Center for Frontiers in Nuclear Science

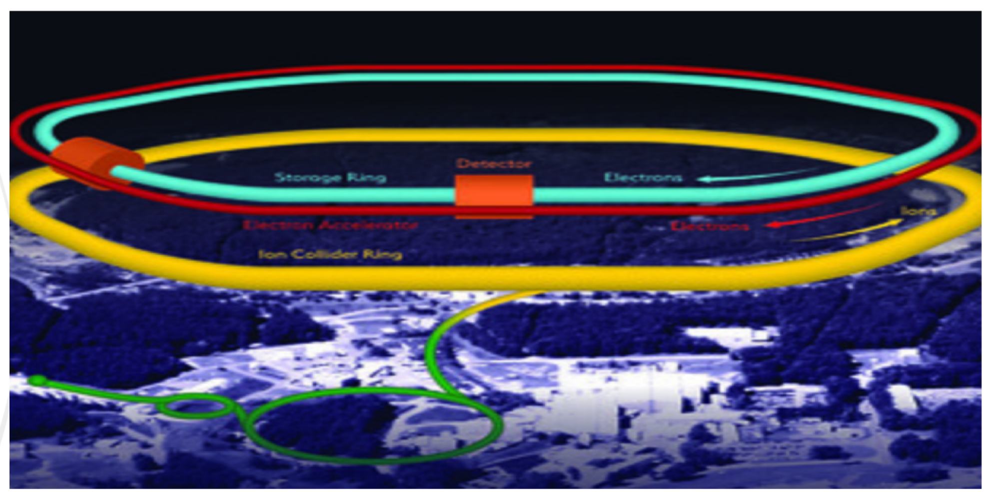
The 2021 CFNS Summer School on the Physics of the Electron-Ion Collider

August 9-20, 2021

Introduction to QCD

- Lec. 1: Fundamentals of QCD
- Lec. 2: Matching observed hadrons to quarks and gluons
- Lec. 3: QCD for cross sections with identified hadrons
- Lec. 4: QCD for cross sections with polarized beam(s)

Jianwei Qiu
Theory Center
Jefferson Lab



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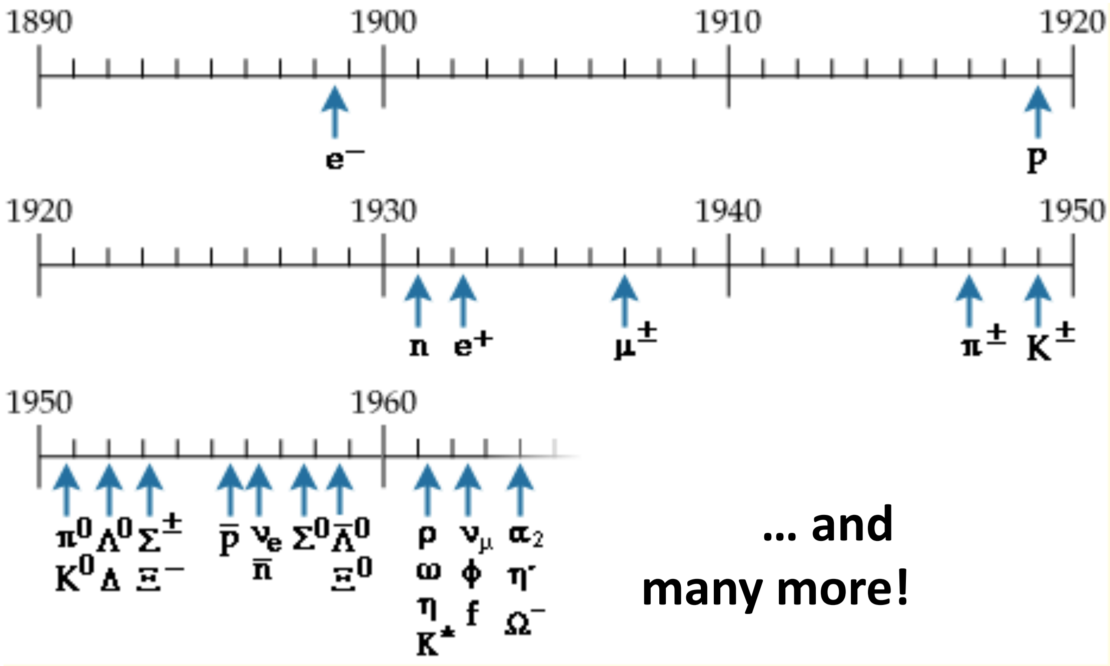
JSA

New particles, new ideas, and new theories

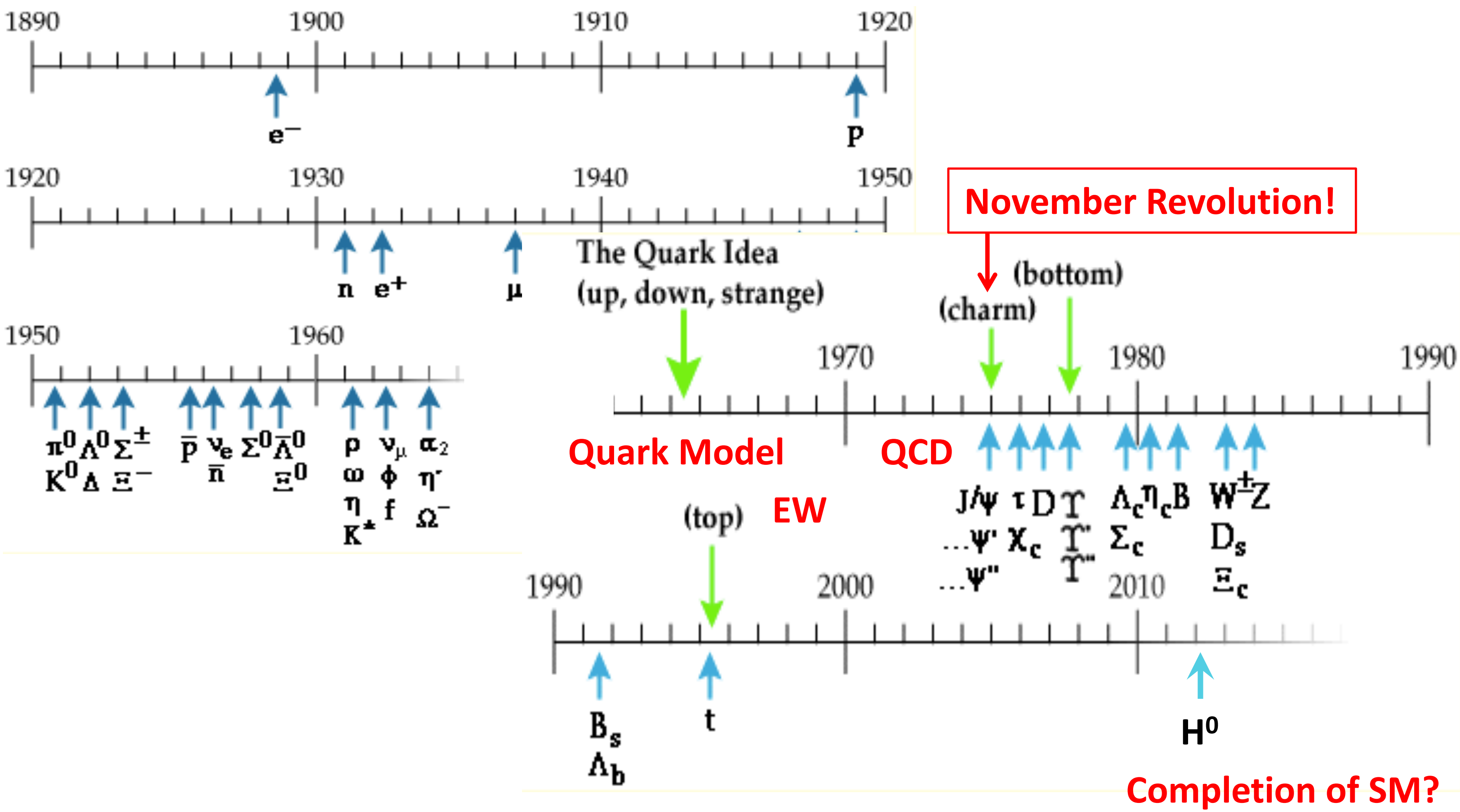
- Early proliferation of new hadrons – "particle explosion":



… and
many more!

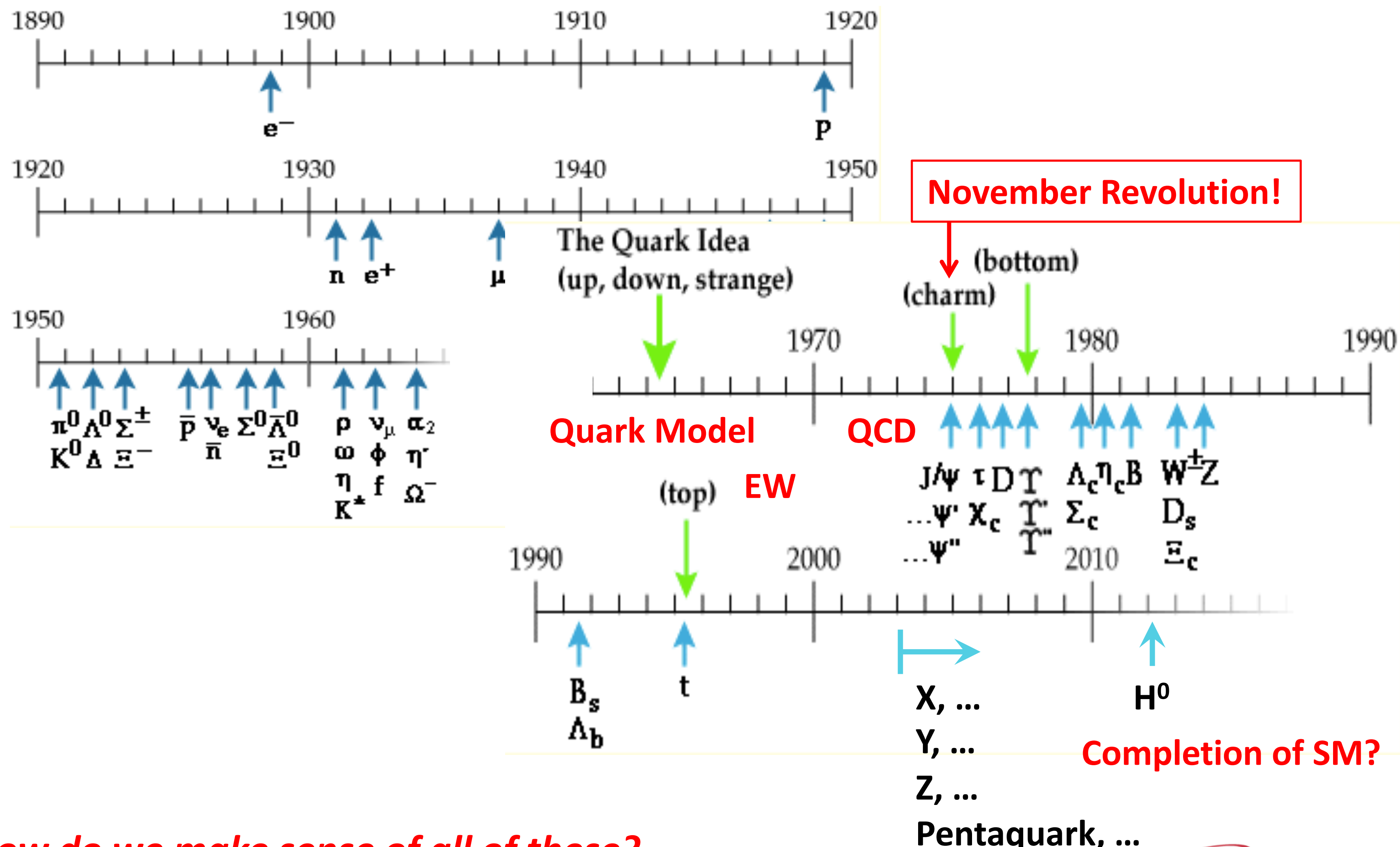
New particles, new ideas, and new theories

❑ **Early proliferation of new hadrons – "particle explosion":**



New particles, new ideas, and new theories

- Early proliferation of new hadrons – "particle explosion":

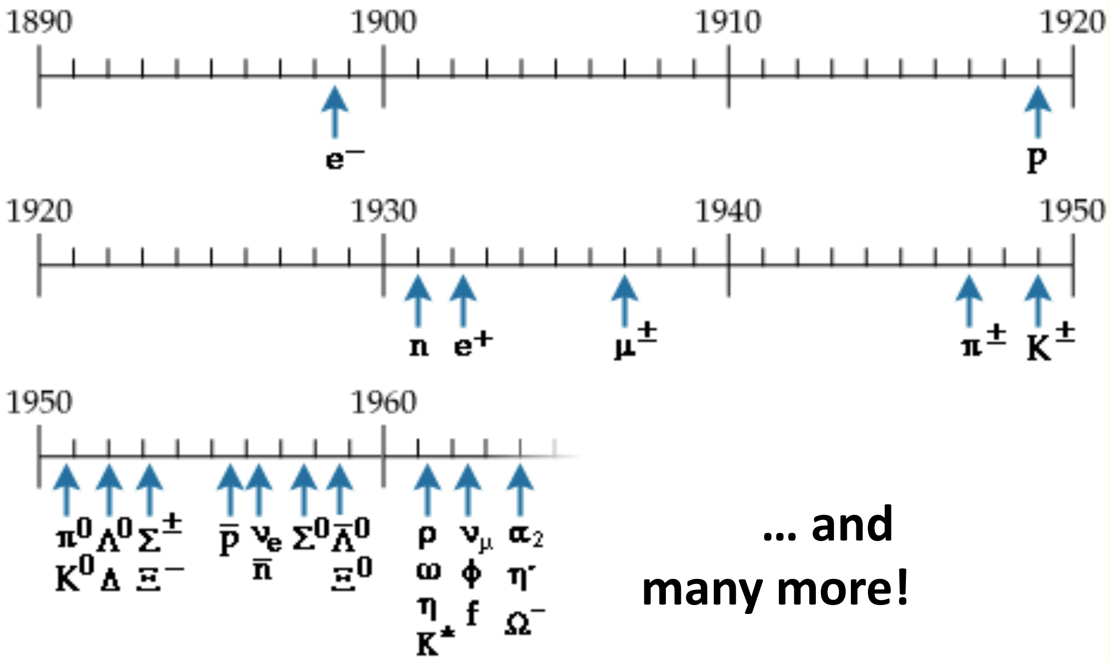


How do we make sense of all of these?

Another particle explosion?

New particles, new ideas, and new theories

- **Early proliferation of new hadrons – "particle explosion":**



- **Nucleons has internal structure!**

1933: Proton's magnetic moment

Otto Stern

Nobel Prize 1943

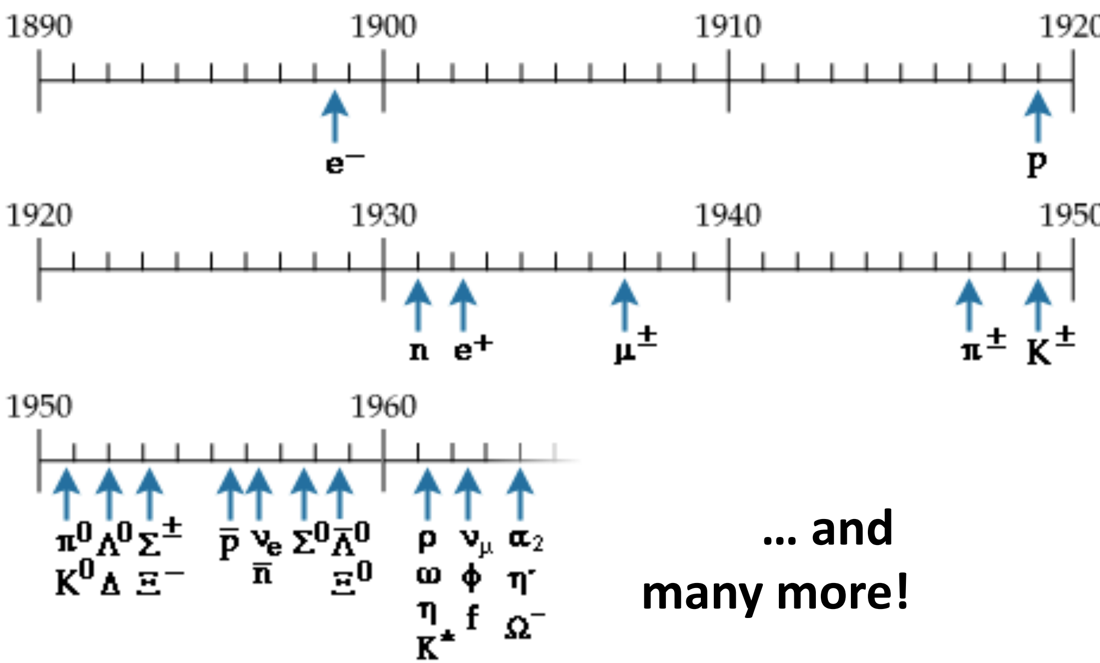
$$\mu_p = g_p \left(\frac{e\hbar}{2m_p} \right)$$

$$g_p = 2.792847356(23) \neq 2!$$

$$\mu_n = -1.913 \left(\frac{e\hbar}{2m_p} \right) \neq 0!$$

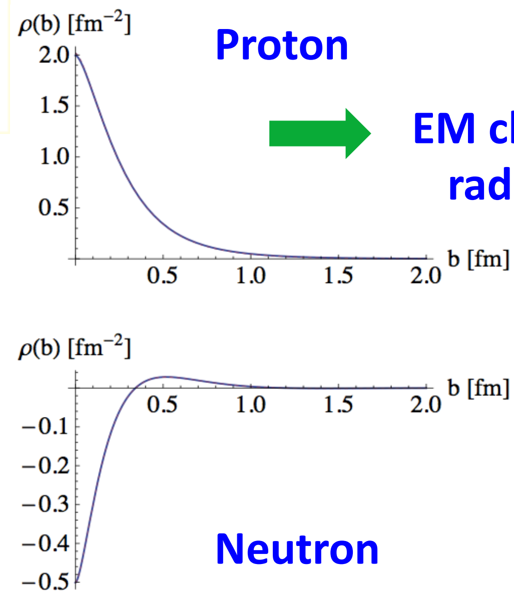
New particles, new ideas, and new theories

- ❑ **Early proliferation of new hadrons – "particle explosion":**



… and many more!

Electric charge distribution

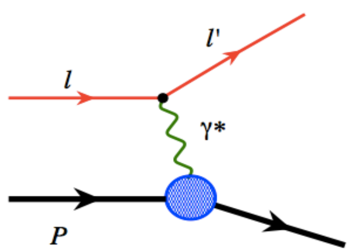


EM charge radius!

- ❑ **Nucleons has internal structure!**

1960: Elastic e-p scattering

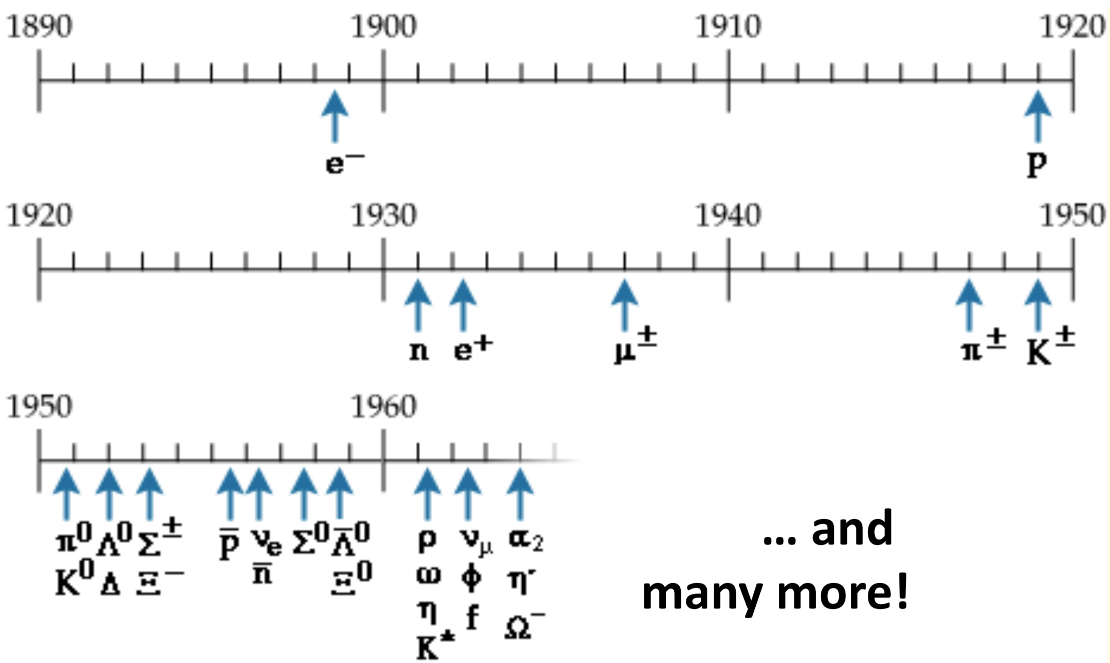
Robert Hofstadter
Nobel Prize 1961



Form factors

New particles, new ideas, and new theories

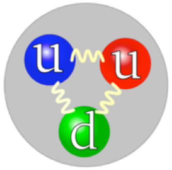
- ❑ **Early proliferation of new hadrons – "particle explosion":**



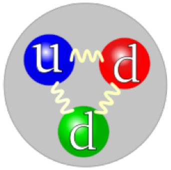
… and many more!

- ❑ **Nucleons are made of quarks!**

Proton



Neutron



→ **Quark Model**

Murray Gell-Mann
Nobel Prize, 1969

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The naïve Quark Model

- **Flavor SU(3) – assumption:**

  Physical states for u, d, s, neglecting any mass difference, are represented by 3-eigenstates of the fund'l rep'n of flavor SU(3)

- **Generators for the fundamental rep'n of SU(3) – 3x3 matrices:**

$$J_i = \frac{\lambda_i}{2} \quad \textbf{with} \quad \lambda_i, i = 1, 2, ..., 8 \quad \textbf{Gell-Mann matrices}$$

- **Good quantum numbers to label the states:**

$$J_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad J_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

simultaneously diagonalized

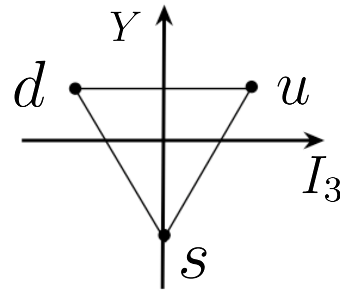
Isospin: $\hat{I}_3 \equiv J_3$ **, Hypercharge:** $\hat{Y} \equiv \frac{2}{\sqrt{3}} J_8$

- **Basis vectors – Eigenstates:** $|I_3, Y\rangle$

$$v^1 \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad v^2 \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad v^3 \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \Rightarrow s = |0, \text{-}\tfrac{2}{3}\rangle$$

The naïve Quark Model

❑ Quark states:



$$u = |\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad d = |\text{-}\tfrac{1}{2}, \tfrac{1}{3}\rangle \quad s = |0,\text{-}\tfrac{2}{3}\rangle$$

Spin: **½**

Baryon #: ***B* = ⅓**

Strangeness: ***S = Y – B*** **Electric charge:** $Q \equiv I_3 + \frac{Y}{2}$

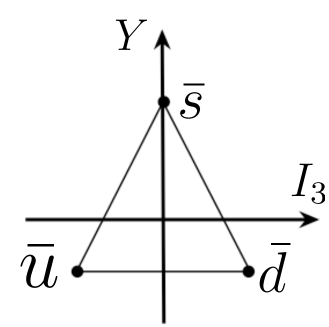
$$u \begin{cases} Q = 2/3\, e \\ s = 1/2 \\ I_3 = 1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad d \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = -1 \\ Y = 1/3 \\ B = 1/3 \\ S = 0 \end{cases} \qquad s \begin{cases} Q = -1/3\, e \\ s = 1/2 \\ I_3 = 0 \\ Y = -2/3 \\ B = 1/3 \\ S = -1 \end{cases}$$

❑ Antiquark states: $v_i \equiv \epsilon_{ijk}\, v^j v^k$

$$\hat{I}_3 v_1 = \epsilon_{123}[(\hat{I}_3 v^2)v^3 + v^2(\hat{I}_3 v^3)] + \epsilon_{132}[(\hat{I}_3 v^3)v^2 + v^3(\hat{I}_3 v^2)] = -\frac{1}{2} v_1$$

$$\hat{Y} v_1 = \epsilon_{123}[(\hat{Y} v^2)v^3 + v^2(\hat{Y} v^3)] + \epsilon_{132}[(\hat{Y} v^3)v^2 + v^3(\hat{Y} v^2)] = -\frac{1}{3} v_1$$

$$u \longrightarrow \bar{u} = |\text{-}\tfrac{1}{2},\text{-}\tfrac{1}{3}\rangle$$



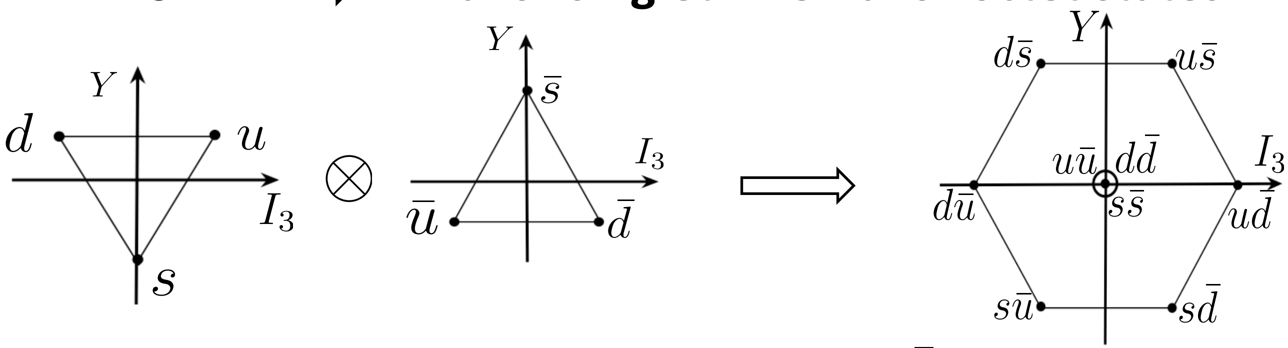
Mesons – Quark Model

Quark-antiquark $q\bar{q}$ flavor states:

❑ Group theory says:

$$q(u,d,s) = \mathbf{3}, \quad \bar{q}(\bar{u},\bar{d},\bar{s}) = \bar{\mathbf{3}}, \quad \text{of flavor SU(3)}$$

$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ $\Longrightarrow$ **1 flavor singlet + 8 flavor octet states**



There are three states with $I_3 = 0, Y = 0$: $u\bar{u}, d\bar{d}, s\bar{s}$

❑ Physical meson states (L=0, S=0):

✧ **Octet states:**

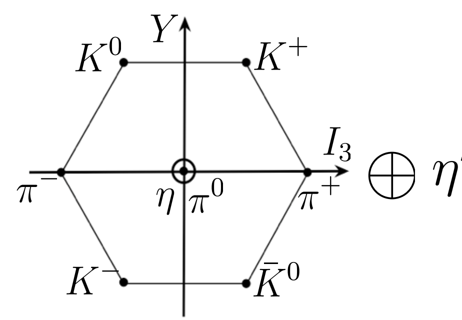
$$A = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \Longrightarrow \pi^0$$

$$B = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \Longrightarrow \eta_8$$

✧ **Singlet states:**

$$C = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \Longrightarrow \eta_1$$

$\eta_8, \eta_1 \Rightarrow \eta, \eta'$



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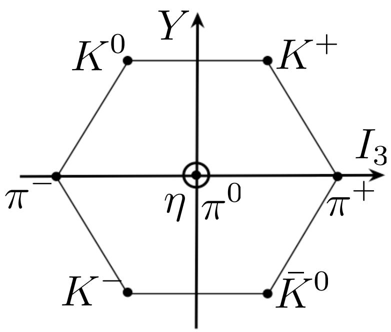
Quantum Numbers

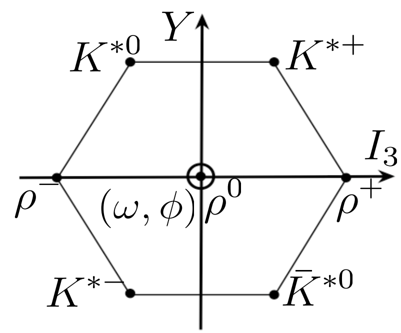
- **Meson states:** J^{PC}
  - **Spin of $q\bar{q}$ pair:** $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \;\longrightarrow\; S = 0, 1$
  - **Spin of mesons:** $J = S + L$
  - **Charge conjugation:** $C = (-1)^{L+S}$
- **L=0 states:**

$J^{PC} = 0^{-+}$: ***(Y=S)***



$\oplus\ \eta'$

$J^{PC} = 1^{--}$: ***(Y=S)***



$\oplus\ \mathrm{mix}(\omega, \phi)$

Flavor singlet, spin octet

Flavor octet, spin octet

- **Color:**

No color was introduced!

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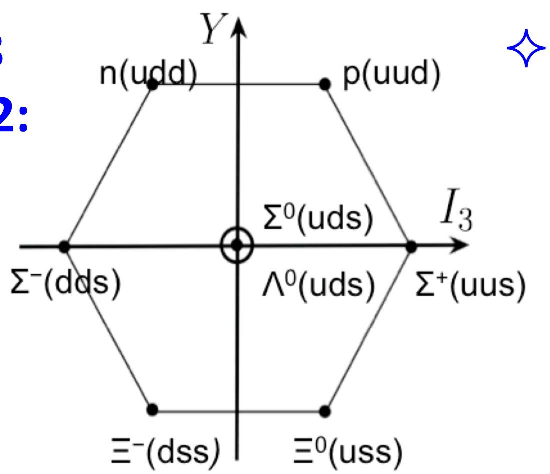
Baryons – Quark Model

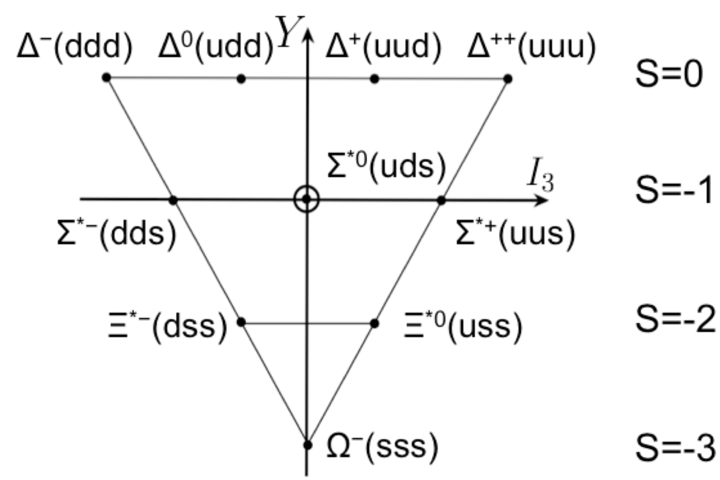
3 quark qqq **states:** $B=1$

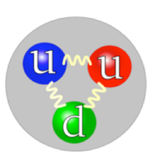
- **Group theory says:**
  - ✧ **Flavor:** $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{10}_S\oplus\mathbf{8}_{M_S}\oplus\mathbf{8}_{M_A}\oplus\mathbf{1}_A$

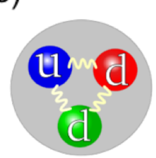
    S: symmetric in all 3 q, M_S: symmetric in 1 and 2,
    M_A: antisymmetric in 1 and 2, A: antisymmetric in all 3
  - ✧ **Spin:** $\mathbf{2}\otimes\mathbf{2}\otimes\mathbf{2}=\mathbf{4}_S\oplus\mathbf{2}_{M_s}\oplus\mathbf{2}_{M_A}\Rightarrow S=\frac{3}{2},\frac{1}{2},\frac{1}{2}$
- **Physical baryon states:**
  - ✧ **Flavor-8 Spin-1/2:**



  - ✧ **Flavor-10 Spin-3/2:**



Proton 

Neutron 

Δ^{++}(uuu), …

Violation of Pauli exclusive principle

➡ ***Need another quantum number - color!***

Color

❑ Minimum requirements:

- ✧ Quark needs to carry at least 3 different colors
- ✧ Color part of the 3-quarks' wave function needs to antisymmetric

❑ SU(3) color:

Recall: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$

$$\Longrightarrow c(\text{Red, Green, Blue})$$

$$\psi_{\text{Color}}(c_1, c_2, c_3) = \frac{1}{\sqrt{6}}[\text{RGB-GRB+RBG-BRG+GBR-BGR}]$$

Antisymmetric color singlet state:

❑ Baryon wave function:

$$\Psi(q_1, q_2, q_3) = \psi_{\text{Space}}(x_1, x_2, x_3) \otimes \psi_{\text{Flavor}}(f_1, f_2, f_3) \otimes \psi_{\text{Spin}}(s_1, s_2, s_3) \otimes \psi_{\text{Color}}(c_1, c_2, c_3)$$

Antisymmetric Symmetric Symmetric Symmetric Antisymmetric

A complete example: Proton

- **Wave function – the state:**

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\left[uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) + udu(\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow - 2\uparrow\downarrow\uparrow)\right.$$
$$\left. + duu(\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow - 2\downarrow\uparrow\uparrow)\right]$$

- **Normalization:**

$$\langle p\uparrow|p\uparrow\rangle = \frac{1}{18}[(1+1+(-2)^2)+(1+1+(-2)^2)+(1+1+(-2)^2)] = 1$$

- **Charge:**

$$\hat{Q} = \sum_{i=1}^{3}\hat{Q}_i$$

$$\langle p\uparrow|\hat{Q}|p\uparrow\rangle = \frac{1}{18}[(\tfrac{2}{3}+\tfrac{2}{3}-\tfrac{1}{3})(1+1+(-2)^2)+(\tfrac{2}{3}-\tfrac{1}{3}+\tfrac{2}{3})(1+1+(-2)^2)$$
$$+(-\tfrac{1}{3}+\tfrac{2}{3}+\tfrac{2}{3})(1+1+(-2)^2)] = 1$$

- **Spin:**

$$\hat{S} = \sum_{i=1}^{3}\hat{s}_i$$

$$\langle p\uparrow|\hat{S}|p\uparrow\rangle = \frac{1}{18}\{[(\tfrac{1}{2}-\tfrac{1}{2}+\tfrac{1}{2})+(-\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2})+4(\tfrac{1}{2}+\tfrac{1}{2}-\tfrac{1}{2})]$$
$$+[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]+[\tfrac{1}{2}+\tfrac{1}{2}+4\tfrac{1}{2}]\} = \frac{1}{2}$$

- **Magnetic moment:**

$$\mu_p = \langle p\uparrow|\sum_{i=1}^{3}\hat{\mu}_i(\hat{\sigma}_3)_i|p\uparrow\rangle = \frac{1}{3}[4\mu_u - \mu_d]$$

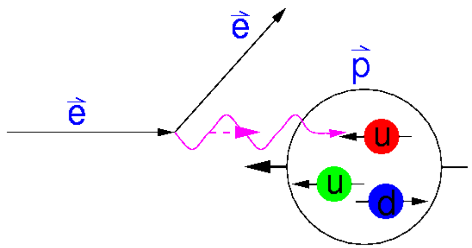
$$\mu_n = \frac{1}{3}[4\mu_d - \mu_u] \qquad \frac{\mu_u}{\mu_d} \approx \frac{2/3}{-1/3} = -2$$

$$\Rightarrow \begin{cases} \left(\dfrac{\mu_n}{\mu_p}\right)_{\mathrm{QM}} = -\dfrac{2}{3} \\ \left(\dfrac{\mu_n}{\mu_p}\right)_{\mathrm{Exp}} = -0.68497945(58) \end{cases}$$

How to "see" substructure of a nucleon?

❑ **A modern "Rutherford" experiment (over 50 years ago):**

SLAC 1968: $e(p) + h(P) \rightarrow e'(p') + X$



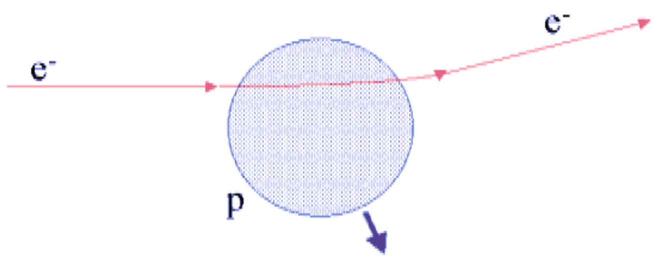
Need a localized probe:

$$Q^2 = -(p - p')^2 \gg 1\ \mathrm{fm}^{-2}$$

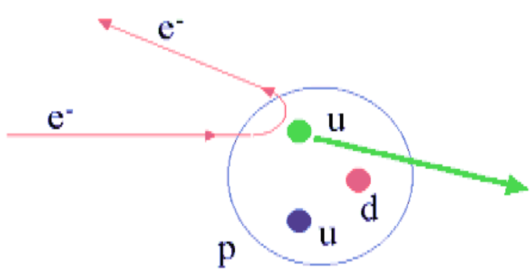
$$\Rightarrow \quad \frac{1}{Q} \ll 1\ \mathrm{fm}$$

Prediction:

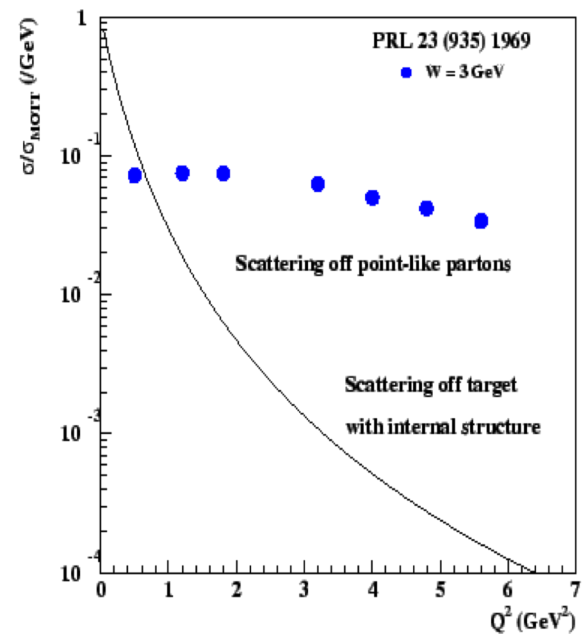
- If proton "charge cloud":



- If proton contains point charges, some of time see:



Discovery:

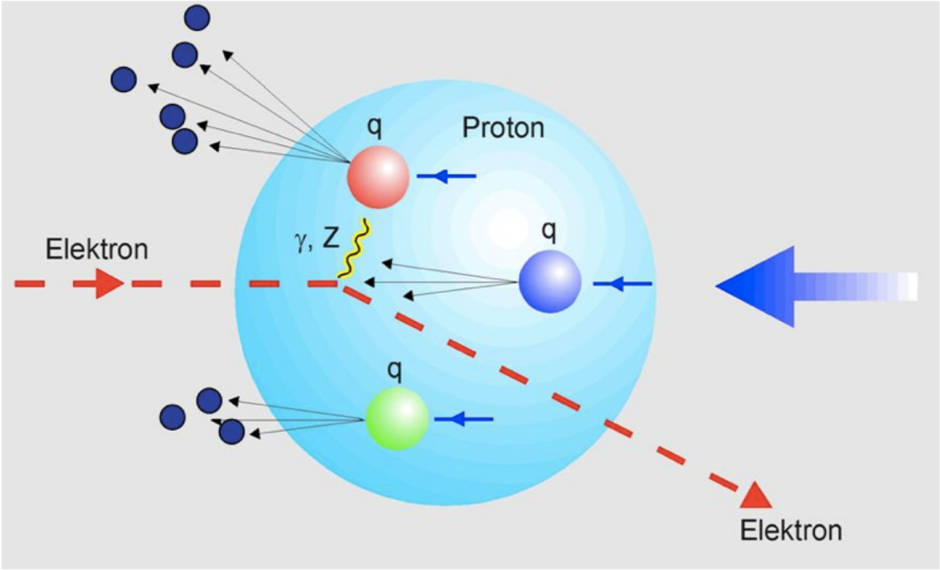


Discovery:
Partons/Quarks

How to "see" substructure of a nucleon?

❑ Lepton-Hadron Deep Inelastic Scattering (DIS):

SLAC 1968: $e(p) + h(P) \to e'(p') + X$



✧ **Localized probe:**

$$Q^2 = -(p-p')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \text{fm}$$

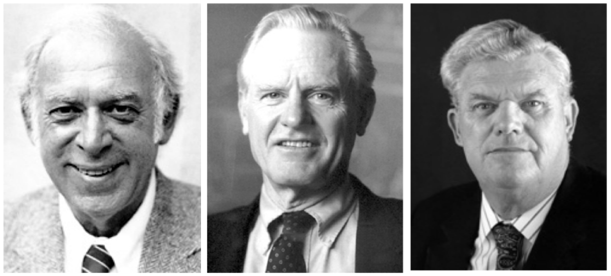
✧ **Two variables:**

$$Q^2 = 4EE'\sin^2(\theta/2)$$

$$x_B = \frac{Q^2}{2m_N\nu}$$

$$\nu = E - E'$$

→ **Discovery of spin ½ quarks, and partonic structure!**



What holds the quarks together?

Nobel Prize, 1990

⇒ **The birth of QCD (1973)**

– Quark Model + Yang-Mill gauge theory

Quantum Chromo-dynamics (QCD)

= A quantum field theory of quarks and gluons =

- **Fields:**

$\psi_i^f(x)$ **Quark fields: spin-½ Dirac fermion (like electron)**

Color triplet: $i = 1, 2, 3 = N_c$

Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$ **Gluon fields: spin-1 vector field (like photon)**

Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

$$\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f - \frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2 + \text{gauge fixing} + \text{ghost terms}$$

- **QED – force to hold atoms together:**

$$\mathcal{L}_{QED}(\phi, A) = \sum_f \overline{\psi}^f \left[(i\partial_\mu - eA_\mu)\gamma^\mu - m_f \right] \psi^f - \frac{1}{4}\left[\partial_\mu A_\nu - \partial_\nu A_\mu\right]^2$$

QCD is much richer in dynamics than QED

Gluons are dark, but, interact with themselves, NO free quarks and gluons

Gauge property of QCD

❑ Gauge Invariance:

$$\psi_i(x) \to \psi'_j(x) = U(x)_{ji}\,\psi_i(x)$$
$$A_\mu(x) \to A'_\mu(x) = U(x)\,A_\mu(x)\,U^{-1}(x) + \frac{i}{g}\left[\partial_\mu U(x)\right]U^{-1}(x)$$

where $$A_\mu(x)_{ij} \equiv A_{\mu,a}(x)(t_a)_{ij}$$

$$U(x)_{ij} = \left[e^{i\,\alpha_a(x)\,t_a}\right]_{ij} \qquad \text{Unitary} \quad [\,\text{det=1, SU(3)}]$$

❑ Color matrices:

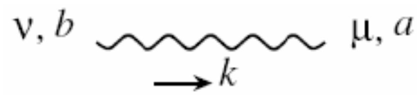
$$[\,t_a, t_b\,] = i\,C_{abc}\,t_c$$

Generators for the fundamental representation of SU3 color

❑ Gauge Fixing:

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}(\partial_\mu A^\mu_a)(\partial_\nu A^\nu_a)$$

Allow us to define the gauge field propagator:



$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

with $\lambda = 1$ **the Feynman gauge**

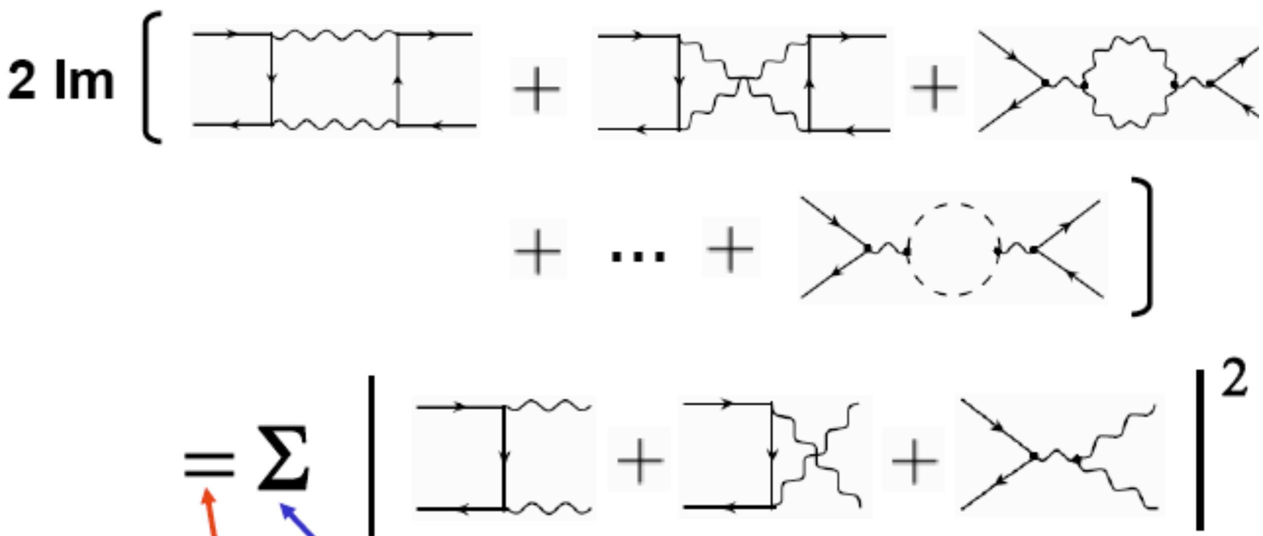
Ghost in QCD

- ❑ **Ghost:**

Ghost

$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a(x))(\partial^\mu \eta_a(x) - g\, C_{abc}\, A_b^\mu(x)\, \eta_c(x))$$

so that the optical theorem (hence the unitarity) can be respected



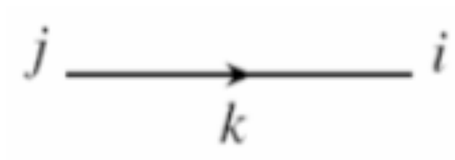
Sum over all physical polarizations

Fail without the ghost loop

Feynman rules in QCD

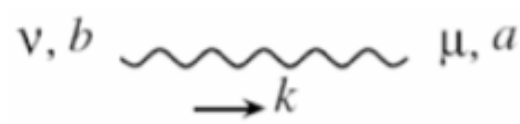
❑ Propagators:

Quark:



$$\frac{i}{\gamma \cdot k - m}\,\delta_{ij}$$

Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

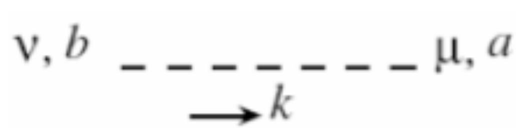
for a covariant gauge

$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n}\right]$$

for a light-cone gauge

$$n \cdot A(x) = 0 \quad \text{with} \quad n^2 = 0$$

Ghost::

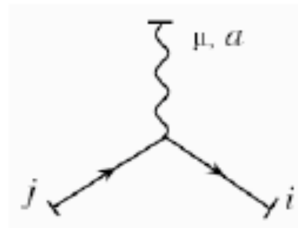


$$\frac{i\delta_{ab}}{k^2}$$

Feynman rules in QCD

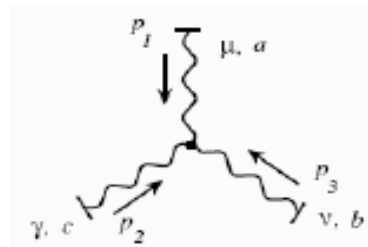
❑ Interactions:

$$-g\bar{\psi}\gamma^{\mu}A_{\mu,a}t_a\psi$$



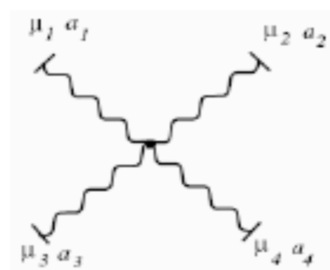
$$-ig(t_a)_{ij}\gamma_{\mu}$$

$$\frac{1}{2}gC_{abc}(\partial_{\mu}A_{\nu,a} - \partial_{\nu}A_{\mu,a})A_b^{\mu}A_c^{\nu}$$



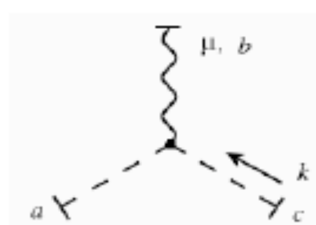
$$-gC_{abc}\left[g_{\mu\nu}(p_1 - p_2)_{\gamma} + g_{\nu\gamma}(p_2 - p_3)_{\mu} + g_{\gamma\mu}(p_3 - p_1)_{\nu}\right]$$

$$-\frac{g^2}{4}C_{abc}C_{ab'c'} * A_b^{\mu}A_c^{\nu}A_{\mu,b'}A_{\nu,c'}$$



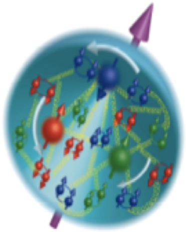
$$-ig^2\left[C_{ea_1a_2}C_{ea_3a_4} * (g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) + \ldots\right]$$

$$\partial_{\mu}\bar{\eta}_a\left(gC_{abc}A_b^{\mu}\right)\eta_c$$

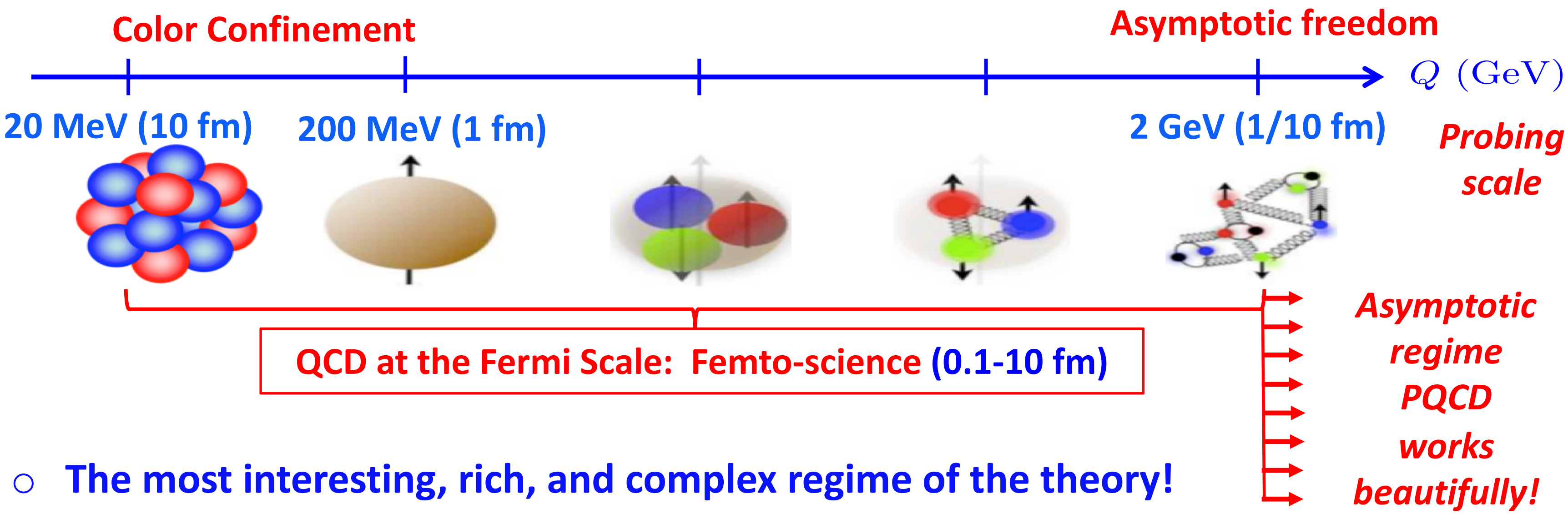


$$gC_{abc}k_{\mu}$$

QCD color is fully entangled



- **QCD color confinement:**
  - ***Do not see any quarks and gluons in isolation***
  - ***The structure of nucleons and nuclei – emergent properties of QCD***

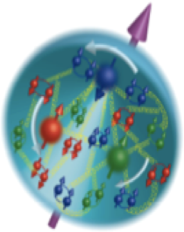


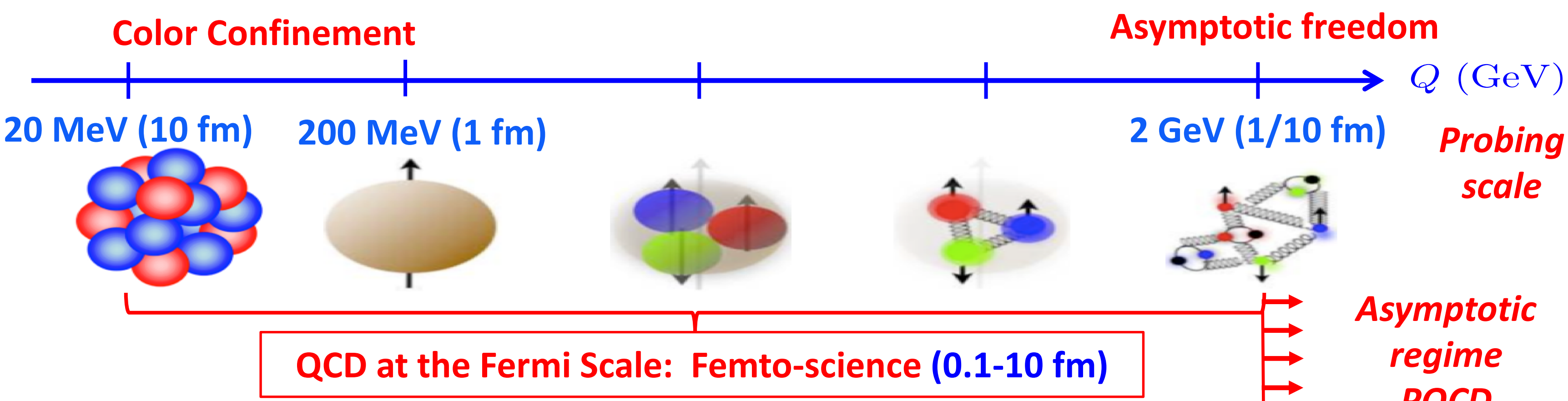
  - **The most interesting, rich, and complex regime of the theory!**
  - **Emergent phenomena depend on the scale at which we probe them!**

QCD color is fully entangled

❑ QCD color confinement:

- *Do not see any quarks and gluons in isolation*
- *The structure of nucleons and nuclei – emergent properties of QCD*





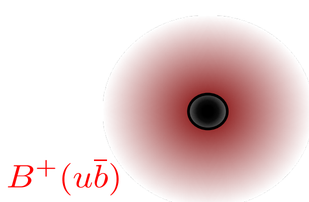
Asymptotic regime PQCD works beautifully!

- The most interesting, rich, and complex regime of the theory!
- Emergent phenomena depend on the scale at which we probe them!

❑ QCD is non-perturbative:

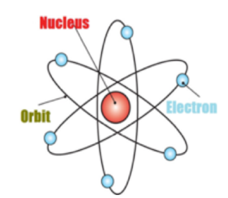
- *Any cross section/observable with identified hadron is not perturbatively calculable!*
- *Color is fully entangled!*

B-meson



Brown-Muck

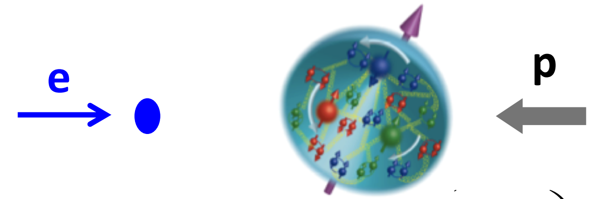
Atomic structure

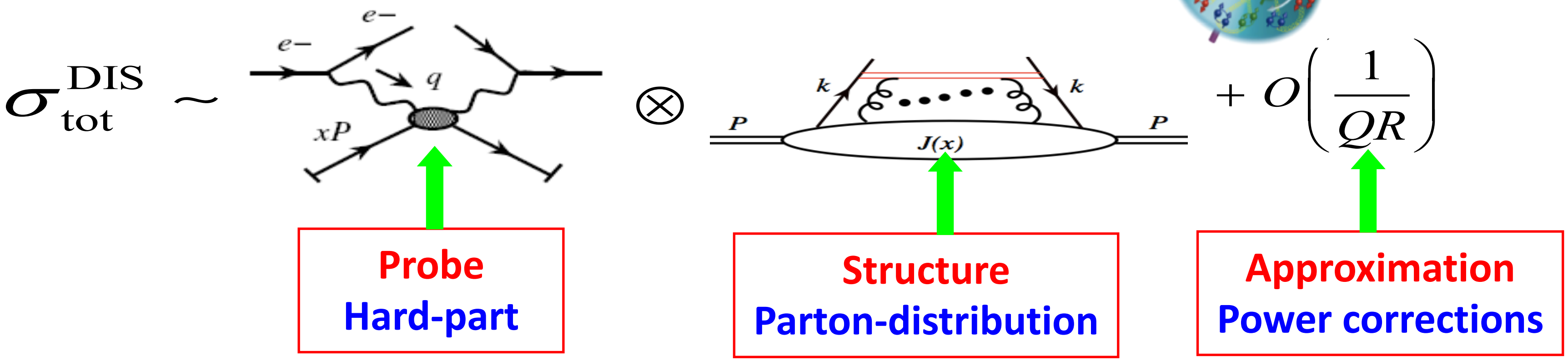


Quantum orbits

Theoretical Approaches - Approximations

- **Perturbative QCD Factorization:**
  – *Approximation at Feynman diagram level*





- **Effective field theory (EFT):**
  – *Approximation at the Lagrangian level*

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD:**
  – *Approximation mainly due to computer power*

  Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

- **Other approaches:**

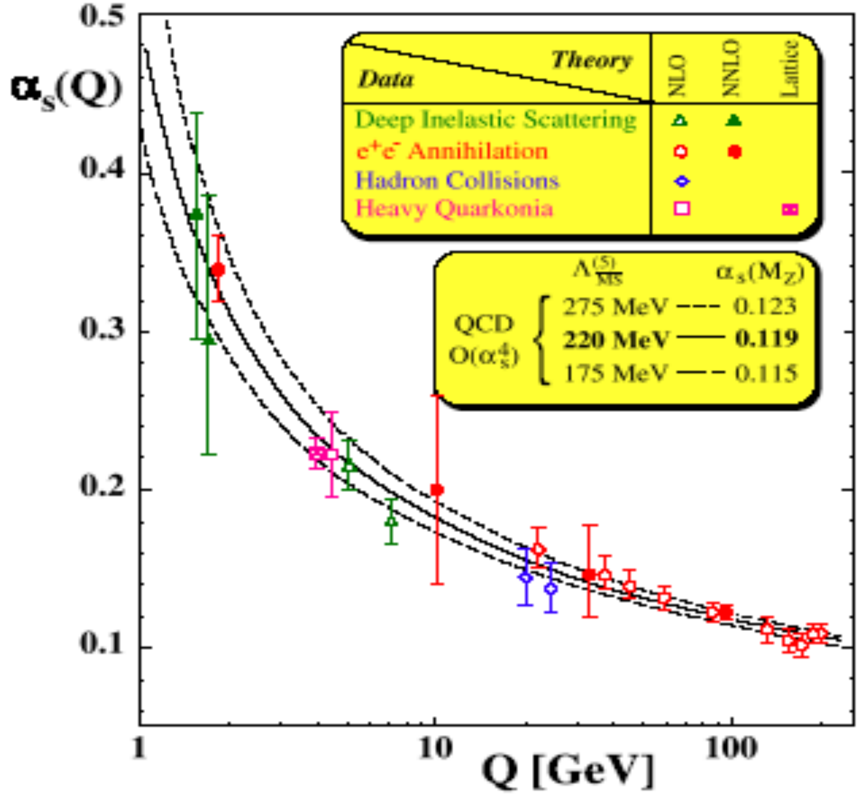
  Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

QCD Asymptotic Freedom

- **Interaction strength:**

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{\mathrm{QCD}}^2}\right)}$$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

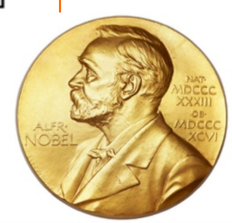
$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

→ **Discovery of QCD Asymptotic Freedom**

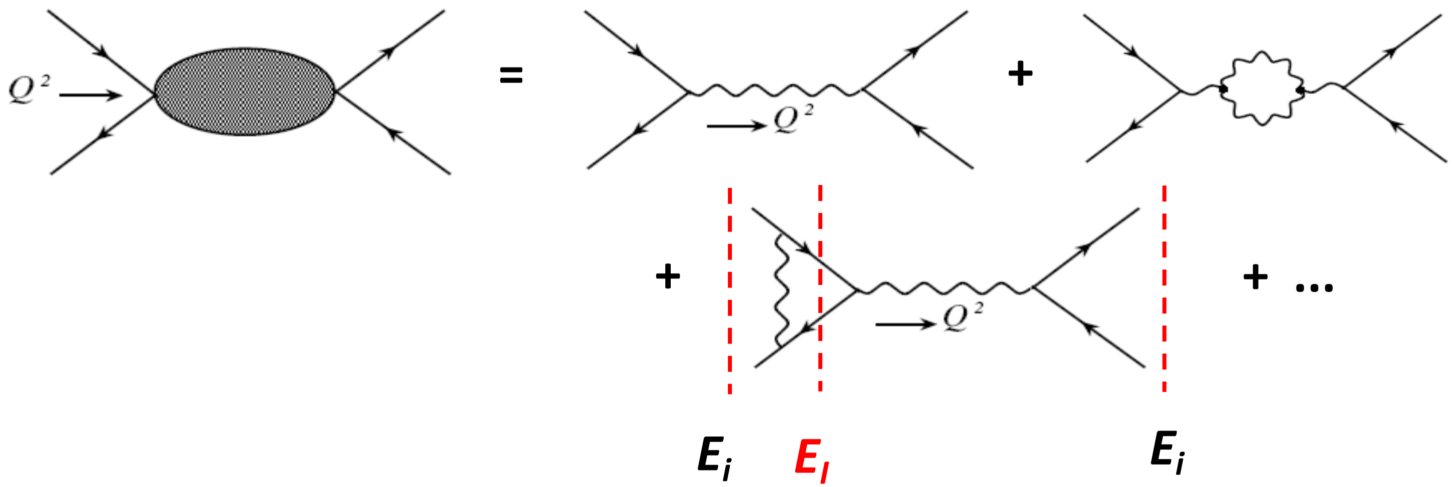


Nobel Prize, 2004

Jefferson Lab

Renormalization, why need?

❑ Scattering amplitude:



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \ldots \right) + \ldots \Rightarrow \infty$$

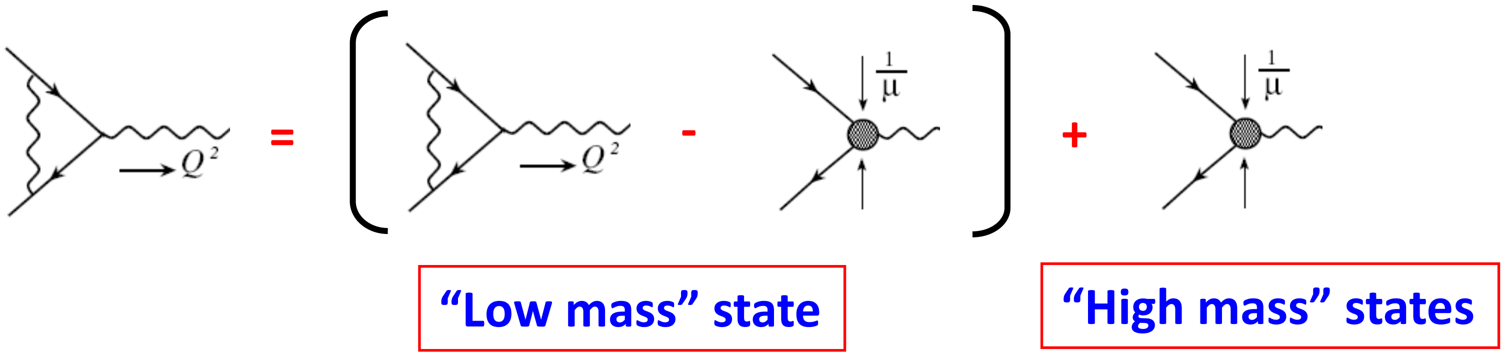
UV divergence: **result of a "sum" over states of high masses**

Uncertainty principle: High mass states = "Local" interactions

No experiment has an infinite resolution!

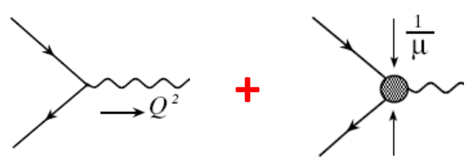
Physics of renormalization

- UV divergence due to "high mass" states, not observed



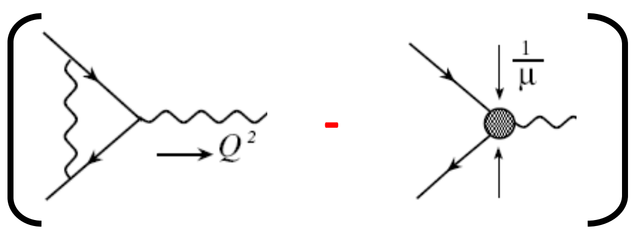
- Combine the "high mass" states with LO

LO:



= $g(\mu)$ ← Renormalized coupling

NLO:



\+ ...

No UV divergence!

- Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantity should not depend on renormalization scale μ ➡ renormalization group equation:

$$\mu^2 \frac{d}{d\mu^2}\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0 \quad \Longrightarrow \quad \sigma_{\rm Phy}(Q^2) = \sum_n \hat{\sigma}^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

- Running coupling constant:

$$\mu\frac{\partial g(\mu)}{\partial\mu} = \beta(g) \qquad \alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- QCD β function:

$$\beta(g) = \mu\frac{\partial g(\mu)}{\partial\mu} = +g^3\frac{\beta_1}{16\pi^2} + \mathcal{O}(g^5) \qquad \beta_1 = -\frac{11}{3}N_c + \frac{4}{3}\frac{n_f}{2} < 0 \quad \text{for } n_f \le 6$$

- QCD running coupling constant:

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \quad \text{as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Effective Quark Mass

❑ **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

❑ **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since } \gamma_m(g(\lambda)) > 0$$

❑ **Choice of renormalization scale:**

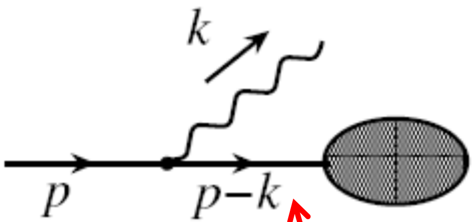
$\mu \sim Q$ **for small logarithms in the perturbative coefficients**

❑ **Light quark mass:** $m_f(\mu) \ll \Lambda_{\text{QCD}} \quad \text{for } f = u, d, \text{ even } s$

QCD perturbation theory ($Q \gg \Lambda_{QCD}$)
is effectively a massless theory

Infrared and collinear divergences

- **Consider a general diagram:**



Singularity

$p^2 = 0, \quad k^2 = 0$ **for a massless theory**

✧ $k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$

$\Longrightarrow$ **Infrared (IR) divergence**

✧ $k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1$

$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2 p^2 = 0$

$\Longrightarrow$ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

Infrared Safety

❑ Infrared safety:

$$\sigma_{\text{Phy}}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right)\Rightarrow\hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+\mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = κ > 0

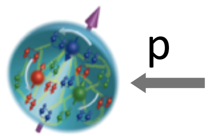
Asymptotic freedom is useful
only for
quantities that are infrared safe

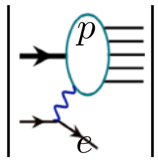
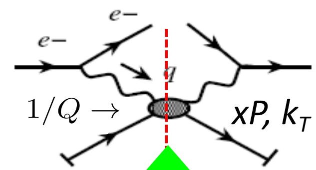
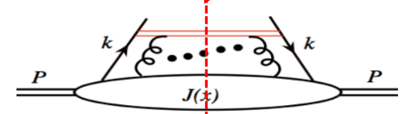
❑ Cross section with identified hadron(s):

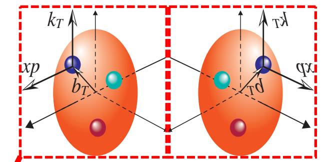
- ***Can not be calculated perturbatively!***
- ***Solution – QCD factorization:***
  - ***to isolated what can be calculated perturbatively,***
  - ***to represent the leading non-perturbative information by universal functions***
  - ***to justify the approximation to neglect other nonperturbative information***

QCD Factorization

- Factorization is an approximation!



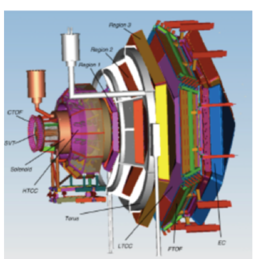
$\sigma_{\mathrm{DIS}}(x, Q^2) = \Big|$  $\Big|^2 \approx$  $\otimes$  $+ O\left(\frac{1}{QR}\right)$



Factorization Theory	Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

- QCD global analyses:

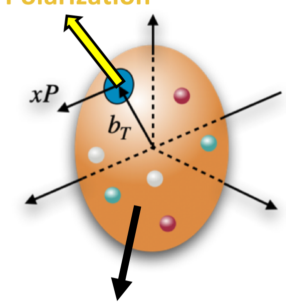
Experimental data



Jefferson Lab, KEK, CERN, BROOKHAVEN NATIONAL LABORATORY, Fermilab

Factorization Theory

$= \int \cdots \int$ Theory Calculation $\otimes$ Quantum Correlation Functions Unknown



Bayesian inference
Theory/Phenomenology
AI/ML, Computation

Extract QCFs

Jefferson Lab

Foundation of QCD perturbation theory

- ❑ **Renormalization**
  - **– QCD is renormalizable**

Nobel Prize, 1999
't Hooft, Veltman

- ❑ **Asymptotic freedom**
  - **– weaker interaction at a shorter distance**

Nobel Prize, 2004
Gross, Politzer, Welczek

- ❑ **Infrared safety and factorization**
  - **– calculable short distance dynamics**
  - **– pQCD factorization – connect the partons to physical cross sections**

J. J. Sakurai Prize, 2003
Mueller, Sterman

Look for infrared safe and factorizable observables!

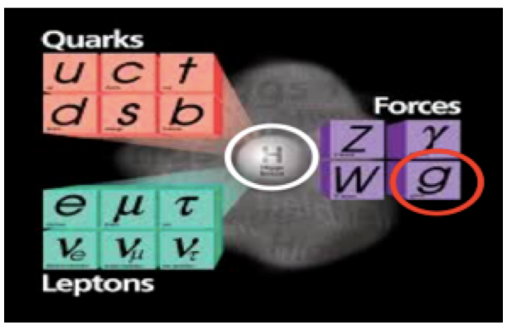
QCD is everywhere in our universe

- ❑ **Understanding where did we come from?**

Global Time: →

  - ○ **QCD at high temperature, high densities, phase transition, …**
  - ○ ***Facilities – Relativistic heavy ion collisions: SPS. RHIC. the LHC. …***

- ❑ **Understanding the visible world at 3°K – what are we made of?**



  - ○ **How to understand the emergence and properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
  - ○ **How does the glue bind us all?**
  - ○ ***Facilities – CEBAF, EIC, …***

Nuclear Femtography

Search for answers to these questions at a Fermi scale!