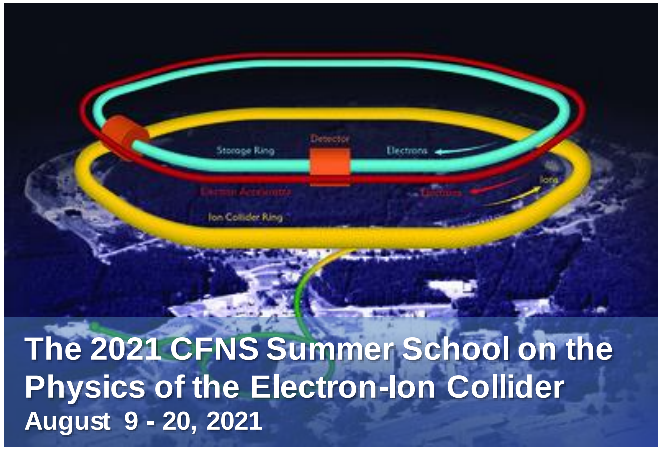
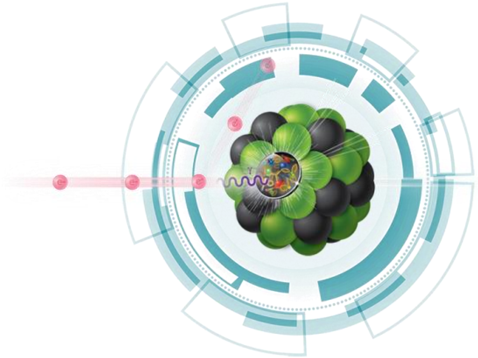


The 2021 CFNS Summer School on the Physics of the Electron-Ion Collider
August 9 - 20, 2021



Three-dimensional structure at the EIC (2)

Cédric Lorcé

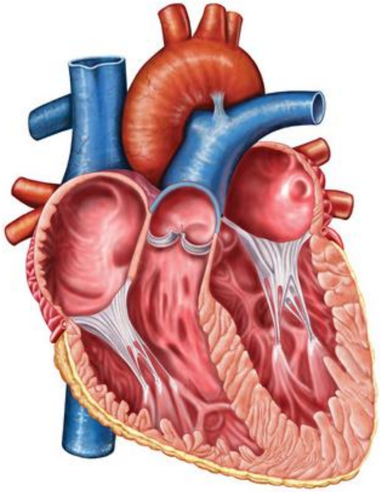
CPHT Centre de Physique Théorique

August 17

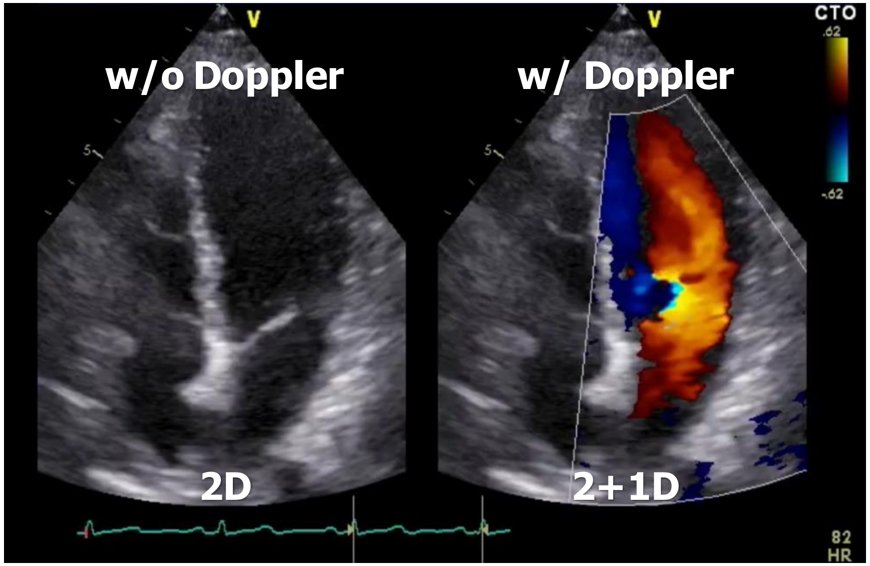
Outline

- Lecture 1 : Spatial distributions
- Lecture 2 : Parton distributions
- Lecture 3 : Wigner distributions
- Tutorial

Adding momentum to the picture



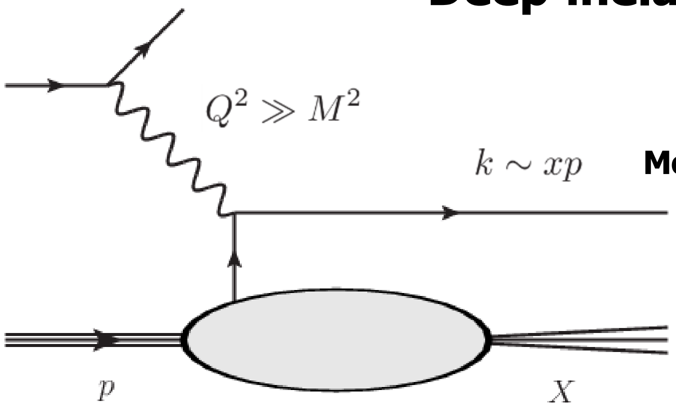
Echography



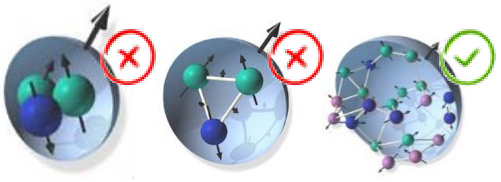
Fluxes provide key complementary information to spatial structure

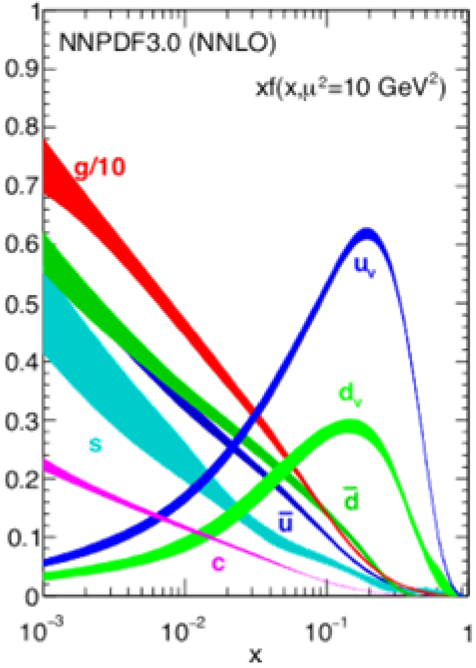
Parton distribution functions (PDFs)

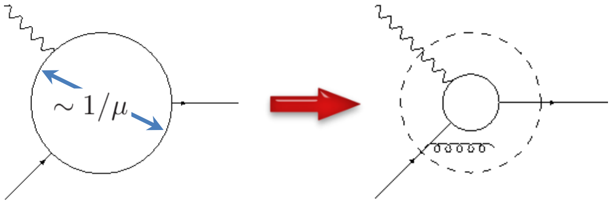
Deep inelastic scattering (DIS)



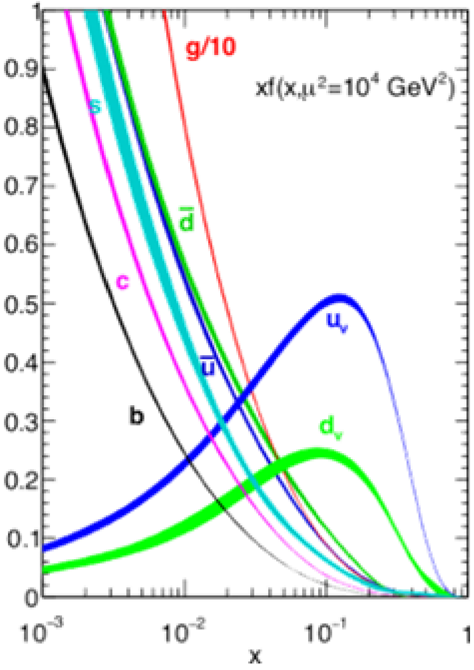
Momentum fraction carried by parton





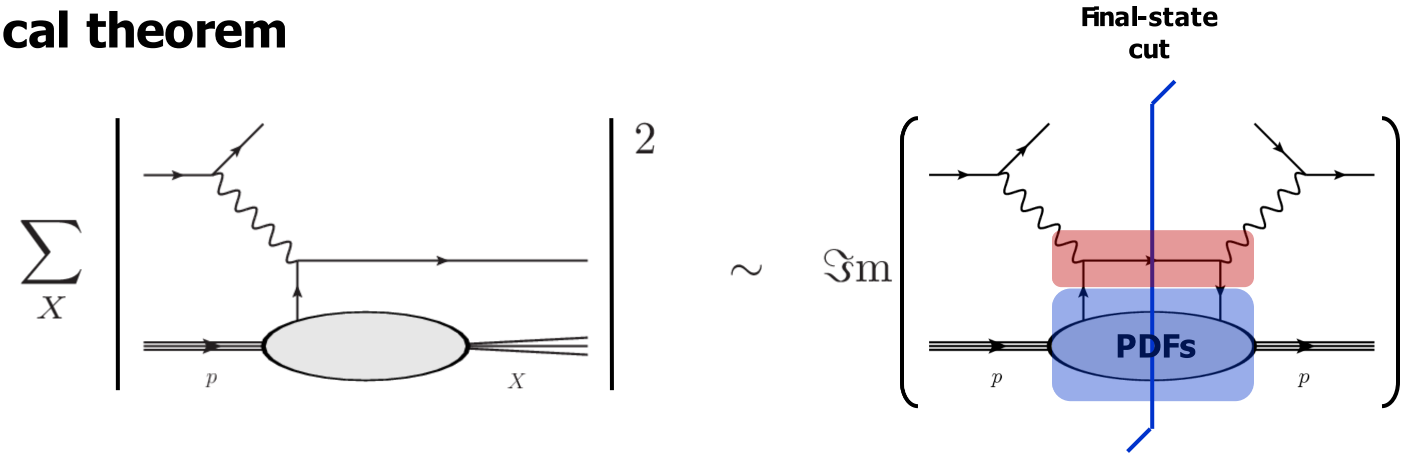


Perturbative QCD evolution (DGLAP)



Parton distribution functions (PDFs)

Optical theorem



$$\mathrm{d}\sigma^{lp\to lX} \sim \sum_i \int_x^1 \frac{\mathrm{d}y}{y}\, C_i\!\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}; \alpha_S(\mu_R)\right) f_i(y, \mu_F^2)$$

$i = q_f, \overline{q}_f, g$

$\mu_F \approx \mu_R$

Perturbative
Process-dependent

Non-perturbative
Process-independent

Parton density (in $A^+ = 0$ gauge)

$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$

$$f(x) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle P|\overline{\psi}(-\tfrac{z}{2})\gamma^+\psi(\tfrac{z}{2})|P\rangle\Big|_{z^+=z_\perp=0}$$

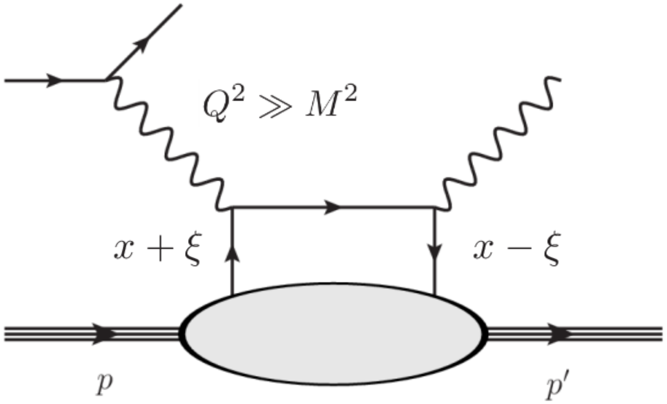
$k^+ = xP^+$

$$\sim \frac{1}{2x(2\pi)}\sum_\lambda \int \frac{\mathrm{d}^2k_\perp}{(2\pi)^2}\, \frac{\langle P|b^\dagger_{k,\lambda} b_{k,\lambda}|P\rangle}{\langle P|P\rangle}$$

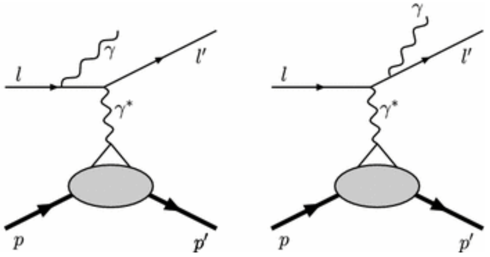
$\langle p'|p\rangle = 2p^+(2\pi)^3\delta(p'^+ - p^+)\delta^{(2)}(\vec{p}'_\perp - \vec{p}_\perp)$

Generalized PDFs

Deeply virtual Compton scattering (DVCS)



interferes with



Bethe-Heitler

Correlator (in $A^+ = 0$ gauge)

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle p'|\overline{\psi}(-\tfrac{z}{2})\gamma^+\psi(\tfrac{z}{2})|p\rangle\big|_{z^+=z_\perp=0}$$

$$= \frac{1}{2P^+}\,\overline{u}(p')\left[\gamma^+\,H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\,E(x,\xi,t)\right]u(p)$$

Momentum transfer variables

$$\xi = -\frac{\Delta^+}{2P^+} = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad t = \Delta^2 = (p' - p)^2$$

Generalized PDFs

Link with other non-perturbative functions

$$H(x,0,0)=f(x)$$

PDF

$$\int \mathrm{d}x\, H(x,\xi,t)=F_1(t)$$

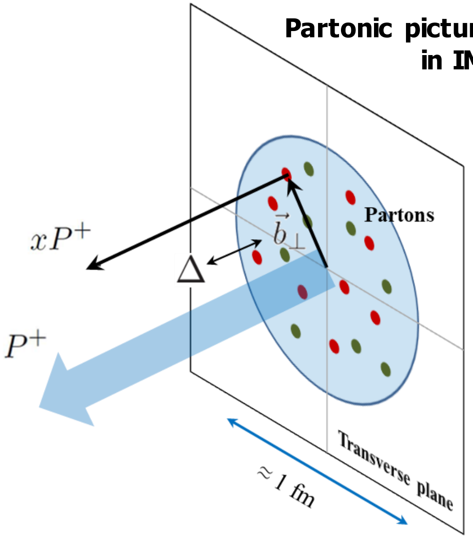
$$\int \mathrm{d}x\, E(x,\xi,t)=F_2(t)$$

Electromagnetic form factors

2+1D imaging (in $A^+=0$ gauge)

$$\rho(x,\vec{b}_\perp)=P^+\int\frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\,e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,\frac{\langle p'|j^+(xP^+)|p\rangle}{2P^+}\Big|_{\Delta^+=0}$$

$$j^\mu(k^+)=\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ik^+z^-}\,\overline{\psi}(-\tfrac{z}{2})\gamma^\mu\psi(\tfrac{z}{2})\big|_{z^+=z_\perp=0}$$


Partonic picture in IMF

Generalized PDFs

Link with gravitational form factors

$$a^{\{\mu}b^{\nu\}} = \tfrac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$$
$$a^{[\mu}b^{\nu]} = \tfrac{1}{2}(a^\mu b^\nu - a^\nu b^\mu)$$

$$\begin{aligned}\langle p'|T^{\mu\nu}(0)|p\rangle = \overline{u}(p')\Bigg[&P^{\{\mu}\gamma^{\nu\}}A(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M}B(t)\\ &+\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C(t) + Mg^{\mu\nu}\bar{C}(t) + \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{2M}D(t)\Bigg]u(p)\end{aligned}$$

$$\begin{aligned}T_q^{++}(0) &= (\overline{\psi}\gamma^+\tfrac{i}{2}\overleftrightarrow{D}^+\psi)(0)\\ &= 2(P^+)^2\int \mathrm{d}x\, x\,\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}\,e^{ixP^+z^-}\,\overline{\psi}(-\tfrac{z}{2})\gamma^+\psi(\tfrac{z}{2})\Big|_{z^+=z_\perp=0}\end{aligned}$$

$$\boxed{\begin{aligned}&\int \mathrm{d}x\,x\,H(x,\xi,t) = A(t) + 4\xi^2C(t)\\ &\int \mathrm{d}x\,x\,E(x,\xi,t) = B(t) - 4\xi^2C(t)\end{aligned}}$$

Generalized PDFs

Poincaré constraints

$$\sum_{a=q,g} A_a(0) = 1$$

$$\sum_{a=q,g} B_a(0) = 0$$

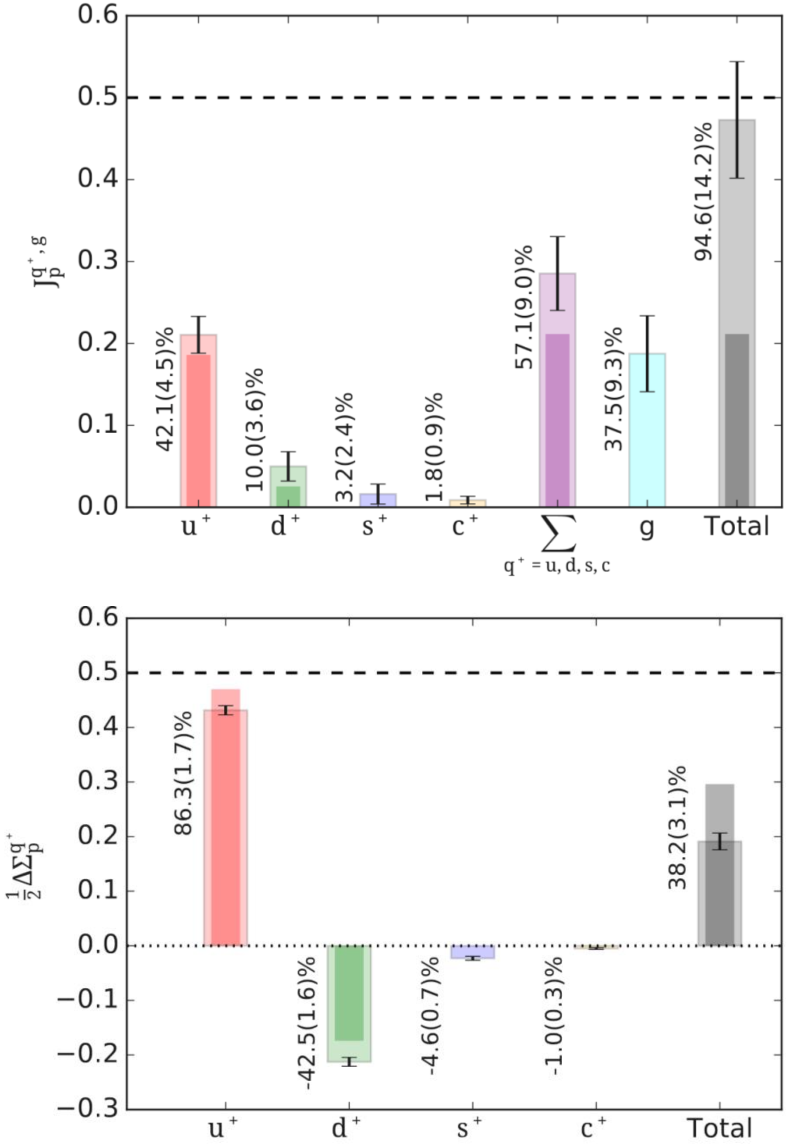
$$\sum_{a=q,g} \bar{C}_a(t) = 0$$

$$D_q(t) = -G_A(t)$$

Angular momentum

$$\langle J_q^z \rangle = \frac{A_q(0) + B_q(0)}{2}$$

$$\langle S_q^z \rangle = \frac{G_A(0)}{2} = \frac{\Delta\Sigma}{2}$$



[Alexandrou *et al.*, PRD101 (2020) 094513]

Transverse-momentum dependent PDFs

Nucleon Wigner distribution

$$\rho_\psi(\vec{R},\vec{P}) = \int \mathrm{d}^3z\, e^{-i\vec{P}\cdot\vec{z}}\, \psi^*(\vec{R}-\tfrac{\vec{z}}{2})\psi(\vec{R}+\tfrac{\vec{z}}{2})$$

Quark Wigner operator (contour gauge)

$$j^\mu(r,k) = \int \frac{\mathrm{d}^4z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(r-\tfrac{z}{2})\gamma^\mu\psi(r+\tfrac{z}{2})$$

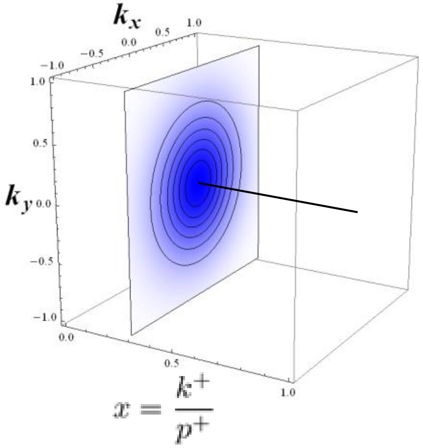
Charge current operator

$$j^\mu(r) = \int \mathrm{d}^4k\, j^\mu(r,k)$$

Unintegrated distributions

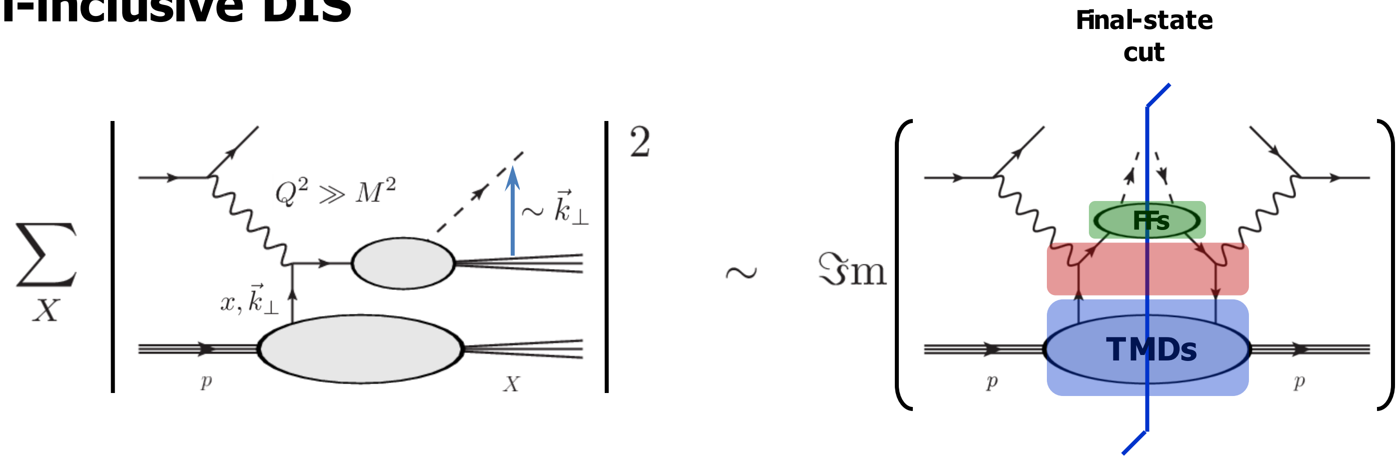
$$\mathrm{PDF}(x) \sim \frac{1}{2}\int \mathrm{d}k^-\, \mathrm{d}^2k_\perp\, \langle p|j^+(0,k)|p\rangle$$

$$\mathrm{TMD}(x,\vec{k}_\perp) \sim \frac{1}{2}\int \mathrm{d}k^-\, \langle p|j^+(0,k)|p\rangle$$



Transverse-momentum dependent PDFs

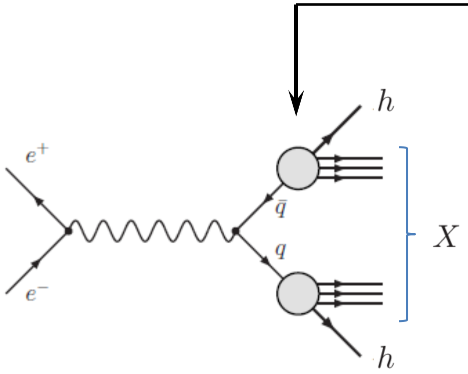
Semi-inclusive DIS



$$d\sigma \sim \sum \mathrm{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \mathrm{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Fragmentation Function

$e^+e^- \to hh'X$



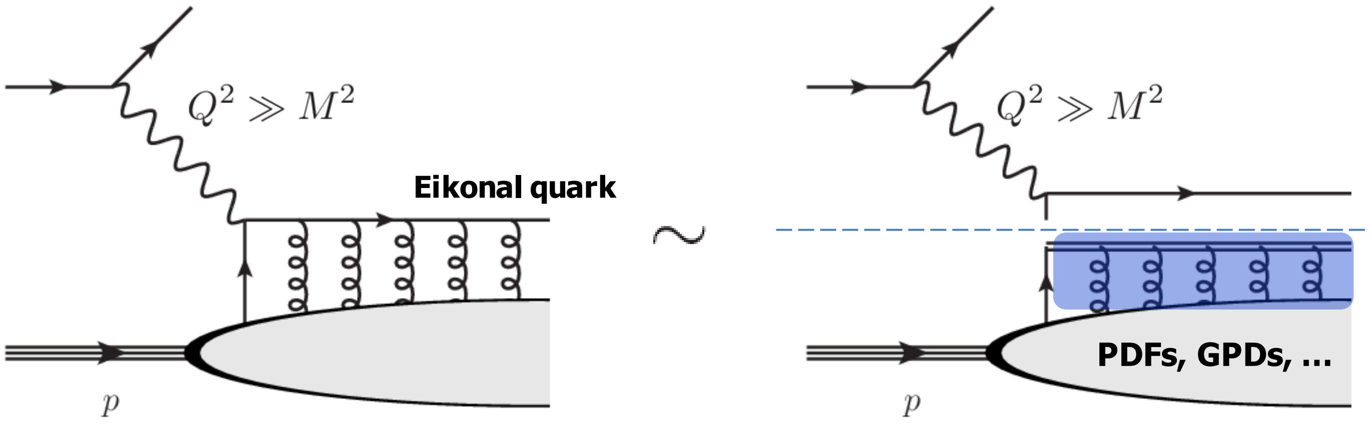
BELLE

BaBar

Transverse-momentum dependent PDFs

Gauge symmetry

System **« Factorized » system**



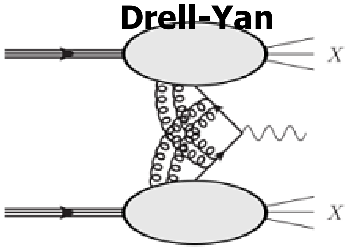
Local **Non-local**

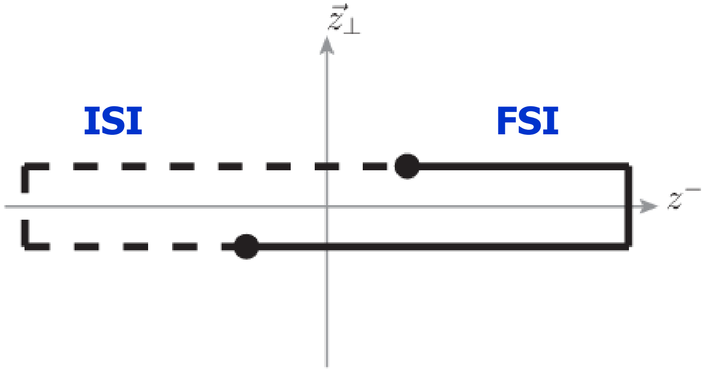
$$j^{\mu}(r,k) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(r-\tfrac{z}{2})\gamma^{\mu}\mathcal{W}(r-\tfrac{z}{2}, r+\tfrac{z}{2})\psi(r+\tfrac{z}{2})$$

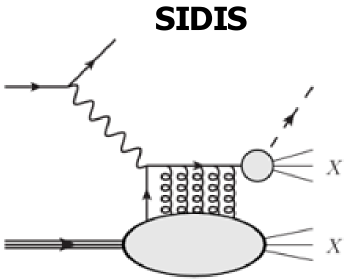
Wilson line $$\mathcal{W}(b,a) = \mathcal{P}\left[e^{ig\int_a^b A^{\mu}\,\mathrm{d}z_{\mu}}\right]$$

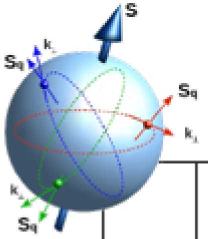
Transverse-momentum dependent PDFs

Process dependence









Quark polarization

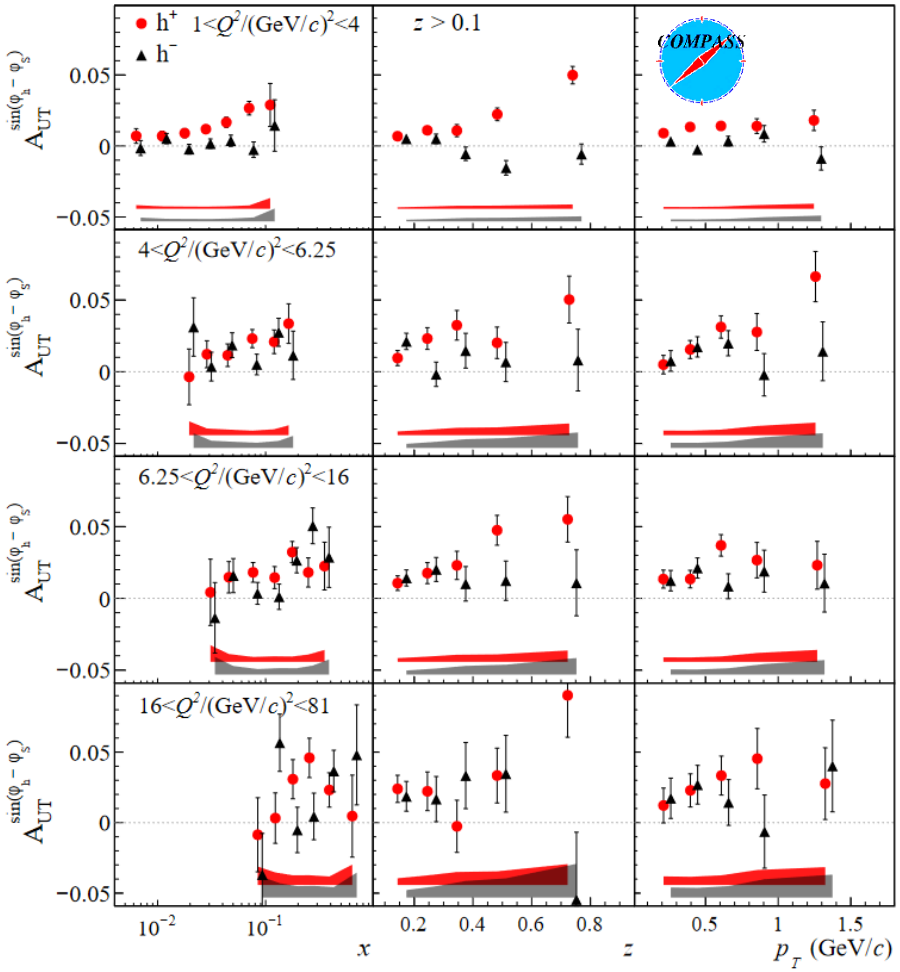
Nucleon polarization

	U	T_x	T_y	L
U	f_1	🟥 $\frac{k_y}{M}\, h_1^\perp$	🟥 $-\frac{k_x}{M}\, h_1^\perp$	
T_x	🟥 $\frac{k_y}{M}\, f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$\frac{k_x}{M}\, g_{1T}$
T_y	🟥 $-\frac{k_x}{M}\, f_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_y}{M}\, g_{1T}$
L		$\frac{k_x}{M}\, h_{1L}^\perp$	$\frac{k_y}{M}\, h_{1L}^\perp$	g_{1L}

🟥 **Changes sign under ISI ↔ FSI (naive T-odd)**

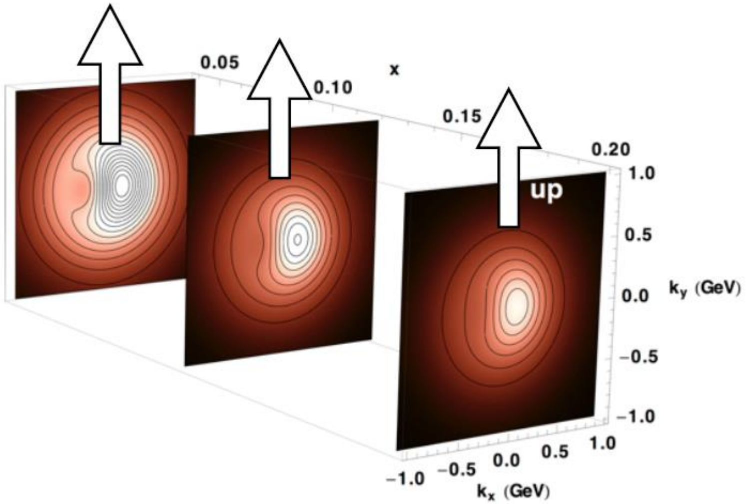
Transverse-momentum dependent PDFs

Clear SIDIS experimental signal



[Adolph *et al.*, PLB770 (2017) 138]

Transversely polarized proton

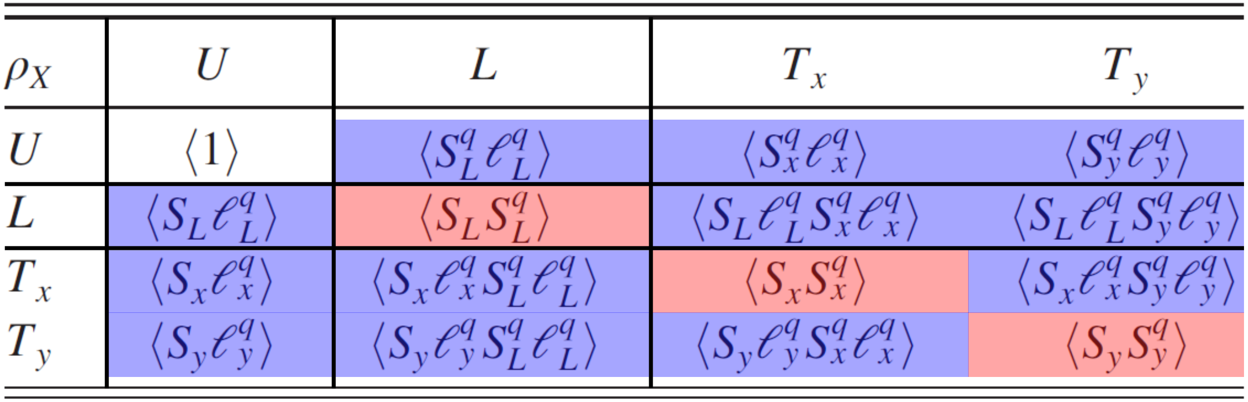


Courtesy of A. Bacchetta

Rich spin structure !

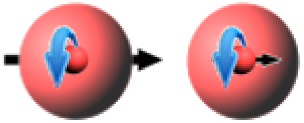
Quark polarization

Nucleon polarization

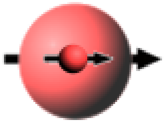


ρ_X	U	L	T_x	T_y
U	$\langle 1\rangle$	$\langle S^q_L \ell^q_L\rangle$	$\langle S^q_x \ell^q_x\rangle$	$\langle S^q_y \ell^q_y\rangle$
L	$\langle S_L \ell^q_L\rangle$	$\langle S_L S^q_L\rangle$	$\langle S_L \ell^q_L S^q_x \ell^q_x\rangle$	$\langle S_L \ell^q_L S^q_y \ell^q_y\rangle$
T_x	$\langle S_x \ell^q_x\rangle$	$\langle S_x \ell^q_x S^q_L \ell^q_L\rangle$	$\langle S_x S^q_x\rangle$	$\langle S_x \ell^q_x S^q_y \ell^q_y\rangle$
T_y	$\langle S_y \ell^q_y\rangle$	$\langle S_y \ell^q_y S^q_L \ell^q_L\rangle$	$\langle S_y \ell^q_y S^q_x \ell^q_x\rangle$	$\langle S_y S^q_y\rangle$

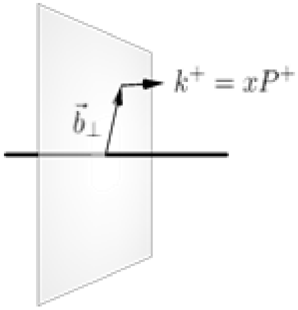
Spin-orbit



Spin-spin



GPDs

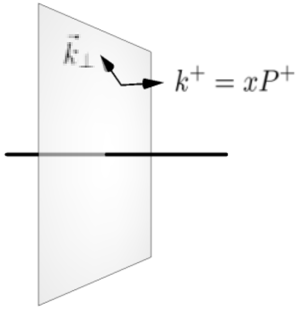


	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$

2+1D imaging

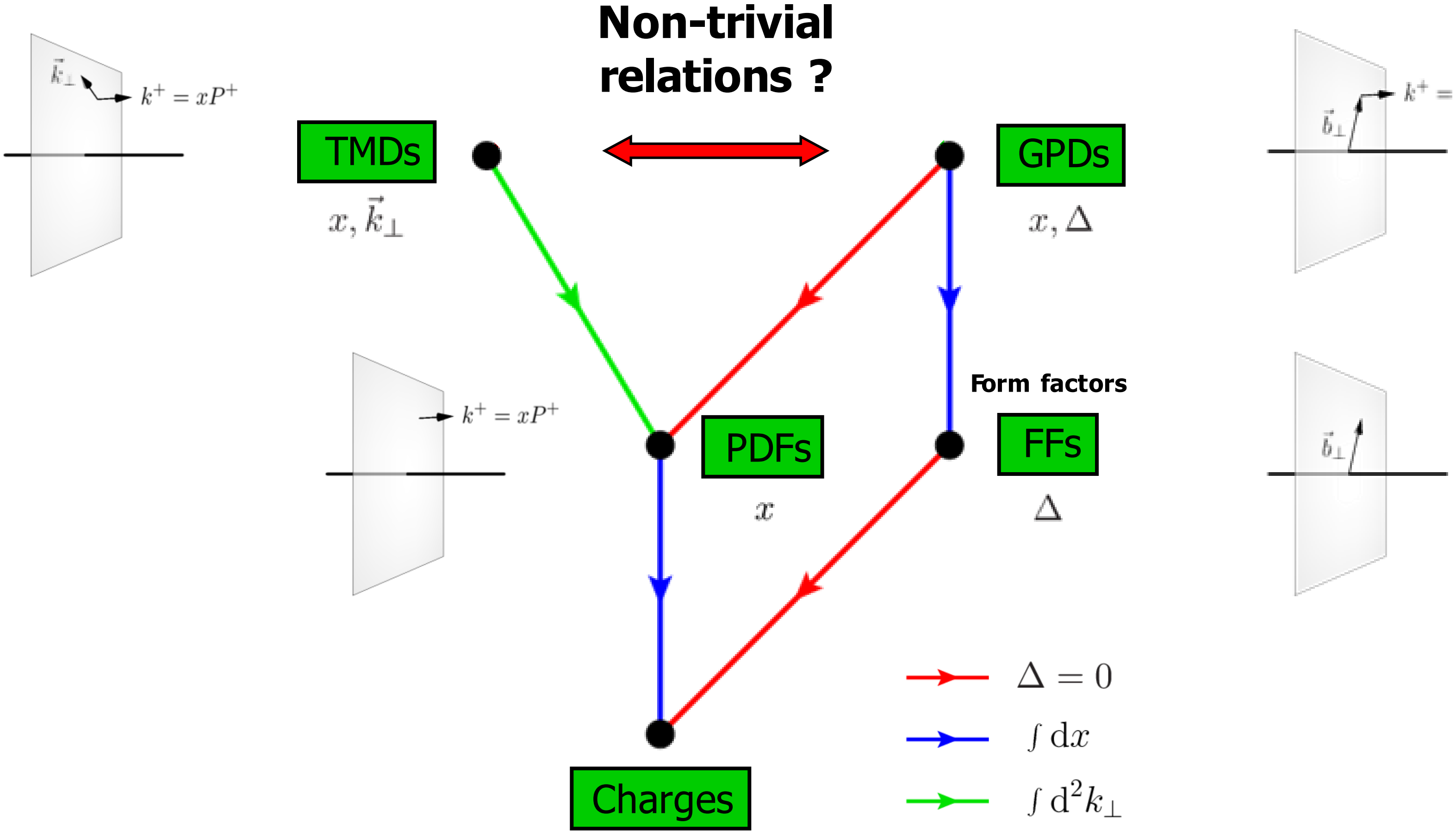
TMDs

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$



0+3D imaging

Parton distribution zoo



Non-trivial relations ?

TMDs

$x, \vec{k}_\perp$

GPDs

x, Δ

Form factors

FFs

Δ

PDFs

x

Charges

$\Delta = 0$

$\int \mathrm{d}x$

$\int \mathrm{d}^2k_\perp$

see next episode !

Some references

- **Ji, PRL78 (1997) 610**
- **Burkardt, PRD62 (2000) 071503**
- **Diehl, PR388 (2003) 41**
- **Belitsky, Radyushkin, PR418 (2005) 1**
- **Bacchetta *et al.*, JHEP02 (2007) 093**
- **Angeles-Martinez *et al.*, APPB46 (2015) 12, 2501**
- **Rogers, EPJA52 (2016) 6, 153**