CFNS Summer School 2021
Accelerator Physics for EIC

Mei Bai, SLAC

August 13, 2021

U.S. DEPARTMENT OF ENERGY | Stanford University
SLAC NATIONAL ACCELERATOR LABORATORY

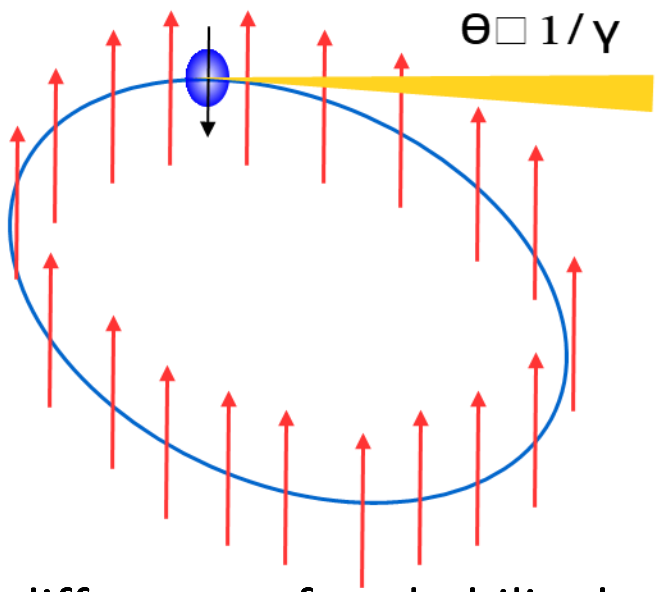
Content

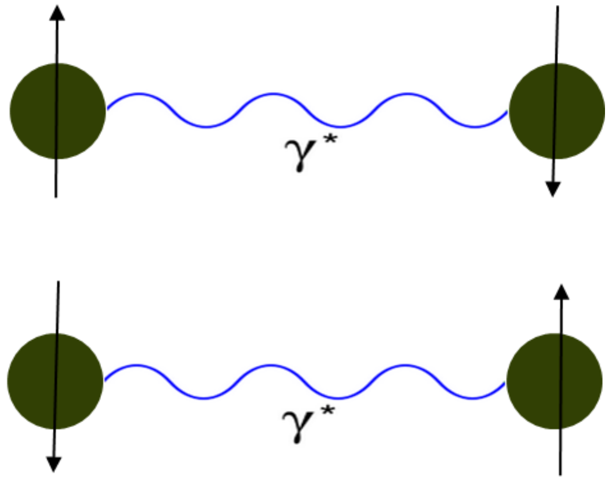
- **Introduction and accelerator fundamental**
  - Overview of US EIC current design
  - Accelerator physics fundamentals
- **Collider accelerator physics**
  - Luminosity, beam-beam effect
- **Polarized Beams in Collider**
  - Spin dynamics in circular accelerators
- **Synchrotron radiation and its applications**

Colliders with polarized beams

Polarized e+e- colliders

- As early as early 70s like ACO, VEPP-2
- Most are circular and the polarization was built up during the store time via Sokolov-Ternov effect (ST effect)





The difference of probability between the two scenarios allows the radiative polarization build up .

In a planar circular accelerator

SLAC

- The ST induced radiative polarization buildup is given

$$P(t) = P_{ST}(1 - e^{-t/\tau_{ST}}),$$

where $P_{ST} = 8/5\sqrt{3} \approx 0.9237$

and $\tau_{ST}^{-1} = \frac{\frac{5\sqrt{3}}{8} c\lambda_e r_e \gamma^5}{\rho^3} = 3654 \frac{R/\rho}{B[T]^3 E[GeV]^2}$ [sec^{-1}]

S. Mane et al, Spin-polarized charged particle bams

- For HERA, the estimated ST polarization buildup time for its 26.7 GeV electrons is about 43 mins

In a planar circular accelerator

SLAC

- In reality, the emission of a photon can yield a sudden change of the particle's energy and induce a spin diffusion mechanism that leads to loss of polarization. The equilibrium polarization is the combination of the two effects

$$P_{eq} = \frac{8}{5\sqrt{3}} \frac{\left\langle |\rho^{-3}| \hat{b} \cdot \left[\hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma} \right] \right\rangle}{\left\langle |\rho^{-3}| \left[1 - \frac{2}{9} (\hat{\beta} \cdot \hat{n})^2 + \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right] \right\rangle}$$

and the subsequent polarization buildup time is

$$\tau_{eq}^{-1} = \tau_{ST}^{-1} + \tau_d^{-1}$$

with

$$\tau_d^{-1} = \tau_{ST}^{-1} \left[-\frac{2}{9} (\hat{\beta} \cdot \hat{n})^2 + \frac{11}{18} \left| \gamma \frac{\partial \hat{n}}{\partial \gamma} \right|^2 \right]$$

In a planar circular accelerator

SLAC

- The radiative polarization buildup in HERA
  - **Best achieved polarization is around 75%**
  - **Polarization buildup time ~ 1.5 hours**

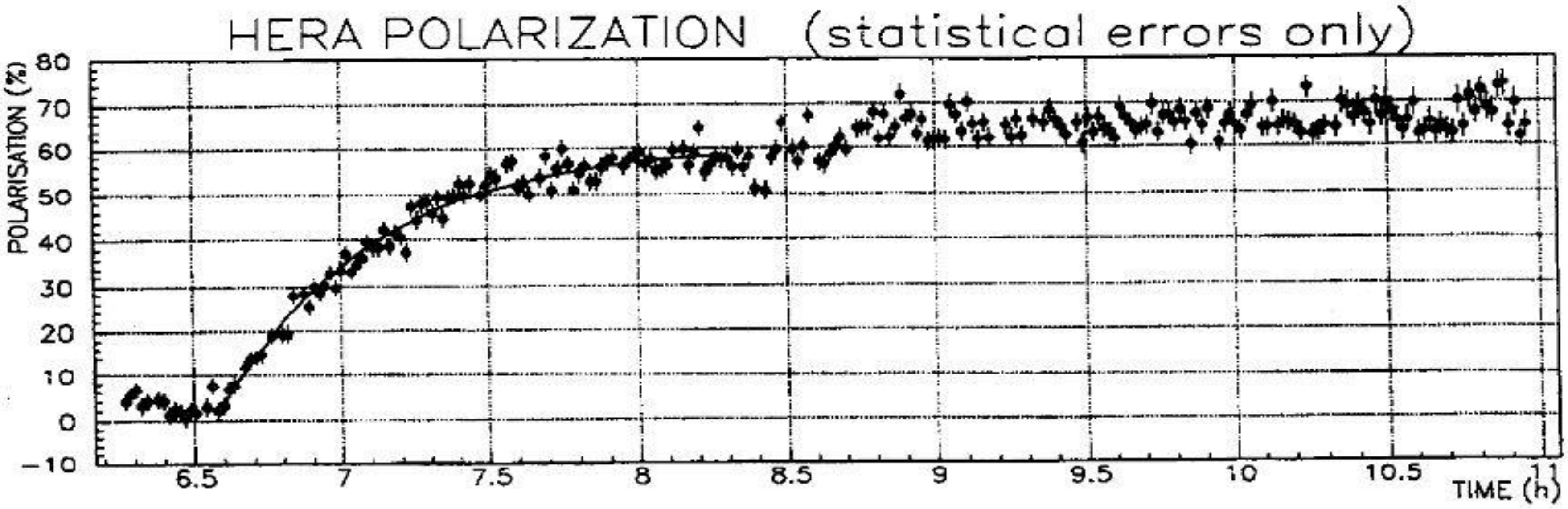


Fig. 19: Polarization P versus the time t in the storage ring HERA at 26.7 GeV.

J. Buon, J. P. Koutchouk, Polarization of Electron and Proton Beams

Spin Orbit Coupling

Thomas BMT Equation: (1927, 1959)

L. H. Thomas, Phil. Mag. 3, 1 (1927); V. Bargmann, L. Michel, V. L. Telegdi, Phys, Rev. Lett. 2, 435 (1959)

Spin vector in particle's rest frame

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1 + G\gamma)\vec{B}_{\perp} + (1 + G)\vec{B}_{\parallel} + \left(G - \frac{\gamma}{\gamma^2 - 1}\right)\frac{\vec{E} \times \vec{\beta}}{c}]$$

Magnetic field perpendicular to the particle's velocity

Magnetic field perpendicular to the particle's velocity

$$\frac{d\vec{S}}{ds} = \boldsymbol{\Omega}(x, p_x, y, p_y, z, \delta)\hat{\boldsymbol{n}} \times \vec{\boldsymbol{S}}$$

- stable spin direction $\hat{n}$, an invariant direction that spin vector aligns to, when the particle returns to the same phase space

$$\hat{n}(I_z, \phi_z, \theta) = \hat{n}(I_z, \phi_z + 2\pi, \theta)$$

Here, I_z and ϕ_z are the 6-D phase-space coordinates $(x, p_x, y, p_y, z, \delta)$

- For particles on closed orbit, stable spin direction can be computed through one-turn spin transfer matrix. $\hat{n}$ is also know as $\hat{n}_0$

Stable Spin Direction

- $\hat{n}_{co}(\vec{I}_z, f_z, q)$ is function of phase space
- For particles on closed orbit, stable spin direction can be computed through one-turn spin transfer matrix. $\hat{n}_{co}$ is also know as $\hat{n}_0$
- For particles not on closed orbit, since in general the betatron tune is non-integer, the stable spin direction is no longer the eigen vector of one turn spin transfer matrix. Algorithms like SODOM[1,2], SLIM[3], SMILE[4] were developed to compute the stable spin direction

[1] K. Yokoya, Non-perturbative calculation of equilibrium polarization of stored electron beams, KEK Report 92-6, 1992

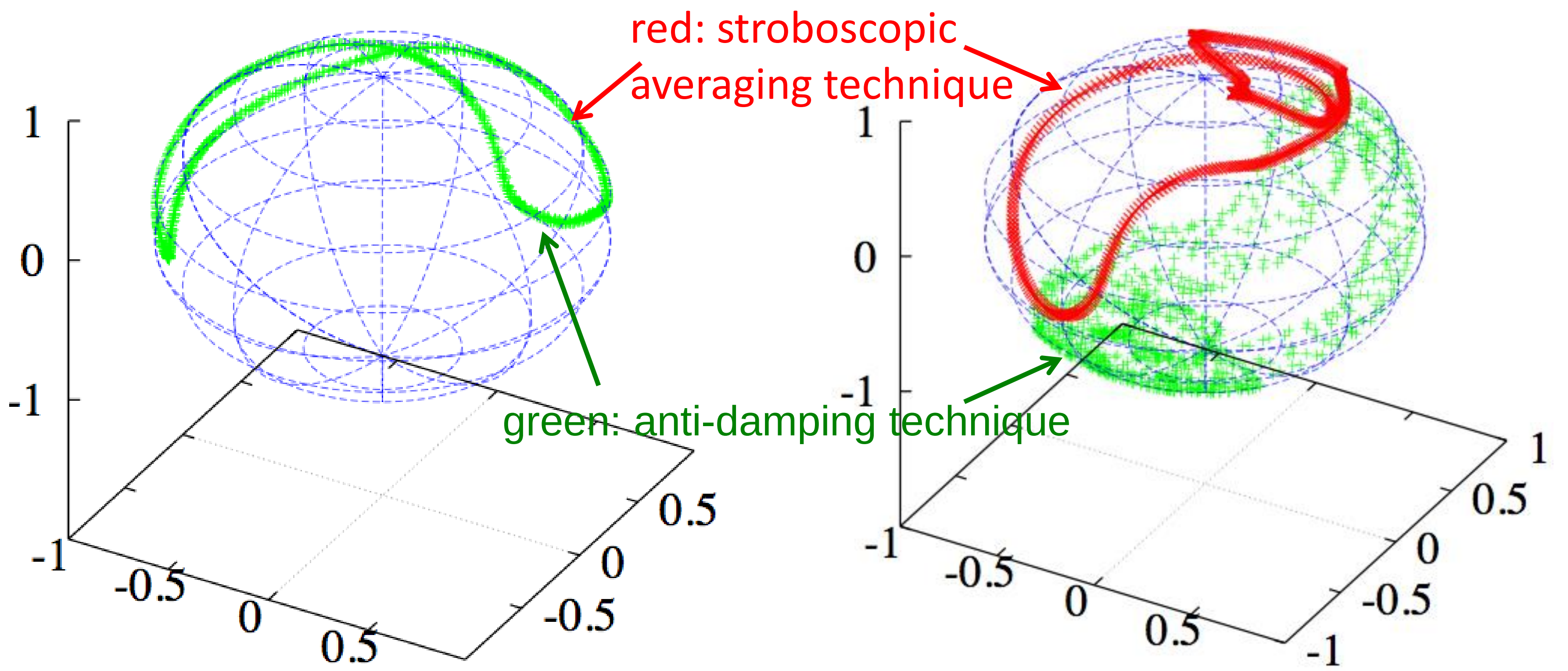
[2] K. Yokoya, An Algorithm for Calculating the Spin Tune in Accelerators, DESY 99-006, 1999

[3] A. Chao, Nucl. Instr. Meth. 29 (1981) 180

[4] S. R. Mane, Phys. Rev. A36 (1987) 149

Stable Spin Direction

- Particles on a 20π mm-mrad phase space
- Particles on a 40π mm-mrad phase space



D. P. Barber, M. Vogt, The Amplitude Dependent Spin Tune and The Invariant Spin Field in High Energy Proton Accelerators, Proceedings of EPAC98

Spin Orbit Coupling

Thomas BMT Equation: (1927, 1959)

L. H. Thomas, Phil. Mag. 3, 1 (1927); V. Bargmann, L. Michel, V. L. Telegdi, Phys, Rev. Lett. 2, 435 (1959)

Spin vector in particle's rest frame

$$\frac{d\vec{S}}{dt} = \frac{e}{\gamma m}\vec{S} \times [(1 + G\gamma)\vec{B}_{\perp} + (1 + G)\vec{B}_{\parallel} + \left(G - \frac{\gamma}{\gamma^2 - 1}\right)\frac{\vec{E} \times \vec{\beta}}{c}]$$

Magnetic field perpendicular to the particle's velocity

Magnetic field perpendicular to the particle's velocity

$$\Rightarrow \quad \frac{d\vec{S}}{ds} = \boldsymbol{\Omega}(x, p_x, y, p_y, z, \delta)\hat{\boldsymbol{n}} \times \vec{\boldsymbol{S}}$$

- stable spin direction $\hat{n}$, an invariant direction that spin vector aligns to, when the particle returns to the same phase space

$$\hat{n}(\boldsymbol{I_z}, \boldsymbol{\phi_z}, \boldsymbol{\theta}) = \hat{\boldsymbol{n}}(\boldsymbol{I_z}, \boldsymbol{\phi_z} + \boldsymbol{2\pi}, \boldsymbol{\theta})$$

Here, I_z and ϕ_z are the 6-D phase-space coordinates $(x, p_x, y, p_y, z, \delta)$

- Spin tune Q_s: # of spin precession in one orbital revolution

$$Q_s = \mathrm{G}\gamma$$

Depolarizing mechanism in a synchrotron

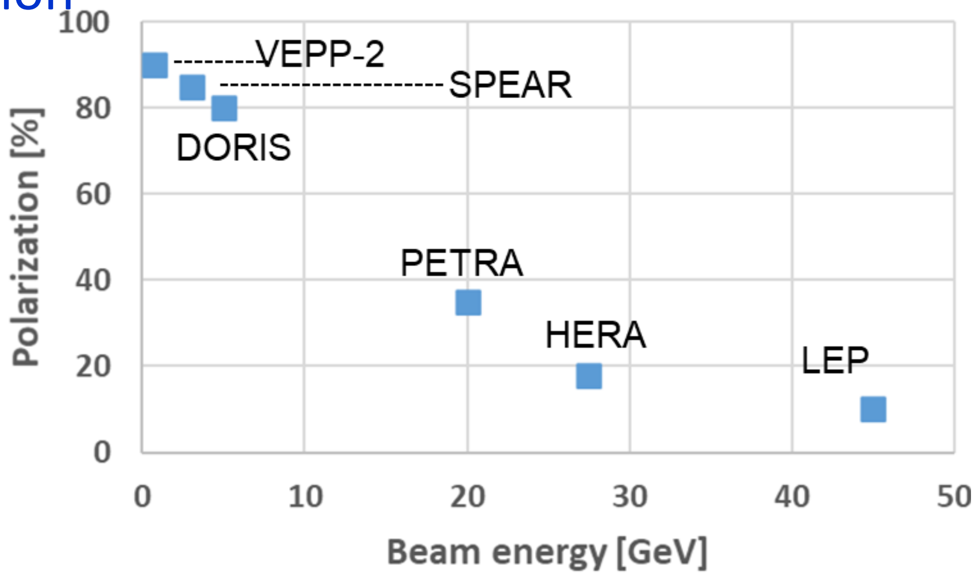
SLAC

- For particles not on closed orbit, since the betatron tunes are typically non-integer, $\hat{n}$ can be significantly away from $\hat{n}_0$ when

$$Q_s = k + k_x Q_x + k_y Q_y + k_z Q_z$$

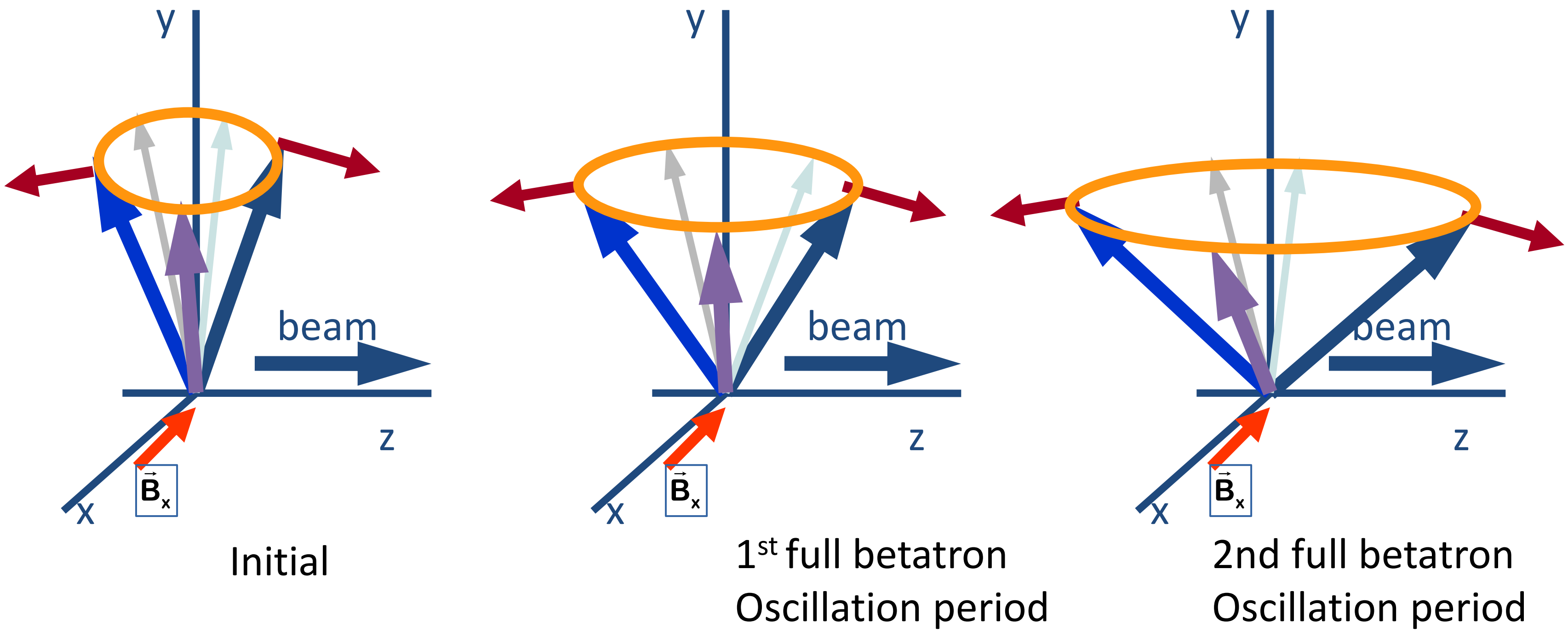
where k_x, k_y, k_z are horizontal, vertical and synchrotron tunes, respectively.

- These resonances contribute to the depolarization time and result to much less equilibrium polarization



Depolarizing mechanism in a synchrotron

- horizontal field kicks the spin vector away from its vertical direction, and can lead to polarization loss



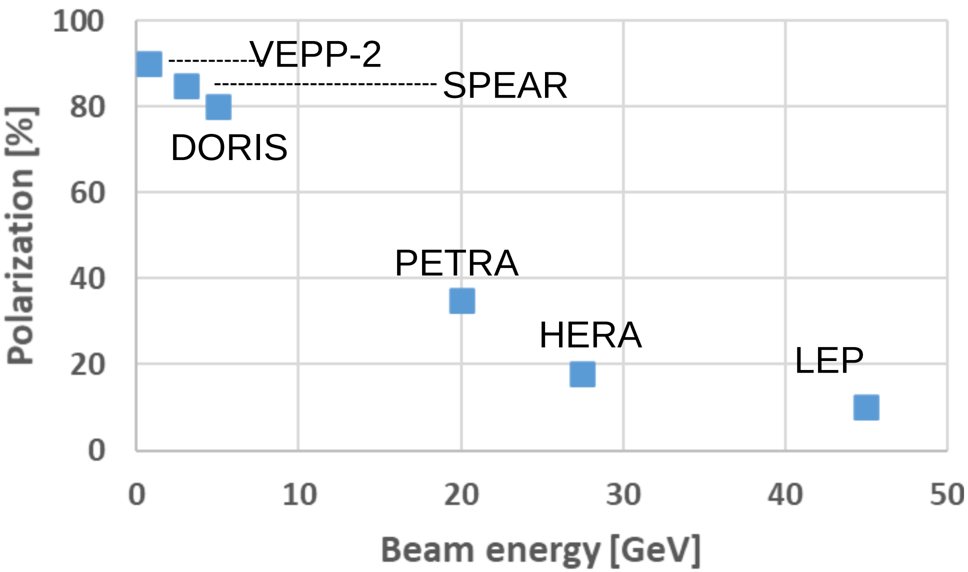
Depolarizing mechanism in a synchrotron

- For particles not on closed orbit, since the betatron tunes are typically non-integer, $\hat{n}$ can be significantly away from $\hat{n}_0$ when

$$Q_s = k + k_x Q_x + k_y Q_y + k_z Q_z$$

where k_x, k_y, k_z are horizontal, vertical and synchrotron tunes, respectively.

- These resonances contribute to the depolarization time and result to much less equilibrium polarization
- Sources of these resonances
  - Miss-alignment of quadrupole
  - Devices that deviate $\hat{n}$ from $\hat{n}_0$
  - Other high order fields



Overcome depolarizing mechanism

- In general, the effect of these resonances grows with energy. For planar electron storage rings, a simply scaling law*

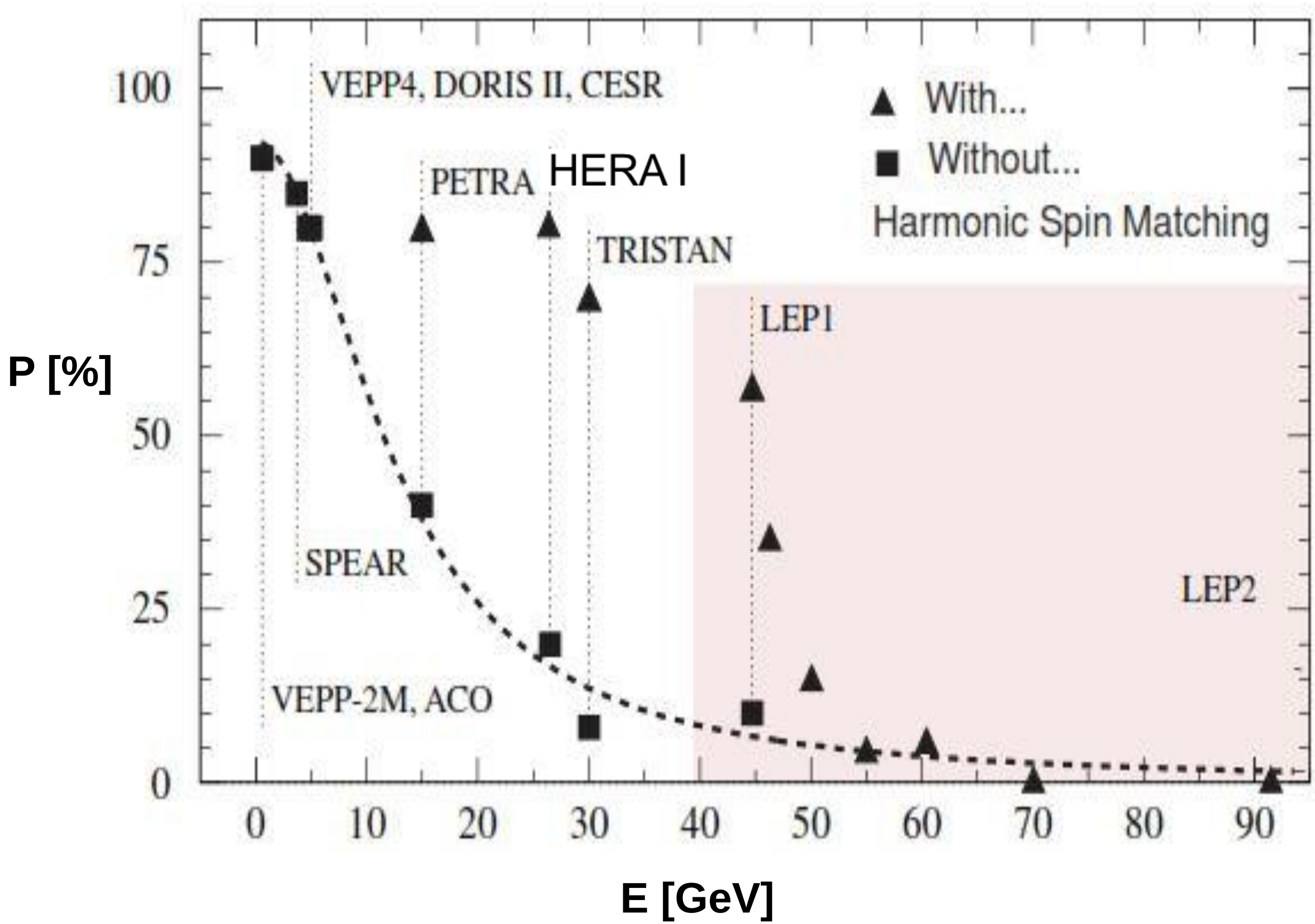
$$p_{eq} \approx \frac{92.4\%}{1+\alpha^2 E^2}$$

Where α is the lattice related factor

- To overcome these resonances in a storage ring, it is critical to either break the resonance condition such as utilizing Siberian snakes, or adapt the lattice optics to minimize the spin orbit coupling strength $\left|\gamma \frac{\partial \hat{n}}{\partial \gamma}\right|^2 \sim (1+G\gamma)^2 \sum_k |c_k|^2/(G\gamma - k)^2$ via spin matching
  - Strong spin matching: full spin transparent at all harmonics
    - Practically very difficult
  - Harmonic spin matching: minimize the driving term at the nearby harmonics
    - Has been implemented in various rings

* S R Mane, Yu M Shatunov and K Yokoya, *Spin-polarized charged particle beams in high-energy accelerators*, Rep. Prog. Phys. 68 (2005) 1997–2265

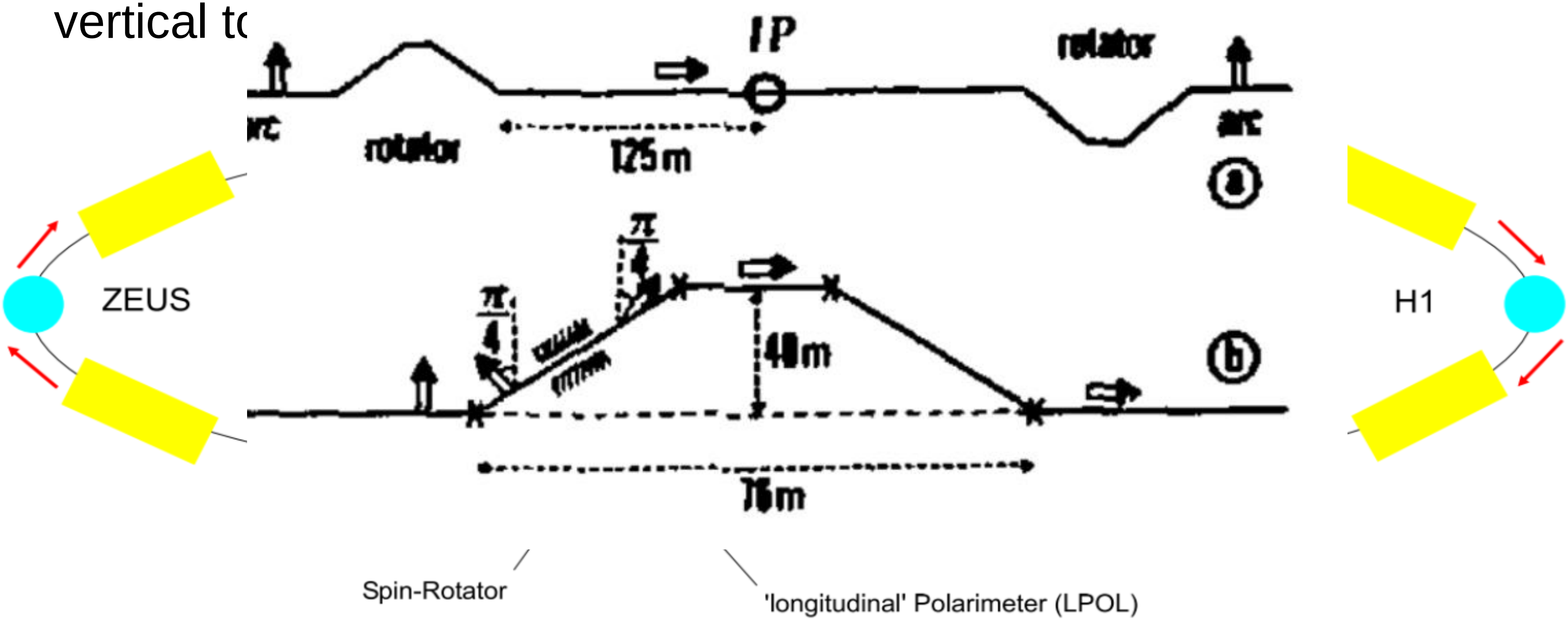
Achieved Performance of Polarized e Beams



A Brief History of the LEP Collider, R. Assmann, M. Lamont, S. Myers for the LEP team

HERA polarization

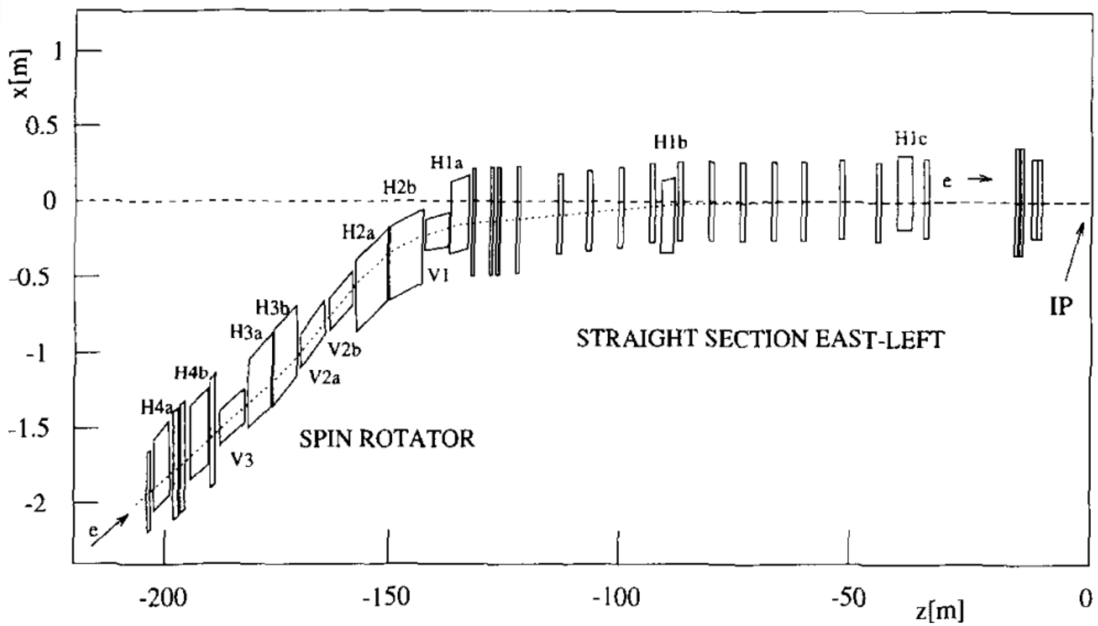
- HERA was the 1st high energy collider, that employed local spin rotators to provide longitudinally polarized electron
- A spin rotator consists of a sequence of horizontal and vertical orbit correctors that interleaves with each other to precess spin vector from vertical t



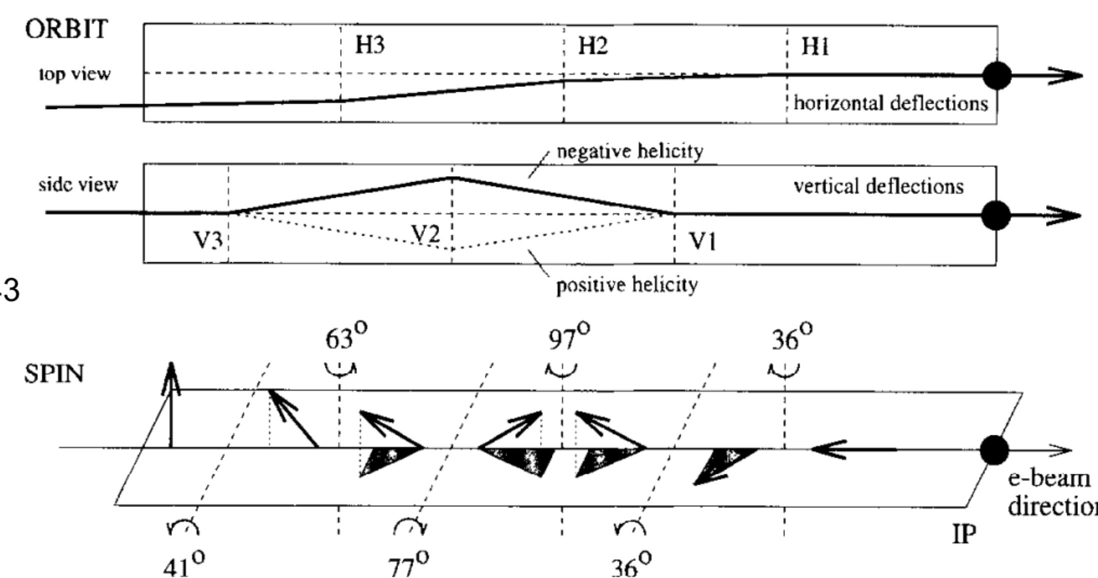
HERA polarization

- A spin rotator induces large orbital excursions in both planes and tilts the $\hat{n}$ away from vertical

HERA polarization



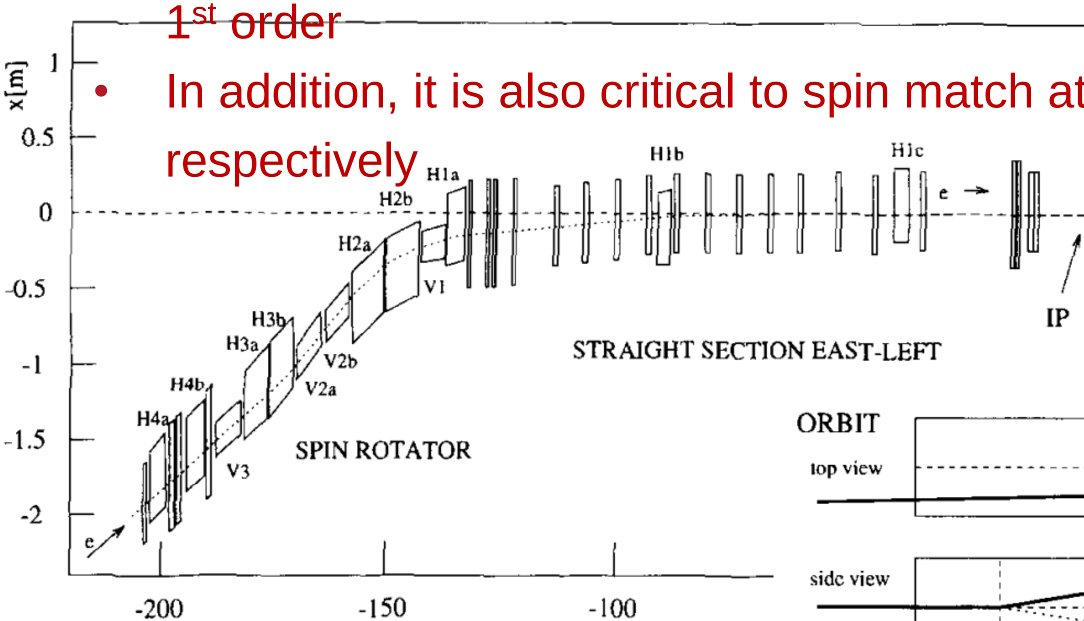
Vertical orbital bump ~ 20mm



D.P. Barber et al. /Physics Letters B 343 (1995) 436-443

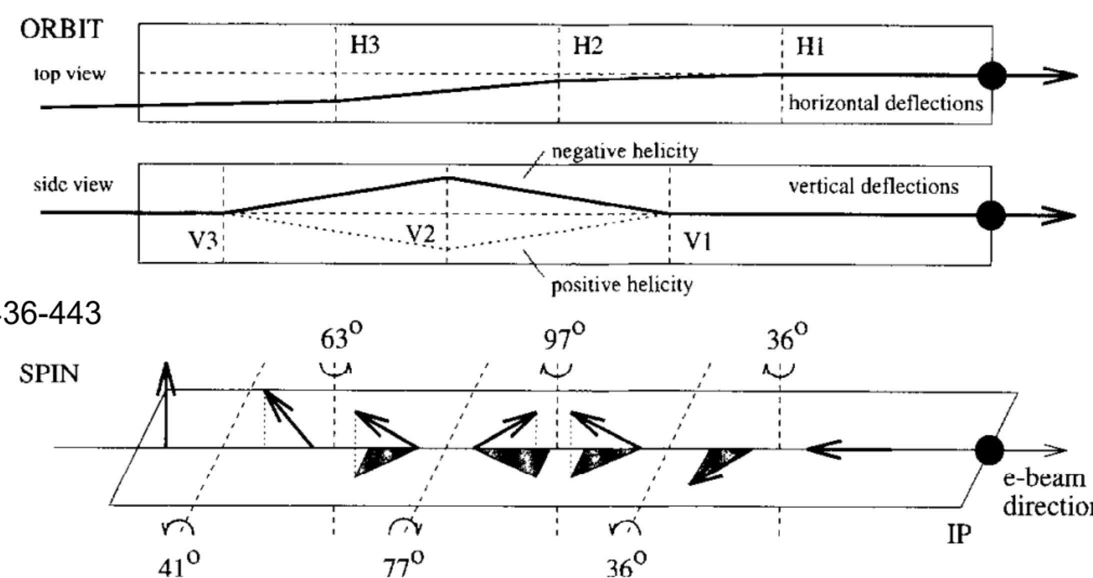
HERA polarization

- A spin rotator induces large orbital excursions in both planes and tilts the $\hat{n}$ away from vertical
  - Spin matching to make the section between spin rotators spin transparent to the 1st order
  - In addition, it is also critical to spin match at the entrance and exit of the rotator, respectively



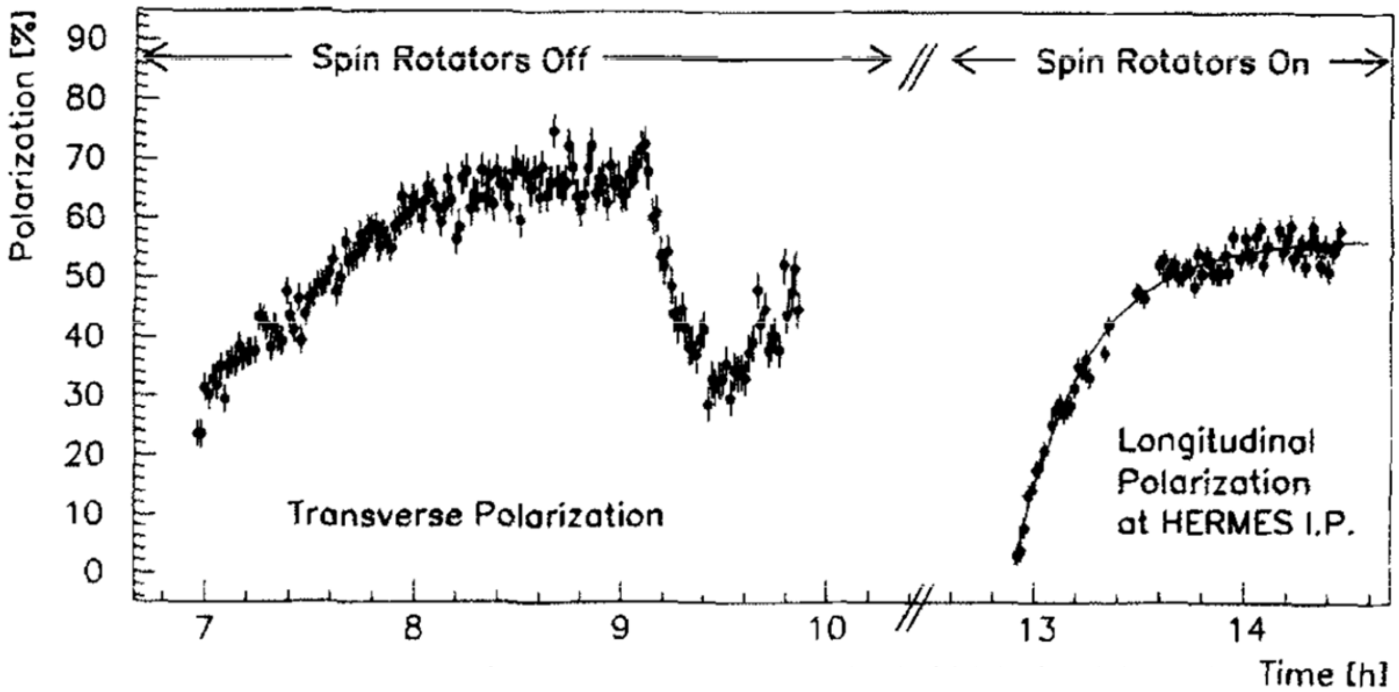
D.P. Barber et al. /Physics Letters B 343 (1995) 436-443

Vertical orbital bump ~ 20mm



HERA polarization

- With the HEAR mini-rotator



•Polarization was later-on improved to 65% after a dedicated spin-match optics was implemented

D.P. Barber et al. /Physics Letters B 343 (1995) 436-443

HERA polarization

- With 3 pairs of rotators

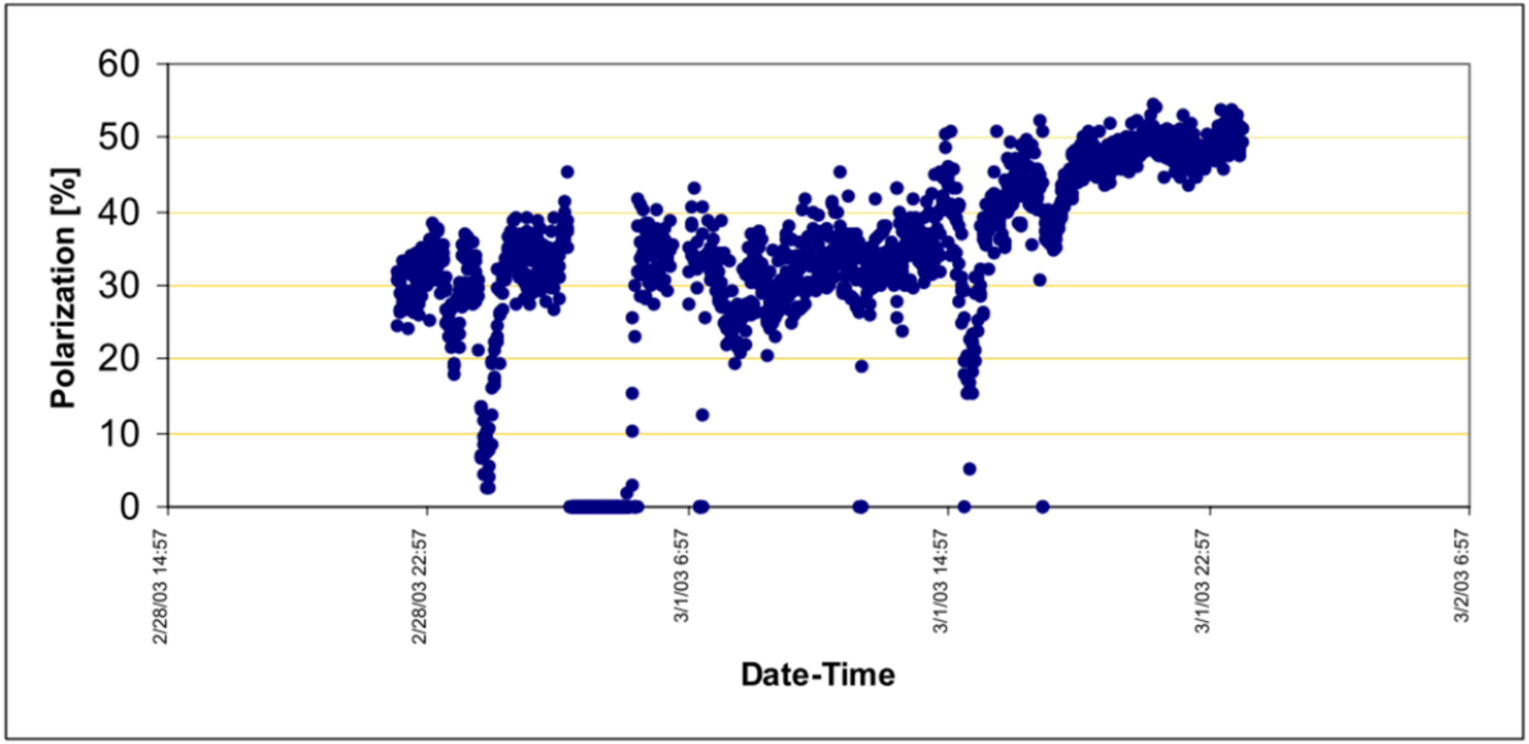


Figure 1: Polarization optimizations with 3 pairs of spin rotators in HERA-e on the 1st of March 2003. A polarization of 54% was ultimately obtained.

Georg Hoffstaetter et al, Experiences with the HERA beams, ICFA Newsletter May 2003

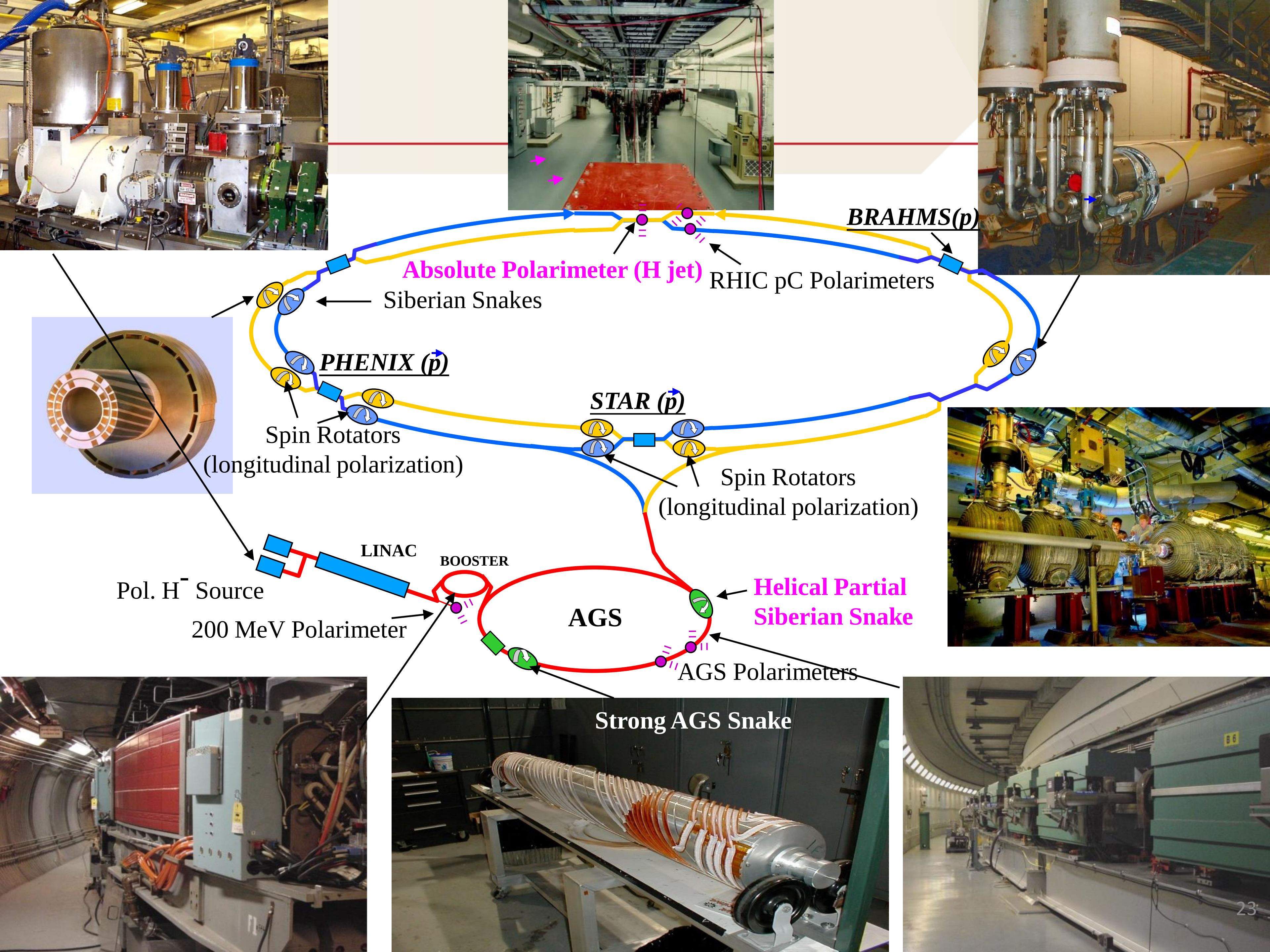
Colliders with polarized beams

Polarized hadron colliders:

- RHIC@BNL: polarized protons

Unlike the e+e- colliders, polarized beam starts from the source, and polarization need to survive through acceleration chain

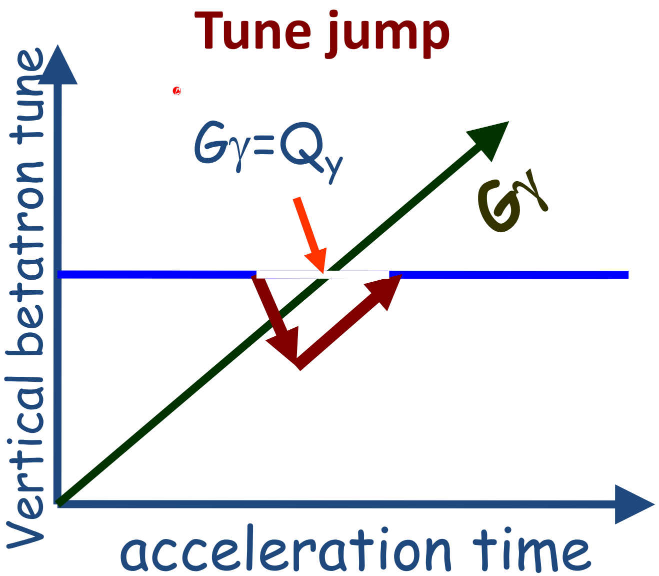
- Polarized ion source
- Pre-Injector: LINAC, booster
- Injector
- Collider


Absolute Polarimeter (H jet)
RHIC pC Polarimeters
BRAHMS(p)
Siberian Snakes
PHENIX (p)
STAR (p)
Spin Rotators
(longitudinal polarization)
Spin Rotators
(longitudinal polarization)
LINAC
BOOSTER
Pol. H⁻ Source
200 MeV Polarimeter
AGS
Helical Partial
Siberian Snake
AGS Polarimeters
Strong AGS Snake

Overcoming Depolarizing Resonance

- **Harmonic orbit correction**
  - to minimize the closed orbit distortion at all imperfection resonances
  - Operationally difficult for high energy accelerators

- **Tune Jump**

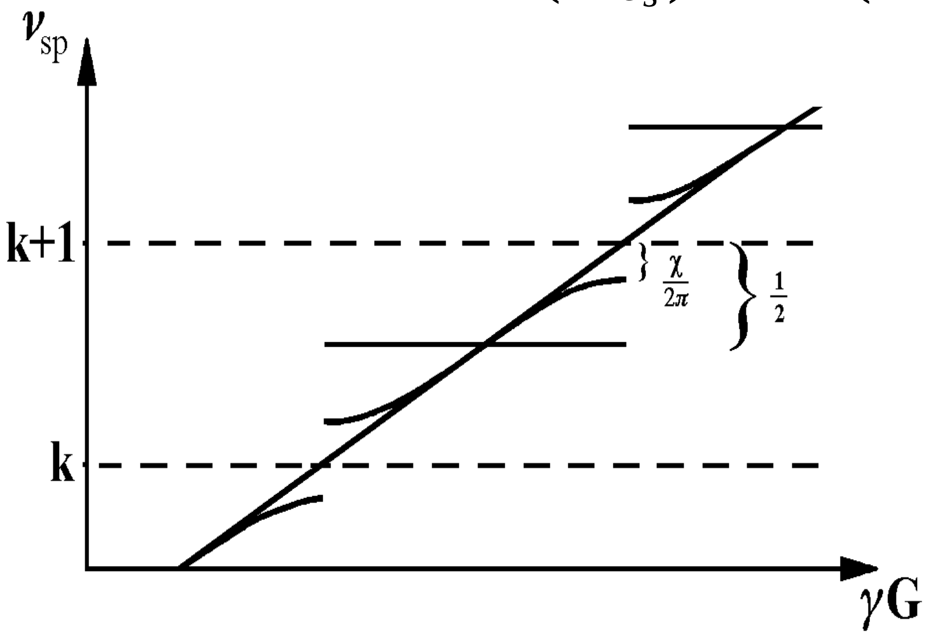


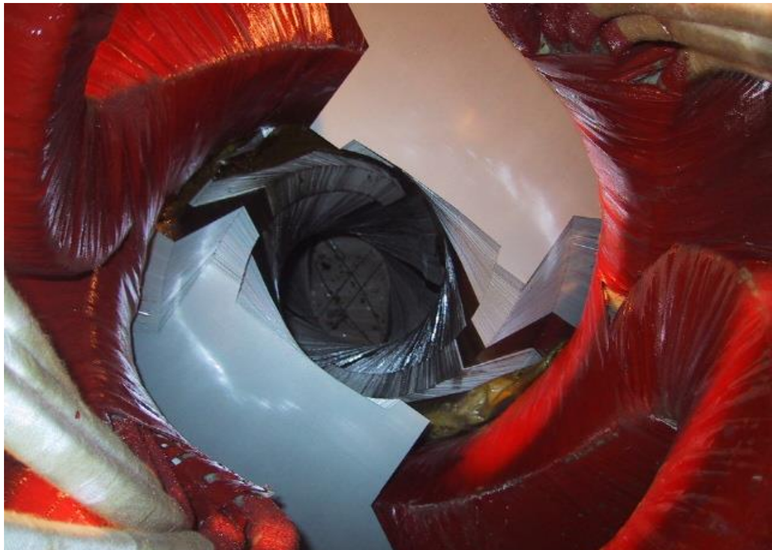
- *Operationally difficult because of the number of resonances*
- *Also induces emittance blowup because of the non-adiabatic beam manipulation*

Partial Siberian Snake

- rotates spin vector by an angle of ψ<180º
- Keeps the spin tune away from integer
- Primarily for avoiding imperfection resonance
- Can be used to avoid intrinsic resonance as demonstrated at the AGS, BNL.

$$\cos(\pi Q_s) = \cos(G\gamma\pi)\cos(\frac{\psi}{2})$$





Dual partial snake configuration

SLAC

- For two partial snakes apart from each other by an angle of ϑ, spin tune the becomes

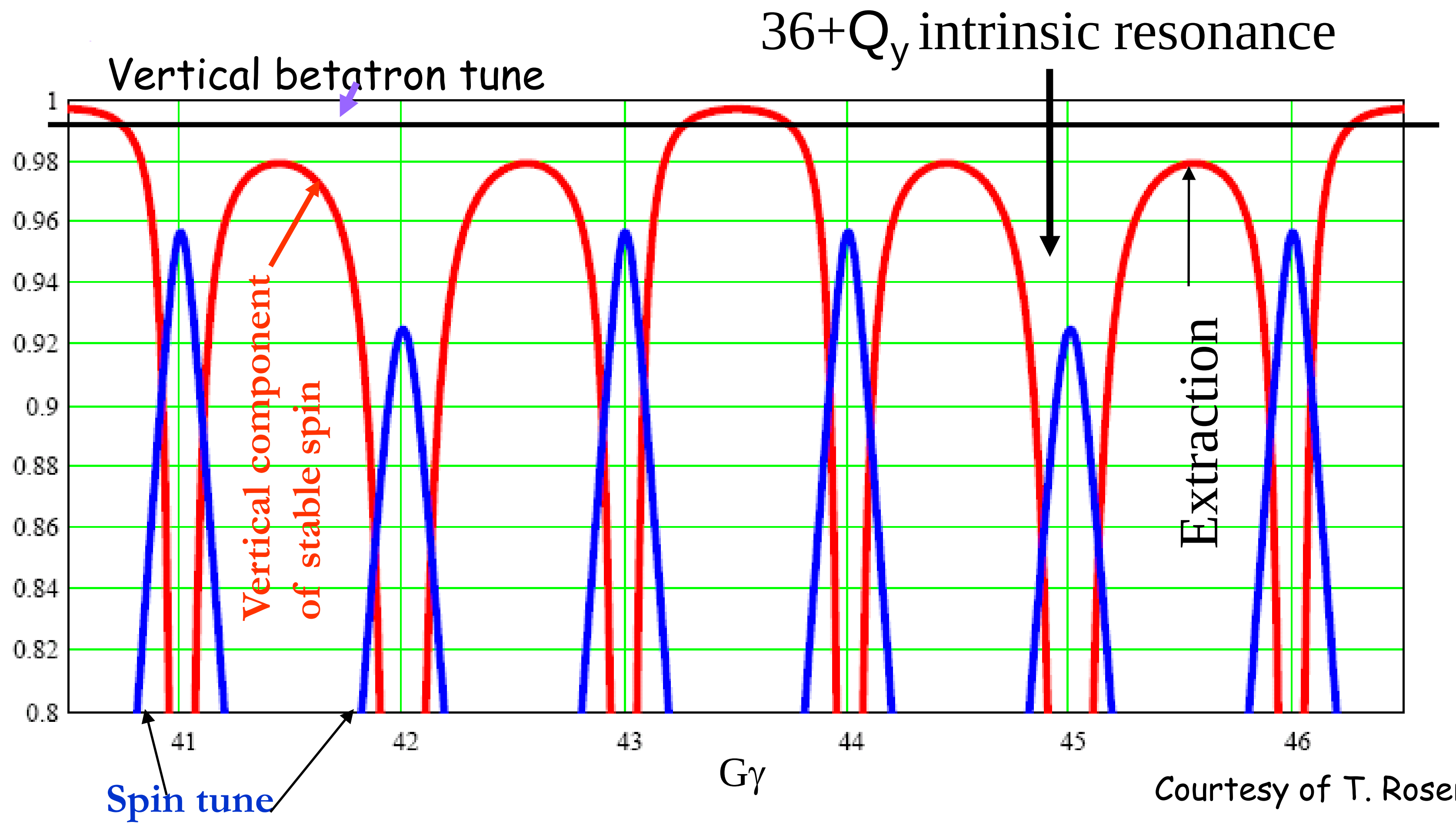
$$\cos\pi Q_s = \cos G\gamma\pi \cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \cos(G\gamma(\pi - \theta))\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}$$

- Spin tune is no-longer integer, and stable spin direction is also tilted away from vertical

- The distance between spin tune and integer is modulated with Int[360/ϑ]. For every integer of Int[360/ϑ] of Gγ, the two partial snakes are effectively added. This provides a larger gap between spin tune and integer, which can be wide enough to have the vertical tune inside the gap to avoid both intrinsic and imperfection resonance

- Stable spin direction is also modulated

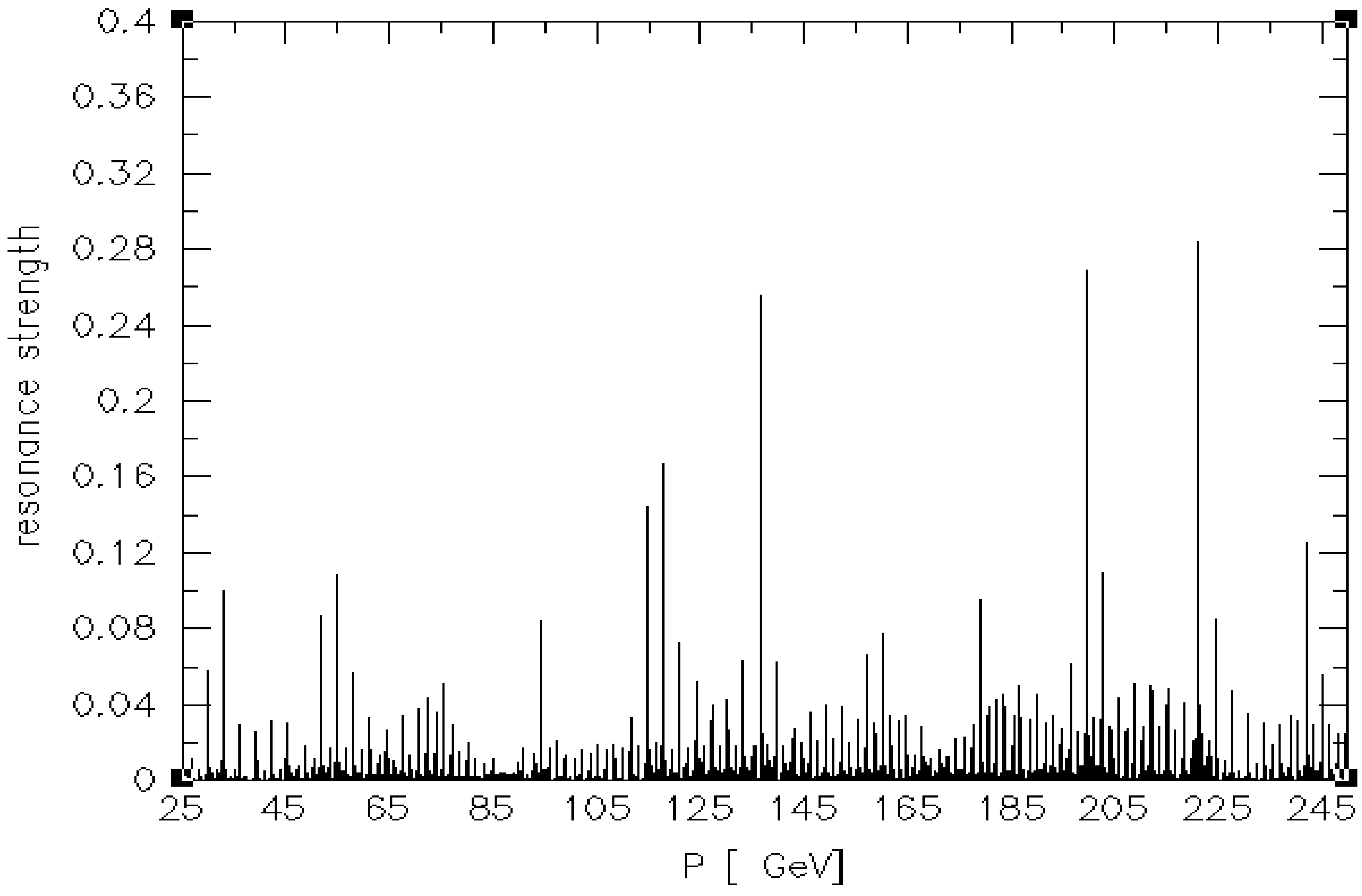
Spin tune with two partial snakes



Courtesy of T. Roser

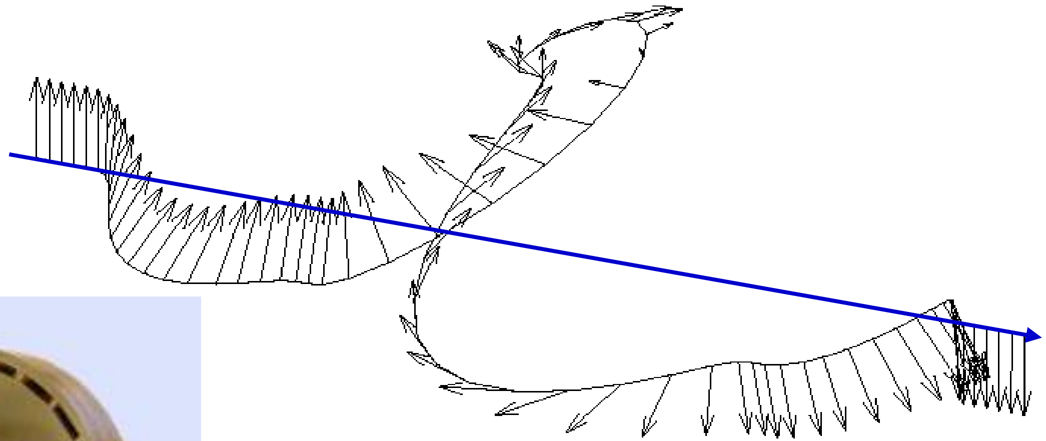
$$\cos\pi Q_s = \cos G\gamma\pi \cos\frac{\Psi_w}{2}\cos\frac{\Psi_c}{2} - \cos G\gamma\frac{\pi}{3}\sin\frac{\Psi_w}{2}\sin\frac{\Psi_c}{2}$$

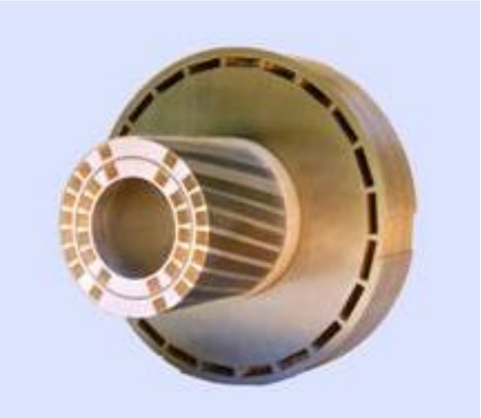
RHIC Intrinsic Spin Depolarizing Resonance



Full Siberian Snake

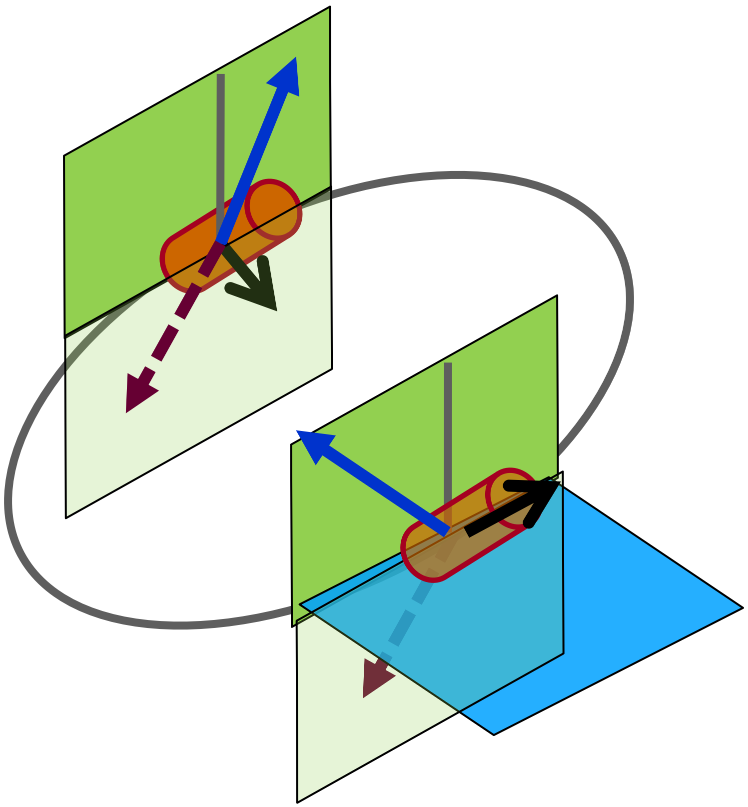
- A magnetic device to rotate spin vector by 180°
- Invented by Derbenev and Kondratanko in 1970s [*Polarization kinematics of particles in storage rings, Ya.S. Derbenev, A.M. Kondratenko* (Novosibirsk, IYF) . Jun 1973. Published in Sov.Phys.JETP 37:968-973,1973, Zh.Eksp.Teor.Fiz 64:1918-1929]
- Keep the spin tune independent of energy



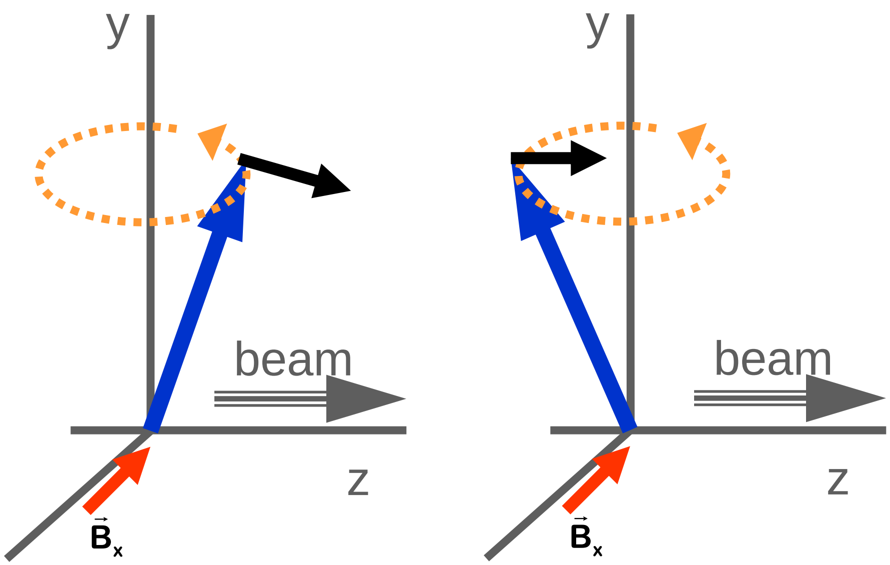


Principle of full Siberian snake

- Use one or a group of snakes to make the spin tune to be at ½



- Break the coherent build-up of the perturbations on the spin vector



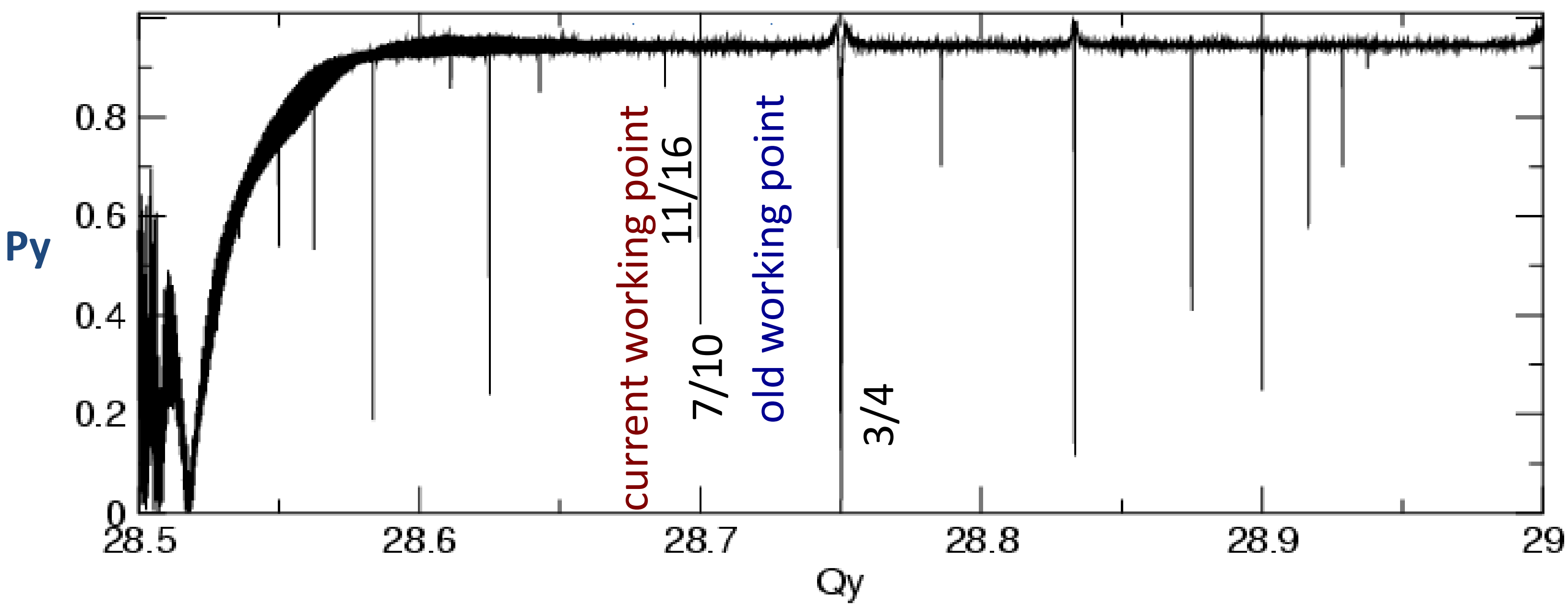
Snake Depolarization Resonance

- **Condition**

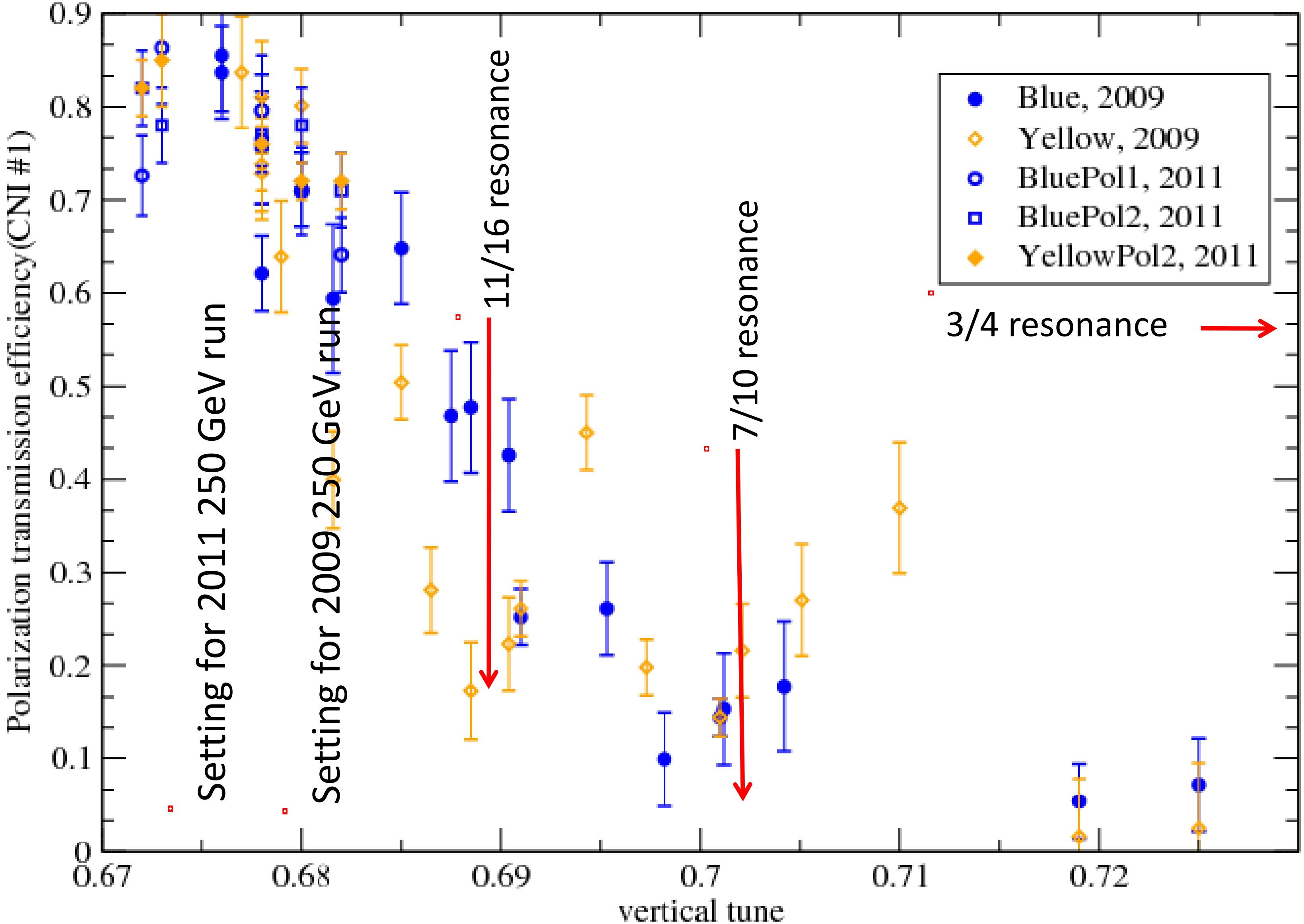
- S. Y. Lee, Tepikian, Phys. Rev. Lett. 56 (1986) 1635
- S. R. Mane, NIM in Phys. Res. A. 587 (2008) 188-212

$$mQ_y = Q_s + k$$

- **even order resonance**
  - Disappears in the two-snake case if the closed orbit is perfect
- **odd order resonance**
  - Driven by the intrinsic spin resonances



Snake resonance observed in RHIC



How to avoid a snake resonance?

SLAC

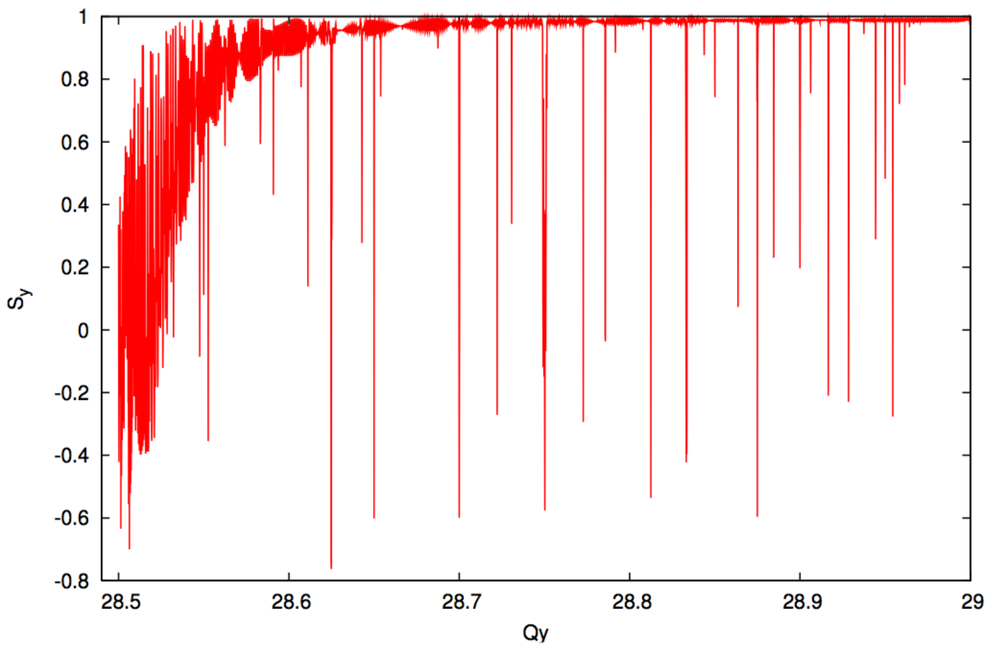
- Adequate number of snakes

$$N_{snk} > 4\left|\epsilon_{k,\max}\right| \qquad Q_s = \sum_{k=1}^{N_{snk}} (-1)^k \phi_k$$

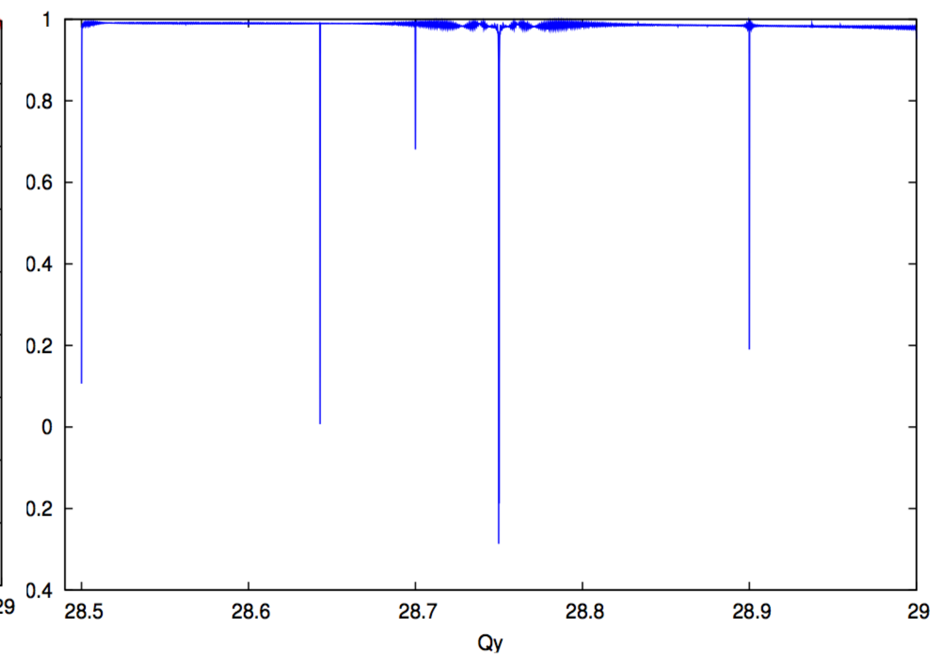
ϕ_k is the snake axis relative to the beam direction

- Minimize number of snake resonances to gain more tune spaces for operations

He-3 with dual snake



He-3 with six-snake



Avoid polarization losses due to snake resonance

- Adequate number of snakes

$$N_{snk} > 4|\epsilon_{k,\max}| \qquad Q_s = \sum_{k=1}^{N_{snk}} (-1)^k \phi_k$$

ϕ_k is the snake axis relative to the beam direction

- Keep spin tune as close to 0.5 as possible
  - **Source of spin tune deviation**
    - Snake configuration
    - Local orbit at snakes as well as other spin rotators. For RHIC,

angle between two snake axes

$$\Delta Q_s = \frac{|\Delta\phi|}{\pi} + (1+G\gamma)\frac{\Delta\theta}{\pi}$$

H orbital angle between two snakes

  - **Source of spin tune spread**
    - momentum dependence due to local orbit at snakes
      - equalize the dispersion primes at both snakes
    - betatron amplitude dependence

How to avoid a snake resonance?

- Adequate number of snakes

- Keep spin tune as close to 0.5 as possible

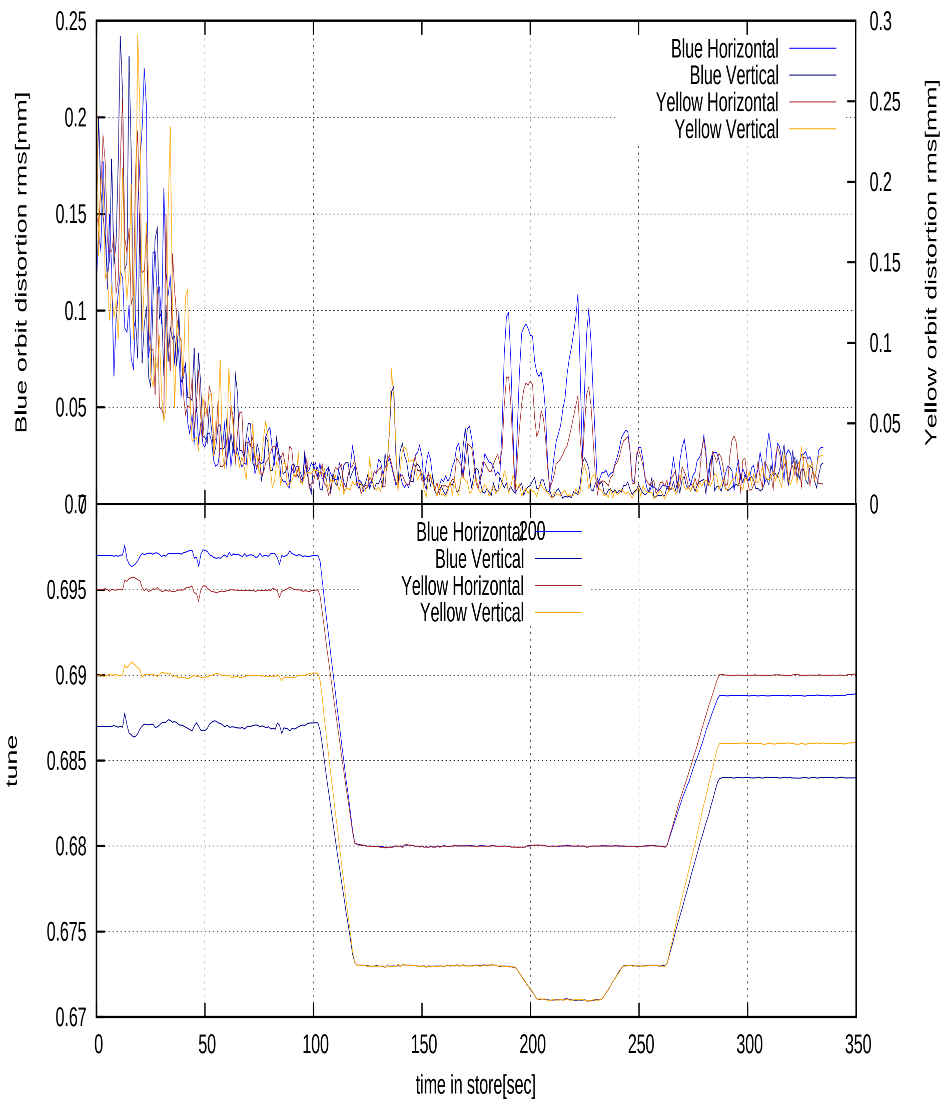
- Precise control of the vertical closed orbit

- Precise optics control
  - Choice of working point to avoid snake resonances
  - Minimize the linear coupling to avoid the resonance due to horizontal betatron oscillation

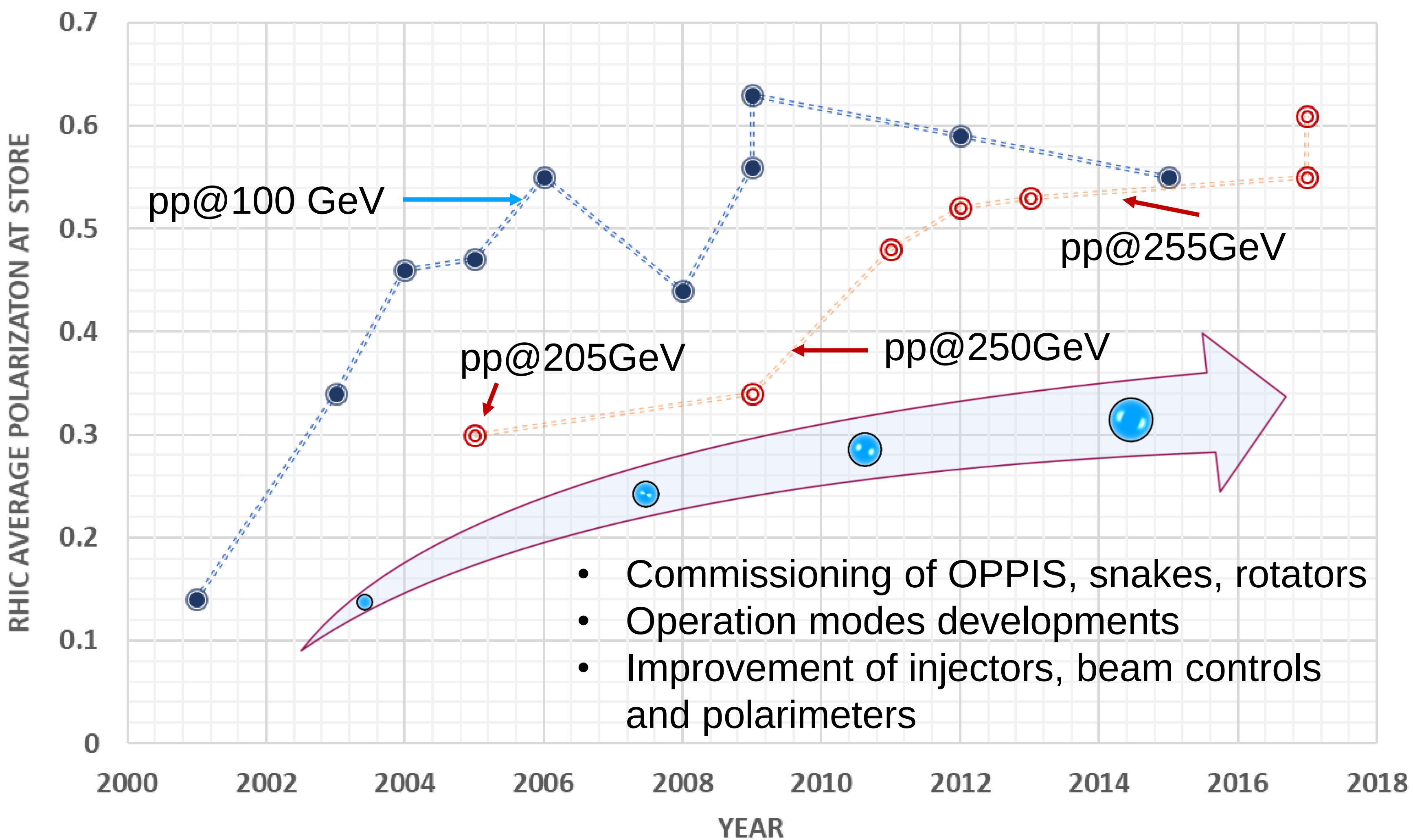
Precise Beam Control

- Tune/coupling feedback system: acceleration close to 2/3 orbital resonance

- Orbit feedback system: rms orbit distortion less than 0.1mm

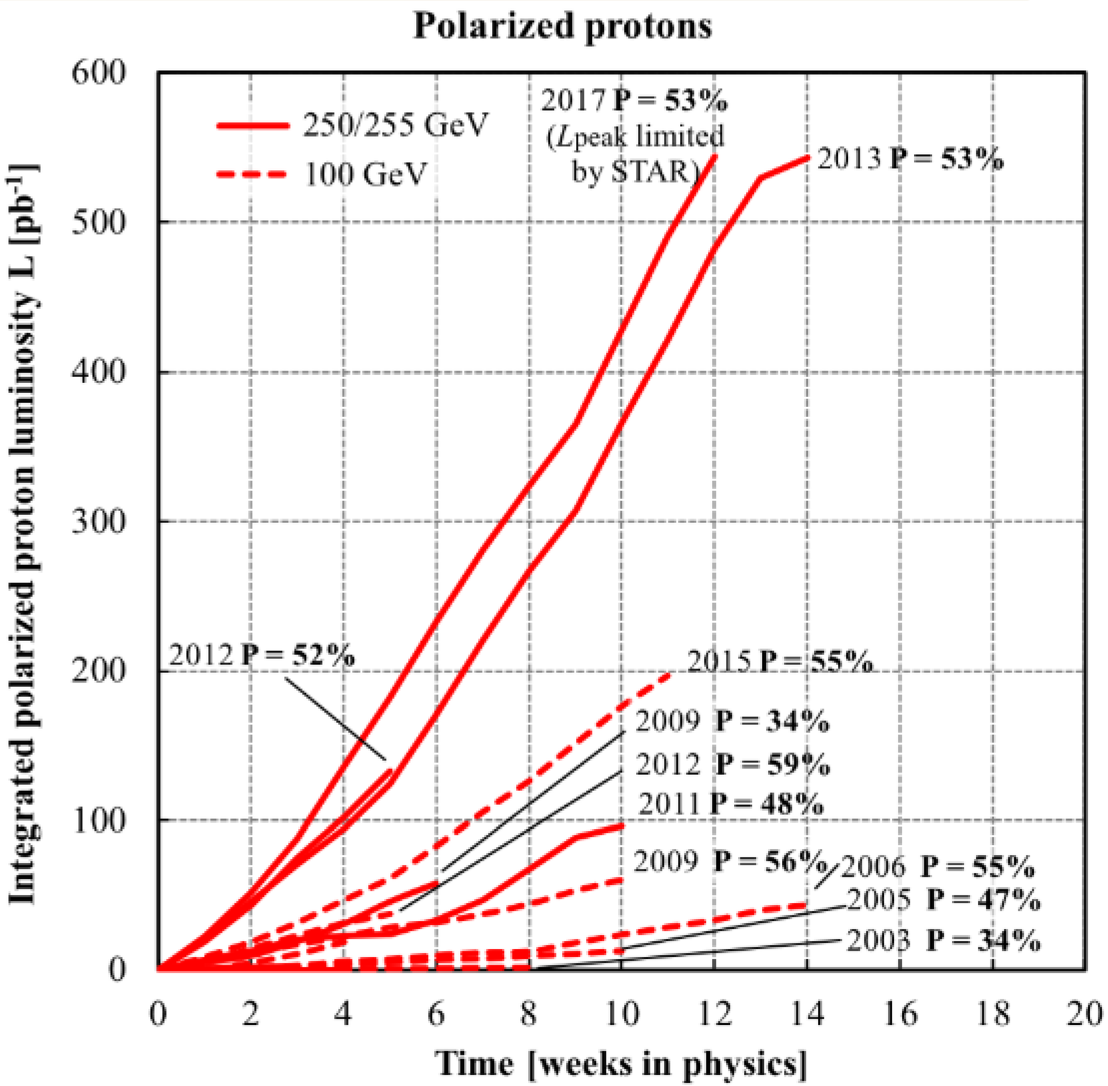


RHIC Polarization Performance



RHIC, the world's 1st high energy pp collider

SLAC



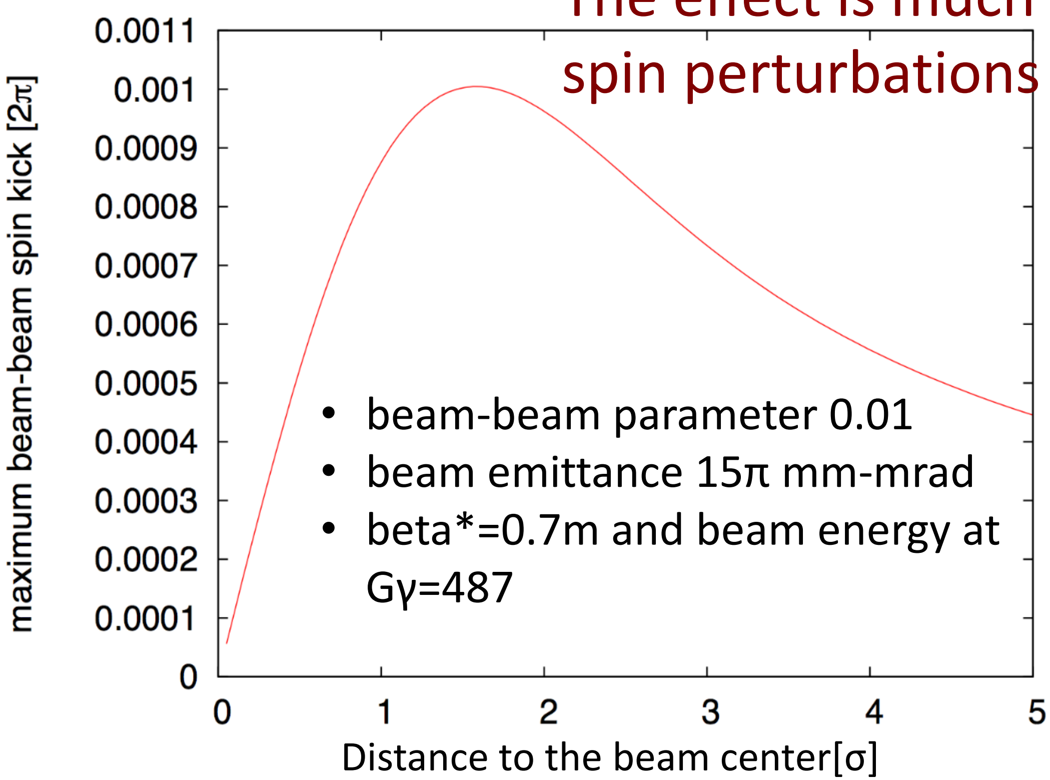
https://www.agsrhichome.bnl.gov/RHIC/Runs/

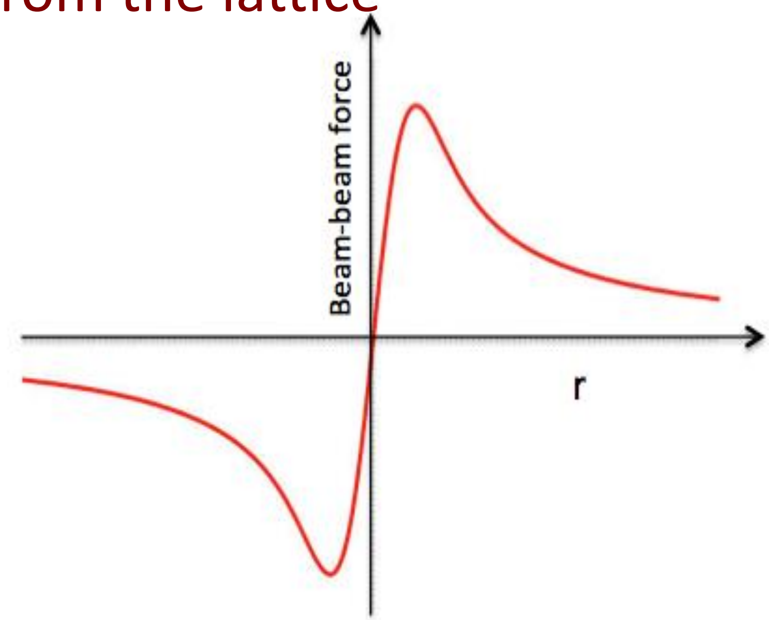
Beam-beam Effect on Polarization

- Beam-Beam force on spin motion
  - For a Gaussian round beam, particle from the other beam sees

$$\vec{E} = \frac{qN}{2\pi\epsilon_0 l r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]\hat{r} \qquad \vec{B} = \frac{1}{c}\vec{\beta}\times\vec{E}$$

The effect is much weaker than the spin perturbations from the lattice



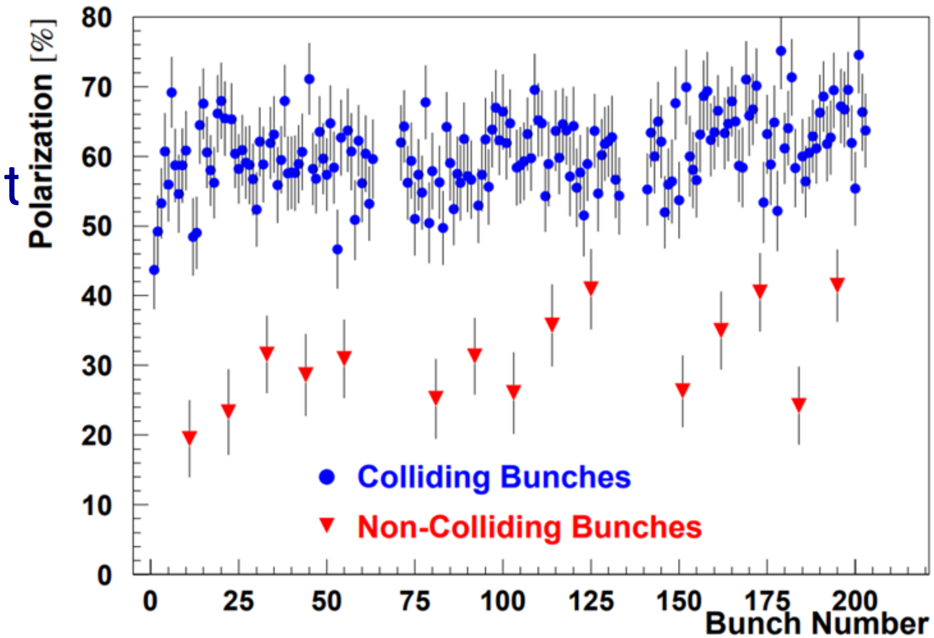


Polarization Performance and Beam-beam

- Beam-Beam induces tune shift of $x = \frac{N r_0 b^*}{4 p g s^2}$, as well as incoherent tune spread

- Both HERA and LEP observed the beam-beam effect on the electron beam polarization

- RHIC has observed very mild t during store



polarization of positrons colliding/not colliding with protons at HERA.

D.P. BARBER, arXiv:physics/9901040v1

Summary

SLAC

- Polarized beams have been successfully used for exploring high energy particle and nuclear physics
- The upcoming EIC, as well as future high energy collider proposals (FCC-ee, ILC, CEPC, etc) requires
  - High luminosity with high polarized lepton and hadron beams
  - Polarized beams at very high energy
- The challenges ahead
  - Novel techniques in overcoming depolarizing effects
    - Existing spin orbit tracking and simulation codes, i.e. SLIM, SITROS, SLICKTRACK, PTC@Bmad, zgoubi etc met challenges in balancing computation power and accuracy
    - Innovative spin orbit tracking and simulation such as the latest discovery of a complete system of spin-orbit stochastic ODEs by K. Heinemann et al
      - More robust and fast spin matching algorithms
  - Novel techniques in spin manipulation