Small-x physics at EIC, a few simple exercises

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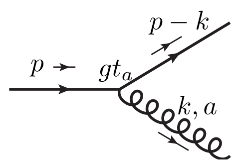
1. Calculate
$$\int_{-\infty}^{\infty} \mathrm{d}k \frac{e^{ikx}}{k^2+m^2}.$$
This can be done the easy way using the theorem of residues; there is probably also a hard way without it.

   Then calculate the two dimensional transform needed in the virtual photon-dipole wave function
$$\int \mathrm{d}^2\mathbf{k}_T \frac{e^{i\mathbf{k}_T\cdot\mathbf{r}_T}}{\mathbf{k}_T^2+m^2}$$
and compare. One way to do this is to integrate over the angle first, which leaves you with the Bessel $J_0(|\mathbf{k}_T||\mathbf{r}_T|) = J_0(kr)$. The integral over k then gives a modified Bessel K_0. Mathematica will do the k-integral for you. A more generalizable way is to use a Schwinger parametrization for the propagator
$$\frac{1}{k^2+m^2} = \int_0^{\infty} \mathrm{d}t e^{-t(k^2+m^2)},$$
then do the (Gaussian) Fourier-transform and identify the integral representation
$$\int \frac{\mathrm{d}t}{t} e^{-tA} e^{-B/t} = 2K_0(2\sqrt{AB}).$$
When you are calculating the virtual photon-to-dipole wave function, what is m? What is the asymptotic behavior for $mr \gg 1$? Compare the asymptotic behavior to the 1-dimensional case. What does this tell you about the size of the dipole in a DIS event?

2. (Kovchegov & Levin, exercise 5.1) The gluon field radiated from a fast-moving quark can be written using usual covariant theory Feynman rules as



$$A_\mu^a(k) = -igt^a \frac{-ig_{\mu\nu}}{k^2+i\varepsilon} \bar{u}_\sigma(p-k)\gamma^\nu u_\sigma(p)(2\pi)\delta((p-k)^2) \tag{1}$$
The incoming quark is on shell, with $p^\mu = (p^+, 0, \mathbf{0}_T)$.

   (a) **The eikonal vertex:** assuming $p^+ \approx (p-k)^+ \gg k^+$ show or convince yourself (e.g. using the Gordon decomposition) that the leading high energy behavior is
$$\bar{u}_{\sigma'}(p-k)\gamma^\nu u_\sigma(p) \approx 2p^+\delta^{\nu+}\delta_{\sigma\sigma'} \tag{2}$$

(b) Then Fourier-transform

$$A^a_\mu(x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{-ik\cdot x} A^a_\mu(k) \tag{3}$$

to get the field in coordinate space

$$A^{+a}_{\mathrm{COV}} = \frac{g}{\pi} t^a \delta(x^-) \ln \frac{1}{|\mathbf{x}_T|\Lambda} \tag{4}$$

You may use the result

$$\int \frac{\mathrm{d}^2\mathbf{k}_T}{(2\pi)^2} \frac{e^{-ik\cdot x}}{\mathbf{k}_T^2} = \frac{1}{2\pi} \ln \frac{1}{|\mathbf{x}_T|\Lambda}. \tag{5}$$

Here Λ is some IR cutoff that you need to introduce to make sense of the logarithmically divergent integral; the UV ($k_T \to \infty$) is finite because of the oscillatory factor in the Fourier transform

(c) In Light Cone Gauge, one introduces an axial gauge fixing four-vector n^μ, with $n^2 = 0$. The LC gauge for a right-moving nucleus has $n^- = 1, n^+ = \mathbf{n}_T = 0$. One then replaces the Feynman propagator as:

$$g_{\mu\nu} \to g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k\cdot n} \tag{6}$$

Calculate now the coordinate space color field $A^\mu_{\mathrm{LC}}(x)$ with the same approximations

3. (Kovchegov & Levin, 4.5 b) Solve the BK equation in zero transverse dimensions:

$$\partial_y N = \alpha_{\mathrm{s}} N - \alpha_{\mathrm{s}} N^2, \quad N(y=0) = N_0 \ll 1 \tag{7}$$