Center for Frontiers in Nuclear Science

The 2021 CFNS Summer School on the Physics of the Electron-Ion Collider

August 9-20, 2021

Introduction to QCD

- Lec. 1: Fundamentals of QCD
- Lec. 2: Matching observed hadrons to quarks and gluons
- Lec. 3: QCD for cross sections with identified hadrons
- Lec. 4: QCD for cross sections with polarized beam(s)

Jianwei Qiu

Theory Center

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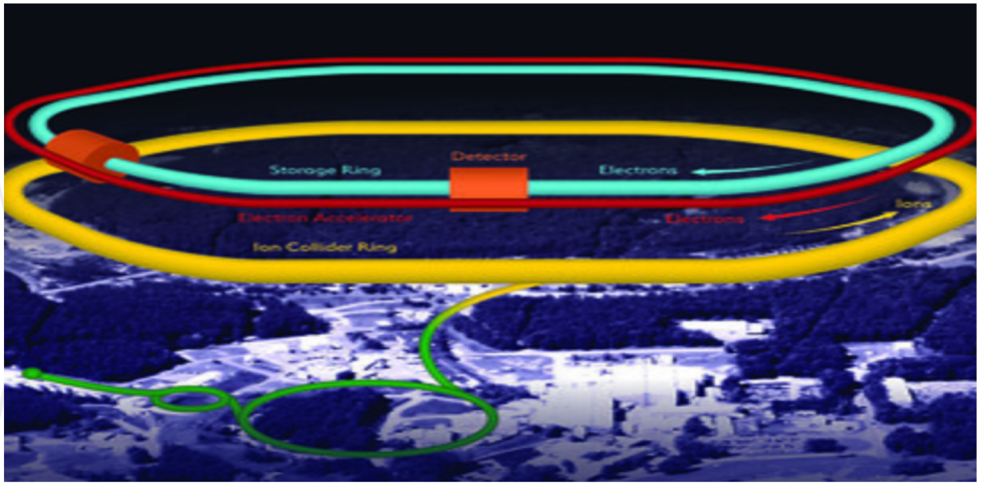

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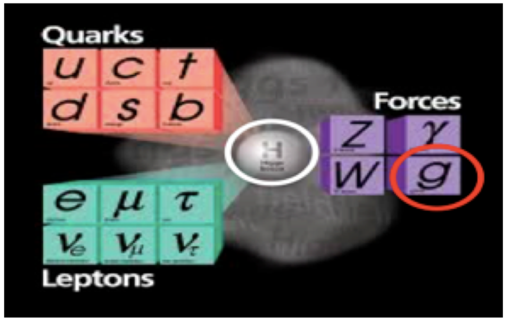
QCD is everywhere in our universe

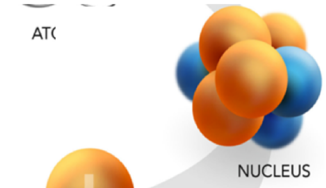
- ❑ **Understanding where did we come from?**

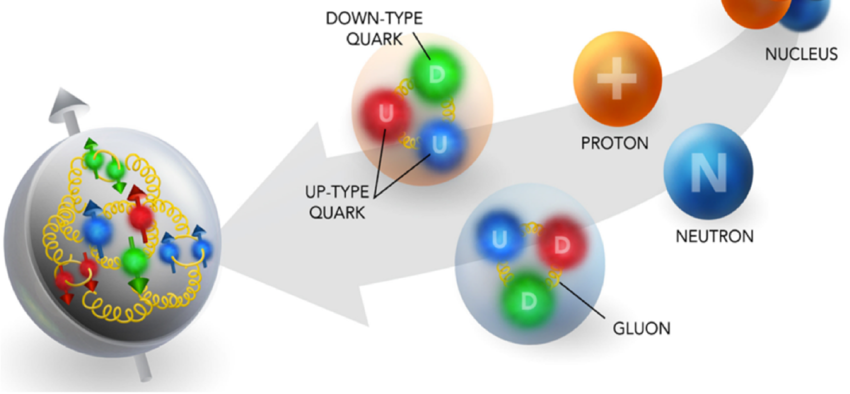
Global Time: →

  - ○ **QCD at high temperature, high densities, phase transition, …**
  - ○ ***Facilities – Relativistic heavy ion collisions: SPS. RHIC. the LHC. …***

- ❑ **Understanding the visible world at 3°K – what are we made of?**







  - ○ **How to understand the emergence and properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
  - ○ **How does the glue bind us all?**
  - ○ ***Facilities – CEBAF, EIC, …***

Nuclear Femtography

Search for answers to these questions at a Fermi scale!

Physical observables

Cross sections with identified hadron(s)
are
non-perturbative!

Hadronic scale ~ 1/fm ~ 200 MeV is not a perturbative scale

Follow a two-step approach:

1) Purely infrared safe quantities

2) Observables with identified hadron(s)

Fully infrared safe observables – I

Fully inclusive, without any identified hadron!

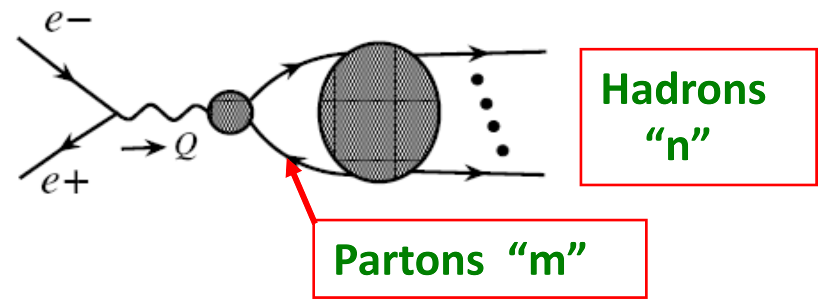
$$\sigma^{\text{total}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{total}}_{e^+e^-\to\text{partons}}$$

The simplest observable in QCD

$e^+e^- \rightarrow$ hadrons inclusive cross sections

❑ $e^+e^- \rightarrow$ hadron total cross section – not a specific hadron!

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \left| \quad \right|^2$$



If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} \propto \sum_n {\rm P}_{e^+e^-\to n} = \sum_n \sum_m {\rm P}_{e^+e^-\to m} {\rm P}_{m\to n} = \sum_m {\rm P}_{e^+e^-\to m} \sum_n {\rm P}_{m\to n}$$

=1

$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}} \propto \sum_m {\rm P}_{e^+e^-\to m}$$

$$\Rightarrow \quad \sigma^{\rm tot}_{e^+e^-\to {\rm hadrons}} = \sigma^{\rm tot}_{e^+e^-\to {\rm partons}}$$

Finite in perturbation theory – KLN theorem

❑ $e^+e^- \rightarrow$ parton total cross section:

$$\sigma^{\rm tot}_{e^+e^-\to {\rm partons}}(s=Q^2) = \sum_n \sigma^{(n)}(Q^2,\mu^2)\left(\frac{\alpha_s(\mu^2)}{\pi}\right)^n$$

Calculable in pQCD

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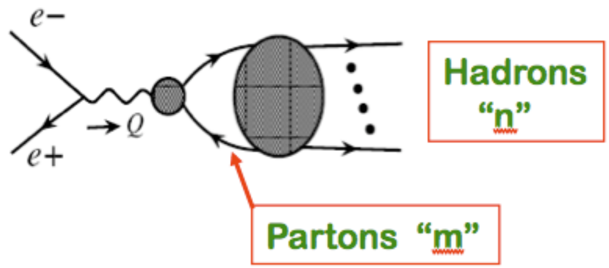
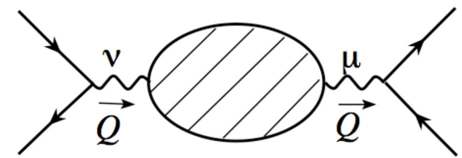
Infrared safety of e$^+$e$^-$ total cross sections

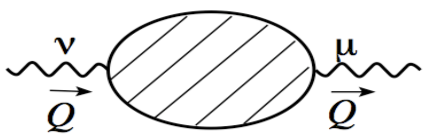
❑ Optical theorem:

$$\sigma^{\rm tot}_{e^+e^-} = \frac{1}{2S}\left|\quad\right|^2 \propto \mathrm{Im}$$





❑ Time-like vacuum polarization:



$$= \left(Q^\mu Q^\nu - Q^2 g^{\mu\nu}\right)\Pi(Q^2)$$

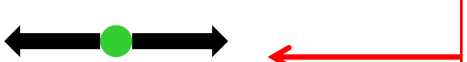
IR safety of $\sigma^{\rm tot}_{e^+e^-\to{\rm partons}}$ = **IR safety of** $\Pi(Q^2)$ **with** $Q^2 > 0$

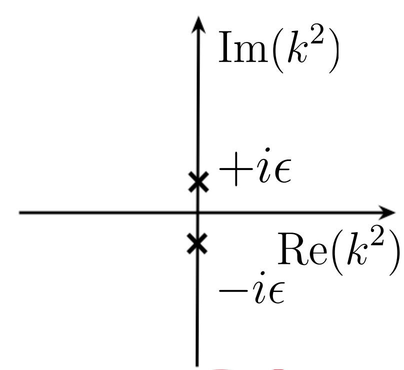
❑ IR safety of $\Pi(Q^2)$:

If there were pinched poles in Π(Q^2),

✧ **real partons moving away from each other**

✧ **cannot be back to form the virtual photon again!**

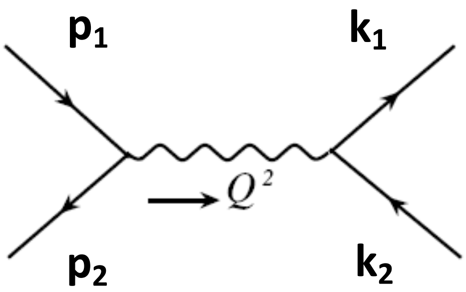




Rest frame of the virtual photon

Lowest order (LO) perturbative calculation

- Lowest order Feynman diagram:



- Invariant amplitude square:

$$|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2}\frac{1}{2^2}\mathrm{Tr}\left[\gamma\cdot p_2\gamma^\mu\gamma\cdot p_1\gamma^\nu\right]$$
$$\times\mathrm{Tr}\left[\left(\gamma\cdot k_1+m_Q\right)\gamma_\mu\left(\gamma\cdot k_2-m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2-t)^2+(m_Q^2-u)^2+2m_Q^2 s\right]$$

$$s=(p_1+p_2)^2$$
$$t=(p_1-k_1)^2$$
$$u=(p_2-k_1)^2$$

- Lowest order cross section:

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt}=\frac{1}{16\pi s^2}|\bar{M}_{e^+e^-\to Q\bar{Q}}|^2 \qquad \text{where } s=Q^2$$

Threshold constraint

$$\sigma_2^{(0)}=\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}=\sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

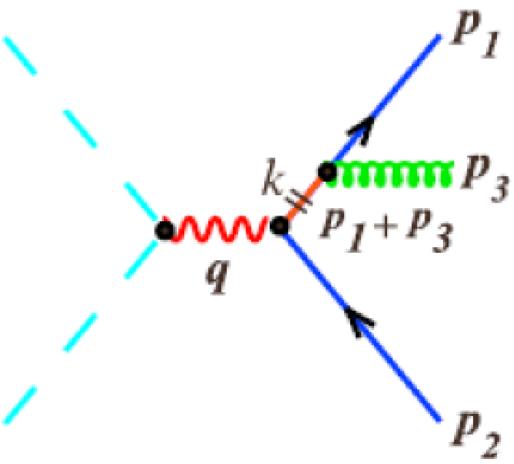
Next-to-leading order (NLO) contribution

❑ Real Feynman diagram:

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2 p_i . q}{s} \qquad \text{with } i = 1,2,3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2\left(1-x_1\right) = x_2 x_3 \left(1-\cos\theta_{23}\right), \quad cycl.$$



+ crossing

❑ Contribution to the cross section:

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^-\to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{\left(1-x_1\right)\left(1-x_2\right)}$$

IR as $x3 \to 0$

CO as $\theta_{13} \to 0$

$\theta_{23} \to 0$

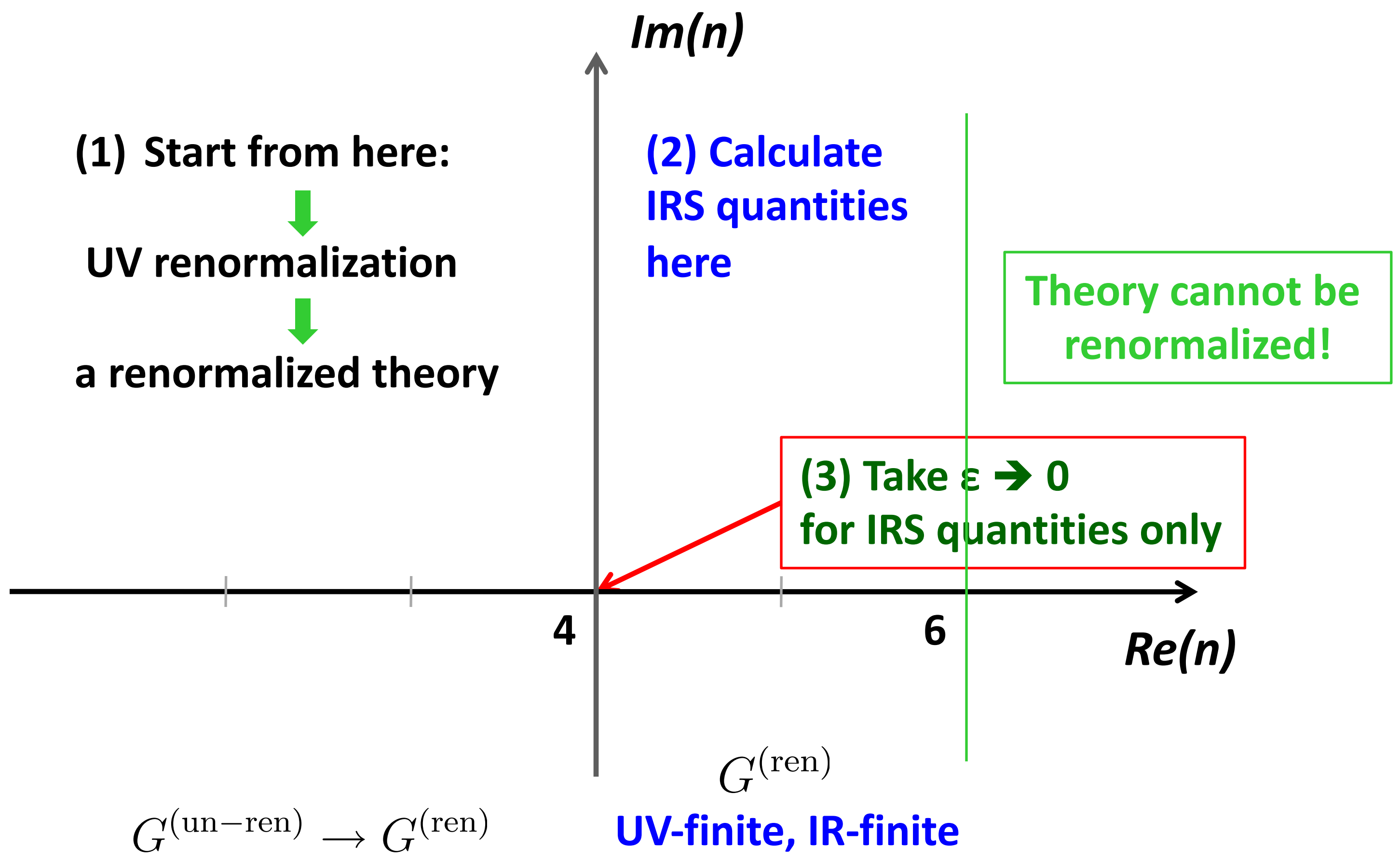
Divergent as $x_i \to 1$

Need the virtual contribution and a regulator!

How does dimensional regularization work?

❑ **Complex *n*-dimensional space:**

$$\int d^n k \, F(k, Q)$$



Dimensional regularization for both IR and CO

- **NLO with a dimensional regulator:**

  - **Real:** $$\sigma^{(1)}_{3,\varepsilon} = \sigma^{(0)}_{2,\varepsilon}\frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

  - **Virtual:** $$\sigma^{(1)}_{2,\varepsilon} = \sigma^{(0)}_{2,\varepsilon}\frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

  - **NLO:** $$\sigma^{(1)}_{3,\varepsilon}+\sigma^{(1)}_{2,\varepsilon} = \sigma^{(0)}_{2}\left[\frac{\alpha_s}{\pi}+O(\varepsilon)\right]$$

    No ε dependence!

  - **Total:** $$\sigma^{\text{tot}} = \sigma^{(0)}_{2}+\sigma^{(1)}_{3,\varepsilon}+\sigma^{(1)}_{2,\varepsilon}+O(\alpha_s^2) = \sigma^{(0)}_{2}\left[1+\frac{\alpha_s}{\pi}\right]+O(\alpha_s^2)$$

σ^{tot} is Infrared Safe!

σ^{tot} is independent of the choice of IR and CO regularization

Go beyond the inclusive total cross section?

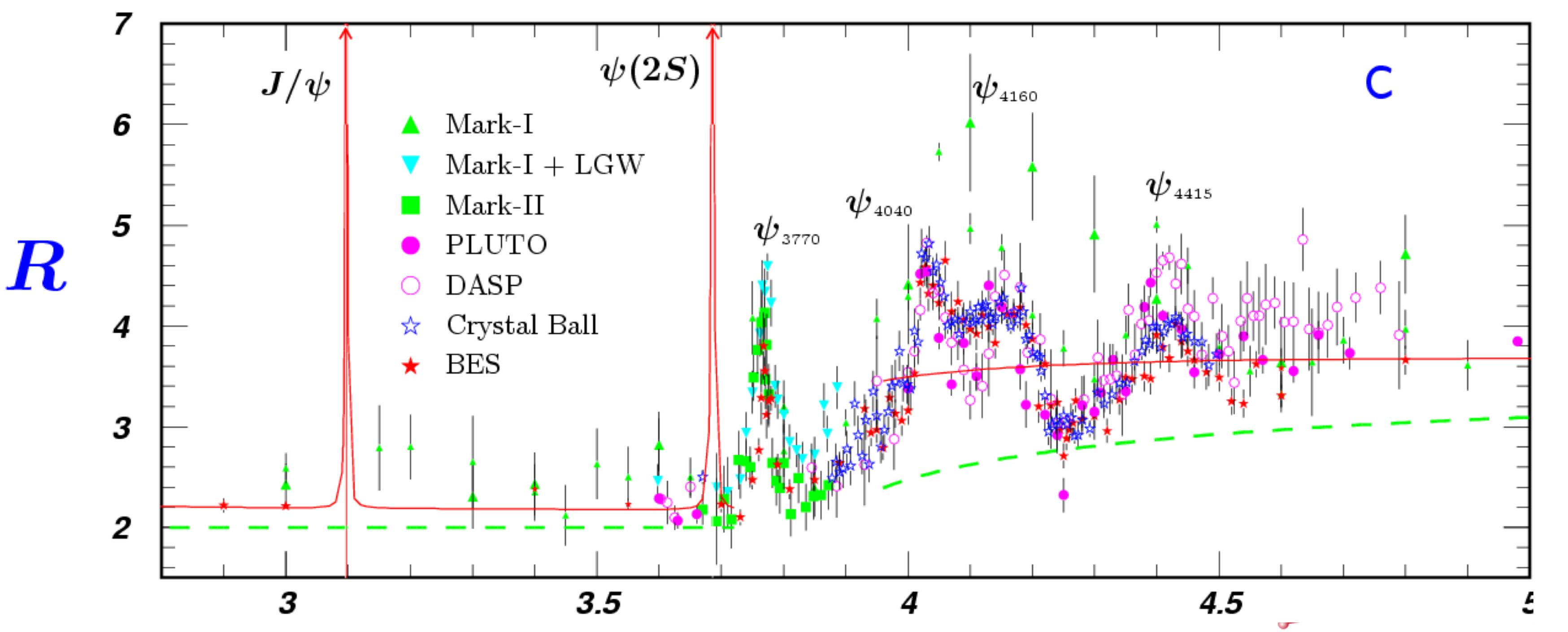
Hadronic cross section in e+e- collision

❑ Normalized hadronic cross section:

$$R_{e^+e^-}(s) \equiv \frac{\sigma_{e^+e^-\to\text{hadrons}}(s)}{\sigma_{e^+e^-\to\mu^+\mu^-}(s)}$$

$$\approx N_c \sum_{q=u,d,s} e_q^2 \left[1 + \frac{\alpha_s(s)}{\pi} + \mathcal{O}(\alpha_s^2(s))\right] \xrightarrow{N_c = 3} 2\left[1 + \frac{\alpha_s(s)}{\pi} + \ldots\right]$$

$$+N_c \sum_{q=c,\ldots} e_q^2 \left[\left(1 + \frac{2m_q^2}{s}\right)\sqrt{1 - \frac{4m_q^2}{s}} + \mathcal{O}(\alpha_s(s))\right]$$



Fully infrared safe observables - II

No identified hadron, but, with phase space constraints

$$\sigma^{\text{Jets}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{Jets}}_{e^+e^-\to\text{partons}}$$

Jets – "trace" or "footprint" of partons

Thrust distribution in e^+e^- collisions

etc.

Jets – trace of partons

- ❑ **Jets – "total" cross-section with a limited phase-space**

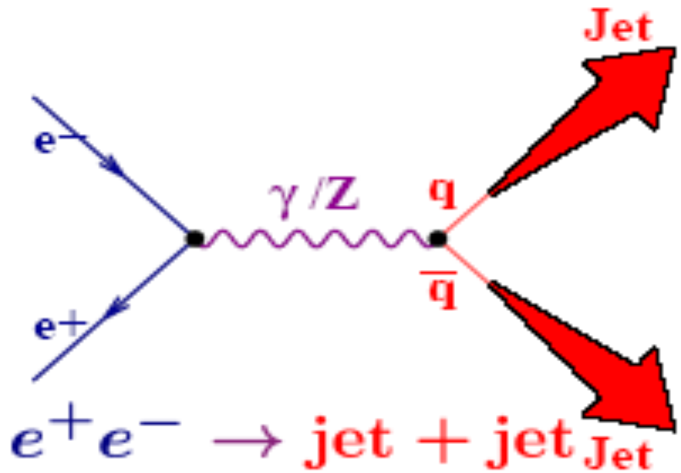
  Not any specific hadron!

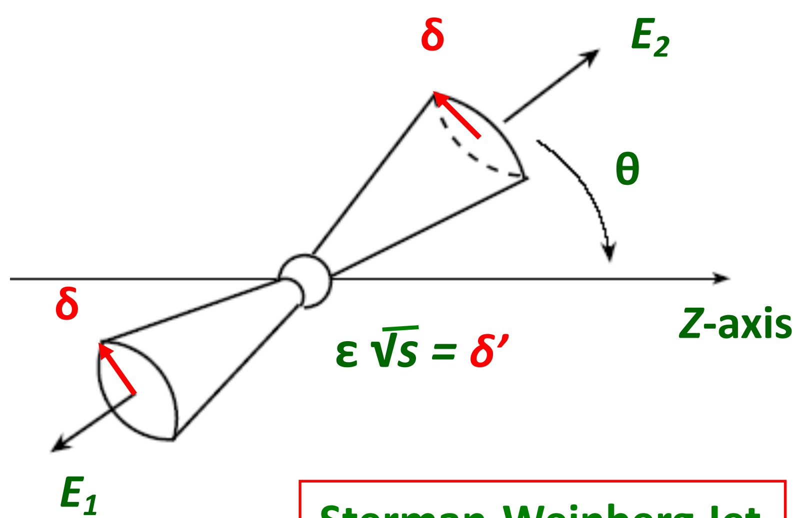
- ❑ **Q: will IR cancellation be completed?**

  - ✧ **Leading partons are moving away from each other**

  - ✧ **Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton**

- ❑ **Many Jet algorithms**





Sterman-Weinberg Jet

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Infrared safety for restricted cross sections

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of dσ(Γ):

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu,\lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right) \quad \text{with} \quad 0 \le \lambda \le 1$$

Physical meaning:

Measurement cannot distinguish a state with a zero/collinear momentum parton from a state without this parton – inclusiveness!

Special case: $\Gamma_n\left(k_1,k_2,\ldots,k_n\right) = 1 \;\text{ for all } n \quad \Rightarrow \quad \sigma^{(\text{tot})}$

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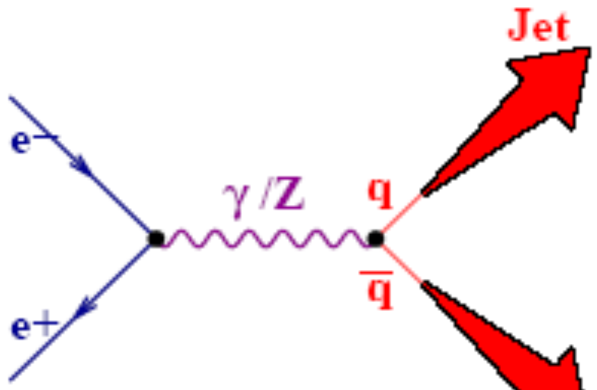
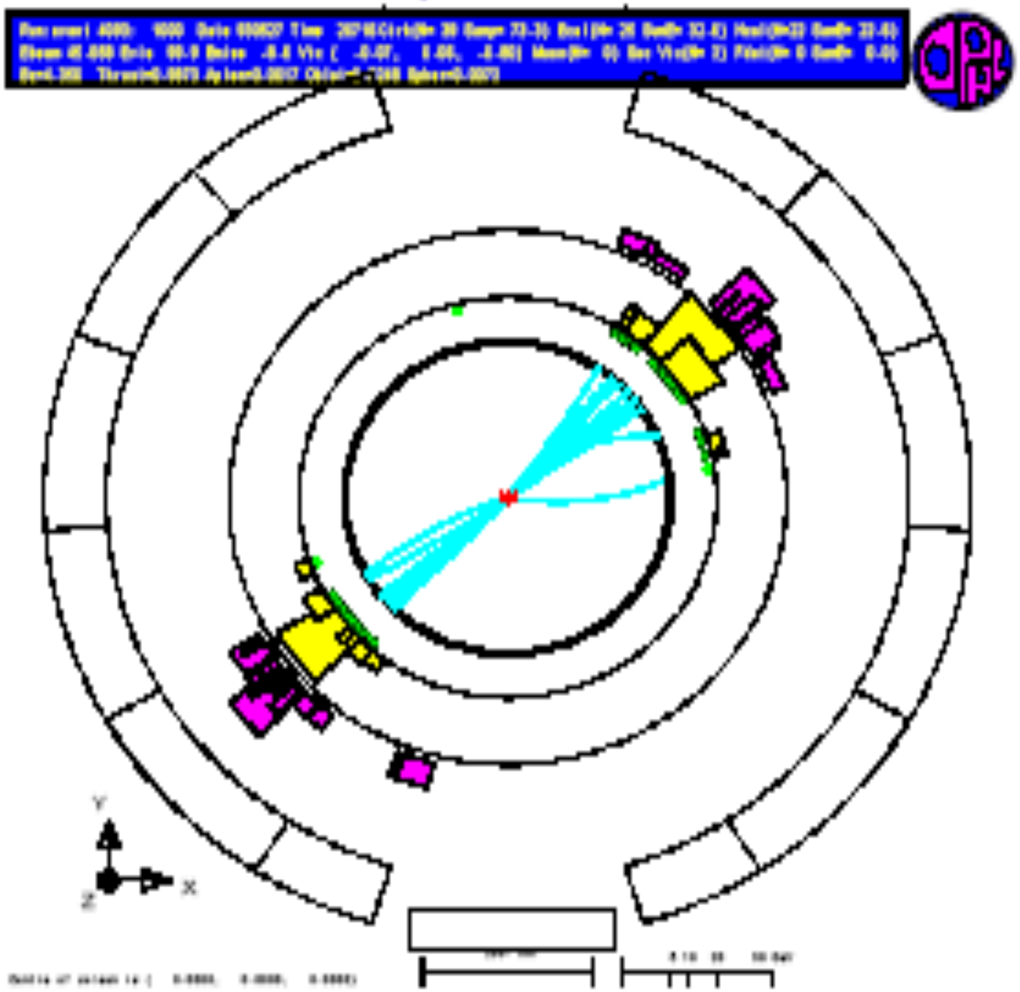
An early clean two-jet event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \to \text{jet} + \text{jet}$

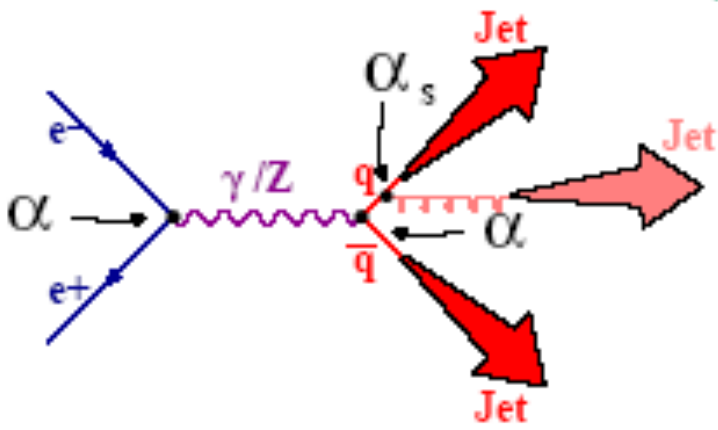
LEP ($\sqrt{s} = 90 - 205$ GeV)



A clean trace of two partons – a pair of quark and antiquark

Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

TASSO Collab., Phys. Lett. B86 (1979) 243

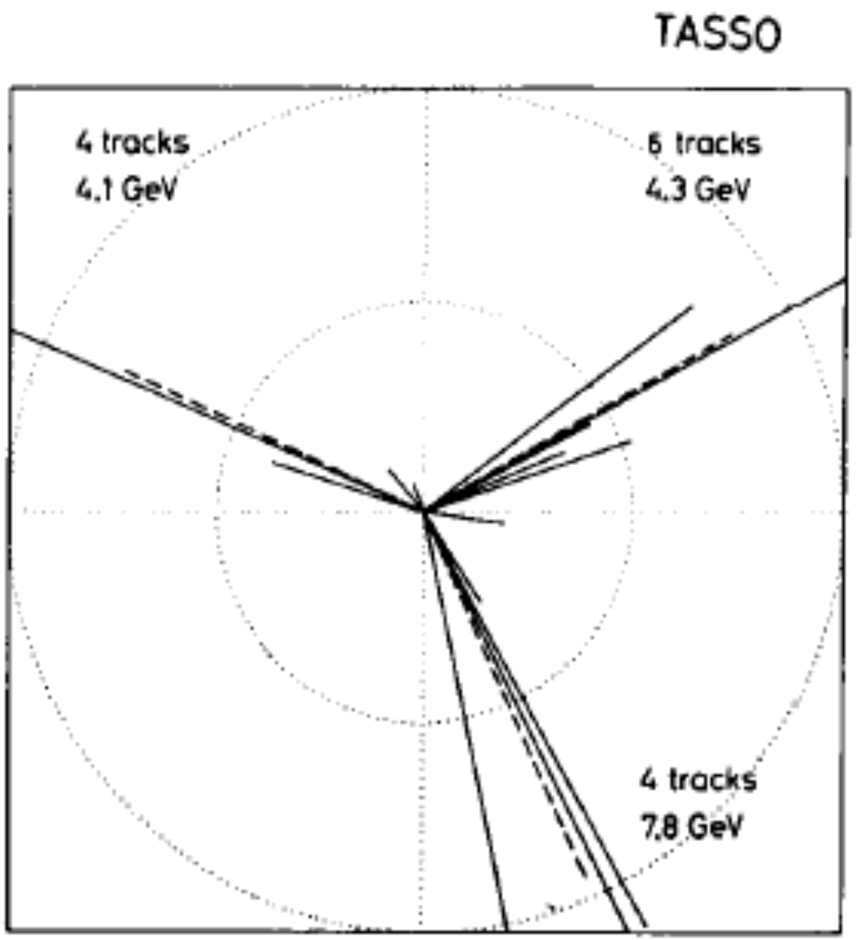
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

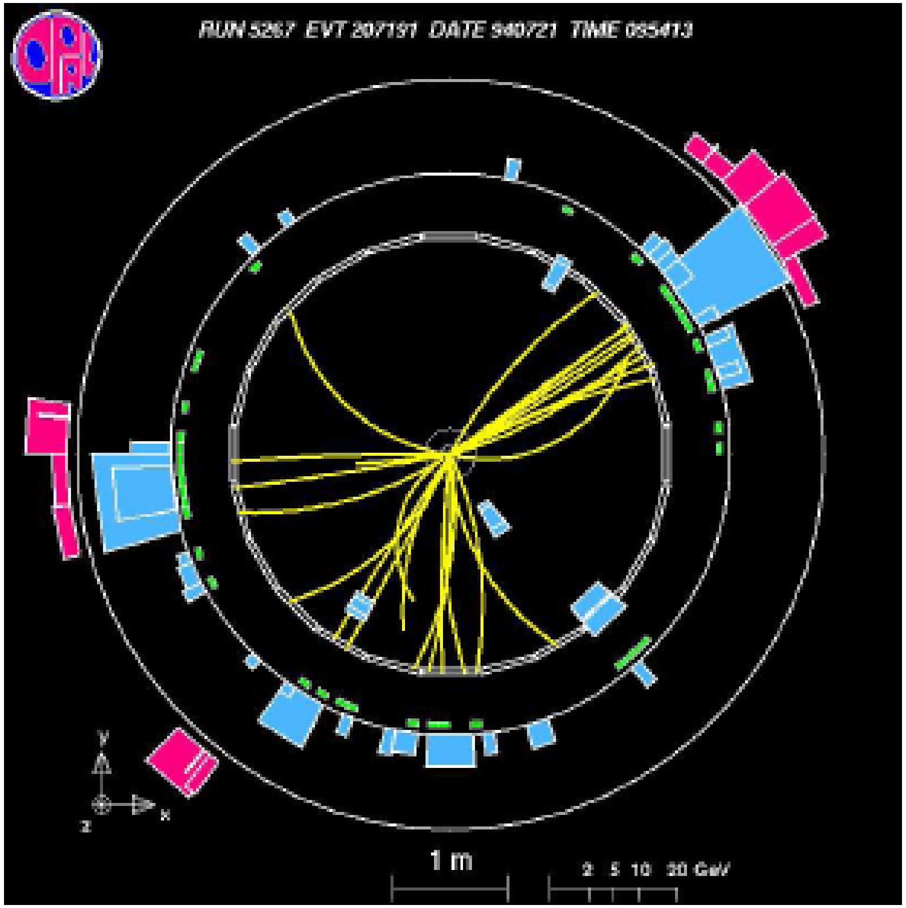
JADE Collab., Phys. Lett. B91 (1980) 142

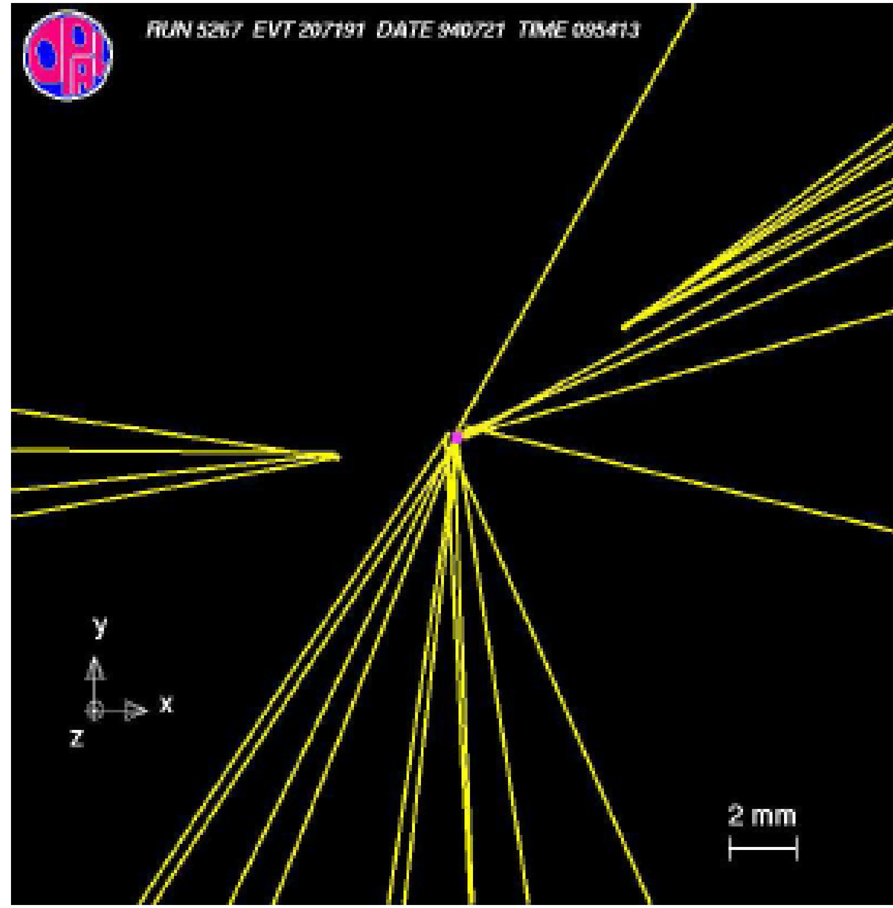
PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Tagged three-jet event from LEP



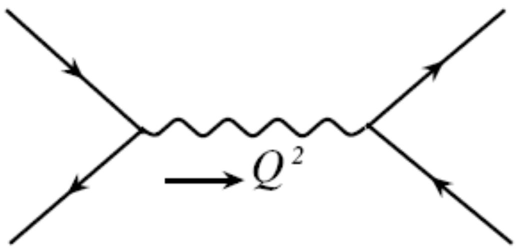


Gluon Jet

Two-jet cross section in e+e- collisions

- **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



- **Two-jet in pQCD:**

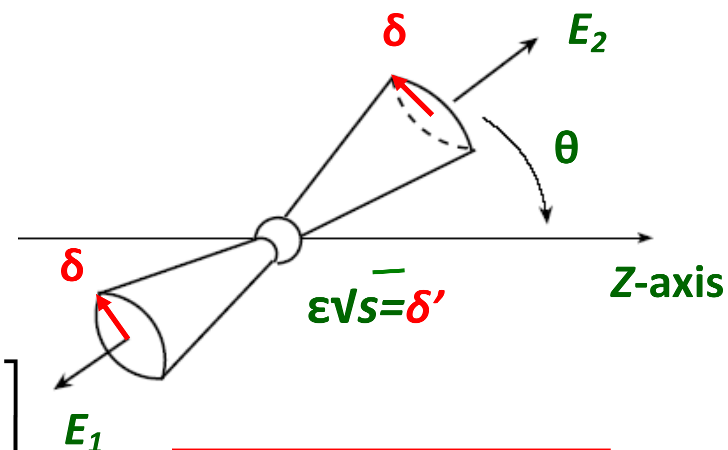
$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $C_n = C_n\left(\delta\right)$

- **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$

$$\text{x}\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln\left(\delta\right)\ln\left(\delta'\right)+3\ln\left(\delta\right)+\frac{\pi^2}{3}+\frac{5}{2}\right)\right]$$



Sterman-Weinberg Jet

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as} \quad Q\to\infty$$

Basics of jet finding algorithms

❑ Recombination jet algorithms (almost all e+e- colliders):

Recombination metric: $y_{ij} = \dfrac{M_{ij}^2}{E_{c.m.}^2}$ $\quad M_{ij}^2 = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$

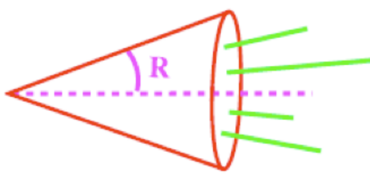
for Durham k_T

✧ different algorithm = different choice of M_{ij}^2:

✧ Combine the particle pair (i, j) with the smallest : y_{ij} $\quad (i, j) \to k$

$$e.g.\ \text{E scheme}: \ p_k = p_i + p_j$$

✧ iterate until all remaining pairs satisfy: $y_{ij} > y_{cut}$



❑ Cone jet algorithms (CDF,LHC, …, colliders):

✧ Cluster all particles into a cone of half angle R to form a jet:

✧ Require a minimum visible jet energy: $E_{jet} > \epsilon$

Recombination metric: $d_{ij} = \min\left(k_{T_i}^{2p}, k_{T_j}^{2p}\right) \dfrac{\Delta_{ij}^2}{R^2}$

✧ Classical choices: ***p=1* – “*k*”**

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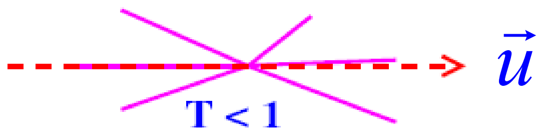
Thrust distribution

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|} \right)$$



- **Phase space constraint:**

$$\frac{d\sigma_{e^+e^-\to \text{hadrons}}}{dT} \quad \text{with} \quad \Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

  - ✧ Contribution from p=0 particles drops out the sum
  - ✧ Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

and

$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

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The harder question

❑ Question:

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

❑ Facts:

Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

Cross section involving identified hadron(s) is not IR safe and is NOT perturbatively calculable!

❑ Solution – Factorization:

✧ Isolate the calculable dynamics of quarks and gluons

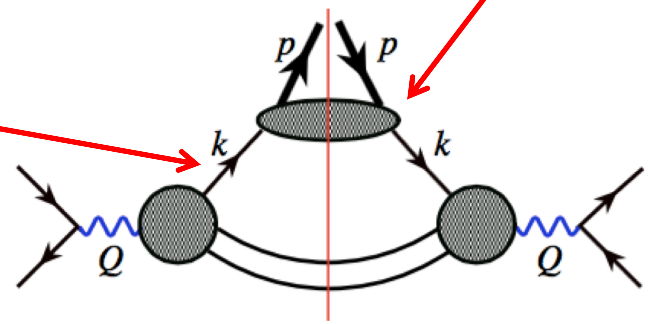
✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
– provide information on the partonic structure of the hadron

Observables with ONE identified hadron

- **Creation of an identified hadron:**

Not necessary to be dominated by one parton, which is always virtual!

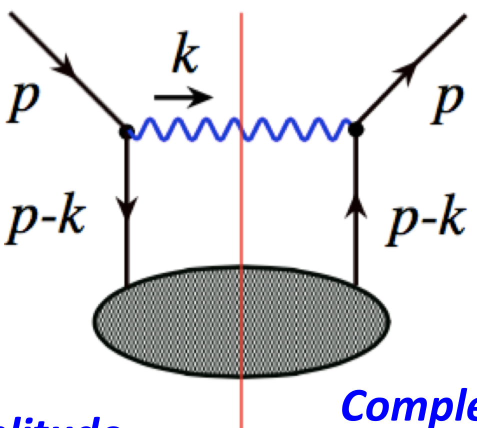
Non-perturbative!



– in a "cut-diagram" notation

- **"Square" of the diagram with a *"unobserved gluon"*:**

"Cut-line" – final-state



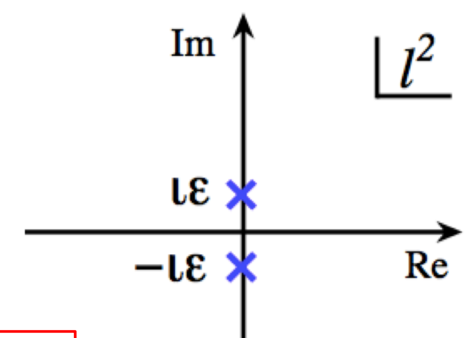
Amplitude

Complex conjugate of the Amplitude

$$\propto \int \mathcal{T}(p-k,Q)\,\frac{1}{(p-k)^2+i\epsilon}\,\frac{1}{(p-k)^2-i\epsilon}\,d^4k\,\delta(k^2)_+$$

$$\propto \int \mathcal{T}(l,Q)\,\frac{1}{l^2+i\epsilon}\,\frac{1}{l^2-i\epsilon}\,dl^2$$

$$\Rightarrow \infty$$



Pinch singularity & pinch surface

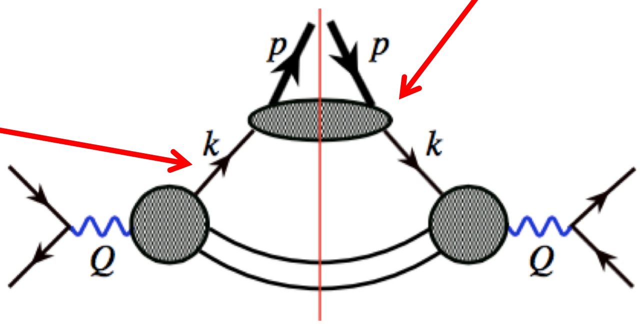
Two parts connected by a "classical" parton

Observables with ONE identified hadron

- **Creation of an identified hadron:**

Not necessary to be dominated by one parton, which is always virtual!

Non-perturbative!



– *in a "cut-diagram" notation*

- **On-shell approximation:**

$$\sigma_{e^+e^-\to h(p)X} \approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,k;\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$\hat{k}^2 = 0$

$$\approx \sum_f \int \frac{d^4k}{(2\pi)^4} \mathcal{H}_{e^+e^-\to f(k)}(Q,\hat{k};\sqrt{S})\, \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \mathcal{O}(\frac{\langle k^2\rangle}{Q^2}) + \ldots$$

$$\approx \sum_f \int dz\, \mathcal{H}_{e^+e^-\to f(k)}(Q,\frac{p}{z};\sqrt{S}) \int \frac{d^4k}{(2\pi)^4} \delta(z - \frac{p\cdot n}{k\cdot n}) \mathcal{F}_{f(k)\to h(p)X}(k,p;\Lambda_{\rm QCD}) + \ldots$$

$$\approx \sum_f \int dz\, \hat{\sigma}_{e^+e^-\to f(k)}(Q,z;\sqrt{S})\; D_{f(k)\to h(p)X}(z,p;\Lambda_{\rm QCD}) + \ldots$$

Hard collision to produce an on-shell parton
– Perturbatively calculable!

FF: Probability for the parton to become the observed hadron
– Non-perturbative, universal!

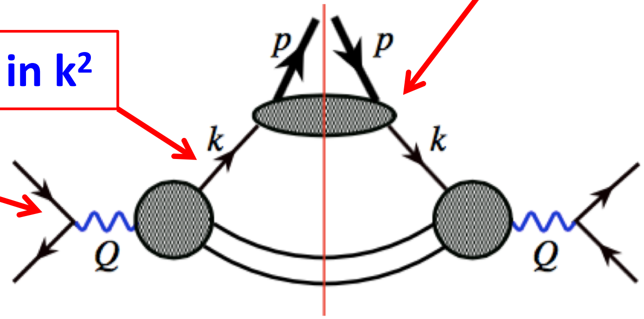
Observables with ONE identified hadron

- **Need more observables to fix the "FFs":**

Non-perturbative!

Pinch in k^2

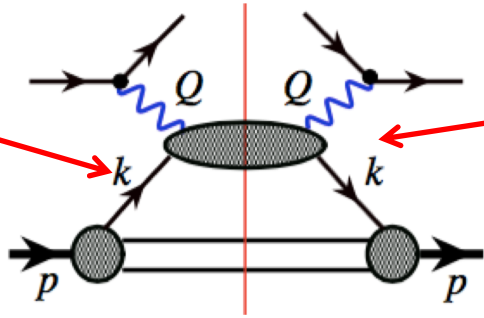
Perturbative!



– in a "cut-diagram" notation

- **Identified initial hadron:**

Pinch in k^2



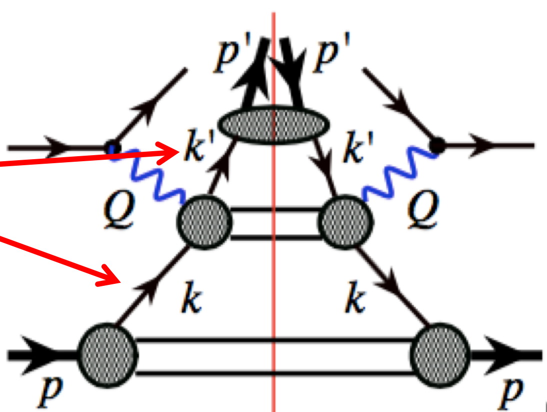
Perturbative!

Non-perturbative!

- **Identified initial + created hadron(s):**

Pinch in both k^2 and k'^2

Quantum interference between dynamics at the HARD and hadronic scales is powerly suppressed!



Predictive power of QCD factorization

❑ Universality of non-perturbative hadron structure:

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\rm EXP}_{l+P\to l+X} = \boxed{C_{l+k\to l+X}} \otimes \boxed{{\rm PDF}_P} + \mathcal{O}(Q_s^2/Q^2)$$

$$\sigma^{\rm EXP}_{l+P\to l+H+X} = \boxed{C_{l+k\to l+k+X}} \otimes \boxed{{\rm PDF}_P} \otimes \boxed{{\rm FF}_H} + \mathcal{O}(Q_s^2/Q^2)$$

- hadron-hadron reactions (LHC)

$$\sigma^{\rm EXP}_{P+P\to l+\bar{l}+X} = \boxed{C_{k+k\to l+\bar{l}+X}} \otimes \boxed{{\rm PDF}_P} \otimes \boxed{{\rm PDF}_P} + \mathcal{O}(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\rm EXP}_{l+\bar{l}\to H+X} = \boxed{C_{l+\bar{l}\to k+X}} \otimes \boxed{{\rm FF}_H} + \mathcal{O}(Q_s^2/Q^2)$$

❑ Hadron structure = Theory + Experiment + Phenomenology:

- **Factorization – Identify "Good" observables (Theory)**
- **Measurement – Get "Reliable" data (Experiment)**
- **Global analysis – Extract "Universal" structure information (Phenomenology)**

QCD global analysis of experimental data

❑ It is an inverse problem:

Experimental data

Future EIC, Jefferson Lab, KEK, CERN, Brookhaven National Laboratory, DESY, Fermilab

QCD Factorization Theorem

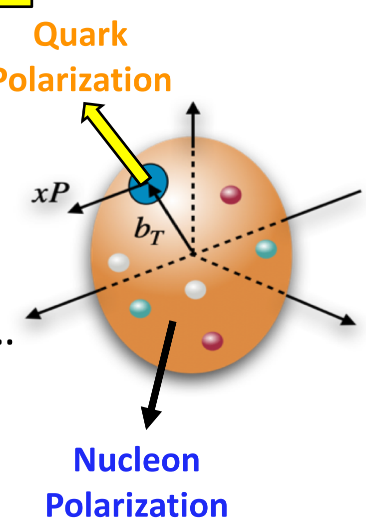
Flavors: u, d, s, …; g
Spin (quark, gluon hadron)
Momentum/position
(Before/after the collision)

$= \int \cdots \int$ **Analytical Coefficient functions derived from QCD perturbation theory** $\otimes$ **Quantum Correlation Functions of quarks and gluons *Unknown*** $+$ **Systematic error from theory**

Bayesian inference
Theory/Phenomenology
AI/ML, Computation

Extract QCFs

PDF_P , FF_H , …



❑ Input for QCD Global analysis/fitting:

PDFs, FFs at an input scale:

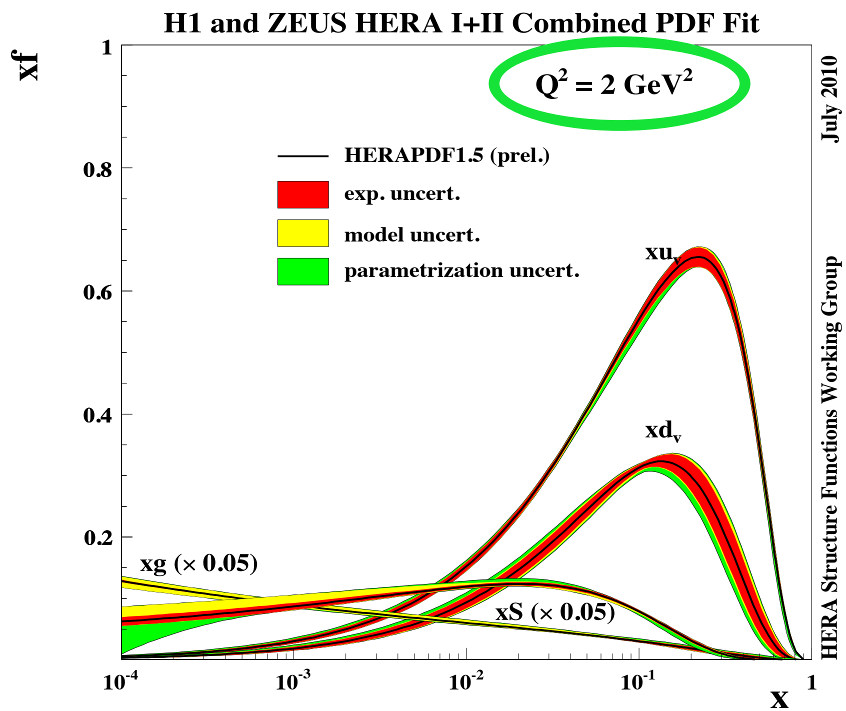
Fitting paramters

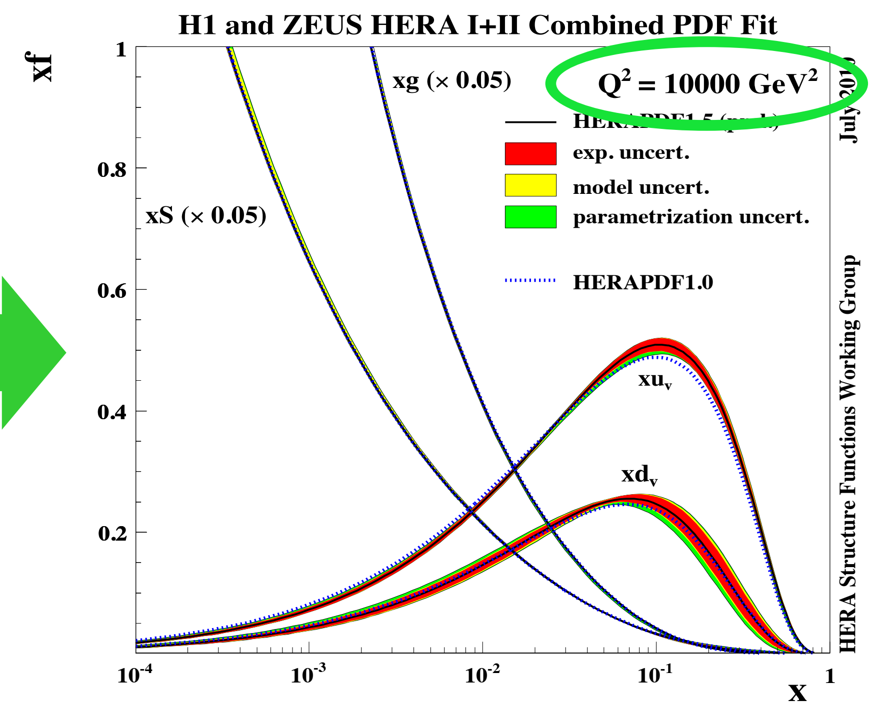
$$\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$$

Input scale ~ GeV

PDFs from DIS

❑ Q2-dependence is a prediction of pQCD calculation:





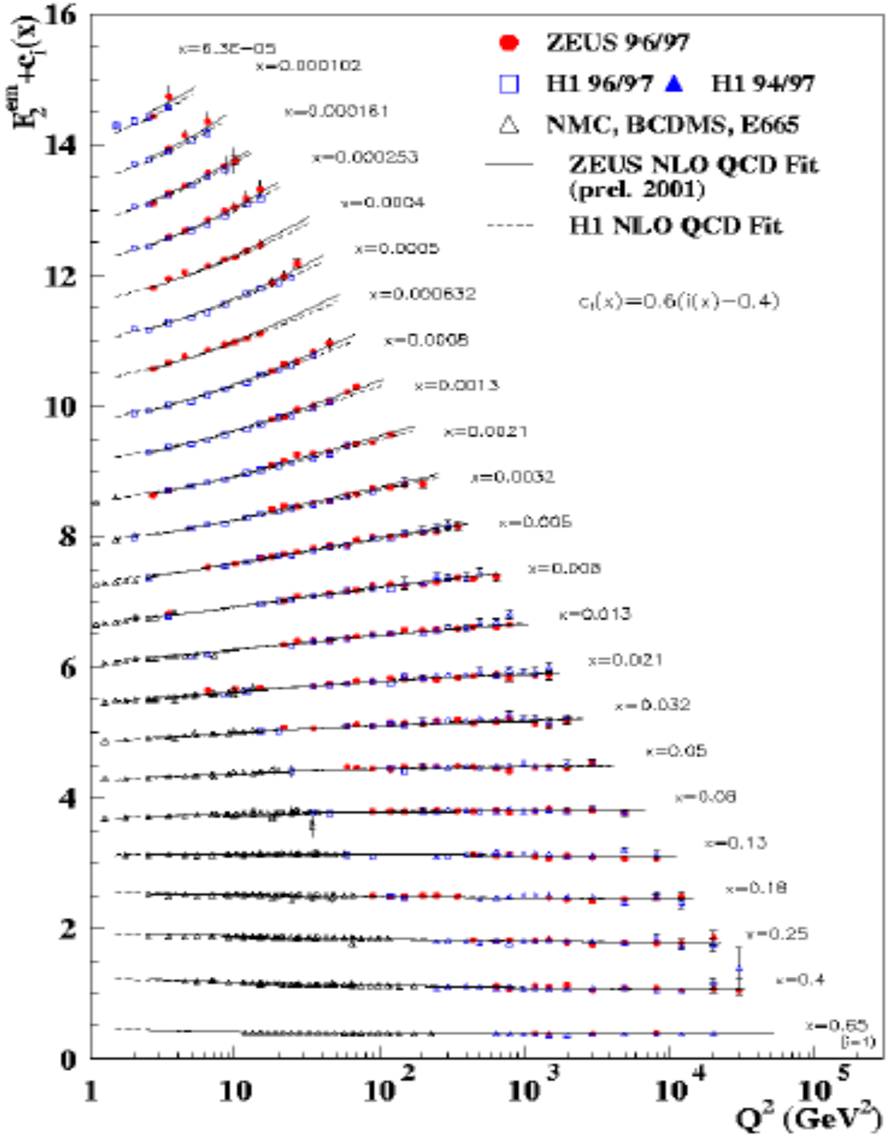
❑ Physics interpretation of PDFs:

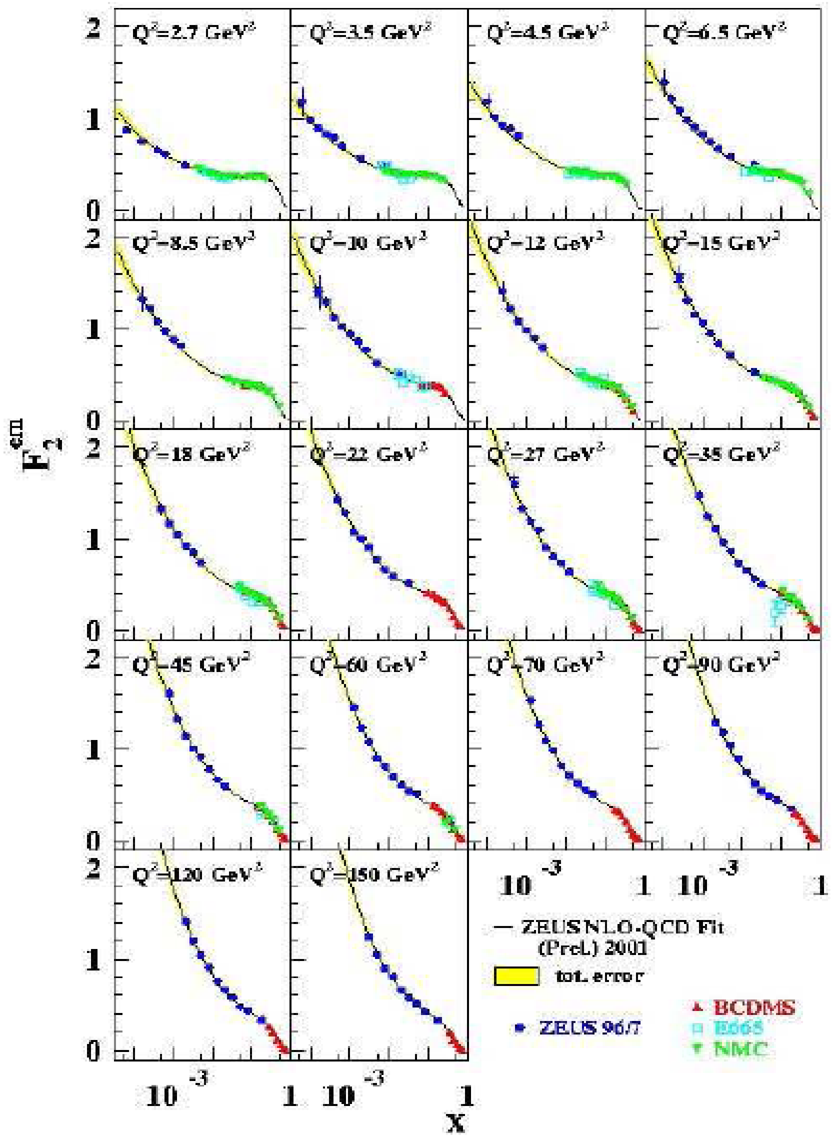
$f(x, Q^2)$: **Probability density to find a parton of flavor "f" carrying momentum fraction "x", probed at a scale of "Q^2"**

✧ **Number of partons:** $$\int_0^1 dx\, u_v(x, Q^2) = 2, \quad \int_0^1 dx\, d_v(x, Q^2) = 1$$

✧ **Momentum fraction:** $$\langle x(Q^2)\rangle_f = \int_0^1 dx\, x\, f(x, Q^2) \quad \Longrightarrow \quad \sum_f \langle x(Q^2)\rangle = 1$$

Scaling and scaling violation





Q^2-dependence is a prediction of pQCD calculation

Jefferson Lab

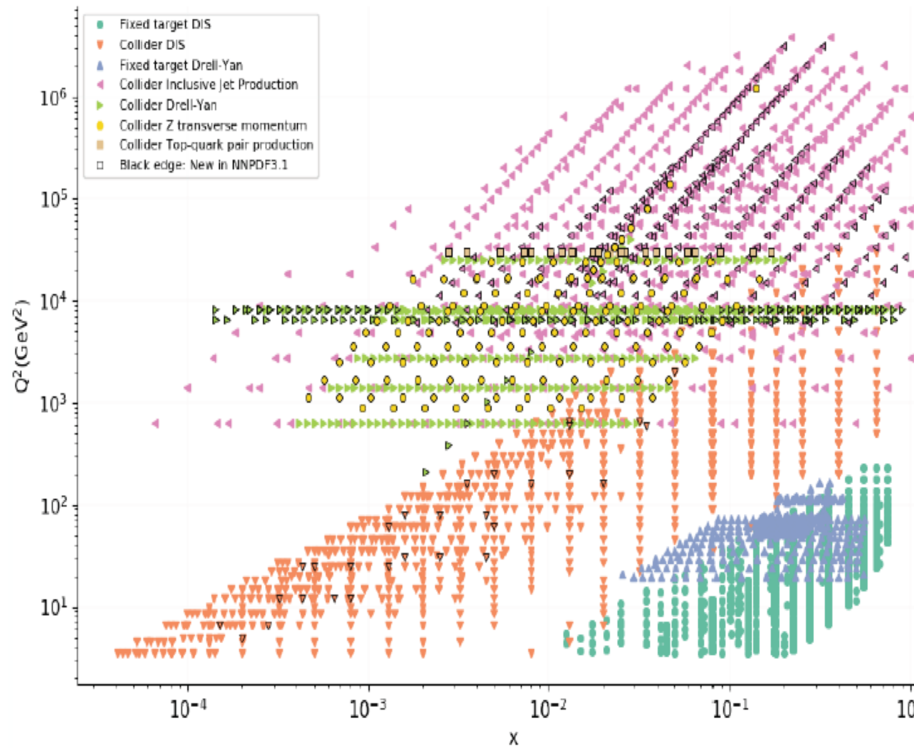
QCD factorization works to the precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^\pm\{p,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu})N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
Collider DIS	$e^\pm p \to e^\pm + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+\{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \to e^\pm c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to e^\pm b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^\pm \to \ell^\pm\nu) + X$	$u\bar{d} \to W^+, \bar{u}d \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$u\bar{u}, d\bar{d} \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$q\bar{q} \to t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^\pm \to \ell^\pm\nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_\perp$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^-c$	$sg \to W^+c, \bar{s}g \to W^-\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic Coverage:

NNPDF3.1

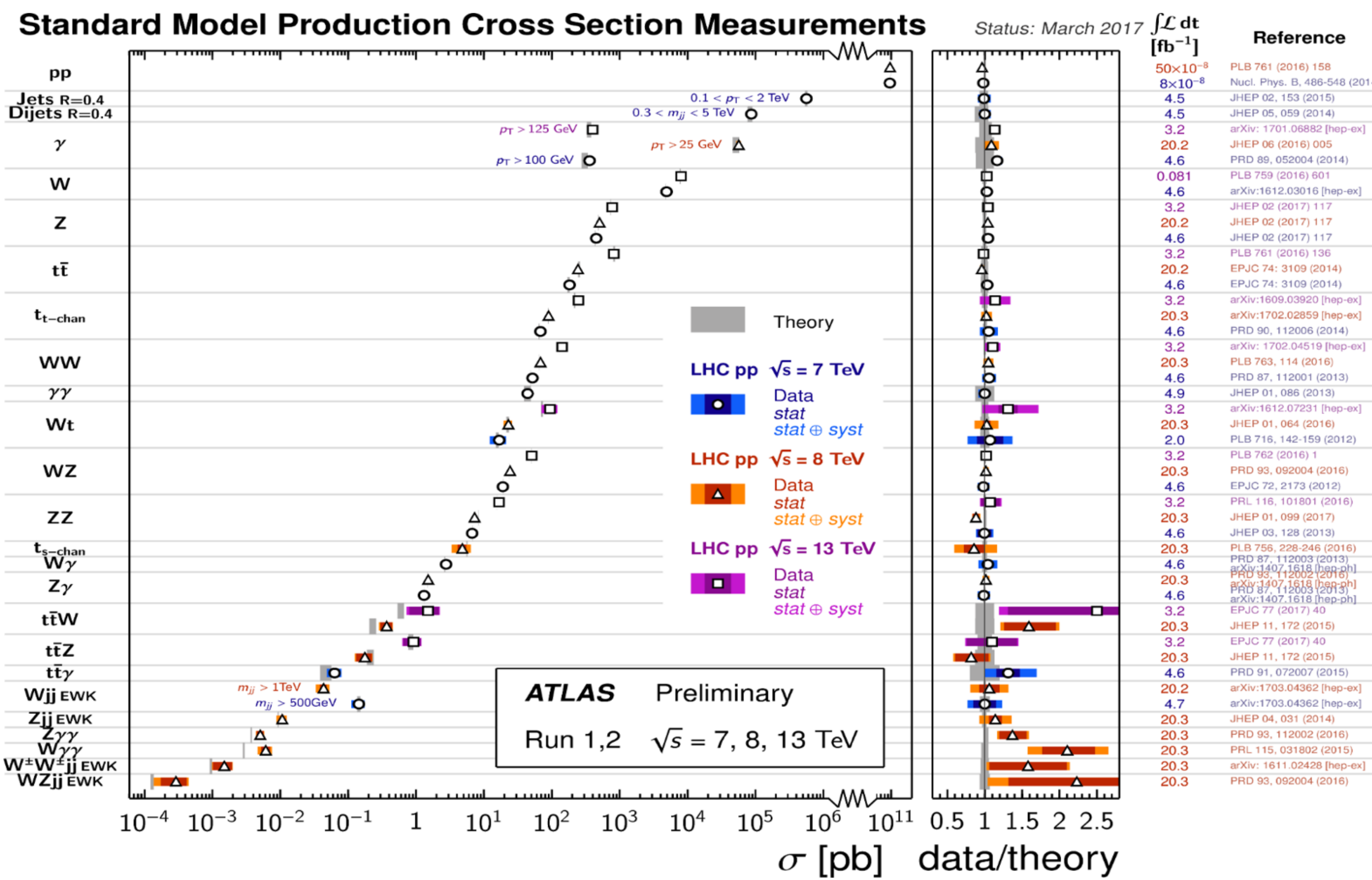


❑ Fit Quality:

$\chi^2/\text{dof} \sim 1 \ \Rightarrow$ Non-trivial check of QCD

All data sets	3706 / 2763	3267 / 2996	2717 / 2663
	LO	NLO	NNLO

Unprecedent Success of QCD and Standard Model

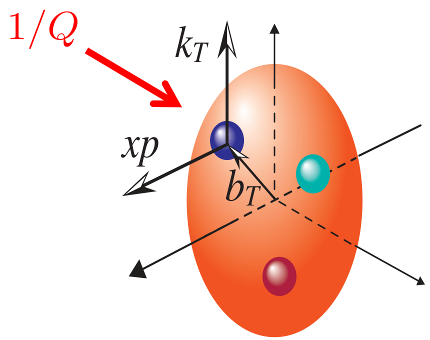


SM: Electroweak processes + QCD perturbation theory + PDFs works!

Jefferson Lab

Probes for 3D hadron structure

- **Single scale hard probe is too "localized":**



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> $1/Q$

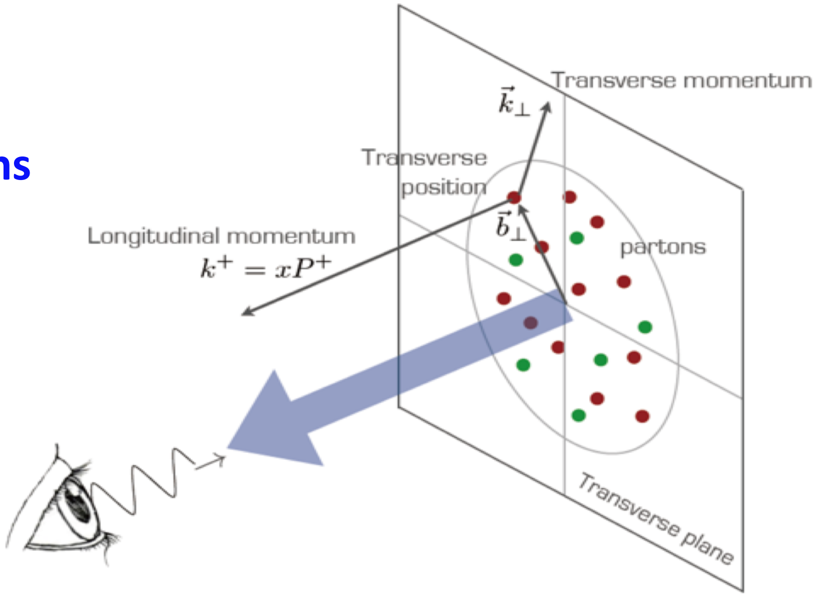
- **Need new type of "Hard Probes" – Physical observables with TWO Scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

Hard scale: Q_1 To localize the probe particle nature of quarks/gluons

"Soft" scale: Q_2 could be more sensitive to the hadron structure ~ 1/fm

Hit the hadron "very hard" without breaking it, clean information on the structure!



"See" hadron's 3D partonic structure?

❑ Two-scale observables in hadron-hadron collisions:

Drell-Yan process: $\sigma^{\rm EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \text{PDF}_P \otimes \text{PDF}_P + \mathcal{O}(Q_s^2/Q^2)$

One-scale case:

$$\frac{d\sigma^{\rm DY}_{hh'}(p_A, p_B, q)}{dQ^2} = \sum_{f,f'} \int_0^1 dx \int_0^1 dx' \, \phi_f(x) \, \frac{d\hat{\sigma}^{\rm el}_{ff'}(xp_A, x'p_B, q)}{dQ^2} \, \phi_{f'}(x')$$

Hard scale – invariant mass of the lepton-pair: $Q^2 \equiv q^2 = (l+\bar{l})^2 \gg \Lambda^2_{\rm QCD} \sim 1/R_h^2$

Two-scale case:

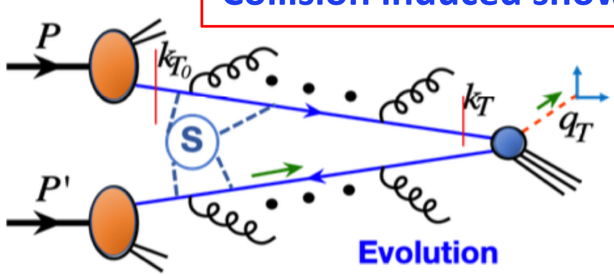
TMDs

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

Hard scale: Q^2 **Soft scale:** q_T^2 **when** $Q^2 \gg q_T^2$ $\quad d^4q = dy\, dQ^2 dq_T^2 d\phi_q$

❑ Confined motion vs. collision effects:

Collision induced shower



QCD Evolution – could be non-perturbative!

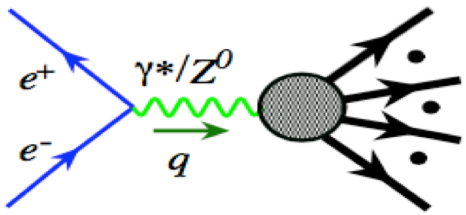
TMDs: $\mathcal{F}(x, k_T) \neq \mathcal{F}(x, k_{T_0})$

Structure + Collision effect

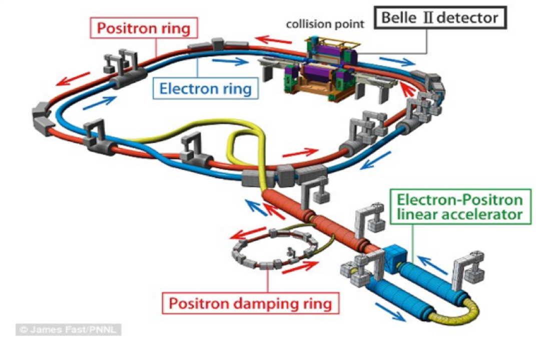
Confined motion

QCD & hadron structure needs lepton-hadron facility

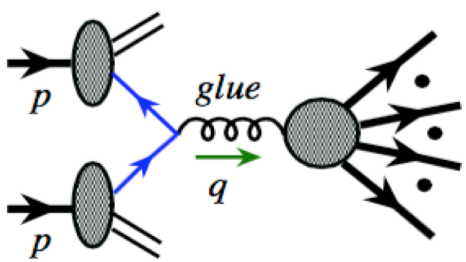
❑ Hadrons are produced from the energy in e+e- collisions:



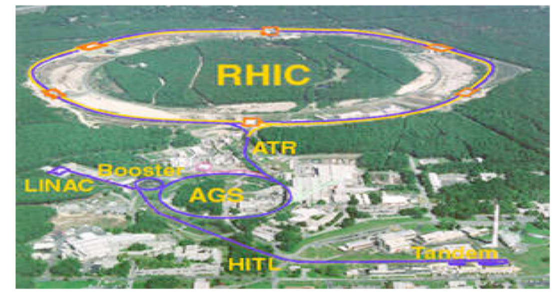
- *No hadron to start with*
- *Emergence of hadrons*



❑ Hadrons are produced in hadron-hadron collisions:

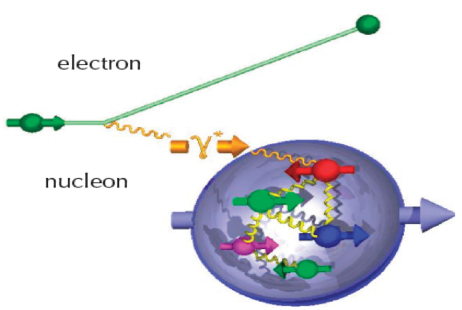


- *Partonic structure*
- *Emergence of hadrons*
- *Heavy ion target or beam(s)*



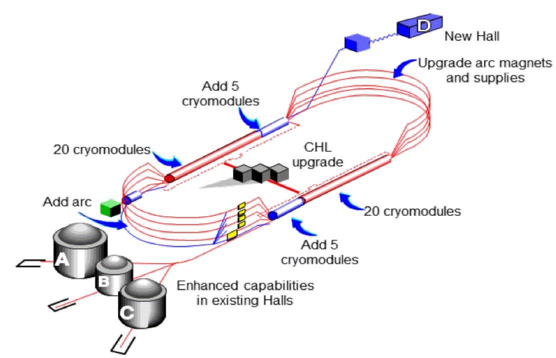
Also at the LHC

❑ Hadrons are produced in lepton-hadron collisions:



- *Colliding hadron can be broken or stay intact!*
- *Imaging partonic structure*
- *Emergence of hadrons*
- *Heavy ion target or beam*

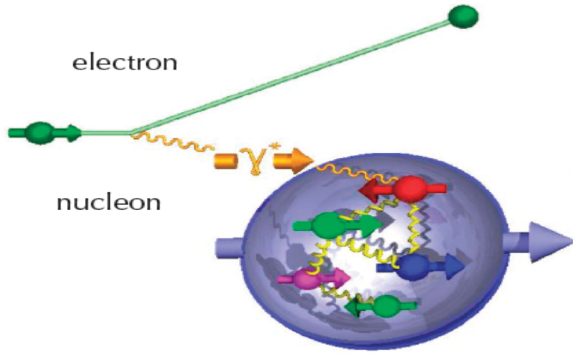
One facility covers all!



Also at COMPASS & future EIC

Why a lepton-hadron facility is special?

- **The new generation of "Rutherford" experiment:**

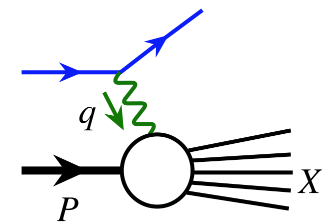


- ✧ A controlled "probe" – virtual photon
- ✧ Can either break or not break the hadron

One facility covers all!
(JLab, COMPASS, EIC, …)

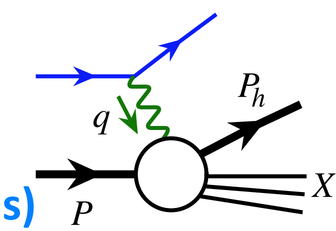
✧ **Inclusive events:** e+p/A → e'+X

Detect only the scattered lepton in the detector
(Modern Rutherford experiment!)



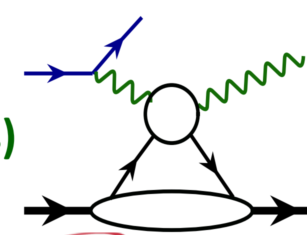
✧ **Semi-Inclusive events:** e+p/A → e'+h(p,K,p,jet)+X

Detect the scattered lepton in coincidence with identified hadrons/jets
(Initial hadron is broken – confined motion! – cleaner than h-h collisions)



✧ **Exclusive events:** e+p/A → e'+ p'/A'+ h(p,K,p,jet)

Detect every things including scattered proton/nucleus (or its fragments)
(Initial hadron is NOT broken – tomography!
– almost impossible for h-h collisions)



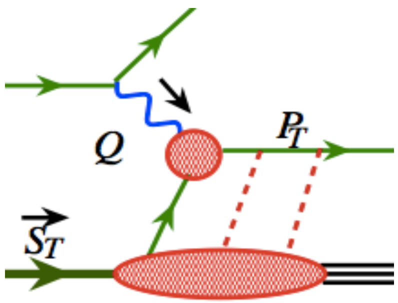
Jefferson Lab

"See" hadron's 3D partonic structure?

- ❑ **Two-scale observables are natural in lepton-hadron collisions:**

See also lectures by
Olga Evdokimov
Renee Fatemi
Xiangdong Ji
Cédric Lorcé
…

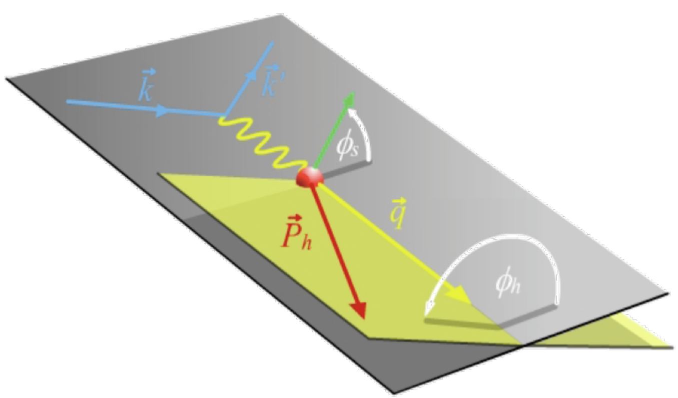
✧ **Semi-inclusive DIS:**



+ …

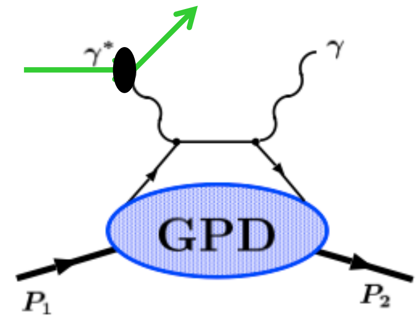
SIDIS: $Q \gg P_T$

Parton's confined motion encoded into TMDs



Two scales, two planes,
Angular modulation, …

✧ **Exclusive DIS:**

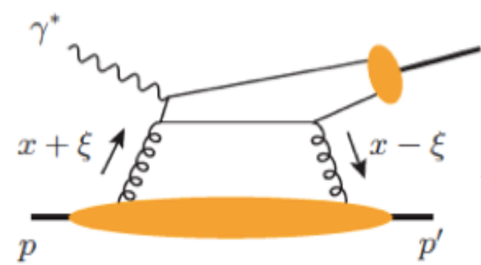


+ …

Imaging quarks

DVCS: $Q^2 \gg |t|$

Parton's spatial imaging from Fourier transform of GPDs' t-dependence



J/Ψ, Φ, …

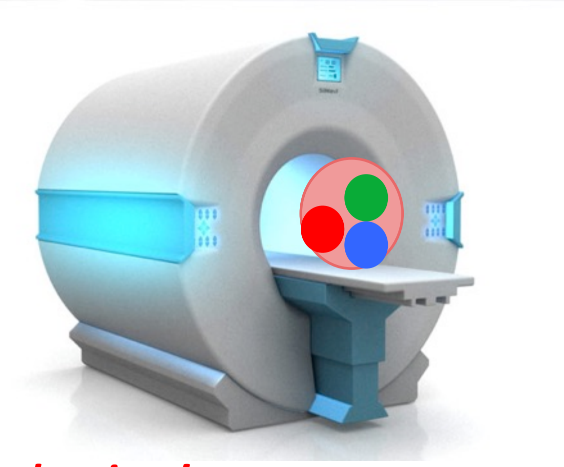
Imaging gluons

Heavy quarkonium: $Q^2+M^2 \gg |t|$

Imaging the glue only at EIC

The Electron-Ion Collider (EIC) – the Future!

❑ **A sharpest "CT"** – "imagine" quark/gluon structure without breaking the hadron

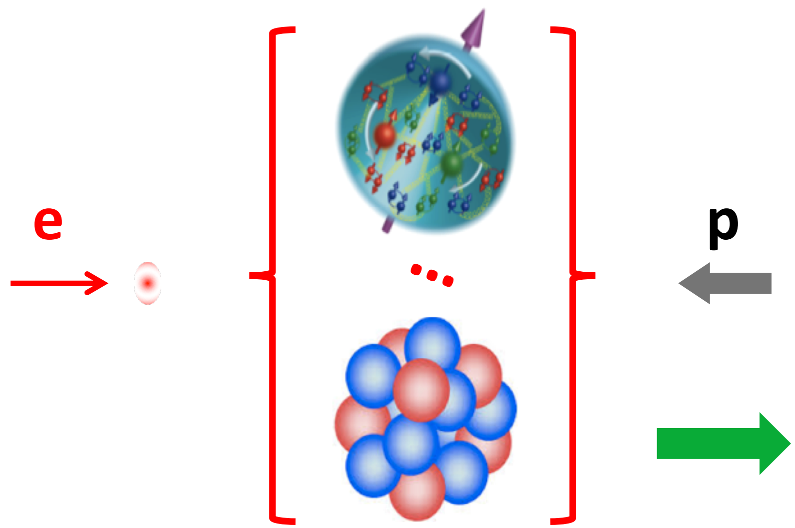


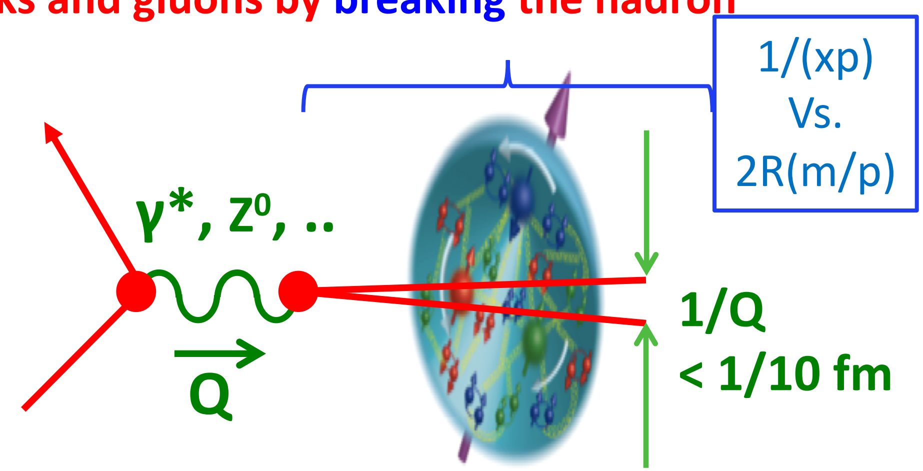
– "cat-scan" the nucleon and nuclei with a better than 1/10 fm resolution

– "see" proton "radius" of quark/gluon density comparing with the radius of EM charge density

➡ *To discover color confining radius, hints on confining mechanism!*

❑ **A giant "Microscope"** – "see" quarks and gluons by breaking the hadron





➡ *To discover/study color entanglement of the non-linear dynamics of the glue!*