Introduction to QCD

- Lec. 1: Fundamentals of QCD
- Lec. 2: Matching observed hadrons to quarks and gluons
- Lec. 3: QCD for cross sections with identified hadrons
- Lec. 4: QCD for cross sections with polarized beam(s)

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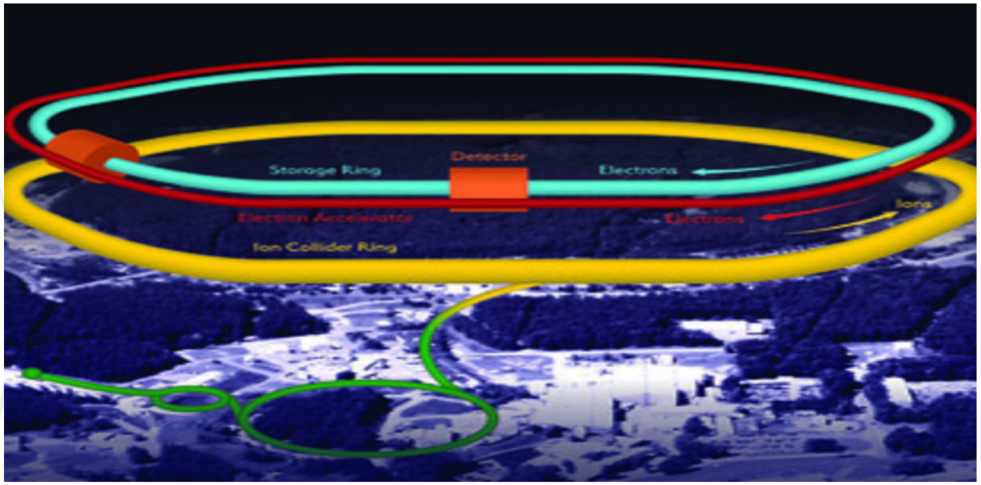
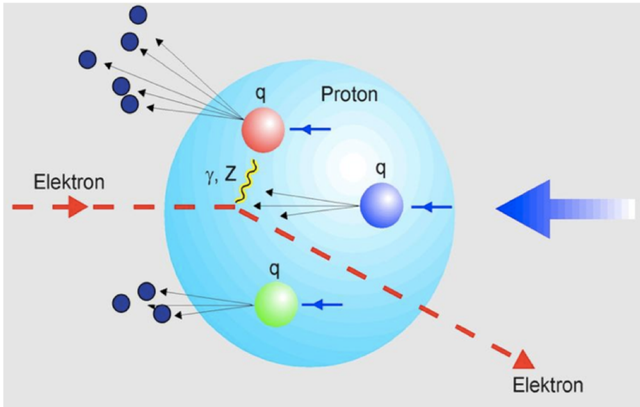


Inclusive lepton-hadron DIS – one hadron

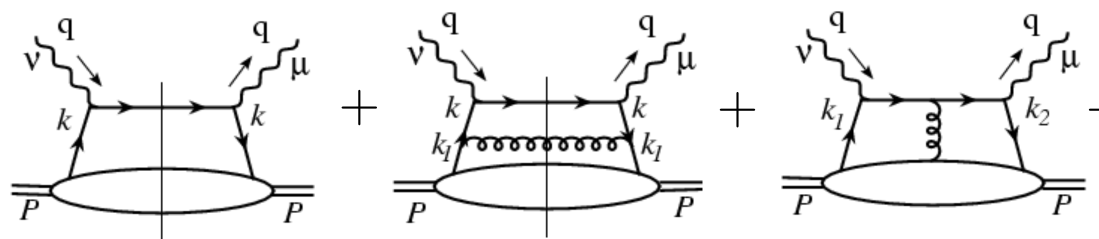


$$\sigma^{\rm DIS}_{\ell p\to\ell' X({\rm everything})}$$

Identified initial-state hadron-proton!

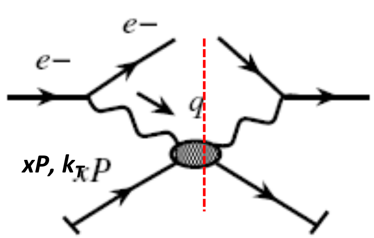
- DIS cross section is infrared divergent, and nonperturbative!

$$\sigma^{\rm DIS}_{\ell p\to\ell' X({\rm everything})} \propto$$



- QCD factorization (approximation!)

$$\sigma^{\rm DIS}_{\ell p\to\ell' X({\rm everything})} =$$



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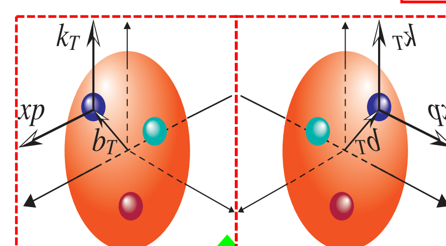


$$+ \, O\left(\frac{1}{QR}\right)$$

Color entanglement Approximation

Physical Observable

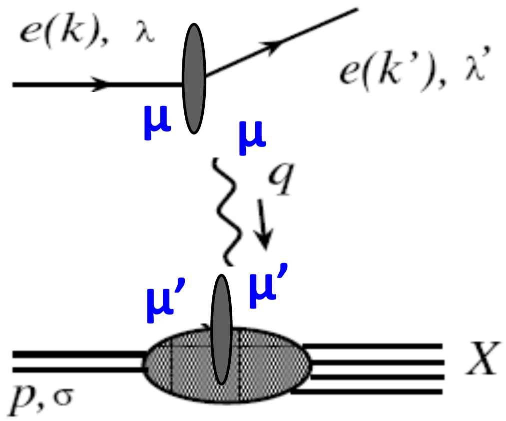
Controllable Probe

Quantum Probabilities Structure

Inclusive lepton-hadron DIS – one hadron

- **Scattering amplitude:**

$$M(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_\mu\right]u_\lambda(k)$$
$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$
$$* \left\langle X\left|eJ^{em}_{\mu'}(0)\right|p,\sigma\right\rangle$$



- **Cross section:**

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|M(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3\, 2E_i}\right]\frac{d^3k'}{(2\pi)^3\, 2E'}(2\pi)^4\,\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\Rightarrow \quad E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')\,W_{\mu\nu}(q,p)$$

- **Leptonic tensor:**

– known from QED

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^\mu k'^\nu + k^\nu k'^\mu - k\cdot k' g^{\mu\nu}\right)$$

DIS structure functions

❑ Hadronic tensor:

$$W_{\mu\nu}(q,p,S)=\frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\,\langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

❑ Symmetries:

- ✧ **Parity invariance (EM current)** ⟶ $W_{\mu\nu}=W_{\nu\mu}$ sysmetric for spin avg.
- ✧ **Time-reversal invariance** ⟶ $W_{\mu\nu}=W_{\mu\nu}^*$ real
- ✧ **Current conservation** ⟶ $q^\mu W_{\mu\nu}=q^\nu W_{\mu\nu}=0$

$$W_{\mu\nu}=-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right)+\frac{1}{p\cdot q}\left(p_\mu-q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu-q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right)+\frac{(p.q)S_\sigma-(S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

$$Q^2=-q^2 \qquad x_B=\frac{Q^2}{2p\cdot q}$$

❑ Structure functions – infrared sensitive:

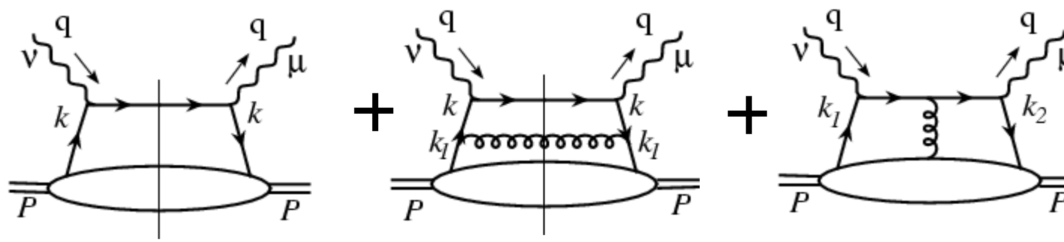
$$F_1\left(x_B,Q^2\right),F_2\left(x_B,Q^2\right),g_1\left(x_B,Q^2\right),g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

Long-lived parton states

❑ Feynman diagram representation of the hadronic tensor:

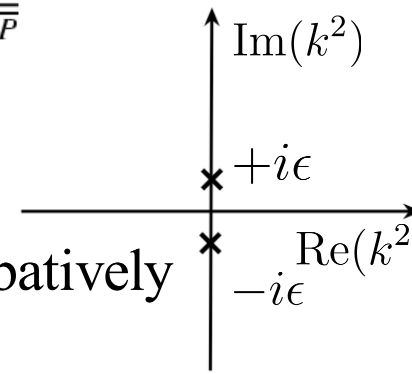
$$W^{\mu\nu} \propto$$



$+ \ldots$

❑ Perturbative pinched poles:

$$\int d^4k \; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$



❑ Perturbative factorization:

Light-cone coordinate:

$$v^\mu = (v^+, v^-, v^\perp),\; v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$$

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

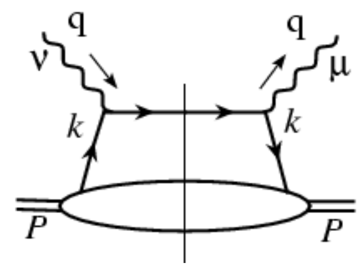
$$\int \frac{dx}{x} d^2k_T \; \mathrm{H}(Q,k^2=0) \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

Short-distance

Nonperturbative matrix element

Collinear factorization – further approximation

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}$



  – **Lowest order:** $\delta\left((k+q)^2\right) = \frac{1}{2P\cdot q}\delta(x-\xi) = \frac{1}{2P\cdot q}\delta\left(x-\frac{k^+}{P^+}\right)$

$$W^{\mu\nu}_{\gamma^* p} = \sum_f \int \frac{d^4k}{(2\pi)^4} \sum_{ij} \left(\gamma^\mu \gamma\cdot(k+q)\gamma^\nu\right)_{ij} (2\pi)\delta((k+q)^2) \int d^4y e^{iky} \langle p|\overline{\psi}_j(0)\psi_i(y)|p\rangle + \ldots$$

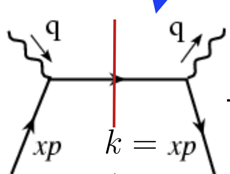
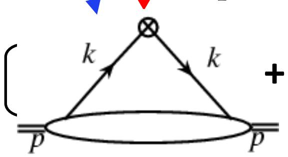
$$\equiv \sum_f \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k)\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\approx \sum_f \int dx\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k\approx xp)\int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k\cdot n}{p\cdot n})\mathcal{F}_{f/p}(k,p)\right] + \mathcal{O}(\frac{\langle k^2\rangle}{Q^2},\frac{\langle k_T^2\rangle}{Q^2}) + \ldots$$

$$\approx \sum_f \int \frac{dx}{x}\ \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right] \int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k\cdot n}{p\cdot n})\mathrm{Tr}\left[\frac{\gamma\cdot n}{2p\cdot n}\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\approx \sum_f \int \frac{dx}{x}\ \widehat{W}^{\mu\nu}_{\gamma^* f}(x,Q^2/\mu^2)\ \phi_{f/p}(x,\mu^2) + \ldots$$

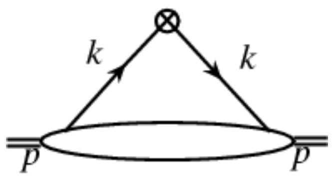
$$\int \frac{dx}{x} \qquad \frac{\gamma\cdot n}{2p\cdot n}\delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$\approx$ ( $+O\left(\frac{k_T^2}{Q^2}\right)$) $\otimes$ ( **+UVCT(μ)**)

$\frac{1}{2}\gamma\cdot(xp)$ $\qquad \widehat{W}^{\mu\nu}_{\gamma^* f}(x,Q^2/\mu^2) = \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,xp)\frac{1}{2}\gamma\cdot(xp)\right]$

Parton distribution functions (PDFs)

❑ **PDFs as matrix elements of two parton fields:**



– **combine the amplitude & its complex-conjugate**

$$\phi_{q/h}(x,\mu^2) = \int \frac{p^+ dy^-}{2\pi} e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \frac{\gamma^+}{2p^+} \psi_q(y^-)|h(p)\rangle$$

$|h(p)\rangle$ **can be a hadron, or a nucleus, or a parton state!**

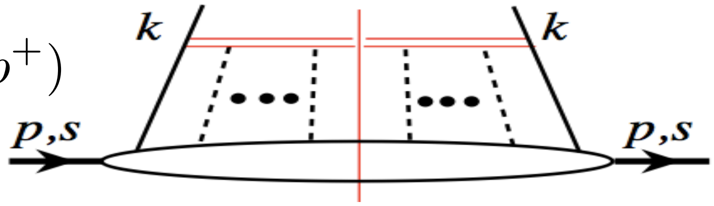
But, it is NOT gauge invariant! $\psi(x) \to e^{i\alpha_a(x)t_a}\psi(x) \quad \bar{\psi}(x) \to \bar{\psi}(x)e^{-i\alpha_a(x)t_a}$

– **need a gauge link:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0) \left[\mathcal{P}e^{-ig\int_0^{y^-} d\eta^- A^+(\eta^-)}\right] \frac{\gamma^+}{2p^+} \psi_q(y^-)|h(p)\rangle \mathcal{Z}_{\mathcal{O}}(\mu^2)$$

– **corresponding diagram in momentum space:**

$$\int \frac{d^4k}{(2\pi)^4} \delta(x - k^+/p^+)$$



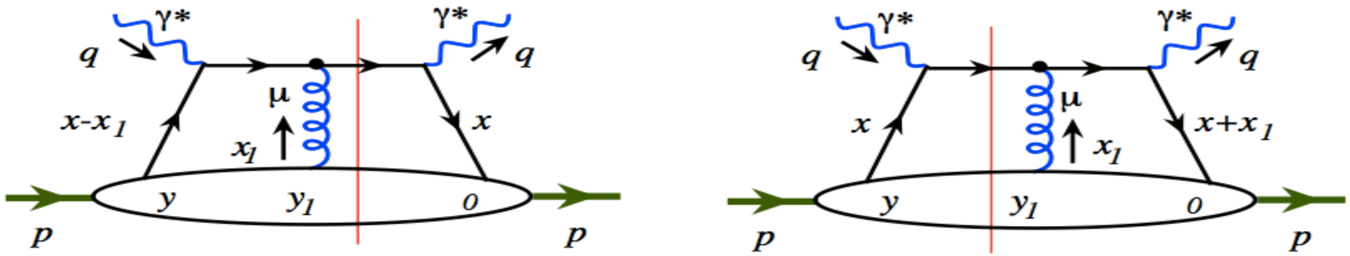
$+\,\mathrm{UVCT}(\mu^2)$

μ-dependence

Universality – process independence – predictive power

Gauge link – 1st order in coupling "g"

❑ Longitudinal gluon:



❑ Left diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)} n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi} n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{-x_1+i\epsilon}\, \mathrm{e}^{ix_1p^+(y_1^- - y^-)}\right] \mathcal{M} = -ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}$$

❑ Right diagram:

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, \mathrm{e}^{ix_1p^+y_1^-} n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_Bp^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi} n\cdot A^a(y_1^-)t^a \left[\int dx_1 \frac{1}{x_1-i\epsilon}\, \mathrm{e}^{ix_1p^+y_1^-}\right] \mathcal{M} \quad = ig\int_0^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}$$

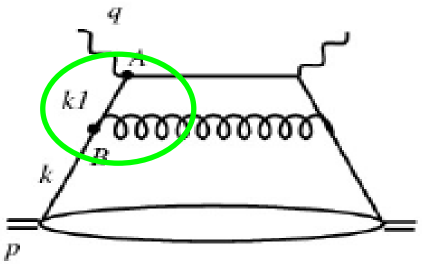
❑ Total contribution:

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

QCD high order corrections

- NLO partonic diagram to structure functions:

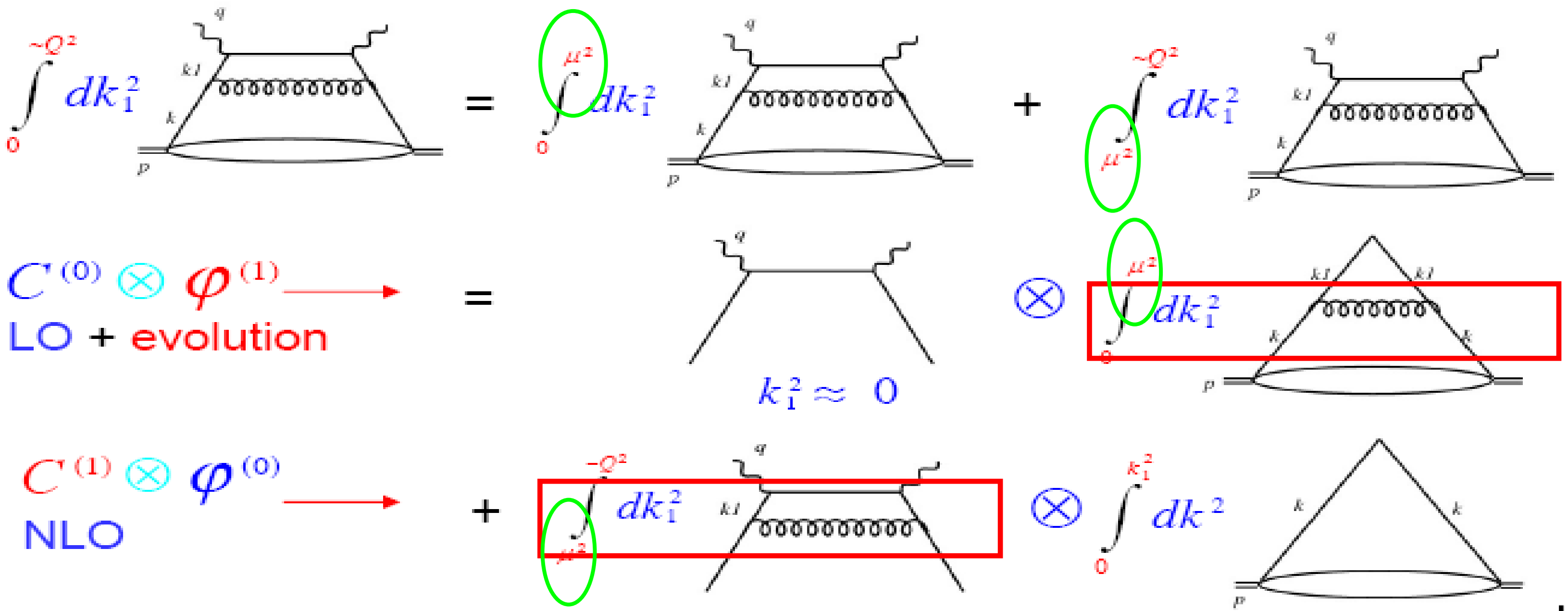


$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by $\begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$

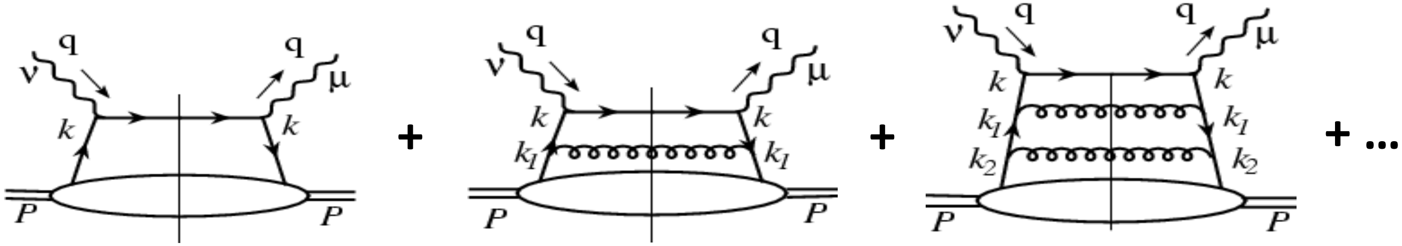
Diagram has both long- and short-distance physics

- Factorization, separation of short- from long-distance:

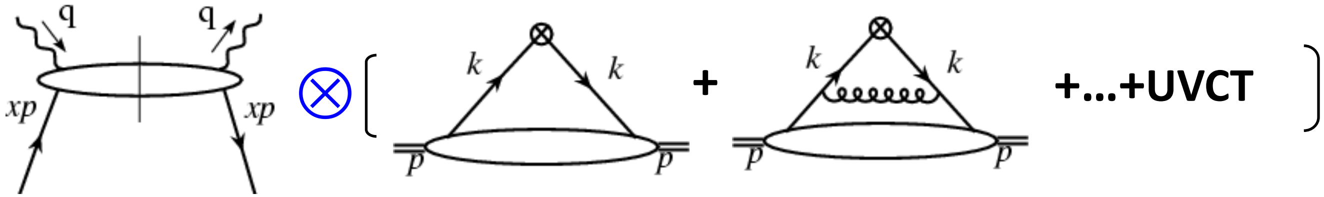


QCD leading power factorization

- **QCD corrections: pinch singularities in** $\int d^4 k_i$



- **Logarithmic contributions into parton distributions:**



$$F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- **Factorization scale:** μ_F^2

→ **To separate the collinear from non-collinear contribution**

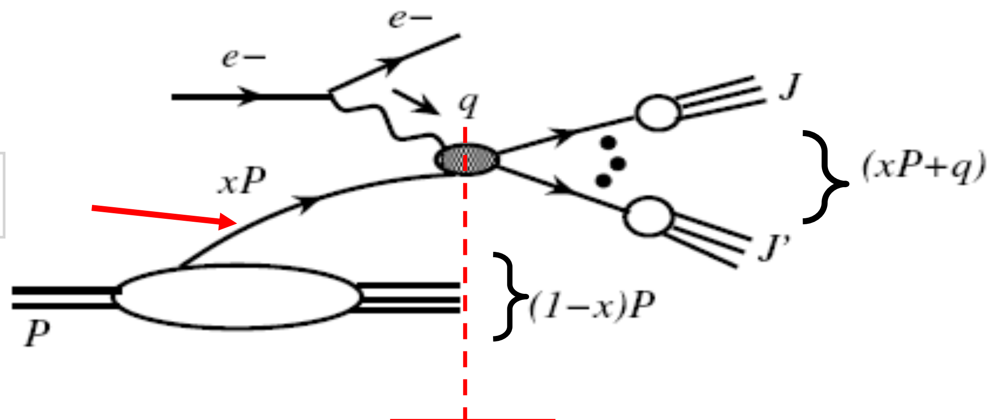
Recall: renormalization scale to separate local from non-local contribution

Jefferson Lab

Picture of factorization for DIS

- ❑ **Time evolution:**

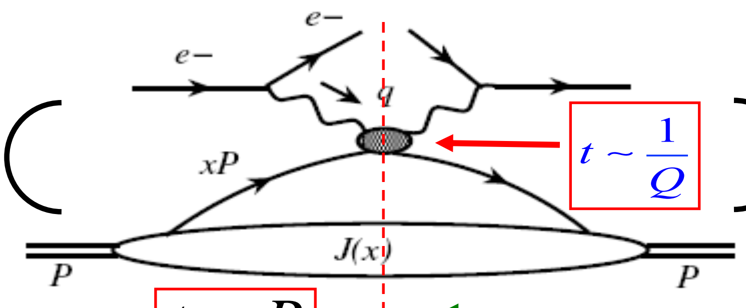
Long-lived parton state



Time: "Past" **Now** "Future"

- ❑ **Unitarity – summing over all hard jets:**

$$\sigma^{\text{DIS}}_{\text{tot}} \propto \text{Im}\Big($$



$$\Big)$$

$t \sim \frac{1}{Q}$

$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

How to calculate the perturbative parts?

❑ **Use DIS structure function F_2 as an example:**

$$F_{2h}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

✧ **Apply the factorized formula to parton states:** $h\to q$

Feynman diagrams → $$F_{2q}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$$ ← **Feynman diagrams**

✧ **Express both SFs and PDFs in terms of powers of α_s:**

0th order:

$$F^{(0)}_{2q}(x_B,Q^2)=C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$

$$\Rightarrow\quad C^{(0)}_q(x)=F^{(0)}_{2q}(x)\qquad \varphi^{(0)}_{q/q}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F^{(1)}_{2q}(x_B,Q^2)=C^{(1)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)+C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

$$\Rightarrow\quad C^{(1)}_q(x,Q^2/\mu^2)=F^{(1)}_{2q}(x,Q^2)-F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

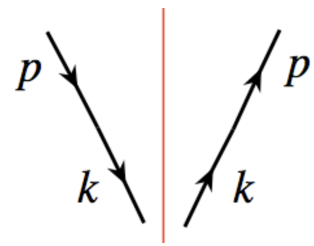
PDFs of a parton

❑ **Change the state without changing the operator:**

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\,\psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow |\text{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

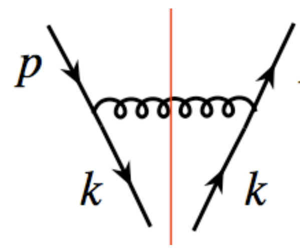
❑ **Lowest order quark distribution:**



✧ **From the operator definition:**

$$\phi^{(0)}_{q'/q}(x) = \delta_{qq'} \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)$$
$$= \delta_{qq'}\;\delta(1-x)$$

❑ **Leading order in α_s quark distribution:**



$+\ldots$

✧ **Expand to $(g_s)^2$ – logarithmic divergent:**

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \text{UVCT}$$

UV and CO divergence

Partonic cross sections

❑ Projection operators for SFs:

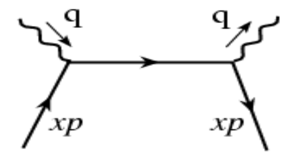
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x,Q^2\right)$$

$$F_1(x,Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

$$F_2(x,Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}(x,Q^2)$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = xg^{\mu\nu}W_{\mu\nu,q}^{(0)} = xg^{\mu\nu}\left[\frac{1}{4\pi}\right.$$



$$\left.\vphantom{\frac{1}{4\pi}}\right]$$

$$= \left(xg^{\mu\nu}\right)\frac{e_q^2}{4\pi}\mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p\gamma_\mu\gamma\cdot\left(p+q\right)\gamma_\nu\right]2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x\delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x\delta(1-x)$$

NLO coefficient function – complete example

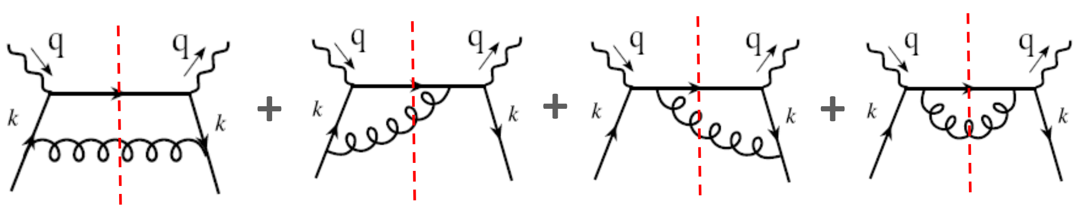
$$C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}\left(x, \mu^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

$$\left(1-\varepsilon\right) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}$$

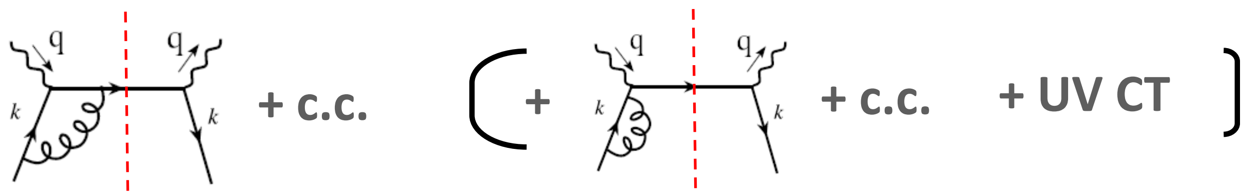
- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$



} **Real**

Virtual { +



- **Calculation:** $-g^{\mu\nu} W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of $W_{\mu\nu}$

Lowest order in n-dimension:

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

NLO virtual contribution:

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

NLO real contribution:

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

- The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ell n(1-z) f(1)$$

- One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2 (1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+ C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$
$$\left.\left. +3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

- Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

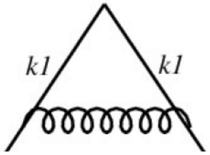
❑ One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon\ell n(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \;\; \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

Renormalization group improvement

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

$$F_2(x_B, Q^2) = \sum_f C_f(x_B/x, Q^2/\mu_F^2, \alpha_s)\, \phi_f(x, \mu_F^2)$$

→ **Evolution (differential-integral) equation for PDFs**

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\text{QCD}}^2\right)$

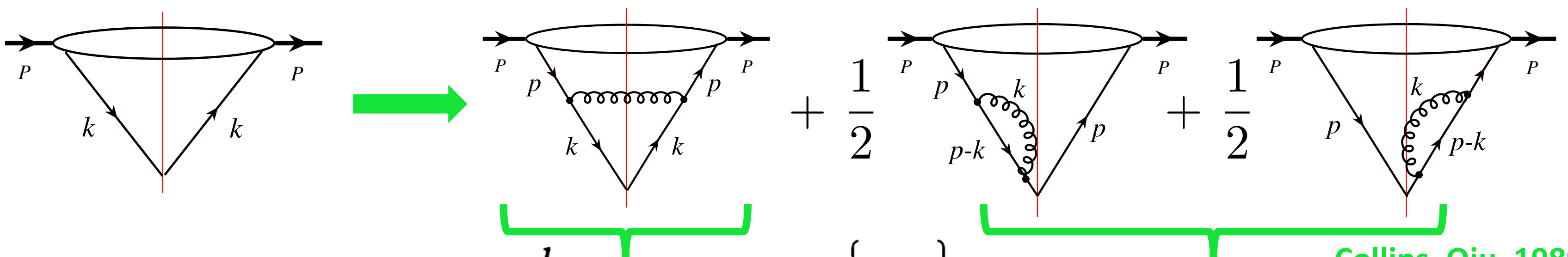
Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ **DGLAP evolution equation:**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

Calculation of evolution kernels

- Evolution kernels are process independent
  - Parton distribution functions are universal
  - Could be derived in many different ways
- Extract from calculating parton PDFs' scale dependence



Collins, Qiu, 1989

$$Q^2\frac{d}{dQ^2}q_i(x,Q^2)=\frac{\alpha_s}{2\pi}\int_x^1\frac{dx_1}{x_1}q_i(x_1,Q^2)\gamma_{qq}\left(\frac{x}{x_1}\right)-\frac{\alpha_s}{2\pi}q_i(x,Q^2)\int_0^1 dz\,\gamma_{qq}(z)$$

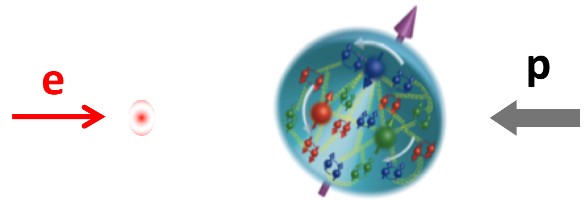
Change "Gain" "Loss"

  - Same is true for gluon evolution, and mixing flavor terms
- One can also extract the kernels from the CO divergence of partonic cross sections

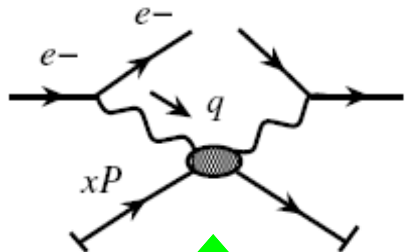
Jefferson Lab

From one hadron to two hadrons

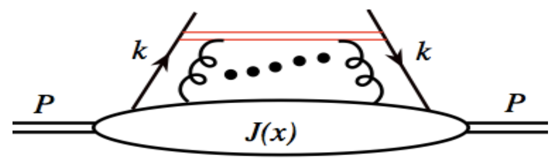
- **One hadron:**



$$\sigma_{\text{tot}}^{\text{DIS}} \sim$$



$$\otimes$$



$$+\,O\left(\frac{1}{QR}\right)$$

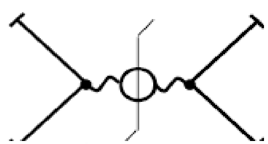
Hard-part
Probe

Parton-distribution
Structure

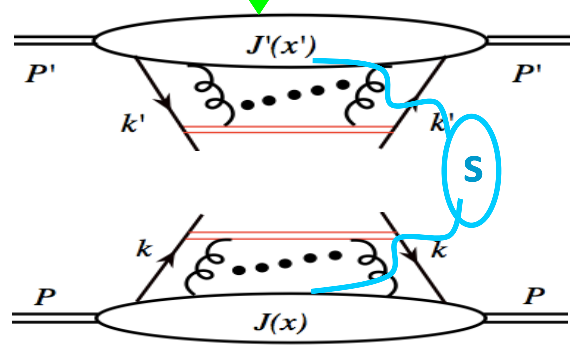
Power corrections
Approximation

- **Two hadrons:**

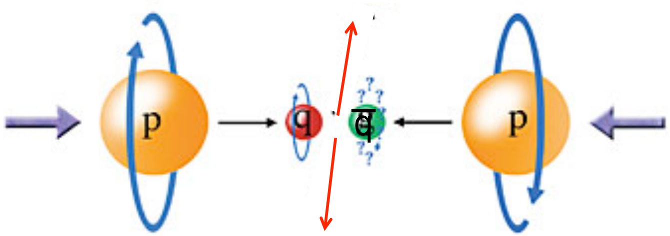
$$\sigma_{\text{tot}}^{\text{DY}} \sim$$



$$\otimes$$



$$+\,O\left(\frac{1}{QR}\right)$$



Predictive power:
Universal Parton Distributions

Drell-Yan process – two hadrons

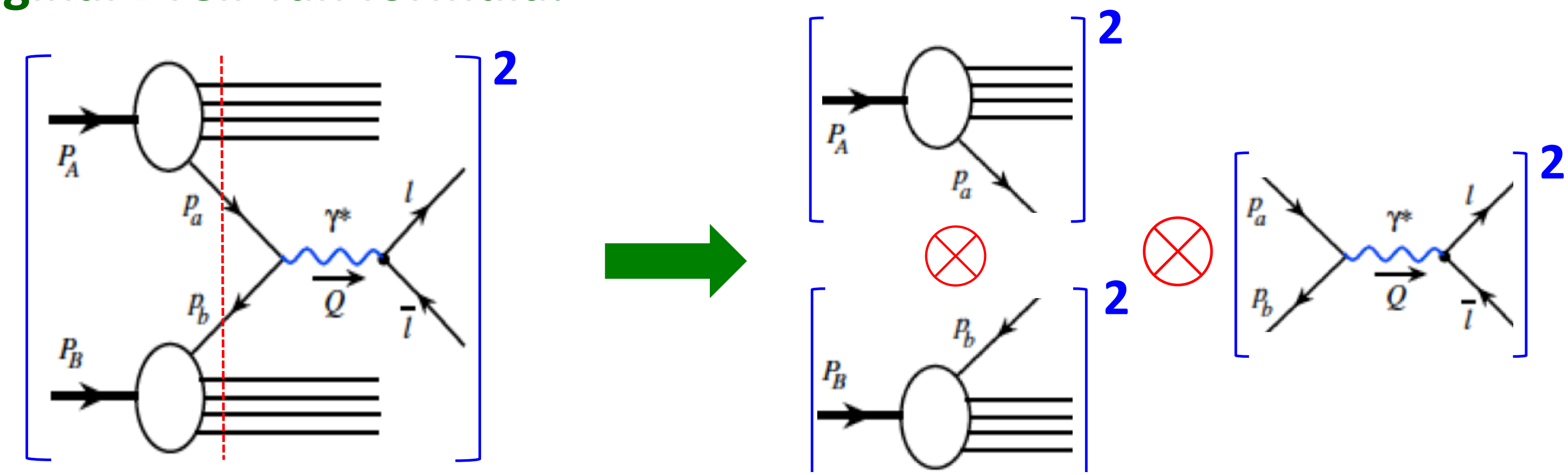
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)

❑ Drell-Yan mechanism:

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \quad \textbf{with} \quad q^2 \equiv Q^2 \gg \Lambda_{\rm QCD}^2 \sim 1/{\rm fm}^2$$

Lepton pair – from decay of a virtual photon, or in general, a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

❑ Original Drell-Yan formula:



$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha_{em}^2}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B)$$

No color yet!

Rapidity: $y = \frac{1}{2}\ln(x_A/x_B)$

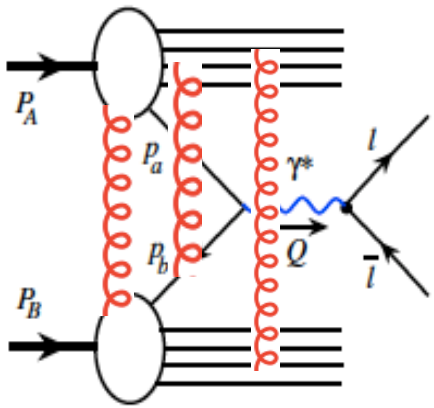
$$x_A = \frac{Q}{\sqrt{S}} e^{y} \quad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

Right shape – But – not normalization

Drell-Yan process in QCD – factorization

Collins, Soper and Sterman, Review in QCD, edited by AH Mueller 1989

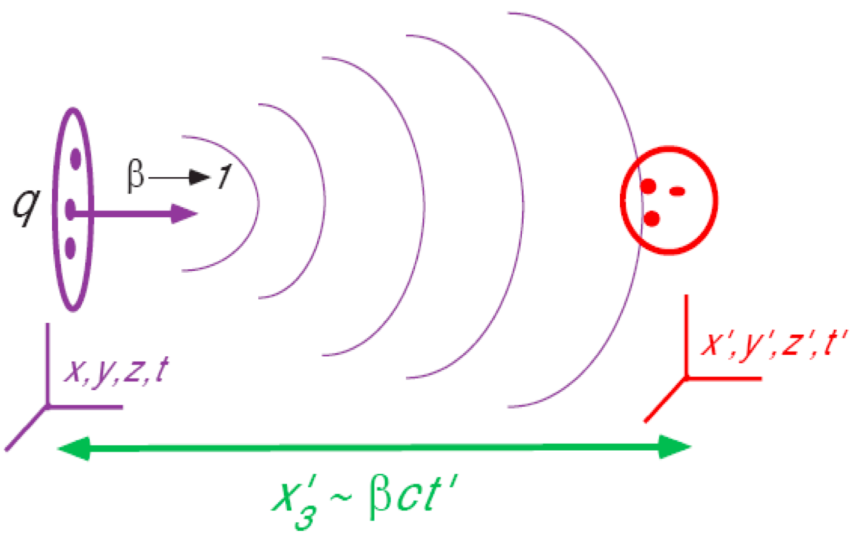
❑ Beyond the lowest order:



- ✧ Soft-gluon interaction takes place all the time
- ✧ Long-range gluon interaction before the hard collision

➡ Break the Universality of PDFs
Loss the predictive power

❑ Factorization – power suppression of soft gluon interaction:



x-Frame	x'-Frame
$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2+\gamma^2\Delta^2)^{1/2}}$
	$\Longrightarrow 1$ "not contracted!"
$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
	$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

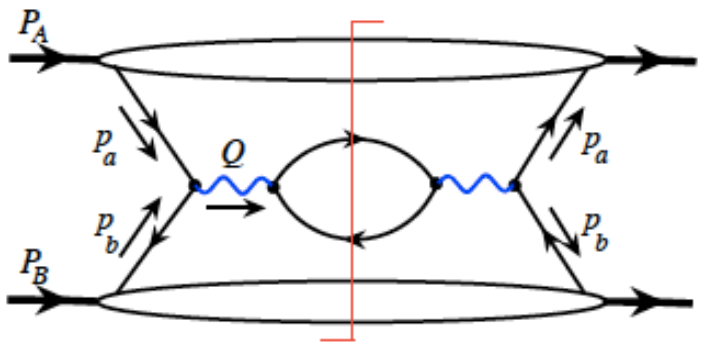
Drell-Yan process in QCD – factorization

❑ Factorization – approximation:

Collins, Soper, Sterman, 1988

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ $1/\Lambda_{QCD}$) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

$$\begin{aligned} &\text{on-shell:} && p_a^2,\ p_b^2 \ll Q^2; \\ &\text{collinear:} && p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ &\text{higher-power:} && p_a^- \ll q^-; \quad \text{and} \\ & && p_b^+ \ll q^+ \end{aligned}$$

✧ Infrared safe of partonic parts:

Cancelation of IR behavior

Absorb all CO divergences into PDFs

Factorized Drell-Yan cross section

- **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

- **Spin dependence:**

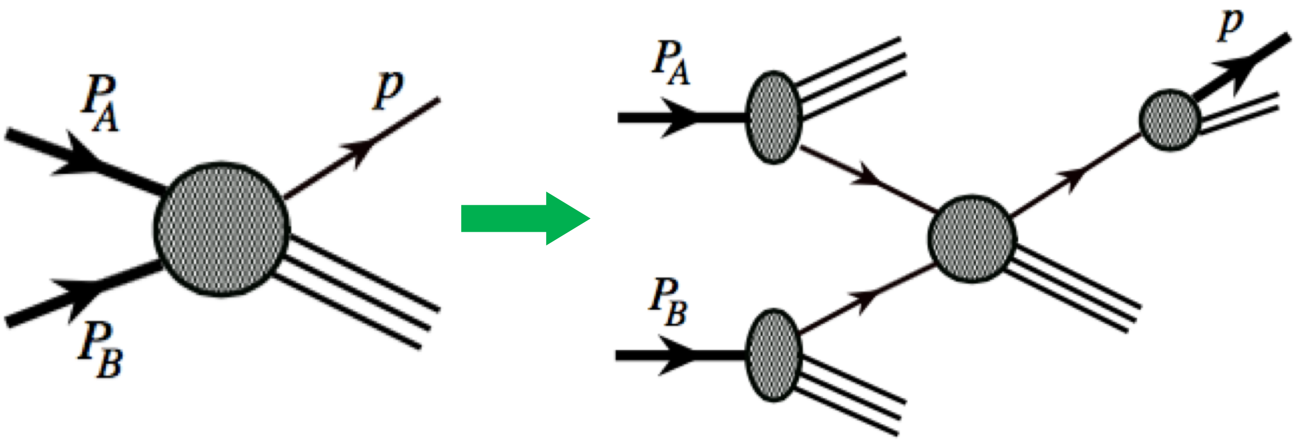
The factorization arguments are independent of the spin states of the colliding hadrons

→ same formula with polarized PDFs for γ^*,W/Z, H^0…

Factorization for more than two hadrons

❑ Factorization for high p_T single hadron:

Nayak, Qiu, Sterman, 2006



$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O $(1/P_T^2)$

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x,\mu_F^2) \otimes \phi_{B\to b}(x',\mu_F^2)$$
$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x,x',z,y,p_T^2,\mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z,\mu_F^2)$$

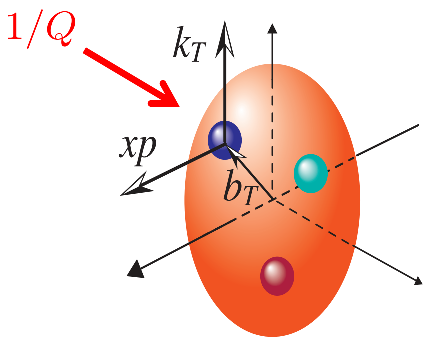
✧ **Fragmentation function:** $D_{c\to C}(z,\mu_F^2)$

✧ **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

Probes for 3D hadron structure

- **Single scale hard probe is too "localized":**



- It pins down the particle nature of quarks and gluons
- But, not very sensitive to the detailed structure of hadron ~ fm
- Transverse confined motion: k_T ~ 1/fm << Q
- Transverse spatial position: b_T ~ fm >> $1/Q$

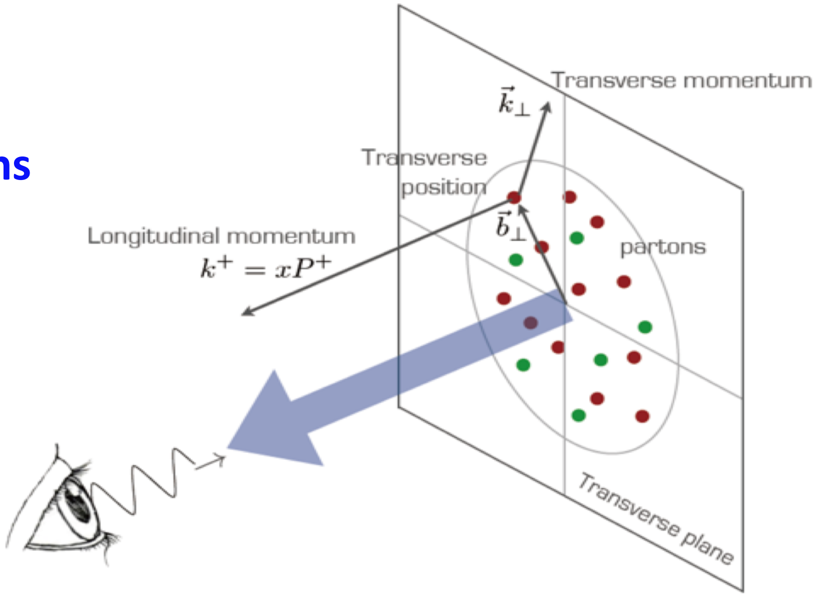
- **Need new type of "Hard Probes" – Physical observables with TWO Scales:**

$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\rm QCD}$$

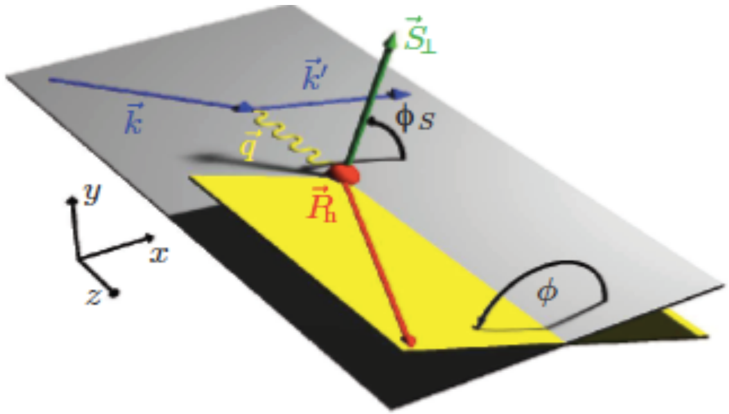
Hard scale: Q_1 **To localize the probe particle nature of quarks/gluons**

"Soft" scale: Q_2 **could be more sensitive to the hadron structure ~ 1/fm**

Hit the hadron "very hard" without breaking it, clean information on the structure!



Semi-inclusive DIS (SIDIS)



- **Process:**

$$e(k) + N(p) \longrightarrow e'(k') + h(P_h) + X$$

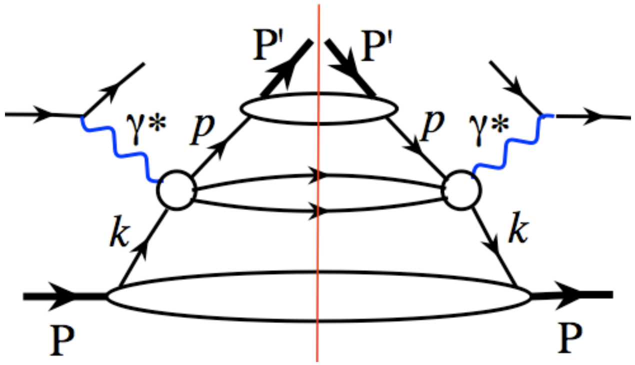
- **Natural event structure:**

In the photon-hadron frame: $P_{h_T} \approx 0$

Semi-Inclusive DIS is a natural observable with TWO very different scales

$Q \gg P_{h_T} \gtrsim \Lambda_{\rm QCD}$ **Localized probe sensitive to parton's transverse motion**

- **Collinear QCD factorization holds if P_{hT} integrated:**



$$d\sigma_{\gamma^* h \to h'} \propto \phi_{f/h} \otimes d\hat{\sigma}_{\gamma^* f \to f'} \otimes D_{f' \to h'}$$

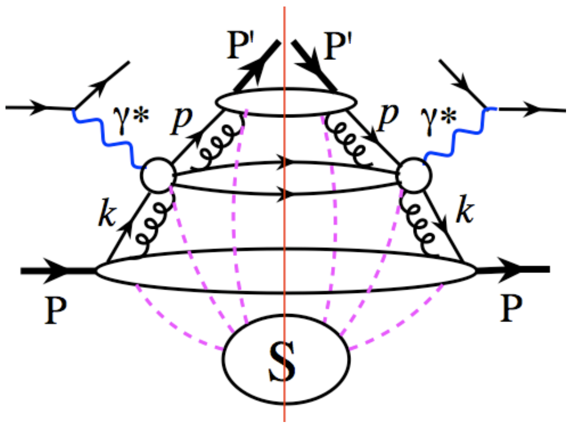
$$z = \frac{P_h \cdot p}{q \cdot p} \qquad y = \frac{q \cdot p}{k \cdot p}$$

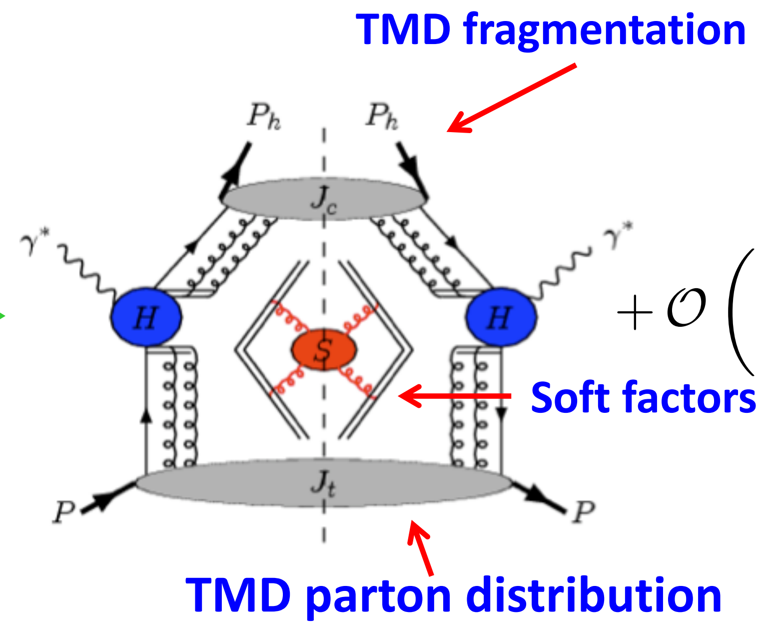
- **"Total c.m. energy":** $s_{\gamma^* p} = (p+q)^2 \approx Q^2 \left[\frac{1-x_B}{x_B}\right] \approx \frac{Q^2}{x_B}$

Semi-inclusive DIS (SIDIS)

❑ Perturbative definition – in terms of TMD factorization:

SIDIS as an example:





$$+\,\mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2},\frac{\langle p^2\rangle}{Q^2}\right)$$

❑ Low P_{hT} – TMD factorization:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q)\otimes\Phi_f(x,k_\perp)\otimes\mathcal{D}_{f\to h}(z,p_\perp)\otimes\mathcal{S}(k_{s\perp})+\mathcal{O}\left[\frac{P_{h\perp}}{Q}\right]$$

❑ High P_{hT} – Collinear factorization:

$$\sigma_{\rm SIDIS}(Q,P_{h\perp},x_B,z_h)=\hat{H}(Q,P_{h\perp},\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{P_{h\perp}},\frac{1}{Q}\right)$$

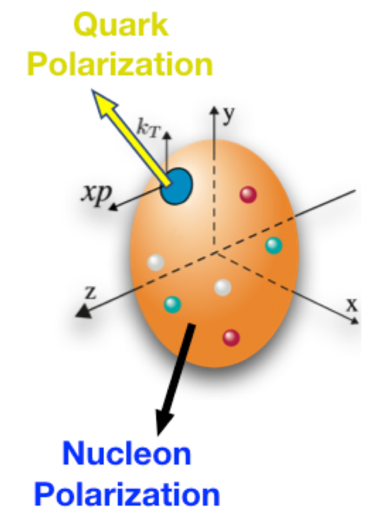
❑ P_{hT} Integrated - Collinear factorization:

$$\sigma_{\rm SIDIS}(Q,x_B,z_h)=\tilde{H}(Q,\alpha_s)\otimes\phi_f\otimes D_{f\to h}+\mathcal{O}\left(\frac{1}{Q}\right)$$

Transverse momentum dependent PDFs (TMDs)

❑ Quark TMDs with polarization:

Nucleon Polarization		Quark Polarization: Unpolarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	$f_1(x,k_T^2)$		$h_1^{\perp}(x,k_T^2)$ Boer-Mulders
	L		$g_1(x,k_T^2)$ Helicity	$h_{1L}^{\perp}(x,k_T^2)$ Long-Transversity
	T	$f_1^{\perp}(x,k_T^2)$ Sivers	$g_{1T}(x,k_T^2)$ Trans-Helicity	$h_1(x,k_T^2)$ Transversity $h_{1T}^{\perp}(x,k_T^2)$ Pretzelosity



Analogous tables for:

- Gluons $f_1 \to f_1^g$ etc
- Fragmentation functions
- Nuclear targets $S \neq \frac{1}{2}$

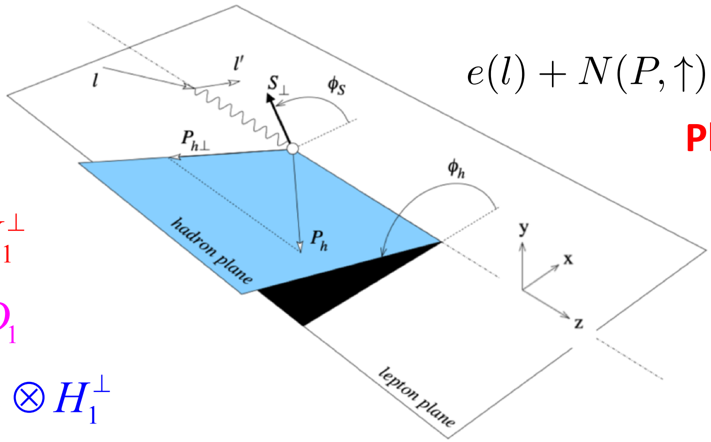
❑ Semi-Inclusive DIS (SIDIS):

$$A_{UT} = \frac{1}{P}\frac{\sigma_{lN(\uparrow)} - \sigma_{lN(\downarrow)}}{\sigma_{lN(\uparrow)} + \sigma_{lN(\downarrow)}}$$

$$A_{UT}^{Collins} \propto \langle \sin(\phi_h + \phi_S) \rangle_{UT} \propto h_1 \otimes H_1^{\perp}$$

$$A_{UT}^{Sivers} \propto \langle \sin(\phi_h - \phi_S) \rangle_{UT} \propto f_{1T}^{\perp} \otimes D_1$$

$$A_{UT}^{Pretzelosity} \propto \langle \sin(3\phi_h - \phi_S) \rangle_{UT} \propto h_{1T}^{\perp} \otimes H_1^{\perp}$$

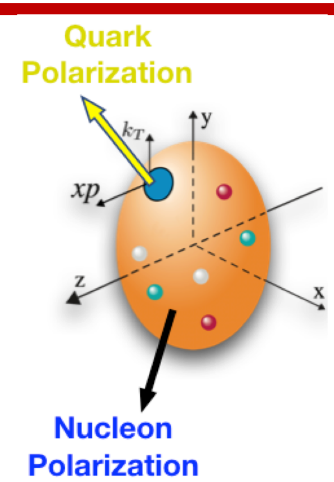


$$e(l) + N(P,\uparrow) \to e(l') + h(P_h) + X$$

Photon-hadron frame

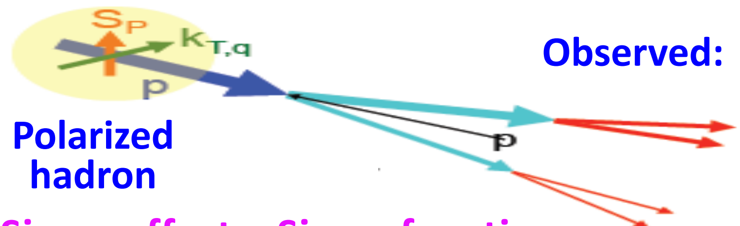
Two planes
Leptonic plane
Hadronic plane

What can we learn from TMDs?



- **Intrinsic & confined parton motion:**
  - ✧ Fundamental information sensitive to how partons are bound together
  - ✧ Responsible for dynamical contribution to emergent hadron properties, such as spin, mass, ..

- **Quantum correlation between hadron spin and parton motion:**



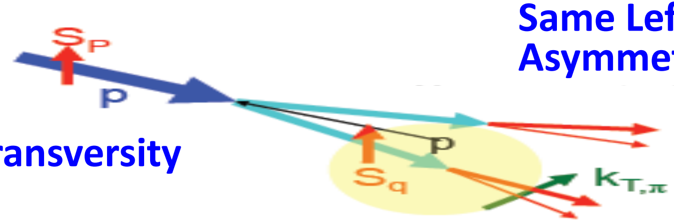
Observed: Left-Right Asymmetry

- ✧ Sivers effect – Sivers function

  Hadron spin influences parton's transverse motion

- **Quantum correlation between parton's spin and its hadronization:**

Same Left-Right Asymmetry



Transversity

- ✧ Collins effect – Collins function

  Parton's transverse polarization influences its hadronization

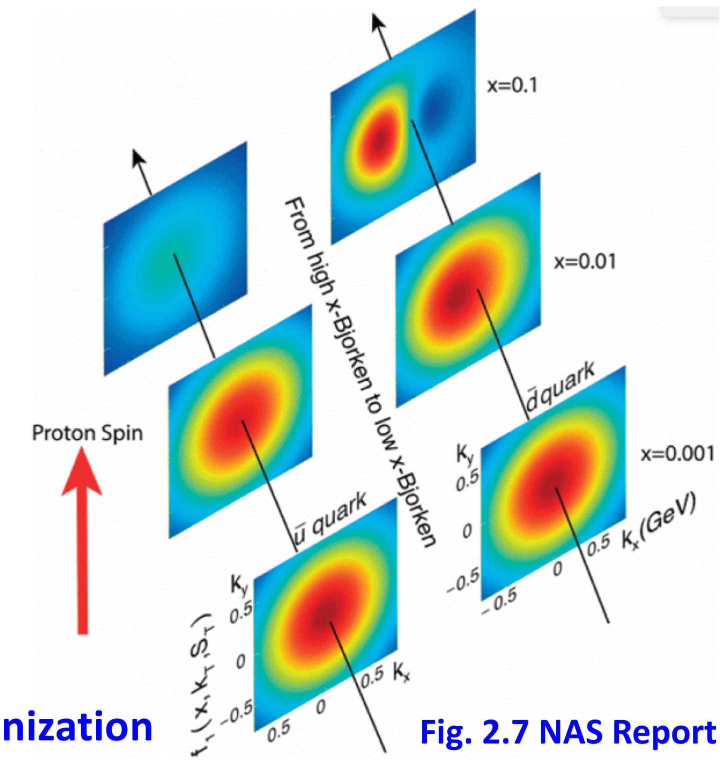


Fig. 2.7 NAS Report

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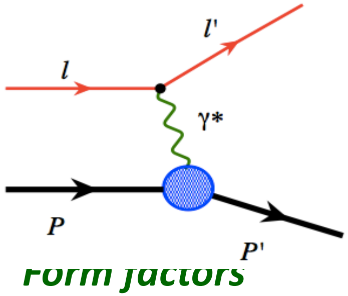
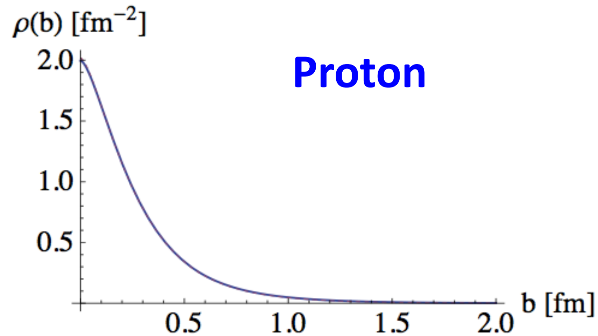
Exclusive lepton-hadron – Spatial imaging

❑ Elastic e-p scattering – Electric charge distribution:

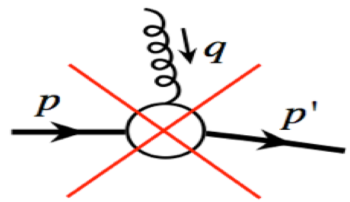


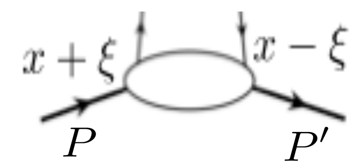


Proton
EM Charge
radius!

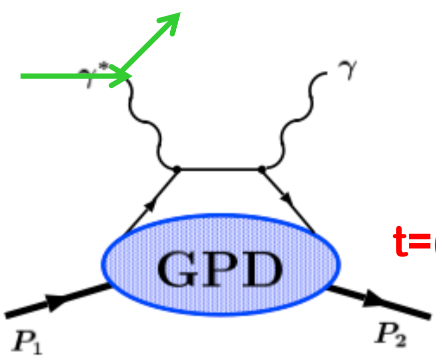
❑ No color nucleon elastic form factor!

No proton color charge radius!



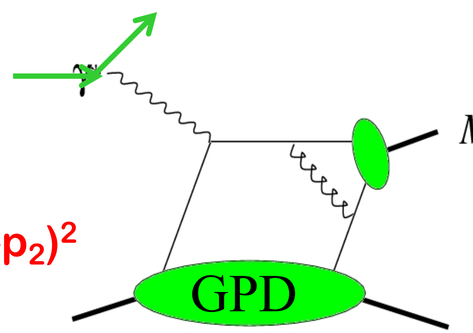


❑ Spatial quark/gluon density distributions – imaging:



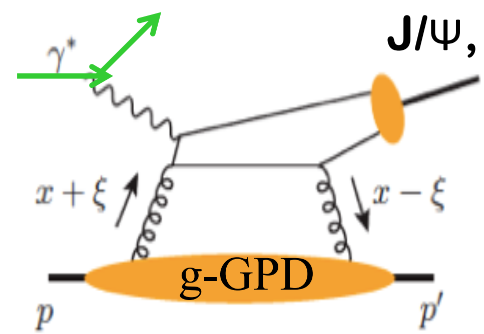
+

$t=(p_1-p_2)^2$



+

J/Ψ, Φ, …

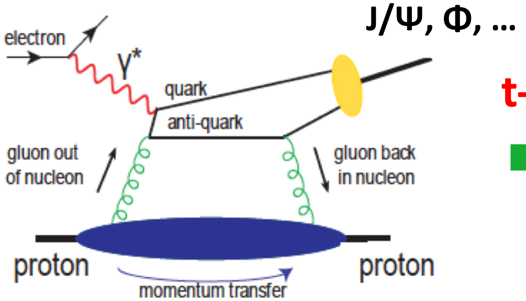


+ …

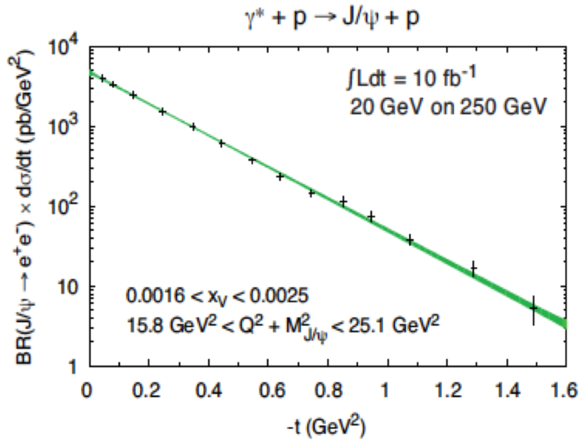
DVCS: $Q^2 \gg |t|$ DVEM: $Q^2 \gg |t|$ EHMP: $Q^2 \gg |t|$

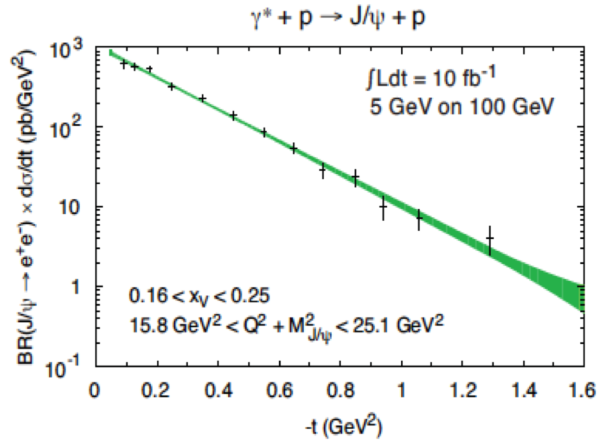
Spatial imaging of nucleon

- "Seeing" the glue at EIC:



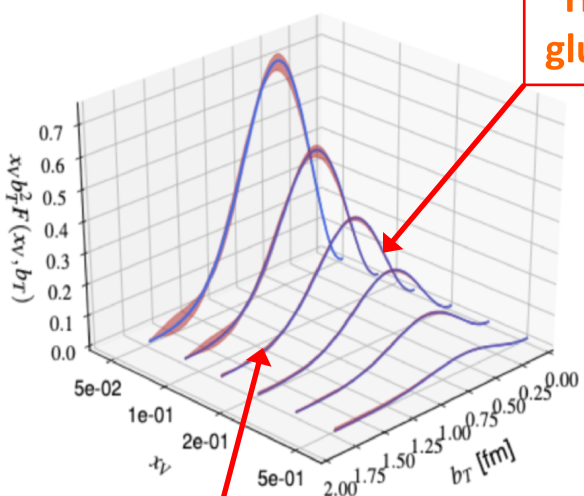
t-dep

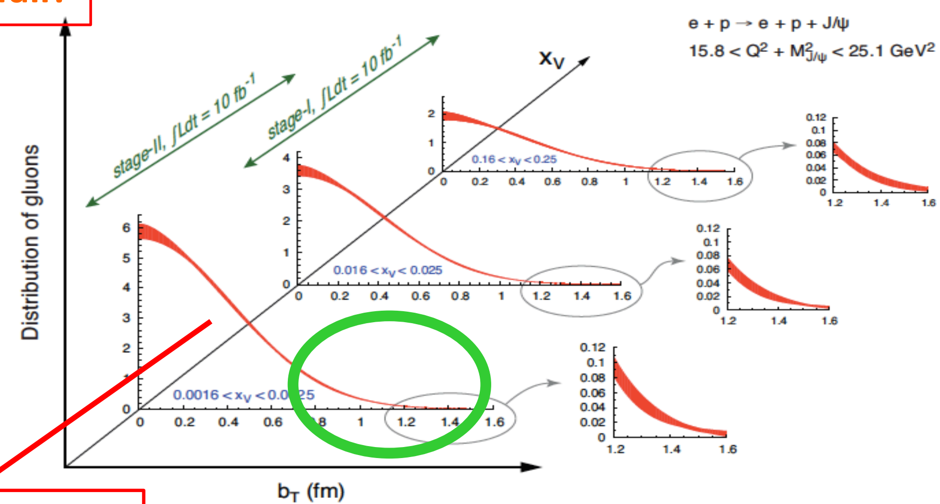




F.T.

How fast does glue density fall?





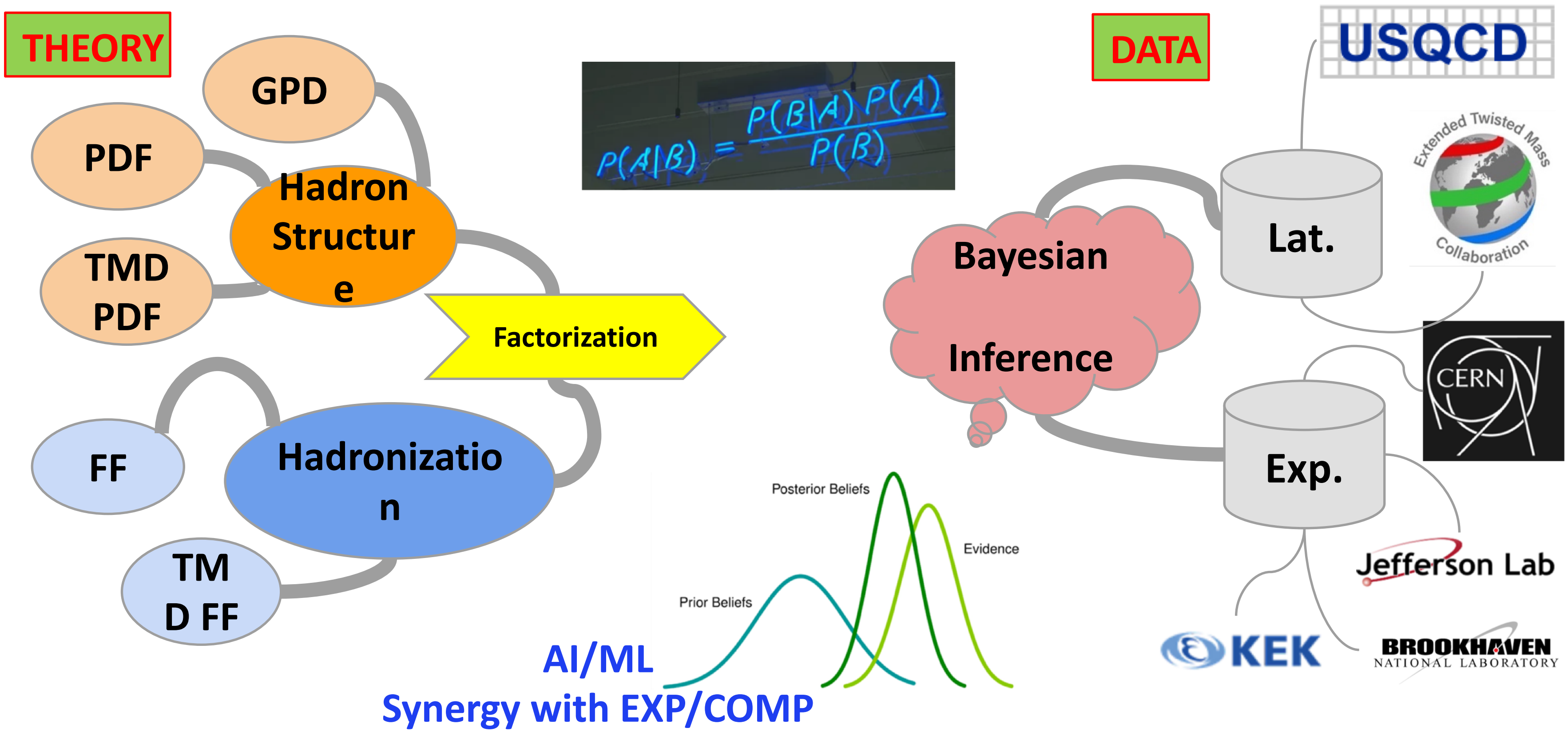
How far does glue density spread?

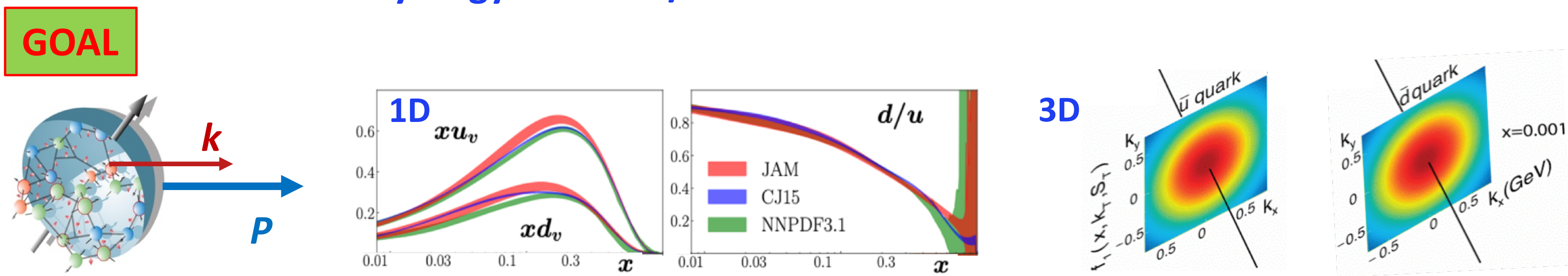
Proton radius of gluons (x)!

Only possible at EIC!

Observables with identified hadrons – Phenomenology

❑ **Need QCD global analyses of all data on factorizable cross sections!**





Backup slides

Drell-Yan Factorization

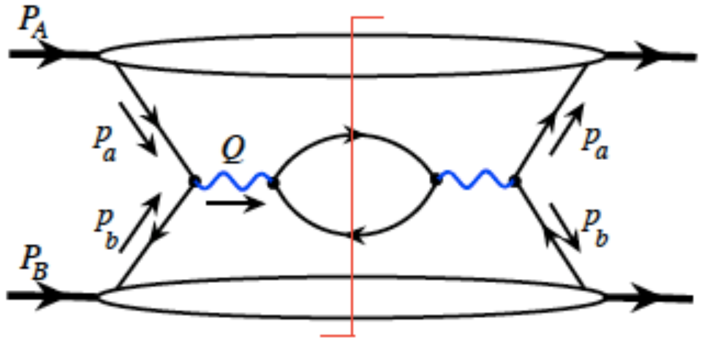
Drell-Yan process in QCD – factorization

❑ Factorization – approximation:

Collins, Soper, Sterman, 1988

✧ Suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ $1/\Lambda_{QCD}$) physics

➡ Need "long-lived" active parton states linking the two



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \rightarrow \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

$$\begin{aligned} \text{on-shell:} \quad & p_a^2,\ p_b^2 \ll Q^2; \\ \text{collinear:} \quad & p_{aT}^2,\ p_{bT}^2 \ll Q^2; \\ \text{higher-power:} \quad & p_a^- \ll q^-; \quad \text{and} \\ & p_b^+ \ll q^+ \end{aligned}$$

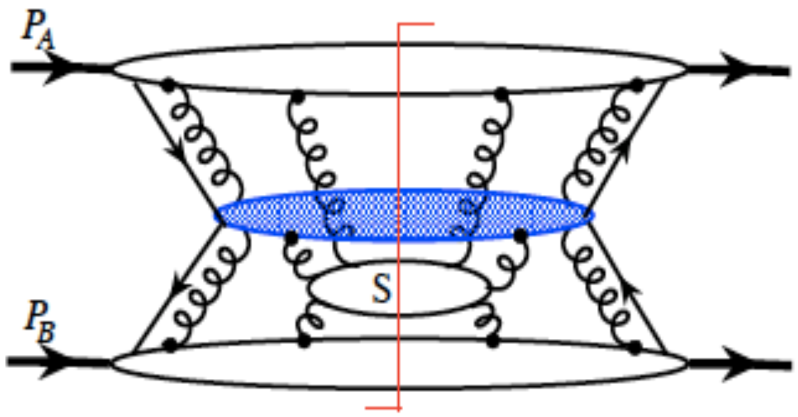
✧ Infrared safe of partonic parts:

Cancelation of IR behavior

Absorb all CO divergences into PDFs

Drell-Yan process in QCD – factorization

❑ Leading singular integration regions (pinch surface):



Hard: all lines off-shell by Q

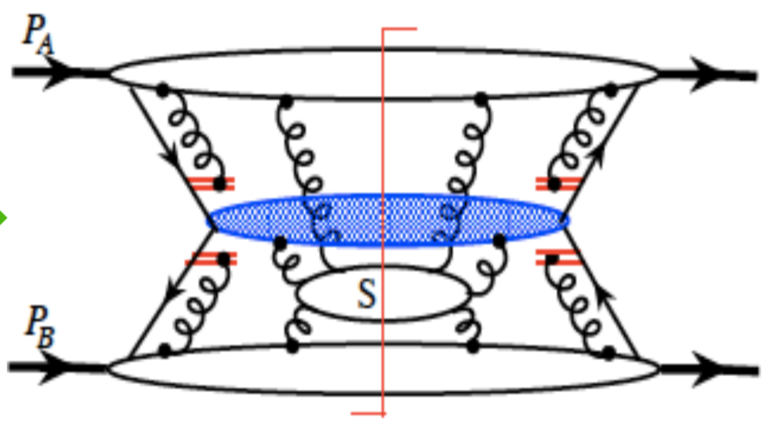
Collinear:

- ✧ lines collinear to A and B
- ✧ One "physical parton" per hadron

Soft: all components are soft

❑ Collinear gluons:

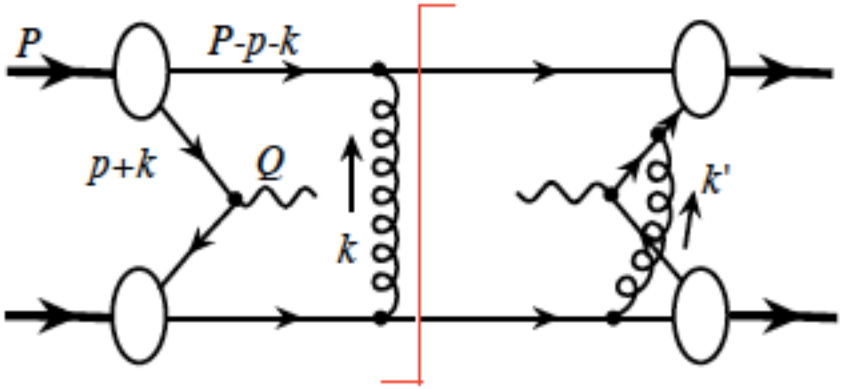
- ✧ Collinear gluons have the polarization vector: $\epsilon^\mu \sim k^\mu$
- ✧ The sum of the effect can be represented by the eikonal lines,



which are needed to make the PDFs gauge invariant!

Drell-Yan process in QCD – factorization

❑ **Trouble with soft gluons:**



$$(xp+k)^2 + i\epsilon \propto k^- + i\epsilon$$
$$((1-x)p-k)^2 + i\epsilon \propto k^- - i\epsilon$$

✧ **Soft gluon exchanged between a spectator quark of hadron B and the active quark of hadron A could rotate the quark's color and keep it from annihilating with the antiquark of hadron B**

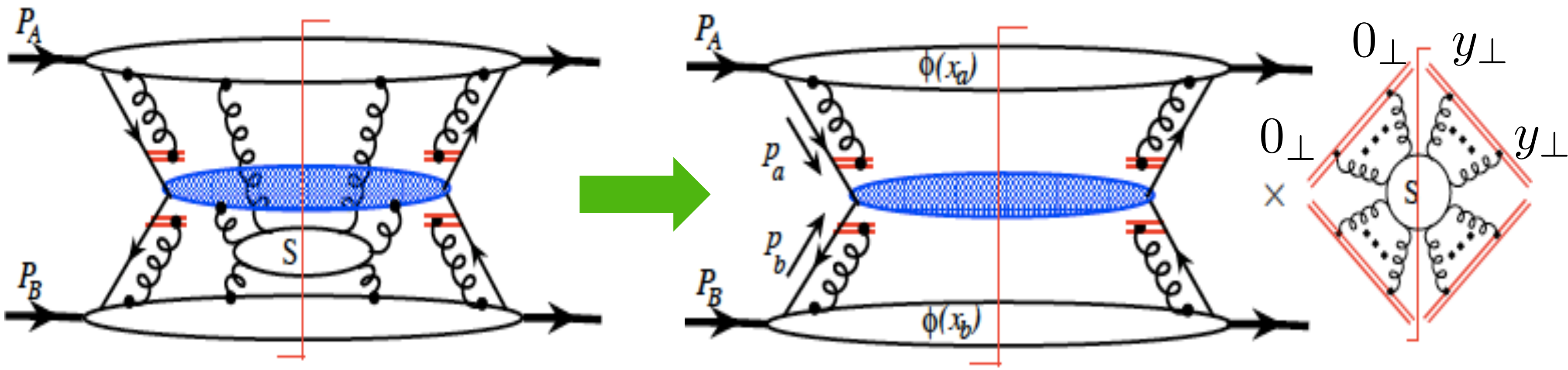
✧ **The soft gluon approximations (with the eikonal lines) need $k^\pm$ not too small. But, $k^\pm$ could be trapped in "too small" region due to the pinch from spectator interaction:** $k^\pm \sim M^2/Q \ll k_\perp \sim M$

Need to show that soft-gluon interactions are power suppressed

Jefferson Lab

Drell-Yan process in QCD – factorization

❑ **Most difficult part of factorization:**



- ✧ **Sum over all final states to remove all poles in one-half plane – no more pinch poles**
- ✧ **Deform the $k^{\pm}$ integration out of the trapped soft region**
- ✧ **Eikonal approximation ⟹ soft gluons to eikonal lines – gauge links**
- ✧ **Collinear factorization: Unitarity ⟹ soft factor = 1**

All identified leading integration regions are factorizable!