Four Lectures on non-perturbative QCD, hadron structure and parton physics

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Outline of Lectures

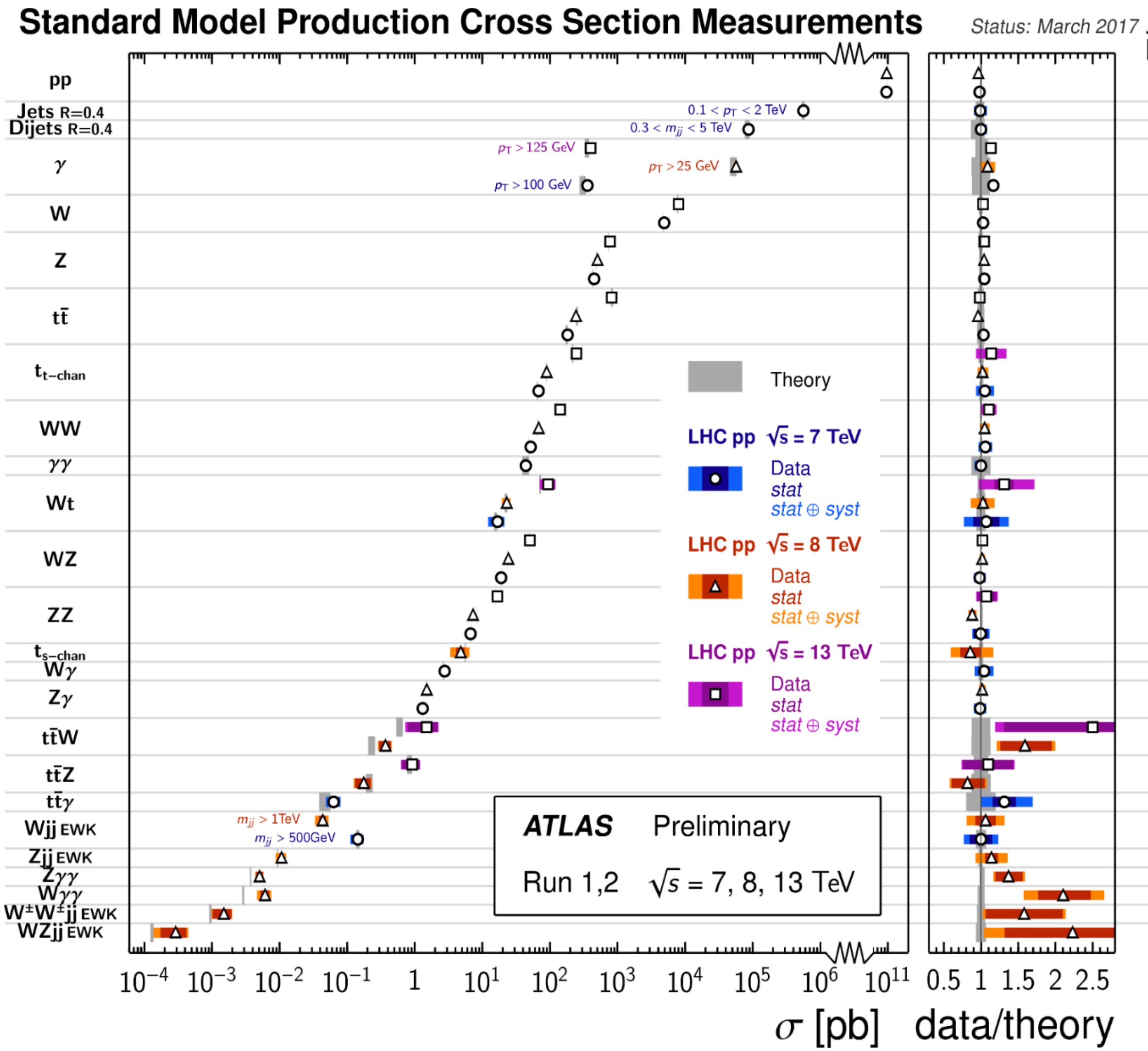
- Lecture 1: What is the fundamental reason for building EIC? Or, why QCD is so difficult?
- Lecture 2: An amateur's introduction to lattice QCD
- Lecture 3: What is parton physics? (at EIC)
- Lecture 4: Large momentum effective theory for calculating partons from first principles

Lecture 1: What is the fundamental reason to build EIC? Or why QCD is so difficult?

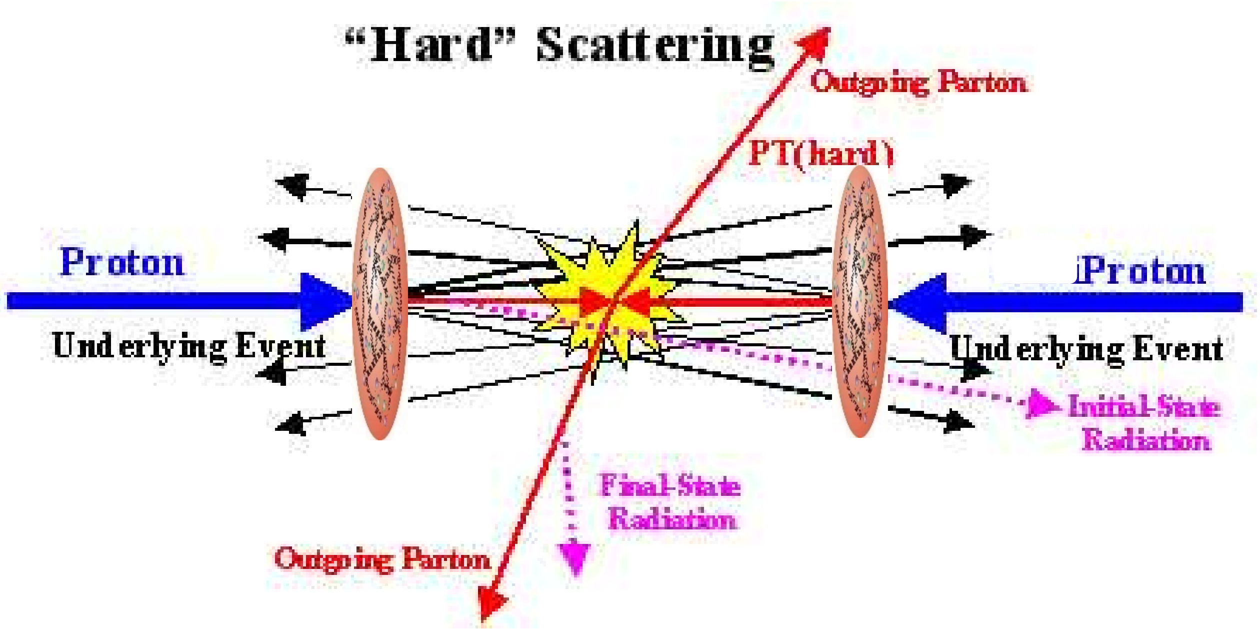
Standard model successes:

- The standard model itself has been hugely successful in explaining many physics phenomena
  - Electroweak processes
  - High-energy QCD processes

Perturbation theory works! (LHC)



Hard scattering theory

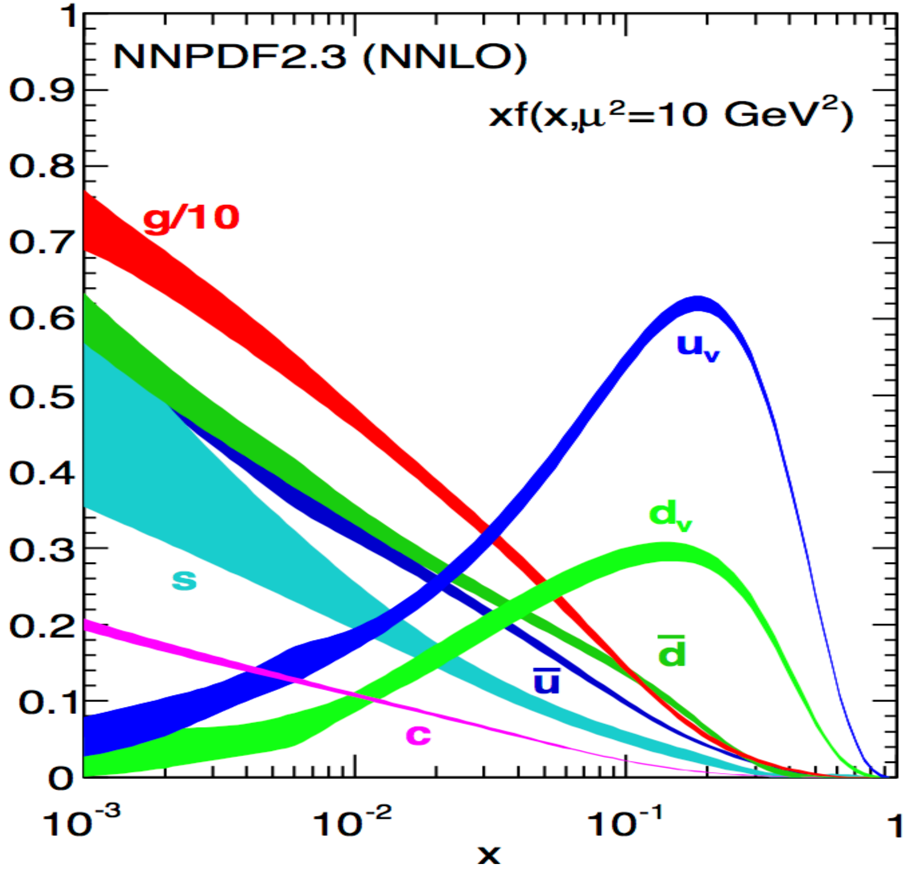


- Factorization theorems: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

Phenomenological PDFs

- Use experimental data (~50 yrs) to extract PDFs



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Structure of the proton

- To calculate PDFs from QCD, we need to understand the structure of the proton, a quantum mechanical bound state.
- Our experiences with bound states in QM are limited to mainly non-relativistic systems:

Atomic systems: electrons + Coulomb int

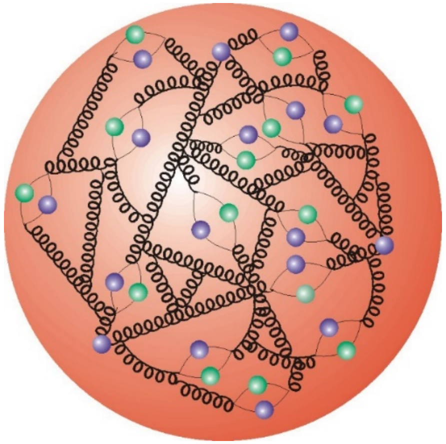
Nuclear systems: protons + neutrons

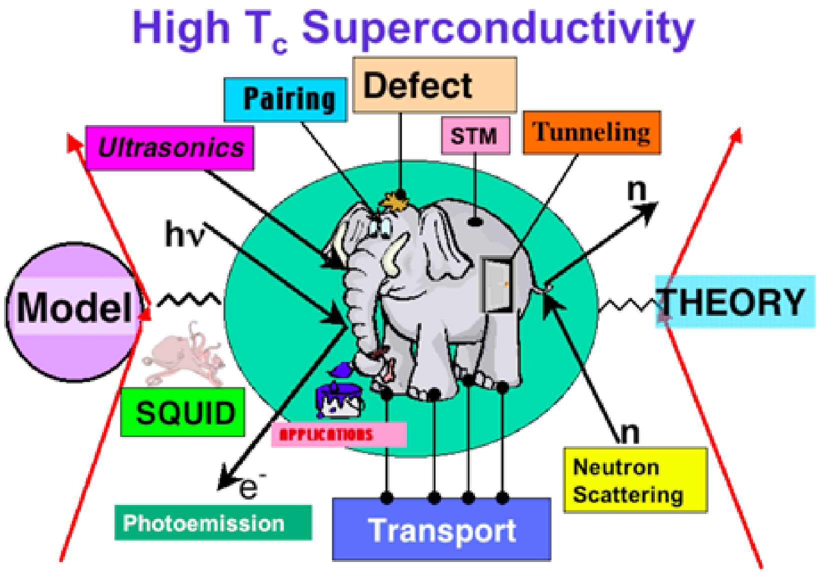
A lot of experience has been accumulated on non-relativistic many-body problems.

QCD bound states

- However, it is an unprecedented challenge to understand how QCD works at low energy, where theory becomes non-perturbative:
  - Guts of Strong Interactions!
- Similar in nature to Condensed Matter Physics: the Lagrangian is known, but the solution is hard

High T_c, Hall effects, strongly coupled electron systems, etc

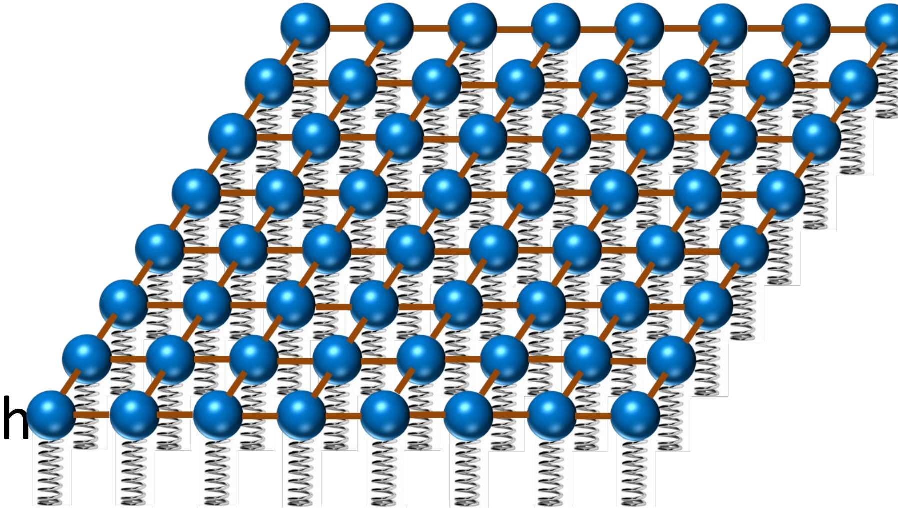




QCD (& SM) is a quantum field theory

- QFT = QM + Relativity
- The fundamental degrees of freedoms are quantized fields (quarks + gluons)
- The fields fill in all space and time. Every point of space and time has a few quantum mechanical degrees of freedom (DOF), which are interacting with their neighboring DOFs (local QFT).
- Extremely complicated (infinite DOF) systems…..

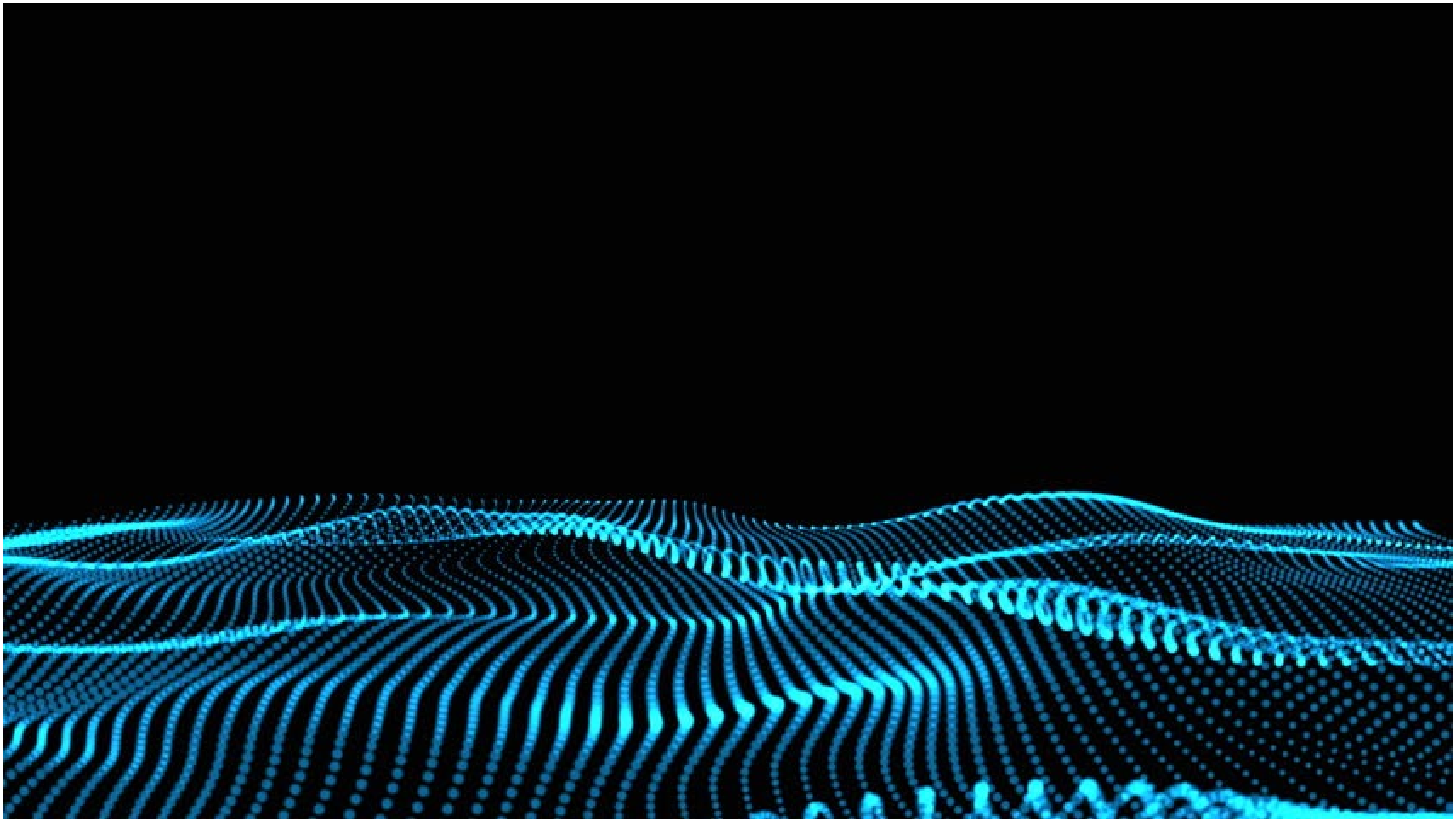
  Infinity is a problem



What is a particle?

- A particle is a quantized wave in a quantum field.

“Physical particles in QFT”



Two “easy” cases

- Non-relativistic limit:

Masses of the particles are much larger than the kinetic and potential energies.

Atomic and condensed matter physics are a non-relativistic limit of QED.

Nuclei physics is the non-relativistic limit of QCD.

Particle number is conserved. (For neutral systems, IR photon decouples)

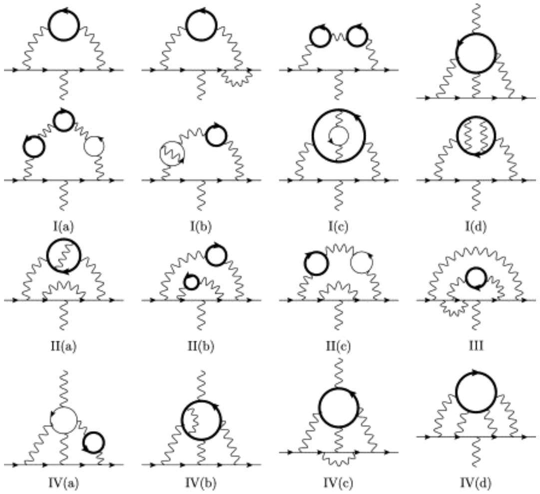
QFT -> Hamiltonian dynamics

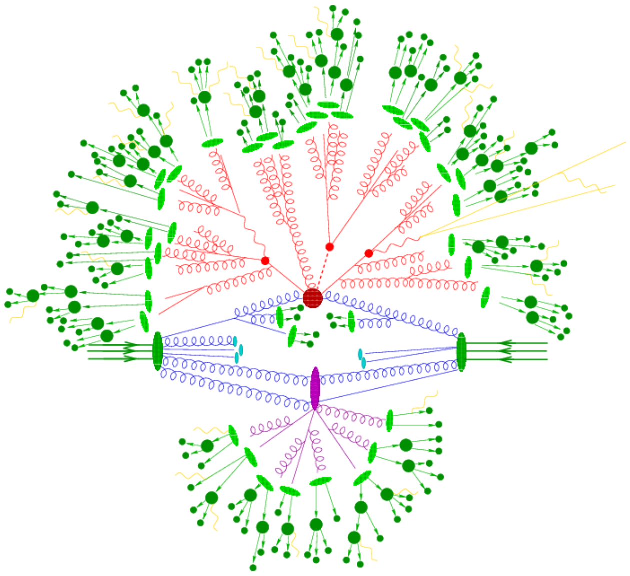
Two “easy” cases

- Weak coupling limit

In the week coupling limit, one can use perturbation theory developed by Feynman et al.

High-loop calculations for g-2 Parton Shower





A remark about the non-relativistic limit

- Not all cases of the non-relativistic limit are easy.
- Heavy quarkonium systems involve non-perturbative potentials which cannot be easily calculated.
- Some level of perturbative expansion in the interactions is needed. (multiple photon effects are perturbative)

High-precision spectroscopy in H-atom

Why strong QCD is so difficult?

- **Strongly coupled: Similar to NR electron systems**
  - Non-perturbative approximation methods must be devised beyond fermi liquid theory.
  - Ab initio numerical simulations
  - Effective degrees of freedom?
- **Extra difficulty: Relativity**
  1. Center-of-mass and internal motions coupled
  2. The QCD vacuum (ground state)
  3. Number of particles not fixed (going to infinity)

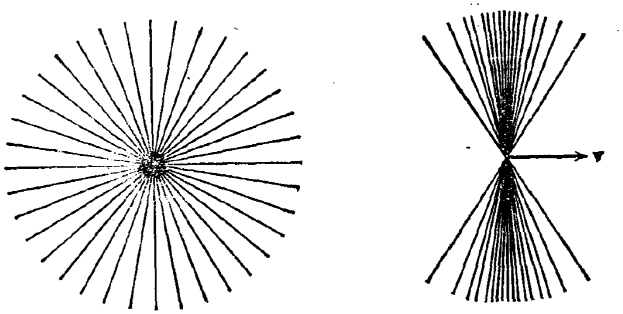
Relativity: internal states are frame-dependent

- The center-of-mass motion is part of the physics: the bound state has definite total momentum
- Because the boost operator is dynamical, the internal states are different at different momenta!

$$|p'\rangle = U(L)|p\rangle$$

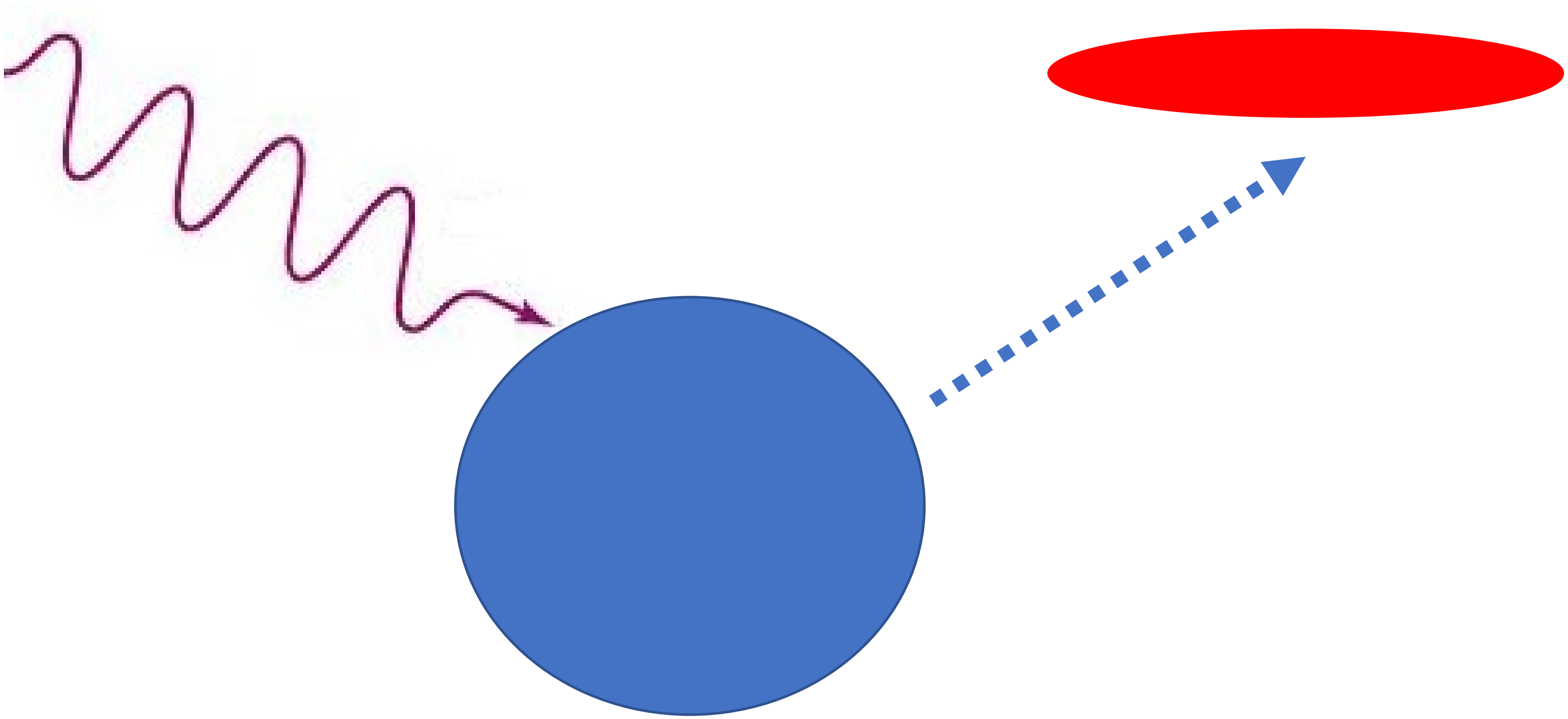
where $|p'\rangle$ is different from $|p\rangle$ dynamically!

  - The electromagnetic fields of a moving charge depends on its velocity or $\beta = v/c$



Elastic scattering: form factors

hadron states (wave functions) depending on the external momenta



The QCD vacuum (ground state)

- Hadron systems are built upon the QCD vacuum which in itself is extraordinary complex
  - Similar to a strongly-interacting fermi sea in Condensed Matter Systems, where Landau’s fermi liquid theory breaks down!
- And the hadron physics phenomena occur as complex excitations of this vacuum.
- It will be difficult to understand the excitations without understanding the ground state.

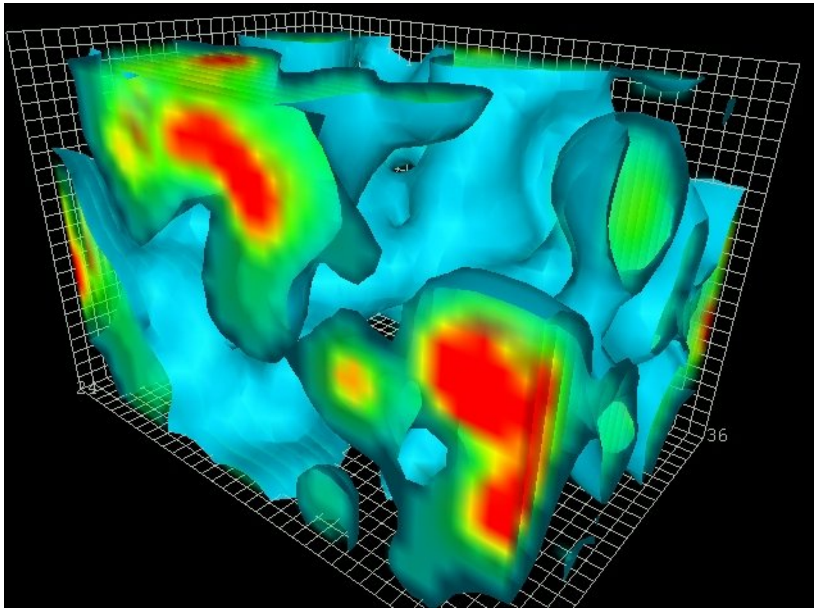
Understanding the water waves

Hadron physics that we try to understand!

QCD vacuum that we don't observe

What does QCD vacuum look like?

- In semi-classical approximation, the vacuum is filled with interacting instantons/anti-instantons, a particular classical gluon configuration.



- The BPST instanton is an essentially non-perturbative classical solution of the Yang–Mills field equations
- $\overrightarrow{iE} = \vec{B}, or\ \overrightarrow{iE} = -\vec{B}$
- Instanton has zero energy density and zero angular momentum density

Vacuum properties: effective scalar field or gluon condensate

- Vacuum scalar field

$\phi = \langle 0 | F^2 | 0 \rangle$ ~density of instantons

The vacuum has a scalar field density which generates a dimension, Λ^4, which sets the strong interaction scale parameter.

- The real QCD physics is independent of this scale parameter, unless there is a new scale introduced, such as quark masses.
- This scalar field is similar to the Higgs field in electroweak theory.

Vacuum properties: chiral symmetry & spontaneous breaking

- When Nf massless quarks q_i are introduced, there is a chiral symmetry

$$U_L(N_f) \times U_R(N_f) = U_V(1) \times U_A(1) \times SU_L(N_f) \times SU_R(N_f)$$

acting on the spaces of $q_{iL}(x)$ and $q_{iR}(x)$

- In the QCD vacuum, the symmetry is broken by instantons

$$U_L(N_f) \times U_R(N_f) \rightarrow U_V(1) \times SU_V(N_f)$$

($U_A(1)$ symmetry is broken explicitly by instantons)

Smoking gun for chiral symmetry breaking

- Non-zero chiral condensate in the instanton vacuum

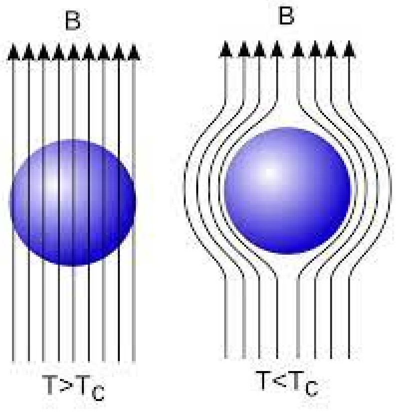
$$\langle 0|\bar{q}_L q_R|0\rangle \neq 0 \quad \text{(density of instantons)}$$

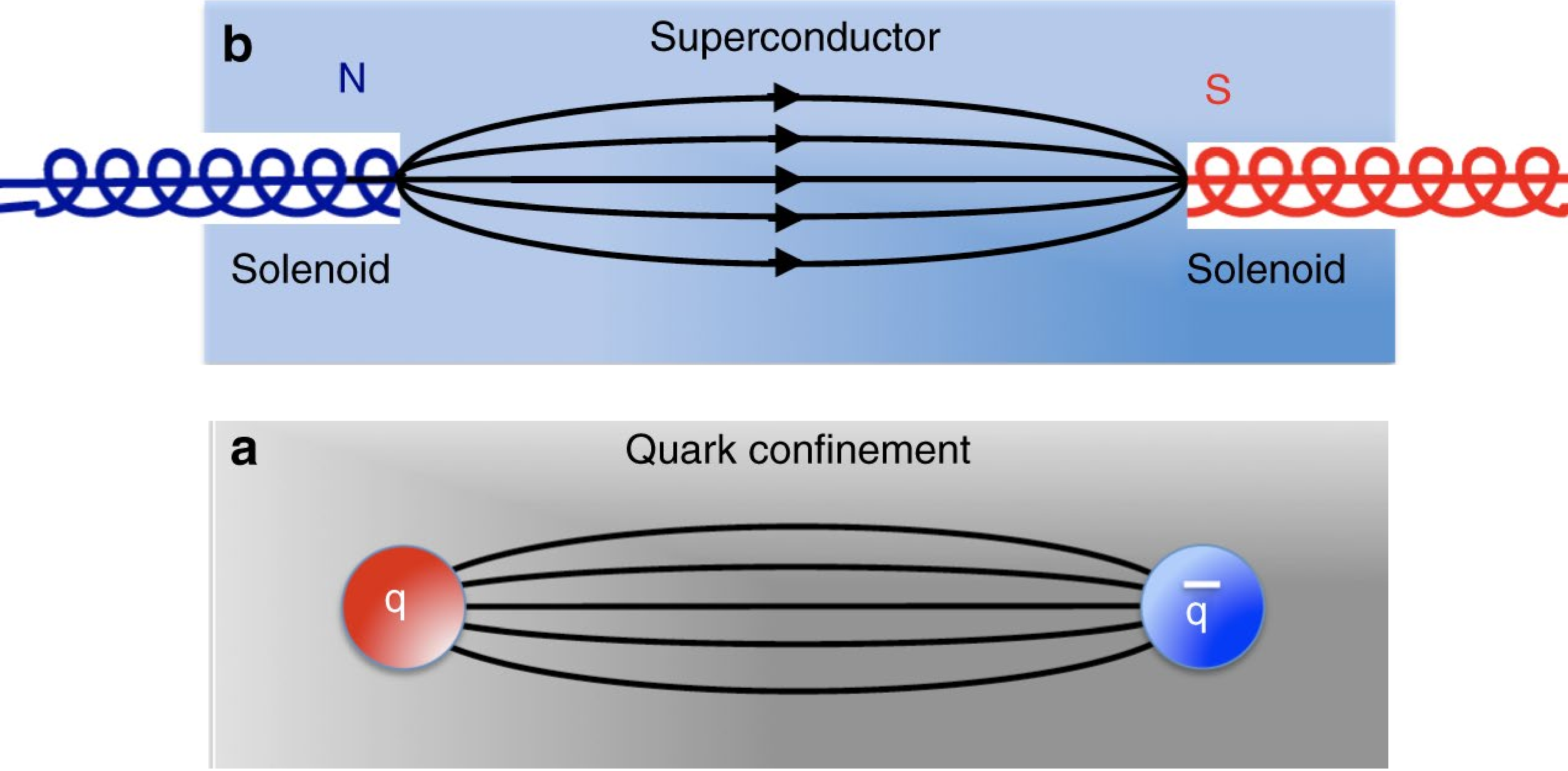
- Appearance of massless particles: Goldstone bosons as vacuum excitations.
- When quarks have small masses, we have pseudo-Goldstone bosons π, K which has mass-squared proportional to quark mass

$$m_\pi^2 \sim m_q$$

Color confinement

- An isolated colored charge has infinite energy in the QCD vacuum. Thus, colored states are not part of the QCD spectrum.
- The QCD vacuum expels the color electric field, just like the superconductor expels the magnetic field




b
Superconductor
N
S
Solenoid
Solenoid
a
Quark confinement
q
$\bar{q}$

Hadrons

- Is it possible to understand the structure of hadrons without understanding the QCD vacuum?
- Is it possible that there are effective degrees of freedom using which one can describe the hadrons?
  - Basis for model building (quark models, bag models)
  - QFT-> simple few body problem.
- In hadron physics, a universal effective description of hadrons has not been found
  - Existing ones are partially effective in limited domains.
  - We are forced to start from scratch

More on water-wave analogy

- We know the basic interactions between water molecules
  - but we don’t know how the state of water is formed, or how to calculate the properties of water.
  - how are the wave excitations formed on the top of it?

Low-energy effective theory: Navier-Stokes equation

  - To understand the waves, we just need to solve Novier-Stocks equation
  - Turbulences?

Ab initio calculations

- The only first principles approach to solve QCD is lattice method, first proposed by Ken Wilson in 1974
- Formulated in Euclidean space-time, allowing calculating Eulidean correlation functions.
- The vacuum properties can be calculated.
- The certain properties of hadrons can be calculated through the Euclidean correlation functions.
- Methods of quantum fields, not particles!

Understanding proton structure in non-perturbative QCD

- Analytically solving QCD is a long shot (AdS/CFT is hardly a controlled approximation)
- Calculations using lattice QCD have made important progress, but is difficult to produce penetrating insight.
- What are the deep insights about the structure of the proton in nonperturbative QCD?
- This the most important reason for EIC!

To theorists:

- Don’t tell us how good your predictions match with experimental data (a lot of them roughly do!)
- Tell us what are the systematic errors of your theory predictions or calculations in non-perturbative QCD (QFT)!
- QCD sum rules, Bethe-Salpeter equations, AdS/CFT, etc are not yet controlled approximations.