CFNS Summer School 2021
Accelerator Physics for EIC

Collider Accelerator Physics

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  - Overview of US EIC current design
  - Accelerator physics 612
- **Collider accelerator physics**
  - Beam dynamics of colliding beams
- **Spin dynamics**
  - Spin dynamics …
- **Synchrotron radiation and its applications**

Motivation of Colliders

The advantage of a collider is to reach higher energy. The center of mass energy of a two head on collision particles is

$$E_{cm} = \sqrt{m_1^2 + m_2^2 + 2E_1E_2(1 + \beta_{1\beta_2})}$$

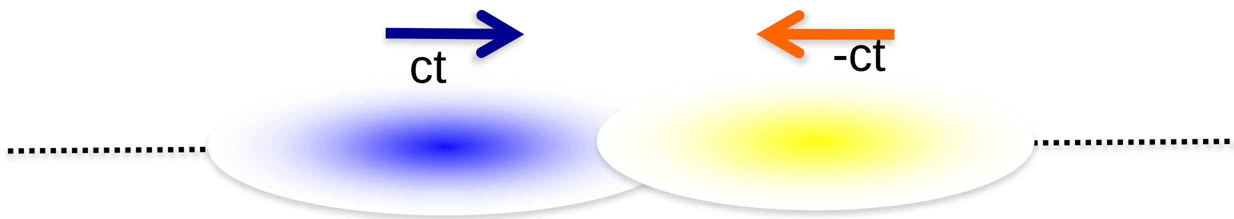
where E_1, E_2 are the energy of the two colliding beams, respectively. And, β_1 and β_2 are the Lorentz of each beam. For two relativistic beam of the same particle with the same energy of E, the effective energy is simply $\sqrt{S} \simeq 2E$.

Typically, collider is used to

- To discover new particles: HiggsLHC, Top quarkTevatron
- To explore the inner structure of matter quark-gluon
  - plasma@RHIC, proton spin structure@RHIC and HERA

Why High Energy Colliders?

Speed up two beams of charged particles to collide against each other



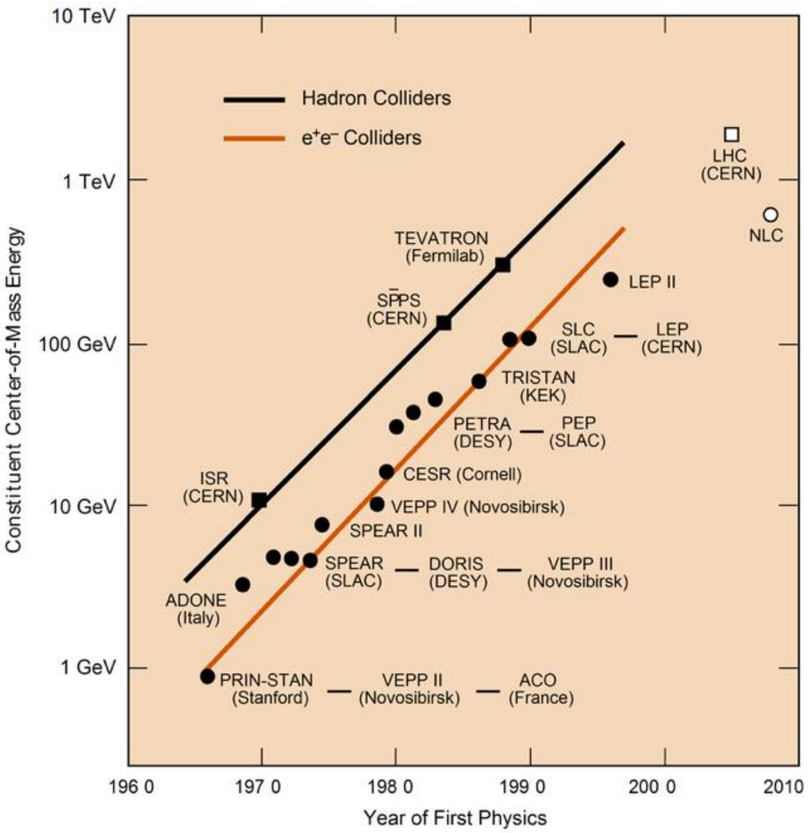
- A powerful tool for hunting of new particles
  - Einstein's $E=mc^2$
  - The heavier the particles, the higher the energy is required
  - For symmetric collision

Head-on collision	**Fixed target**
• C.M. energy is twice of the beam energy	• C.M. energy is ~ $\sqrt{2E_i m_i}$ if $E_i >> m_i$
• C.M. stays still w.r.t. to the detector	• C.M. moves forward w.r.t. to the detector

History of collider

- orders of magnitude increase of energy over the past 40 yearsto
  - explore the fine structure of matter
  - to discover/produce heavier particles

- more lepton colliders than hadron colliders



List of Colliders

Facility	Location	Type of collision	Energy	Year of Operation	Legacy
ISR	CERN	p	31.5 GeV	1971-1984	Stochastic cooling
SppS	CERN	pbar	270-315GeV	1981-1984	W, Z boson
PEP II	SLAC	e- e+	9 -3.1 GeV	1998 -2008	BaBar
SLC	SLAC	e- e+	45 GeV	1988 - 1998	1^{st} LC
CESR	Cornell	e- e+	6 GeV	1979-2002	1^{st} evidence B decay
B-factory	KEK	e- e+	8 – 3.5 GeV	1999 -	Belle
Tevatron	FNAL	p - pbar	900 – 980 GeV	1992 -2001	Top quark
HERA	DESY	e - p	e: 27 GeV p: 920 GeV	1992-2007	Spin physics
RHIC	BNL	$p^{\uparrow}$, d, Au. U	255 – 100 GeV/u	2000 -	Quark gluon

Figure of merit of a typical collider

- **Peak Luminosity**: # of collisions per unit area and per unit time
- For the case of ultra relativistic head-on collisions

$$L = \frac{f_{rev} N_{col} N_1 N_2}{2\pi\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}}$$

where f_{rev} is the orbital revolution frequency, N_{col} is the number of bunches in collision and $N_{1,2}$ is the bunch intensity f for the two colliding beams, respectively. $\sigma_{x1,2,y1,2}$ is the transverse beam size of the two beams, respectively.

	f_{rev}[kHz]	P[GeV/c]	N_{col}	$N_{1,2}$	$\epsilon_{1,2}$ [mm-mrad]	β^* [m]	L [$cm^{-2}\ s^{-1}$]
RHIC	78	250	110	1.5e11	15	1	3.8e32
LHC	11.25	1000	2808	1.2e11	14	0.5	1e34
HERA	47.273	920/27.5	174	5.9e11/ 2.1e11	5.1/5.1, 40/4.0	7/0.5 1/0.7	4e31

Figure of merit of a typical collider

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- **Integrated luminosity:** total number of collisions within a duration of period such as store

$$L_{int} = \int_0^{T_{store}} L(t)dt$$

- The unit of integrated luminosity is the inverse of cross-section unit, and typically expressed in inverse barn ($10^{-24}cm^{-2}s^{-1}$). For instance, RHIC delivered about 540 pb^{-1} of about 4 month polarized proton operation in 2013. In additional to the direct burn-out rate of collisions, the integrated luminosity is directly affected by
  - how effective is the detector: vertex distribution, detector ramp-up time, etc.
  - beam emittance growth during store due to various diffusion mechanisms such as intra-beam scattering, beam-beam effect, orbital resonance, etc.
  - overall percentage of time-in-store

Ways to increase luminosity

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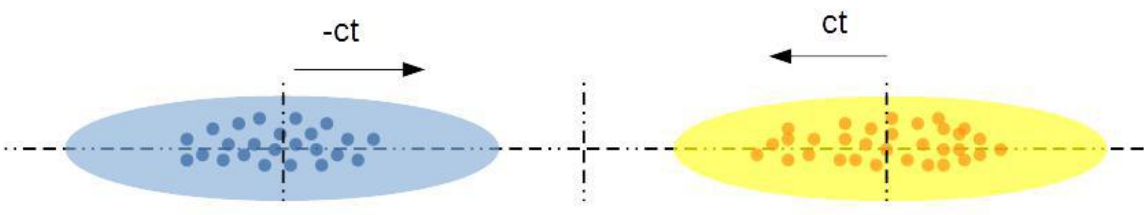
- Increase # of particles in each beam, i.e. bunch intensity
- Increase # of bunches
- Make each bunch more bright, i.e. shrink the size of the bunch at collision point
- Improve luminosity lifetime

Beam-beam force

- EM field that a particle experiences in the colliding bunch



$$2\pi E_r = \frac{1}{\epsilon_0}\int_0^r 2\pi r'\rho(r')dr'$$

$$2\pi B_\phi = \mu_0\int_0^r 2\pi r'\beta c\rho(r')dr'$$

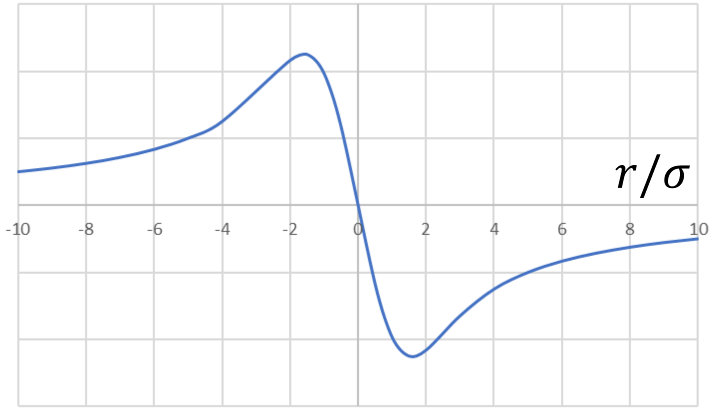
- In the case of round beam with Gaussian distribution $\rho(x,y) = \frac{Nq}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}$

$$2\pi r E_r = \frac{1}{\epsilon_0}\int_0^r 2\pi r' \frac{Nq}{2\pi\sigma^2}e^{-\frac{r'^2}{2\sigma^2}}dr'$$

Beam-beam force

- The beam-beam force is then given by

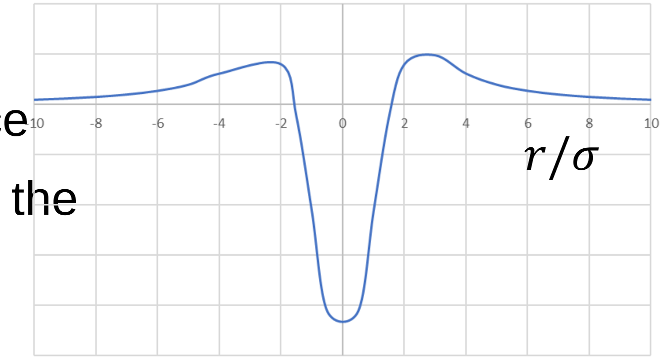
$$F_r(r) = -\frac{Nq^2(1+\beta^2)}{2\pi\epsilon_0 r}\left(1 - e^{-\frac{r^2}{2\sigma^2}}\right)$$



- **It is highly amplitude dependent!**
  - Near beam center, i.e. for $r \ll \sigma$,

$$F_r(r) = -\frac{Nq^2(1+\beta^2)}{4\pi\epsilon_0\sigma^2} r$$

beam-beam force is a linear, quadrupole-like force



  - This becomes quite non-linear for particles not in the beam center and the derivative of the force even changes sign!

Impact of beam-beam force on beam dynamics

- For the particles close to the core of the beam, this results to a tune change of $\Delta Q = -\frac{N r_0 \beta}{4\pi\gamma\sigma^2}$, where $r_0 = \frac{q^2}{4\pi\epsilon_0 m c^2}$, and β is the amplitude function at the collision point

- In general, tune change as function of betratron amplitude

$$\Delta Q(J) = \frac{2}{J}\left(1 - I_0\left(\frac{J}{2}\right)e^{-\frac{J}{2}}\right)$$

where $I_0(x)$ is modified Bessel function, $\mathrm{J} = \epsilon\beta/2\sigma^2$, and ϵ is the emittance

  - Particles with small amplitude, i.e. close to the beam center, experiences larger tune shift
  - particles with very large amplitude barely see any tune change
  - Hence, a beam in collision occupied an area in the tune diagram, aka tune foot print

Tune shift and tune footprint

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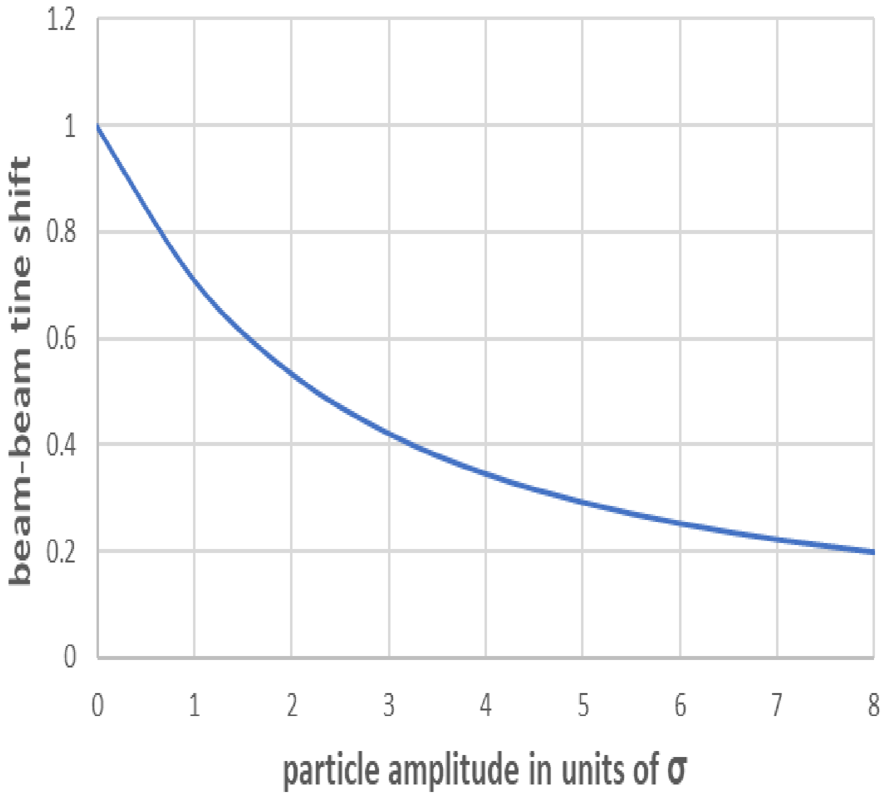
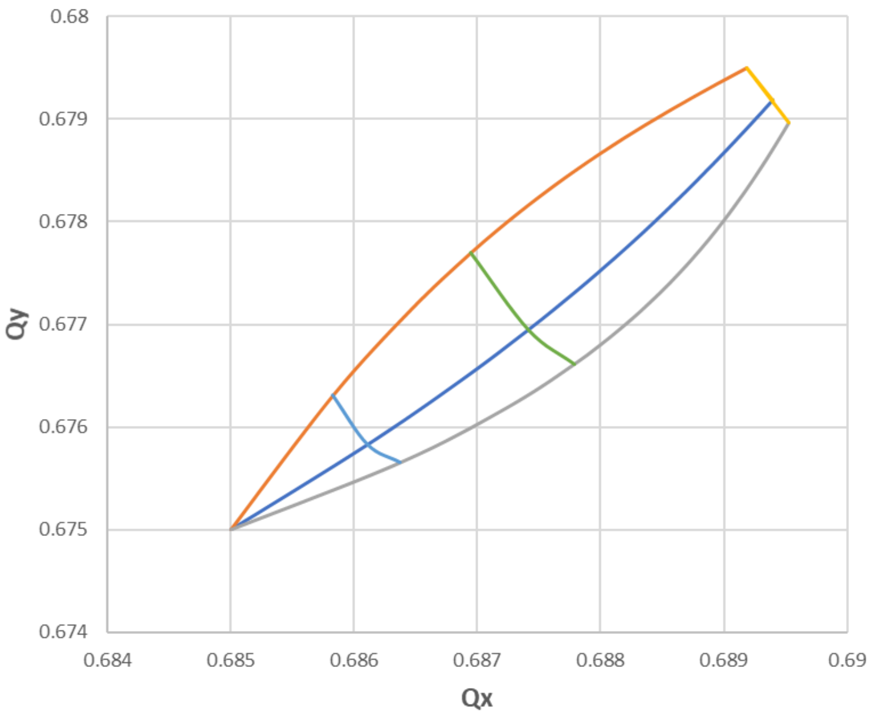
Beam-beam tune shift in units of beam-beam parameter



Tune footprint

$Q_{x0} = 0.69,\ Q_{y0} = 0.68$



Beam-beam parameter

- Recall the peak luminosity for the case of head-on collision of two round beams

$$L = \frac{f_{rev} N_{col} N_1 N_2}{2\pi\sigma^2}$$

- The coherent tune shift in this case is $\frac{N r_0 \beta}{4\pi\gamma\sigma^2}$ for round beam. In general, $\frac{N r_0 \beta_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}$ is horizontal/vertical tune change

- Hence, $\xi_{x,y} = \frac{N r_0 \beta_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x+\sigma_y)}$ is defined as beam-beam parameter for scaling the luminosity performance. In general, the larger the beam-beam parameter, the higher the luminosity

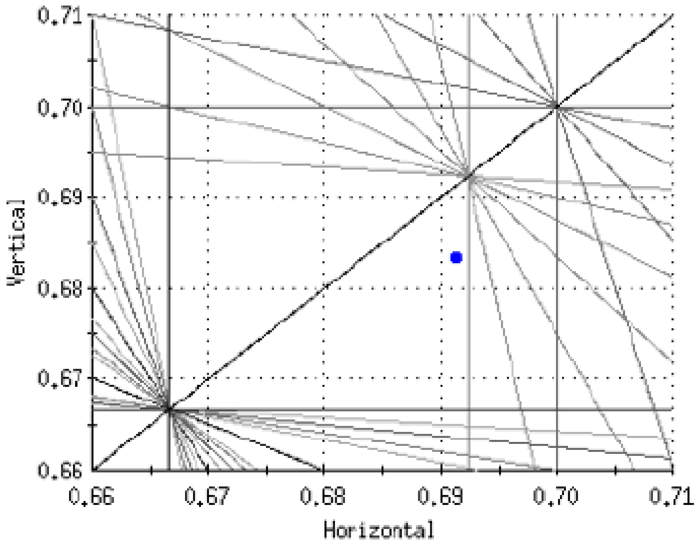
Limitation on beam-beam parameter

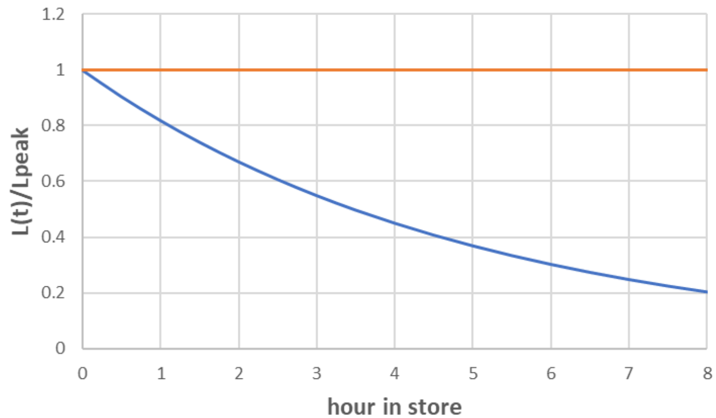
- Orbital resonance

$$MQ_x + NQ_y = k$$

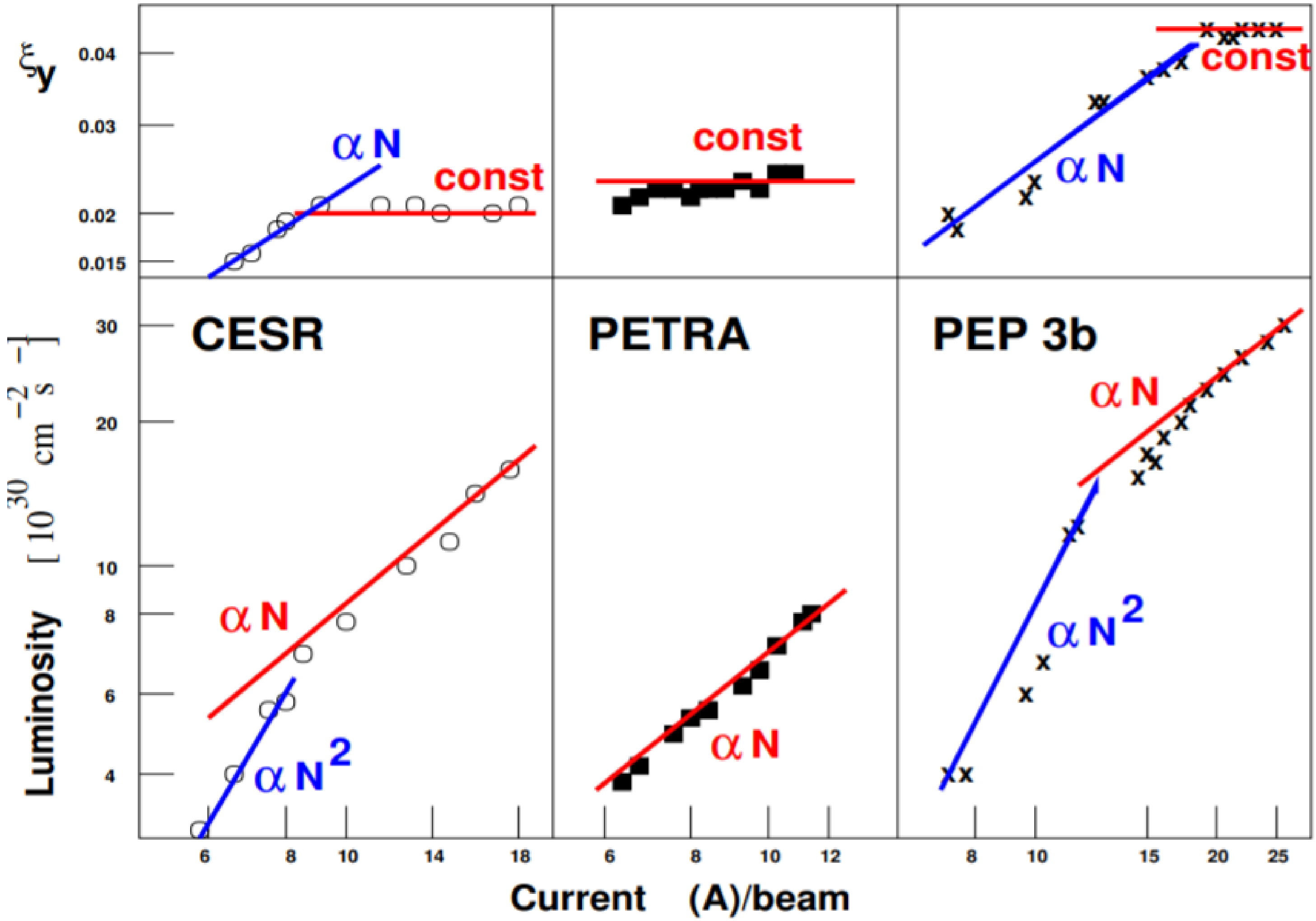
- Beam stability
  - When beam-beam tune shift pushes towards a major orbital resonance such as the 3rd order resonance

- Degradation of luminosity lifetime
  - Large beam-beam parameter corresponds to large tune footprint, which can lead particles in different part of the phase space experience weak orbital resonances and leads to growth of betatron oscillation amplitudes. This in turns can result to emittance growth that leads to limited luminosity lifetime





Beam-beam limit



Werner Herr, Beam Beam Interactions, https://cds.cern.ch/record/941319/files/p379.pdf

Hourglass

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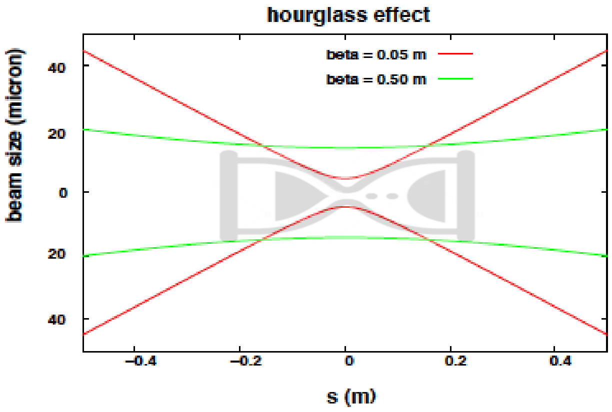
- In reality, bunch has non-zero length longitudinally.

  The transverse size evolves as

$$\sigma(s) = \sigma^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2}$$



- In the case of head-on collision of two round beam with Gaussian distribution. The effective luminosity then becomes

$$L = 2N_1 N_2 f_{rev} N_{col} \int\int\int\int_{-\infty}^{\infty} \rho_1 \rho_{1s}(s - s_0) \rho_2 \rho_{2s}(s + s_0)\, dx\, dy\, ds\, ds_0$$

Where, s and s_0 are the particle and center longitudinal distance from the collision point, respectively.

Hourglass

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Where, s and s_0 are the particle and center longitudinal distance from the collision point, respectively. With

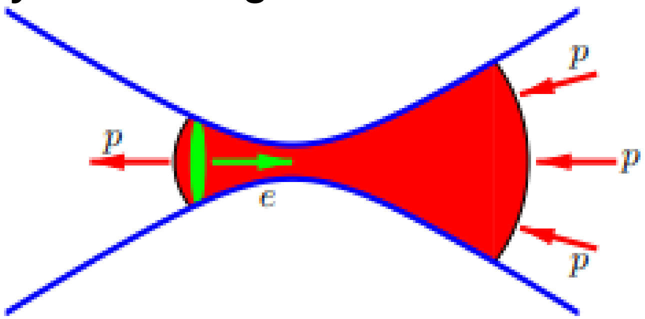
$$\rho_{1,2s}(s\pm s_0) = \frac{1}{\sqrt{2\pi}\sigma_s}\mathrm{e}^{-\frac{(s\pm s_0)^2}{2\sigma_s^2}} \text{ and } \rho_{1,2}(r,s) = \frac{1}{\sqrt{2\pi}\sigma_{1,2}(s)}\mathrm{e}^{-\frac{r^2}{2\sigma_{1,2}(s)^2}}$$

The effective luminosity becomes $L = \sqrt{\pi}\frac{\beta^*}{\sigma_s}e^{\left(\frac{\beta^*}{\sigma_s}\right)^2}\mathrm{erfc}\left(\frac{\beta^*}{\sigma_s}\right)L_0$, where $L_0 =$ is the luminosity without hourglass.

  - This effect is negligible if bunch is significantly shorter than β^*
  - Once bunch length gets comparable with β^*, effective luminosity will then suffer reduction

Hourglass for asymmetric collision

For electron ion colliders like HERA and US-EIC, the bunch length of the two incoming beams at the IP can be substantially different. For HERA, the proton bunch length is 19cm while the electron beam is only 1 cm long.



The corresponding luminosity then becomes

$$\frac{L^{hg}}{L} = \iint \rho_1(s_1)\rho_2(s_2)\frac{\Sigma_x(0)\Sigma_y(0)}{\Sigma_x(s)\Sigma_y(s)}\Big|_{s=\frac{s_1+s_2}{2}} ds_1 ds_2$$

where $\rho_{1,2}(s_{1,2})$ is the longitudinal density of the either beams, L is the luminosity without hourglass, and $\Sigma_{x,y}(0)$ and $\Sigma_{x,y}(s)$ are the transverse size factor for without and with hour-glass effect, respectively.

Assuming the hourglass effect only significant for one plane, the beam-beam parameter becomes

$$\frac{\xi_{x,y}^{hg}(s_1)}{\xi_{x,y}} = \int \rho_2(s_2)\frac{\beta(s)\sigma_{x,y}^*(s)[\sigma_x^*(0)+\sigma_y^*(0)]}{\beta(0)\sigma_{x,y}^*(s)[\sigma_x^*(s)+\sigma_y^*(s)]}\Big|_{s=\frac{s_1+s_2}{2}} ds_2$$

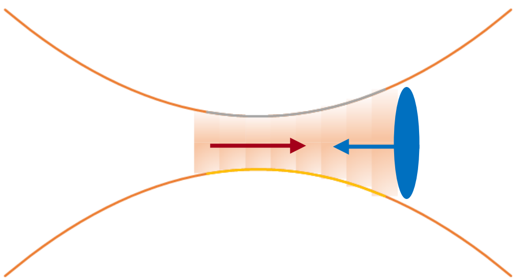
G. Hoffstaedtter, F. Willeke, EPAC’02, https://www.classe.cornell.edu/~hoff/hoff/papers/02epac_beambeam.pdf

Hourglass for asymmetric collision (con't)

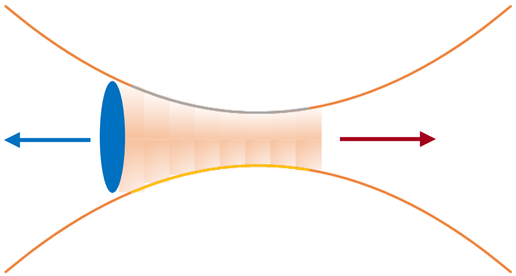
Assuming the hourglass effect only significant for one plane, the beam-beam parameter becomes

$$\frac{\xi_{x,y}^{hg}(s_1)}{\xi_{x,y}} = \int \rho_2(s_2) \frac{\beta(s)\sigma_{x,y}^*(s)[\sigma_x^*(0) + \sigma_y^*(0)]}{\beta(0)\sigma_{x,y}^*(s)[\sigma_x^*(s) + \sigma_y^*(s)]}\Big|_{s=\frac{s_1+s_2}{2}} ds_2$$

- The beam-beam parameter changes during the duration of the two-opposing bunch traveling through the IP region



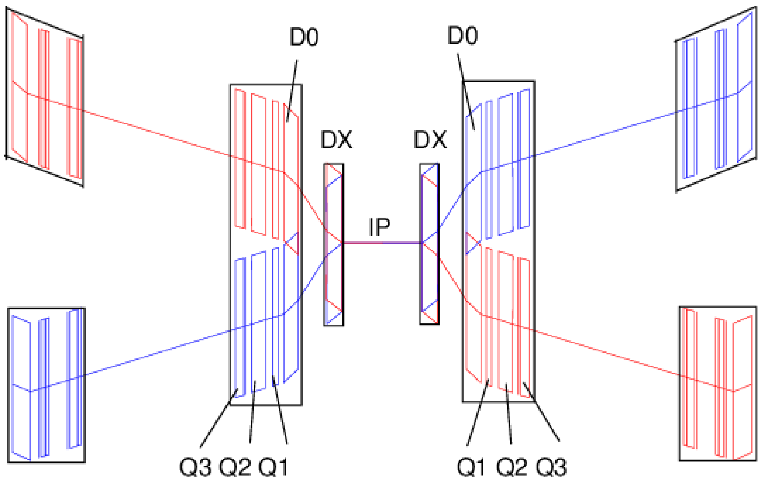
entering collision

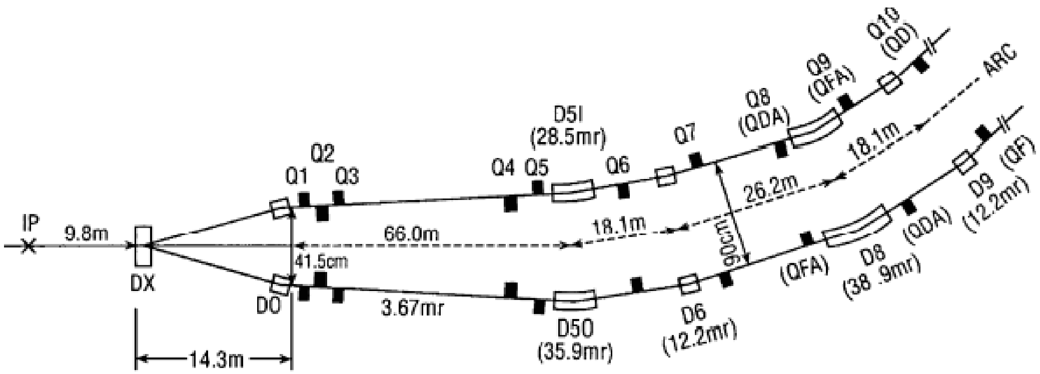


leaving collision

- The details of this beam-beam impact on the beam dynamics are more complicated than what this formula shows
  - The distribution of the two beams can be significantly impacted by the electromagnetic effect of the beam-beam, which in turn changes beam-beam parameter
  - The vertex distribution for the detectors

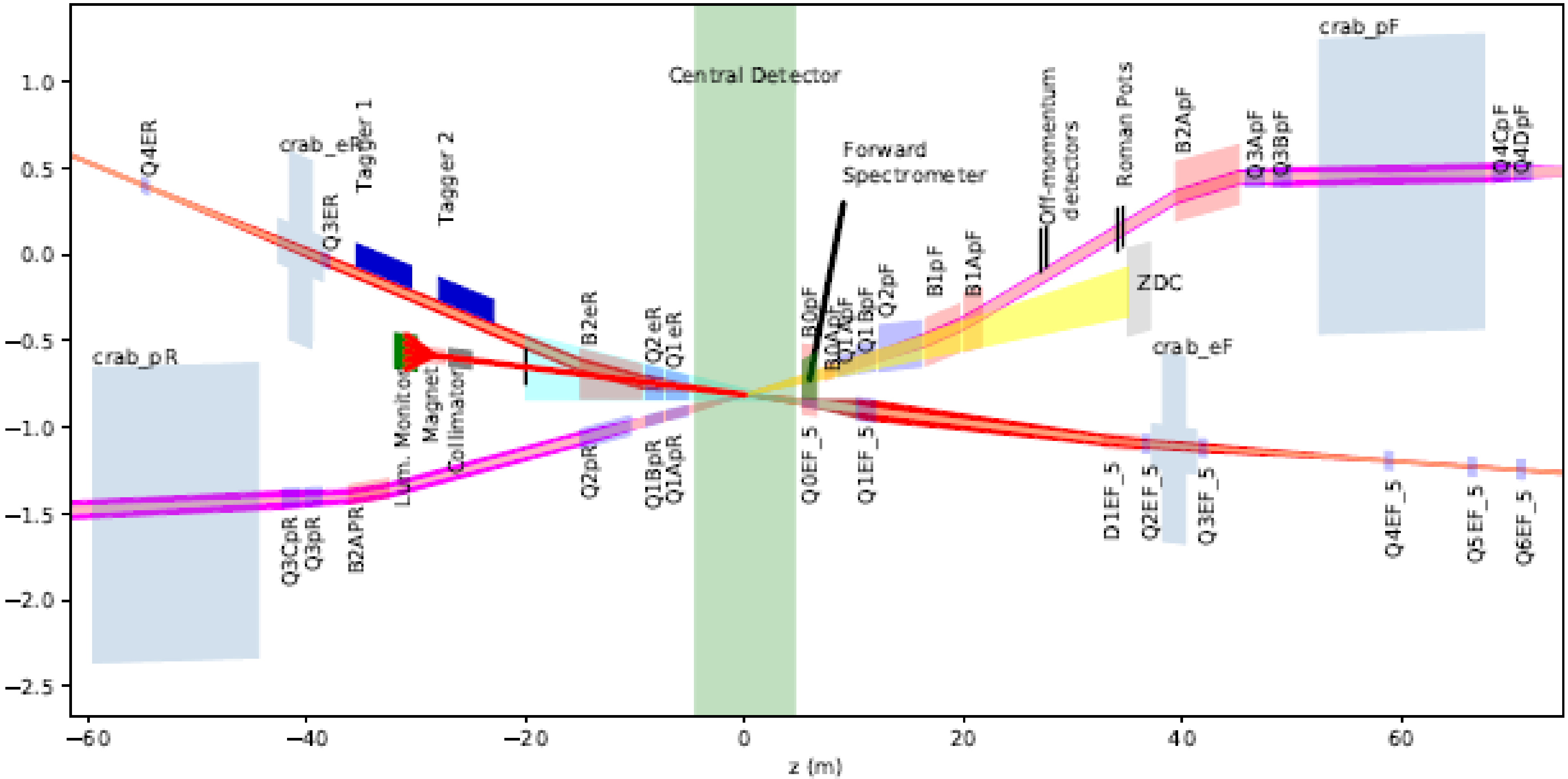
RHIC Interaction Region





Head-on collision. The two colliding beams are longitudinally separated

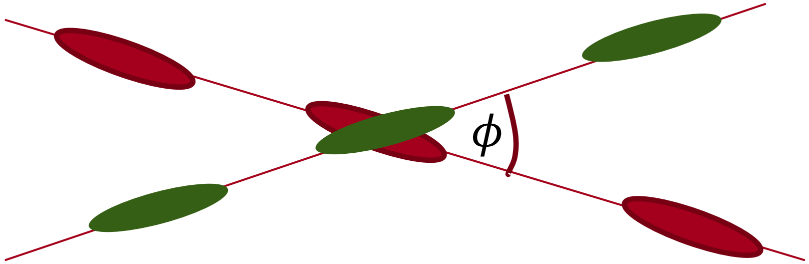
EIC Interaction Region



- Large crossing angle (25 mrad) to minimize the beam-beam effect
- The two crossing bunches are also transversely separated

Luminosity for crossing angle

In the presence of large crossing angle, the luminosity becomes



In this case, the luminosity becomes $L = L_0 S$, where S is the geometric factor. For small cross angle and bunch is significantly shorter than its transverse size,

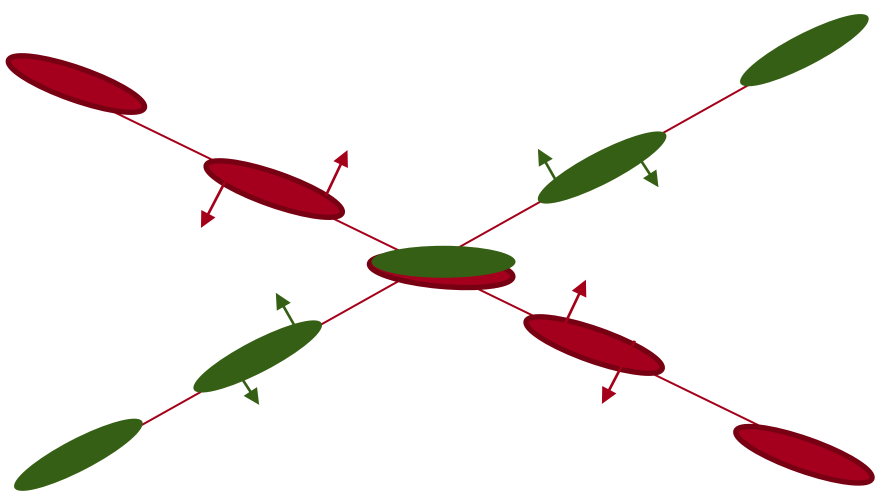
$$S = \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \tan\frac{\phi}{2}\right)^2}}$$

S is usually less than 1. Hence, non-zero cross angle results in luminosity loss

Compensation of large crossing angle luminosity

Crab crossing:

- use RF cavity on either side of the collision point to align the bunch shape of the two beams to recover luminosity reduction due to geometric factor. Such a cavity is called crab cavity

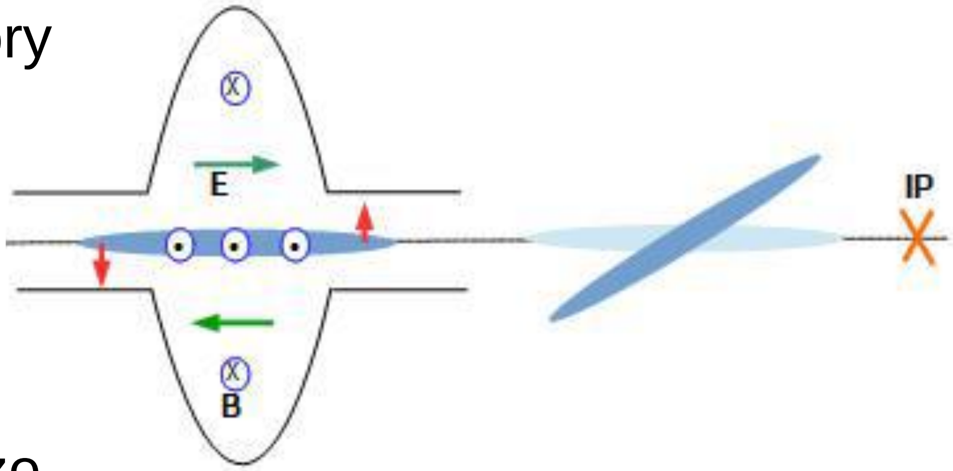


Compensation of large crossing angle luminosity

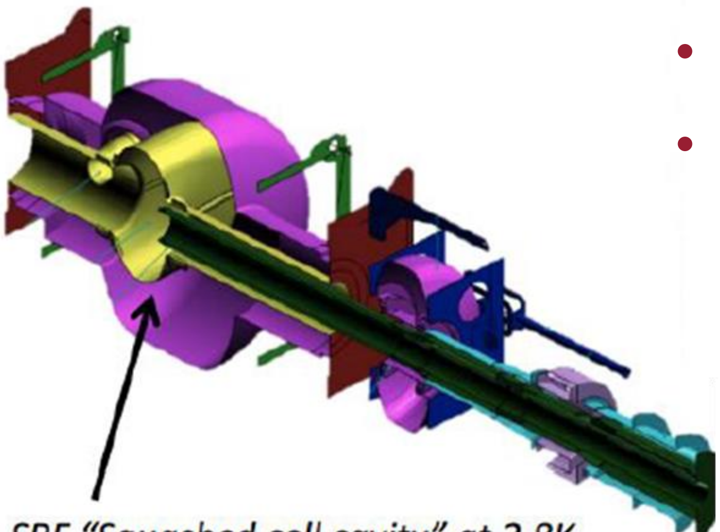
Crab cavity:

- First introduced by Dr. R. Palmer (BNL) in 1988 and first demonstrated at KEK B-factory in 2007

- An RF device operates at TM110 mode that provides phase dependent transverse kicks to tilt the bunch. The size of the tilt is proportionally to the strength of the maximum field of the cavity and distance between cavity to collision point

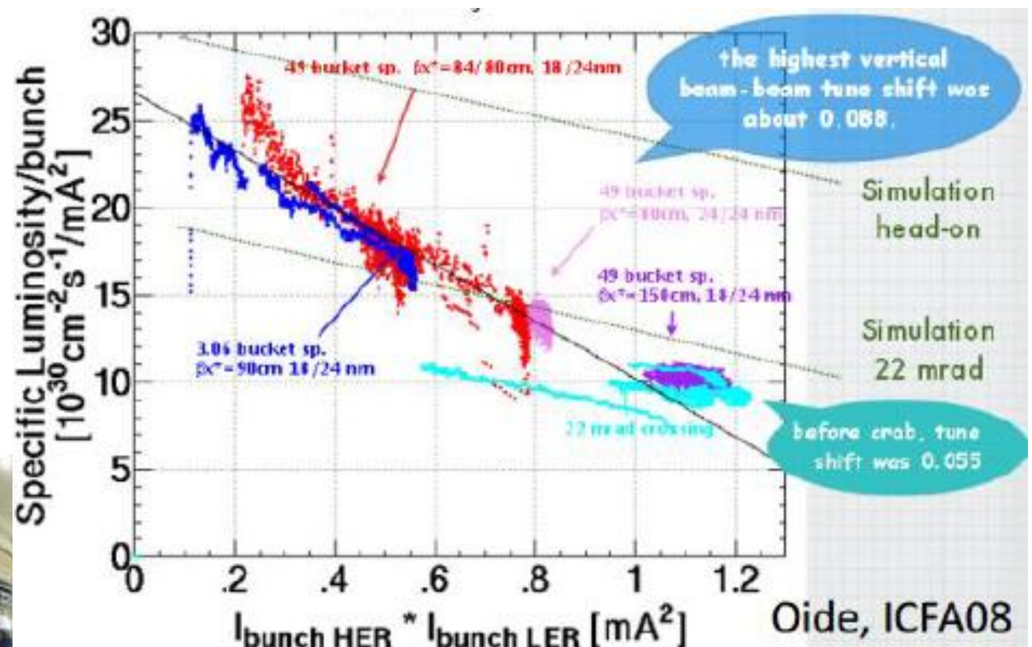


KEKB Crab Cavity



SRF "Squashed cell cavity" at 2.8K
with crabbing mode at 500 MHz
(2.8 MV defl voltage)

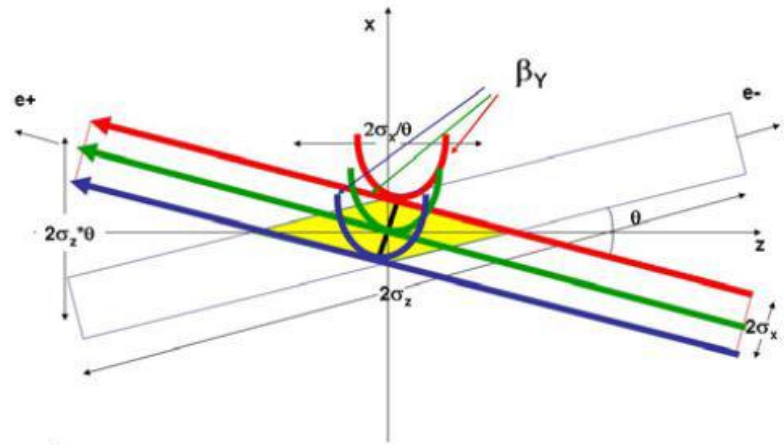
- Applied in operation with high current
- A peak luminosity of $21.1 \times 10^{33}\ cm^{-2}s^{-1}$ with crab cavity was reached



Oide, ICFA08

Compensation of large crossing angle luminosity

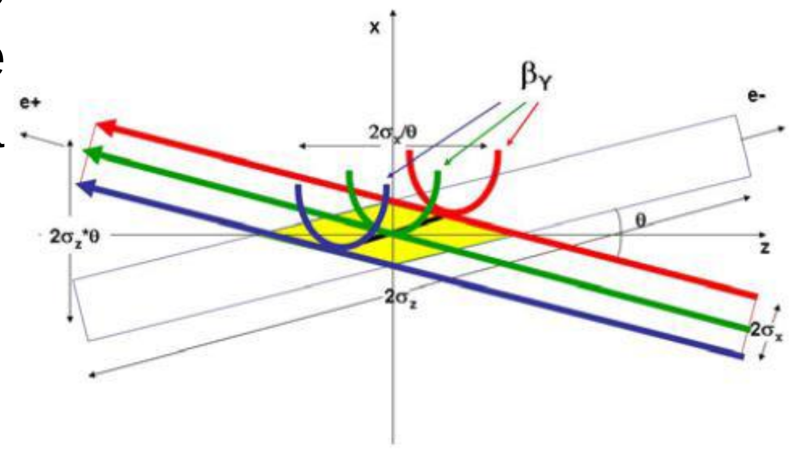
- For a collider with two flat beams in collision, one can use large horizontal crossing angle to reduce parasitic collision as well as beam-beam tune shift



a)

- one can use a sextupole on either side of the collision point to re-distribute the beta squeeze waist in the overlap area of the two beams. The beta function at the waist then becomes

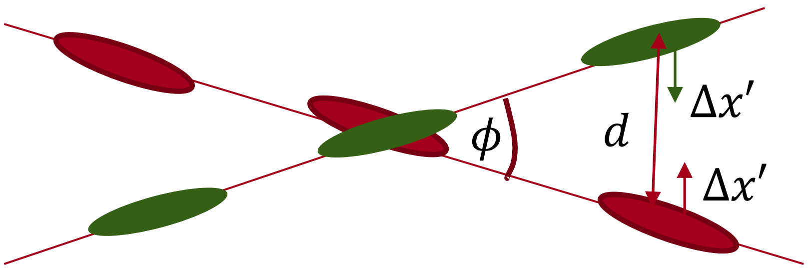
$$\beta(s) = \beta^* + \frac{\left(s - \frac{x}{\theta}\right)^2}{\beta^*}$$



b)

Long range beam-beam effect

In the interaction region beyond the collision point, the two beams are separated either transversely and/or longitudinally. In the case of transversely separation, the crossing bunches experience orbital kick due to beam-beam interaction, aka long range beam-beam force.



The corresponding beam-beam orbital kick from a horizontal separation of d

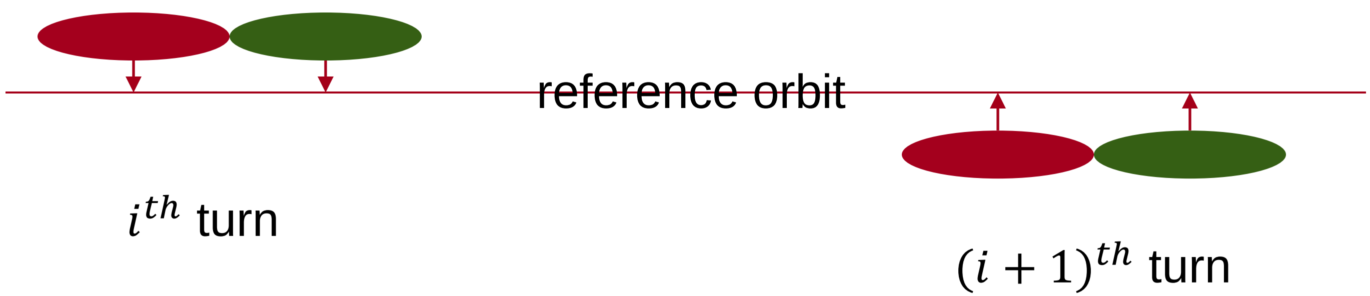
$$\Delta x' = \frac{2Nr_0}{\gamma}\frac{x+d}{r^2}\left[1 - e^{-\frac{r^2}{2\sigma^2}}\right] \qquad \Delta y' = \frac{2Nr_0}{\gamma}\frac{y}{r^2}\left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

with $r^2 = (x+d)^2 + y^2$ and $r_0 = q^2/4\pi\epsilon_0 mc^2$

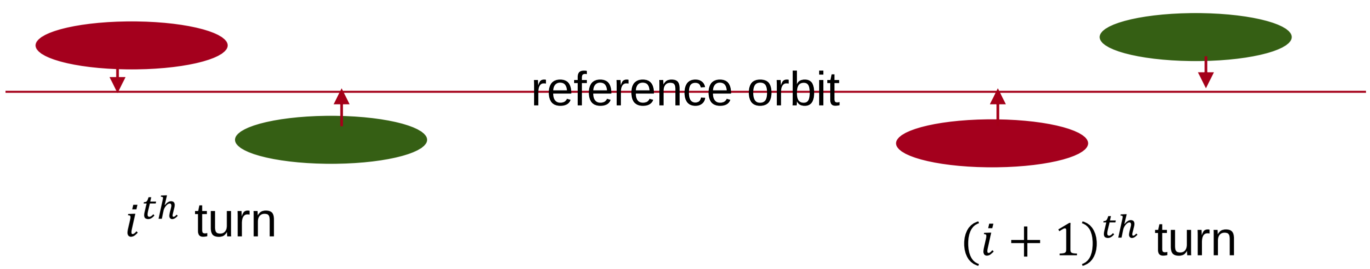
Coherent beam-beam effect

This beam-beam orbital kick in turn perturbs both beams periodically and can drive coherent dipole oscillation. This driven oscillation can be decomposed to two fundamental modes, ie.

- 0-mode, i.e. the two colliding bunches move completely in-phase as illustrated below. In



- π-mode, i.e.



Coherent beam-beam mode observed

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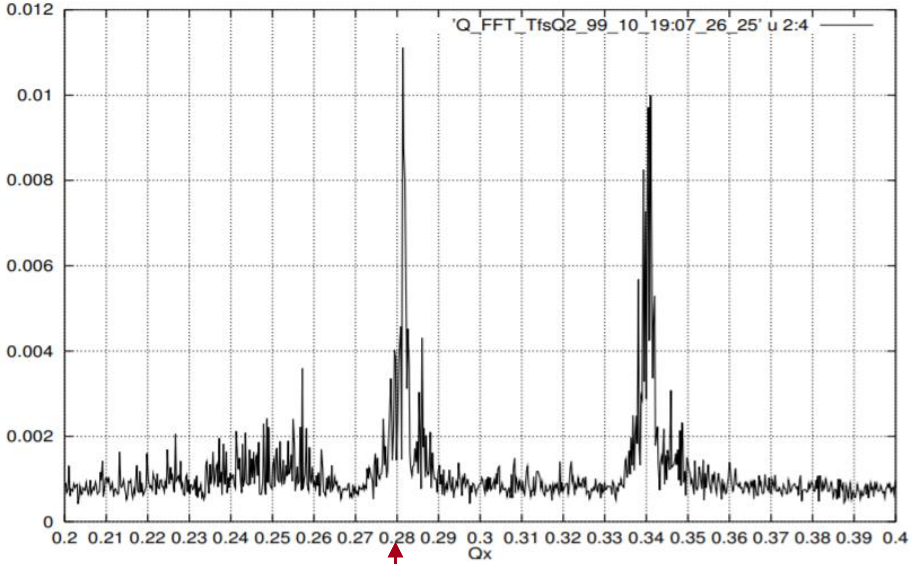
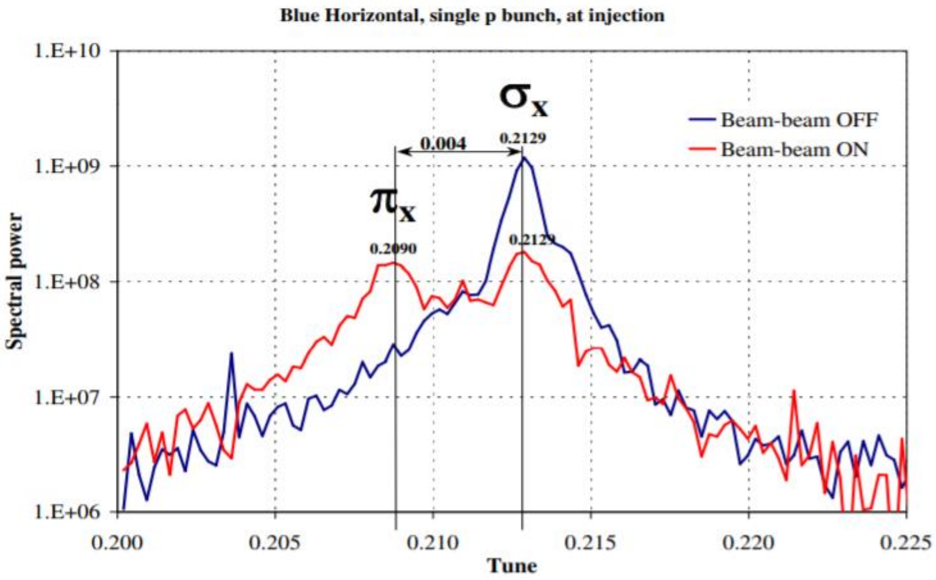
coherent beam-beam mode at LEP



unperturbed tune

coherent beam-beam mode at RHIC



Source of the plots: Werner Herr, Beam-Beam Interactions, https://cds.cern.ch/record/941319/files/p379.pdf?version=1

Other long-range beam-beam effect on beam dynamics

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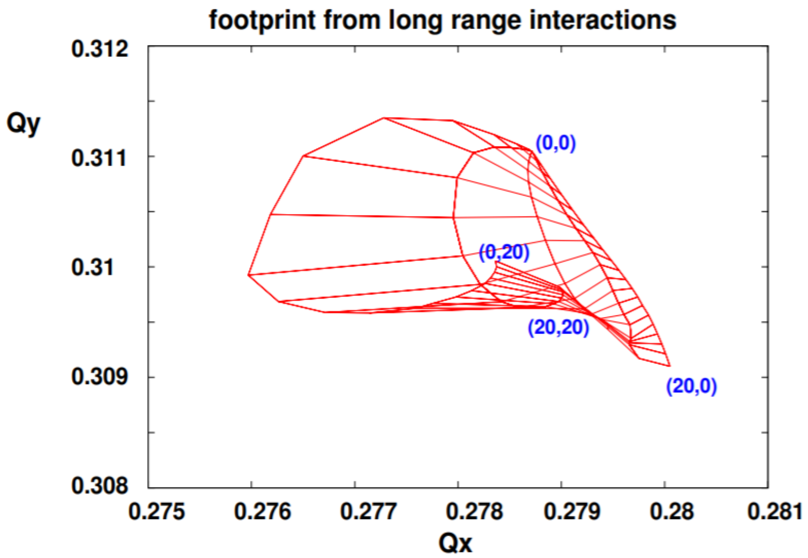
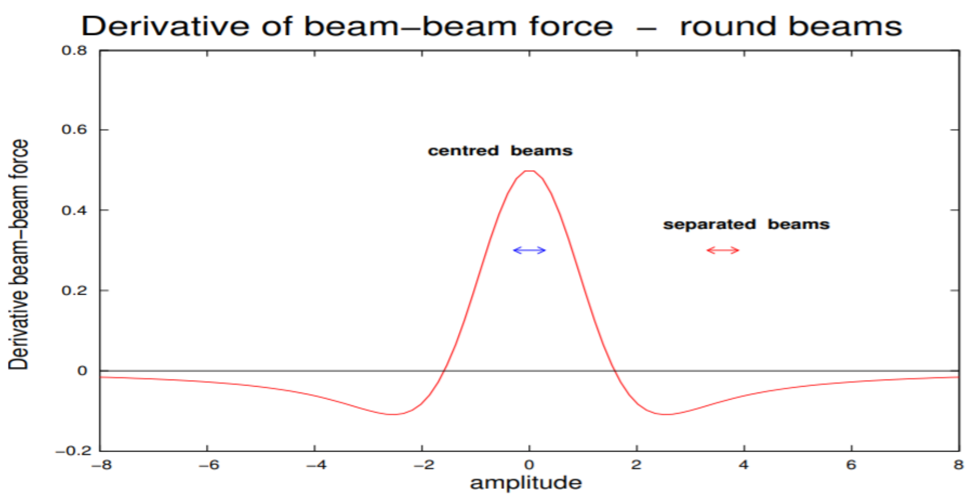
Coherent and in-coherent effect on betatron tune

Impact more on particles at large amplitude vs. head-on beam beam induces largest tune shift for particles at smaller amplitude



The long-range beam-beam driven tune shift has opposite sign in the plane to separation in comparison to the head-on beam beam tune shift



Source of the plots: Werner Herr, Beam-Beam Interactions, https://cds.cern.ch/record/941319/files/p379.pdf?version=1

Figure of merit of a typical collider

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$$L_{int} = \int_0^{T_{store}} L(t)dt$$

- The unit of integrated luminosity is the inverse of cross-section unit, and typically expressed in inverse barn ($10^{-24} cm^{-2} s^{-1}$). For instance, RHIC delivered about 540 pb^{-1} of about 4 month polarized proton operation in 2013. In additional to the direct burn-out rate of collisions, the integrated luminosity is directly affected by
  - how effective is the detector: vertex distribution, detector ramp-up time, etc.
  - beam emittance growth during store due to various diffusion mechanisms such as intra-beam scattering, beam-beam effect, orbital resonance, etc.
  - overall percentage of time-in-store

Integrated luminosity

Assuming

- Luminosity lifetime τ
- Store length T_{store}
- Store to store time t_{s2s}
    - Injection setup
    - Acceleration and collision setup
    - Collision optimization

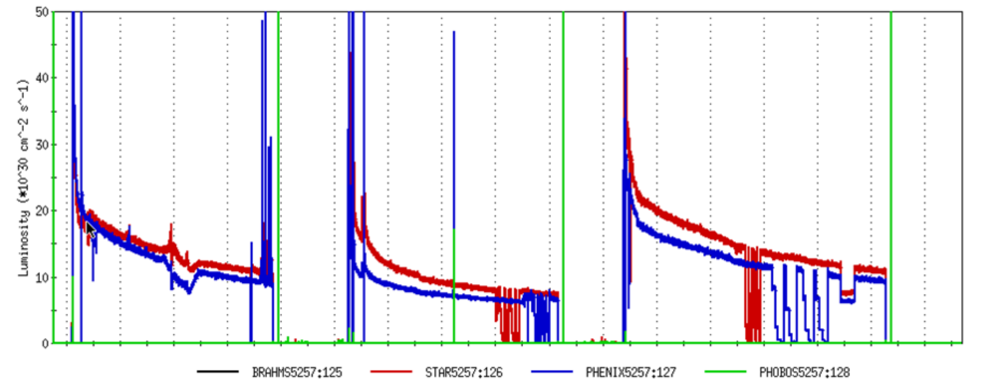
The store average luminosity

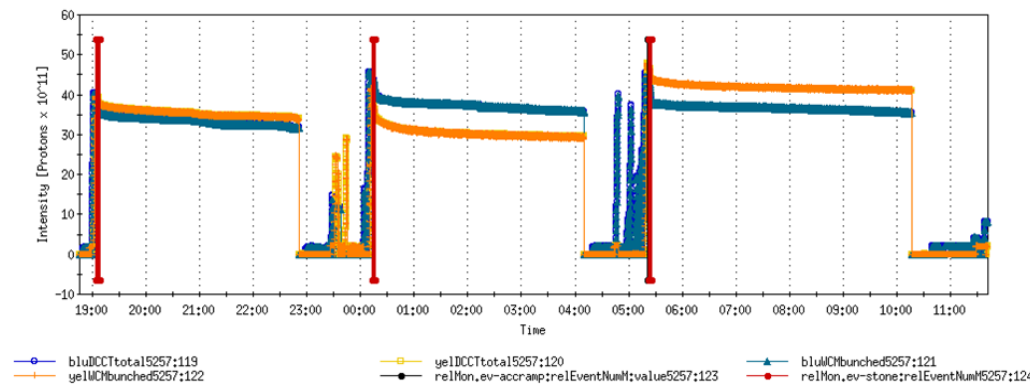
$$< L >_{store} = 1/T_{store} \int_0^{T_{store}} L_p e^{-t/\tau} dt$$

$$= L_p \tau (1 - e^{-\frac{T_{store}}{\tau}})$$

The weekly average luminosity

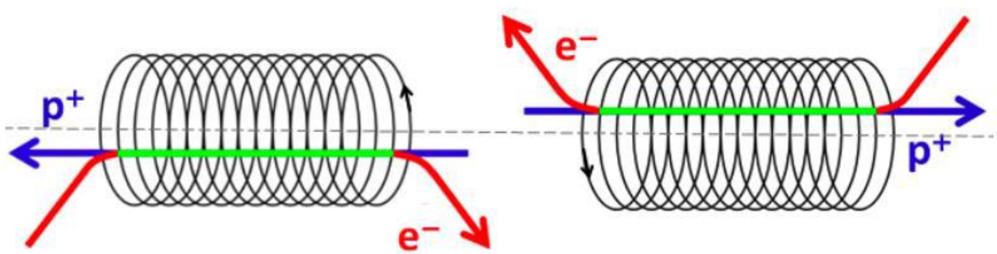
$< L > = N_{store} < L >_{store}$, where $N_{store} = T_{week}/(T_{store} + t_{s2s})$

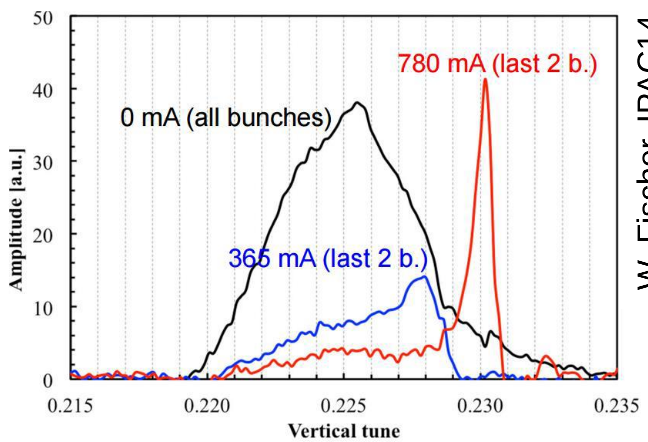




Mitigation towards beam-beam limit

- Careful choice of working point
- Minimize the resonance strength
- Deploy beam-beam compensation
  - E-lens for symmetric collision
    - Demonstrated at RHIC with polarized protons





W. Fischer, IPAC14

  - Large crossing angle + crab cavity
- Beam cooling
  - To keep the beam emittance from growing

Beam cooling methods

- Acceleration: the divergence in the phase space shrinks as the energy of the particle increases, i.e. physical beam emittance goes with $1/\sqrt{\beta\gamma}$

- Synchrotron radiation: damping time
  - $\tau_s = 2T_0/W$, and $W = \frac{dU}{dE}$, where $U \sim E^4/\rho$ is the power loss per turn. T_0 is the revolution time.

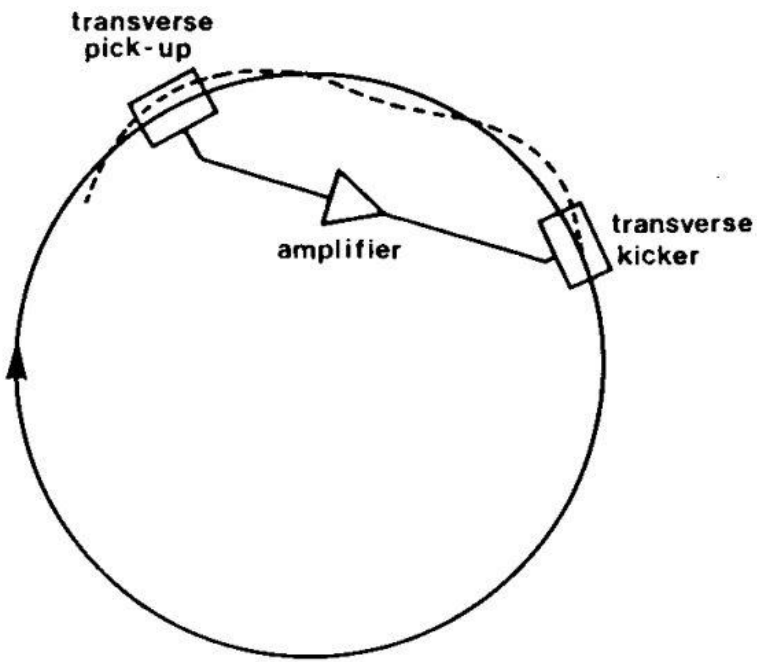
- Electron Cooling: using low temperature electron beams parallel to the ion beam to reduce the phase space area
  - first proposed by Gersh Budker in 1966
  - have been applied to heavy ions, anti protons

- Stochastic Cooling:
  - first proposed and demonstrated by Simon van der Meer at ISR@CERN in 1972

- Laser cooling

Stochastic cooling

Detects the information about individual particle's position, and directly apply the correction accordingly



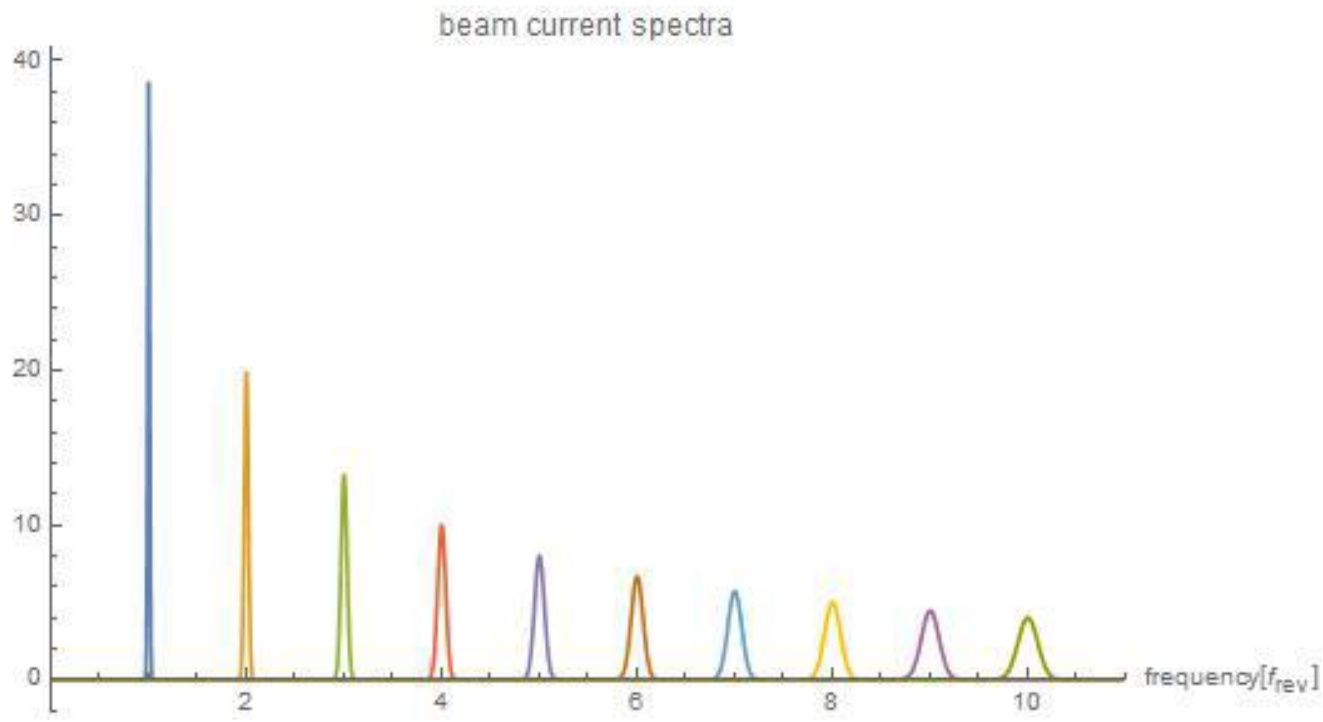
S. van der Meer, Nobel Lecture

- pickup: an RF structure to pick up the individual particle's position, aka, Schottky noise. Required bandwidth directly promotional to the number of particles in a beam as well as beam energy

- kicker: an RF structure to generate E&M fields to apply kick to individual particles and push them towards the center of the distribution

Schematic Schottky Spectrum

for a beam with Gaussian distributed longitudinal profile. Its frequency distribution is given by $\frac{\Delta f}{f} = -\eta \frac{\Delta p}{p}$. One can see that the higher harmonic, the lower the amplitude and the wider band.



Cooling rate

For the transverse cooling, the cooling rate is given by

$$\frac{1}{\tau_{x^2}} = \frac{2W}{N}(2g - g^2 M)$$

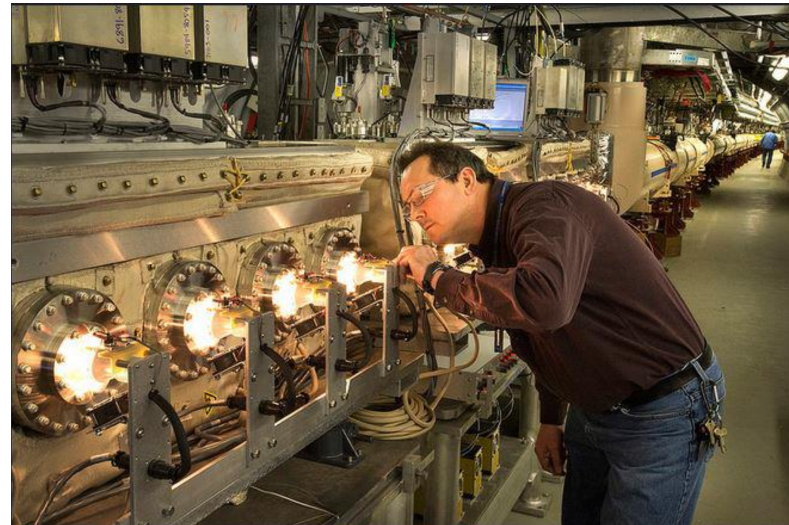
where N is the number of particle, W is the bandwidth, g is the gain factor and M is the mixing factor.

Example: Stochastic cooling at COSY

- two pairs of pickup and kicker with bandwidth 1-1.8 GHz (band 1) and 1.8-3.0 GHz (band 2). Both operate from 1.5 GeV to 3.3 GeV

Stochastic cooling at RHIC

- high energy bunched heavy ion, both transverse and longitudinal
- significantly improved RHIC heavy ion operation luminosity performance
- typical performance: cooling 2×10^9 Au or other heavy ion beam
- For longitudinal cooling, only band 1 is used in sum mode for Notch filtering



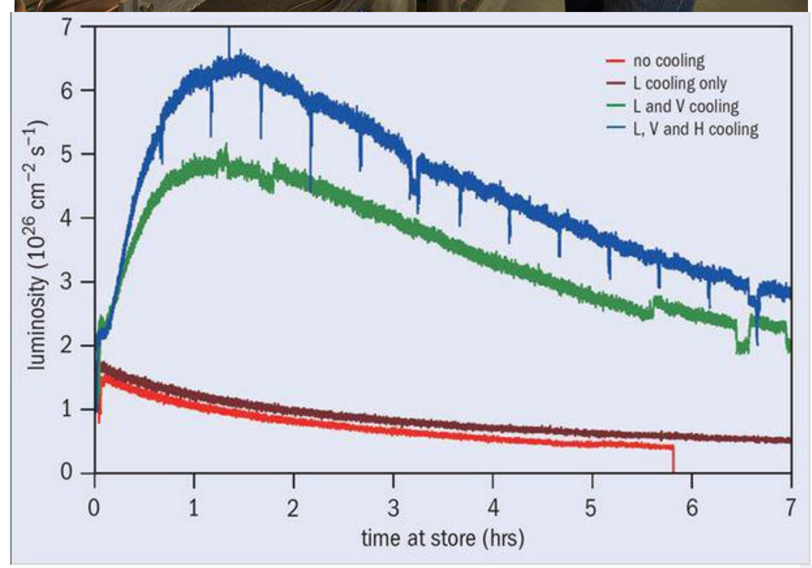


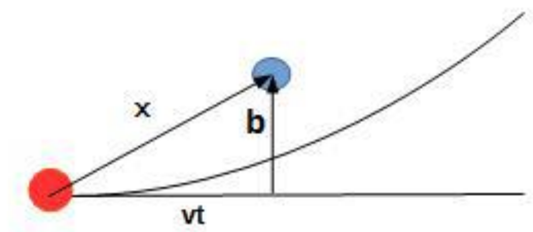
Fig. 2. Luminosity (collision rate) for stores without cooling; with longitudinal cooling only; with longitudinal and vertical cooling; and with cooling in all planes.

Electron Cooling

The idea of electron cooling was first proposed by Prof. Gersh Budker in 1966. It is to use a low to medium energy semi-monochromatic electron beams co-moving with a hadron beam to reduce its phase space.

For a traditional electron cooling, i.e. shooting ion beams through a beam of co-moving electrons, the force experienced by the ion is

$$\vec{F} = -\frac{Ze^2}{4\pi\epsilon_0|\vec{x}|^3}\vec{x}$$



where $\vec{x} = \vec{b} + \vec{v}t$, and $\vec{v}$ is the relative velocity between the electron and ion. The momentum change of the ion can then be given by

$$\Delta p_\perp = \int_{-\infty}^{\infty} -\frac{Ze^2}{4\pi\epsilon_0}\frac{b}{(vt)^2 + b^2}dt$$

Electron Cooling

Cooling force at small $\vec{v}$ is rather linear, and cooling is at its maximum strength

At large relative velocity between e-beam and ion beam, cooling force becomes rather nonlinear, and cooling rate goes down with beam energy as $1/\gamma^2$. However, at low energy, there is large probability of the recombination of electrons and charge ions can significantly impact beam lifetime

Electron cooling rate linearly dependent of electron density and cooler length

Electron cooling rate is more effective for highly charged heavy ions(A/Z^2), and independent of ion beam intensity

Electron cooling is also more effective when the velocity distribution of ion beam and electron beam overlaps

Advanced Electron cooling for EIC

Coherent electron cooling