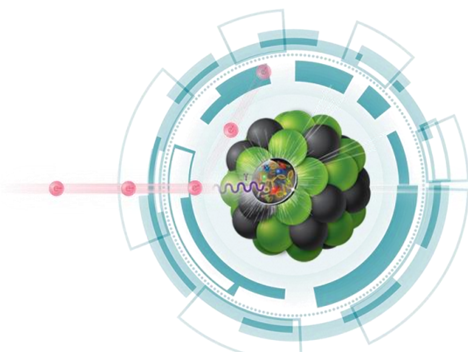


Three-dimensional structure at the EIC (I)

Cédric Lorcé

CPHT Centre de Physique Théorique

August 17

Outline

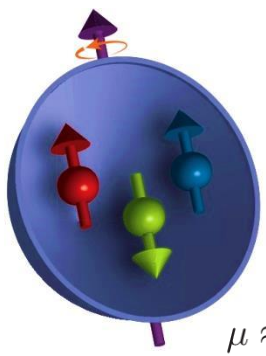
- Lecture 1 : Spatial distributions
- Lecture 2 : Parton distributions
- Lecture 3 : Wigner distributions
- Tutorial

Nucleon structure

Non-relativistic picture

dominated by constituents

Spectroscopy



$\mu \approx \Lambda_{\mathrm{QCD}}$

Mass

$$M_N c^2 \sim \sum_Q M_Q c^2 + E_{\mathrm{binding}}$$

~ 102 %   ~ - 2 %

Spin

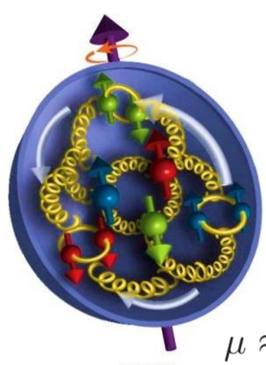
$$J_z^N \sim \sum_Q S_z^Q$$

~ 100 %

Relativistic picture

dominated by dynamics

High-energy scattering



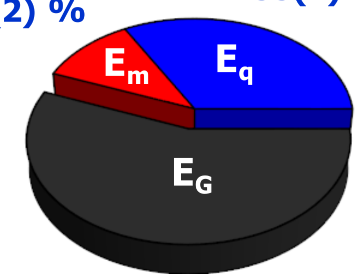
$\mu \approx 2\,\mathrm{GeV}$

Quark mass & QCD condensate

~ 11(2) %

Quark kinetic+potential energies

~ 33(2) %



?

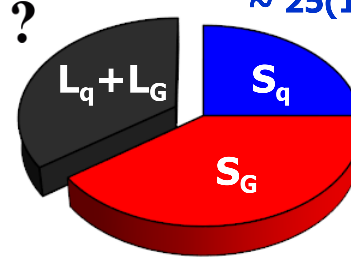
Gluon kinetic+potential energies

Orbital angular momentum

?

Quark spin

~ 25(10) %



~ 40(?) %

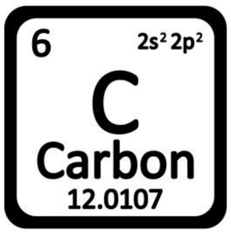
Gluon spin

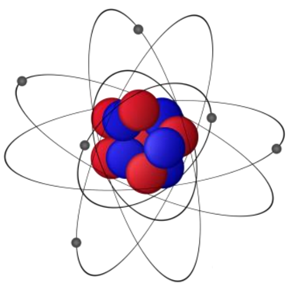
→ **EIC !**

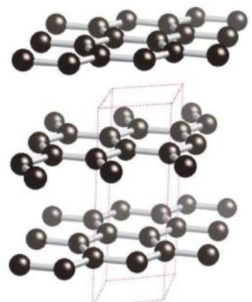
Spatial structure

3D structure is fundamental to understand physical properties

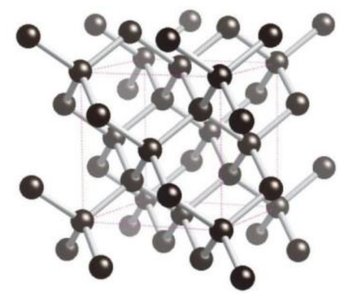
i.e. thermal, electrical, mechanical, ...



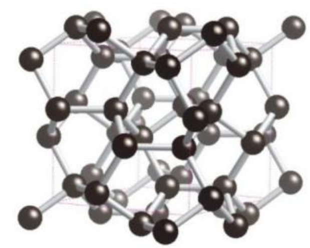




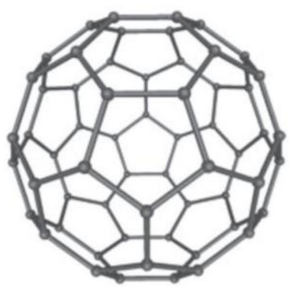
graphite



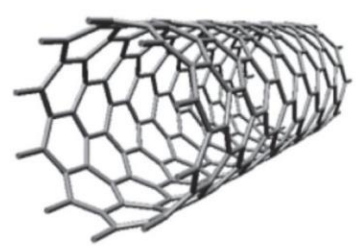
diamond



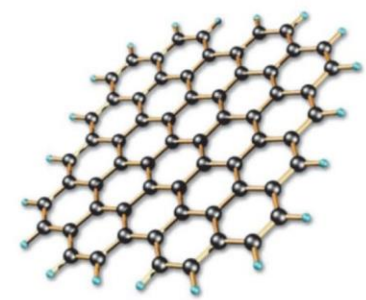
BC8



fullerene



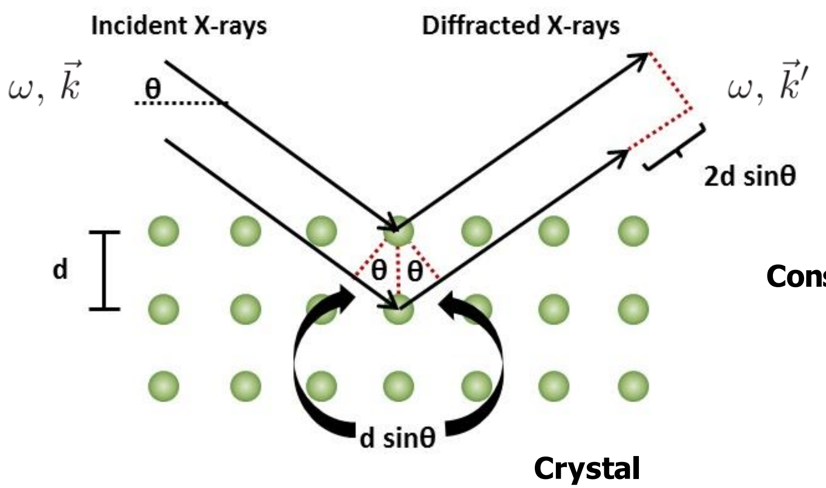
nanotube



graphene

Spatial distributions - Principles

Example: X-ray diffraction

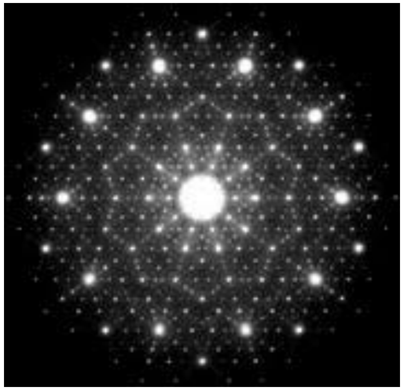


Constructive interference (Bragg's law)

$$2d\,\sin\theta = n\lambda$$

$$\lambda \sim d$$

Diffraction pattern



$$\propto |A_{\mathrm{scatt}}|^2$$

Scattered amplitude

$$A_{\mathrm{scatt}} \propto F(\vec{q}) = \int \mathrm{d}^3r\, e^{i\vec{q}\cdot\vec{r}}\, \rho(\vec{r}) \qquad \vec{q} = \vec{k} - \vec{k}'$$

Form factor

Scatterer distribution

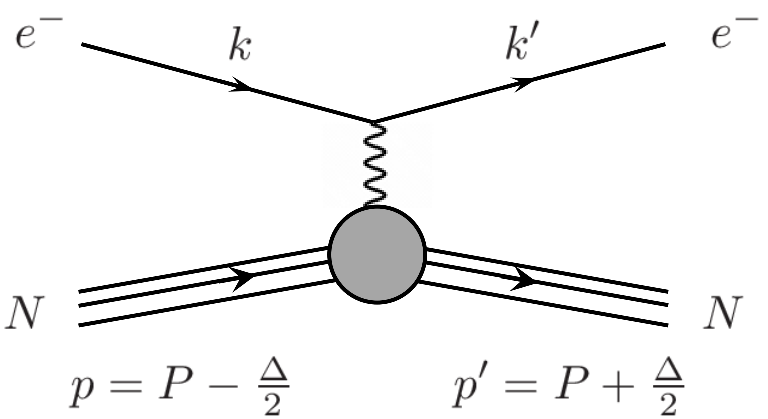
Spatial distributions - Principles

Crystals, atoms $d \approx 10^{-10}\,\mathrm{m} \quad \Rightarrow \quad \hbar\omega \approx 10^4\,\mathrm{eV}$ ➡ **X-rays**

Nuclei, nucleons $d \approx 10^{-15}\,\mathrm{m} \quad \Rightarrow \quad \hbar\omega \approx 10^9\,\mathrm{eV}$ ➡ **High-energy electron beams**

⚠ **Large recoil for light nuclei!**

Relativistic treatment
in Born approximation



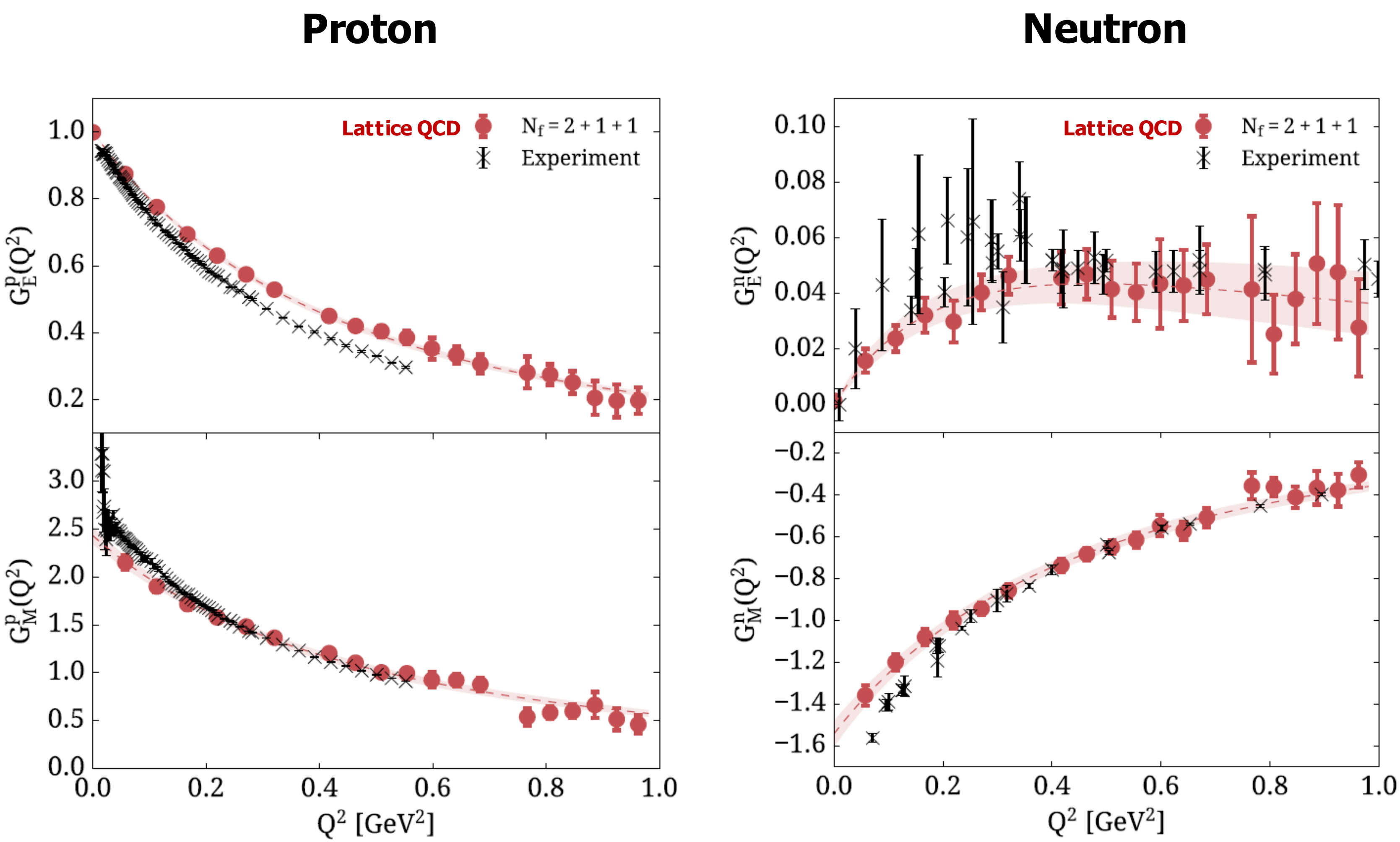
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\mathrm{pointlike}} = [F(Q^2)]^2$$ *Spin-0 target*

$$= \left\{ \left[G_E(Q^2)\right]^2 + \frac{\tau}{\epsilon}\left[G_M(Q^2)\right]^2 \right\} \frac{1}{1+\tau}$$ *Spin-1/2 target*

Electric form factor **Magnetic form factor**

$$Q^2 = -\Delta^2$$
$$\tau = Q^2/4M_N^2$$
$$\epsilon = (1 + 2(1+\tau)\tan^2\tfrac{\theta_e}{2})^{-1}$$

Nucleon electromagnetic form factors



[Alexandrou *et al.*, PRD100 (2019) 014509]

Spatial distributions - Formalism

Expectation value

$$\langle\psi|j^\mu(\vec r)|\psi\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,\tilde\psi^*(\vec P+\tfrac{\vec\Delta}{2})\tilde\psi(\vec P-\tfrac{\vec\Delta}{2})\,\langle\vec P+\tfrac{\vec\Delta}{2}|j^\mu(\vec r)|\vec P-\tfrac{\vec\Delta}{2}\rangle$$

Normalization $\langle\vec p\,'|\vec p\rangle = (2\pi)^3\,\delta^{(3)}(\vec p\,'-\vec p)$

Wave packet $\tilde\psi(\vec p) = \langle\vec p|\psi\rangle$

Probabilistic interpretation $\tilde\psi(\vec P\pm\tfrac{\vec\Delta}{2})\approx\tilde\psi(\vec P)$

$$\langle\psi|j^\mu(\vec r)|\psi\rangle \approx \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,|\tilde\psi(\vec P)|^2\,\underbrace{\mathcal{J}^\mu_{\vec P}(\vec r)}$$

$$= \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\,e^{-i\vec\Delta\cdot\vec r}\langle\vec P+\tfrac{\vec\Delta}{2}|j^\mu(\vec 0)|\vec P-\tfrac{\vec\Delta}{2}\rangle$$

Internal distribution

Validity domain $1/D \ll |\vec\Delta| \ll |\delta\vec p| \ll P^0$

Hydrogen $M_H D_H \approx 10^5$

Nucleon $M_N D_N \approx 4$ ⚠

Rest frame $|\vec P| = 0 \quad\Rightarrow\quad P^0\approx M$

Infinite momentum frame $|\vec P|\gg M \quad\Rightarrow\quad P^0\gg M$

Spatial distributions - Formalism

Phase-space approach

$$\langle\psi|j^\mu(\vec{r})|\psi\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\mathrm{d}^3R\;\rho_\psi(\vec{R},\vec{P})\,\langle j^\mu\rangle_{\vec{R},\vec{P}}(\vec{r})$$

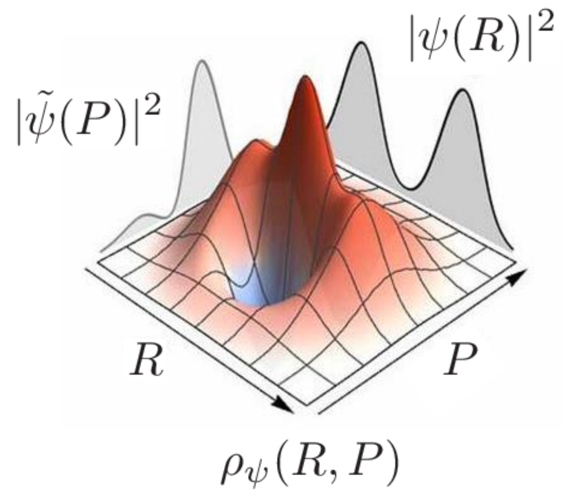
Nucleon Wigner distribution

$$\rho_\psi(\vec{R},\vec{P}) = \int \mathrm{d}^3z\, e^{-i\vec{P}\cdot\vec{z}}\,\psi^*(\vec{R}-\tfrac{\vec{z}}{2})\psi(\vec{R}+\tfrac{\vec{z}}{2})$$
$$= \int \frac{\mathrm{d}^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{R}}\,\tilde{\psi}^*(\vec{P}+\tfrac{\vec{q}}{2})\tilde{\psi}(\vec{P}-\tfrac{\vec{q}}{2})$$

$$\psi(\vec{r}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, e^{-i\vec{p}\cdot\vec{r}}\,\tilde{\psi}(\vec{p})$$

***Quasi*-probabilistic interpretation**

$$\int \mathrm{d}^3R\,\rho_\psi(\vec{R},\vec{P}) = |\tilde{\psi}(\vec{P})|^2$$
$$\int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\rho_\psi(\vec{R},\vec{P}) = |\psi(\vec{R})|^2$$



Spatial distributions - Formalism

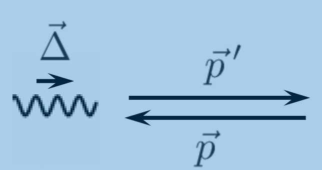
Internal distribution (for a state localized in phase-space)

$$\langle j^\mu \rangle_{\vec{R},\vec{P}}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot(\vec{r}-\vec{R})} \langle \vec{P} + \tfrac{\vec{\Delta}}{2} | j^\mu(0) | \vec{P} - \tfrac{\vec{\Delta}}{2} \rangle$$

NB: $\mathcal{J}^\mu_{\vec{P}}(\vec{r}) = \langle j^\mu \rangle_{\vec{0},\vec{P}}(\vec{r})$

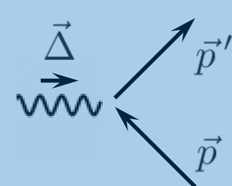
Elastic frames $\Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} \stackrel{!}{=} 0$ (no energy transfer ➡ same initial and final boost factor)

$|\vec{P}| = 0$

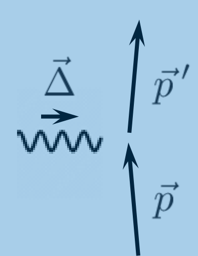


Breit (or brick-wall or rest) frame

$|\vec{P}| \neq 0$



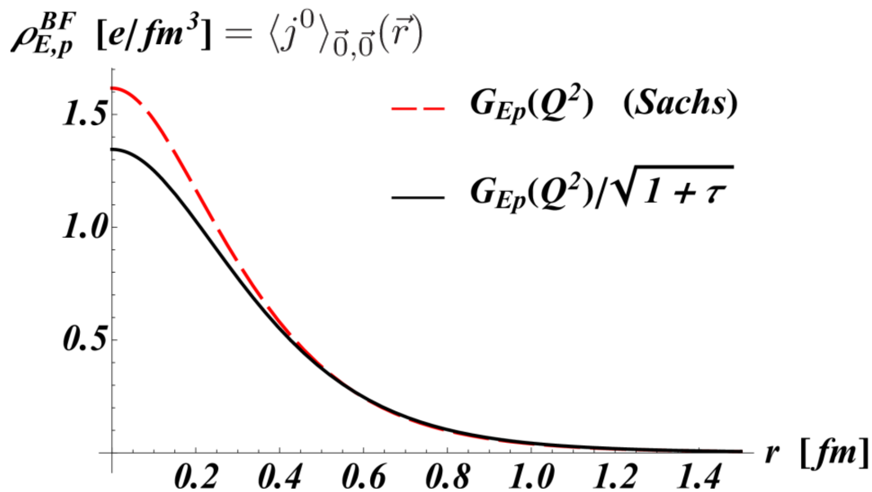
$|\vec{P}| \gg M$



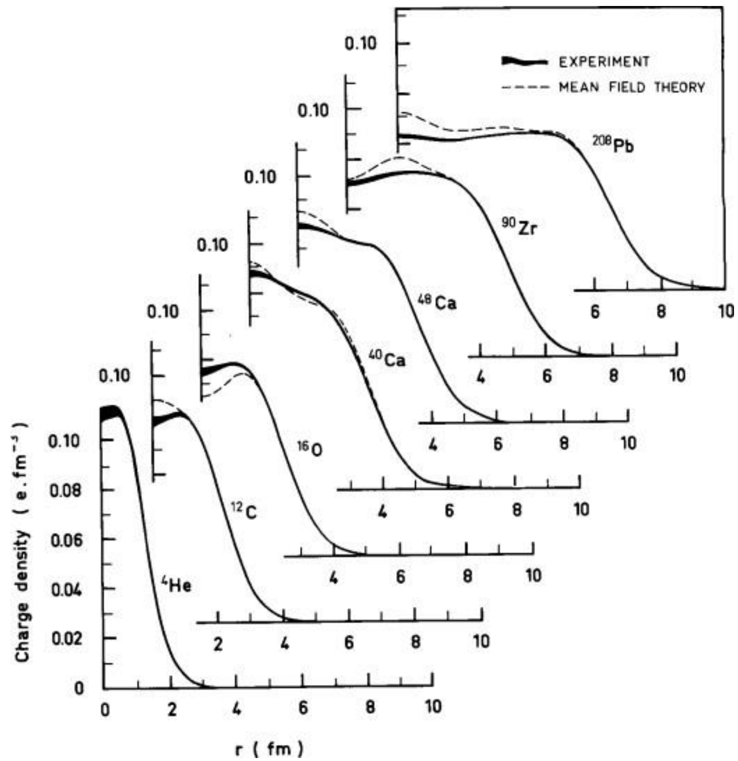
Infinite-momentum Frame (IMF)

Nucleon charge distributions (3D Breit frame)

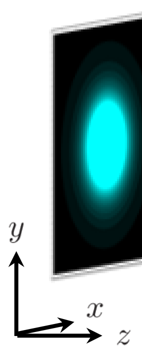
Proton

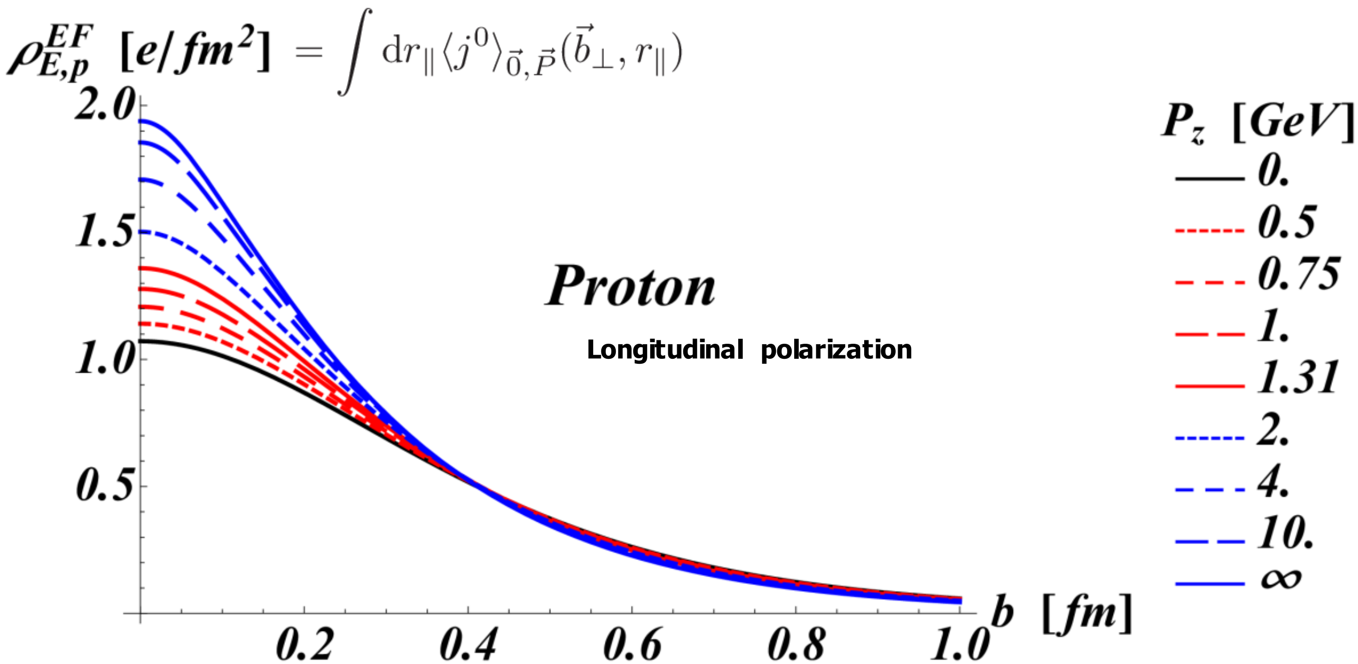


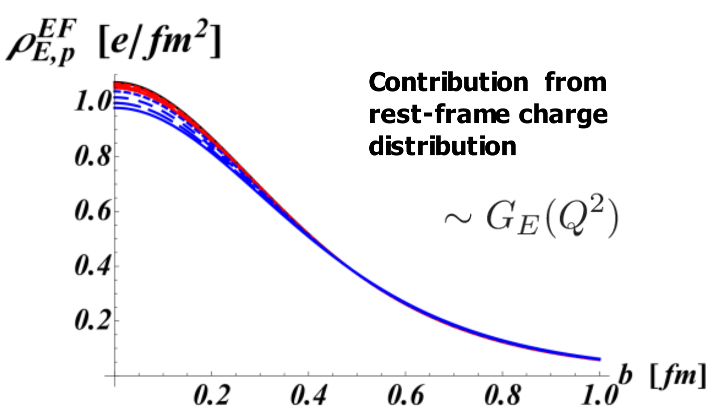
Nuclei

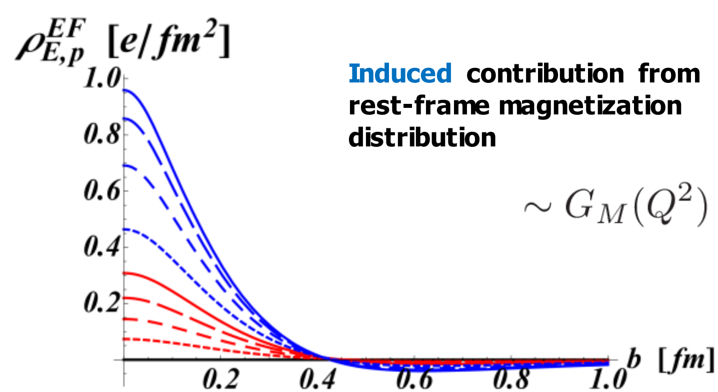


Nucleon charge distributions (2D elastic frame)



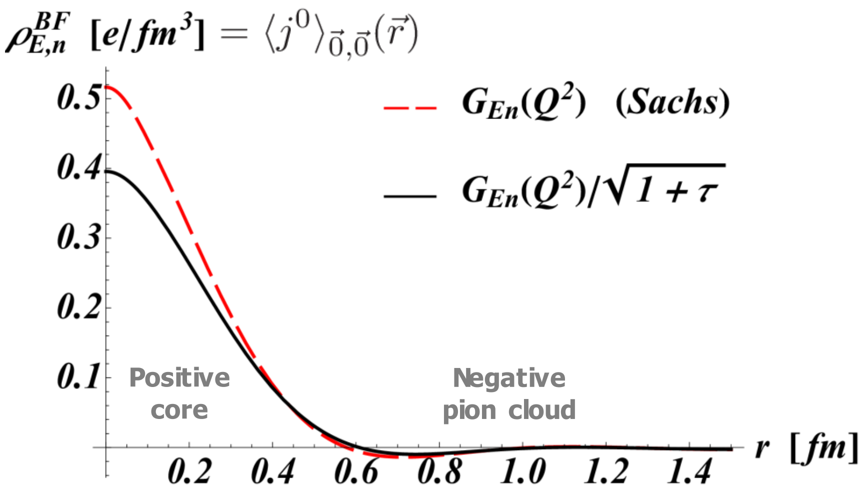




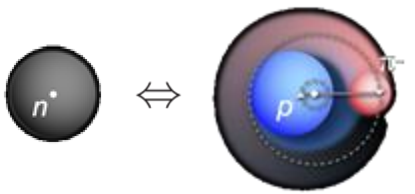


Nucleon charge distributions (3D Breit frame)

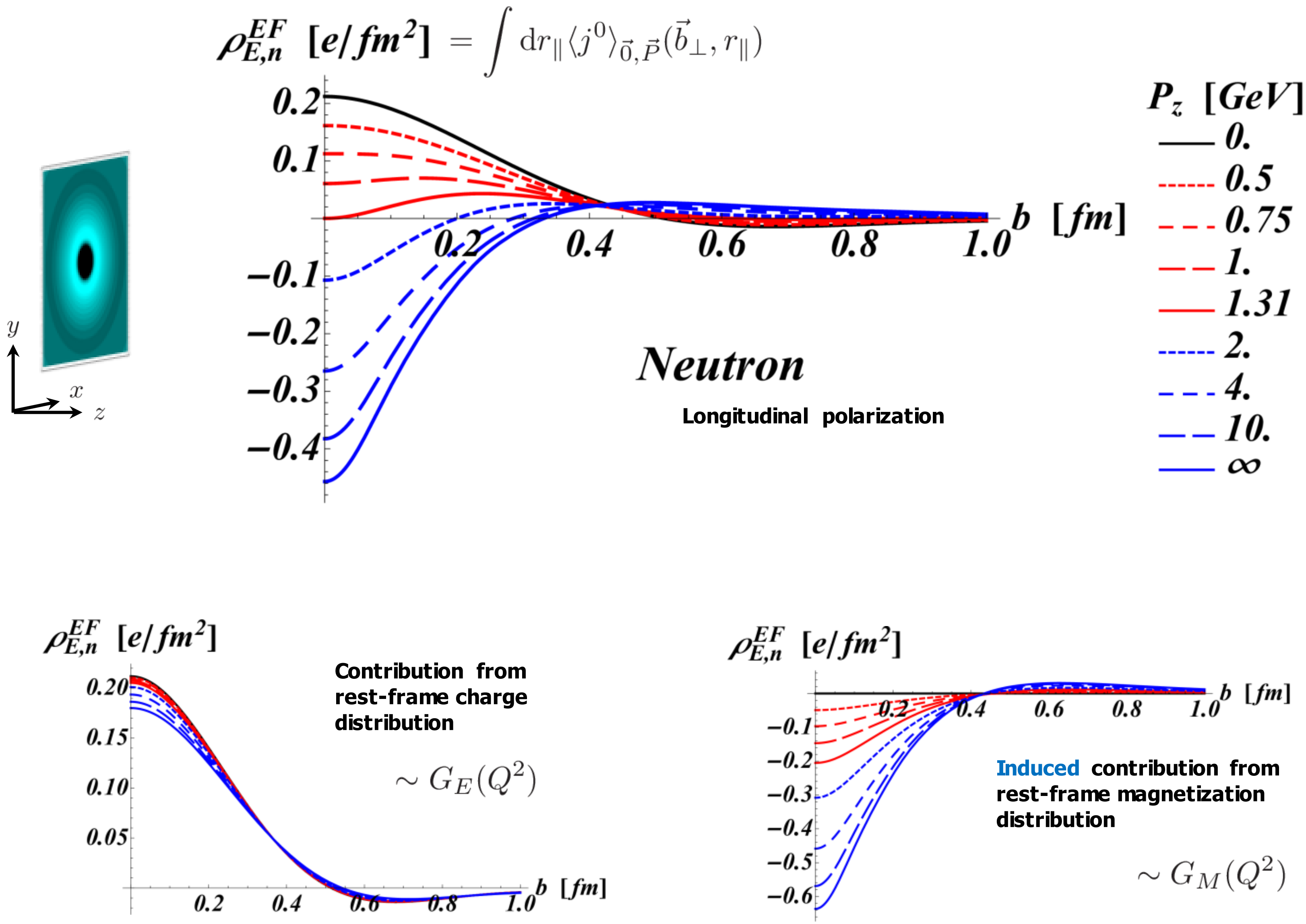
Neutron



Proton-pion fluctuation



Nucleon charge distributions (2D elastic frame)



Energy-momentum tensor

Instead of charge current, we can look at four-momentum current

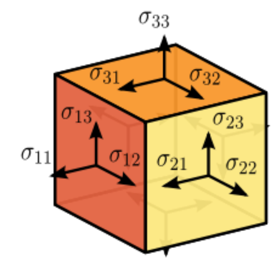
Energy density — Momentum density

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Energy flux — Momentum flux

Shear stress

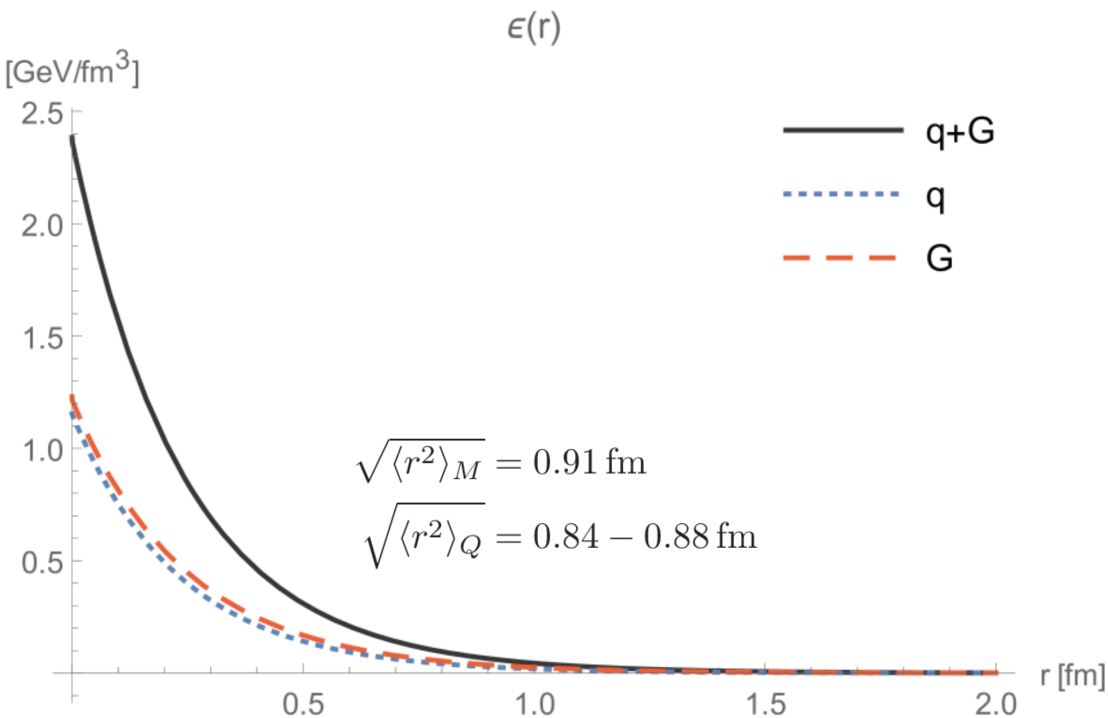
Normal stress (pressure)



Four-momentum operator $$P^\mu_a = \int \mathrm{d}^3 r\, T^{0\mu}_a(r) \qquad a = q, g$$

Energy distribution (3D Breit frame)

$$\langle T^{00}\rangle_{\vec{0},\vec{0}}(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\langle \tfrac{\vec{\Delta}}{2}|T^{00}(0)| -\tfrac{\vec{\Delta}}{2}\rangle$$

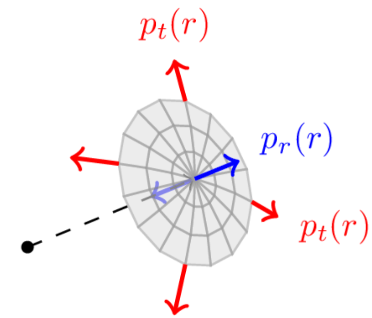


Multipole model for the gravitational form factors

$$F(t) = \frac{F(0)}{(1+t/\Lambda^2)^n}$$

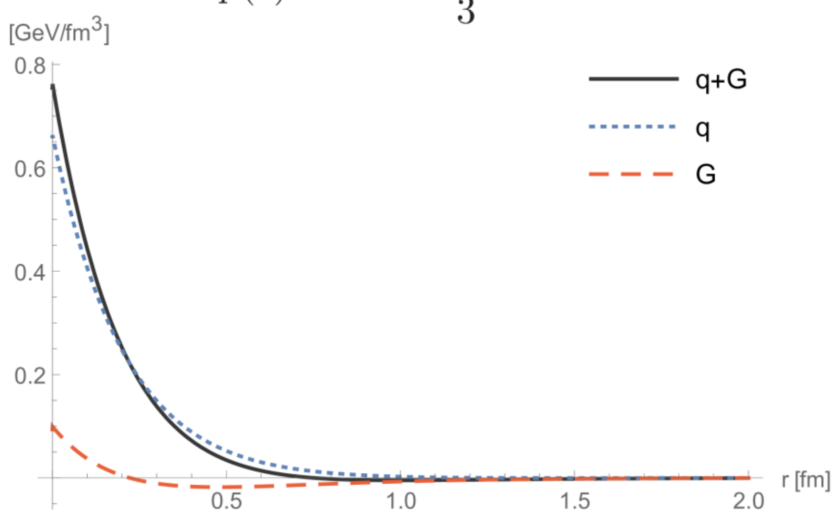
Pressure distributions (3D Breit frame)

$$\langle T^{ij}\rangle_{\vec{0},\vec{0}}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} \langle \tfrac{\vec{\Delta}}{2}|T^{ij}(0)| - \tfrac{\vec{\Delta}}{2}\rangle$$



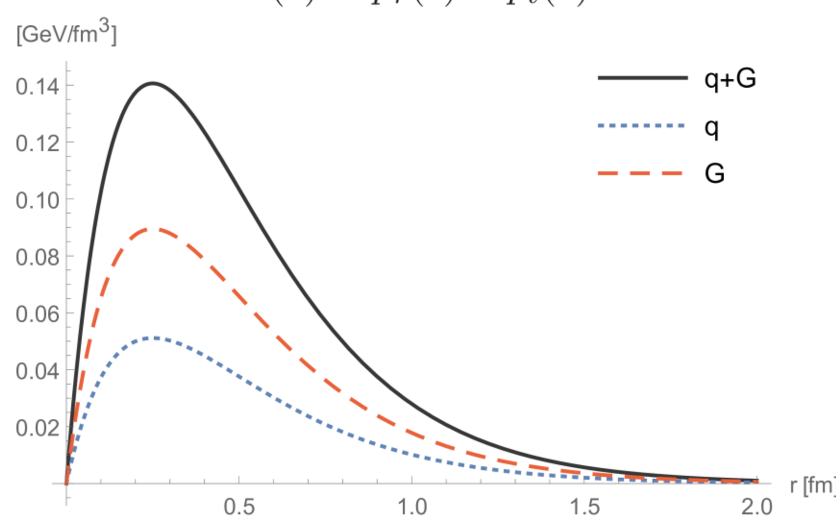
Isotropic pressure

$$p(r) = \frac{p_r(r) + 2p_t(r)}{3}$$



Pressure anisotropy

$$s(r) = p_r(r) - p_t(r)$$

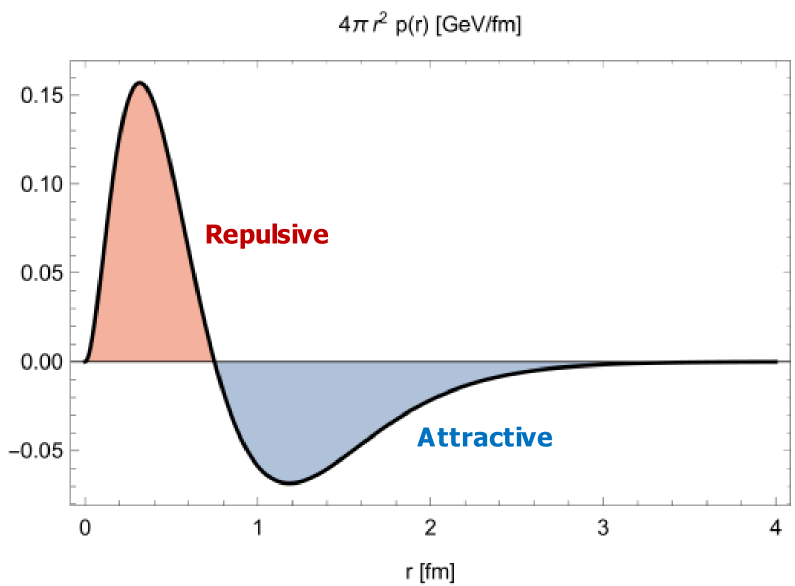


Mechanical equilibrium

$$\nabla^i \langle T^{ij} \rangle_{\vec{0},\vec{0}}(\vec{r}) = 0 \quad \Rightarrow \quad \frac{\mathrm{d}p_r(r)}{\mathrm{d}r} = -\frac{2s(r)}{r}$$

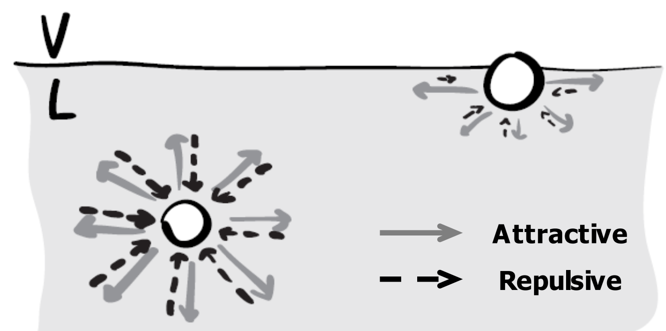
von Laue relation

$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$



Surface tension

$$\gamma = \int \mathrm{d}r\, s(r)$$

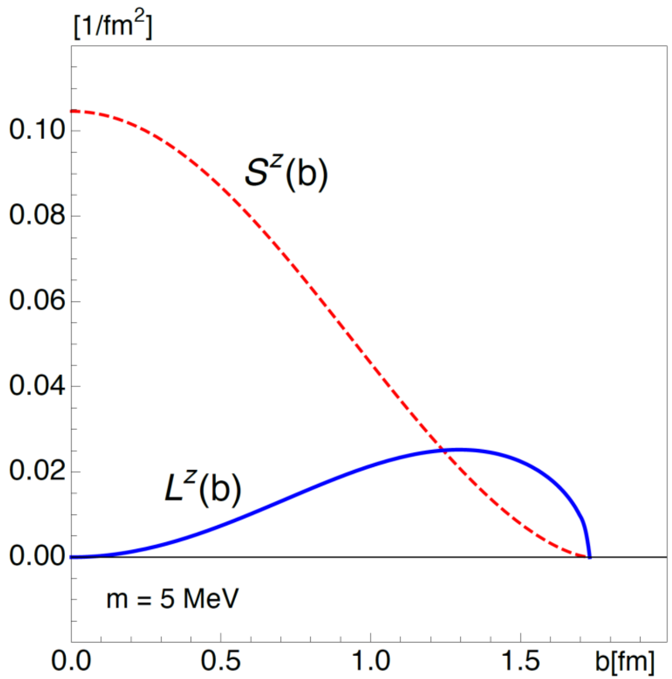


Angular momentum distributions

Orbital *vs* intrinsic

$$L^i(\vec{r}) = \epsilon^{ijk} r^j \langle T^{0k} \rangle_{\vec{0},\vec{0}}(\vec{r})$$

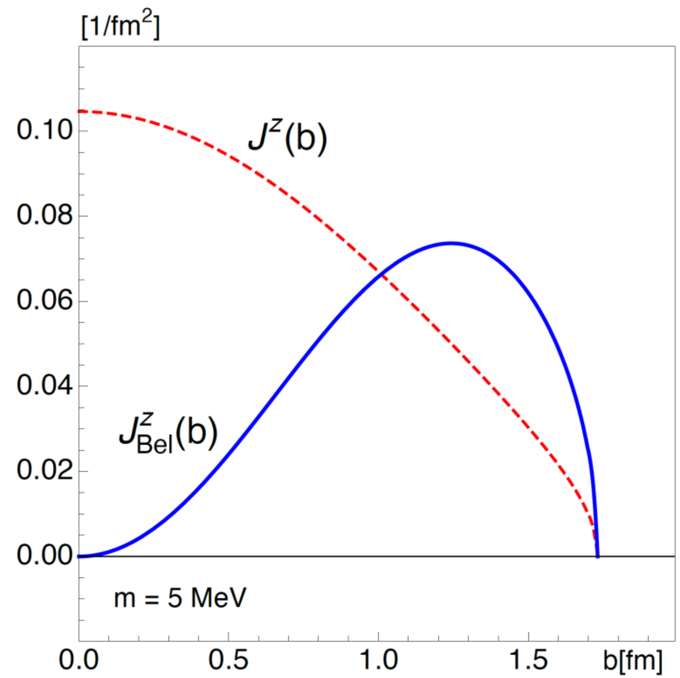
$$S^i(\vec{r}) = \langle \overline{\psi} \gamma^i \gamma_5 \psi \rangle_{\vec{0},\vec{0}}(\vec{r})$$



Kinetic *vs* Belinfante

$$J^i(\vec{r}) = L^i(\vec{r}) + S^i(\vec{r})$$

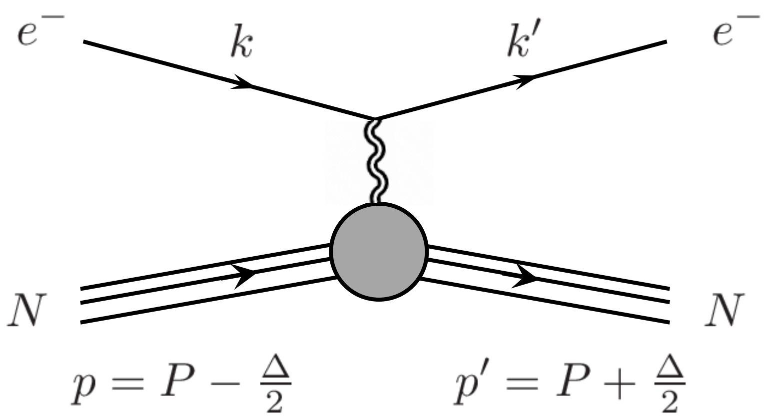
$$J^i_{\rm Bel}(\vec{r}) = \epsilon^{ijk} r^j \langle \tfrac{1}{2}(T^{0k} + T^{k0}) \rangle_{\vec{0},\vec{0}}(\vec{r})$$



Large-N_c bag model

How to access gravitational form factors ?

Graviton exchange



Waaaaaayyy too weak in practice

→ **see next episode !**

Some references

- **Burkardt, PRD62 (2000) 071503**
- **Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025**
- **Lorcé, Mantovani, Pasquini, PLB776 (2018) 38**
- **Lorcé, Moutarde, Trawinski, EPJC79 (2019) 89**
- **Lorcé, PRL125 (2020) 232002**