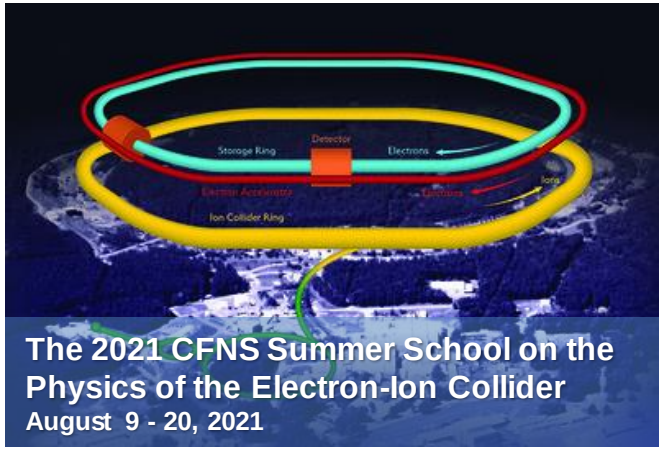
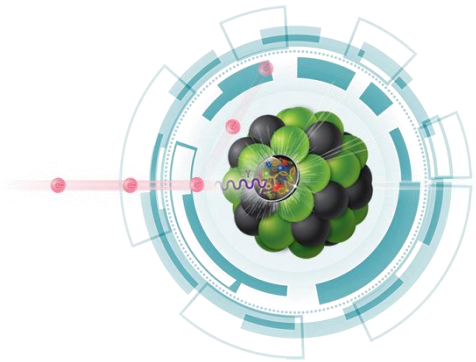




Three-dimensional structure at the EIC (3)

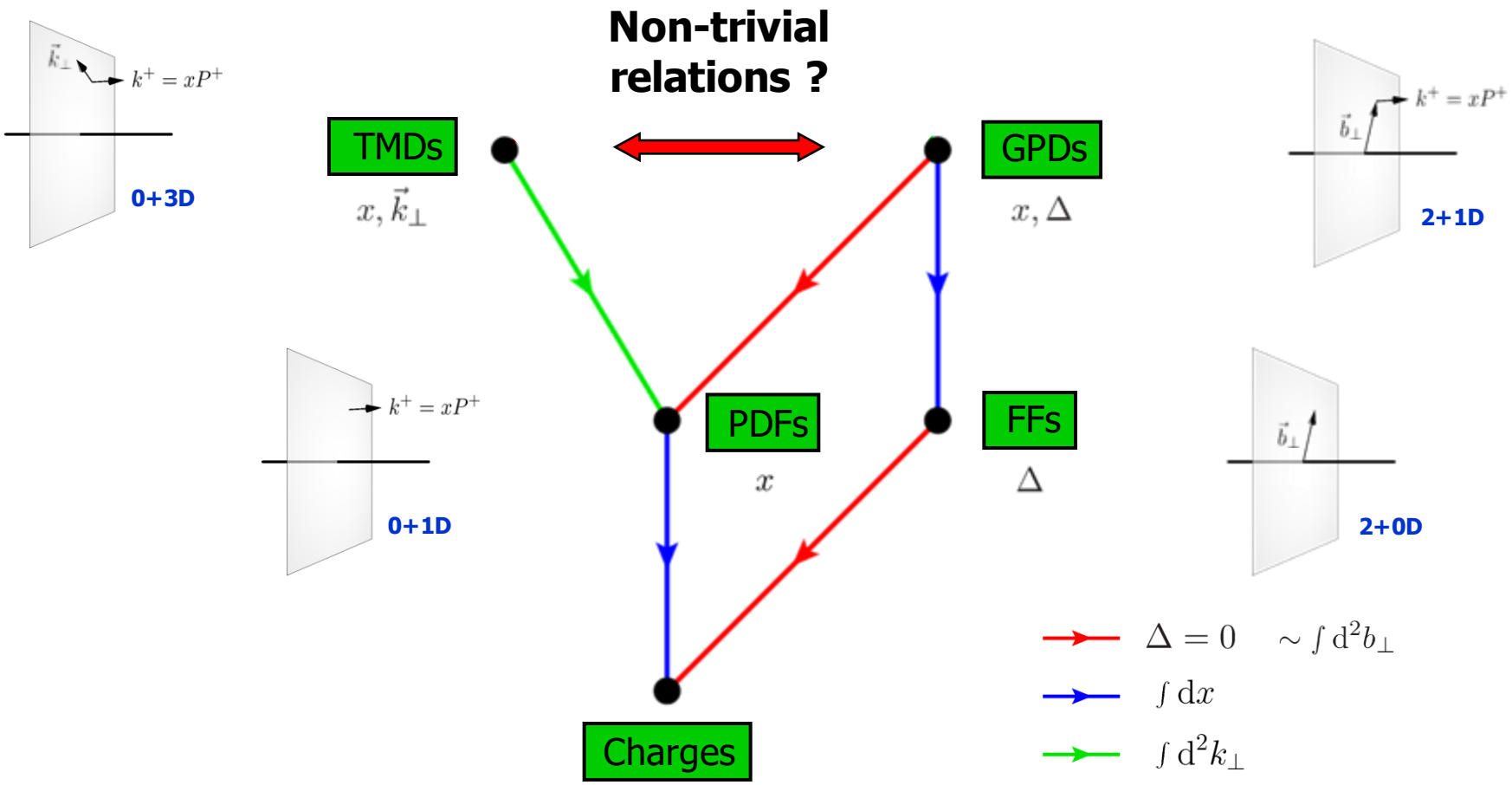
Cédric Lorcé

August 18

Outline

- Lecture 1 : Spatial distributions
- Lecture 2 : Parton distributions
- Lecture 3 : Wigner distributions
- Tutorial

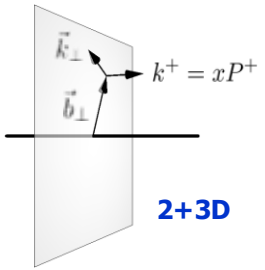
Where we were

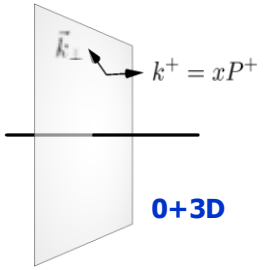


Mother distributions

GTMDs

$x, \vec{k}_\perp, \Delta$



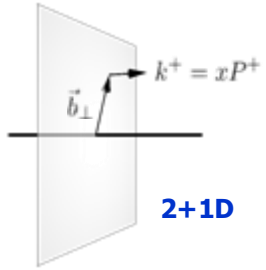


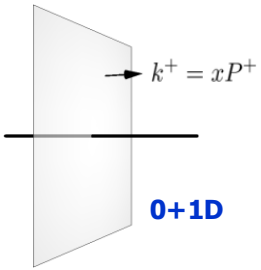
TMDs

$x, \vec{k}_\perp$

GPDs

x, Δ





PDFs

x

FFs

Δ

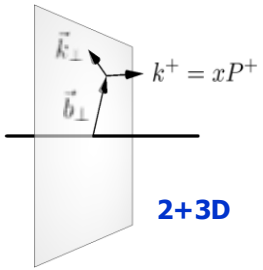
Charges

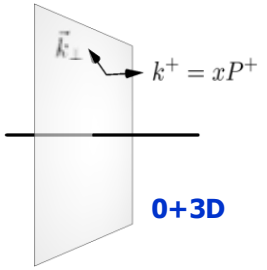
$\Delta = 0 \quad \sim \int \mathrm{d}^2 b_\perp$

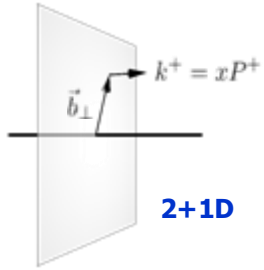
$\int \mathrm{d}x$

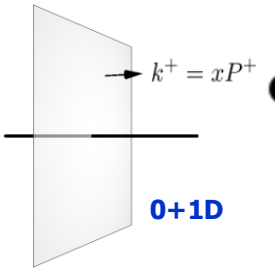
$\int \mathrm{d}^2 k_\perp$

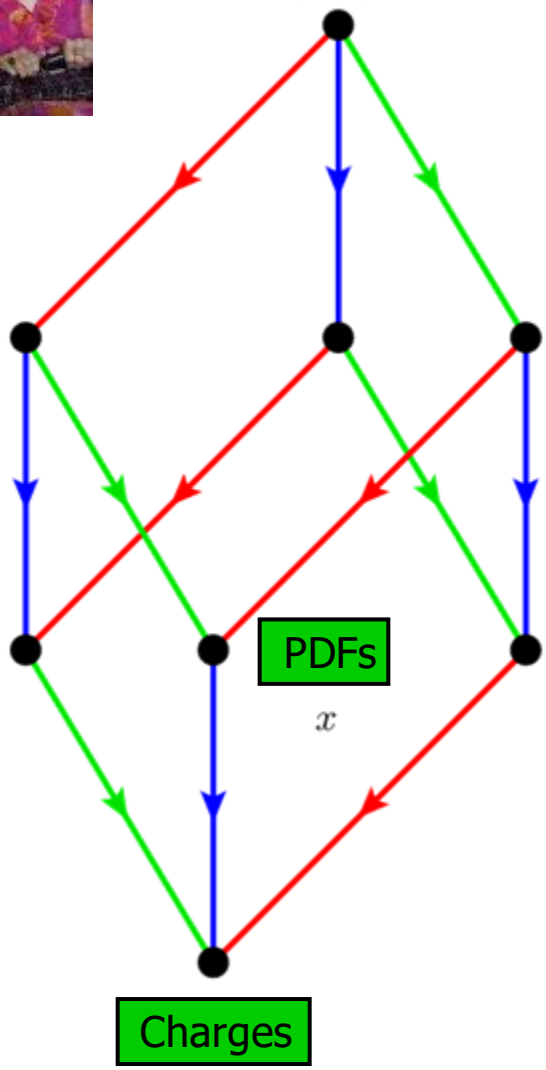
Mother distributions











$\Delta = 0 \quad \sim \int \mathrm{d}^2 b_\perp$

$\int \mathrm{d}x$

$\int \mathrm{d}^2 k_\perp$

Generalized TMDs

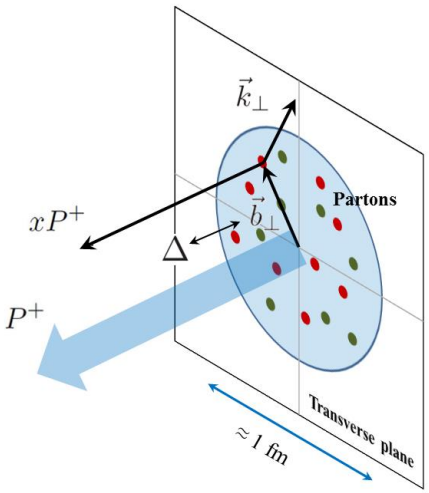
$$\mathrm{PDF}(x) \sim \frac{1}{2}\int \mathrm{d}k^-\,\mathrm{d}^2k_\perp\,\langle p|j^+(0,k)|p\rangle \qquad \mathrm{GPD}(x,\Delta) \sim \frac{1}{2}\int \mathrm{d}k^-\,\mathrm{d}^2k_\perp\,\langle p'|j^+(0,k)|p\rangle$$

$$\mathrm{TMD}(x,\vec{k}_\perp) \sim \frac{1}{2}\int \mathrm{d}k^-\,\langle p|j^+(0,k)|p\rangle \qquad \mathrm{GTMD}(x,\vec{k}_\perp,\Delta) \sim \frac{1}{2}\int \mathrm{d}k^-\,\langle p'|j^+(0,k)|p\rangle$$

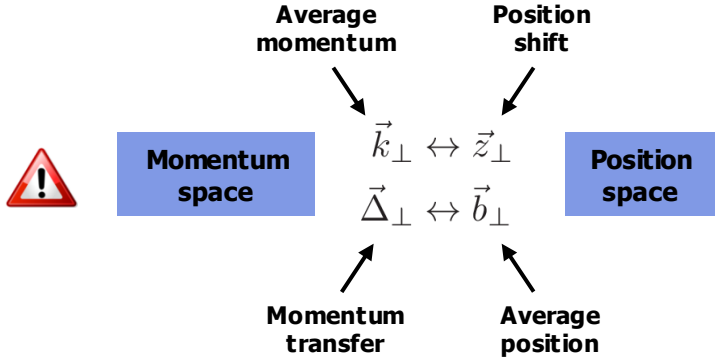
Quark Wigner operator

$$j^\mu(r,k) = \int \frac{\mathrm{d}^4z}{(2\pi)^4}\, e^{ik\cdot z}\,\overline{\psi}(r-\tfrac{z}{2})\gamma^\mu \mathcal{W}(r-\tfrac{z}{2},r+\tfrac{z}{2})\psi(r+\tfrac{z}{2})$$

Wigner (or phase-space) **distribution**



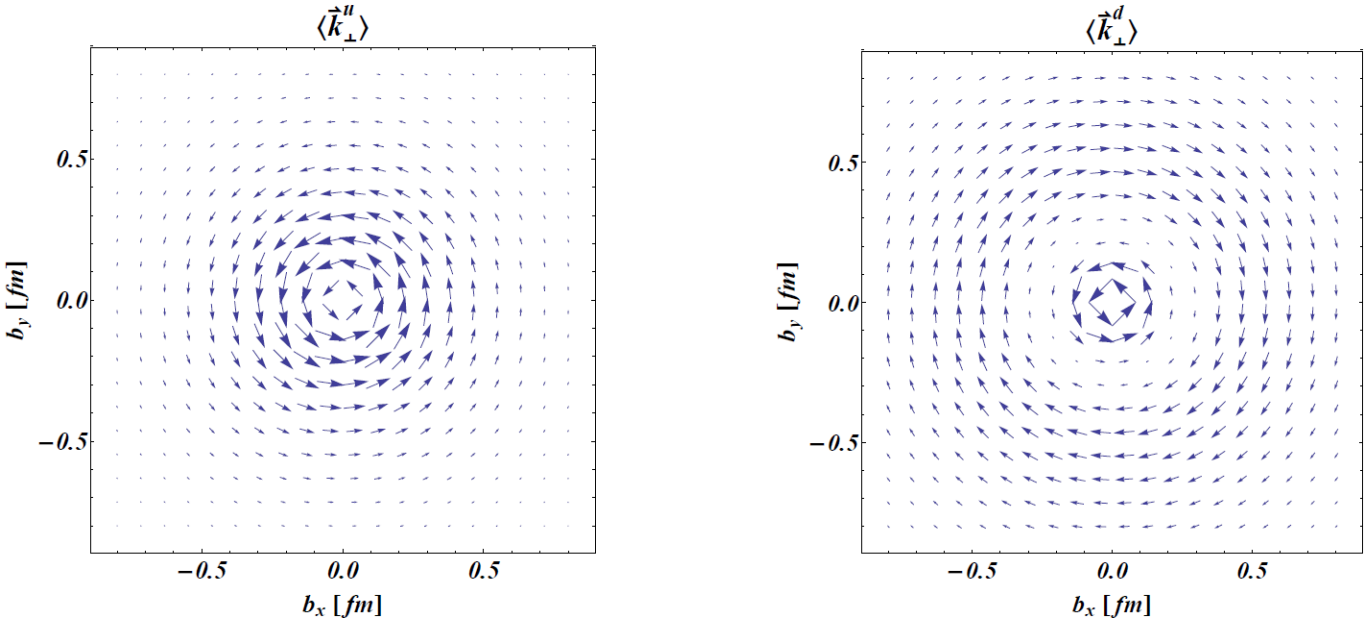
$$\mathrm{WD}(x,\vec{k}_\perp,\vec{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,\mathrm{GTMD}(x,\vec{k}_\perp,\Delta)\big|_{\Delta^+=0}$$



Generalized TMDs

Average transverse quark momentum

$$\langle \vec{k}_\perp \rangle (\vec{b}_\perp) = \int \mathrm{d}x \, \mathrm{d}^2 k_\perp \, \vec{k}_\perp \, \rho_{LU}(x, \vec{k}_\perp, \vec{b}_\perp)$$

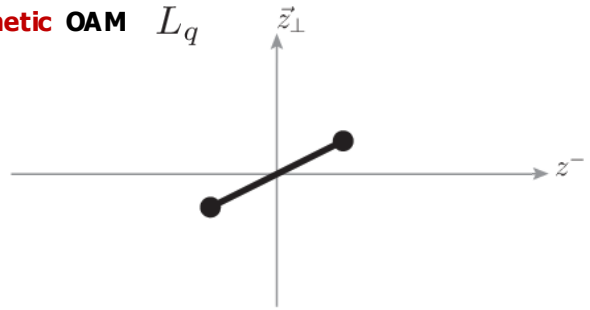


Generalized TMDs

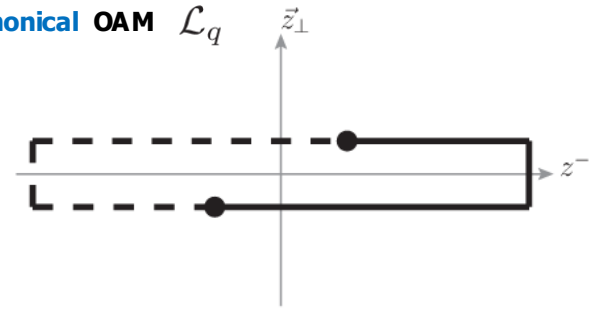
Orbital angular momentum

$$\ell_z = \int \mathrm{d}^2 b_\perp \, \vec{b}_\perp \times \langle \vec{k}_\perp \rangle (\vec{b}_\perp)$$

Kinetic OAM L_q

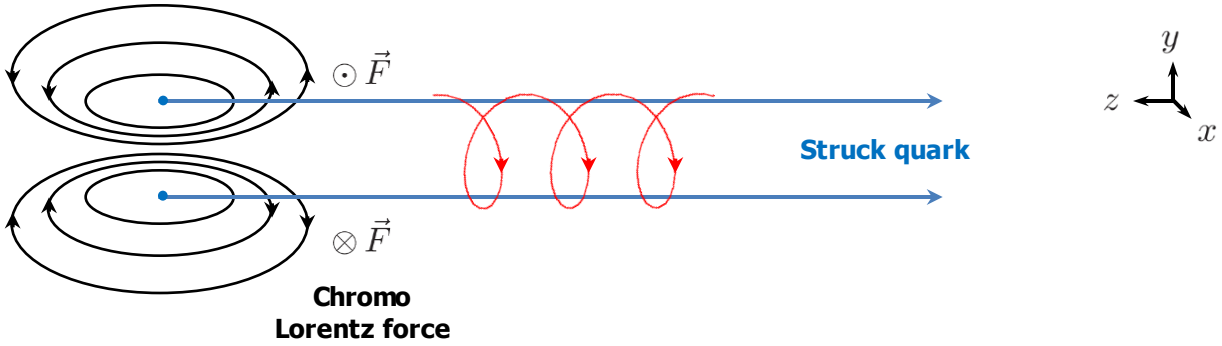


Canonical OAM $\mathcal{L}_q$



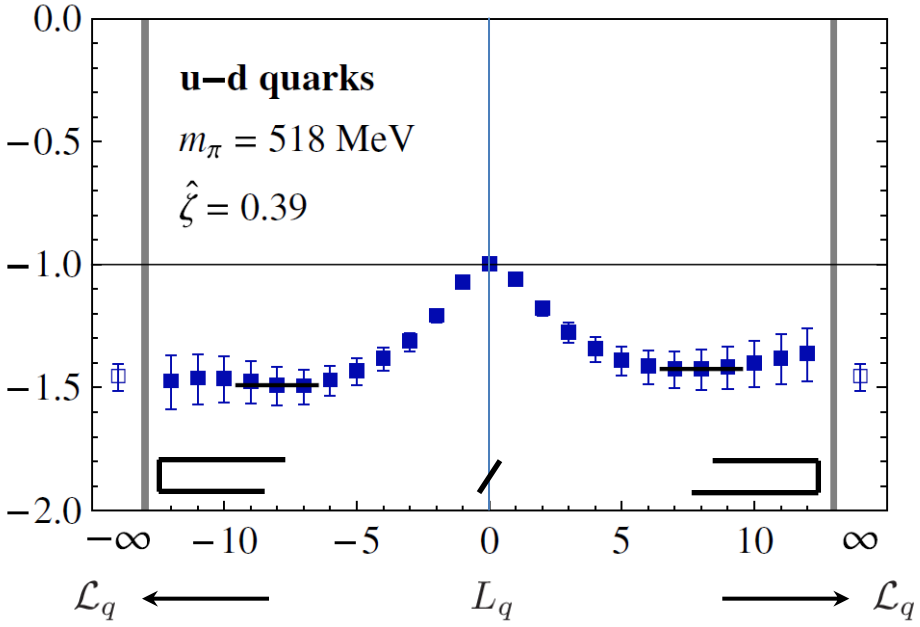
« Inside » the nucleon

« Outside » the nucleon



Chromo Lorentz force

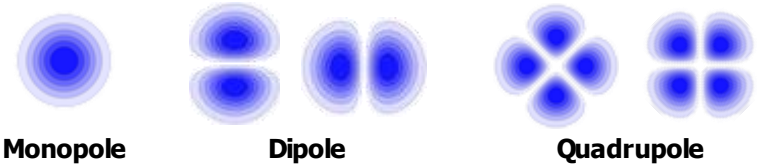
Generalized TMDs



[Engelhardt, PRD95 (2017) 094505]

Correlations

Twist-2



GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\Delta = 0$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

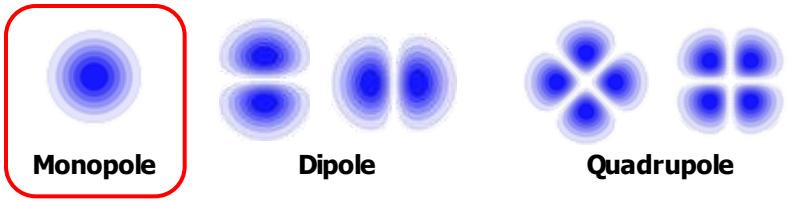
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	
T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Correlations

Twist-2



GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\Delta = 0$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

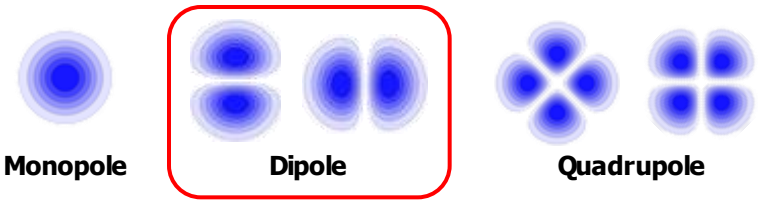
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	
T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Correlations

Twist-2



GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\Delta = 0$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

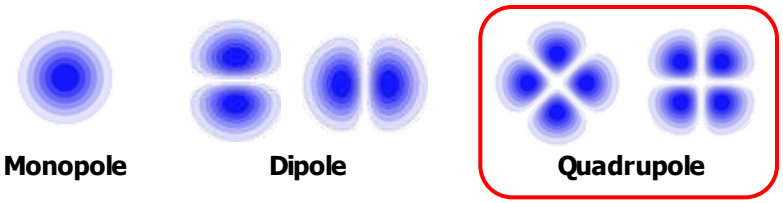
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	
T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Correlations

Twist-2



GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

$\Delta = 0$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

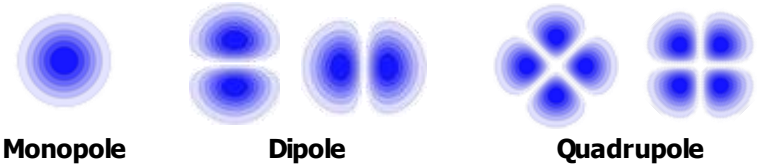
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	
T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Correlations

Twist-2



GTMDs

Quark polarization

Nucleon polarization

	U	T_x	T_y	L
U	F_{11}	$\frac{i}{M}\left(k_y H_{11} + \Delta_y H_{12}\right)$	$-\frac{i}{M}\left(k_x H_{11} + \Delta_x H_{12}\right)$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left(k_y F_{12} + \Delta_y \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_x \bar{G}_{12} + \Delta_x \bar{G}_{13}\right)$
T_y	$-\frac{i}{M}\left(k_x F_{12} + \Delta_x \bar{F}_{13}\right)$	$\cdots$	$\cdots$	$\frac{1}{M}\left(k_y \bar{G}_{12} + \Delta_y \bar{G}_{13}\right)$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M}\left(k_x H_{17} + \Delta_x H_{18}\right)$	$\frac{1}{M}\left(k_y H_{17} + \Delta_y H_{18}\right)$	G_{14}

New !

$\Delta = 0$

$\int \mathrm{d}^2 k_\perp$

$\vec{k}_\perp \leftrightarrow i\vec{\Delta}_\perp$

TMDs

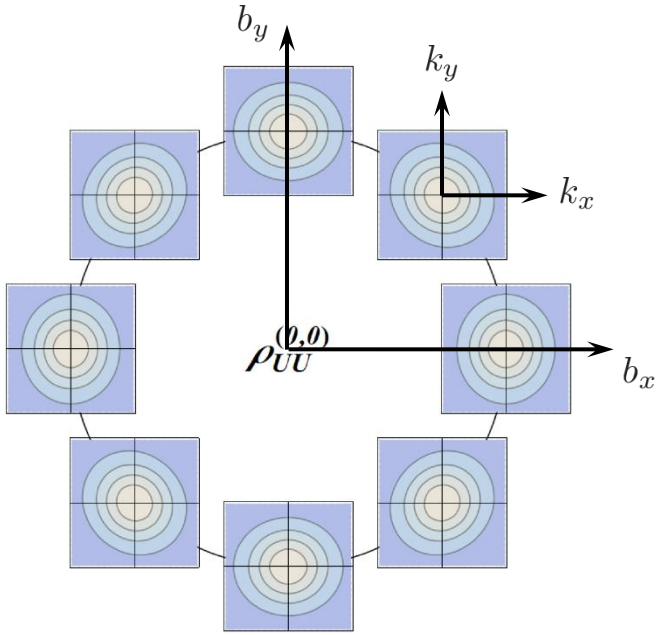
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	✗
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L	✗	$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

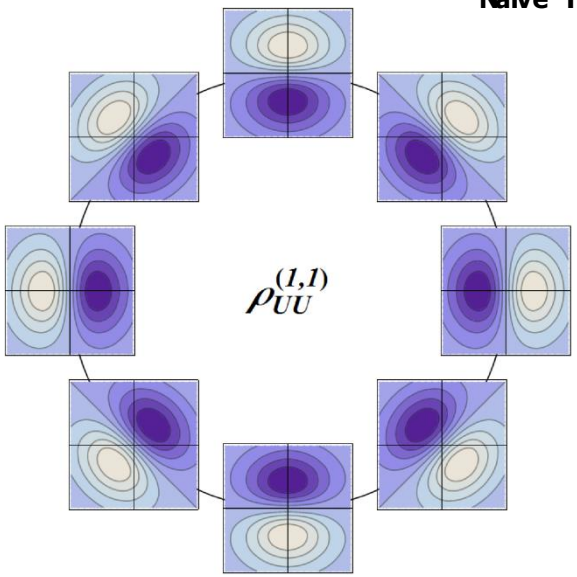
	U	T_x	T_y	L
U	$\mathcal{H}$	$i\frac{\Delta_y}{2M}\mathcal{E}_T$	$-i\frac{\Delta_x}{2M}\mathcal{E}_T$	✗
T_x	$i\frac{\Delta_y}{2M}\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}$
T_y	$-i\frac{\Delta_x}{2M}\mathcal{E}$	$\frac{\Delta_x \Delta_y}{M^2}\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2 - \Delta_y^2}{2M^2}\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}$
L	✗	$\frac{\Delta_x}{2M}\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

Phase-space representation

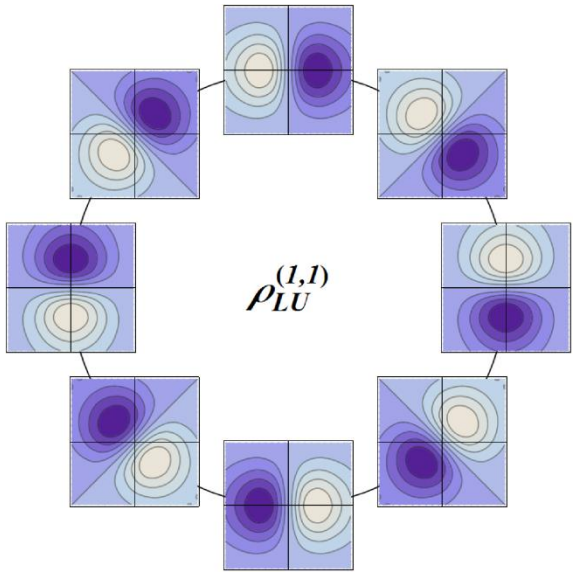
UU



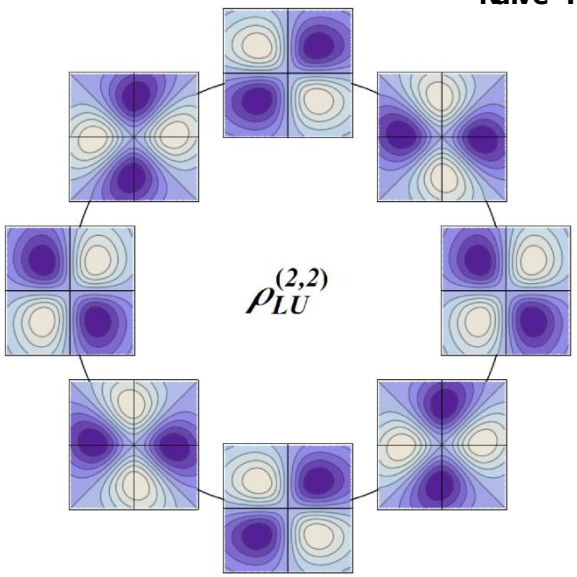
Naive T-odd



LU

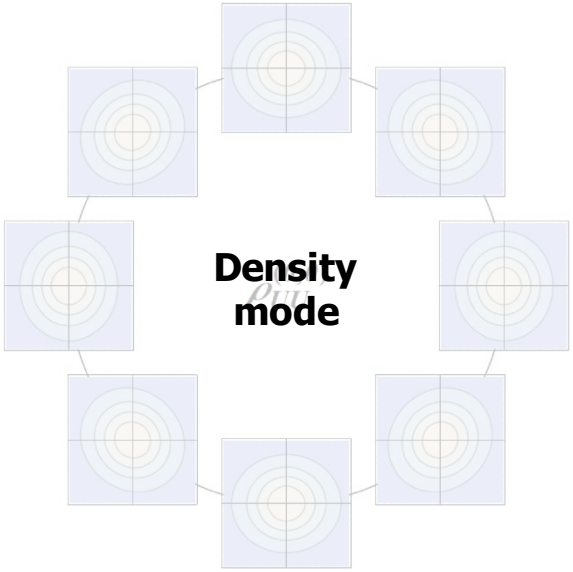


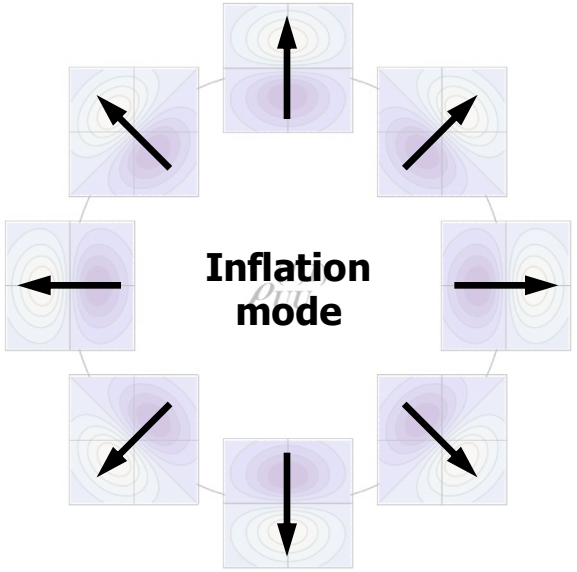
Naive T-odd



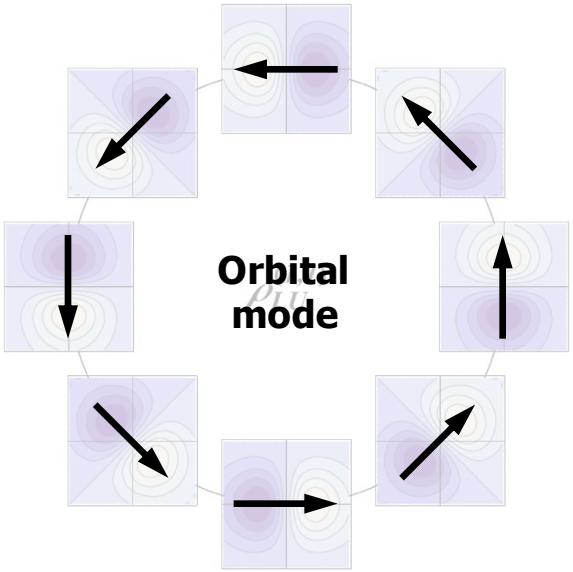
Phase-space representation

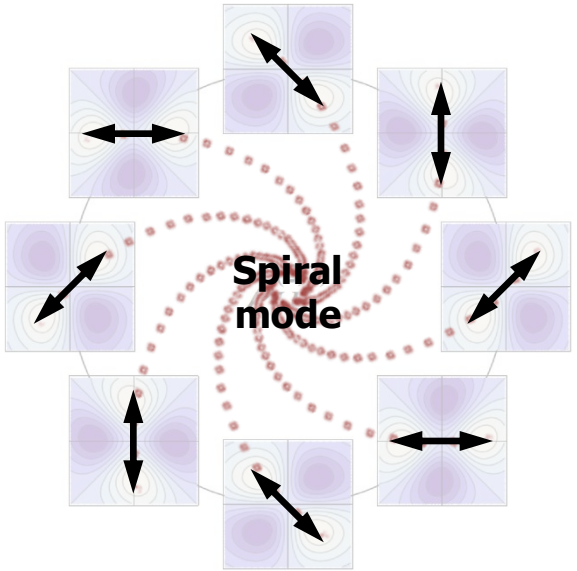
UU





LU

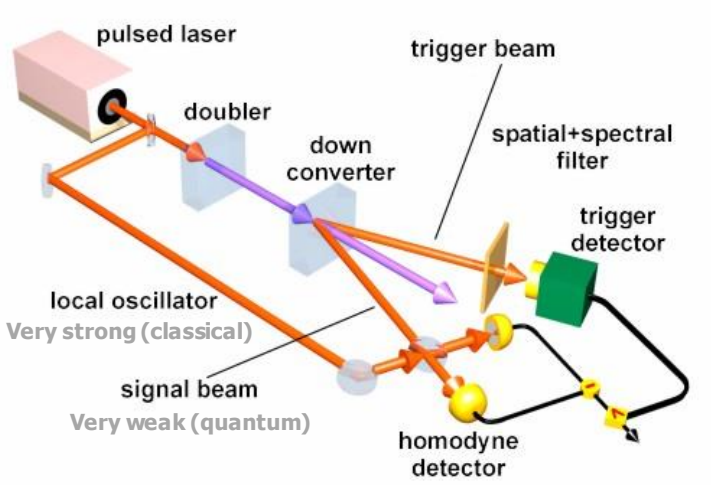




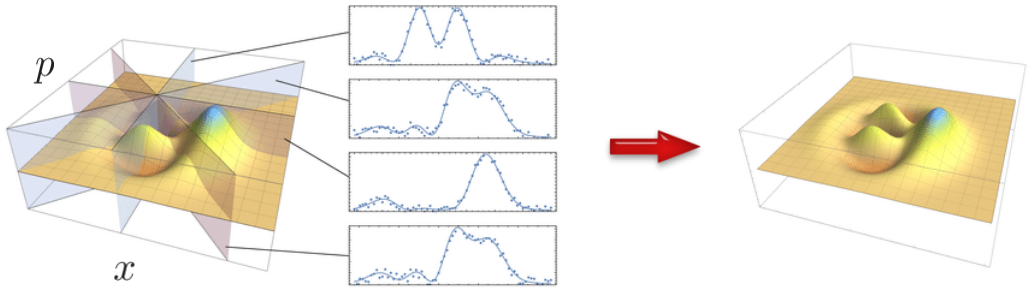
How to measure Wigner distributions

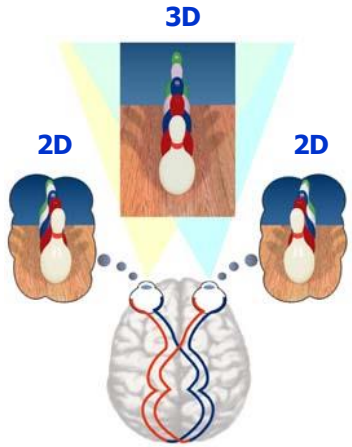
Quantum optics

Wigner distributions can be « measured » using homodyne tomography



<u>Idea :</u> **measure projections of Wigner distributions from different directions in phase space**





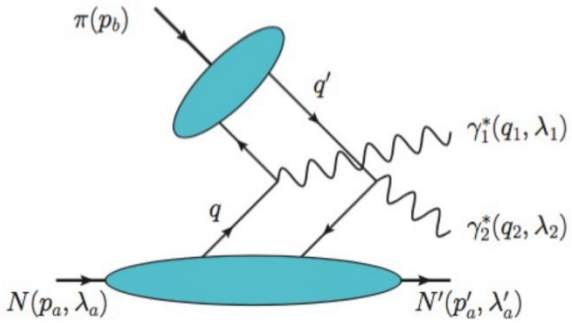
How to measure Wigner distributions

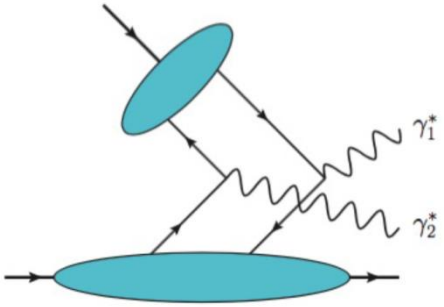
What 3D image is hidden in this stereogram?

How to measure Wigner distributions

Many observables sensitive to GTMDs have recently been proposed

- **eA scattering**
  - **Dijet production** — [Hatta, Xiao, Yuan (2016)]
  - **Longitudinal SSA** — [Hatta, Nakagawa, Yuan, Zhao (2016)] [Ji, Yuan , Zhao (2016)]
- **pA scattering**
  - **Double parton scattering (DPS)** — [Hagiwara, Hatta, Xiao, Yuan (2017)]
  - **Ultra-peripheral collisions (UPCs)** — [Hagiwara *et al.* (2017)]
- **πN scattering**
  - **Exclusive double Drell-Yan** — [Bhattacharya, Metz, Zhou (2017)]





Typically complicated final state ➡ **very small cross sections !**

Some references

- **Meissner, Metz , Schlegel, JHEP08 (2009) 056**
- **Lorcé, Pasquini, Vanderhaeghen, JHEP05 (2011) 041**
- **Lorcé, Pasquini, PRD84 (2011) 014015**
- **Hatta, PLB708 (2012) 186**
- **Lorcé, Pasquini, JHEP09 (2013) 138**
- **Kanazawa *et al.*, PRD 90 (2014) 014028**
- **Lorcé, Pasquini, PRD93 (2016) 034040**