Polarisation Measurement and Spin Tracking at the ILC

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DESY

Beam Polarization and Polarimetry at the EIC, June 26, 2020

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Introduction

Collision Data

Compton Polarimeters

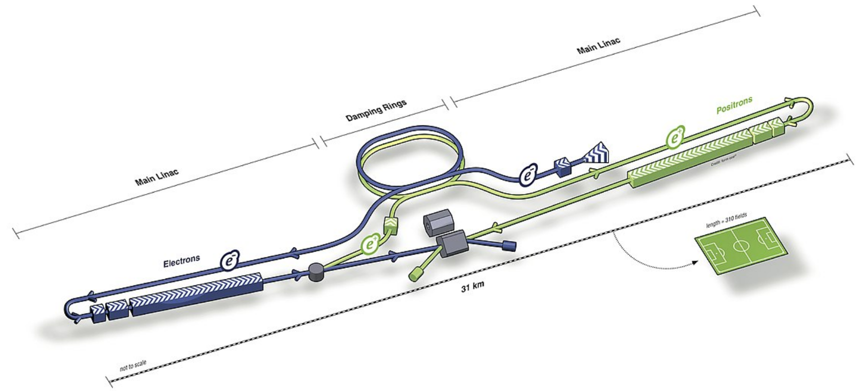
Detector R&D

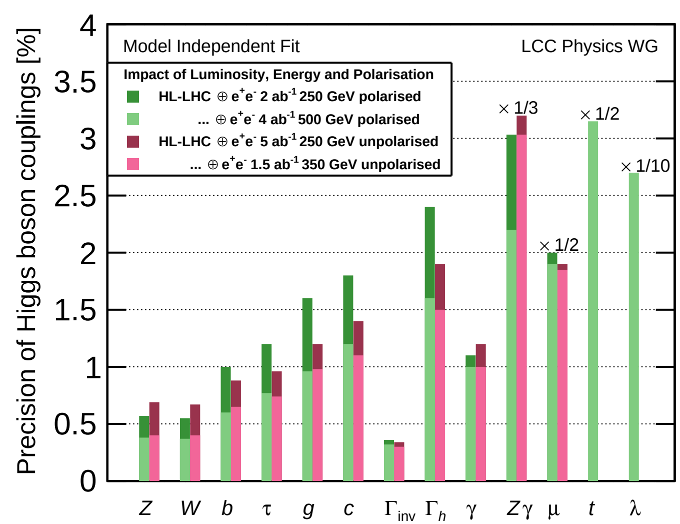
Spin Tracking

Conclusions

The International Linear Collider.

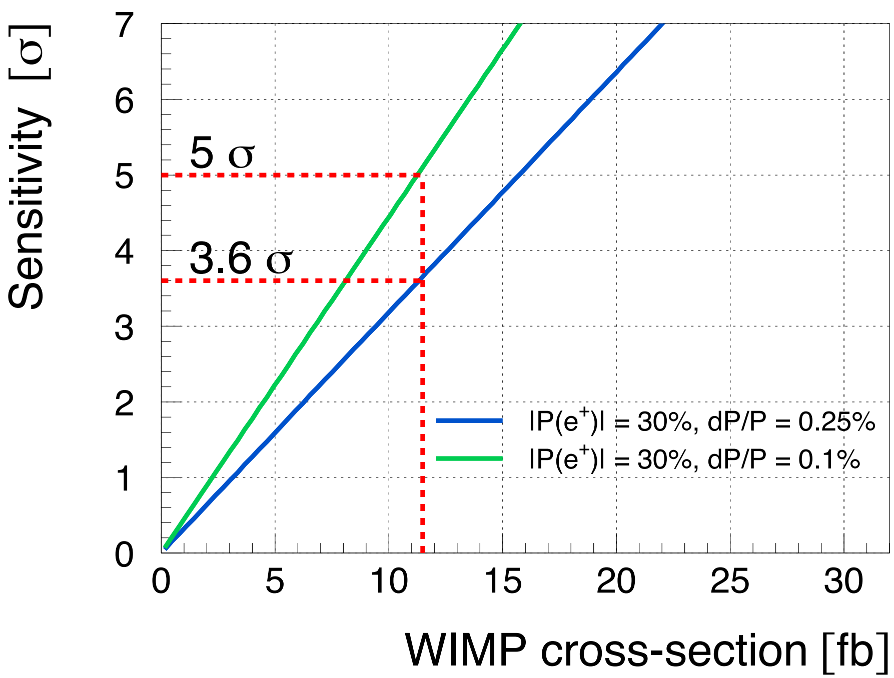
- e^+e^- "Higgs factory" with $\sqrt{s} = 250\,\text{GeV}$, upgradable to up to 1 TeV
- *both* beams polarised: $|P(e^-)| = 80\%$, $|P(e^+)| = 30\%...60\%$
- integral part of physics programme, for Higgs and beyond, c.f. `https://arxiv.org/abs/1801.02840`.
- construction under political consideration in Japan





Impact of Polarisation Uncertainty.

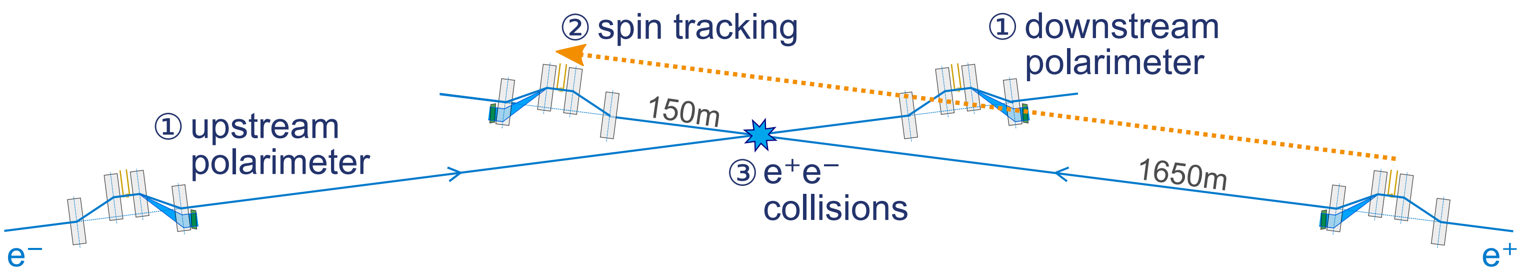
- SM precision measurements, eg. A_{LR} at Z pole will be limited by polarisation knowledge
  $\rightarrow$ simultaneous extraction of A_{LR} and $\langle P_{\mathrm{eff}} \rangle_{IP}$
- BSM example: WIMP Dark Matter Search



- 500 fb^{-1} at 500 GeV, $|P(e^-, e^+)| = (0.8, 0.3)$
- ILD full simulation incl. systematics
- $dP/P = 0.25\%$
  $\rightarrow$ "evidence for"
- $dP/P = 0.1\%$
  $\rightarrow$ "discovery of"

Polarimetry concept for the ILC.

Goal for ILC polarimetry: per-mille level precision on luminosity weighted average polarisation at the IP, $\langle P_z \rangle_{IP} = \frac{\int P_z(t)\mathcal{L}(t)dt}{\int \mathcal{L}(t)dt}$



① Compton polarimeter measurements upstream and downstream of the e^+e^- interaction point

② Spin tracking to relate these measurements to the polarization at the e^+e^- interaction point

③ Long-term average determined from e^+e^- collision data as absolute scale calibration

Polarisation Average from Collision Data.

Direct extraction from collision data

- any abundant, well-known, polarisation dependent process
- total cross-sections only (aka "(modified) Blondel scheme"): $P_+(e^-) = -P_-(e^-)$ and $P_+(e^+) = -P_-(e^+)$
- to lift condition: differential cross-sections *and* polarimeter constraints.

Methods studied so far

- total cross-sections: *WW* at 500 GeV and 1 TeV (ILD)
  single *W* etc at 3 TeV (CLIC)
- single-differential cross-sections: *WW* at 500 GeV & 1 TeV (ILD)
- double-differential cross-sections: *WW* at 1 TeV (SiD)
- **NEW:** global fit incl. differential cross-sections for all $e^+e^- \to f\bar{f}$ and $e^+e^- \to f\bar{f}'f''\bar{f}'''$ processes

Fast helicity reversal: .

... for both beams:

- collect data for all helicity configurations simultaneously
- ensures similar polarisation (absolute) values for all data sets
- enables cancellation of time dependent effects / systematic uncertainties for collider detector!

Counter example HERA:

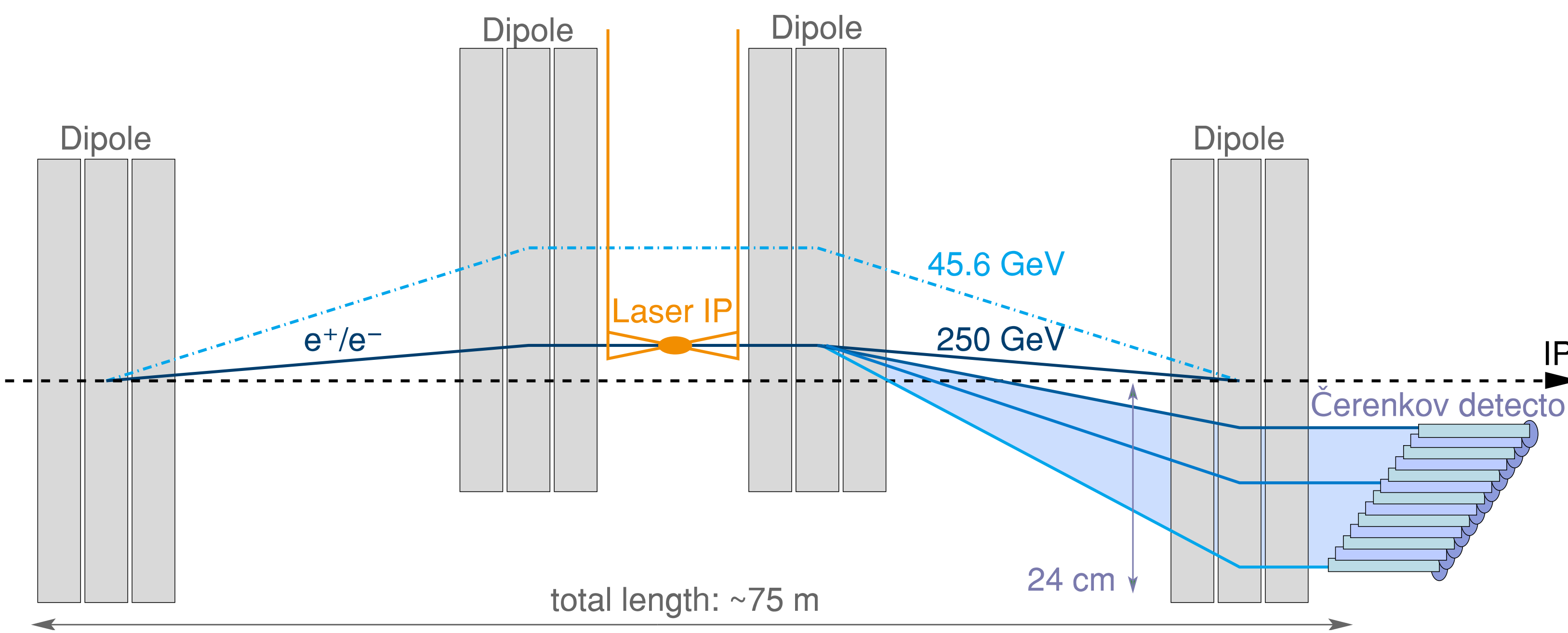
- slow helicity reversal: weeks between flips
- differences in $\langle P_e \rangle_{IP}$: rely on polarimeters
- uncertainty $\sim$ 2%

Collisions	$P_e[\%]$	$\mathcal{L}[\mathrm{pb}^{-1}]$
e^+p	+32	98
e^+p	-38	82
e^-p	+37	46
e^-p	-26	103

Phys. Lett. B704 (2011) 388 [arxiv:1107.3716] (H1 Leptoquarks)

Compton polarimetry.

- fast measurement: $\mathcal{O}(10^3)$ Compton scatterings/bunch
- Energy spectrum of scattered e^+/e^- depends on product of lepton ($\mathcal{P}$) and laser (λ) polarisations
- Magnetic chicane: energy distibution $\rightarrow$ position distribution
- Measure number of e^+/e^- per detector channel



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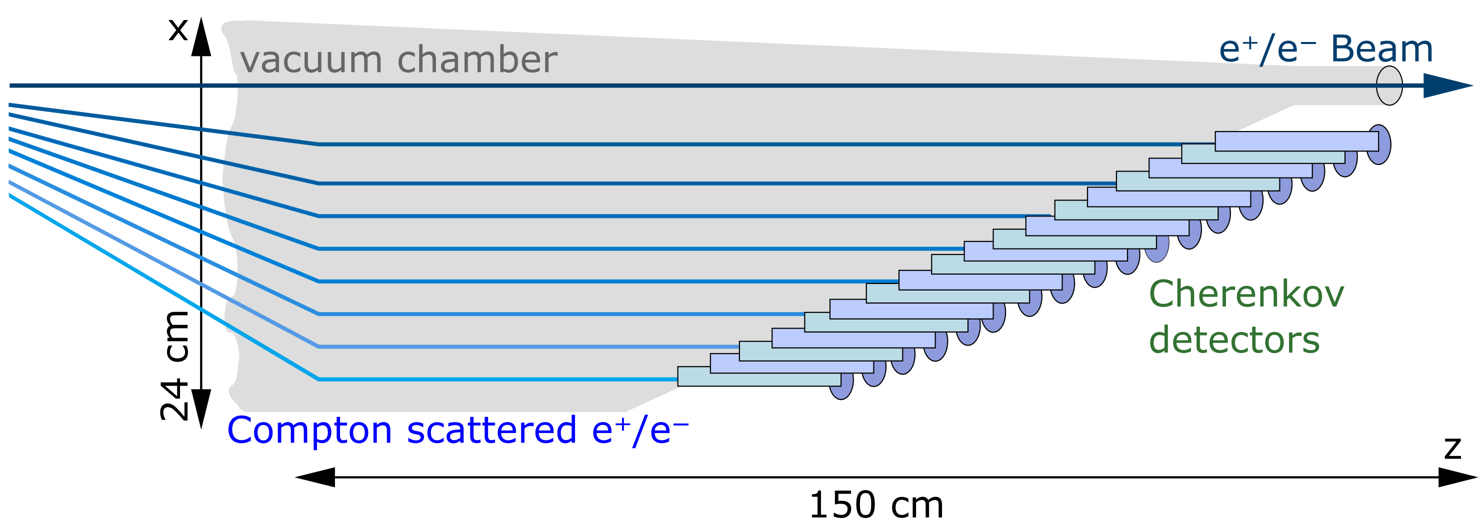
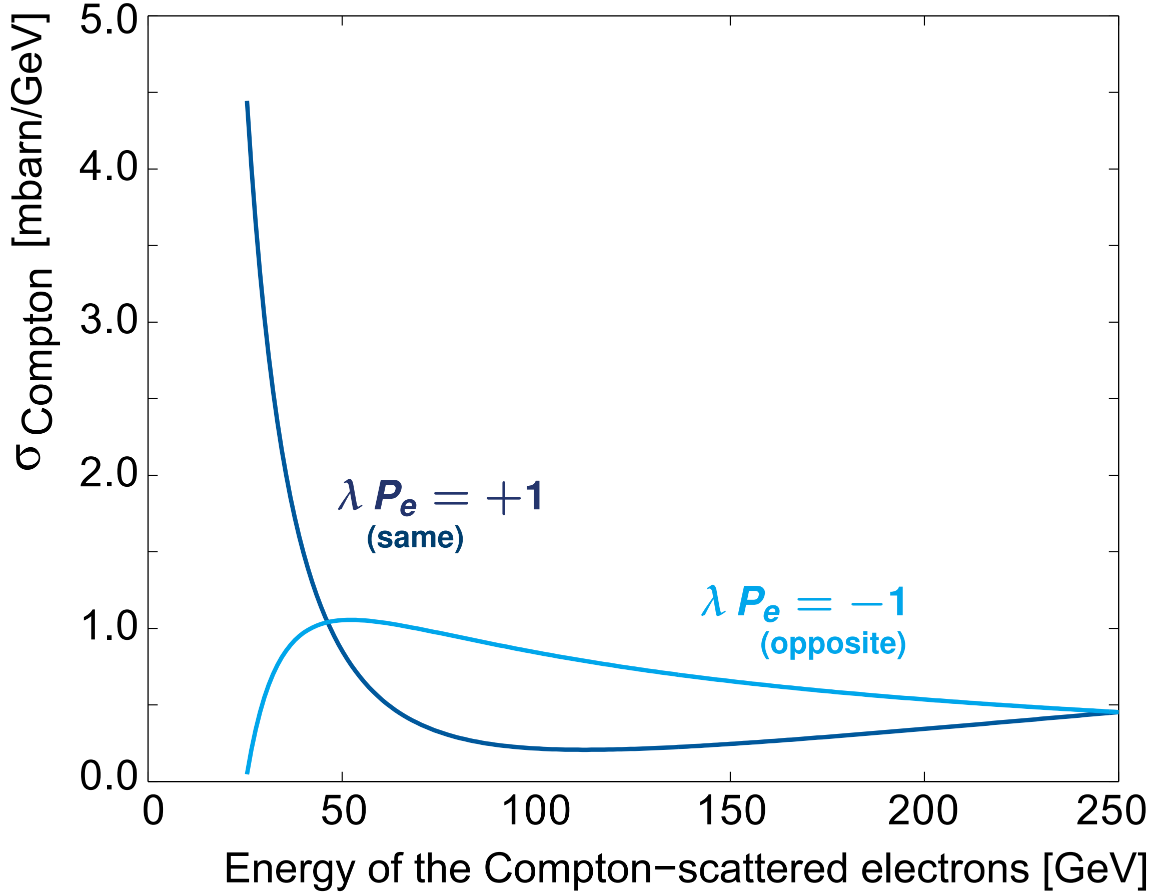


Measurement Principle.

Compton rate asymmetry is proportional to the beam polarisation:



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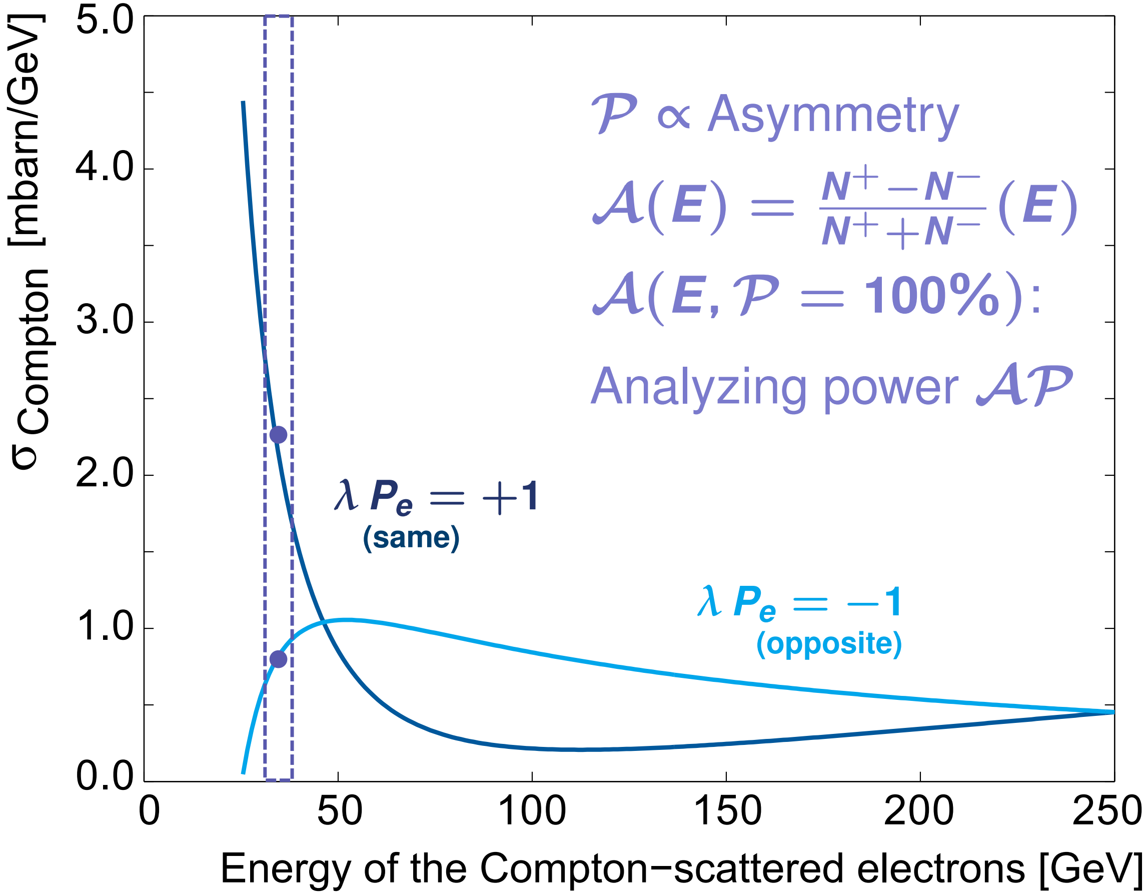

Measurement Precision.

ILC Goal: total uncertainty $\delta P / P \approx 0.25\,\%$ for $|P| \simeq 80\%$

source of uncertainty	$\delta P / P$	
	SLC achieved	ILC goals
laser polarisation	0.1%	0.1%
analyzing power	0.4%	0.15% – 0.2%
detector linearity	0.2%	0.1%
electronic noise and beam jitter	0.2%	0.05%
Total	0.5%	0.25%

- analysing power: prediction of count rate asymmetry per detector channel
  $\Rightarrow$ knowledge of beam parameters, design of chicane, beam-detector alignment, backgrounds
- detector linearity, electronic noise:
  $\Rightarrow$ detector design & calibration
- beam jitter: much smaller at ILC due to luminosity requirements

Complementarity of Up- and Downstream.

Upstream Polarimeter

- 1.8 km upstream of IP
- rather clean environment
- begnin beam cond.
- samples every bunch
- stat. error 1% after few μs
- reference for control of collision effects

Downstream Polarimeter

- 140 m downstream of IP
- high backgrounds
- distrupted beam
- samples one bunch / train
- stat. error 1% after $\simeq$ 1 min
- access to depolarisation in collision

Combination

- without collisions: spin transport in Beam Delivery System and Extraction Line
- with collisions: depolarisation at IP
- **cross check each other!**

  [c.f. "Spin Dance" Exp., Phys. Rev. ST Accel. Beams **7** 042802 (2004)]

Detector Requirements.

Magnetic Chicane...

- transforms energy spectrum into spatial distribution
- behind chicane: $\sim$ 20 cm wide
- detect Compton electrons over this area

Detector requirements:

- Total ionising dose up to 100 Mrad / year
- read out signals of 1000-2000 Compton electrons (25-250 GeV) every bunch crossing
- either very linear response or “counting“ electrons
- alignment to $\sim$ 100 µm and $\sim$ 1 mrad
- suppression of background from low energetic particles

Detector Options.

Simple, robust, fast: Cherenkov detectors

- Cherenkov light emission proportional to number of electrons
- independent of electron energy (once relativistic)
- successfully used in best polarimeter sofar at SLC
- gas or quartz option for Cherenkov medium

Detector Options.

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- Cherenkov light emission proportional to number of electrons
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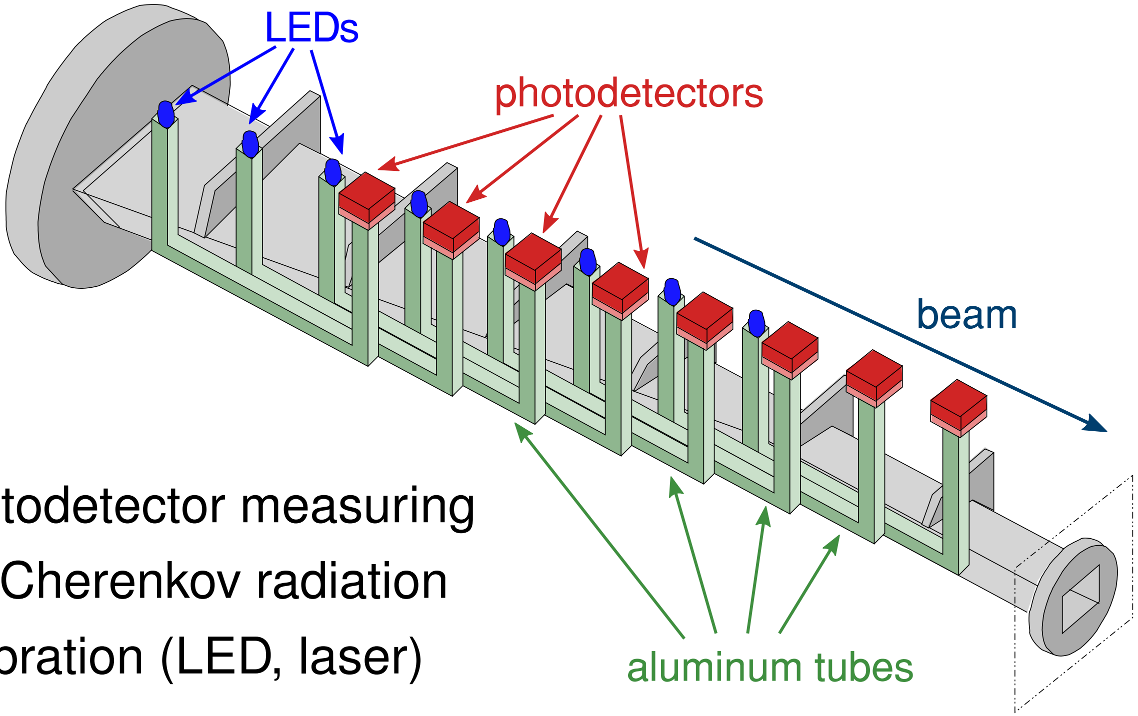
Goal: total uncertainty $\Delta P / P \approx 0.25\,\%$, of which

- laser: 0.1 %
- analysing power (i.e. asymmetry at $\mathcal{P} = 1$): 0.2 %
  $\Rightarrow$ e.g. alignment
- detector linearity: 0.1 % $\Rightarrow$ photodetector calibration

Gas Cherenkov detector.



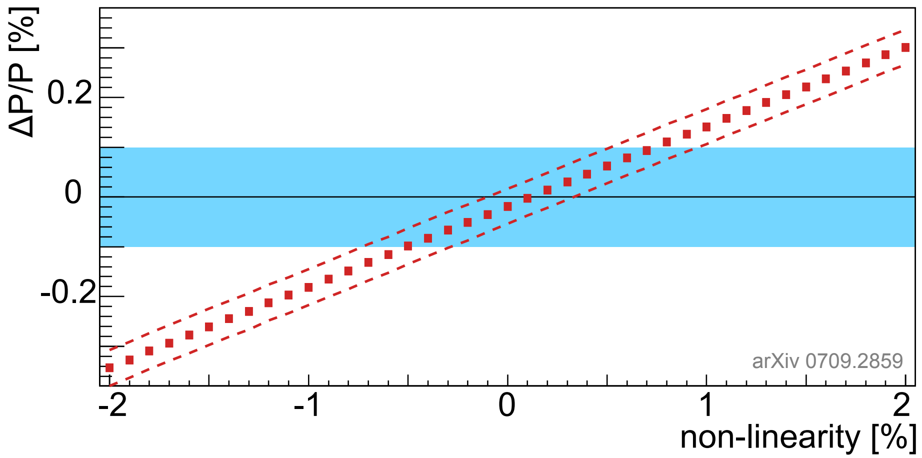
- **hind U-leg:** photodetector measuring the Cherenkov radiation
- **front U-leg:** calibration (LED, laser)

Alignment: locate Compton edge in the spectrometer
Segmented photodetectors: Tilt alignments via asymmetries

2-channel prototype tested at ELSA [JINST 7, P01019 (2012)]
$\Rightarrow$ tilt alignment of 0.1°, nearly fulfils alignment requirements

Calibration of detector non-linearity.

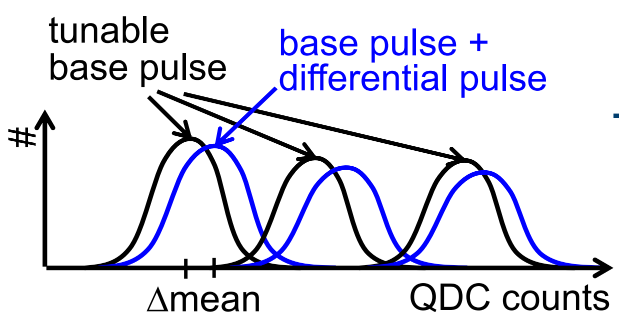
Goal: contribution to overall uncertainty $< 0.1\,\%$

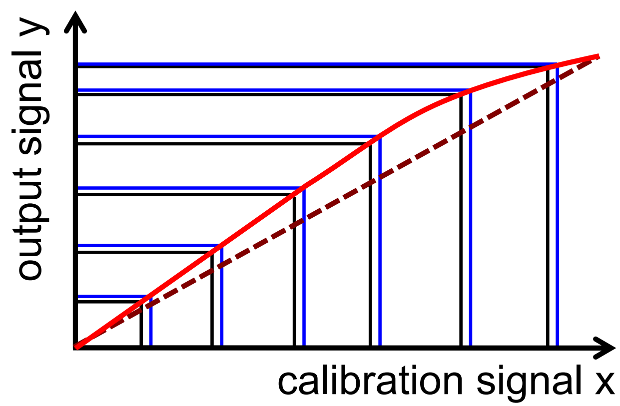


PMTs have to be calibrated to non-linearity $< 0.5\,\%$.

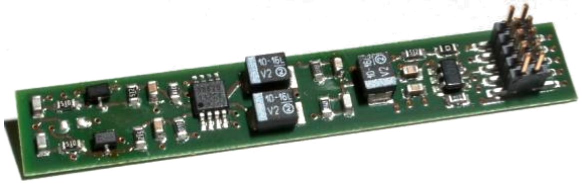
$\mathcal{P} \propto \frac{N^+ - N^-}{N^+ + N^-}$: no absolute calibration needed.

$\rightarrow$ Differential calibration method using two LEDs:



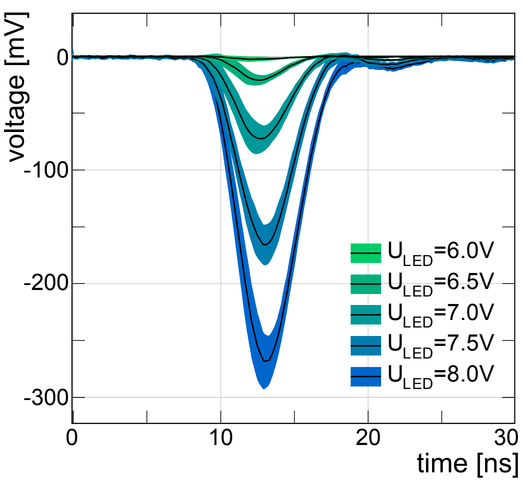


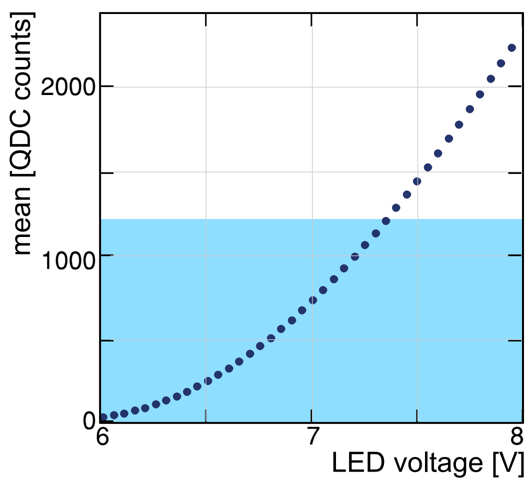
Calibration source requirements.

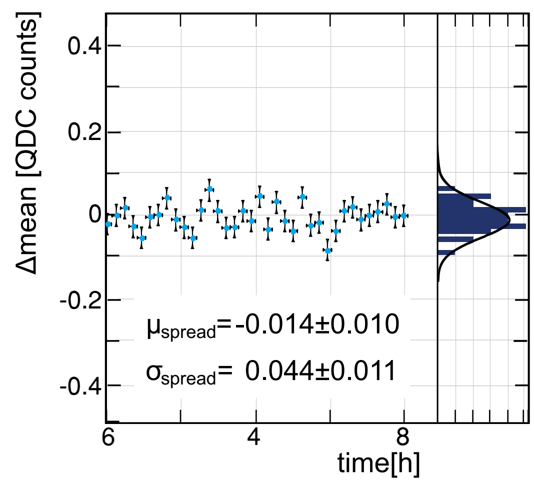


Requirements on the LED driver:

- wave length in UV range ($\lambda = 405\,\text{nm}$)
- applicable in detector design $\rightarrow$ small
- short light pulses ($< 10\,\text{ns}$)
- coverage of the whole dynamic range of the expected signal
- reproducable and stable light pulses

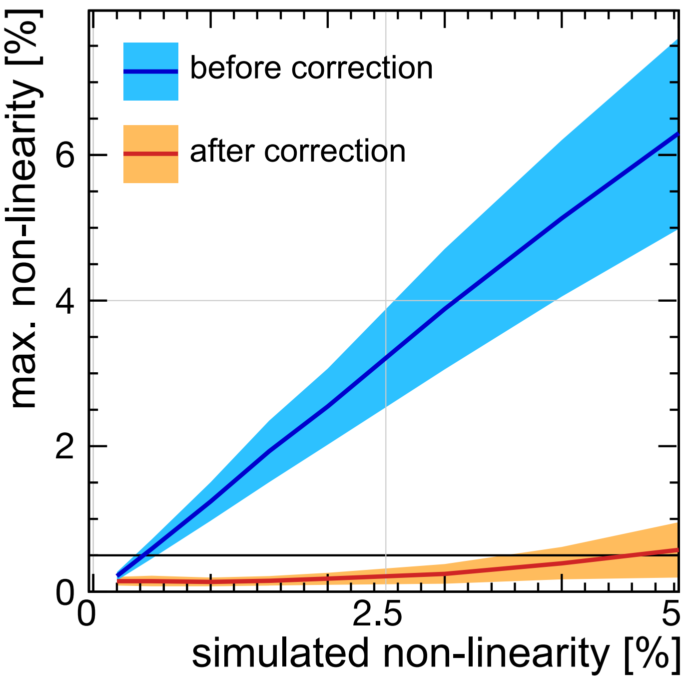






Test of non-linearity correction.

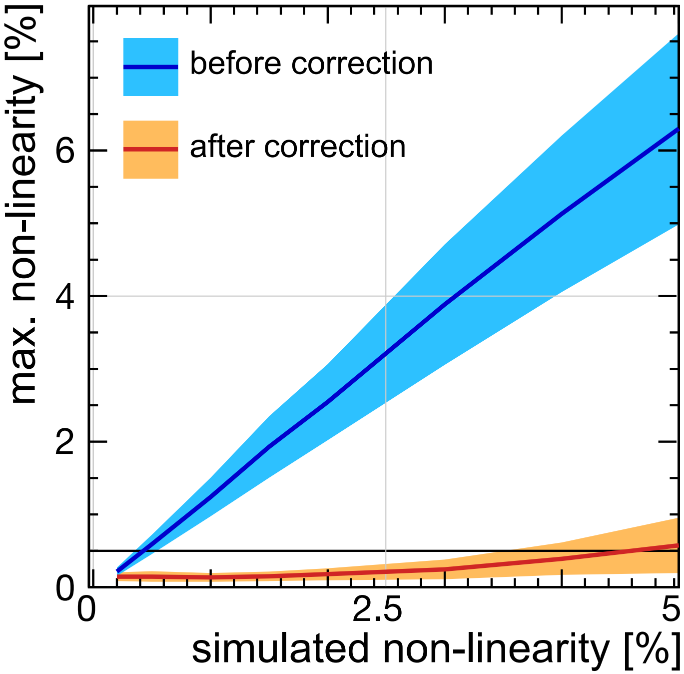
Simulations: Corrections of non-linearities up to 4 % possible.

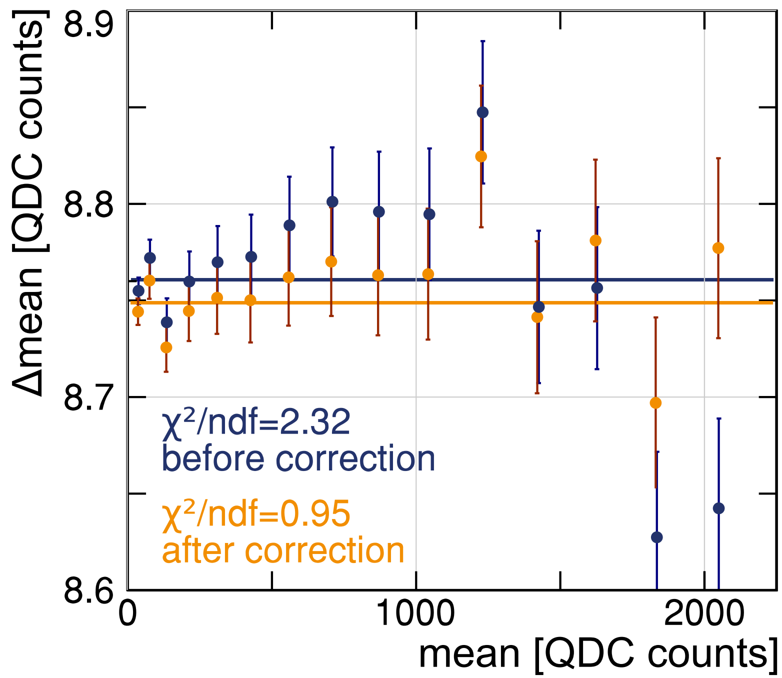


Test of non-linearity correction.

Simulations: Corrections of non-linearities up to 4 % possible.

Applied method to on of the photodetectors used in testbeam:



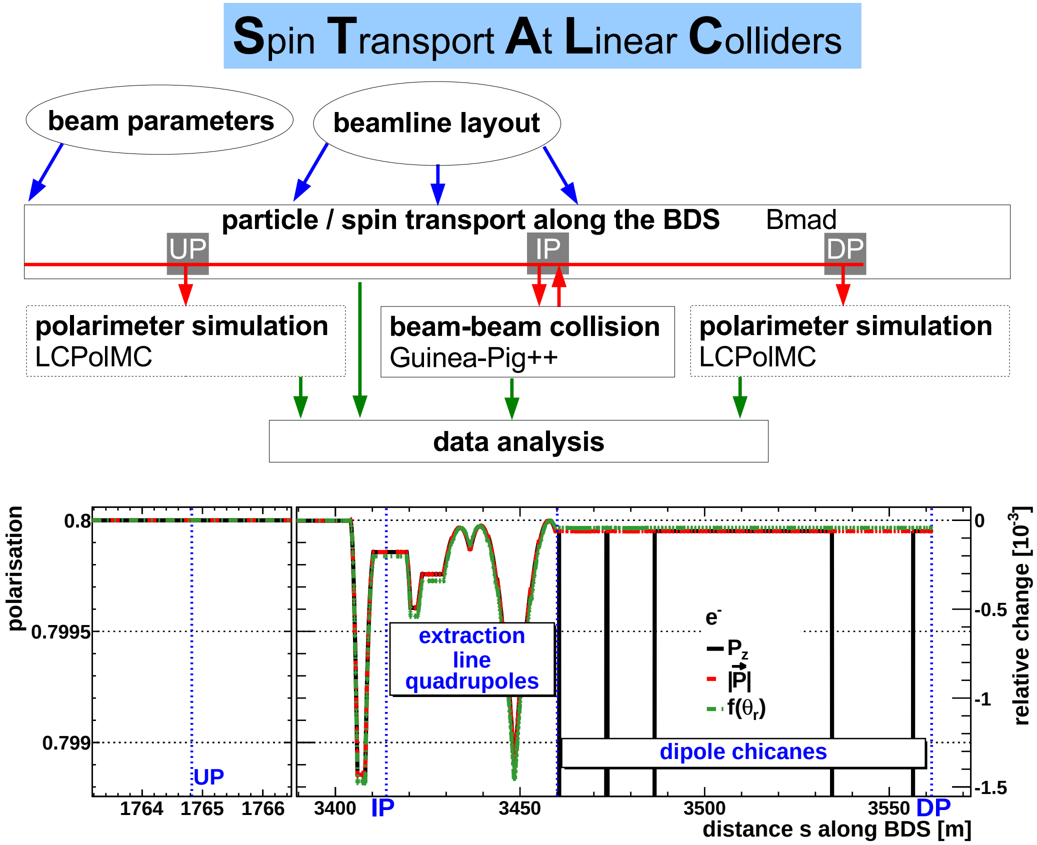


⇒ Reached non-linearity $< 0.2\,\%$ in the expected dynamic range, in single polarimeter channels even smaller.

Spin Tracking in BDS and Extraction Line.

Based on SB2009-Nov10 lattice (PhD Thesis M.Beckmann)

- developed simulation framework STaLC
- without collisions
  ⇒ cross-calibration of polarimeters
- with collisions
  ⇒ what does the downstream polarimeter measure?



Cross-calibration of Polarimeters.

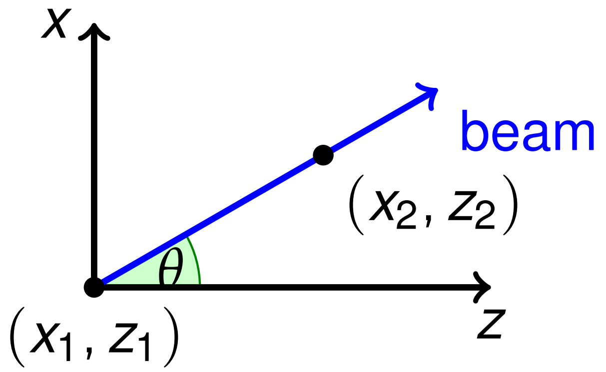
Without Collisions:
predict value at downstream location from upstream measurement

	effect on $P[10^{-3}]$
Beam and detector alignment at polarimeters ($\Delta\theta_{bunch} = 50\,\mu rad$, $\Delta\theta_{pol} = 25\,\mu rad$)	0.72
Variation in emittances	0.03
Crabbing	< 0.01
Detector magnets	0.01
Emission of synchrotron rad.	0.005
random misalignments (10 µm)	0.35
Total	0.80

Beam alignment.

Precision of Polarimeter Cross-Calibration

- is dominated by relative angle between beam directions at UP/DP $\Delta\theta_{bunch}$
- need *pairs* of BPMs at UP, IP and DP $\Rightarrow$ "local" angle
- $\Delta x = 7\,\mu$m, distance along z = a few meters $\Rightarrow 1 - 2\,\mu$rad
- challenge: absolute reference over $\simeq 2\,$km
  $\Rightarrow$ is 50 μrad a realistic number?



$$\theta \approx \frac{x_2 - x_1}{L}; \quad L := z_2 - z_1; \quad L = 8m$$

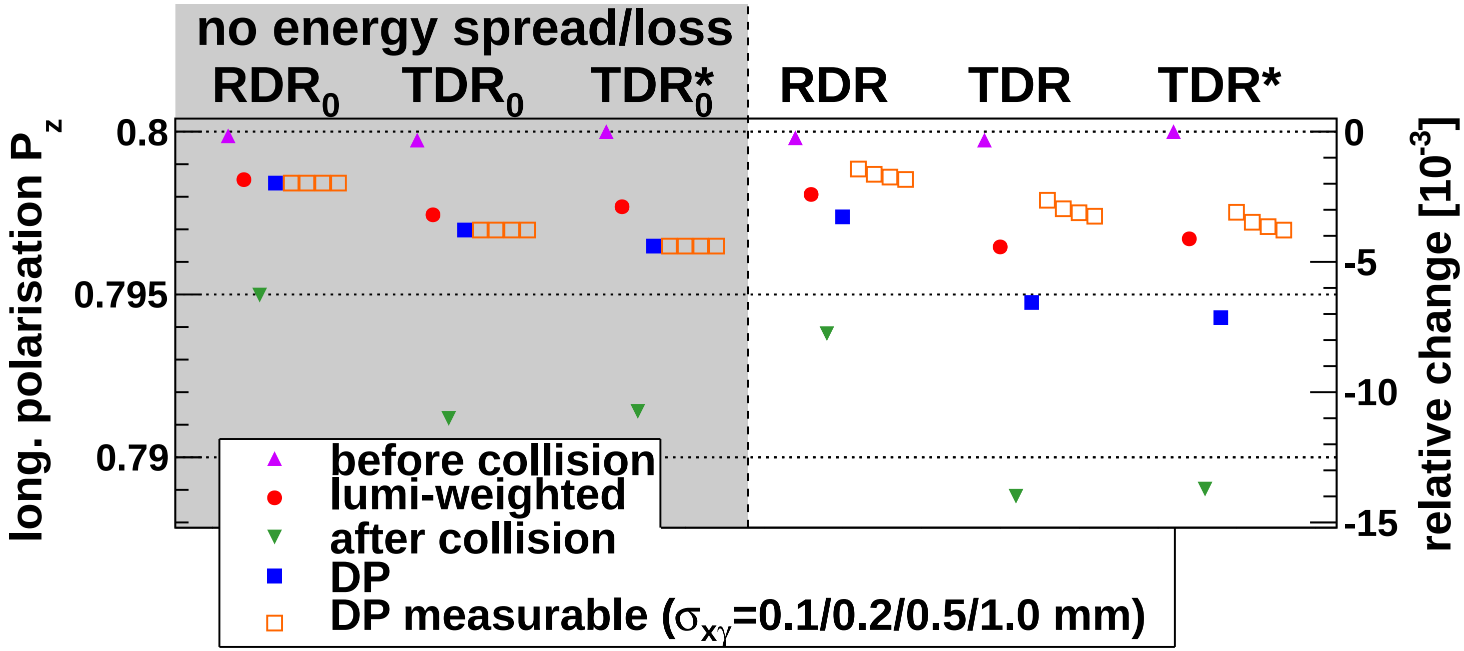
$$\Delta x_i = \Delta y_i = 7\,\mu m; \quad \Delta L \simeq 0$$

$$\Rightarrow \Delta\theta^2 \leq \underbrace{2\left(\frac{\Delta x_i}{L}\right)^2 + 2\left(\frac{\Delta y_i}{L}\right)^2}_{:=(\Delta\theta_{BPM})^2} + \underbrace{\left(\frac{\theta\Delta L}{L}\right)^2}_{:=(\Delta\theta_L)^2}$$

$$\Rightarrow \Delta\theta \simeq 1.7\mu rad$$

Collision Effects & Downstream Polarimeter.

- Extraction line optics designed to retrieve $\langle P \rangle_{IP}$ at downstream polarimeter
- Confirmed by STaLC w/o beamstrahlung (energy loss, grey background)
- With beamstrahlung: few permille difference to $\langle P \rangle_{IP}$
- Measured polarisation depends on laser spot size (here: perfect centering!)
- Effect doubles from RDR → TDR parameters



Conclusions Spin Tracking.

In the presence of significant Beamstrahlung:

- Downstream polarimeter does *not* measure directly any more $\langle P \rangle_{IP}$
- difference DP measurement vs $\langle P \rangle_{IP}$ depends on
  - laser spot size & position
  - luminosity ($\simeq$ energy loss in collision)
- $\Rightarrow$ correcting for this requires
  - absolute reference from upstream polarimeter
  - luminosity & beam parameter monitoring
  - long-term scale of $\langle P \rangle_{IP}$ from collision data

Conclusions.

Permille-level precision on lumi-weighted average polarisation at IP required by physics, needs combination of

- long-term scale calibration from e^+e^- collision data (fast helicity reversal!)
- upstream (UP) and downstream (DP) polarimeters
  - **UP**: time resolution, absolute reference
  - **DP**: collision effects
  - **combined**: cross-calibration, lumi-weighted polarisation @ IP
- spin tracking and understanding of collision effects

Compton Polarimeters:

- beam-detector alignment & detector linearity crucial
- R&D well underway, requirements $\simeq$ reached in prototypes
- cross-calibration without collisions: $\sim 0.1\%$ from alignment
  - esp. orbit and spin at UP and DP locations (2 km apart)

More Details.

- E&P workshop Zeuthen 2008

  http://indico.desy.de/conferenceDisplay.py?confId=585

- its Executive Summary

  arXiv:0903.2959 [physics.acc-ph]

- downstream polarimeter 6-magnet chicane

  http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-12425.pdf

- publication on beam energy and polarisation measurements

  JINST **4** (2009) P10015, arXiv:0904.0122 [physics.ins-det]

- publications on polarimeter detector R&D:

  JINST **7** (2012) P01019, arXiv:1011.6314 [physics.ins-det]

  arXiv:1502.06955 [physics.ins-det], DESY 15-028

- publication on BDS spin tracking

  JINST **9** (2014) P07003, arXiv:1405.2156 [physics.acc-ph]

Backup Slides .

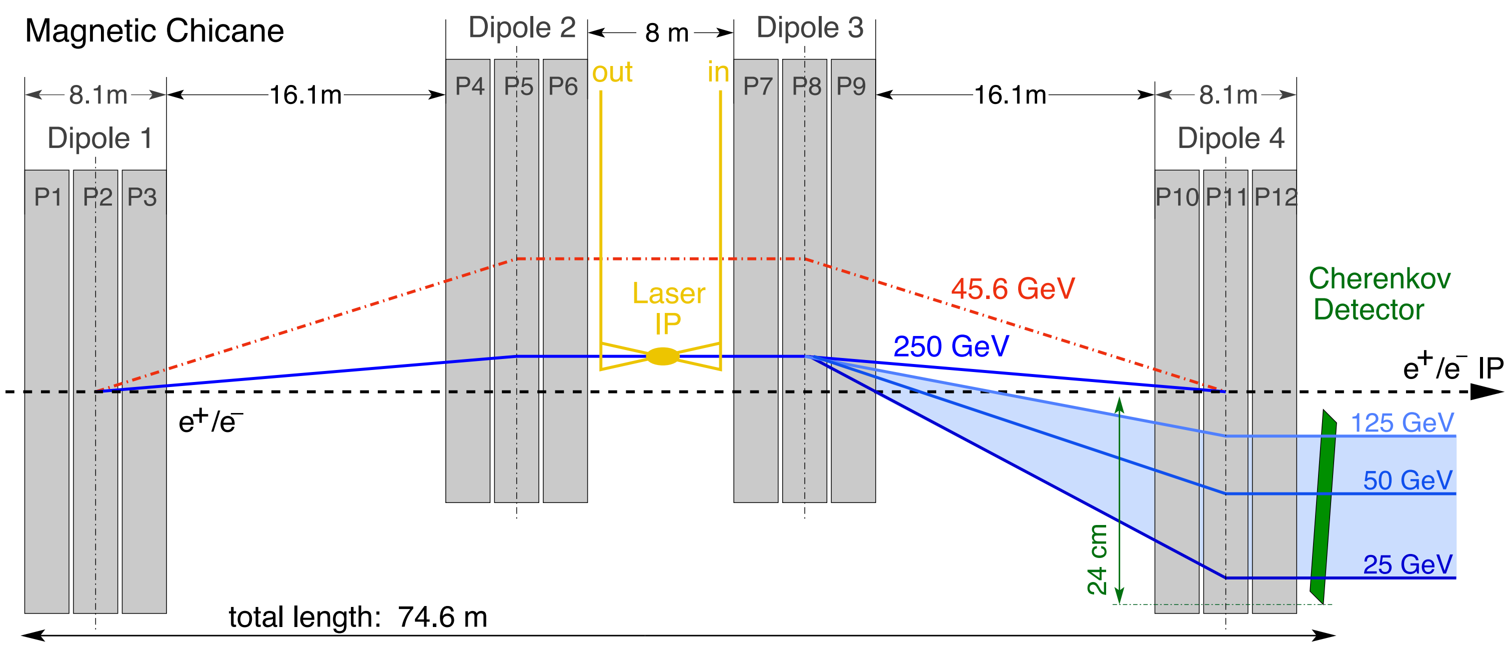
For historic information: Recommendations to GDE and Research Director (2008).

- Separate the functions of the upstream polarimeter chicane. Do not include an MPS energy collimator or laser-wire emittance diagnostics; use instead a separate setup for these two.
- Modify the extraction line polarimeter chicane from a 4-magnet chicane to a 6-magnet chicane to allow the Compton electrons to be deflected further from the disrupted beam line.
- Include precise polarisation and beam energy measurements for Z-pole calibration runs into the baseline configuration.
- Keep an initial positron polarisation of 30-45% for physics, don't reduce to 22% .
- Implement parallel spin rotator beamlines with a kicker system before the damping ring to provide rapid helicity flipping of the positron spin.
- Move the pre-DR positron spin rotator system from 5 GeV to 400 MeV. This eliminates expensive superconducting magnets and reduces costs.
- Move the pre-DR electron spin rotator system to the source area. This eliminates expensive superconducting magnets and reduces costs.

Design of the Upstream Polarimeter Chicane.

Why a 4-Dipole-Chicane?

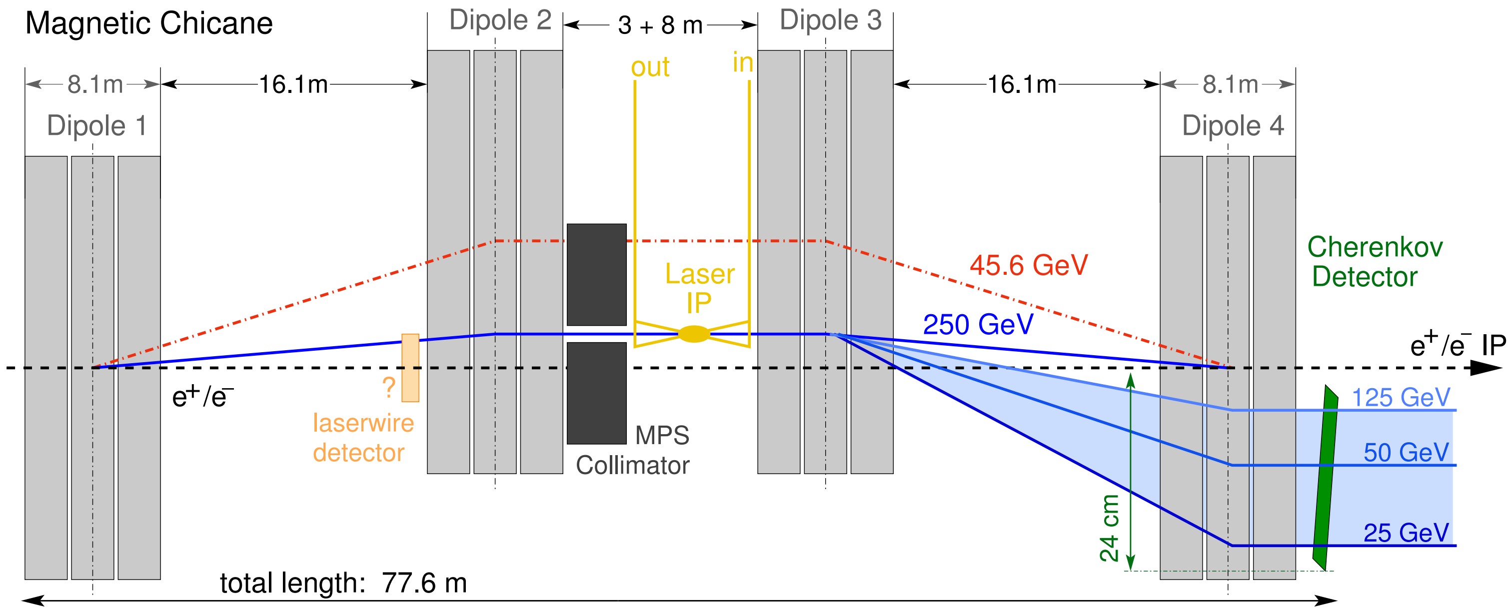
- Compton spectrum position at detector independent of E_{beam} if B-field constant
- price to pay: Compton IP moves laterally with E_{beam}



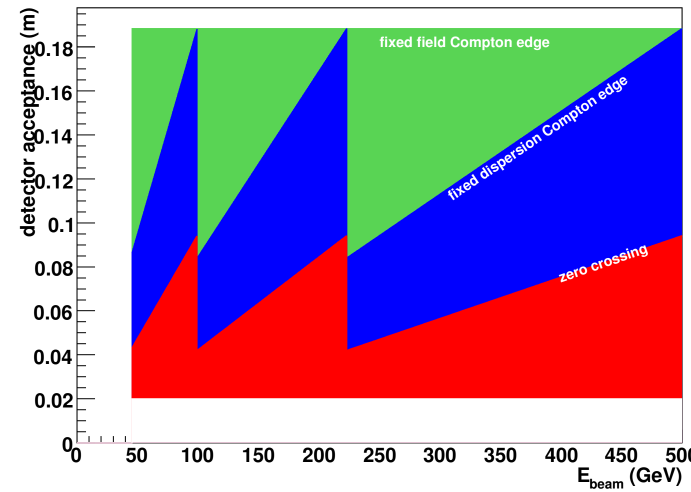
Design of the Upstream Polarimeter Chicane.

RDR Design:

- energy collimation and emittance diagnostics in same chicane
- $\Rightarrow$ laterally fixed Compton IP $\Rightarrow$ scaled field operation!
- collimator & laser-wire will create severe backgrounds

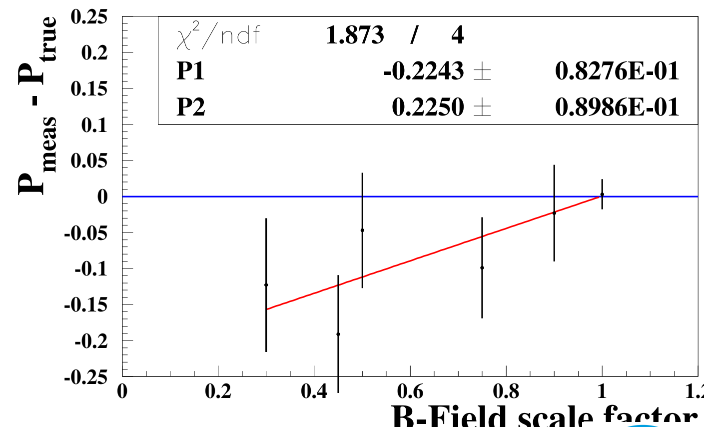


Scaled vs Fixed Field Operation.



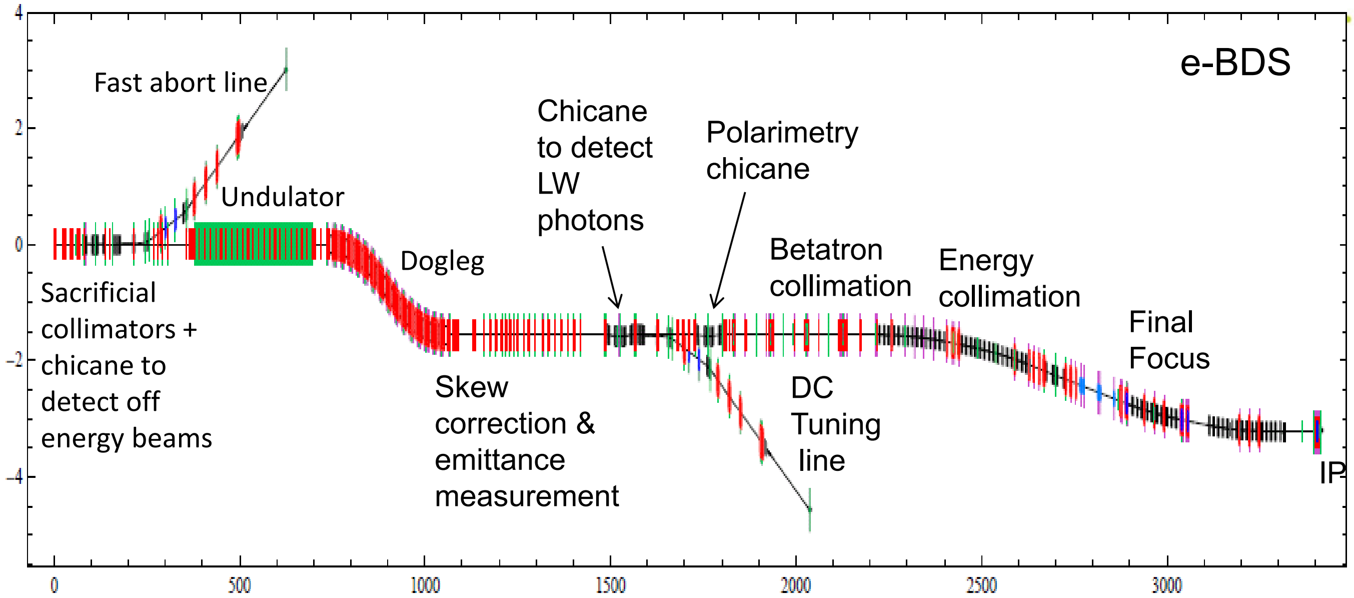
Effects of scaled field on measurement:

- acceptance: varies with E_{beam} $\Rightarrow$ inhomogeneous quality of polarisation measurement
- alignment: via Compton edge position w.r.t. main beam $\Rightarrow$ effect on $\delta\mathcal{P}/\mathcal{P}$ doubles
- systematic deviations for large scale factors



not compatible with extreme precision requirements c.f. ILC-NOTE-2008-047

The Upstream Polarimeter in SB2009-Nov10 lattice.



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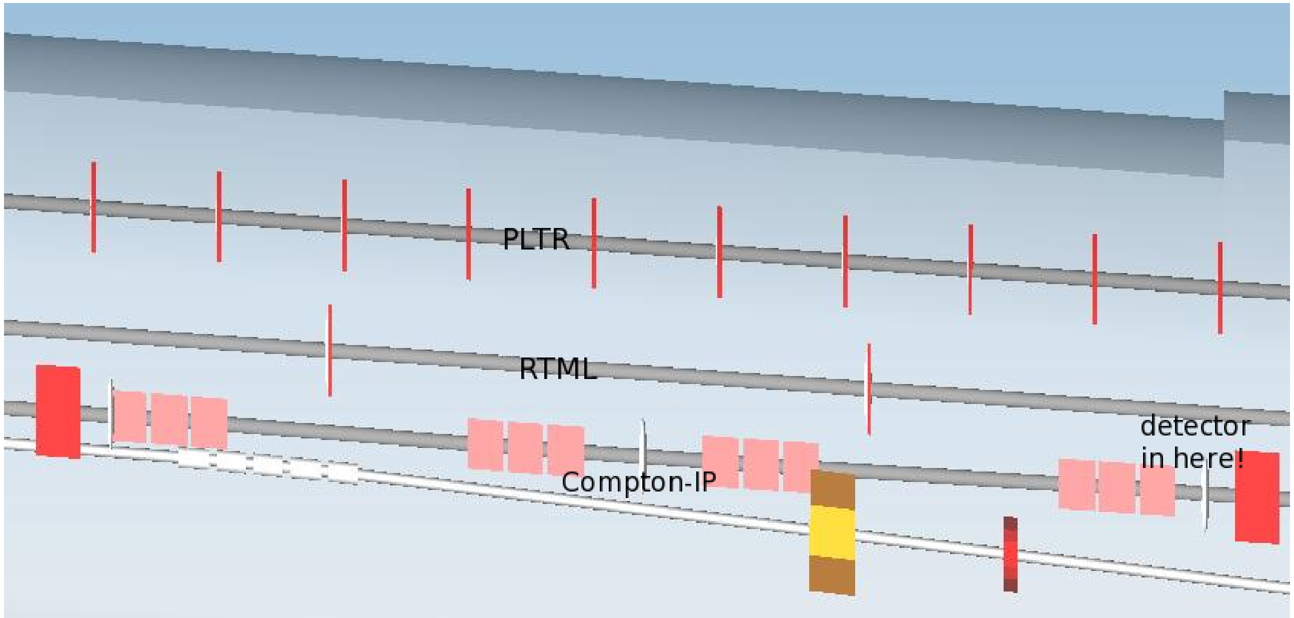

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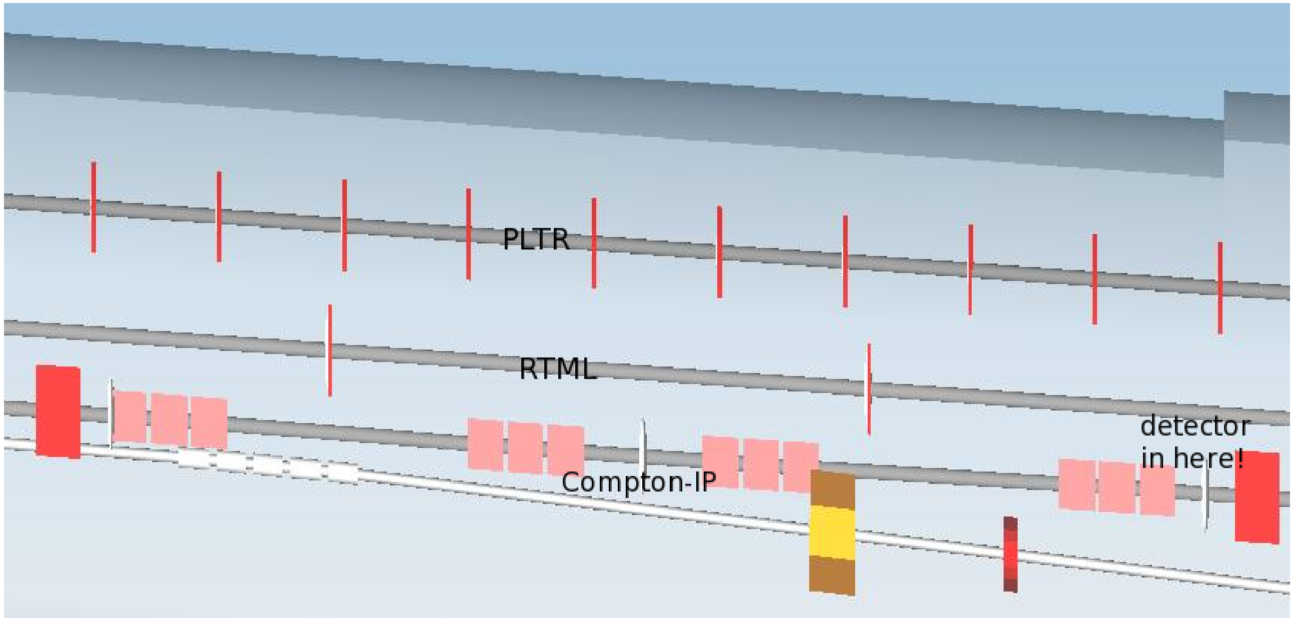


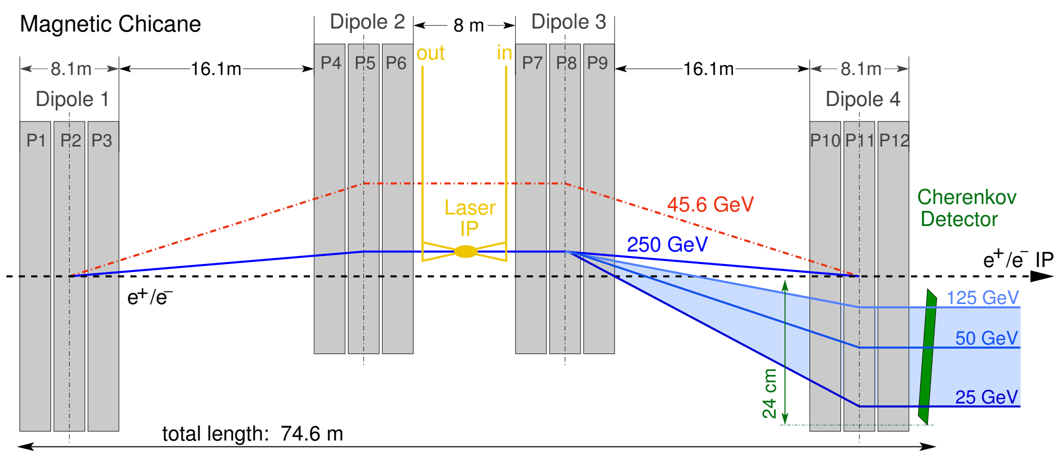
- distance Compton-IP to dump line ca 30 cm at 250 GeV
- down to ca 20 cm at lowest energies - enough?

[c.f. Baseline Technical Review Workshop 2011]

Vacuum Chamber in Chicane Region.

need special beam pipe through out whole chicane

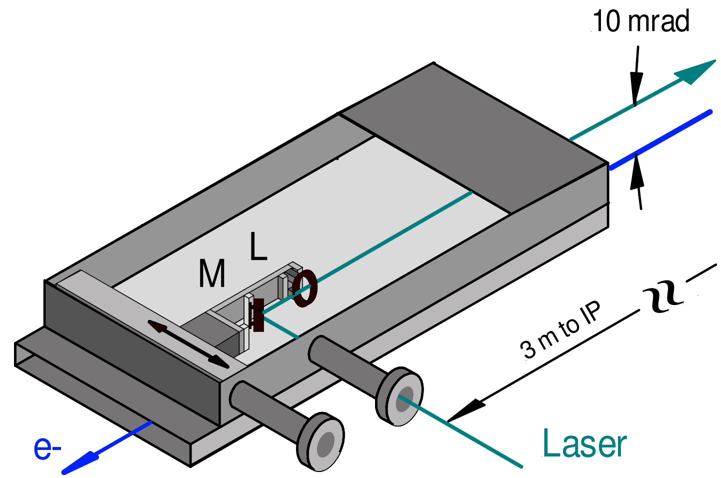
- to allow for varying bending angle
- to guide laser in and out
- to let fan of Compton scattered electrons pass
- to extract Compton fan to detector



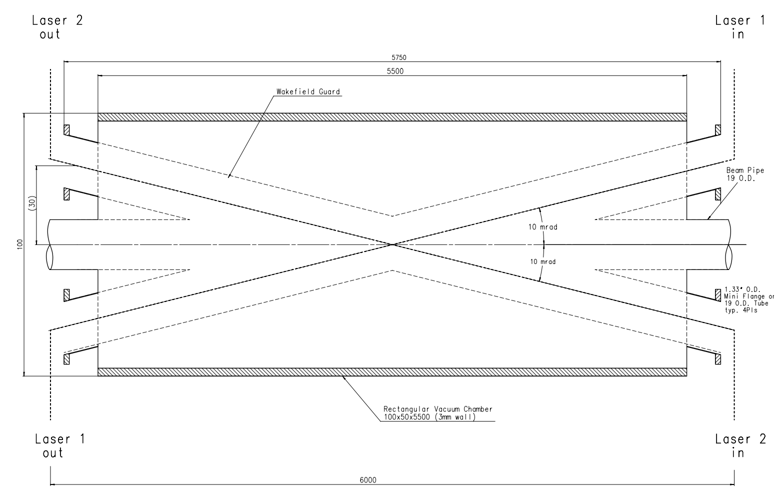
Attention: deflection of chicane the otherway round as on previous page!

Vacuum Chamber: Laser in / out.

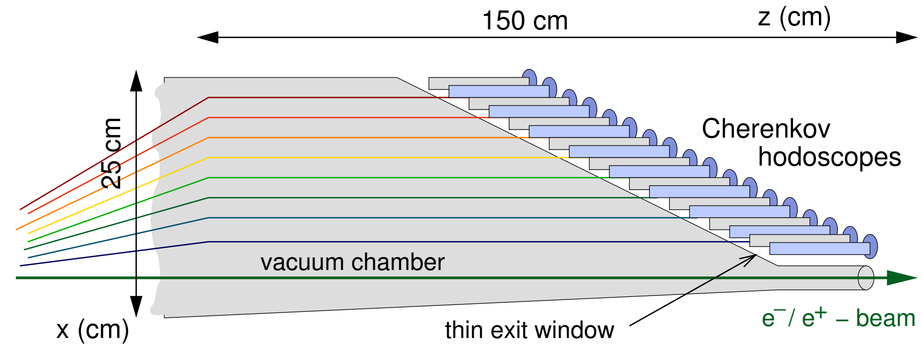
- Laser enters chicane *horizontally* (far side from tune-up dump line!)
- final mirror / lens movable to adjust to e^- beam
- had been designed to some extent for TESLA (!) by N. Meyners, P. Schüler

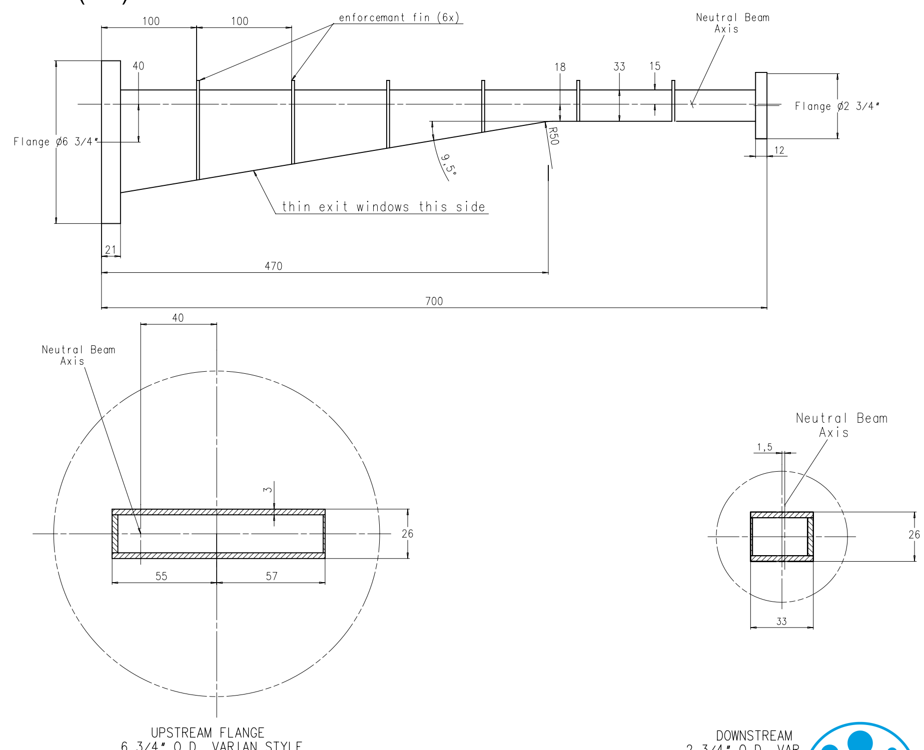


Movable Laser Beam



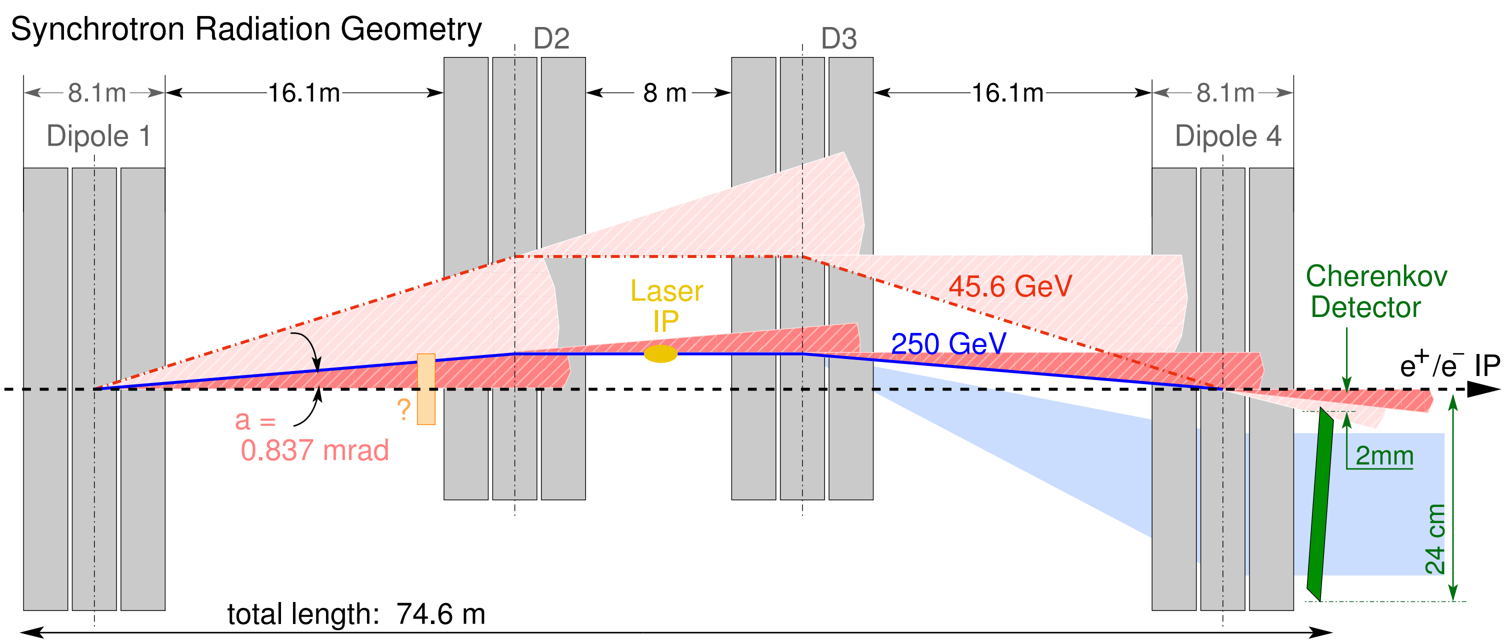
Vacuum Chamber: Compton fan exit.





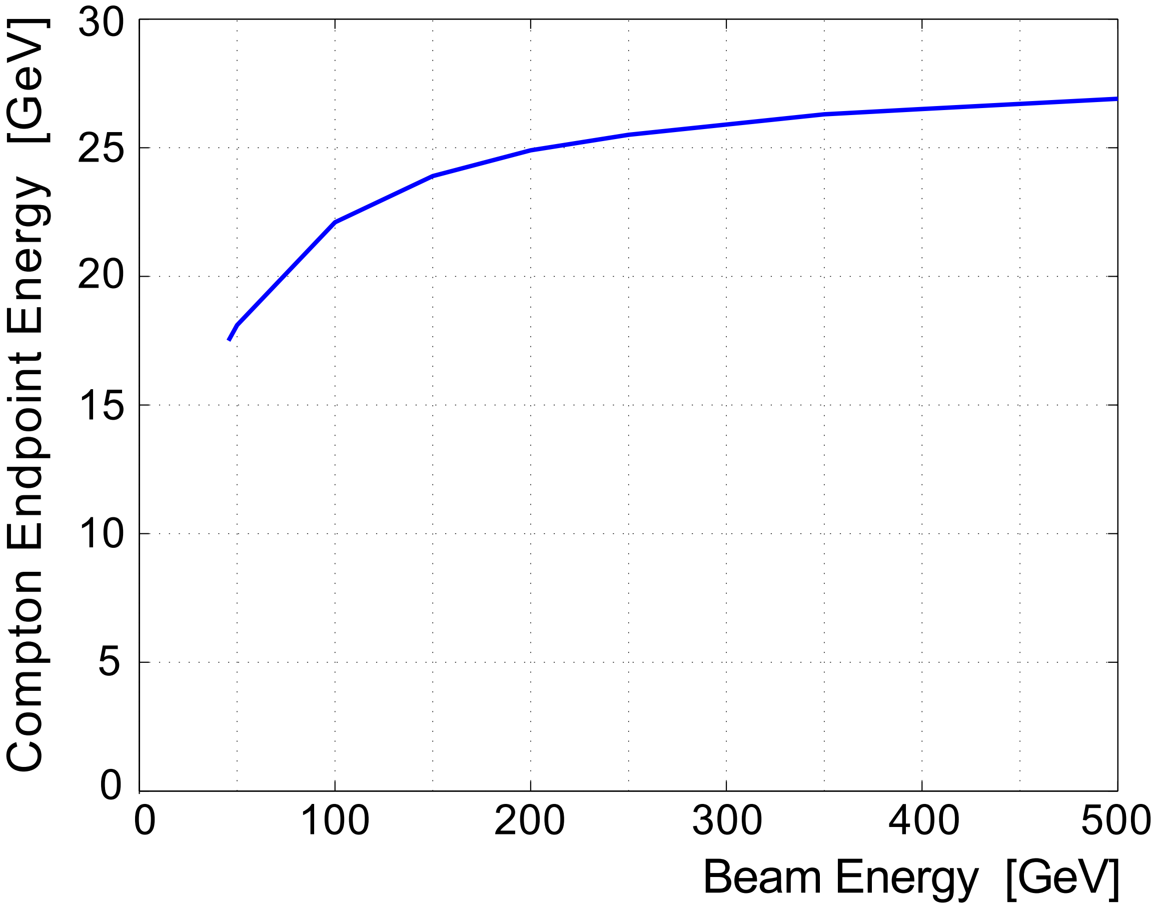
- need tapered exit window to avoid wake fields
- again estimate from TESLA: $\simeq 10^\circ$ is fine (opinions?)
- need $\simeq$ 1.5 m for detector array, make it 2 m for shielding, accessability,...
- fine with SB2009-Nov10 lattice

Synchrotron Radiation.



Compton edge.

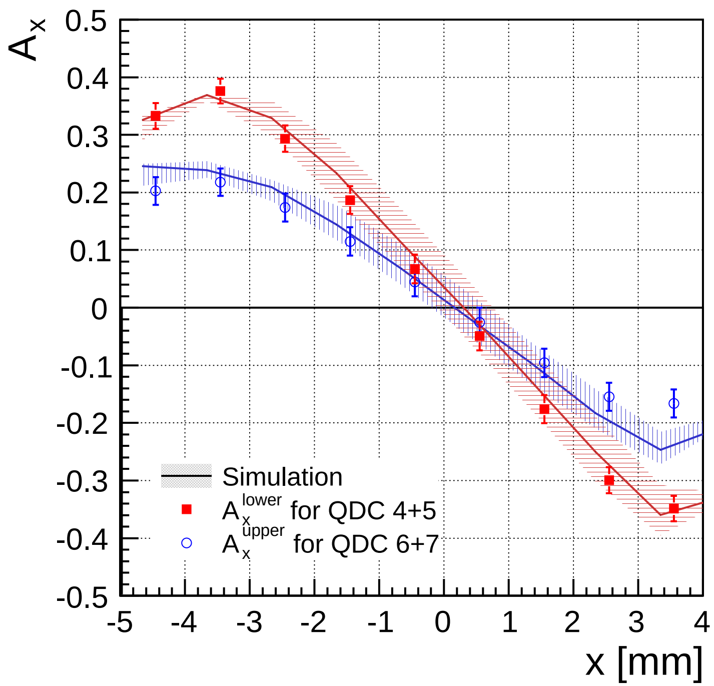
Compton edge position nearly independent of beam energy



Gas Cherenkov detector: Alignment.

If the detector is tilted

- beam path through the detector varies $\Rightarrow$ different light path
- different light pattern on the photocathode
  $\Rightarrow$ alignment via spatial assymetries possible:

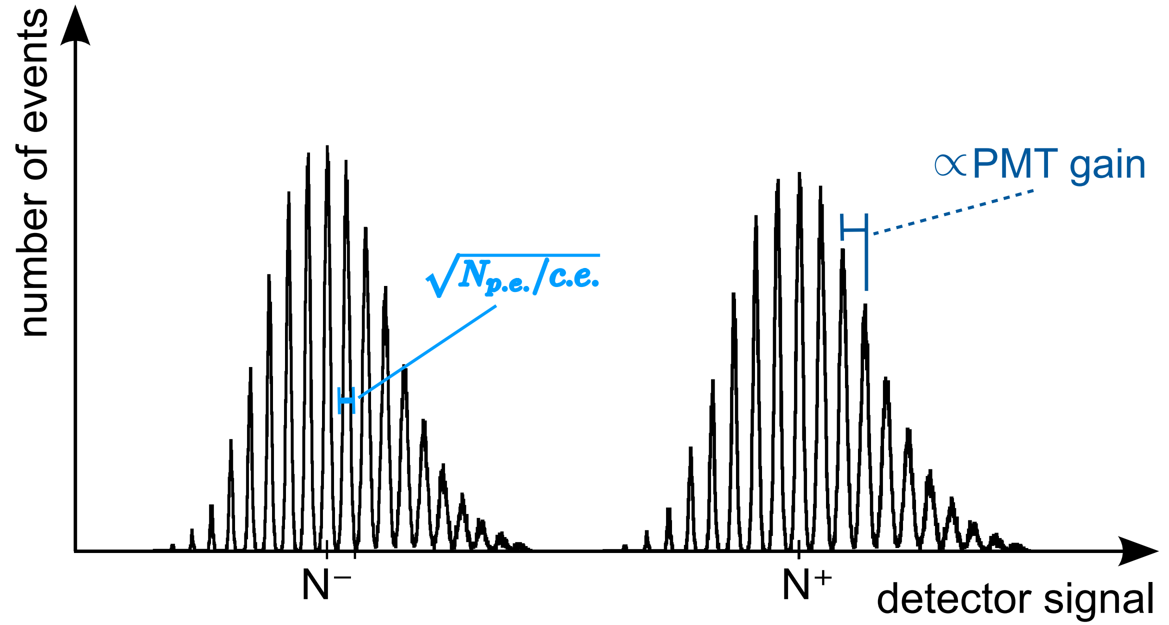


$\Rightarrow$ Reached a tilt alignment of 0.1°. [JINST 7, P01019 (2012)]

Quartz Cherenkov detector.

Alternative detector concept: quartz detector

- Higher refractive index $\rightarrow$ higher photon yield
- For enough photons per Compton e^-:
  $\rightarrow$ calibrate gain directly from the data

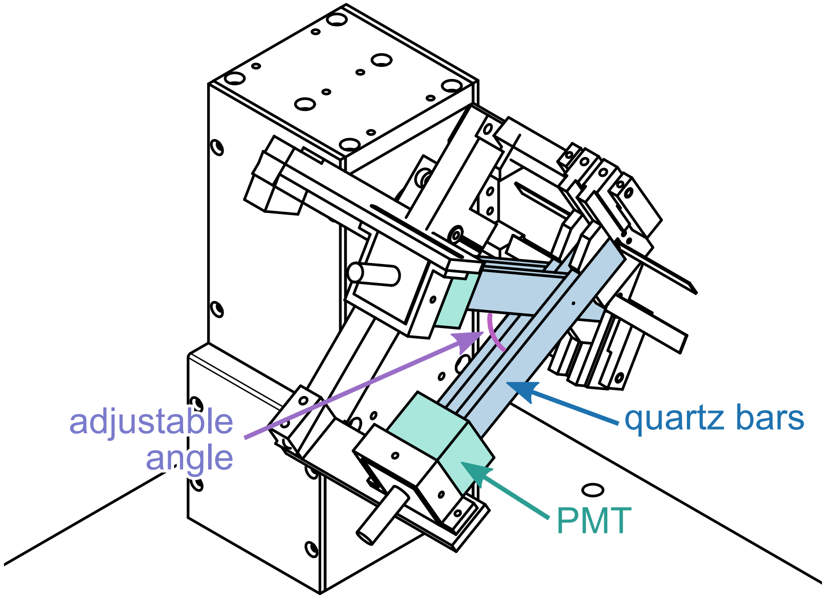


4-channel prototype operated at DESY II testbeam in 2014.

Quartz Cherenkov detector (2).

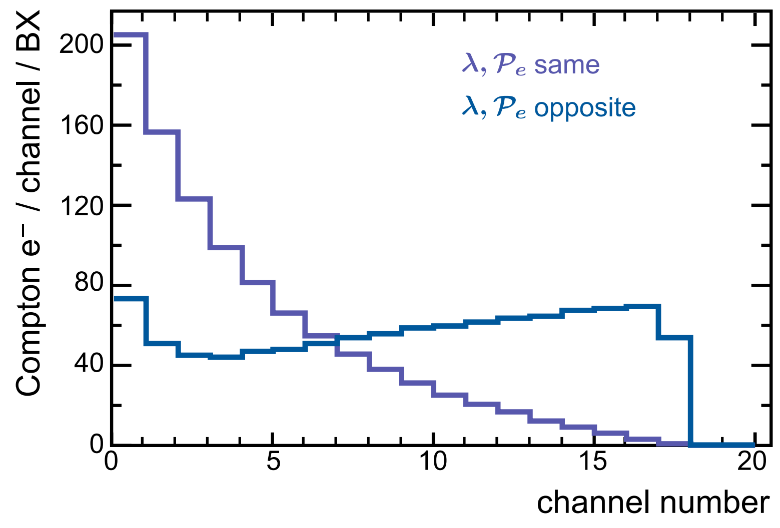
4-channel prototype operated at DESY II testbeam in 2014

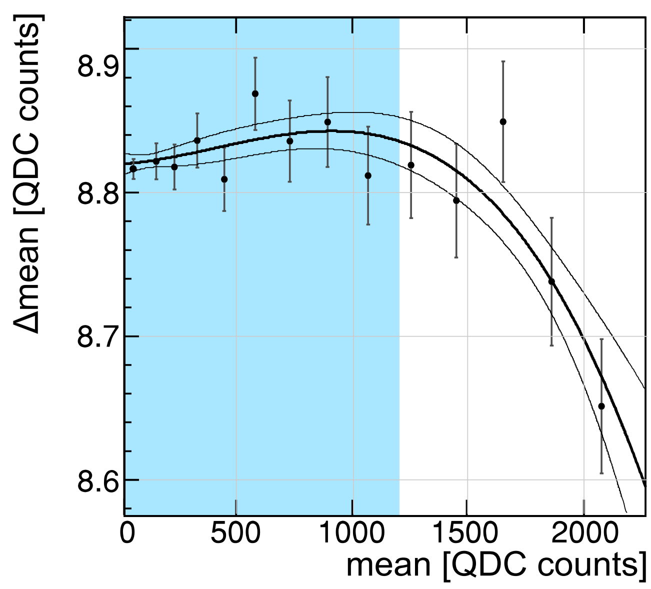
- channels: quartz bars (5 mm x 18 mm x 100 mm)
- qualitative agreement with simulations (angular dependence, etc.)
- light yield smaller than predicted, studies ongoing

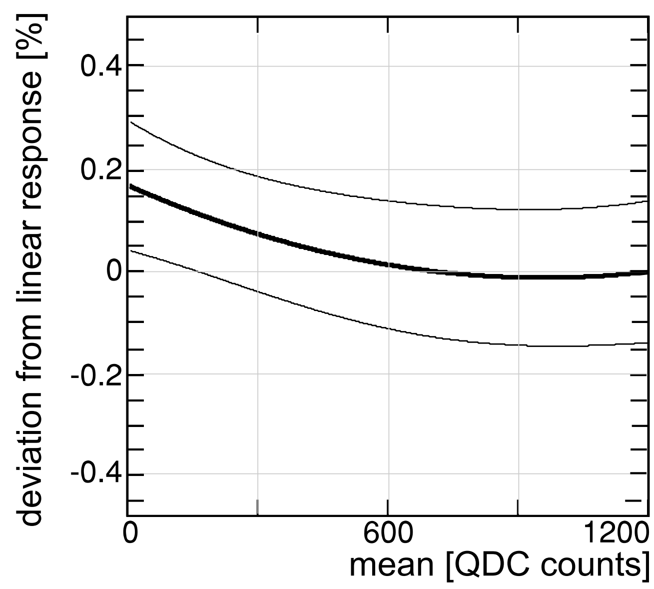


Non-linearity in extreme polarimeter channels.

- up to 210 Compton e^- ($\sim$ 1200 QDC counts)
- overall non-linearity already small in this range (max 0.2 %)
- in single channels even smaller







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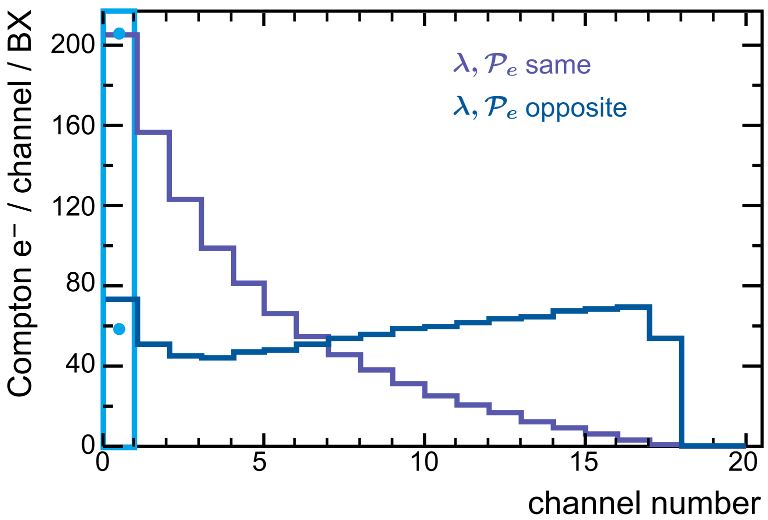

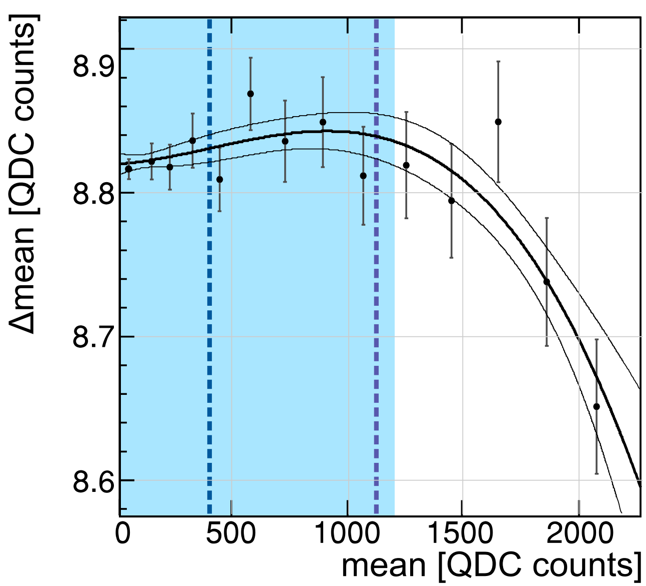

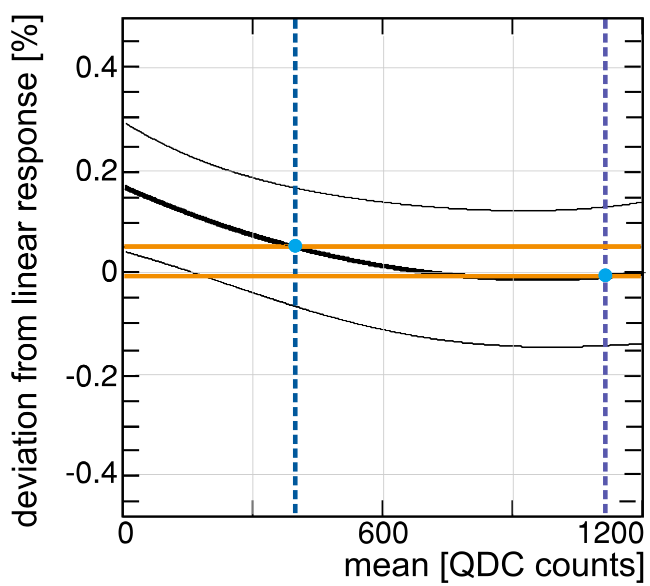

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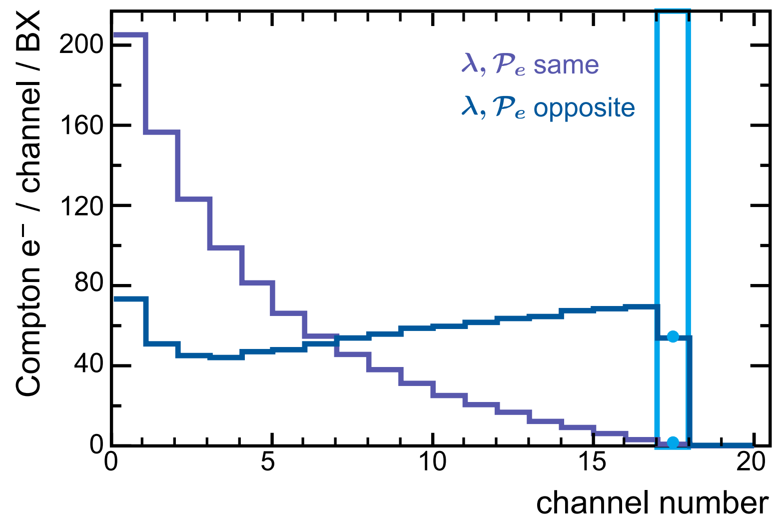
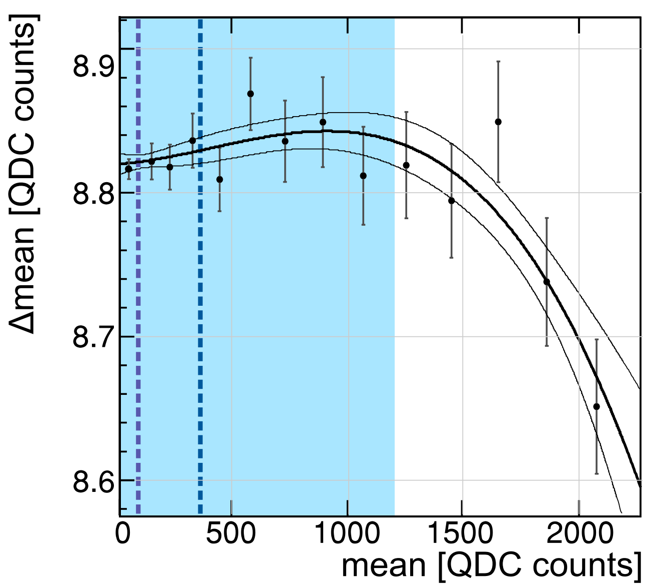
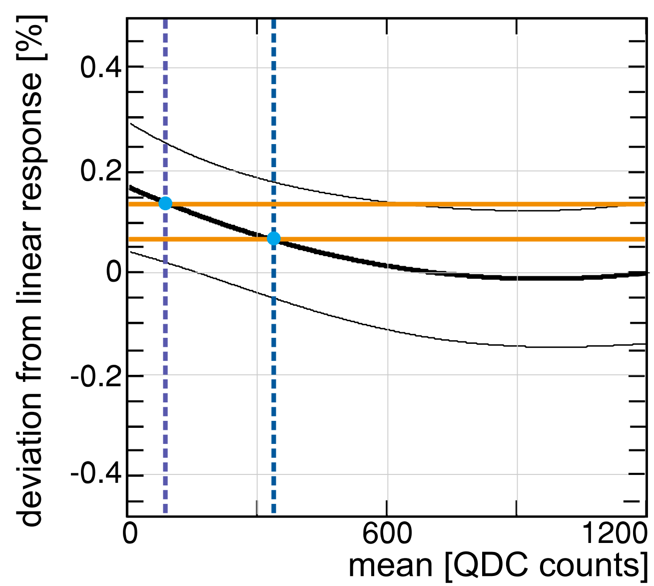


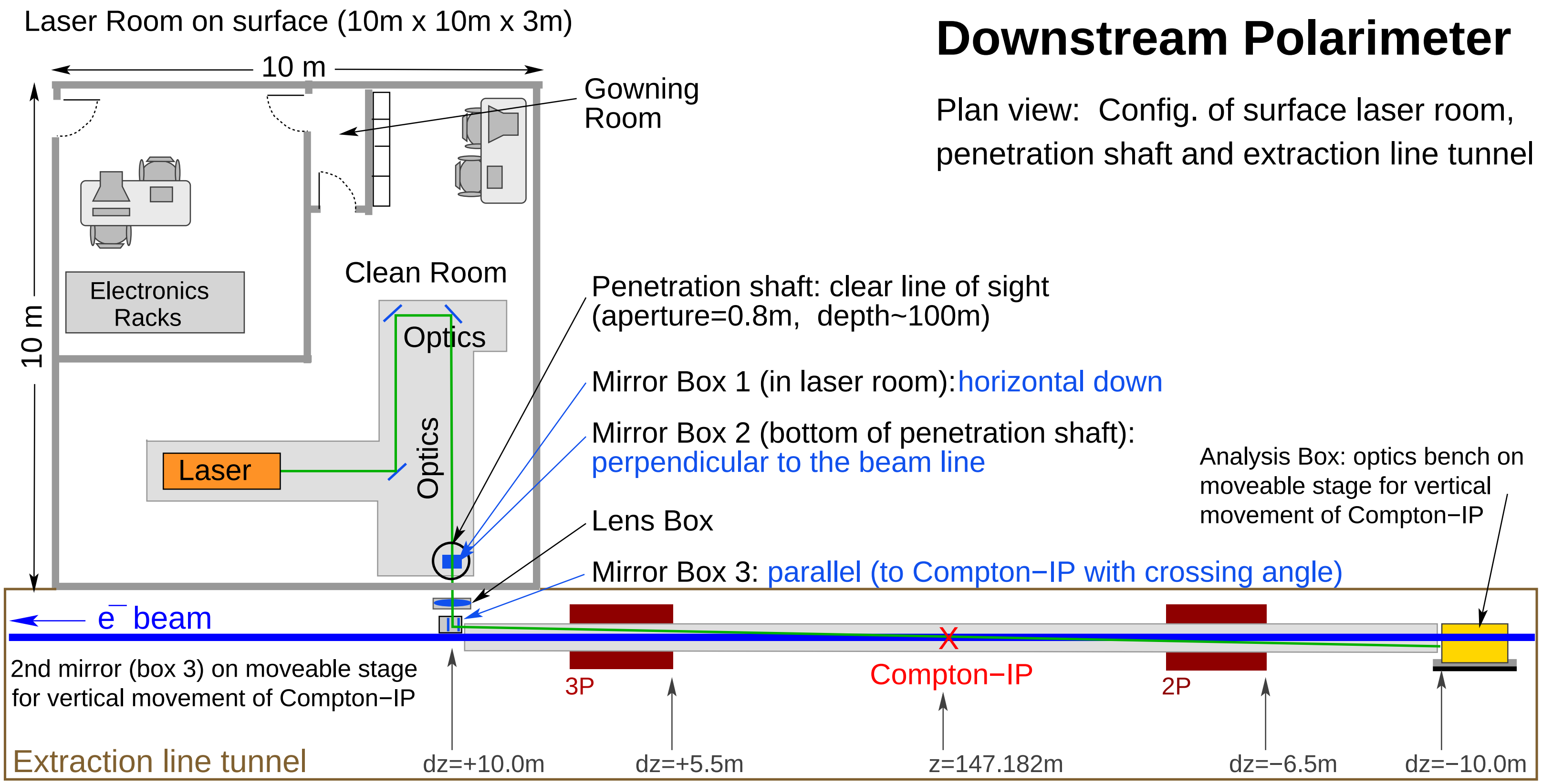




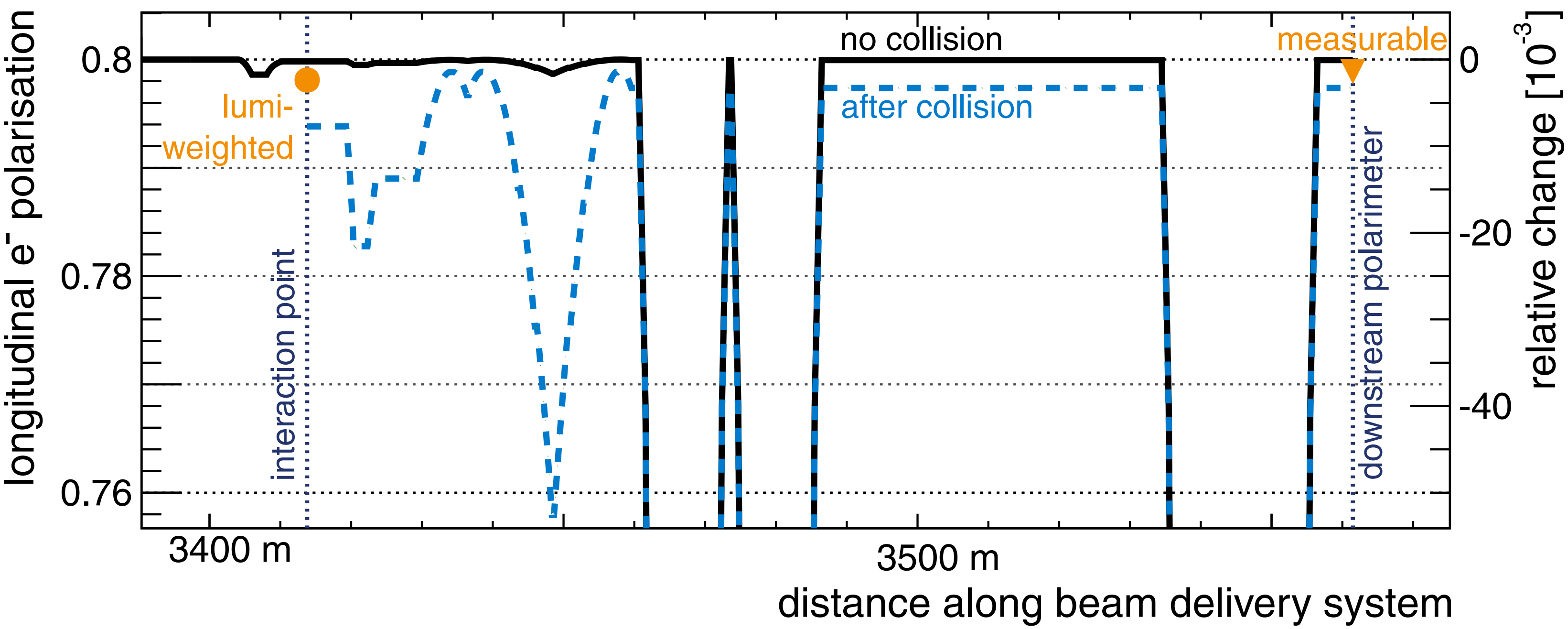
Laser Room.

Downstream Polarimeter

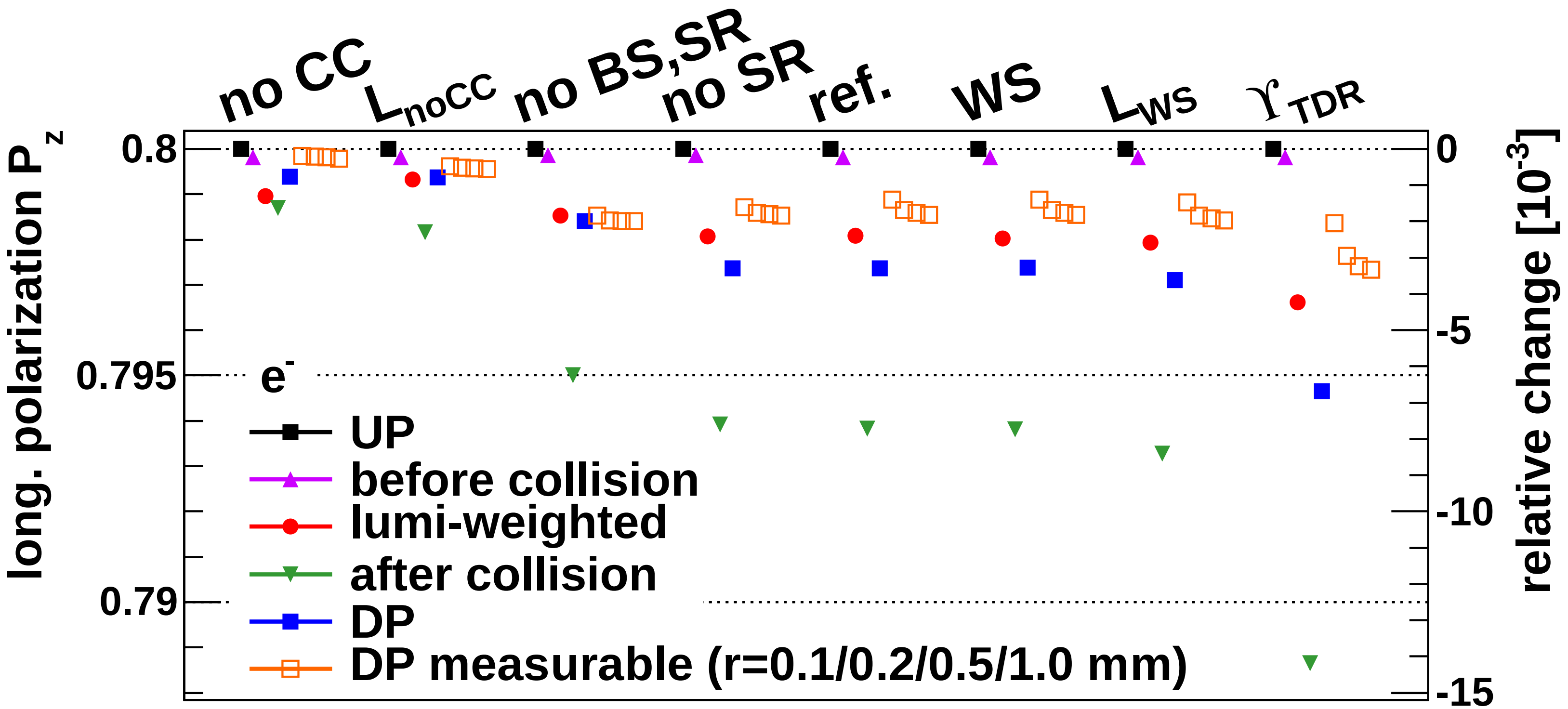
Plan view: Config. of surface laser room, penetration shaft and extraction line tunnel



Spin transport.



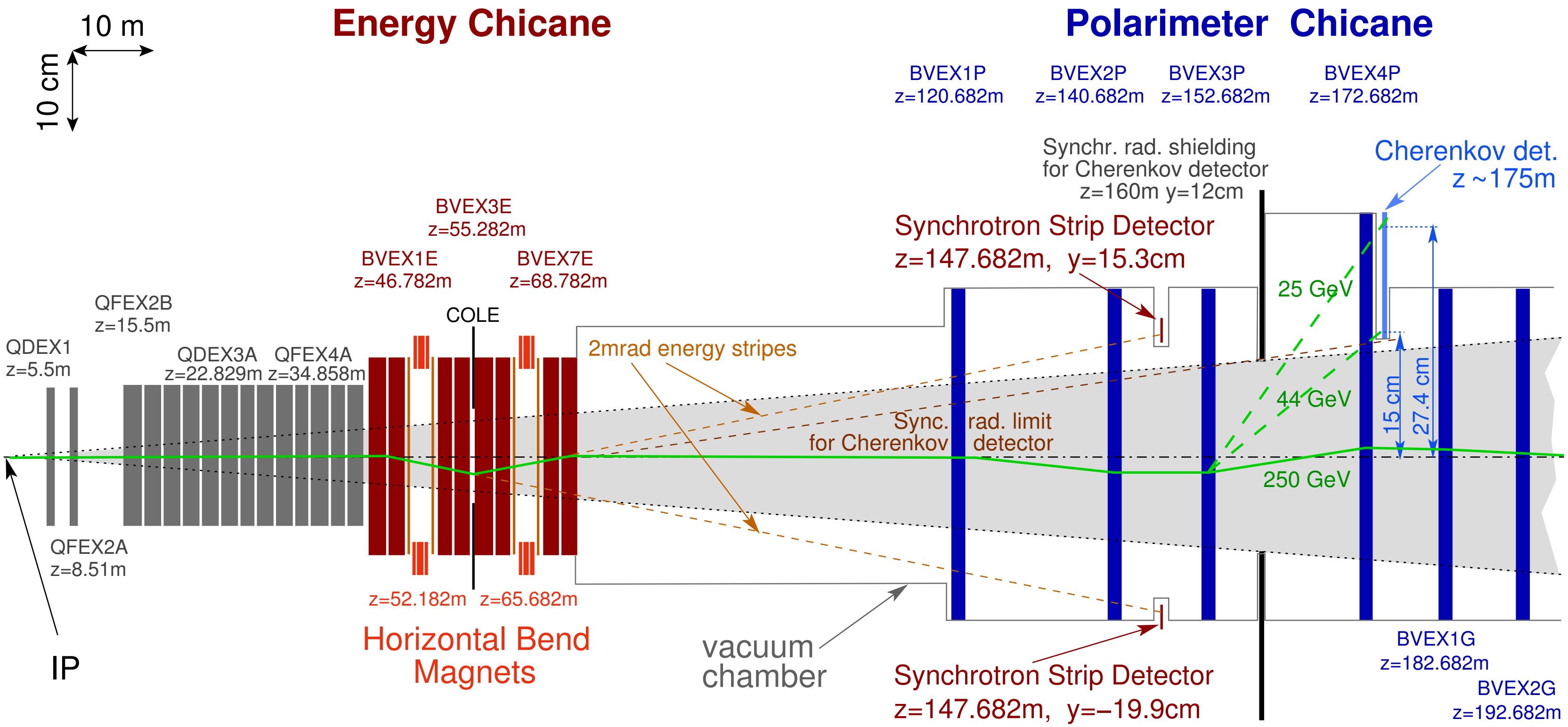
Spin tracking (more).



Downstream Polarimeter.

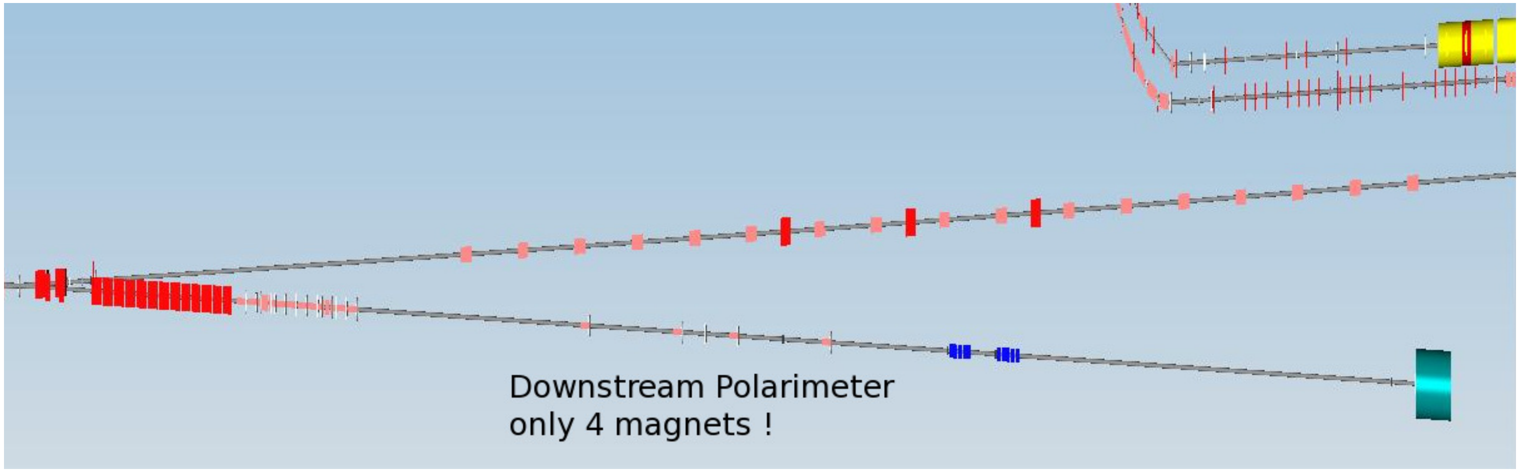
6-magnet chicane suggested in 2007 by Ken Moffeit et al:

- kick Compton e^- further out of the synchrotron radiation fan



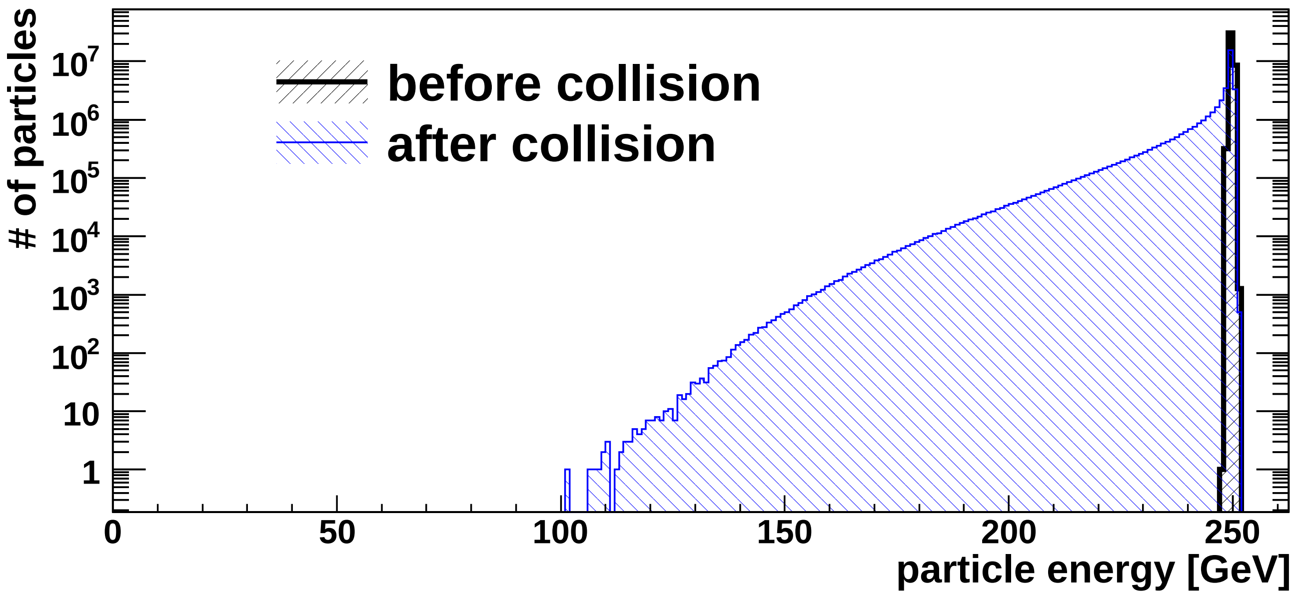
Downstream Polarimeter in SB2009-Nov10 lattice.

- still 4-magnet chicane - should be upgraded to 6-magnet design as propsed in SLAC-PUB-12425
- necessary due to push-pull related changes to the extraction SC quadrupoles
- at the same time gives better shielding of magnets due to additional collimators
- even more impact due to worse spent beam in low power configuration....



Beam Energy Spectrum with Collisions.

GuineaPig++, RDR nominal, $\sqrt{s} = 500\,\text{GeV}$

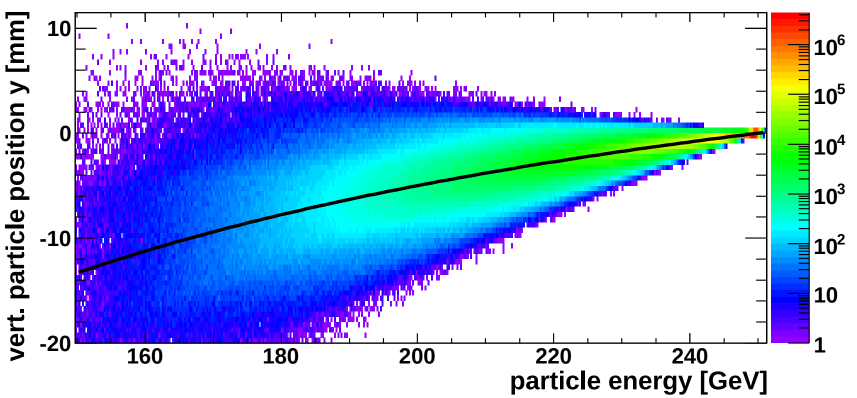


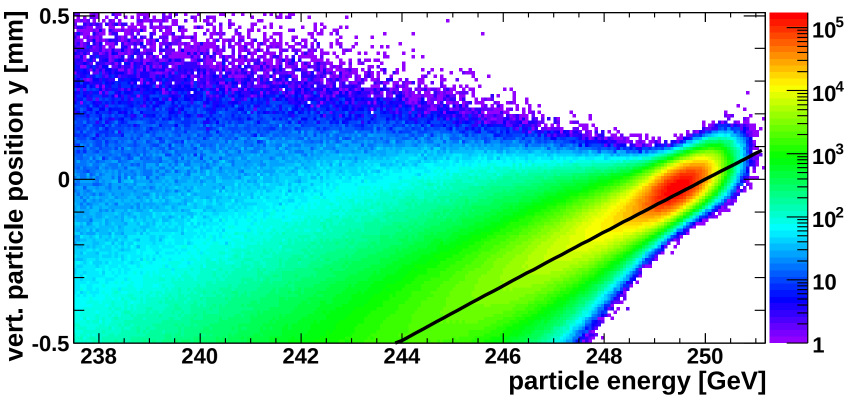
How will this influence the measurement at the downstream polarimeter?

y vs E at DP IP.

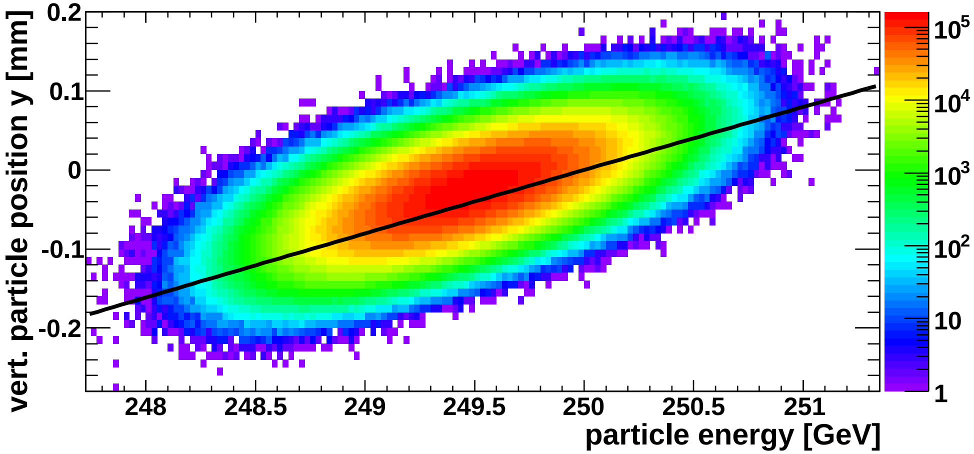
Particle Tracking through SB2009-Nov10 lattice (M.Beckmann)

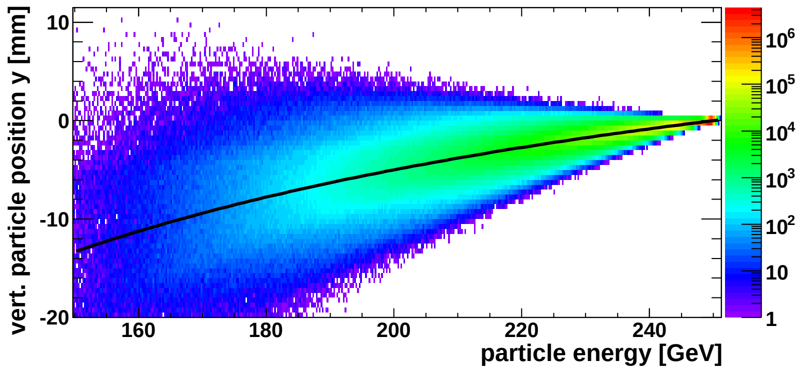
- vertical extension of spent beam at DP IP $\mathcal{O}$(cm)
- “core” size still $\simeq$ 0.5 mm
- sizable correlation of energy and position
- which part will the laser sample?
- expect dependence of measured polarisation on laser spot size and laser-beam alignment

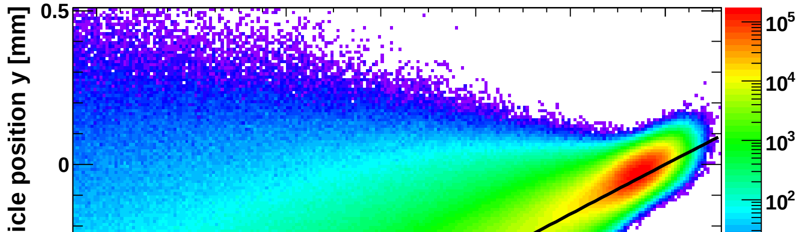




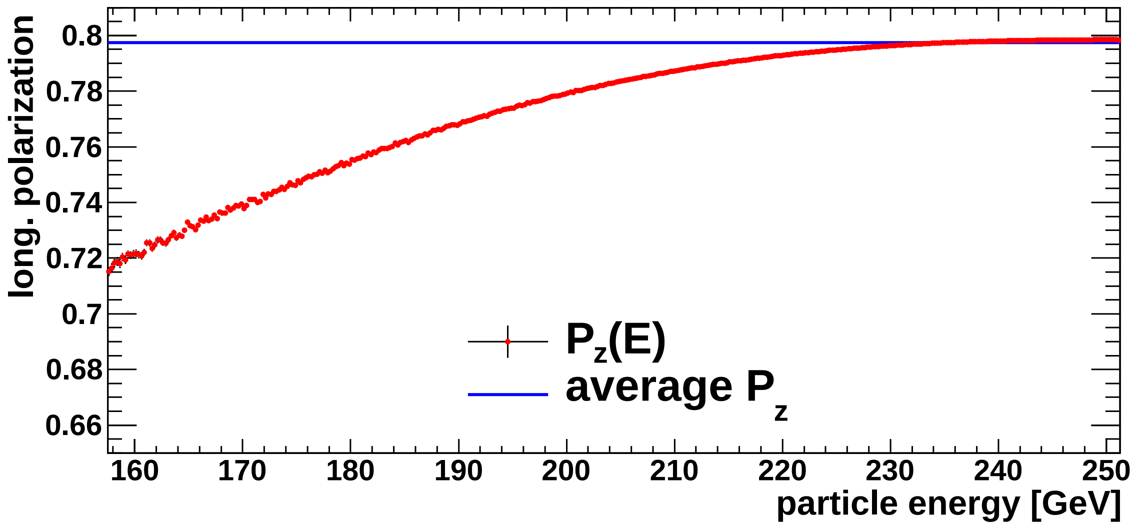
Downstream Polarimeter: y vs E.

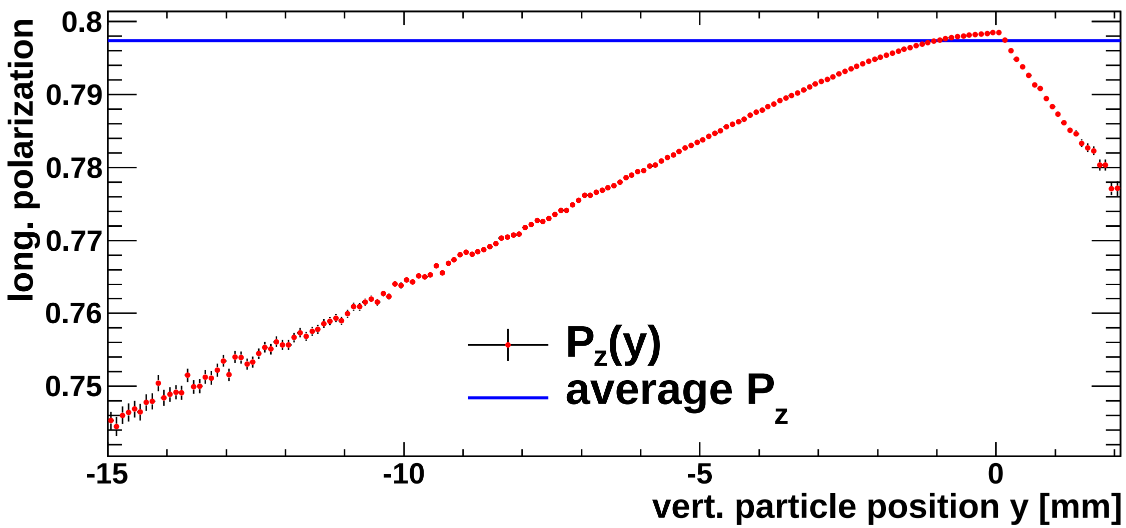




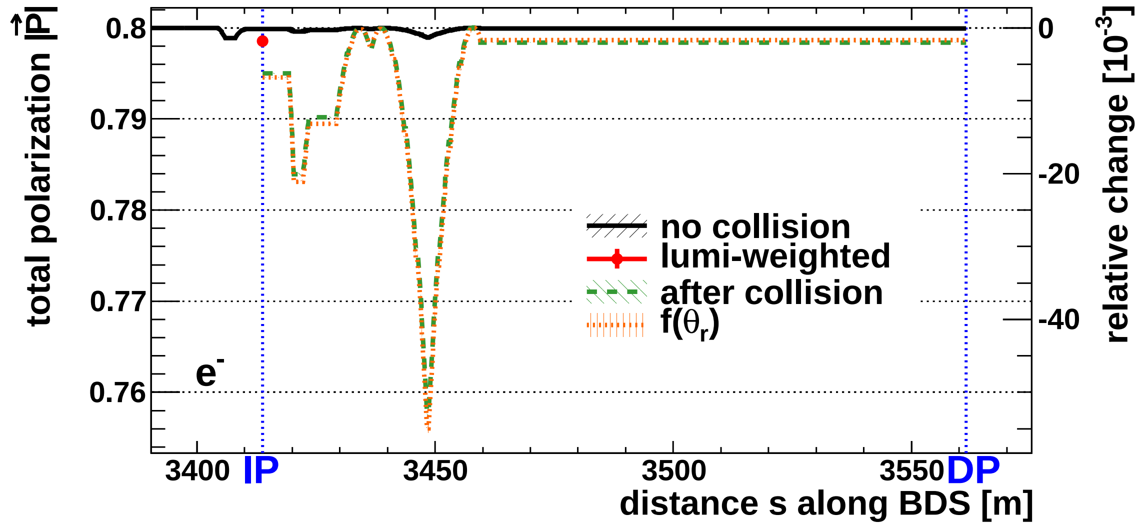


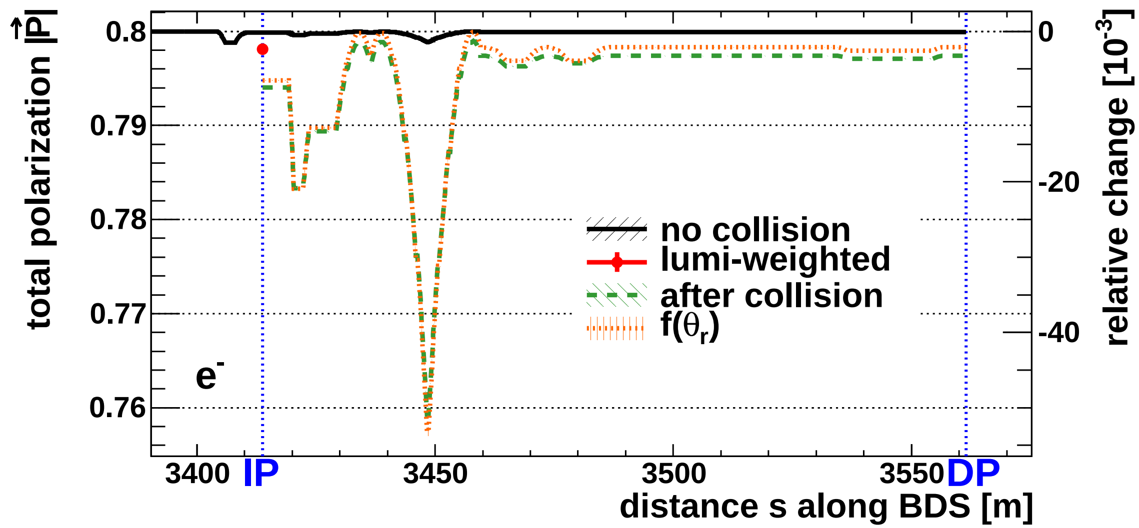
Downstream Polarimeter: P_Z vs E, P_Z vs y.





Total Polarisation IP $\rightarrow$ DP.





Polarisation for Physics.

Longitudinal polarisation $P_z = \frac{N_R - N_L}{N_R + N_L}$
with $N_{R,L}$: number of right-/left-handed particles in bunch

- SM & BSM: left- and righthanded particles couple differently
  - polarised cross-sections are important observables carrying qualitatively new information!
  - beam polarisation can suppress background / enhance signal
- wanted for physics: luminosity weighted average polarisation at the IP, $\langle P_z \rangle_{IP} = \frac{\int P_z(t)\mathcal{L}(t)dt}{\int \mathcal{L}(t)dt}$
- Note: most physics studies sofar assume this average is known exactly and independently for e^- and e^+ beam.

$P \equiv P_z$ in the following.

Polarised Cross-sections.

$$\sigma_{P_{e^-}P_{e^+}} = \frac{1}{4} \quad \{ \quad (1+P_{e^-})(1+P_{e^+})\sigma_{RR} + (1-P_{e^-})(1-P_{e^+})\sigma_{LL}$$
$$+ \quad (1+P_{e^-})(1-P_{e^+})\sigma_{RL} + (1-P_{e^-})(1+P_{e^+})\sigma_{LR}\}$$

processes with s-channel Z/γ exchange only:

- $\sigma_{RR} = \sigma_{LL} = 0$
- $4\sigma_{P_{e^-}P_{e^+}} = (1 - P_{e^-}P_{e^+})(\sigma_{LR} + \sigma_{RL})[1 - P^{-}_{\mathrm{eff}} A_{LR}]$
- with $P^{-}_{\mathrm{eff}} = 1 - \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-}P_{e^+}}$ and $A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$

general case:

- $\sigma_{RR} \neq \sigma_{LL} \neq 0$
- $4\sigma_{P_{e^-}P_{e^+}} = (1 + P_{e^-}P_{e^+})(\sigma_{LL} + \sigma_{RR})[1 + P^{+}_{\mathrm{eff}} A_{LLRR}]$ + above
- with $P^{+}_{\mathrm{eff}} = 1 + \frac{P_{e^-} + P_{e^+}}{1 + P_{e^-}P_{e^+}}$ and $A_{LLRR} = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{LL} + \sigma_{RR}}$

Polarisation Averages.

Absolute cross-section measurements require:

- $\langle P_{e^\pm} \rangle_{IP} = \frac{\int P_{e^\pm}(t)\mathcal{L}(t)dt}{\int \mathcal{L}(t)dt}$
- $\langle P_{e^-} P_{e^+} \rangle_{IP} = \frac{\int P_{e^-}(t) P_{e^+}(t)\mathcal{L}(t)dt}{\int \mathcal{L}(t)dt}$
- correlations between lumi and polarisation?!

Direct extraction from collision data

- any abundant, well-known, polarisation dependent process:
- $\langle | P_{e^\pm} | \rangle_{IP} = \sqrt{\frac{(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++})(\pm\sigma_{-+}\mp\sigma_{+-}+\sigma_{--}-\sigma_{++})}{(\sigma_{-+}+\sigma_{+-}+\sigma_{--}+\sigma_{++})(\pm\sigma_{-+}\mp\sigma_{+-}-\sigma_{--}+\sigma_{++})}}$
- σ_{+-} is total cross-section for $P(e^-, e^+) = (+x\%, -y\%)$, etc.
- assumes $P_+(e^-) = -P_-(e^-)$ and $P_+(e^+) = -P_-(e^+)$

Correction to modified Blondel scheme.

$$P_{+}(e^{\pm}) = P^{\pm} + \epsilon^{\pm} \text{ and } P_{-}(e^{\pm}) = P^{\pm} - \epsilon^{\pm}$$