Introduction to QCD

- Lec. 1: Fundamentals of QCD
- Lec. 2: Matching observed hadrons to quarks and gluons
- Lec. 3: QCD for cross sections with identified hadrons
- Lec. 4: QCD for cross sections with polarized beam(s)

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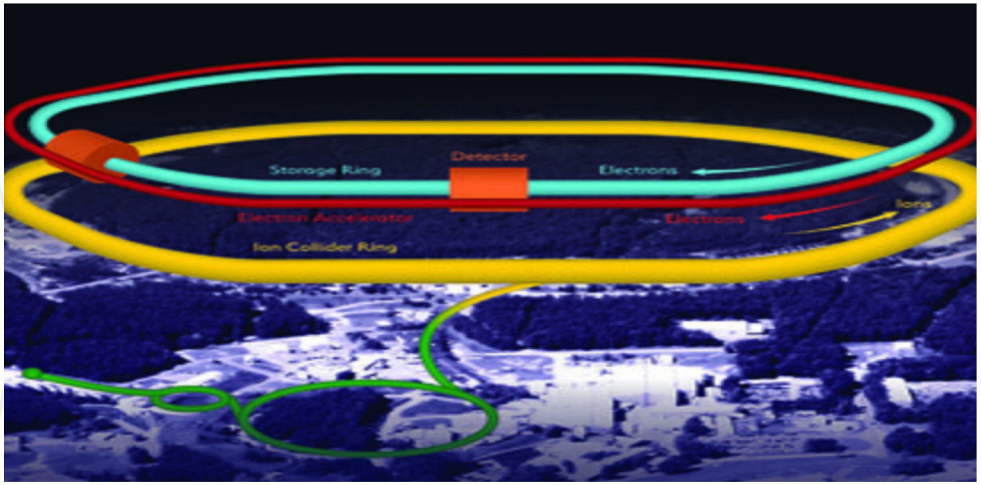


Polarization and spin asymmetry

Explore new QCD dynamics – vary the spin orientation

- **Cross section:**

Scattering amplitude square – Probability – Positive definite

$$\sigma_{AB}(Q,\vec{s}) \approx \sigma^{(2)}_{AB}(Q,\vec{s}) + \frac{Q_s}{Q}\sigma^{(3)}_{AB}(Q,\vec{s}) + \frac{Q_s^2}{Q^2}\sigma^{(4)}_{AB}(Q,\vec{s}) + \cdots$$

- **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$

– Positive definite

- **Asymmetries or difference of cross sections:**

– Not necessary positive!

- **both beams polarized** A_{LL}, A_{TT}, A_{LT}

$$A_{LL} = \frac{[\sigma(+,+) - \sigma(+,-)] - [\sigma(-,+) - \sigma(-,-)]}{[\sigma(+,+) + \sigma(+,-)] + [\sigma(-,+) + \sigma(-,-)]} \quad \text{for } \sigma(s_1,s_2)$$

- **one beam polarized** A_L, A_N

$$A_L = \frac{[\sigma(+) - \sigma(-)]}{[\sigma(+) + \sigma(-)]} \quad \text{for } \sigma(s) \qquad A_N = \frac{\sigma(Q,\vec{s}_T) - \sigma(Q,-\vec{s}_T)}{\sigma(Q,\vec{s}_T) + \sigma(Q,-\vec{s}_T)}$$

Chance to see quantum interference directly

Two roles of the proton spin program

❑ Proton is a composite particle:

Spin is a consequence of internal dynamics of the bound state

For example, the nucleon-nucleon interaction and shell structure determines the observed nuclear spin states

➡ Decomposition of proton spin in terms of quark and gluon d.o.f.

helps understand the dynamics of a fundamental QCD bound state

– Nucleon is a building block all hadronic matter

(> 95% mass of all visible matter)

❑ Use the spin as a tool – asymmetries:

Cross section is a probability – classically measured

Spin asymmetry – the difference of two cross sections involving two different spin states

Asymmetry could be a pure quantum effect!

Spin of a composite particle

- **Spin:**
  - ✧ Pauli (1924): two-valued quantum degree of freedom of electron
  - ✧ Pauli/Dirac: $S = \hbar\sqrt{s(s+1)}$ (fundamental constant ħ)
  - ✧ Composite particle = Total angular momentum when it is at rest
- **Spin of a nucleus:**
  - ✧ Nuclear binding: 8 MeV/nucleon << mass of nucleon
  - ✧ Nucleon number is fixed inside a given nucleus
  - ✧ Spin of a nucleus = sum of the valence nucleon spin
- **Spin of a nucleon – Naïve Quark Model:**
  - ✧ If the probing energy << mass of constituent quark
  - ✧ Nucleon is made of three constituent (valence) quark
  - ✧ Spin of a nucleon = sum of the constituent quark spin

State: $$|p\uparrow\rangle = \sqrt{\frac{1}{18}}\left[u\uparrow u\downarrow d\uparrow + u\downarrow u\uparrow d\uparrow - 2u\uparrow u\uparrow d\downarrow + \text{perm.}\right]$$

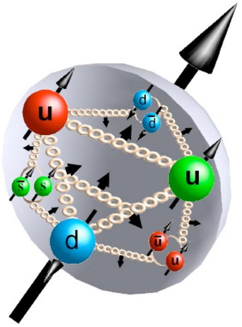
Spin: $$S_p \equiv \langle p\uparrow|S|p\uparrow\rangle = \frac{1}{2}, \qquad S = \sum_i S_i$$

Carried by valence quarks

Spin of a composite particle

❑ Spin of a nucleon – QCD:

- ✧ Current quark mass << energy exchange of the collision
- ✧ Number of quarks and gluons depends on the probing energy



❑ Angular momentum of a proton at rest:

$$S = \sum_f \langle P, S_z = 1/2 |\, \hat{J}_f^z \,| P, S_z = 1/2 \rangle = \frac{1}{2}$$

❑ QCD Angular momentum operator:

Energy-momentum tensor

$$J^i_{\rm QCD} = \frac{1}{2}\,\epsilon^{ijk} \int d^3x \; M^{0jk}_{\rm QCD} \quad \Longleftarrow \quad M^{\alpha\mu\nu}_{\rm QCD} = T^{\alpha\nu}_{\rm QCD}\, x^\mu - T^{\alpha\mu}_{\rm QCD}\, x^\nu$$

Angular momentum density

- ✧ Quark angular momentum operator:

$$\vec{J}_q = \int d^3x \left[\psi_q^\dagger \vec{\gamma}\gamma_5 \psi_q + \psi_q^\dagger (\vec{x} \times (-i\vec{D}))\, \psi_q\right]$$

- ✧ Gluon angular momentum operator:

$$\vec{J}_g = \int d^3x \left[\vec{x} \times (\vec{E} \times \vec{B})\right]$$

Need to have the matrix elements of these partonic operators measured independently

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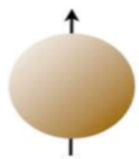
Current understanding for Proton Spin

- **The sum rule:** $$S(\mu) = \sum_f \langle P, S | \hat{J}_f^z(\mu) | P, S \rangle = \frac{1}{2} \equiv J_q(\mu) + J_g(\mu)$$
  - **Infinite possibilities of decompositions – connection to observables?**
  - **Intrinsic properties + dynamical motion and interactions**

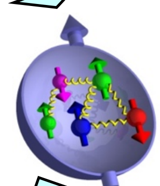
- **An incomplete story:**

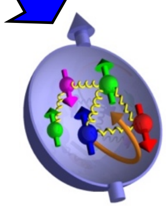


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + (L_q + L_g)$$

Jaffe-Manohar, 90
Ji, 96, …

Proton Spin





Quark helicity
Best known

$$\frac{1}{2}\int dx \left(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}\right)$$

$$\sim 30\%$$

Sea quarks?

Gluon helicity
Start to know

$$\Delta G = \int dx \Delta g(x)$$

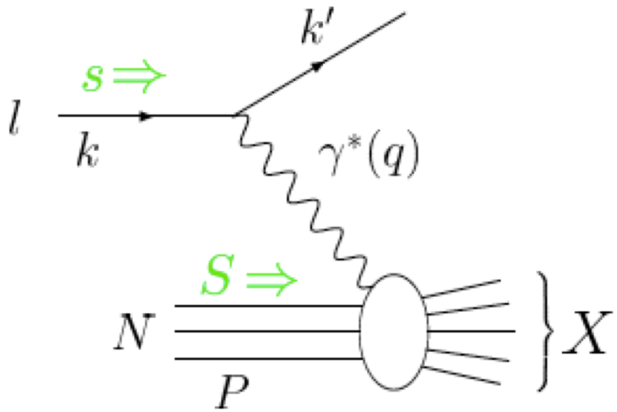
$$\sim 40\% (\text{with RHIC data})$$

Orbital Angular Momentum
of quarks and gluons
Little known

Net effect of partons' transverse motion?

Polarized deep inelastic scattering

❑ DIS with polarized beam(s):



"Resolution" $Q \equiv \sqrt{-q^2}$

$$\frac{\hbar}{Q} = \frac{2 \times 10^{-16}\text{m}}{Q/\text{GeV}} \lesssim 10^{-16}\text{m} = 1/10\text{fm}$$

"Inelasticity" **– known as Bjorken variable**

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{Q^2 + M_X^2 - m^2}$$

✧ **Recall – from lecture 2:**

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

$$+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{p.q} g_1\left(x_B, Q^2\right) + \frac{(p.q) S_\sigma - (S.q) p_\sigma}{(p.q)^2} g_2\left(x_B, Q^2\right)\right]$$

✧ **Polarized structure functions:**

$$g_1(x_B, Q^2), \;\; g_2(x_B, Q^2)$$

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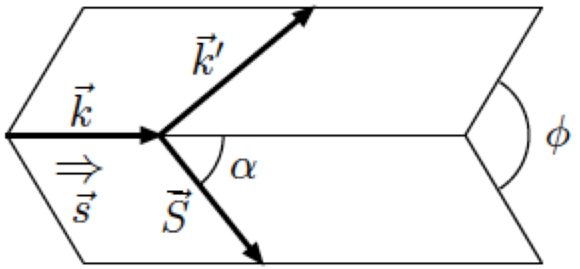
Polarized deep inelastic scattering

❑ Extract the polarized structure functions:

$$\mathcal{W}^{\mu\nu}(P,q,S) \; - \; \mathcal{W}^{\mu\nu}(P,q,-S)$$



✧ **Define:** $\angle(\hat{k},\hat{S}) = \alpha$,
and lepton helicity λ

✧ **Difference in cross sections with hadron spin flipped**

$$\frac{d\sigma^{(\alpha)}}{dx\,dy\,d\phi} - \frac{d\sigma^{(\alpha+\pi)}}{dx\,dy\,d\phi} = \frac{\lambda\, e^4}{4\pi^2 Q^2} \times$$

$$\times \left\{ \cos\alpha \left\{ \left[1 - \frac{y}{2} - \frac{m^2 x^2 y^2}{Q^2} \right] g_1(x,Q^2) \; - \; \frac{2m^2 x^2 y}{Q^2} g_2(x,Q^2) \right\} \right.$$

$$\left. - \sin\alpha \cos\phi \frac{2mx}{Q} \sqrt{\left(1 - y - \frac{m^2 x^2 y^2}{Q^2}\right)} \left(\frac{y}{2} g_1(x,Q^2) \; + \; g_2(x,Q^2) \right) \right\}$$

✧ **Spin orientation:**

$$\alpha \;=\; 0 \;: \;\Rightarrow \quad g_1$$

$$\alpha \;=\; \pi/2 : \;\Rightarrow \quad y\, g_1 + 2\, g_2$$, suppressed m/Q

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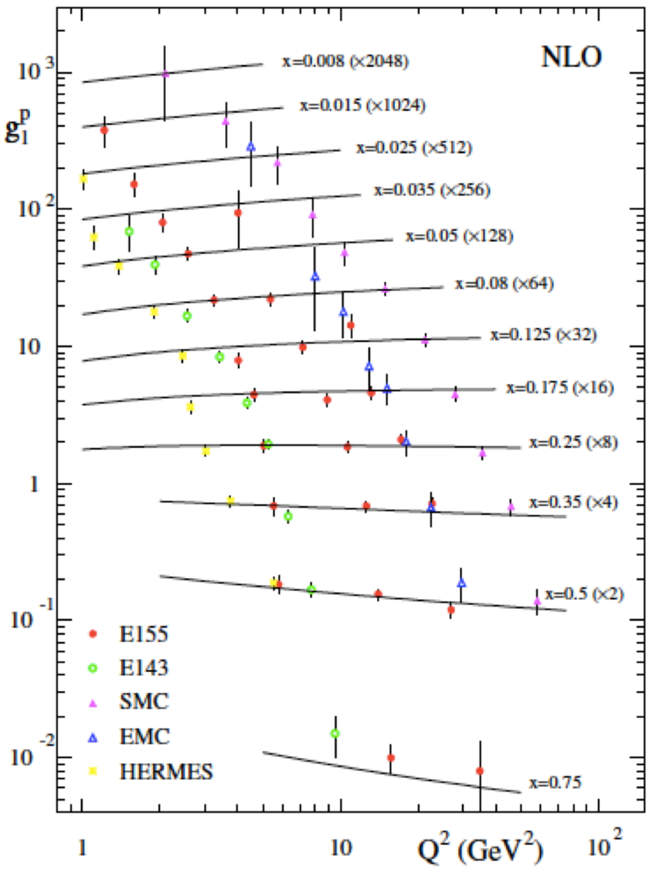
Polarized deep inelastic scattering

❑ Spin asymmetries – measured experimentally:

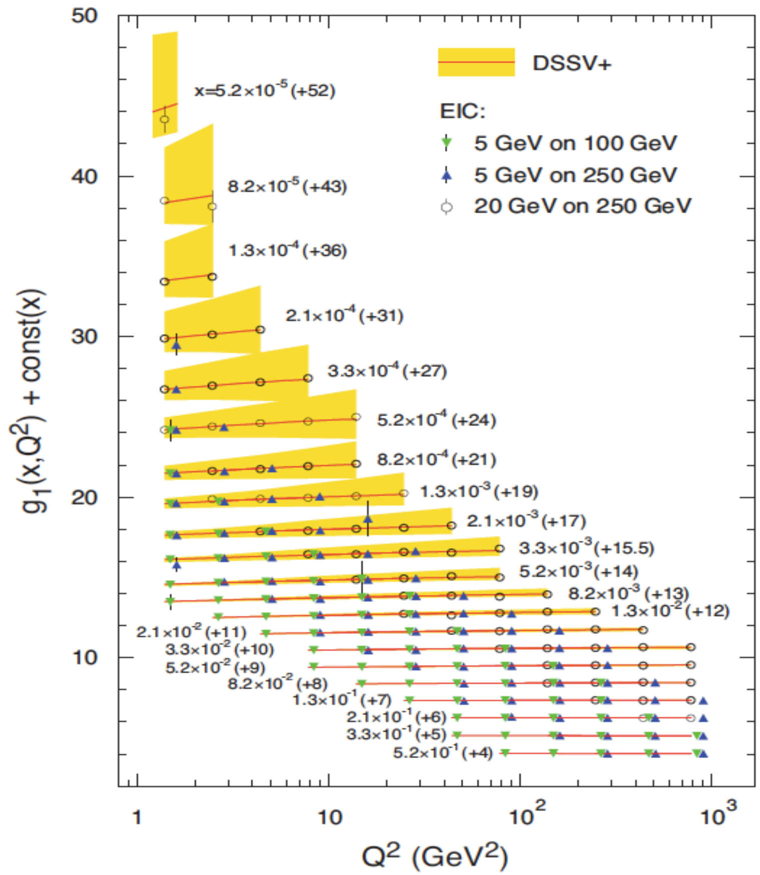
✧ **Longitudinal polarization –** $\alpha = 0$

Known function

$$A_{\parallel} = \frac{d\sigma^{(\rightarrow\Leftarrow)} - d\sigma^{(\rightarrow\Rightarrow)}}{d\sigma^{(\rightarrow\Leftarrow)} + d\sigma^{(\rightarrow\Rightarrow)}} = D(y)\,\frac{g_1(x,Q^2)}{F_1(x,Q^2)} \equiv D(y)\,A_1(x,Q^2)$$

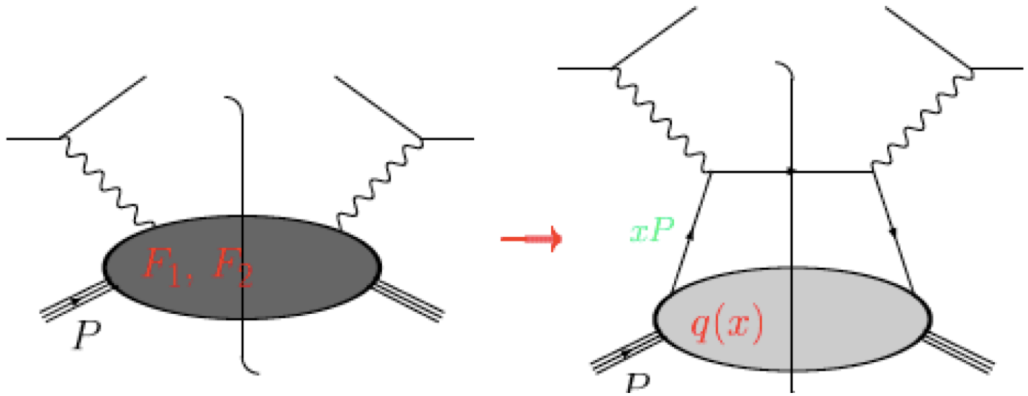


Polarized DIS at EIC



Polarized deep inelastic scattering

❑ Parton model results – LO QCD:



✧ **Structure functions:**

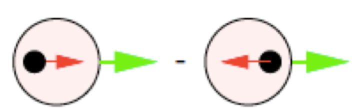
$$F_1(x) = \frac{1}{2}\sum_q e_q^2 \left[q(x) + \bar{q}(x) \right]$$

$$g_1(x) = \frac{1}{2}\sum e_q^2 \left[\Delta q(x) + \Delta\bar{q}(x) \right]$$

$$g_1 = \frac{1}{2}\left[\frac{4}{9}(\Delta u + \Delta\bar{u}) + \frac{1}{9}(\Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}) \right]$$

✧ **Polarized quark distribution:**

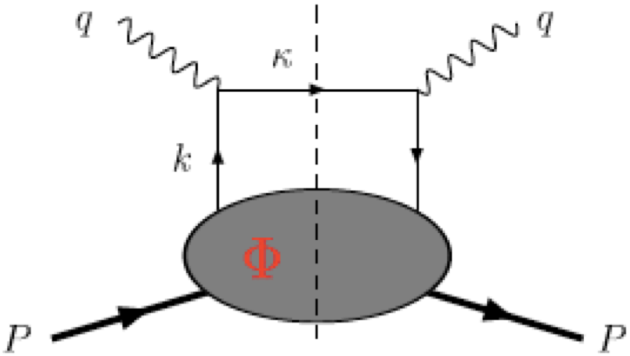
$$\boxed{\Delta f(\xi) \equiv f^{+}(\xi) - f^{-}(\xi)}$$

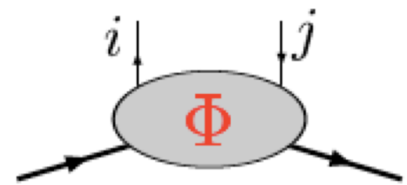


Information on nucleon's spin structure

Polarized deep inelastic scattering

❑ Systematics polarized PDFs – LO QCD:





✧ **Two-quark correlator:**

$$\Phi_{ij}(k,P,S) = \sum_X \int \frac{d^3\mathbf{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4\, \delta^4(P-k-P_X)\, \langle PS|\bar{\psi}_j(0)|X\rangle \langle X|\psi_i(0)|PS\rangle$$

$$= \int d^4z\, e^{ik\cdot z} \langle PS\,|\,\bar{\psi}_j(0)\,\psi_i(z)\,|\,PS\rangle$$

✧ **Hadronic tensor (one –flavor):**

$$\mathcal{W}^{\mu\nu} = e^2 \int \frac{d^4k}{(2\pi)^4}\, \delta\left((k+q)^2\right)\ \mathrm{Tr}\left[\,\Phi\, \gamma^\mu (\not{k}+\not{q})\gamma^\nu\,\right]$$

Polarized deep inelastic scattering

✧ **General expansion of :**

must have general expansion in terms of P, $\not{n}$, $\not{s}$ etc.

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\parallel}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$

✧ **3-leading power quark parton distribution:**

$$q(x) = \frac{1}{4\pi}\int dz^{-}\, \mathrm{e}^{iz^{-}xP^{+}}\, \langle P,S|\bar{\psi}(0)\,\gamma^{+}\,\psi\left(0,z^{-},\mathbf{0}_{\perp}\right)|P,S\rangle$$

$$\Delta q(x) = \frac{1}{4\pi}\int dz^{-}\, \mathrm{e}^{iz^{-}xP^{+}}\, \langle P,S|\bar{\psi}(0)\,\gamma^{+}\,\gamma_5\,\psi\left(0,z^{-},\mathbf{0}_{\perp}\right)|P,S\rangle$$

$$\delta q(x) = \frac{1}{4\pi}\int dz^{-}\, \mathrm{e}^{iz^{-}xP^{+}}\, \langle P,S|\bar{\psi}(0)\,\gamma^{+}\,\gamma_{\perp}\,\gamma_5\,\psi\left(0,z^{-},\mathbf{0}_{\perp}\right)|P,S\rangle$$

"unpolarized" – "longitudinally polarized" – "transversity"

Polarized deep inelastic scattering

❑ Physical interpretation:

$$q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\,\mathcal{P}^+\psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 + \left|\langle X|\,\mathcal{P}^-\psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 \right]$$

$$\Delta q(x) = \frac{1}{2}\sum_X \delta\left(P_X^+ - (1-x)P^+\right) \times \left[\left|\langle X|\,\mathcal{P}^+\psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\,\mathcal{P}^-\psi_+(0)\,|P,\lambda=\frac{1}{2}\rangle\right|^2 \right]$$

$$\delta q(x) = \frac{1}{2}\sum_X \delta(P_X^+ - (1-x)P^+) \times \left[\left|\langle X|\,\mathcal{P}^\uparrow\psi_+(0)\,|P,S_\perp=\frac{1}{2}\rangle\right|^2 - \left|\langle X|\,\mathcal{P}^\downarrow\psi_+(0)\,|P,S_\perp=\frac{1}{2}\rangle\right|^2 \right]$$

Spin projection: $\mathcal{P}^\pm \equiv \frac{1\pm\gamma_5}{2}$ **and** $\mathcal{P}^{\uparrow\downarrow} \equiv \frac{1\pm\gamma_\perp\gamma_5}{2}$

Basics for spin observables

- **Factorized cross section:**

$$\sigma_{h(p)}(Q,s) \propto \langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle$$

$$e.g.\ \mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\hat{\Gamma}\,\psi(y^-) \qquad \text{with } \hat{\Gamma} = I, \gamma_5, \gamma^\mu, \gamma_5\gamma^\mu, \sigma^{\mu\nu}$$

- **Parity and Time-reversal invariance:**

$$\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle$$

- **IF:** $\langle p,-\vec{s}|\mathcal{PT}\mathcal{O}^\dagger(\psi,A^\mu)\mathcal{T}^{-1}\mathcal{P}^{-1}|p,-\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

  or $\langle p,\vec{s}|\mathcal{O}(\psi,A^\mu)|p,\vec{s}\rangle = \pm\langle p,-\vec{s}|\mathcal{O}(\psi,A^\mu)|p,-\vec{s}\rangle$

  Operators lead to the "+" sign ⟹ **spin-averaged cross sections**

  Operators lead to the "-" sign ⟹ **spin asymmetries**

- **Example:**

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\,\psi(y^-) \ \Rightarrow\ q(x)$$

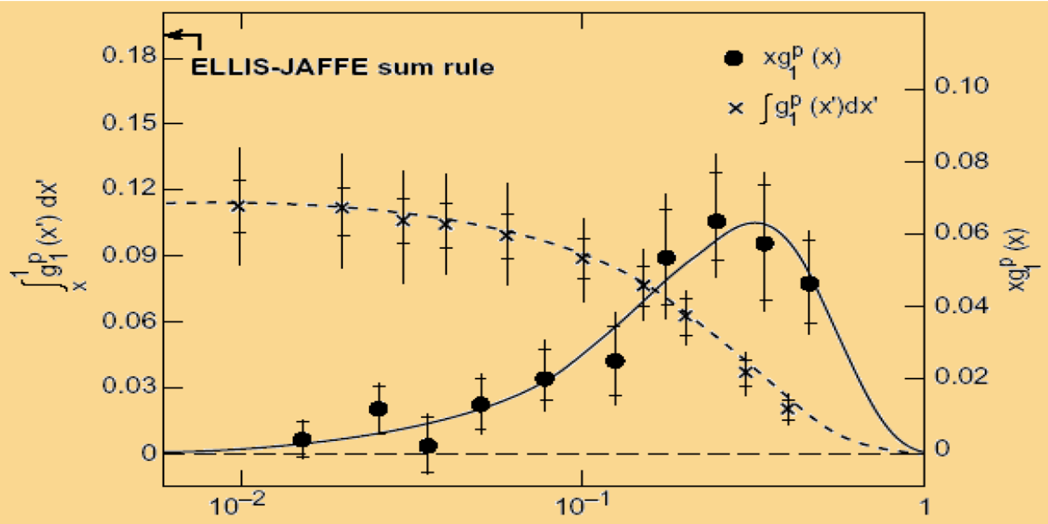
$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma_5\,\psi(y^-) \ \Rightarrow\ \Delta q(x)$$

$$\mathcal{O}(\psi,A^\mu) = \overline{\psi}(0)\,\gamma^+\gamma^\perp\gamma_5\,\psi(y^-) \ \Rightarrow\ \delta q(x) \to h(x)$$

$$\mathcal{O}(\psi,A^\mu) = \frac{1}{xp^+}F^{+\alpha}(0)[-i\varepsilon_{\alpha\beta}]F^{+\beta}(y^-) \Rightarrow\ \Delta g(x)$$

Proton "spin crisis" – excited the field

❑ EMC (European Muon Collaboration '87) – "the Plot":



$$g_1(x) = \frac{1}{2}\sum_q e_q^2\left[\Delta q(x) + \Delta\bar{q}(x)\right] + \mathcal{O}(\alpha_s) + \mathcal{O}\left(1/Q\right)$$

✧ **Combined with earlier SLAC data:** $$\int_0^1 g_1^p(x)dx = 0.126 \pm 0.018$$

✧ **Combined with:** $g_A^3 = \Delta u - \Delta d$ **and** $g_A^8 = \Delta u + \Delta d - 2\Delta s$

from low energy neutron & hyperon β decay

$$\Delta\Sigma = \sum_q \left[\Delta q + \Delta\bar{q}\right] = 0.12 \pm 0.17$$

❑ "Spin crisis" or puzzle:

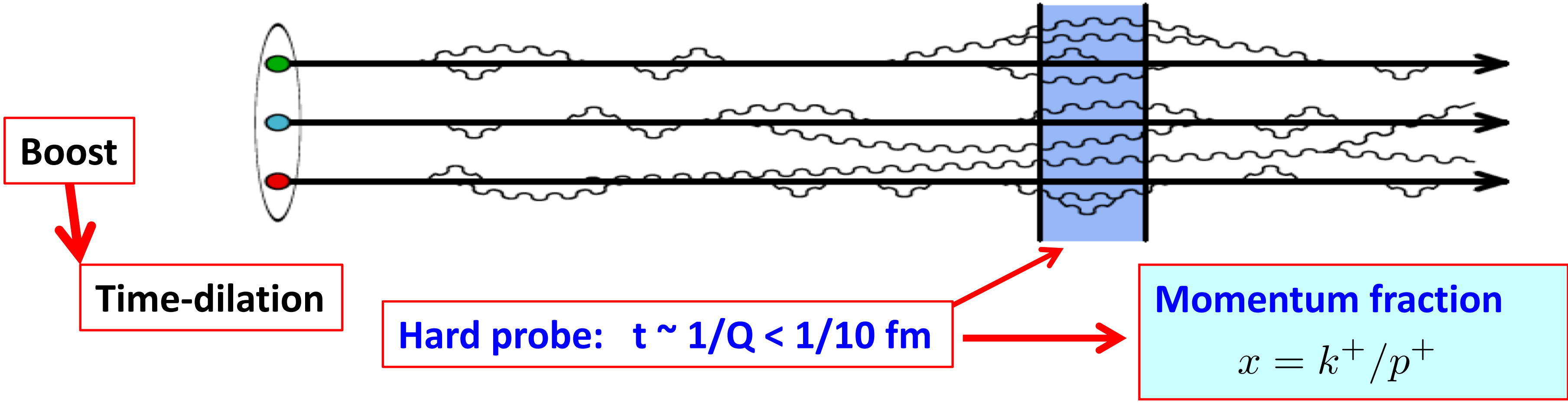
✧ **Strange sea polarization is sizable & negative**

✧ **Very little of the proton spin is carried by quarks**

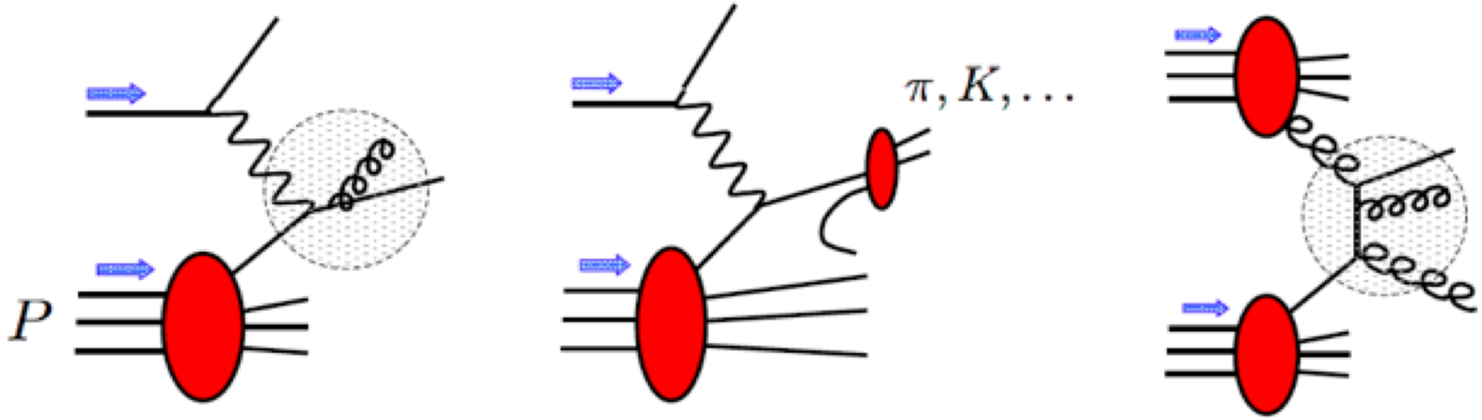
New era of spin physics

Probes and facilities

- **High energy scattering – to see quarks and gluons:**



- **Spin Probes:**



DIS **SIDIS**

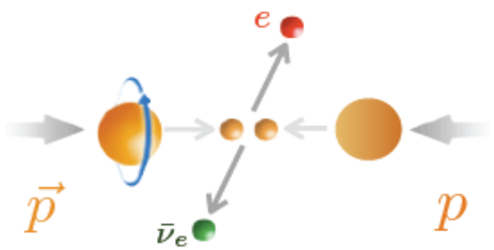
HEMES, COMPASS, JLab, Future EIC, …

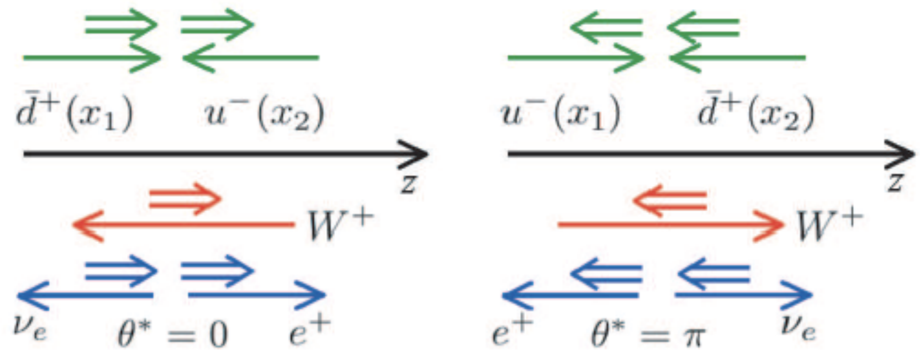
Hadron-hadron

RHIC, FermiLab, JPAC, …

Determination of Δq and $\Delta\bar{q}$

- **W's are left-handed:**





- **Flavor separation:**

Lowest order:

$$A_L^{W^+} = -\frac{\Delta u(x_1)\bar{d}(x_2) - \Delta\bar{d}(x_1)u(x_2)}{u(x_1)\bar{d}(x_2) + \bar{d}(x_1)u(x_2)}$$

$$x_1 = \frac{M_W}{\sqrt{s}}e^{y_W}, \quad x_2 = \frac{M_W}{\sqrt{s}}e^{-y_W}$$

Forward W⁺ (backward e⁺):

$$A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$$

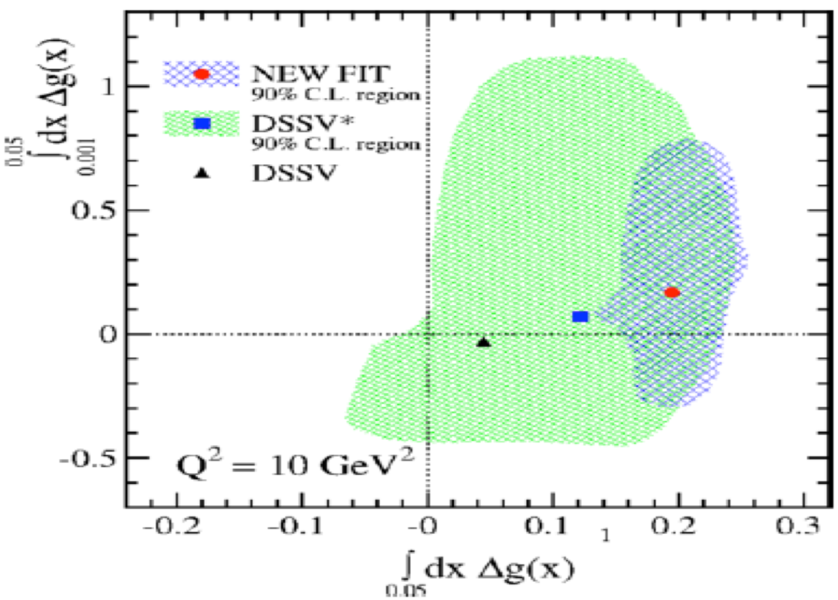
Backward W⁺ (forward e⁺):

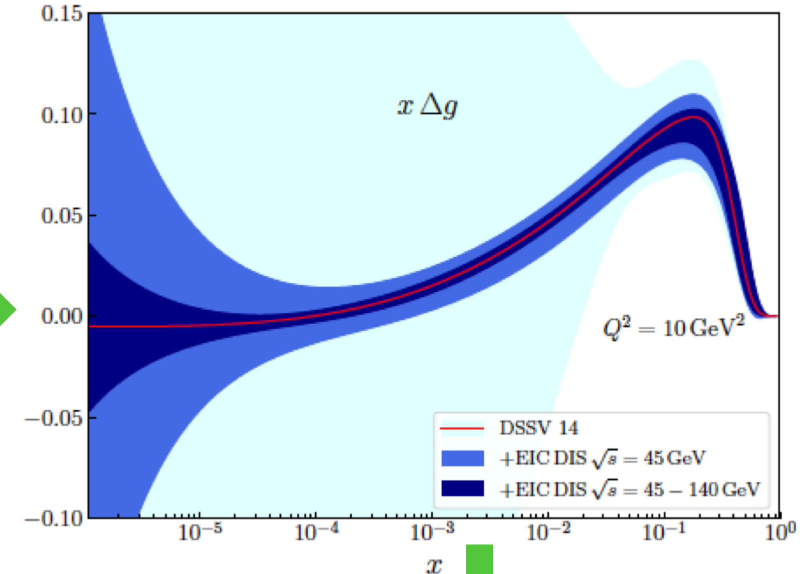
$$A_L^{W^+} \approx -\frac{\Delta\bar{d}(x_2)}{\bar{d}(x_2)} < 0$$

- **Complications:**

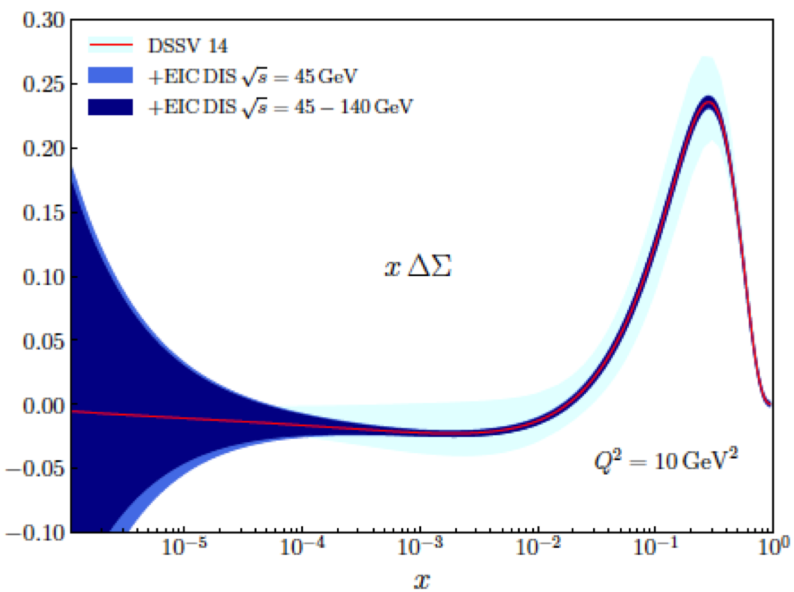
High order, W's p_T-distribution at low p_T

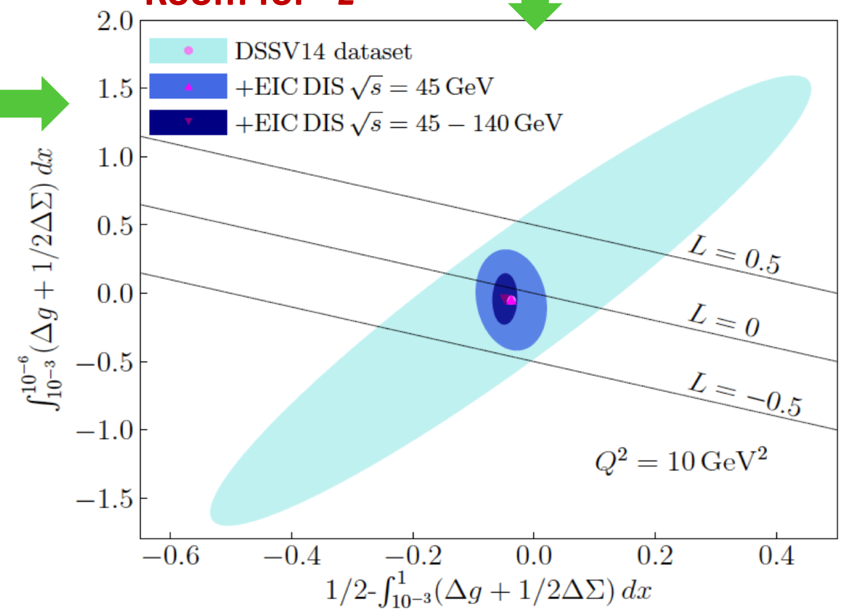
What the EIC can do – EIC Yellow Report?





Room for *"L"*

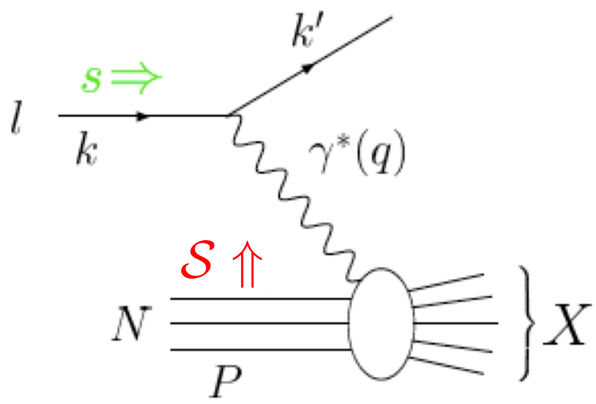




Transverse spin phenomena in QCD

- 40 years ago, Profs. Christ and Lee proposed to use A_N of inclusive DIS to test the Time-Reversal invariance

N. Christ and T.D. Lee, Phys. Rev. 143, 1310 (1966)



Single Transverse-Spin Asymmetry (SSA)

$$A(\ell,\vec{s}) \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

They predicted:

In the approximation of one-photon exchange, A_N of inclusive DIS vanishes if Time-Reversal is invariant for EM and Strong interactions

A_N for inclusive DIS

- **DIS cross section:** $$\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, W_{\mu\nu}(\vec{s}_\perp)$$
- **Leptionic tensor is symmetric:** $L^{\mu\nu} = L^{\nu\mu}$
- **Hadronic tensor:** $$W_{\mu\nu}(\vec{s}_\perp) \propto \langle P, \vec{s}_\perp |\, j_\mu^\dagger(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle$$
- **Polarized cross section:**

$$\Delta\sigma(\vec{s}_\perp) \propto L^{\mu\nu}\, \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp)\right]$$

- **Vanishing single spin asymmetry:**

$$\boxed{\begin{aligned} A_N = 0 \quad \Longleftrightarrow \quad & \langle P, \vec{s}_\perp |\, j_\mu^\dagger(0)\, j_\nu(y)\, | P, \vec{s}_\perp \rangle \\ & \stackrel{?}{=} \langle P, -\vec{s}_\perp |\, j_\nu^\dagger(0)\, j_\mu(y)\, | P, -\vec{s}_\perp \rangle \end{aligned}}$$

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A_N for inclusive DIS

- **Define two quantum states:**

$$\langle\beta| \equiv \langle P,\vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y) \qquad\qquad |\alpha\rangle \equiv |P,\vec{s}_\perp\rangle$$

- **Time-reversed states:**

$$|\alpha_T\rangle = V_T\,|P,\vec{s}_\perp\rangle = |-P,-\vec{s}_\perp\rangle$$

$$\begin{aligned}|\beta_T\rangle &= V_T\left[j_\mu^\dagger(0)\, j_\nu(y)\right]^\dagger|P,\vec{s}_\perp\rangle \\ &= \left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P,-\vec{s}_\perp\rangle\end{aligned}$$

- **Time-reversal invariance:**

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\,V_T^\dagger\, V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

$$\boxed{\begin{aligned}\Longrightarrow\ &\langle -P,-\vec{s}_\perp|\left(V_T j_\nu^\dagger(y) V_T^{-1}\right)\left(V_T j_\mu(0) V_T^{-1}\right)|-P,-\vec{s}_\perp\rangle \\ &= \langle P,\vec{s}_\perp|\, j_\mu^\dagger(0)\, j_\nu(y)\,|P,\vec{s}_\perp\rangle\end{aligned}}$$

A_N for inclusive DIS

- **Parity invariance:**

$$\boxed{1 = U_P^{-1} U_P = U_P^{\dagger} U_P}$$

$$\langle -P, -\vec{s}_\perp | \left(V_T j_\nu^\dagger(y) V_T^{-1}\right) \left(V_T j_\mu(0) V_T^{-1}\right) | -P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \left(U_P V_T j_\nu^\dagger(y) V_T^{-1} U_P^{-1}\right) \left(U_P V_T j_\mu(0) V_T^{-1} U_P^{-1}\right) | P, -\vec{s}_\perp \rangle$$

$$\Downarrow$$

$$\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(-y) \, j_\mu(0) \, | P, -\vec{s}_\perp \rangle$$

Translation invariance:

$$\Downarrow$$

$$\begin{aligned} &\langle P, -\vec{s}_\perp | \, j_\nu^\dagger(0) \, j_\mu(y) \, | P, -\vec{s}_\perp \rangle \\ &= \langle P, \vec{s}_\perp | \, j_\mu^\dagger(0) \, j_\nu(y) \, | P, \vec{s}_\perp \rangle \end{aligned}$$

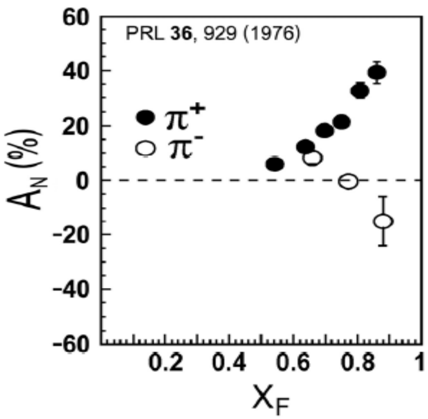
- **Polarized cross section:**

$$\begin{aligned} \Delta\sigma(\vec{s}_\perp) &\propto L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\mu\nu}(-\vec{s}_\perp) \right] \\ &= L^{\mu\nu} \left[W_{\mu\nu}(\vec{s}_\perp) - W_{\nu\mu}(\vec{s}_\perp) \right] = 0 \end{aligned}$$

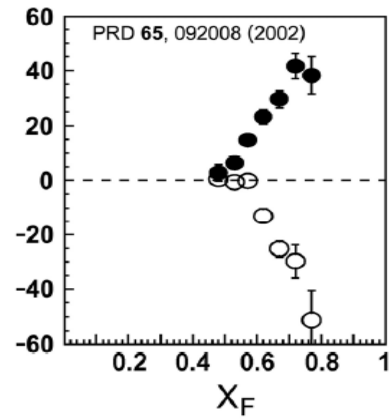
A_N in hadronic collisions

- A_N - consistently observed for over 35 years!

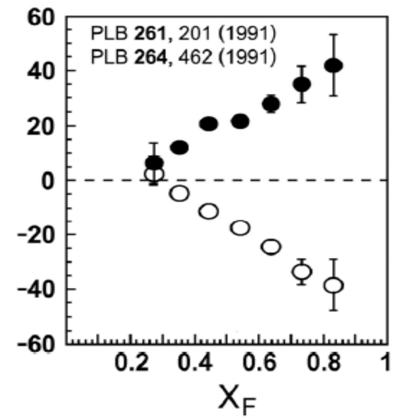
ANL – 4.9 GeV



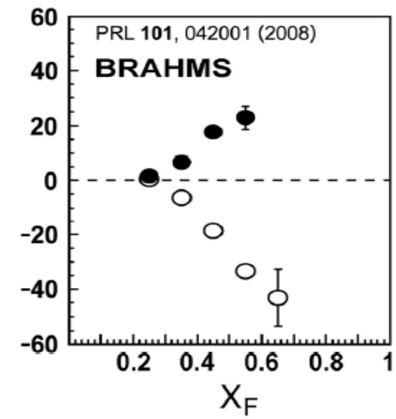
BNL – 6.6 GeV



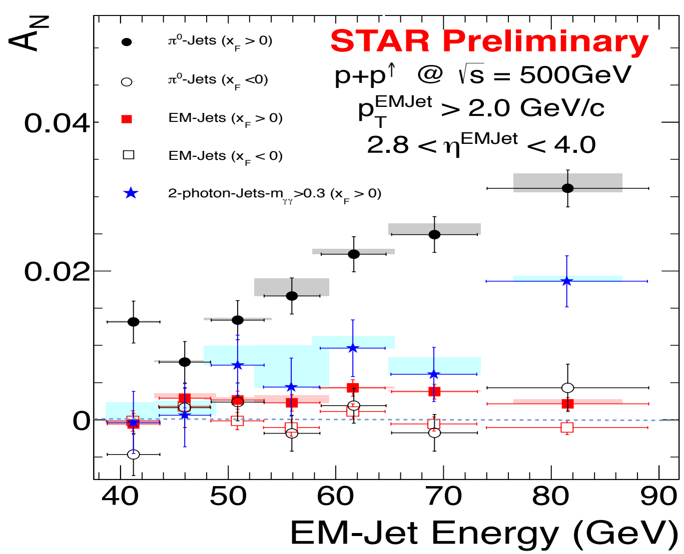
FNAL – 20 GeV

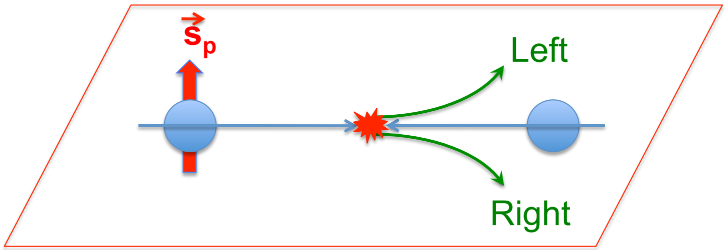


BNL – 62.4 GeV



- Survived the highest RHIC energy:





$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s}) - \sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s}) + \sigma(\ell,-\vec{s})}$$

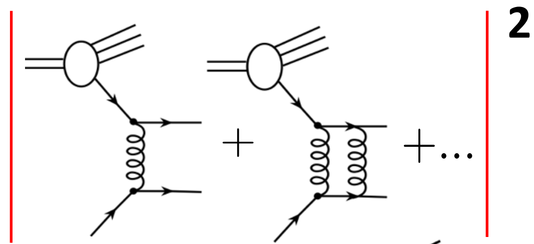
Do we understand this?

A_N in hadronic collisions

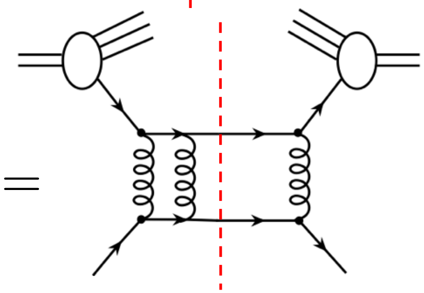
Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T,\vec{s}) \propto$



Asymmetry: $\sigma_{AB}(p_T,\vec{s}) - \sigma_{AB}(p_T,-\vec{s}) =$



$\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

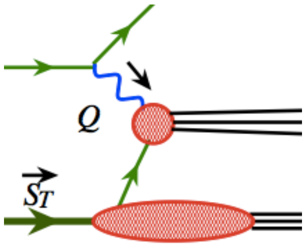
❑ Vanish without parton's transverse motion:

→ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

Current understanding of TSSAs

- **Symmetry plays important role:**

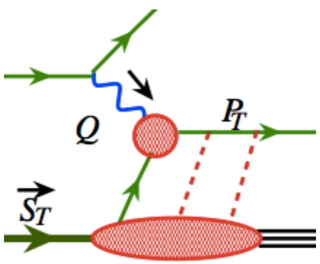


Inclusive DIS
Single scale
Q

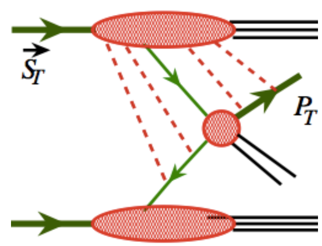
Parity
Time-reversal

→ $A_N = 0$

- **One scale observables – $Q \gg \Lambda_{QCD}$:**



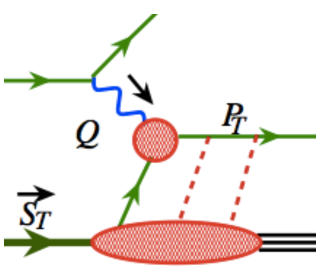
SIDIS: $Q \sim P_T$



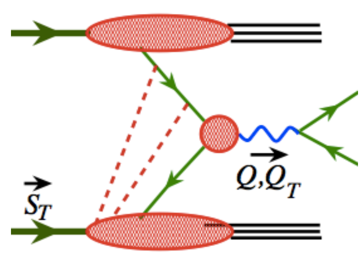
DY: $Q \sim P_T$; Jet, Particle: P_T

→ **Collinear factorization**
Twist-3 distributions

- **Two scales observables – $Q_1 \gg Q_2 \sim \Lambda_{QCD}$:**



SIDIS: $Q \gg P_T$



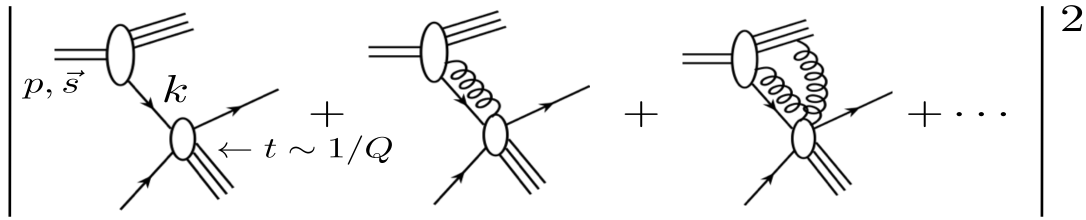
DY: $Q \gg P_T$ or $Q \ll P_T$

→ **TMD factorization**
TMD distributions

How collinear factorization generates TSSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto$



$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ **– Expansion**

$$\sigma(Q,s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

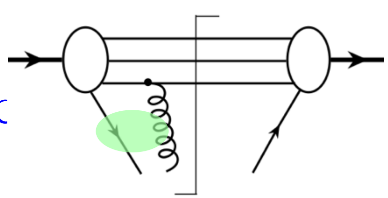
Too large to compete!

Three-parton correlation

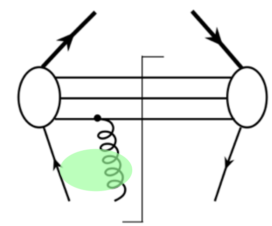
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$ 

Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: **Integration of "dx" using unpinched poles**

Jefferson Lab

Twist-3 distributions relevant to A_N

❑ Twist-2 distributions:

- **Unpolarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

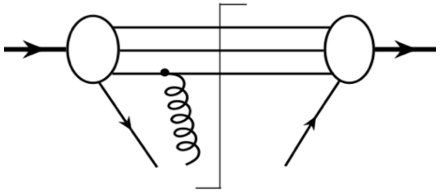
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

- **Polarized PDFs:**

$$\Delta q(x) \propto \langle P,S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P,S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P,S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

❑ Two-sets Twist-3 correlation functions:



No probability interpretation!

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma{}^+(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

❑ Twist-3 fragmentation functions:

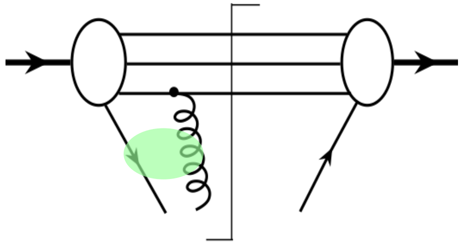
See Kang, Yuan, Zhou, 2010, Kang 2010

"Interpretation" of twist-3 correlation functions

Qiu, Sterman, 1991, …

❑ Measurement of direct QCD quantum interference:

$$T^{(3)}(x, x, S_\perp) \ \propto$$



Interference between a single active parton state and an active two-parton composite state

❑ "Expectation value" of QCD operators:

$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \ \longrightarrow \ \langle P, s|\overline{\psi}(0)\gamma^+ \left[\epsilon_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- \, F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

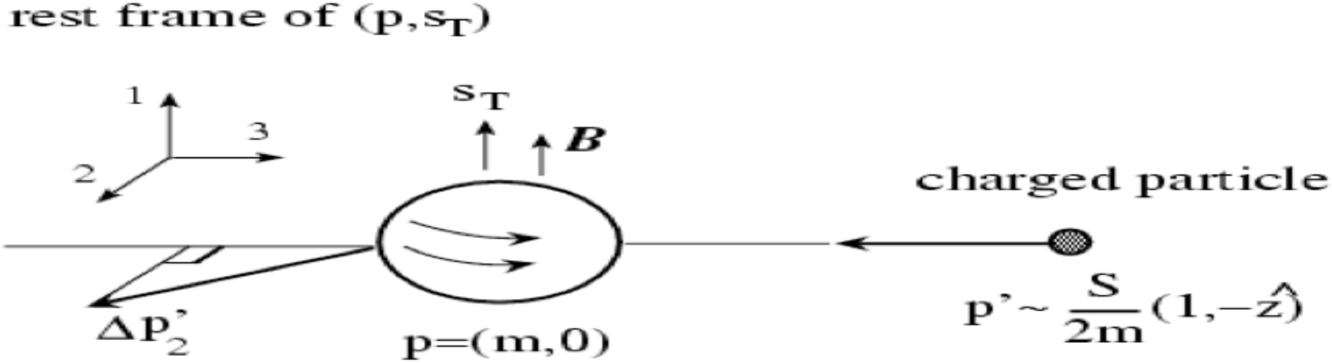
$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \ \longrightarrow \ \langle P, s|\overline{\psi}(0)\gamma^+ \left[i\, g_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- \, F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

Qiu, Sterman, 1998

- The operator in Red – a classical Abelian case:



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

- In the c.m. frame:

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

- The total change:

$$\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Jefferson Lab

Test QCD at twist-3 level

Kang, Qiu, 2009

- Scaling violation – "DGLAP" evolution:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \underbrace{\otimes}_{\int d\xi \int d\xi_2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

- Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

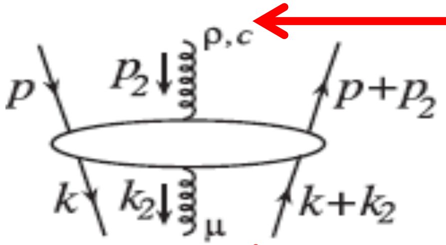
DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

Evolution kernels – an example

Kang, Qiu, 2009

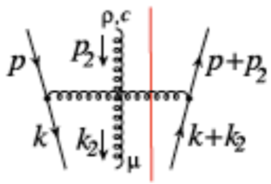
❑ Quark to quark:



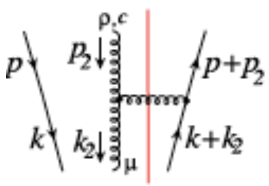
$$\mathcal{P}^{(\mathrm{LC})}_{q,F} = \frac{1}{2}\gamma\cdot P\left(\frac{-1}{\xi_2}\right)(i\epsilon^{s_T\rho n\bar{n}})\tilde{C}_q$$

$$\mathcal{V}^{\mathrm{LC}}_{q,F} = \frac{\gamma^+}{2P^+}\delta\left(x-\frac{k^+}{P^+}\right)x_2\delta\left(x_2-\frac{k_2^+}{P^+}\right)(i\epsilon^{s_T\sigma n\bar{n}}[-g_{\sigma\mu}]C_q$$

❑ Feynman diagram calculation:

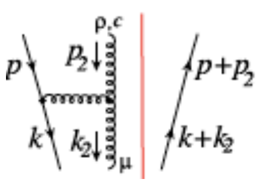


$$\int d\xi \int d\xi_2\, \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\;\delta(\xi_2)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[C_F-\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1+z^2}{1-z}\right)$$



$$\int d\xi \int d\xi_2\, \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\;\delta(\xi-x)\frac{1}{\xi_2}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\,\frac{2x+\xi_2}{x+\xi_2}\right)$$

$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\,\mathcal{T}_{q,F}(x,x)$$



$$\int d\xi \int d\xi_2\, \mathcal{T}_{q,F}(\xi,\xi+\xi_2)\;\delta(\xi+\xi_2-x)\frac{1}{\xi}\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\left(\frac{1}{2}\,\frac{1+z}{1-z}\right)$$

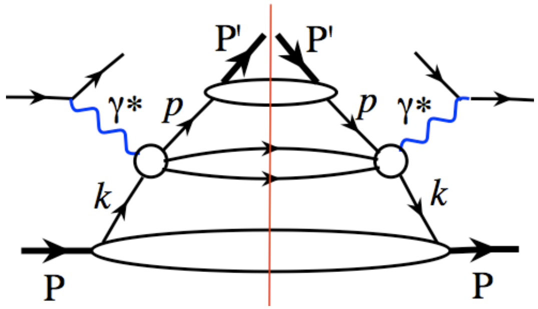
$$-\int^{\mu_F^2}\frac{dk_T^2}{k_T^2}\left[\frac{C_A}{2}\right]\frac{\alpha_s}{2\pi}\,\mathcal{T}_{q,F}(x,x)$$

+ Virtual loop diagrams

How TMD factorization generates TSSA?

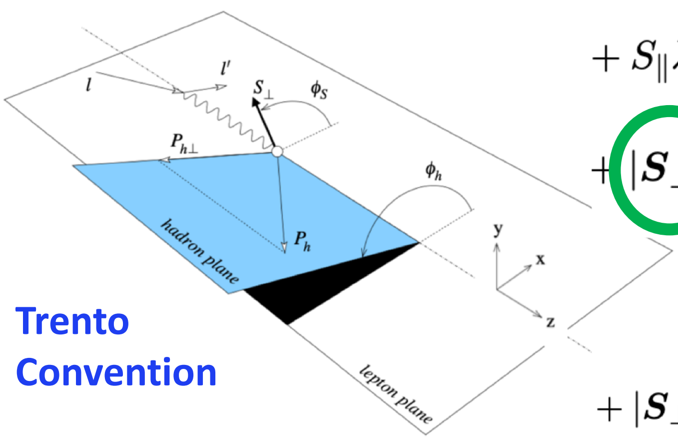
❑ SIDIS – "one-photon approximation":

- 18 Structure functions
- TTSA = at least one of 6 F_{UT} structure functions is finite!



$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+\,S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\,\varepsilon\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$
$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

A. Bacchetta, et al.



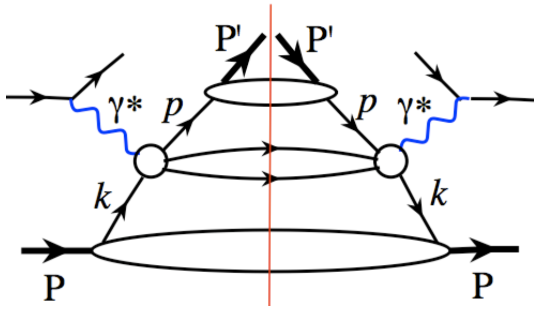
Trento Convention

$$\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

Jefferson Lab

How TMD factorization generates TSSA?

❑ TMD factorization for SIDIS:



In the photon-hadron frame, all 18 structure functions can be factorized in terms of convolution of TMDs

- **Unpolarized**

$$F_{UU,T} = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) f^a(x, p_T^2)\, D^a(z, k_T^2)$$

- **Transverse Single-Spin Asymmetry – Sivers:**

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right] \qquad \hat{\boldsymbol{h}} = \frac{\boldsymbol{P}_{h\perp}}{|\boldsymbol{P}_{h\perp}|}$$

- **Transverse Single-Spin Asymmetry – Collins:**

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

With:

$$\mathcal{C}[w\, f\, D] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

Orbital angular momentum

OAM: Correlation between parton's position and its motion – in an averaged (or probability) sense

- **Jaffe-Manohar's quark OAM density:**

$$\mathcal{L}_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{\partial})\right]^3 \psi_q$$

- **Ji's quark OAM density:**

$$L_q^3 = \psi_q^\dagger \left[\vec{x} \times (-i\vec{D})\right]^3 \psi_q$$

- **Difference between them:**

Hatta, Yoshida, Burkardt, Meissner, Metz, Schlegel, …

✧ generated by a "torque" of color Lorentz force

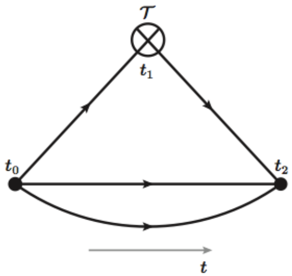
$$\mathcal{L}_q^3 - L_q^3 \propto \int \frac{dy^- d^2 y_T}{(2\pi)^3} \langle P'|\overline{\psi}_q(0)\frac{\gamma^+}{2}\int_{y^-}^{\infty} dz^- \Phi(0,z^-) \times \underbrace{\sum_{i,j=1,2} \left[\epsilon^{3ij} y_T^i F^{+j}(z^-)\right]}_{\text{“Chromodynamic torque”}} \Phi(z^-,y)\psi(y)|P\rangle_{y^+=0}$$

Similar color Lorentz force generates the single transverse-spin asymmetry (Qiu-Sterman function), and is also responsible for the twist-3 part of g_2

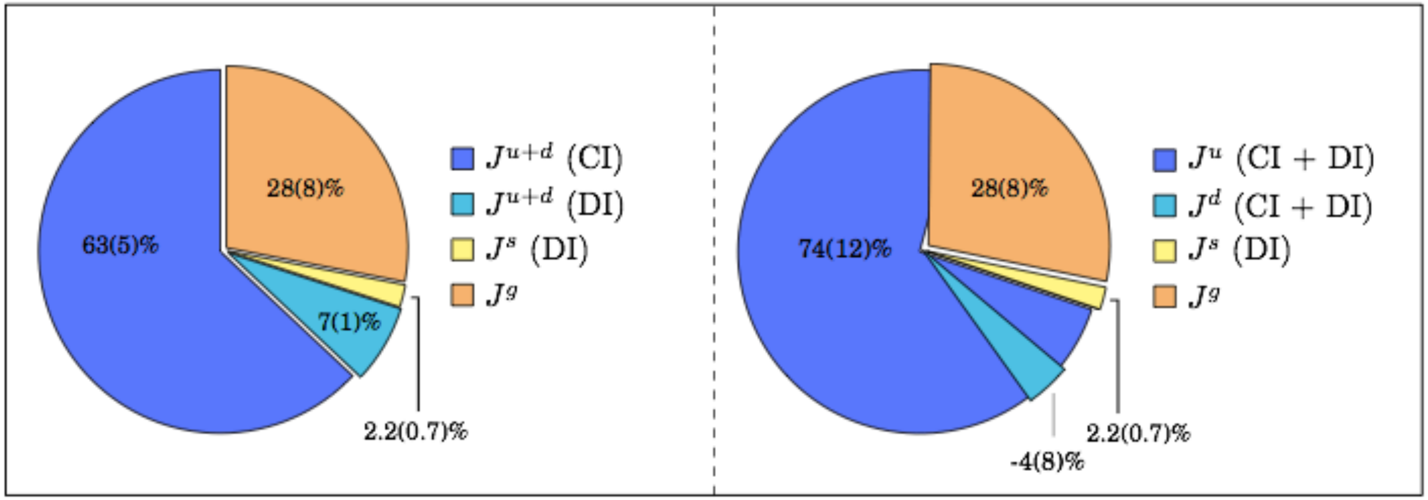
Nucleon spin and OAM from lattice QCD

❑ QCD Collaboration:

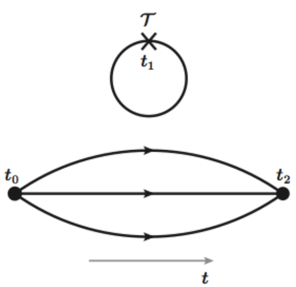
[Deka *et al.* arXiv:1312.4816]



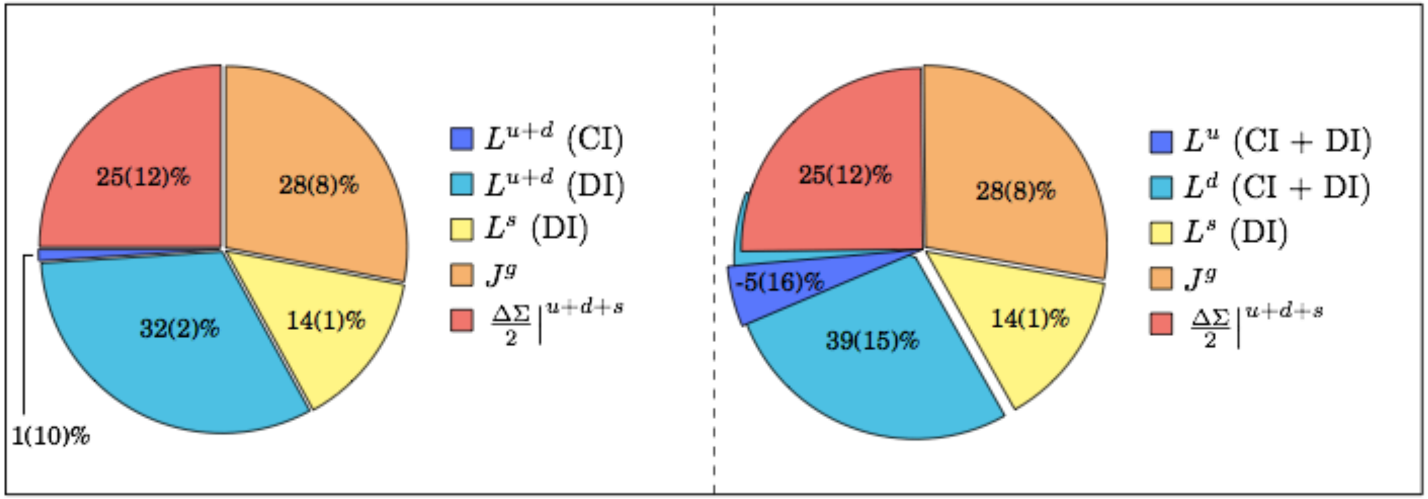
Connected Interaction (CI)



(b)



Disconnected Interaction (DI)



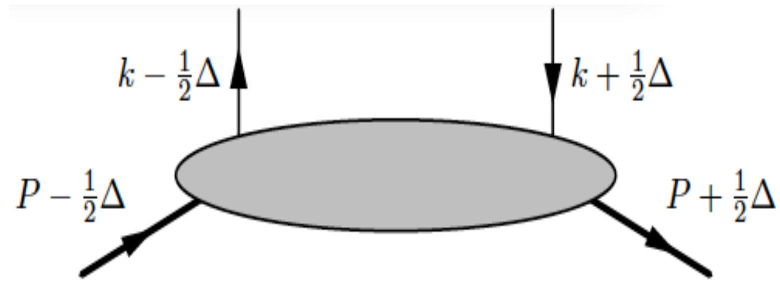
(c)

Partonic motion seen by a hard probe – GTMD

- **Fully unintegrated distribution:** Meissner, Metz, Schiegel, 2009

$$W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) = \frac{1}{2}\int \frac{d^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\Gamma\,\mathcal{W}(-\tfrac{1}{2}z,\tfrac{1}{2}z\,|\,n)\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle$$

– not factorizable in general



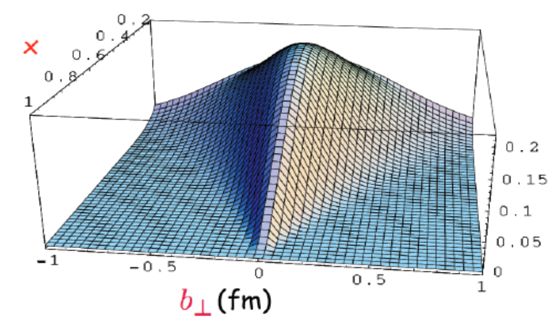
- **Generalized TMDs – hard probe:**

$$\mathcal{W}(x,k_T,\Delta)_\Gamma = \int dk^2\, W(P,k,\Delta)_\Gamma$$

– could be factorized assuming on-shell parton for the hard probe

- **Wigner function:** Belitsky, Ji, Yuan

$$W(x,k_T,b) \propto \int d^3\Delta\, e^{i\vec{b}\cdot\vec{\Delta}}\, \mathcal{W}(x,k_T,\Delta)_{\Gamma=\gamma^+}$$



Burkardt, 2002

- **Connection to all other known distributions:**

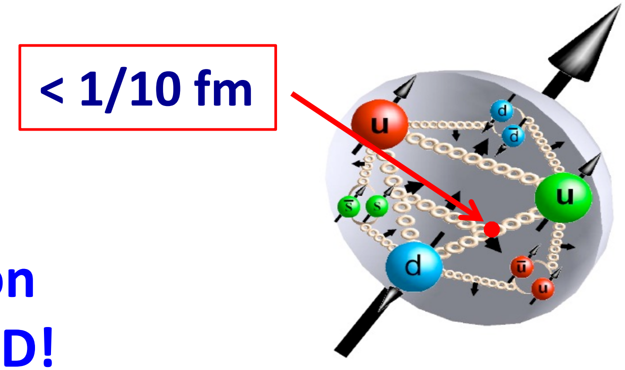
$W(x,k_T,b) \;\Rightarrow\;$ Tomographic image of nucleon

$$q(x,b_\perp) = \int d^2k_T\, db^-\, W(x,k_T,b)_{\gamma^+}$$

$\mathcal{W}(x,k_T,\Delta)_\Gamma \;\Rightarrow\;$ **TMDs** $(\Delta=0)$, **GPDs** $(\int d^2k_T)$, **PDFs** $(\Delta=0, \int d^2k_T)$

Summary and outlook

- QCD has been extremely successful in interpreting and predicting high energy experimental data!



- But, we still do not know much about hadron structure – The emerging phenomena of QCD!

- Nuclear Femtography – QCD at a Fermi scale requires two-scale probes. Major advance in both measurement and factorization of two-scale observables!

- Lepton-Hadron facility, such as EIC, is ideal for two-scale observables

- TMDs and GPDs, accessible by high energy scattering with polarized beams, encode important information on hadron's 3D structure – distributions as well as motions of quarks and gluons

Thank you!

Basic fundamentals about spin

Some fundamentals about spin

❑ Spin in non-relativistic quantum mechanics:

✧ **Spin as an intrinsic angular momentum of the particle**

– **three spin vector:**

$$\vec{\mathcal{S}} = (\mathcal{S}_x, \mathcal{S}_y, \mathcal{S}_z)$$

– **angular momentum algebra:**

$$[\mathcal{S}_i, \mathcal{S}_j] = i\epsilon_{ijk}\mathcal{S}_k \qquad \epsilon_{123} = +1$$

$$\Longrightarrow \quad \left[\vec{\mathcal{S}}^2, \mathcal{S}_j\right] = 0$$

✧ $\vec{\mathcal{S}}^2, \mathcal{S}_z$ **have set of simultaneous eigenvectors:** $|S, m\rangle$

$$\vec{\mathcal{S}}^2 |S, m\rangle = S(S+1)\hbar^2 |S, m\rangle \qquad S = 0, \tfrac{1}{2}, 1, \tfrac{3}{2}, \ldots$$

$$\mathcal{S}_z |S, m\rangle = m\hbar |S, m\rangle \qquad -S \le m \le S$$

✧ **Spin d.o.f. are decoupled from kinematic d.o.f.**

$$\Psi_{\mathsf{Schr}}(\vec{r}) \longrightarrow \Psi_{\mathsf{Schr}}(\vec{r}) \times \chi_m$$

where χ_m is a (2S+1) – component "spinor"

Some fundamentals about spin

❑ Spin–1/2:

- ✧ **Two component spinors:**

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}$$

- ✧ **Operators could be represented by Pauli-matrices:**

$$\mathcal{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathcal{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- ✧ **Eigenstates to** $\vec{\mathcal{S}}^{\,2}$ **d :** $\mathcal{S}_z$

$$\chi_z^{\uparrow} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_z^{\downarrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- ✧ **Eigenvalues:**

$$\mathcal{S}_z\,\chi_z^{\uparrow} = +\frac{1}{2}\chi_z^{\uparrow} \qquad \mathcal{S}_z\,\chi_z^{\downarrow} = -\frac{1}{2}\chi_z^{\downarrow}$$

Particles in these states are "polarized in z-direction"

Some fundamentals about spin

❑ **General superposition:**

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\,\chi_z^{\uparrow} + b\,\chi_z^{\downarrow} \qquad (\chi^{\dagger}\chi = 1)$$

$$\langle \mathcal{S}_z \rangle = \chi^{\dagger}\mathcal{S}_z\chi = \left(+\frac{1}{2}\right)|a|^2 + \left(-\frac{1}{2}\right)|b|^2 = \frac{1}{2}\left[|a|^2 - |b|^2\right]$$

✧ **Example:** $a = b = 1/\sqrt{2}$ $\quad \chi = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \langle \mathcal{S}_z \rangle = 0$

✧ **Notice:** $\langle \mathcal{S}_x \rangle = \chi^{\dagger}\mathcal{S}_x\chi = +\frac{1}{2}$

✧ **Eigenstate to** $\mathcal{S}_x$ $\quad \chi_x^{\uparrow} = \frac{1}{\sqrt{2}}\left[\chi_z^{\uparrow} + \chi_z^{\downarrow}\right]$

✧ **Arbitrary direction** $\vec{n}$ **with** $|\vec{n}| = 1$

$$\mathcal{S}_n = \vec{n}\cdot\vec{\mathcal{S}} = n_x\mathcal{S}_x + n_y\mathcal{S}_y + n_z\mathcal{S}_z = \frac{1}{2}\begin{pmatrix} n_z & n_x - in_y \\ n_x + in_y & -n_z \end{pmatrix}$$

A state that is an eigenstate to this operator: "polarized in $\vec{n}$ ***- direction"***

$\vec{n}$ **= Polarization vector** **Eigenvalues =** $\pm 1/2$

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Some fundamentals about spin

❑ Spin in the relativistic theory:

Physics is invariant under Lorentz transformation:
boost, rotations, and translations in space and time

- **Poincare group – 10 generators:** $\mathcal{P}^{\mu}\,, \quad \mathcal{M}^{\mu\nu}$
- **Pure rotations:** $J_i = -\frac{1}{2}\epsilon_{ijk}\,\mathcal{M}^{jk}$ **re boosts:** $\mathcal{K}_i = \mathcal{M}^{i0}$

  Total angular momentum: $[\,J_i\,, J_j\,] = i\,\epsilon_{ijk}\,J_k$
- **Two group invariants (fundamental observables):**

$$\mathcal{P}_{\mu}\,\mathcal{P}^{\mu} = \mathcal{P}^2 = m^2$$

$\mathcal{W}_{\mu}\,\mathcal{W}^{\mu}$ where $\mathcal{W}_{\mu} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\,\mathcal{M}^{\nu\rho}\,\mathcal{P}^{\sigma}$ Pauli-Lubanski

- **Fact:** $[\,\mathcal{W}_{\mu}\,, \mathcal{W}_{\nu}\,] = i\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{W}^{\rho}\,\mathcal{P}^{\sigma}$ ➡ $[\,\mathcal{W}^{i}\,, \mathcal{W}^{j}\,] = i\,m\,\epsilon_{ijk}\,\mathcal{W}^{k}$

  If acting on states at the rest
- **Spin:** $\mathcal{S}_i \equiv \frac{1}{m}\mathcal{W}^{i} = J_i$

  Note: $\mathcal{W}_{\mu}\,\mathcal{W}^{\mu}$ **has eigenvalues** $m^2\,S\,(S+1)$

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Some fundamentals about spin

- **Recall: constructed eigenstates to $\vec{S}^2$ and $\vec{n}\cdot\vec{S}$:**

$$\mathcal{W}_\mu \mathcal{W}^\mu \, |p,\, S\rangle \;=\; m^2 S(S+1)\, |p,\, S\rangle \qquad S = \tfrac{1}{2}$$

$$-\frac{W\cdot n}{m}\, |p,\, S\rangle \;=\; \pm\frac{1}{2}\, |p,\, S\rangle \qquad W^\mu = \mathcal{W}^\mu|_{\text{at rest}}$$

- **"Polarization operator":**

$$\mathcal{P} \equiv -\frac{W\cdot n}{m}$$

- **"Covariant polarization vector":** n^μ **with** $n^2 = -1, \quad n\cdot p = 0$

- **For Dirac particles:**

$$\mathcal{P} = \frac{1}{2}\gamma_5\, \gamma_\mu n^\mu$$

➡ **Projection operators to project out the eigenstates of :**

$$\tfrac{1}{2}\left(\mathbb{1} \pm \gamma_5 \not{n}\right)$$

- **Longitudinal polarization:** $\vec{n} = \vec{p}/|\vec{p}|\,, \quad n^0 = 0$

➡ $\mathcal{P} = \frac{1}{2}\gamma_5\gamma_\mu n^\mu = \frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}$ **with eigenvalues** $\pm\frac{1}{2}$

$$\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|}\, u_\pm(p) \;=\; \pm\frac{1}{2}u_\pm(p) \;\equiv\; \frac{\lambda}{2}u_\pm(p) \qquad \rightsquigarrow \;\lambda \text{ “helicity”}$$

Massless particle: $\frac{\vec{J}\cdot\vec{p}}{|\vec{p}|} \to \gamma_5$ $\qquad$ helicity = chirality

Some fundamentals about spin

✧ **Transverse polarization:** $n^\mu = (0, \vec{n}_\perp, 0)$ (for $\vec{p}$ in z direction)

➡ $\mathcal{P} = \gamma_0 \vec{J} \cdot \vec{n} = \gamma_0\, J_\perp \neq J_\perp$

✧ **Transversity, not "transverse spin", has the eigenvalue:** $\pm\frac{1}{2}$

$$\gamma_0\, J_\perp\, u_{\uparrow\downarrow}(p) = \pm\frac{1}{2} u_{\uparrow\downarrow}(p)$$

with spinors: $u_\uparrow^{(x)} = \frac{1}{\sqrt{2}}\left[u_+ + u_-\right]$

Same as in non-relativistic theory

➡ **Transverse polarization, or transversity, not "transverse spin", is invariant under the "boosts along $\vec{p}$ "**

✧ **Projection operator with both longitudinal and transverse components:**

$$\frac{1}{2}\not{p}\left[\mathbb{1} - s_\parallel \gamma_5 + \gamma_5 \not{s}_\perp\right]$$ **at high energy**

with $s_\parallel \sim \lambda, \quad s_\perp \sim n_\perp$

Some fundamentals about spin

❑ Back to Spin–1/2:

✧ **A free spin-1/2 particle obeys Dirac equation**

$$(\not{p} - m)\, u(p) = 0 \qquad \text{where} \quad \not{p} = \gamma_\mu p^\mu$$

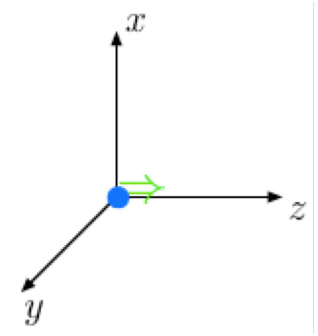
with 4-component solutions:

$$\Psi(x) = \begin{cases} e^{-i\,p\cdot x}\, u(p) & \text{positive energy} \to \text{particle} \\ e^{+i\,p\cdot x}\, v(p) & \text{negative energy} \to \textit{anti}\text{particle} \end{cases}$$

Each with "two" solutions: "spin up/down"

✧ **If it is at rest,**

$$u^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u^- = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$



They are eigenstates to the spin operator : $\mathcal{S}_z$

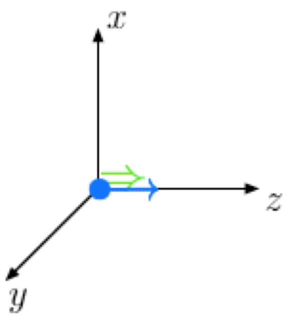
$$\mathcal{S}_z\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

"polarized in z-direction"

Some fundamentals about spin

✧ **Boost the particle to momentum** $p = (E, 0, 0, p_z)$

$$u^+ = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix} \qquad u^- = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix}$$



✧ **Eigenstates of the helicity operator:**

$$\frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

✧ **Also eigenstates of the Pauli-Lubanski (polarization) operator:**

$$\frac{1}{2}\gamma_5 \not{n}\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

where the polarization vector $n = (p_z, 0, 0, E)/m$

✧ **At high energy,** $E \approx p_z$ **also become eigenstates to chirality** γ_5 **:**

$$\gamma_5\, u^\pm = \pm\frac{1}{2}\, u^\pm$$

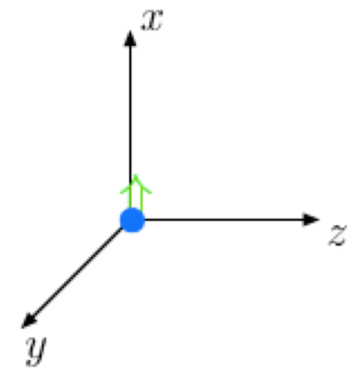
Some fundamentals about spin

❑ Back to rest frame:

✧ **Construct eigenstates to the spin operator** $\mathcal{S}_x$ **:**

$$\mathcal{S}_x\, u^{\uparrow} = +\frac{1}{2}\, u^{\uparrow} \qquad \mathcal{S}_x\, u^{\downarrow} = -\frac{1}{2}\, u^{\downarrow}$$

with $u^{\uparrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} + u^{-}\,\right] \qquad u^{\downarrow} = \frac{1}{\sqrt{2}}\left[\, u^{+} - u^{-}\,\right]$



"polarized along x-direction"

✧ **Boost the particle to momentum** $p = (E,\, 0,\, 0,\, p_z)$

$$u^{\uparrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ \frac{p_z}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix} \qquad u^{\downarrow} = \frac{N}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ \frac{p_z}{E+m} \\ \frac{p_z}{E+m} \end{pmatrix}$$

Still has

$u^{\uparrow} = (u^{+} + u^{-})/\sqrt{2}$

✧ **Still the eigenstates of the Pauli-Lubanski (polarization) operator:**

$$\frac{1}{2}\gamma_5\, \not{n}\; u^{\uparrow\downarrow} = \pm\frac{1}{2}\, u^{\uparrow\downarrow} \qquad \text{where} \quad n = (0,\, 1,\, 0,\, 0)$$

✧ **But, no longer eigenstates of the transverse-spin operator:**

$$\mathcal{S}_x\, u^{\uparrow} \neq +\frac{1}{2}\, u^{\uparrow}$$

Parity and Time-reversal invariance

- In quantum field theory, physical observables are given by matrix elements of quantum field operators

- Consider two quantum states: $|\alpha\rangle \quad |\beta\rangle$

- Parity transformation:

$$|\alpha_P\rangle \equiv U_P\,|\alpha\rangle \qquad\qquad |\beta_P\rangle \equiv U_P\,|\beta\rangle$$

$$\langle\alpha_P|\beta_P\rangle = \langle\alpha|\,U_P^\dagger\,U_P\,|\beta\rangle = \langle\alpha|\beta\rangle$$

- Time-reversal transformation:

$$|\alpha_T\rangle \equiv V_T\,|\alpha\rangle \qquad\qquad |\beta_T\rangle \equiv V_T\,|\beta\rangle$$

$$\langle\alpha_T|\beta_T\rangle = \langle\alpha|\,V_T^\dagger\,V_T\,|\beta\rangle = \langle\alpha|\beta\rangle^* = \langle\beta|\alpha\rangle$$

Parity and Time-reversal invariance

- **Parton fields under P and T transformation:**

$$U_P\,\psi(y_0,\vec{y})\,U_P^{-1} = \gamma^0\,\psi(y_0,-\vec{y})$$

$$V_T\,\psi(y_0,\vec{y})\,V_T^{-1} = (i\gamma^1\gamma^3)\,\psi(-y_0,\vec{y}) \qquad \boxed{\mathcal{J} = i\,\gamma^1\,\gamma^3}$$

$$\Rightarrow \quad \langle P,\vec{s}_\perp|\,\bar{\psi}(0)\,\Gamma_i\,\psi(y^-)\,|P,\vec{s}_\perp\rangle$$

$$= \langle P,-\vec{s}_\perp|\,\bar{\psi}(0)\left[\mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger\right]\psi(y^-)\,|P,-\vec{s}_\perp\rangle$$

- **Quark correlations contribute to polarized X-sections:**

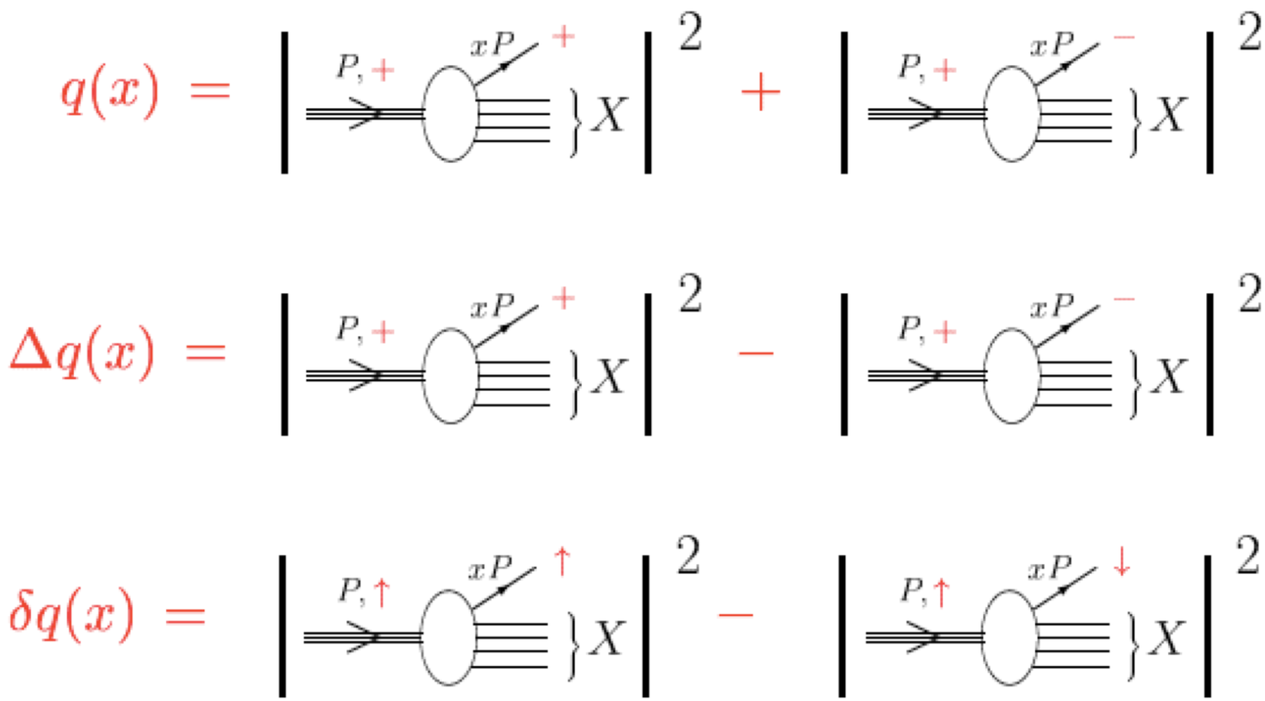
$$T_i(x;\vec{s}_\perp) = -T_i(x;-\vec{s}_\perp) \quad \Rightarrow \quad \mathcal{J}\left(\Gamma_i^\dagger\right)^*\mathcal{J}^\dagger = -\Gamma_i$$

$$\boxed{\Gamma_i = \gamma^\mu\gamma_5,\quad \sigma^{\mu\nu} \quad \text{or} \quad \sigma^{\mu\nu}\,(i\gamma_5)}$$

$$\Gamma_i = I,\quad i\gamma_5,\quad \gamma^\mu$$ **contribute to spin-avg X-sections:**

Polarized deep inelastic scattering

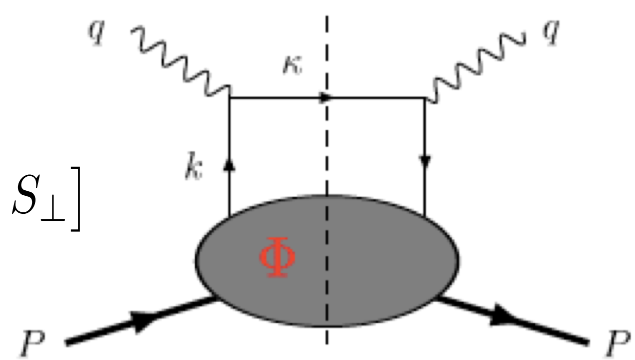
❑ Pictorially:



❑ Note:

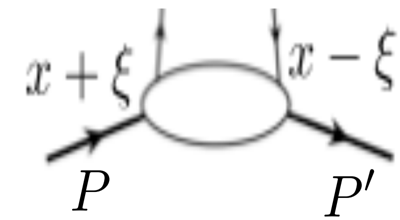
No transversity contribution to inclusive DIS!

$$\phi(x) = \frac{1}{2}\left[q(x)\gamma\cdot P + s_{\parallel}\Delta q(x)\gamma_5\gamma\cdot P + \delta q(x)\gamma\cdot P\gamma_5\gamma\cdot S_{\perp}\right]$$



GPDs – role in solving the spin puzzle

❑ Quark "form factor":



$$
\begin{aligned}
F_q(x,\xi,t,\mu^2) &= \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P'|\bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2)|P\rangle \\
&\equiv \; H_q(x,\xi,t,\mu^2)\,\left[\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P)\right] \frac{n_\mu}{2P\cdot n} \\
&+ \; E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P') \frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M} \mathcal{U}(P)\right] \frac{n_\mu}{2P\cdot n}
\end{aligned}
$$

with $\xi = (P'-P)\cdot n/2$ and $t = (P'-P)^2 \Rightarrow -\Delta_\perp^2$ if $\xi \to 0$

$\tilde{E}_q(x,\xi,t,Q)$ **Different quark spin projection**

❑ Total quark's orbital contribution to proton's spin:

Ji, PRL78, 1997

$$
\begin{aligned}
J_q &= \frac{1}{2} \lim_{t\to 0} \int dx\, x\, [H_q(x,\xi,t) + E_q(x,\xi,t)] \\
&= \frac{1}{2}\Delta q + L_q
\end{aligned}
$$

❑ Connection to normal quark distribution:

$H_q(x,0,0,\mu^2) = q(x,\mu^2)$ **The limit when** $\xi \to 0$

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