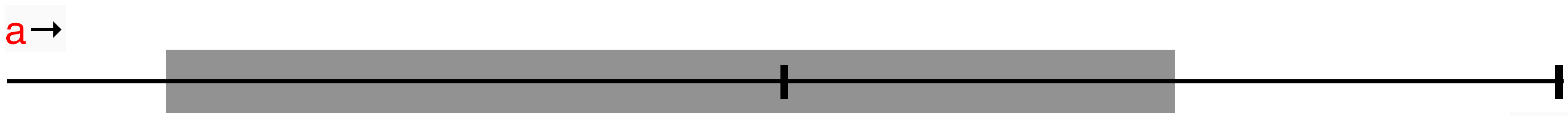
Status



$a \to 0$
Thrust
Up to N3LL'

$a \to 1$
Broadening
at NNLL'

Angularity

e+e- : Hornig, Lee, Ovanesyan'09; Bell, Hornig, Lee, Talbert'18, A.Budhraja, A.Jain and M.Procura'19

Photoproduction:
E.C.Aschenauer, K.Lee, B.S.Page and F.Ringer'19

e+e- : Catani, Trentadue, Turnock, Webber'93; Florian, Grazzini'04; Schwartz'07; Becher, Schwartz'08; Abbate, Fickinger, Hoang, Mateu, Stewart'10 ;Becher,Schwartz'08; Stewart,Tackmann,Waalewijn'10

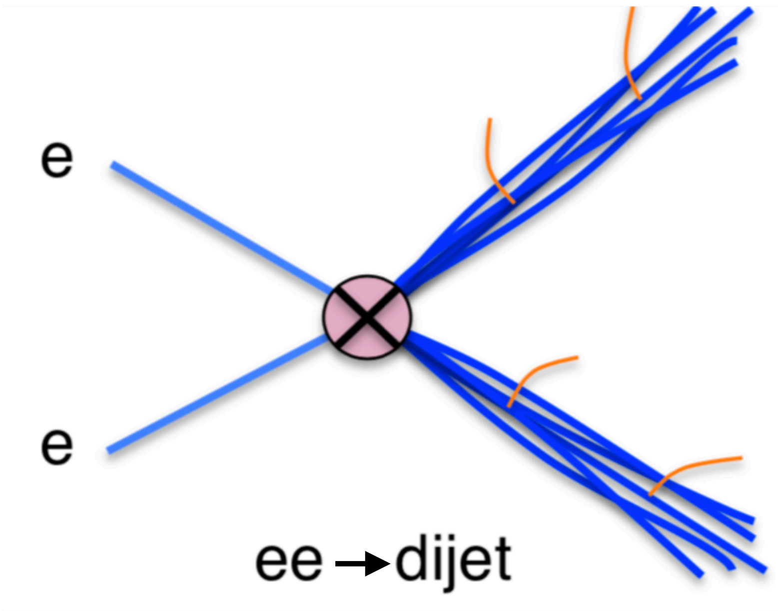
pp : Stewart,Tackmann,PRL'10,'11;PRD'13]

DIS: D. Kang,C. Lee,I. Stewart'13]

Dokshitzer, Lucenti, Marchesini, Salam'98; Becher, Bell, Neubert'11; Chiu, Jain, Neill, Rothstein'11; Becher and Bell'12

DIS angularity??

e+e- Vs eP **Angularity**



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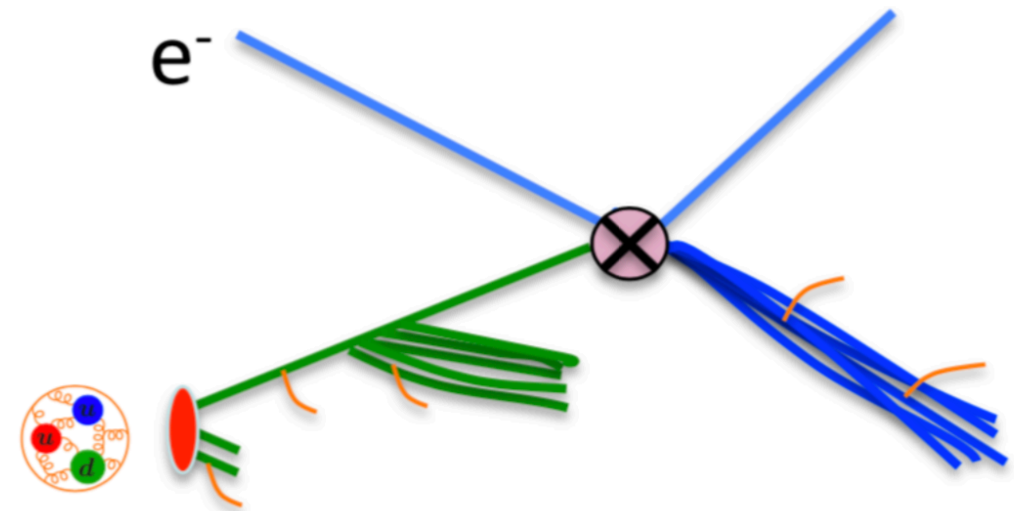


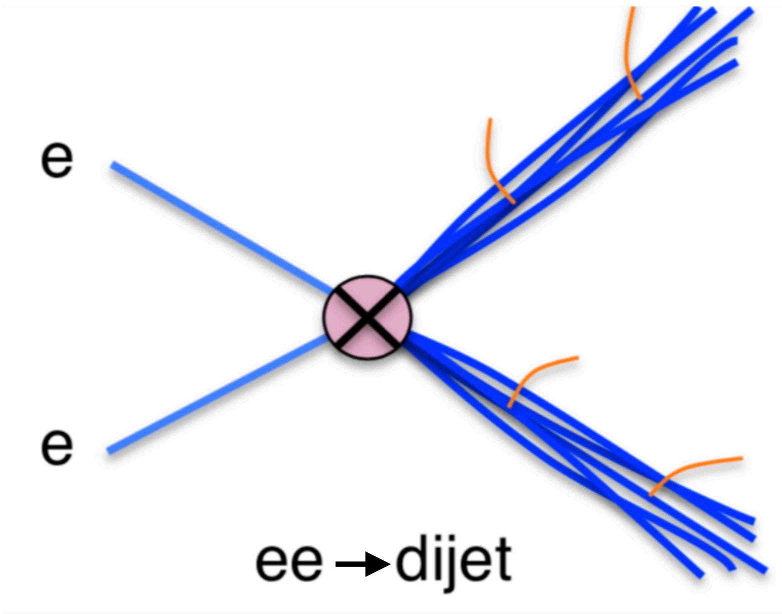
ep → dijet

Beam and jet hemisphere
are not equally divided.

- Transverse momentum is defined with respect to thrust axis.

- In terms of the four-vectors qB along the incident proton beam direction and qJ along the direction of the final state jet one wish to measure

e+e- Vs eP **Angularity**



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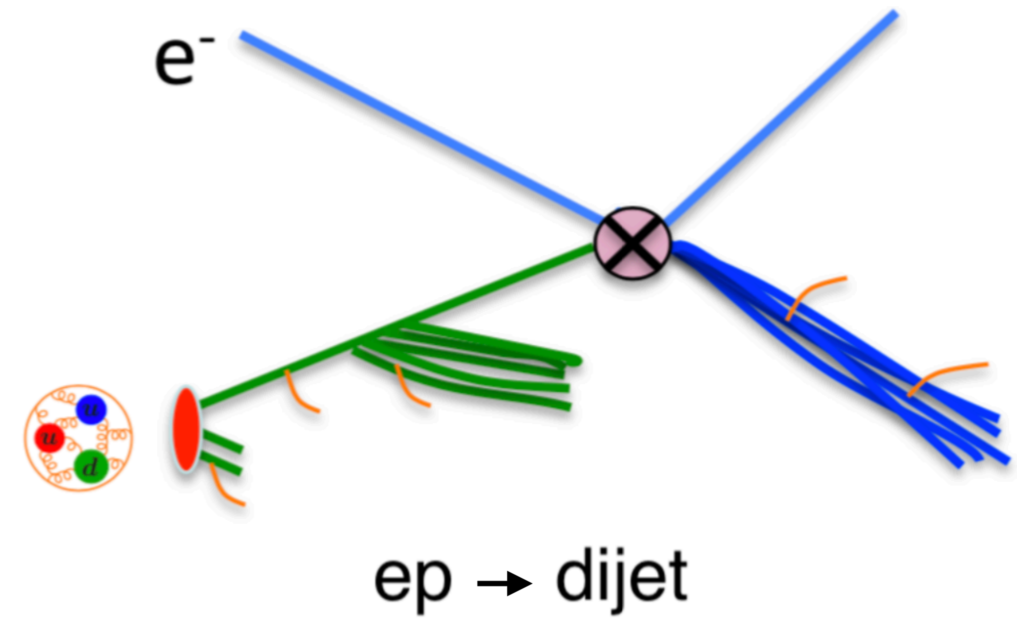

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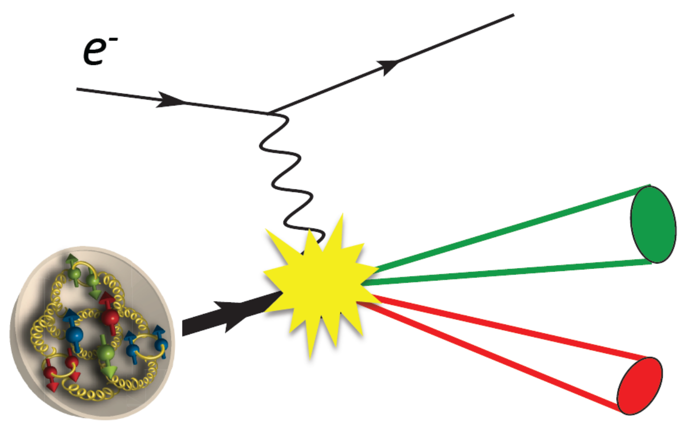
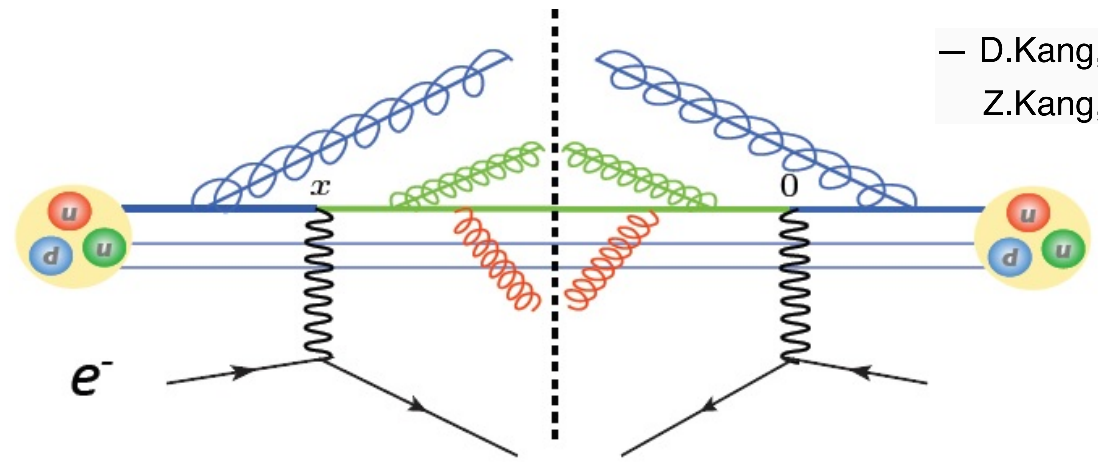
DIS Angularity

$$\boxed{\tau_a = \frac{2}{Q^2}\sum_{i\varepsilon\mathscr{X}} min\left\{(q_B.p_i)\left(\frac{q_B.p_i}{q_J.p_i}\right)^{-a/2},\ (q_J.p_i)\left(\frac{q_J.p_i}{q_B.p_i}\right)^{-a/2}\right\}}$$

Our axis Choice: qB= xP, qJ= jet axis

DIS factorization and Angularity Beam Function





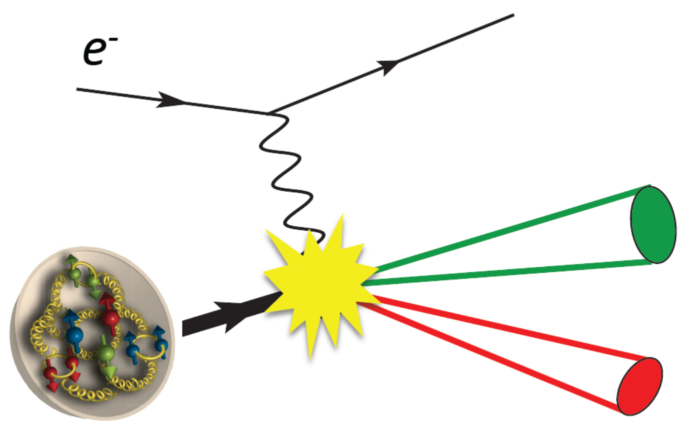
— D.Kang,Lee,Stewart'2013
Z.Kang,Mantry,Qiu'2012

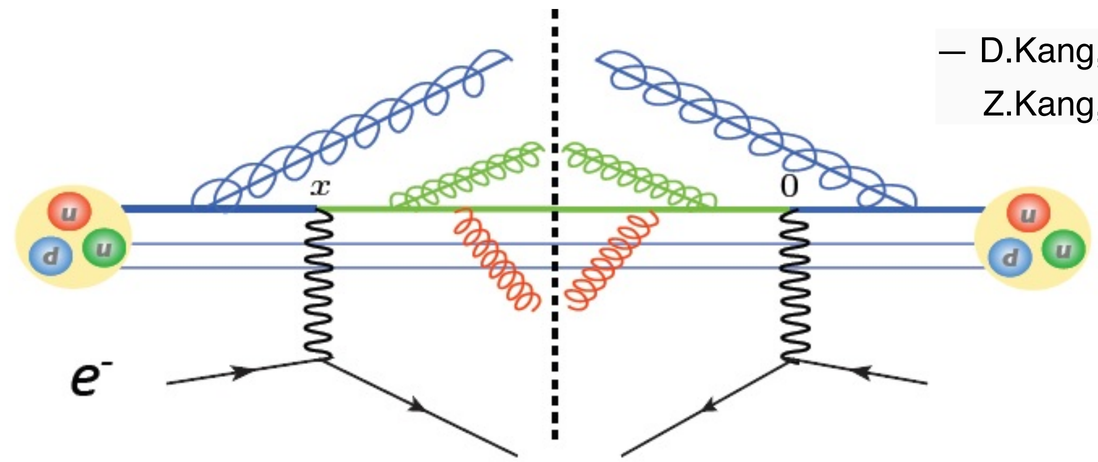
SCET facto.: σeP = Hard × Beam ⊗ Jet ⊗ Soft

Beam func.: $B(\tau_a, x, \mu) \;=\; \text{pdf} \otimes \left[\delta_{qj}\delta(\tau_a) + \tilde{\mathcal{I}}^{(1)}_{qj} + \mathcal{O}(\alpha_s^2) + \ldots\right]$

NP LO NLO $NNLO$

DIS factorization and Angularity Beam Function





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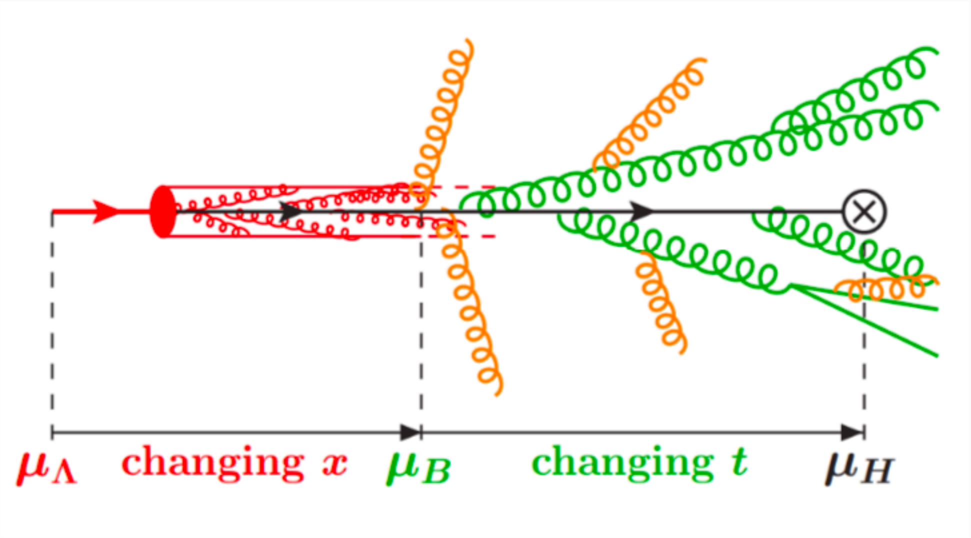
$NP \quad LO \quad NLO \quad NNLO$

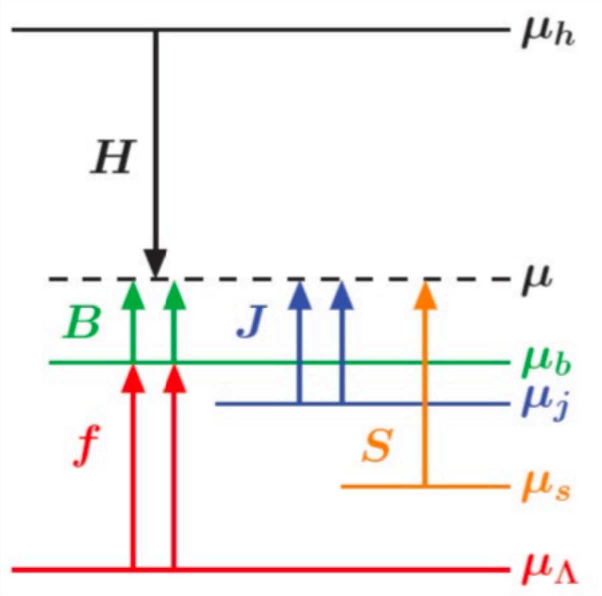
One-loop angularity beam function is presented for the first time!

$$\tilde{\mathcal{I}}_{qj}^{(1)} = \frac{\alpha_s}{4\pi}\left[\left(j_B\kappa_B\frac{\Gamma_0}{2}L_B^2 + \gamma_0^B L_B\right)1 + 4C_{qj}P_{qj}(z)L_B + c_1^{qj}(z)\right]$$

$$L_B(\tau_a) = \log[\frac{Q}{\mu_B}(\tau_a e^{-\gamma_E})^{1/j_B}]$$

Resummation from evolution





-Stewart et. al. PRD81(2010)

Evolution Equation for beam function

$$\mu \frac{d}{d\mu} B(\nu,\mu) \quad = \quad \gamma_G(\mu)\, B(\nu,\mu) \quad ; \qquad\qquad \text{similar to} J, S, H$$

$$\text{Solution}: \; B(\nu,\mu) \quad = \quad B(\nu,\mu_B)\, e^{K_B(\mu_B,\mu)+j_B\eta_B(\mu_B,\mu)L_B} \;,$$

- Jet and beam functions are defined by same collinear operator: $\gamma_J(\mu) = \gamma_B(\mu)$

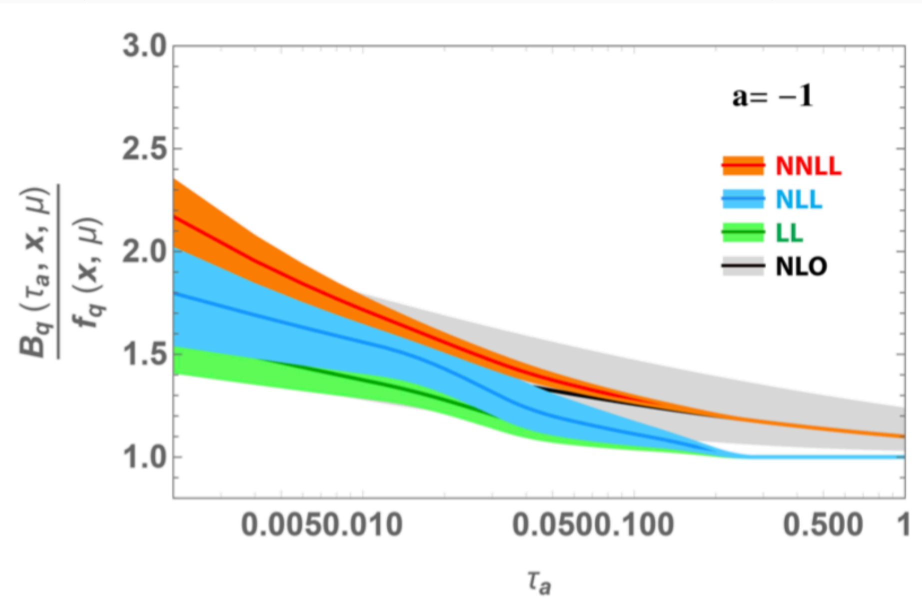
$$K_B(\mu_B,\mu) \quad = \quad L_B \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \sum_{k=1}^{\infty} (\alpha_s L_B)^k + \ldots$$

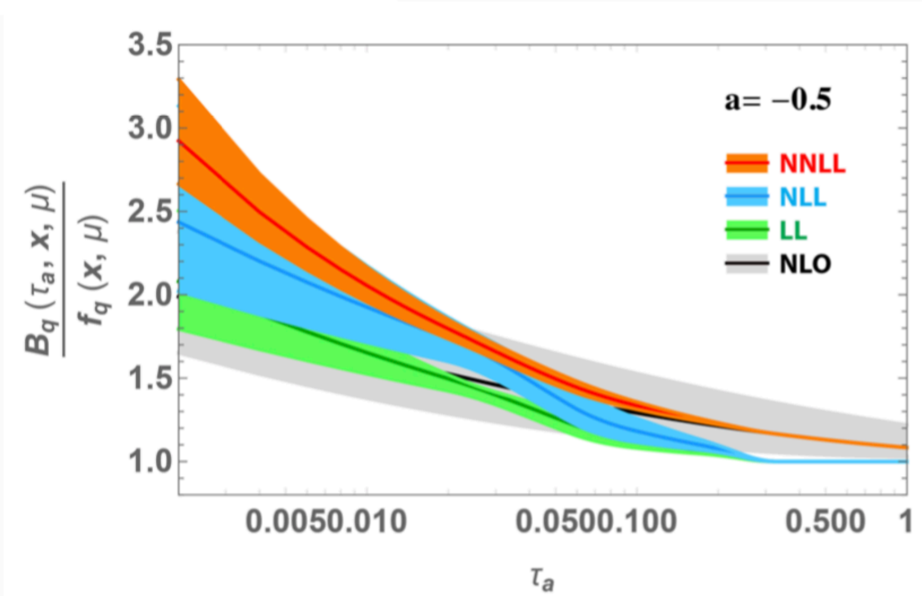
LL $\qquad NLL$

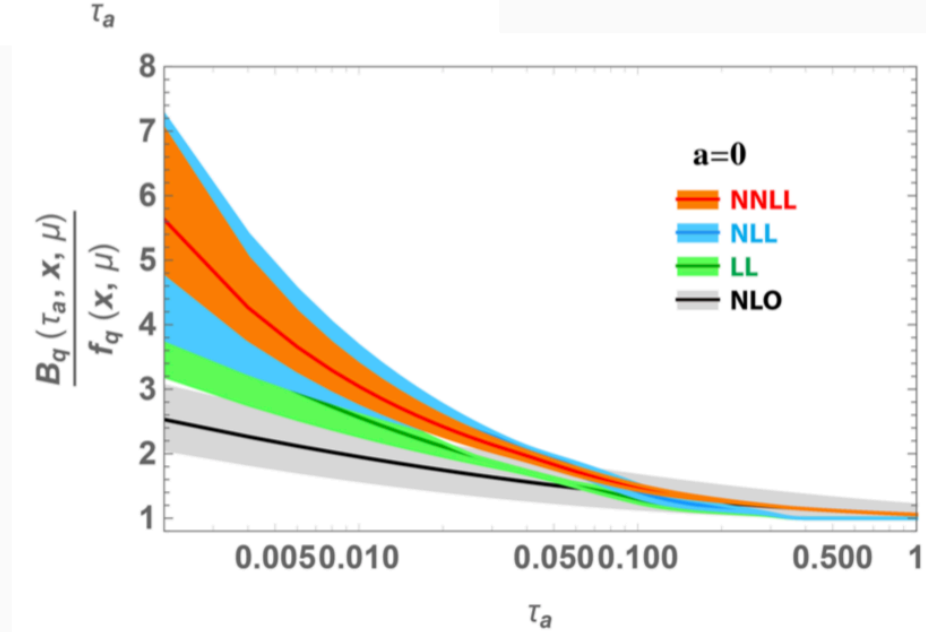
$$L_B = \ln(\mu/\mu_B)$$

- Resummation of large logs provides better perturbative convergence.

DIS angularity Beam Function at NNLL



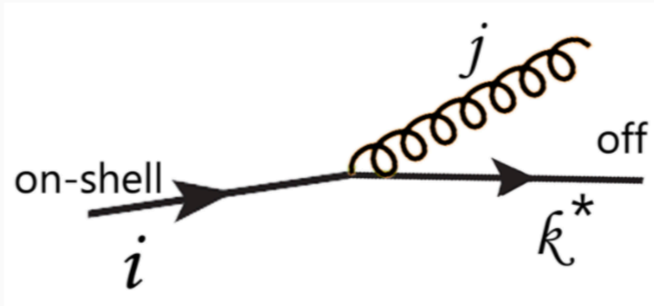




Resummation of large logs provides better perturbative convergence.

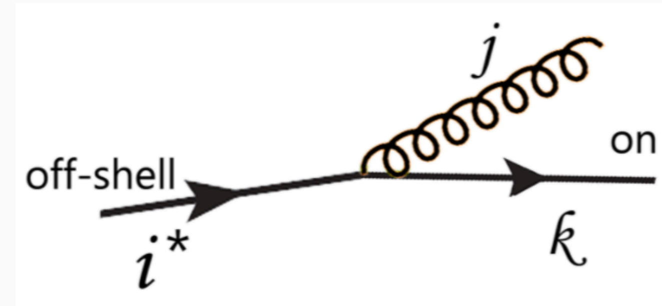
Beam Func. And Fragmentation func.

Beam at NLO: $i \to k^* j$



Crossing Symmetry!

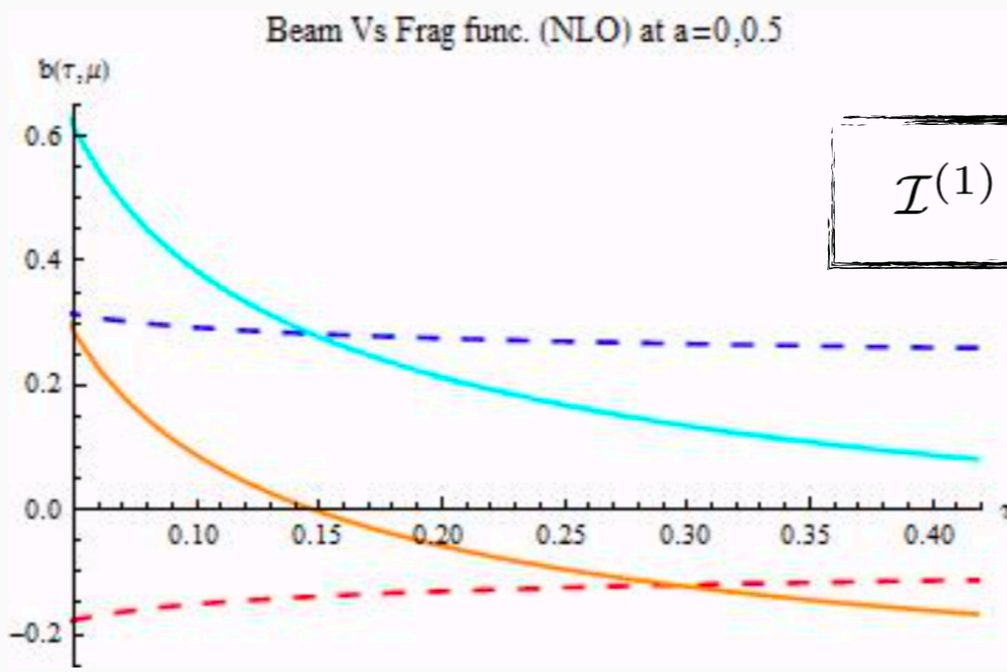
Fragmentation at NLO: $i^* \to kj$



Splitting Function: [M.Ritzmann,W.J.Waalewijn,PRD90(2014)]

$$P_{i\to k^* j}(2pi.pj, x) \equiv (-1)^{\Delta_f} P_{k*\to ij}(-2pi.pj, 1/x)$$

- Change comes only from the two-particle phase-space and effectively change in sign of the $\log(x)$ term in the matching co-efficient $\mathcal{I}^{(1)}$.



$$\boxed{\mathcal{I}^{(1)} \sim \ldots - \frac{\alpha_s C_F}{2\pi}\frac{2(1-a)}{2-a}\frac{1+x^2}{1-x}\log x}$$

Beam match. Coeffi.: $a = 0$

Beam match. Coeffi.: $a = 0.5$

FF match. Coeffi.: $a = 0$

FF match. Coeffi.: $a = 0.5$

- Difference decreases with the increase of angularity parameter a.

Analytical results

Differential Cross-section

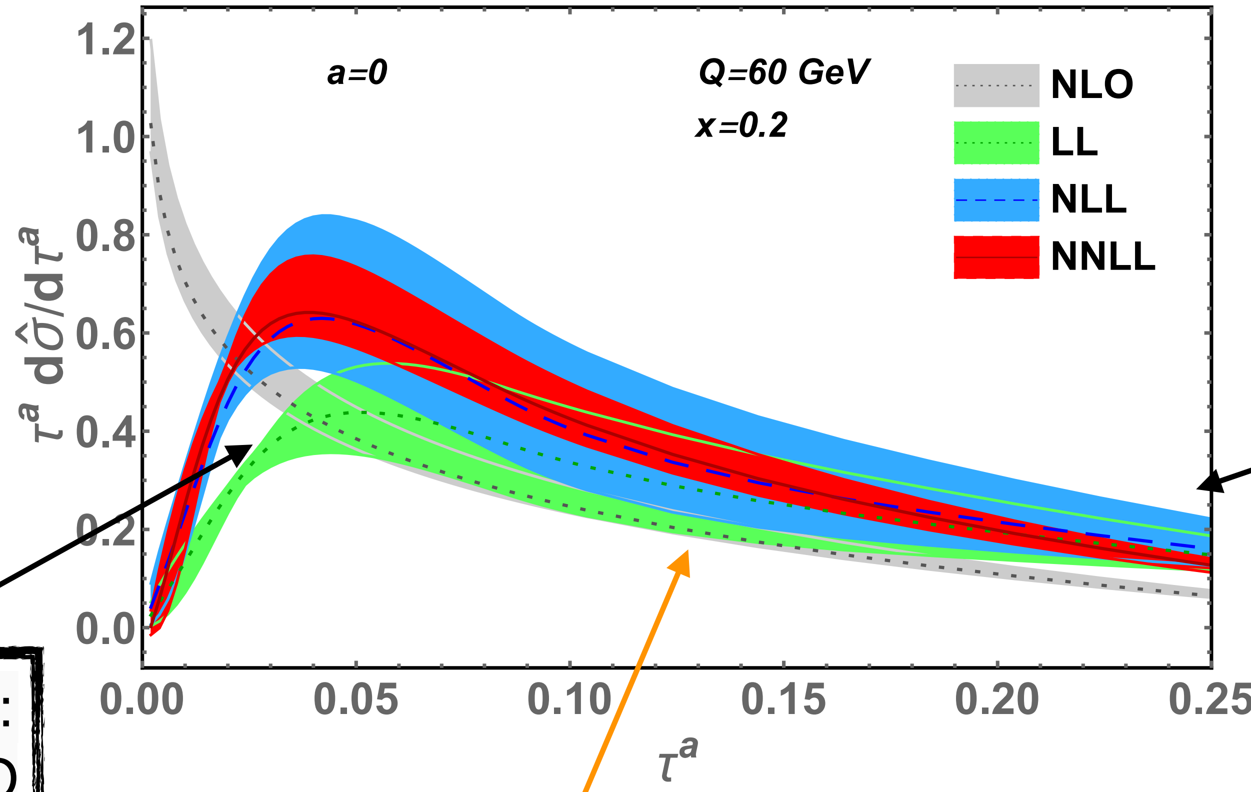
$$\frac{d\sigma}{dx\,dQ^2 d\,\tau_a} = \frac{d\sigma_0}{dxdQ^2}\sum_\nu H_\nu(Q^2,\mu)\int d\tau_a^J\, d\tau_a^B\, dk_S\, J_q(\tau_a^J,\mu) B_{\nu/q}(\tau_a^B,x,\mu) \times S(k_S,\mu)\,\delta\left(\tau_a - \tau_a^J - \tau_a^B - \frac{k_S}{Q_R}\right),$$

Resummed result

$$\begin{aligned}
\sigma(x,Q^2,\tau_a,\mu) \quad = \quad & \sigma_0(x,Q^2)\left(\frac{Q}{\mu_H}\right)^{\eta_H(\mu,\mu_H)} e^{\kappa(\mu_H,\mu_J,\mu_B,\mu_S,\mu)} \\
& \times\left(\left(\frac{Q}{\mu_J}\right)^{2-a}\tau_a e^{-\gamma_E}\right)^{\eta_J(\mu,\mu_J)}\left(\left(\frac{Q}{\mu_B}\right)^{2-a}\tau_a e^{-\gamma_E}\right)^{\eta_B(\mu,\mu_B)}\left(\frac{Q^2}{\mu_S}\tau_a e^{-\gamma_E}\right)^{2\eta_S(\mu,\mu_S)} \\
& \times\tilde{j}_q\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_J^{2-a}}\tau_a e^{-\gamma_E}\right),\mu_J\right)\tilde{s}\left(\frac{1}{Q_R}\left(\partial_\Omega + \log\left(\frac{Q}{\mu_S}\tau_a e^{-\gamma_E}\right)\right),\mu_S\right) \\
& \times\Bigg[H_q(y,Q^2,\mu_H)\tilde{b}_q\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_B^{2-a}}\tau_a e^{-\gamma_E}\right),x,\mu_B\right) \\
& +H_{\bar{q}}(y,Q^2,\mu_H)\tilde{b}_{\bar{q}}\left(\partial_\Omega + \log\left(\frac{Q^{2-a}}{\mu_B^{2-a}}\tau_a e^{-\gamma_E}\right),x,\mu_B\right)\Bigg]\frac{1}{\tau_a\Gamma(\Omega)}
\end{aligned}$$

Numerical result at NNLL

- DIS angularity cross-section at NNLL accuracy

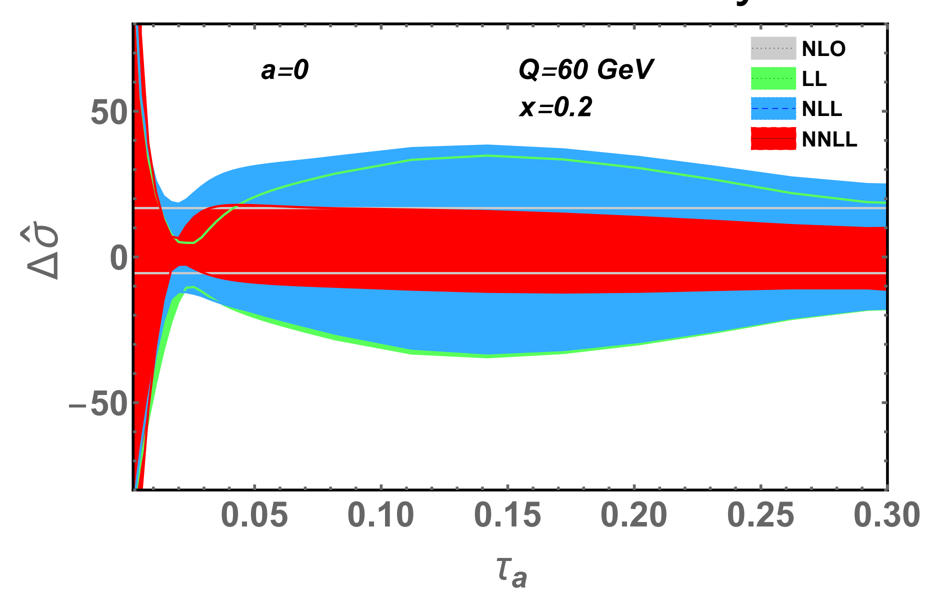


Far-tail region: Need full QCD

Low angularity region: Log singularity at NLO

Tail region: Resummation gives convergence from LL to NNLL

Relative Uncertainty



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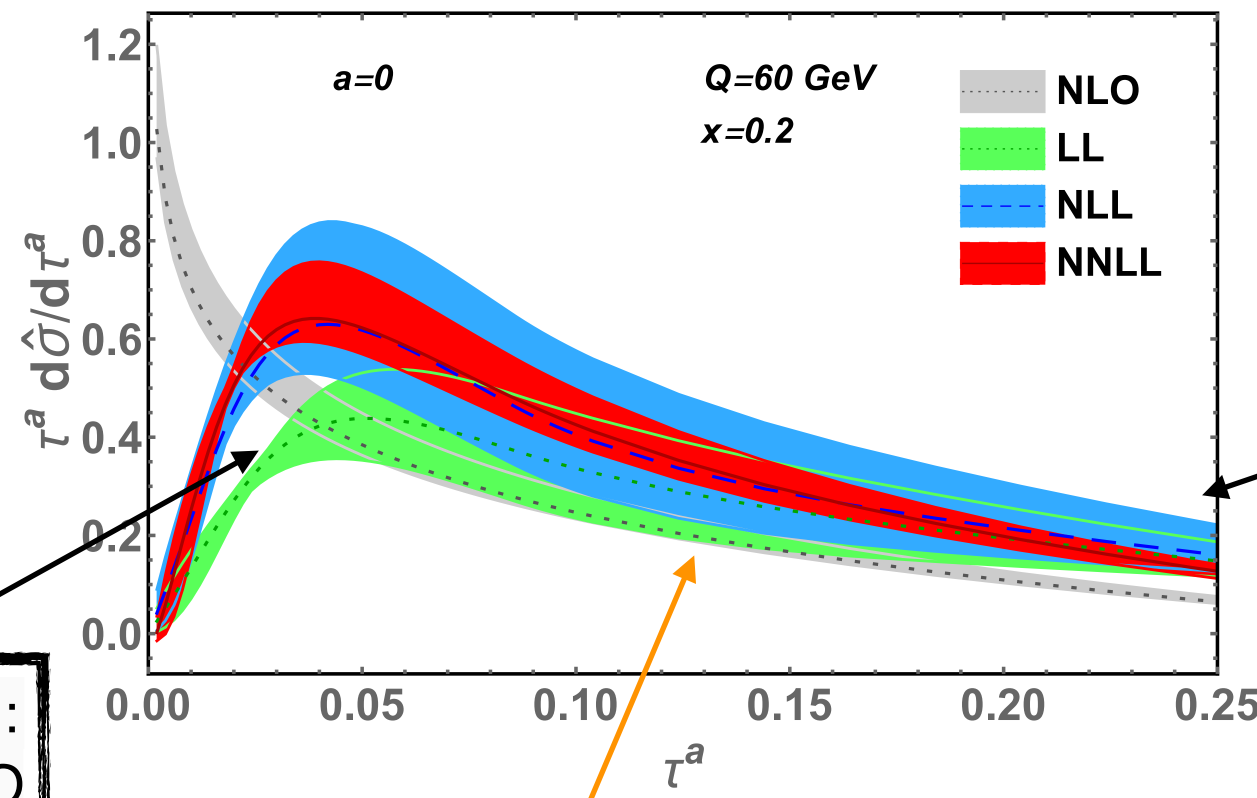

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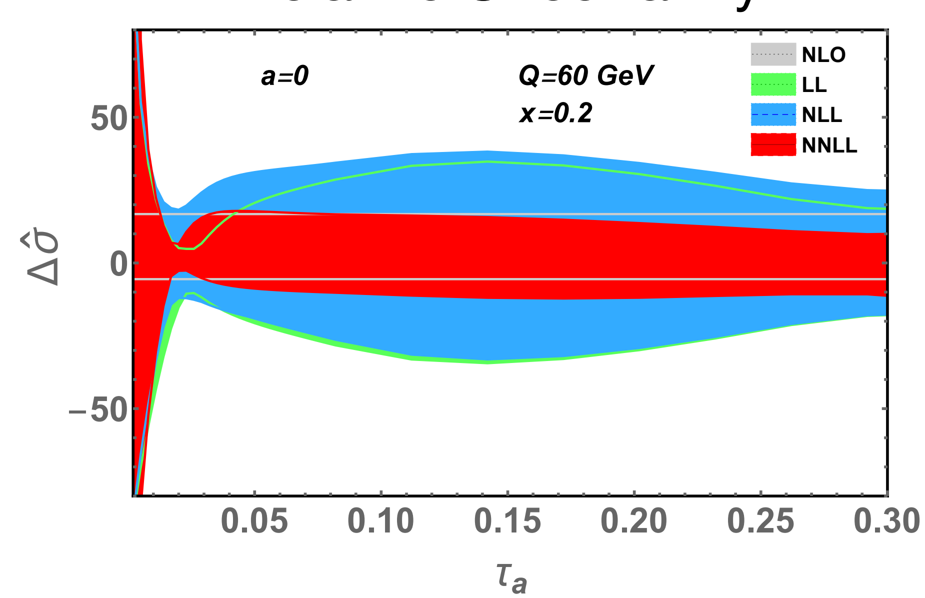
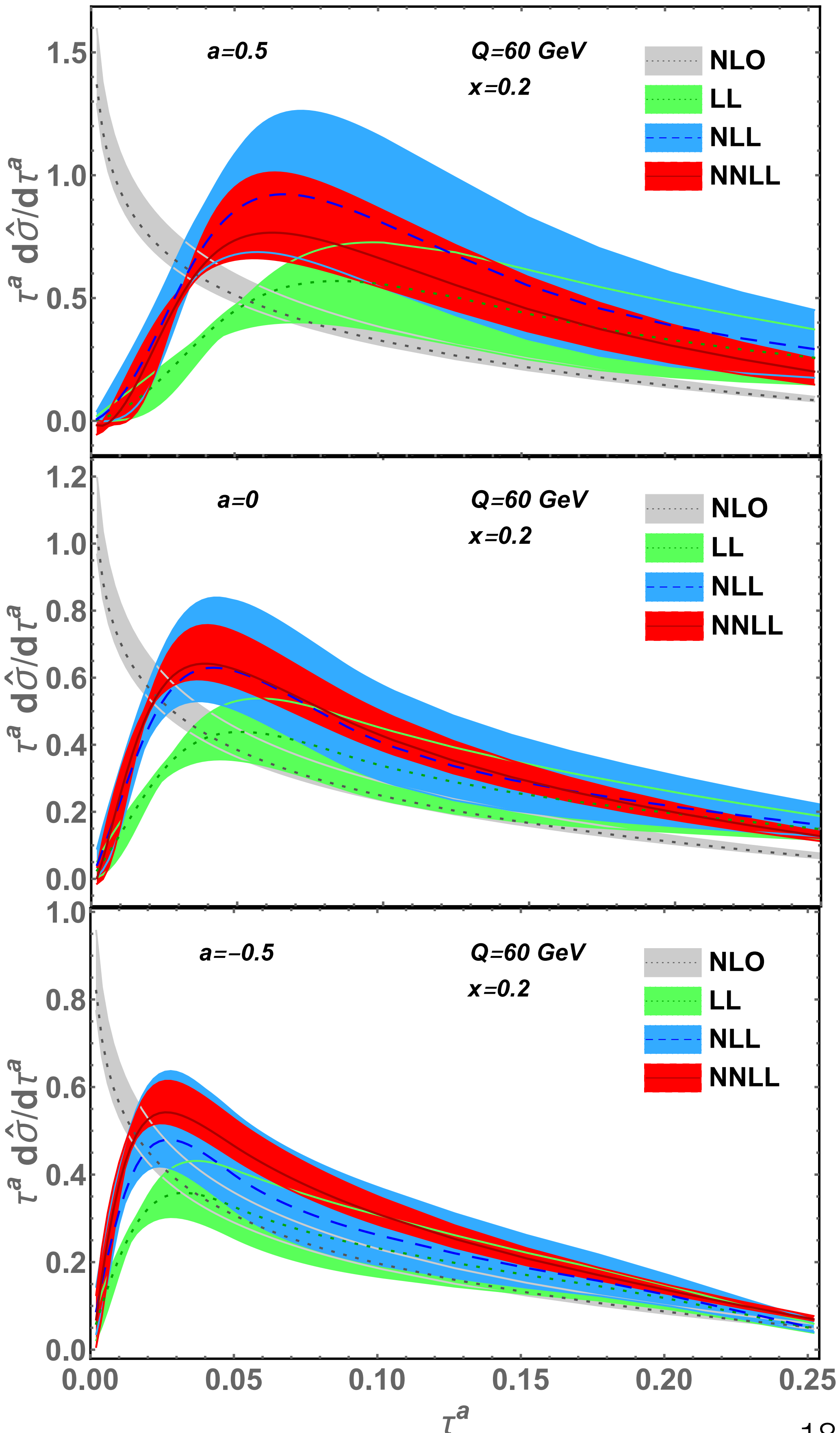
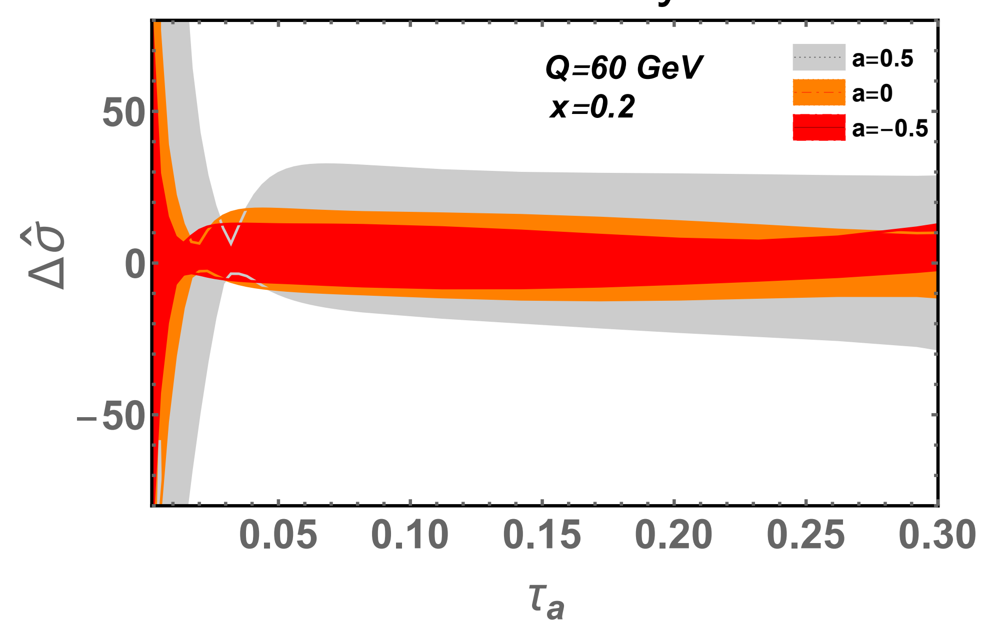


- We adopt electron-positron angularity profile function from Bell, Hornig, Lee, Talbert, 18

‘a’ dependency

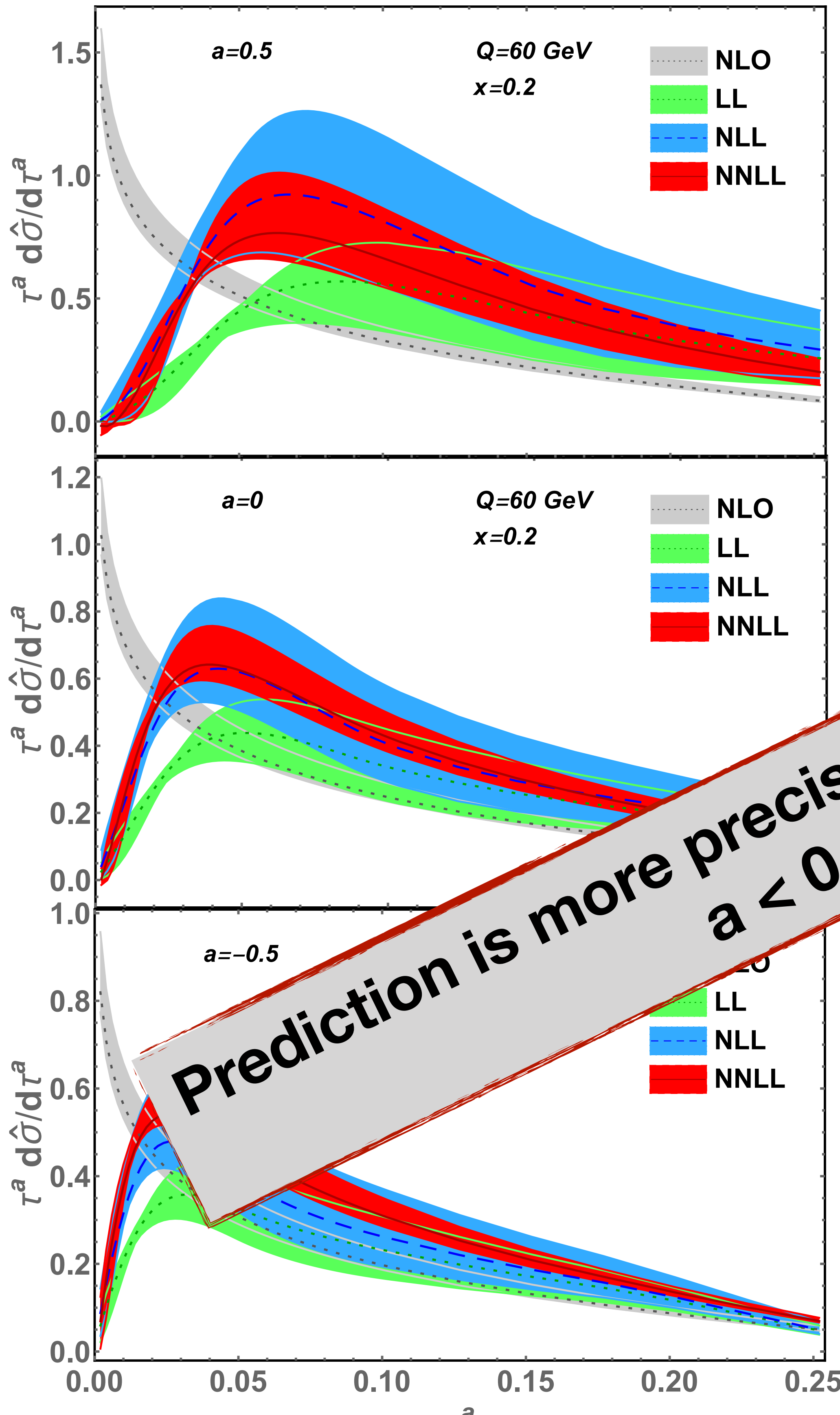


Relative uncertainty at NNLL

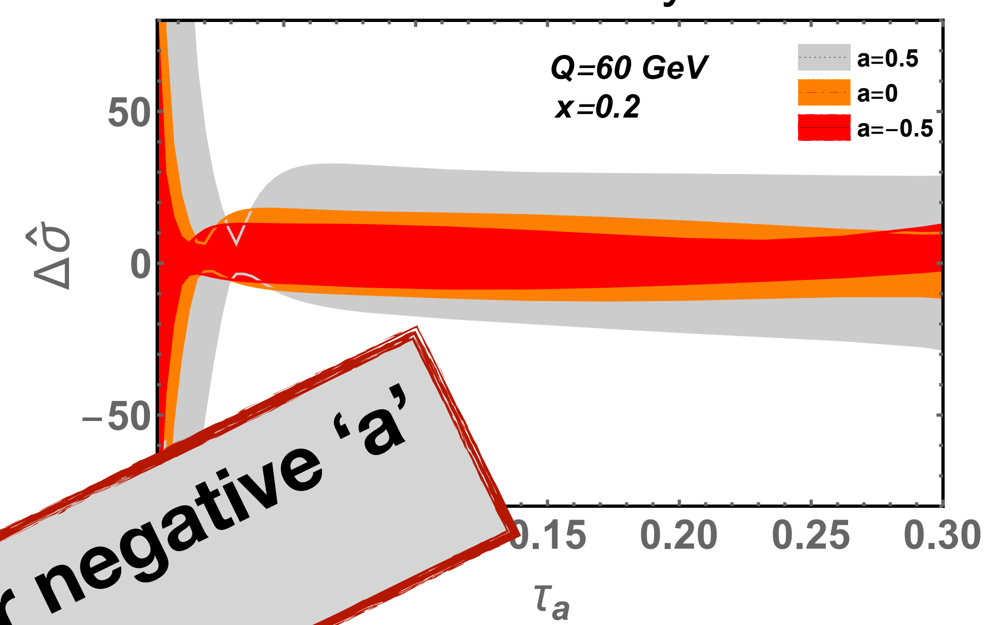


uncertainty in the DIS angularity cross-section depends on the angularity parameter ‘a’

‘a’ dependency



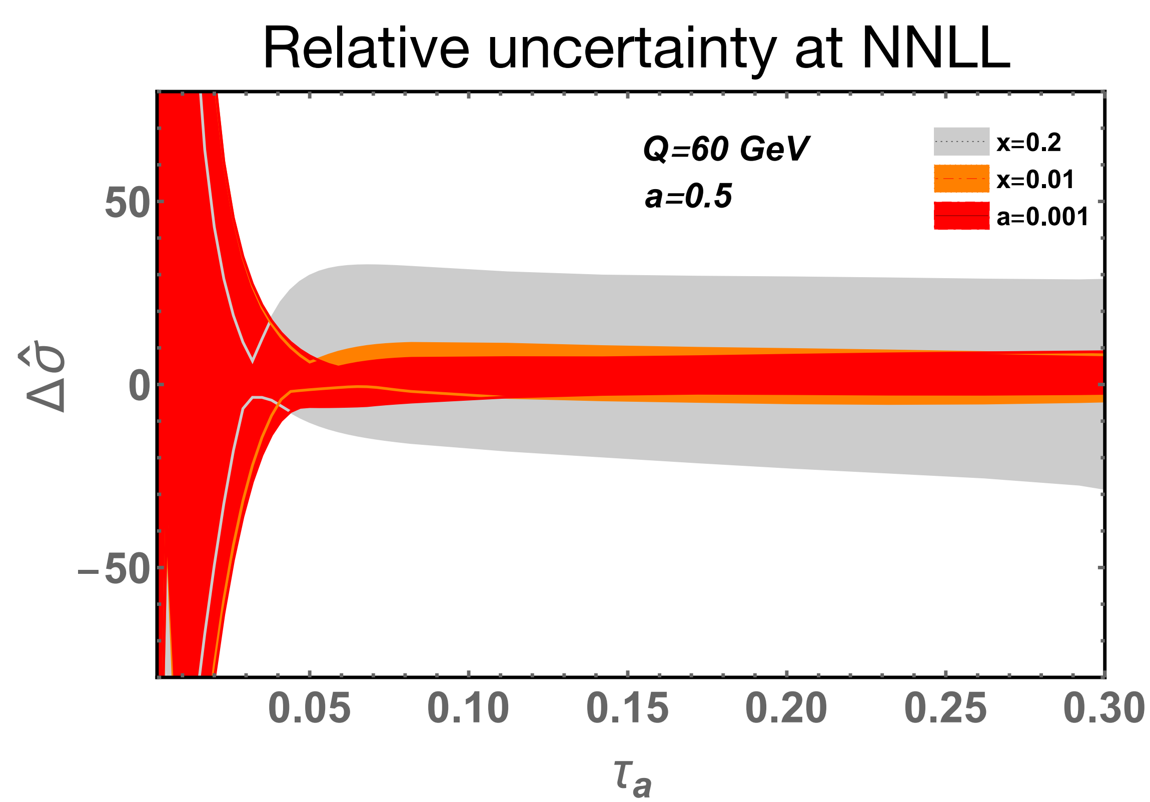
Relative uncertainty at NNLL



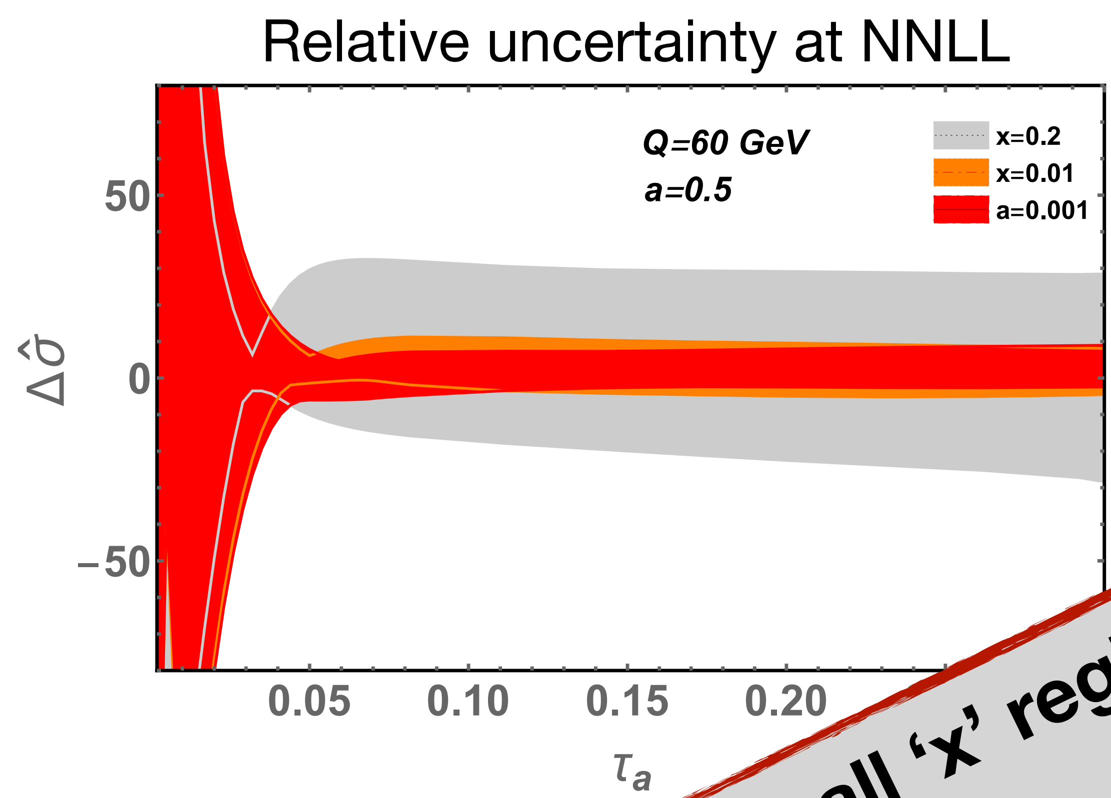
Prediction is more precise for negative ‘a’
a < 0

- uncertainty in the DIS angularity cross-section depends on the angularity parameter ‘a’

‘x’ dependency



- uncertainty in the angularity cross-section depends on the longitudinal momentum fraction (x) of the partons.

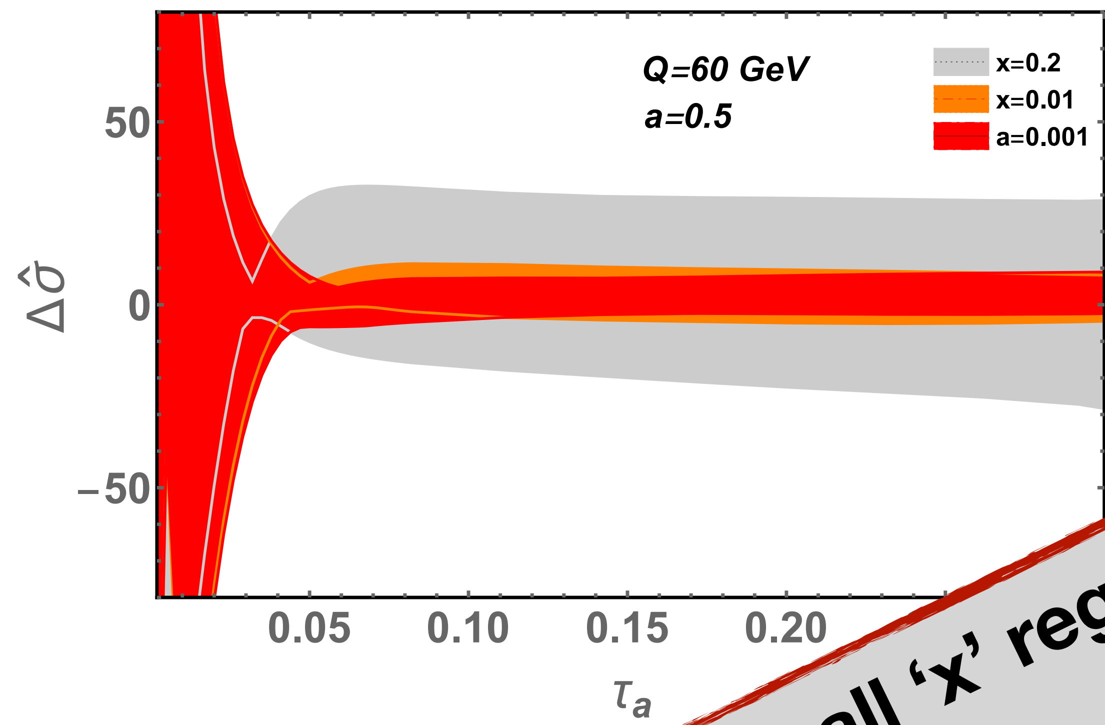


uncertainty in the … section depends on the
longitu… fraction (x) of the partons.

Prediction is more precise for small ‘x’ region. small-x

‘x’ dependency

Relative uncertainty at NNLL



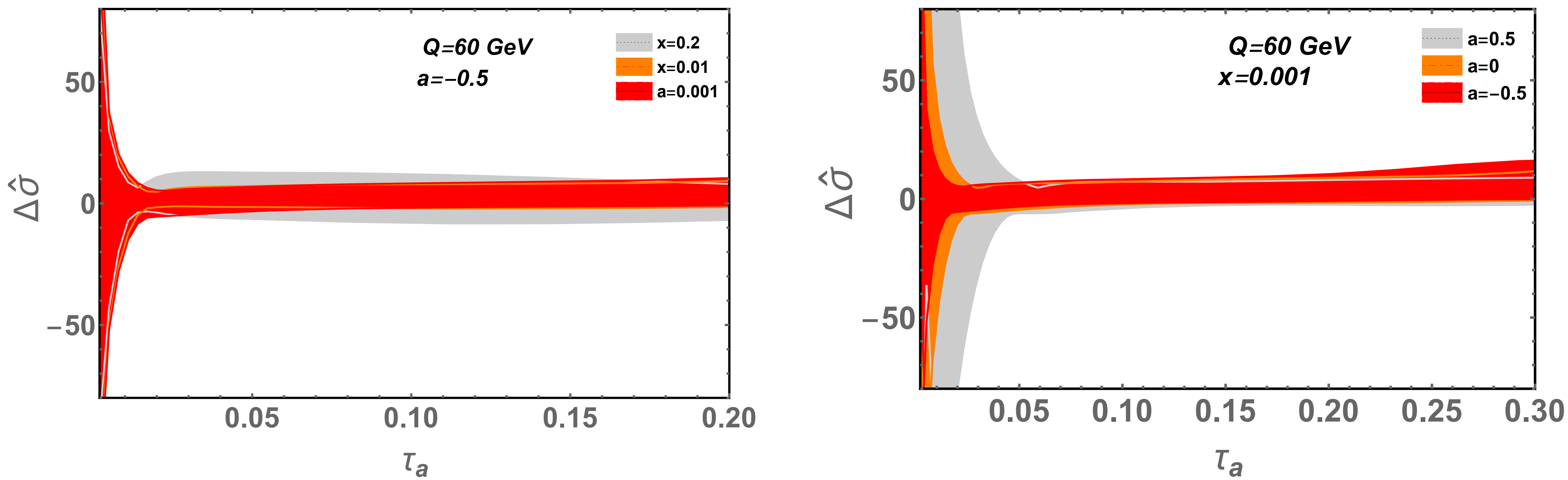
uncertainty in the … section depends on the longitu… fraction (x) of … e partons.

Prediction is more precise for small ‘x’ region.

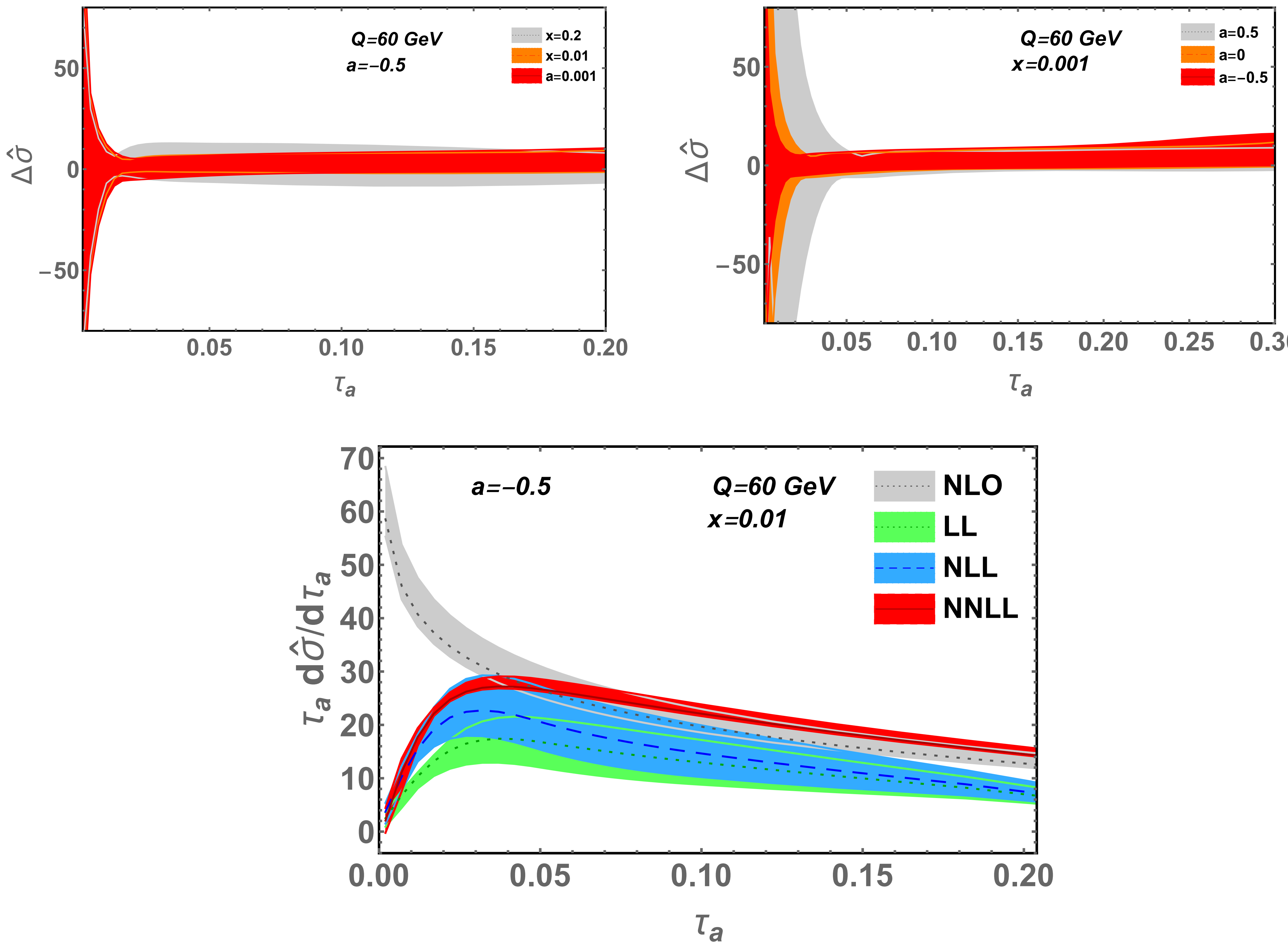
small-x

$a < 0$
&
small-x region

a < 0 and small-x result



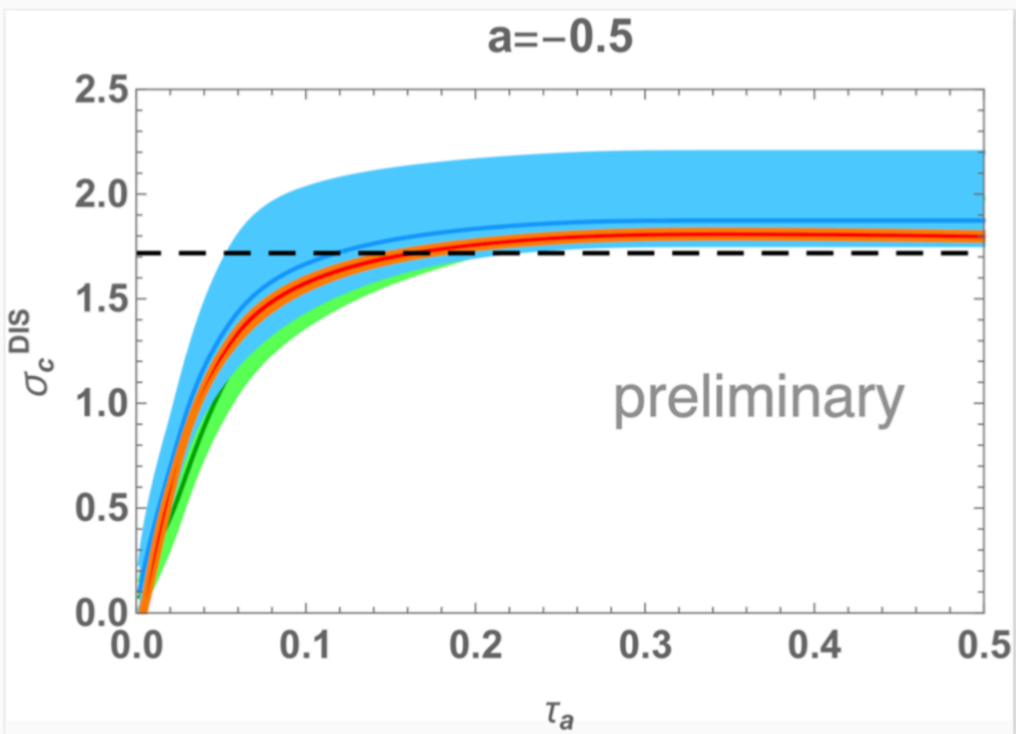
a < 0 and small-x result

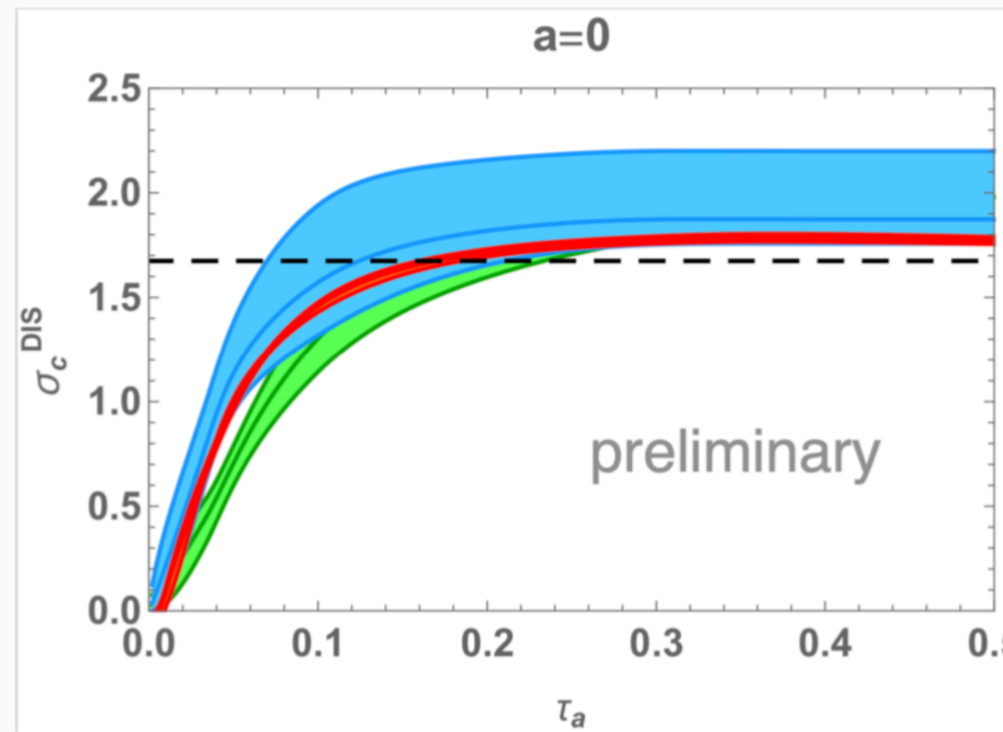


- Gluon contribution to the NLO correction is large at small-x region

Cummulant

$$\sigma_c(x, Q^2, \tau_a) = \frac{1}{\sigma_0}\int_0^{\tau_a} d\tau'_a \frac{d\sigma}{dx dQ^2 d\tau'_a}.$$





- Deviation of NNLL prediction from the total cross-section (at $\mathcal{O}(\alpha_s)$) at large angularity is due to the non singular term that not considered.

Summary and Conclusion

1. Present one-loop angularity beam function for the axis choice along jet axis.
2. Give precision prediction to the DIS angularity cross-section at NNLL accuracy.

uncertainty depends on the angularity parameter ‘a’ as well as on the longitudinal momentum fraction ‘x’ of the partons.

Future direction

1. Calculation of the fixed order terms which includes the contribution from the non-singular part.
2. Uncertainty in the cross-section is sensitive to Q, ’a’ and ‘x’ and we need to find out a reasonable profile function for DIS angularity.
3. Investigation of the non-singular part and prediction to DIS angularity distribution for entire angularity spectrum.

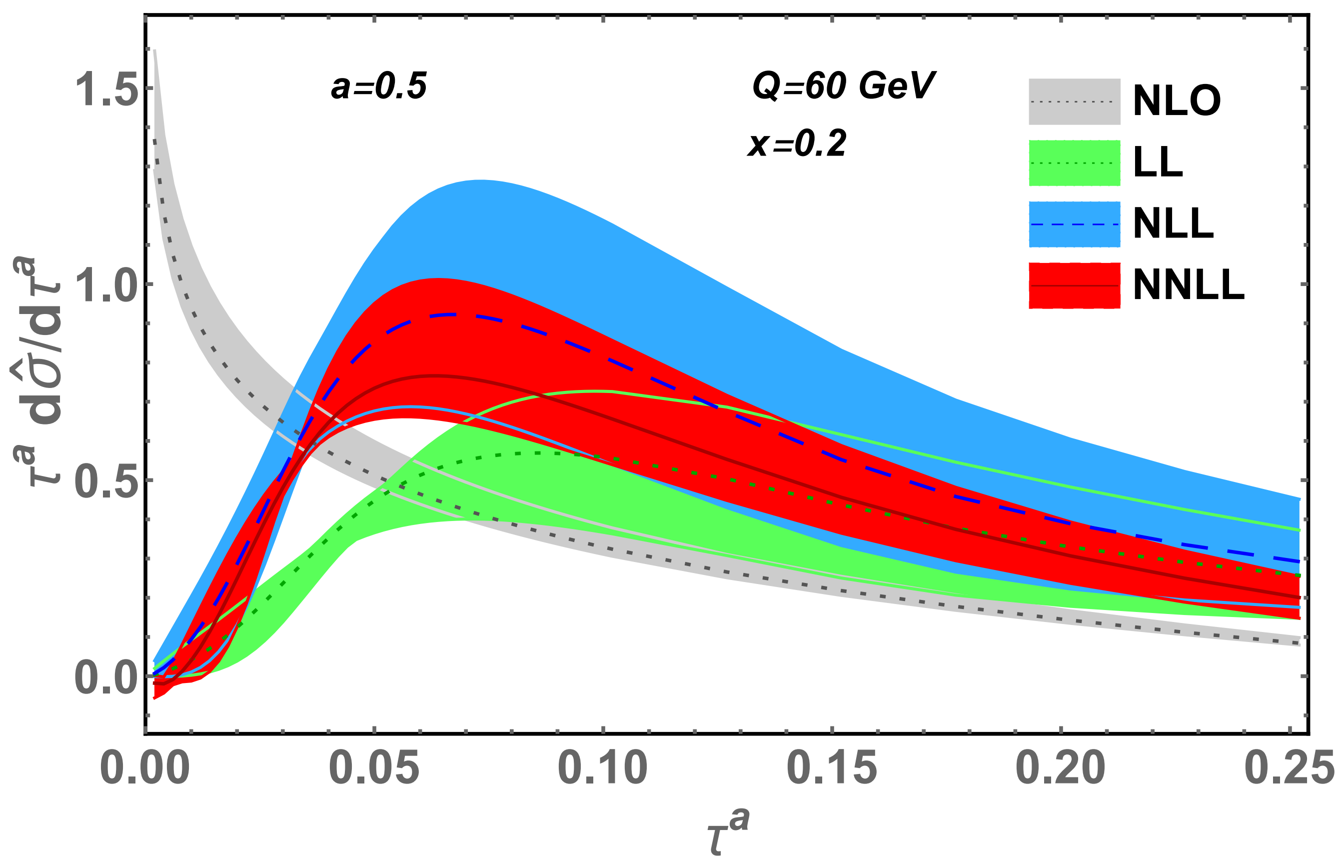
Thank You!

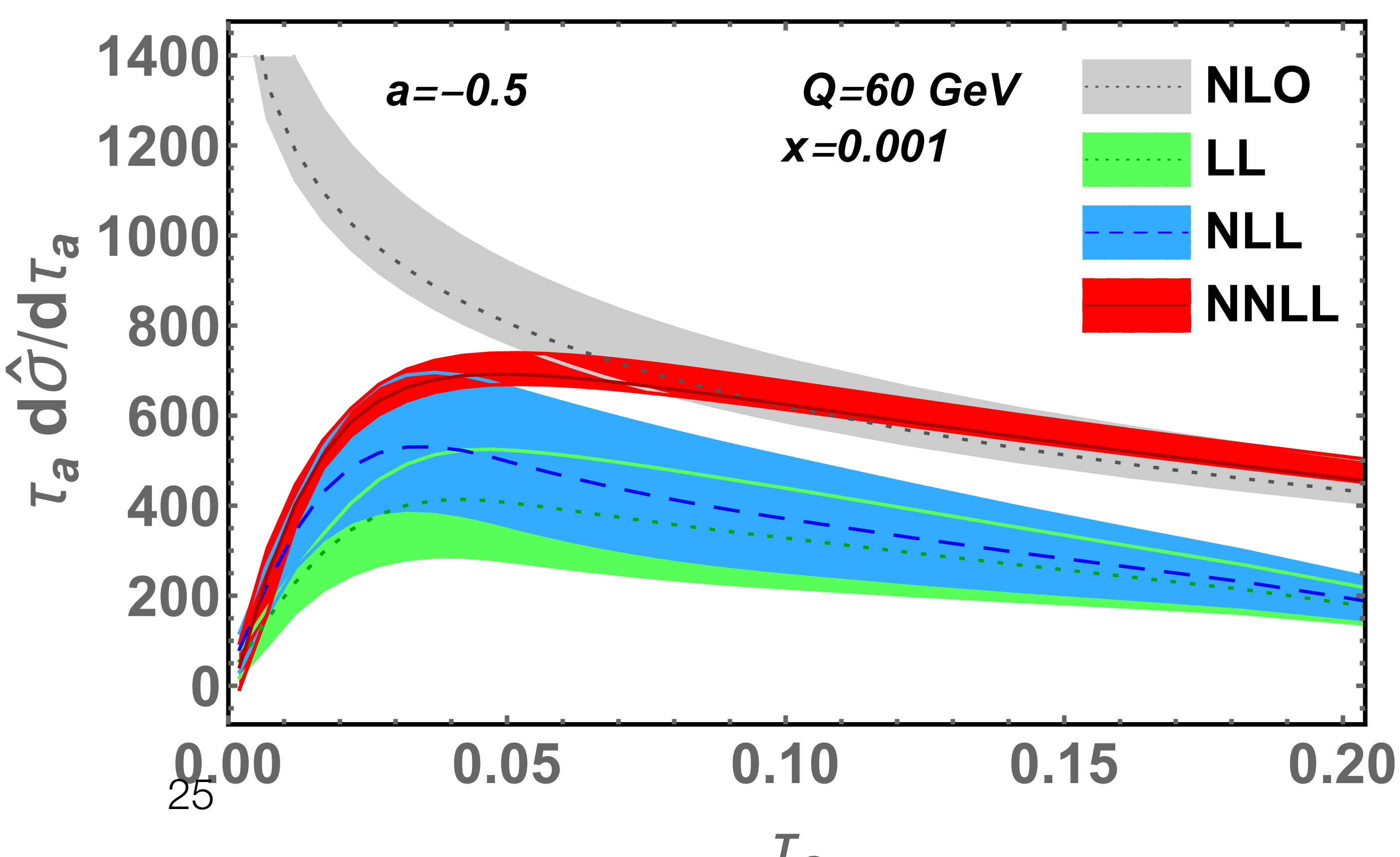
Back up

- Soft-collinear effective theory (SCET) is a systematic expansion of QCD in a small parameter λ which characterizes the scale of collinear and soft radiation from energetic massless partons.

In light-cone coordinates, n-collinear and soft momenta scales (thrust) as

$$
\begin{aligned}
\text{collinear:} \quad & p_n \sim Q(1, \lambda^2, \lambda) \\
\text{soft:} \quad & p_s \sim Q(\lambda^2, \lambda^2, \lambda^2)
\end{aligned}
$$


a=0.5
Q=60 GeV
x=0.2
NLO
LL
NLL
NNLL
$\tau^a\, d\hat{\sigma}/d\tau^a$
τ^a


a=-0.5
Q=60 GeV
x=0.001
NLO
LL
NLL
NNLL
$\tau_a\, d\hat{\sigma}/d\tau_a$