

Elastic e-D Scattering for Deuteron Polarimetry at the EIC

Barak Schmookler

+

Douglas Higinbotham (JLab), Elena Long (UNH), Andrew Puckett (UConn),
Allen Pierre-Louis (SBU), Asia Parker (DUQ)

Tensor Polarization – Briefly

Spin-1
System

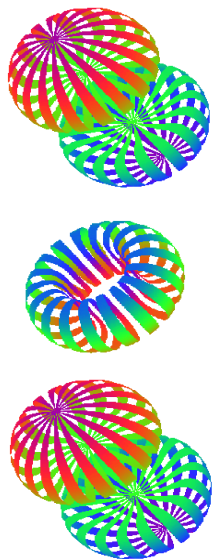
$m = +1$



$m = 0$



$m = -1$



$$P_i = \frac{\langle S_i \rangle}{S}$$

Vector Polarization

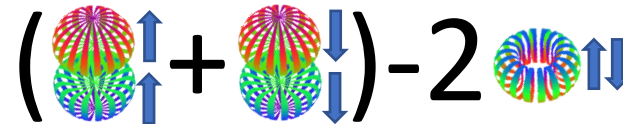
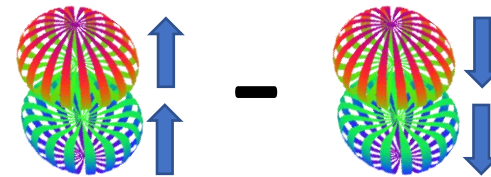
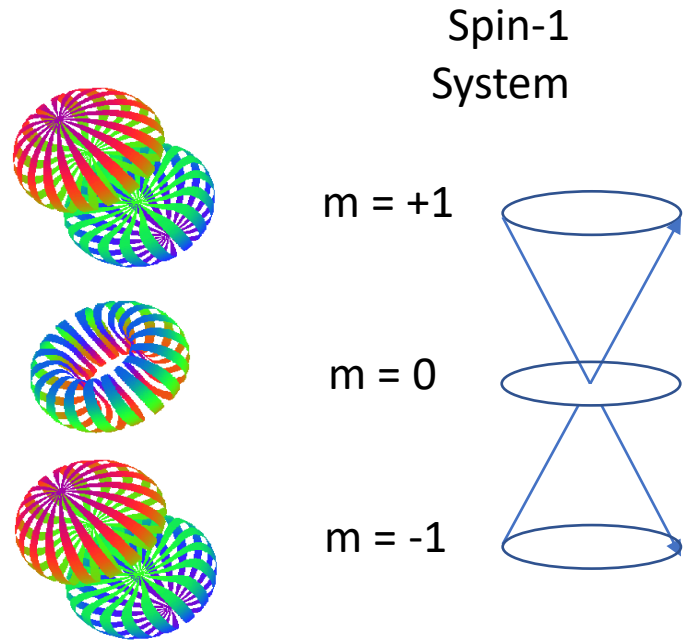
$$P_{ij} = \frac{3\langle S_i S_j + S_j S_i \rangle - 2S(S+1)\delta_{ij}}{2S(2S-1)}$$

Tensor Polarization

arXiv:1811.06377

Tensor Polarization – Briefly

For a Spin-1 System:



Vector Polarization:

$$P_z = \frac{n^+ - n^-}{n^+ + n^0 + n^-}$$

$$-1 \leq P_z \leq 1$$

Tensor Polarization:

$$P_{zz} = \frac{n^+ + n^- - 2n^0}{n^+ + n^0 + n^-}$$

$$-2 \leq P_{zz} \leq 1$$

Tensor Polarization – Spin-1 Examples

1. $n^+ = 100\%$

2. $n^- = 100\%$

3. $n^+ = 50\%$ & $n^- = 50\%$

4. $n^0 = 100\%$

5. $n^+ = 66.67\%$ & $n^0 = 33.33\%$

6. $n^- = 66.67\%$ & $n^0 = 33.33\%$

→ $P_z = +1$ & $P_{zz} = +1$

→ $P_z = -1$ & $P_{zz} = +1$

→ $P_z = 0$ & $P_{zz} = +1$

→ $P_z = 0$ & $P_{zz} = -2$

→ $P_z = +2/3$ & $P_{zz} = 0$

→ $P_z = -2/3$ & $P_{zz} = 0$

Vector Polarization:

$$P_z = \frac{n^+ - n^-}{n^+ + n^0 + n^-}$$

$$-1 \leq P_z \leq 1$$

Tensor Polarization:

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$$P_z = +1 \& P_{zz} = +1$$



$$P_z = -1 \& P_{zz} = +1$$



$$P_z = 0 \& P_{zz} = +1$$



$$P_z = 0 \& P_{zz} = -2$$



$$P_z = +2/3 \& P_{zz} = 0$$



$$P_z = -2/3 \& P_{zz} = 0$$

Vector Polarization:

$$P_z = \frac{n^+ - n^-}{n^+ + n^0 + n^-}$$

$$-1 \leq P_z \leq 1$$

**A high vector polarization
implies a non-zero tensor
polarization**

Tensor Polarization:

$$P_{zz} = \frac{n^+ + n^- - 2n^0}{n^+ + n^0 + n^-}$$

$$-2 \leq P_{zz} \leq 1$$

Tensor Polarization – Spin-1 Examples

$\mathbf{P}_z = +1$ implies a **positive** P_{zz} tensor polarization (along deuteron beam direction):

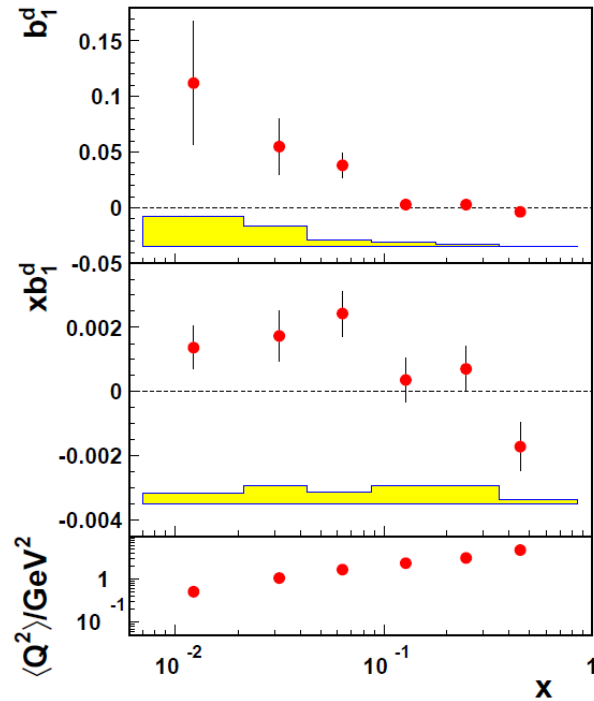
$$\langle +1 \rangle_Z = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow P_Z = +1 \ \& \ P_{ZZ} = +1$$

$\mathbf{P}_x = +1$ implies a **negative** P_{zz} tensor polarization (along deuteron beam direction):

$$\langle +1 \rangle_x = \frac{1}{2} \langle +1 \rangle_Z + \frac{1}{\sqrt{2}} \langle 0 \rangle_Z + \frac{1}{2} \langle -1 \rangle_Z \Rightarrow P_Z = 0 \ \& \ P_{ZZ} = -\frac{1}{2}$$

Why do we care about Tensor-Polarized Deuterium?

Potential measurement of
 b_1^d structure function at
low x with the EIC

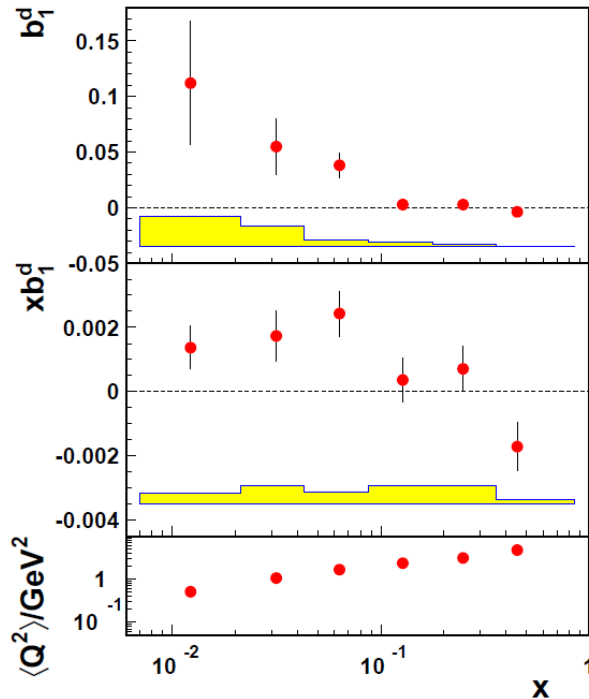


Data from the *HERMES* Collaboration

Phys. Rev. Lett. **95**, 242001

Why do we care about Tensor-Polarized Deuterium?

Potential measurement of b_1^d structure function at low x with the EIC



Data from the *HERMES* Collaboration

Possible contamination of g_1^d structure function measurements due to non-zero tensor polarization

$$\frac{d^2\sigma_P}{dx dQ^2} \simeq \frac{d^2\sigma}{dx dQ^2} \left[1 - P_z P_B D A_1^d + \frac{1}{2} P_{zz} A_{zz}^d \right]$$

$$\frac{g_1^d}{F_1^d} \simeq A_1^d \simeq \frac{c_{zz}}{|P_z P_B| D} \frac{(\sigma^{\vec{\zeta}\vec{\zeta}} - \sigma^{\vec{\zeta}\vec{\sigma}})}{(\sigma^{\vec{\zeta}\vec{\zeta}} + \sigma^{\vec{\zeta}\vec{\sigma}})}$$

$$c_{zz} = \frac{(\sigma^{\vec{\zeta}\vec{\zeta}} + \sigma^{\vec{\zeta}\vec{\sigma}})}{2\sigma_U} = 1 + \frac{(P_{zz}^{\vec{\zeta}\vec{\zeta}} + P_{zz}^{\vec{\zeta}\vec{\sigma}})}{4} A_{zz}^d$$

Phys. Rev. Lett. **95**, 242001

Why do we care about Tensor-Polarized Deuterium?

See last Friday's talk by Ellie Long for more details:

<https://indico.bnl.gov/event/7583/contributions/38664/>

Can we use our knowledge of the Deuteron Form Factors to determine tensor polarization at the EIC?

Measurement of the Tensor Analyzing Powers T_{20} and T_{21} in Elastic Electron-Deuteron Scattering

D. M. Nikolenko,¹ H. Arenhövel,² L. M. Barkov,¹ S. L. Belostotsky,³ V. F. Dmitriev,¹ M. V. Dyug,¹ R. Gilman,^{4,5} R. J. Holt,⁶ L. G. Isaeva,¹ C. W. de Jager,^{7,5} E. R. Kinney,⁸ R. S. Kowalczyk,⁶ B. A. Lazarenko,¹ A. Yu. Loginov,⁹ S. I. Mishnev,¹ V. V. Nelyubin,³ A. V. Osipov,⁹ D. H. Potterveld,⁶ I. A. Rachek,¹ R. Sh. Sadykov,¹ Yu. V. Shestakov,¹ A. A. Sidorov,⁹ V. N. Stibunov,⁹ D. K. Toporkov,¹ V. V. Vikhrov,³ H. de Vries,⁷ and S. A. Zevakov¹

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The tensor analyzing power components T_{20} and T_{21} have been measured in elastic electron-deuteron scattering at the 2 GeV electron storage ring VEPP-3, Novosibirsk, in a four-momentum transfer range from 8.4 to 21.6 fm⁻². A new polarized internal gas target with an intense cryogenic atomic beam source was used. The new data determine the deuteron form factors G_C and G_Q in an important range of momentum transfer where the first node of the deuteron monopole charge form factor is located. The new results are compared with previous data and with some theoretical predictions.

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where N^+ and N^- are the event counts of a detector when the target polarization is P_{zz}^+ and P_{zz}^- , respectively. N^+ and N^- are normalized to the electron beam charge. In accordance with Eq. (1), A^t can be written as a linear combination of tensor analyzing powers (right formula).

We assume that depolarization processes occur identically in both polarization states; therefore P_{zz}^- / P_{zz}^+ is close to -2 (the same as for the ABS beam; see also [9]).

The value of A^t measured by the LQP can be used to calculate the target polarization if the tensor analyzing power is known at small Q^2 . At present, the measure-

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Elastic Electron-Deuteron Scattering

Let's start with unpolarized scattering...

The elastic deuteron structure is described in terms of three form factors: charge monopole $G_C(Q^2)$, charge quadrupole $G_Q(Q^2)$, and magnetic dipole $G_M(Q^2)$

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The cross section (in the **deuteron rest frame**) is

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4E_e^2 \sin^4\left(\frac{\theta_e}{2}\right)} \times \frac{1}{1 + \frac{2E_e}{M_d} \sin^2\left(\frac{\theta_e}{2}\right)} \times [A(Q^2) + B(Q^2) \tan^2\left(\frac{\theta_e}{2}\right)]$$

Elastic Electron-Deuteron Scattering – Unpolarized

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2 \cos^2\left(\frac{\theta_e}{2}\right)}{4E_e^2 \sin^4\left(\frac{\theta_e}{2}\right)} \times \frac{1}{1 + \frac{2E_e}{M_d} \sin^2\left(\frac{\theta_e}{2}\right)} \times [A(Q^2) + B(Q^2) \tan^2\left(\frac{\theta_e}{2}\right)]$$

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Mott Cross Section

Recoil Term

Reduced Cross Section

Elastic Electron-Deuteron Scattering – Unpolarized

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Mott Cross Section

Recoil Term

Reduced Cross Section

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2)$$

$$\eta = \frac{Q^2}{4M_d^2}$$

$$B(Q^2) = \frac{4}{3}\eta(1 + \eta)G_M^2(Q^2)$$

$$\frac{E'_e}{E_e} = \frac{M_d}{M_d + E_e(1 - \cos \theta_e)}$$

Kinematics for Electron-Deuteron Scattering at the EIC

Incoming Electron and Deuteron 4-vectors:

$$p_e = (0, 0, -|p_e|, E_e) \approx (0, 0, -E_e, E_e)$$

$$p_d = (0, 0, |p_d|, E_d) \approx (0, 0, E_d, E_d)$$

The energy of the incoming deuteron nucleus is approximately twice the per-nucleon energy:

$$E_d \approx 2 \times E_N$$

Kinematics for Electron-Deuteron Scattering at the EIC

For elastic e-d scattering, it is preferable to analyze the kinematics assuming the deuteron is a single object (as opposed to composed of individual nucleons). Ignoring the electron and deuteron masses, we define the following variables:

$$x_d = \frac{Q^2}{2p_d \cdot q}$$

$$s_d = 4E_e E_d$$

$$Q^2 = x_d s_d y_d$$

$$\epsilon \approx \frac{1 - y_d}{1 - y_d + y_d^2/2}$$

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$$x_d = \frac{Q^2}{2p_d \cdot q} = 1 \quad \text{for e-d elastic}$$

$$s_d = 4E_e E_d$$

$$Q^2 = x_d s_d y_d$$

$$\epsilon \approx \frac{1 - y_d}{1 - y_d + y_d^2/2}$$

Kinematics for Electron-Deuteron Scattering at the EIC

We can then calculate the energies and angles of the outgoing electron and deuteron in the collider frame:

$$E'_e = (1 - y_d)E_e + x_d y_d E_d$$

$$E'_d = y_d E_e + x_d (1 - y_d) E_d$$

$$\cos \theta_e = \frac{x_d y_d E_d - (1 - y_d) E_e}{x_d y_d E_d + (1 - y_d) E_e}$$

$$\cos \theta_d = \frac{-y_d E_e + (1 - y_d) x_d E_d}{y_d E_e + (1 - y_d) x_d E_d}$$

Kinematics for Electron-Deuteron Scattering at the EIC

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for e-d elastic

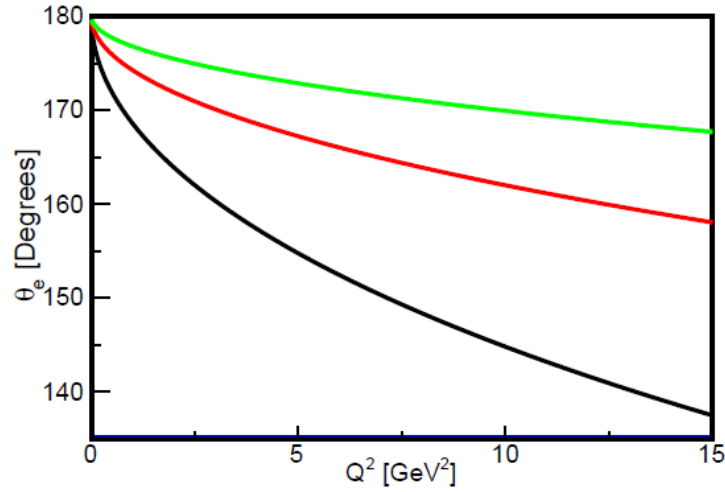
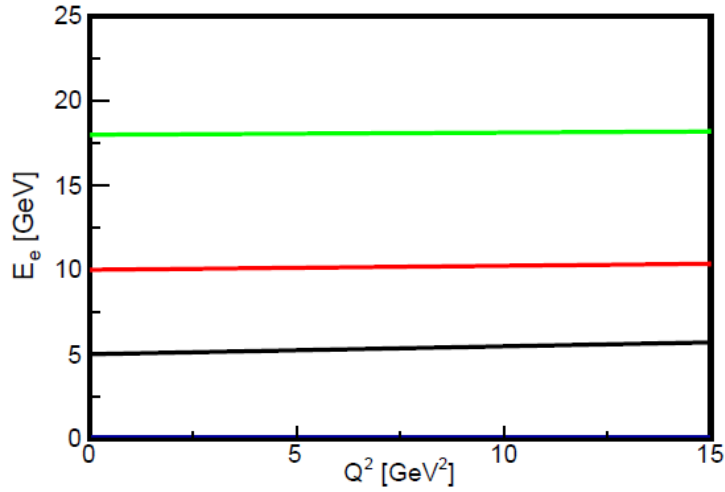
$$E'_e = (1 - y_d)E_e + \cancel{x_d}y_dE_d$$

$$E'_d = y_dE_e + \cancel{x_d}(1 - y_d)E_d$$

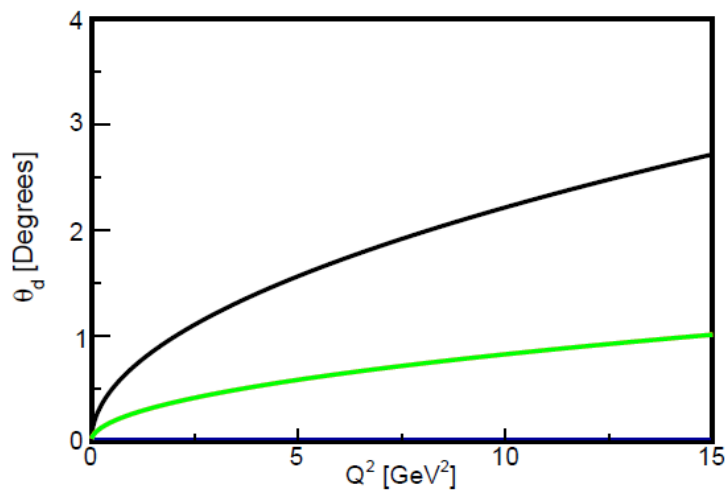
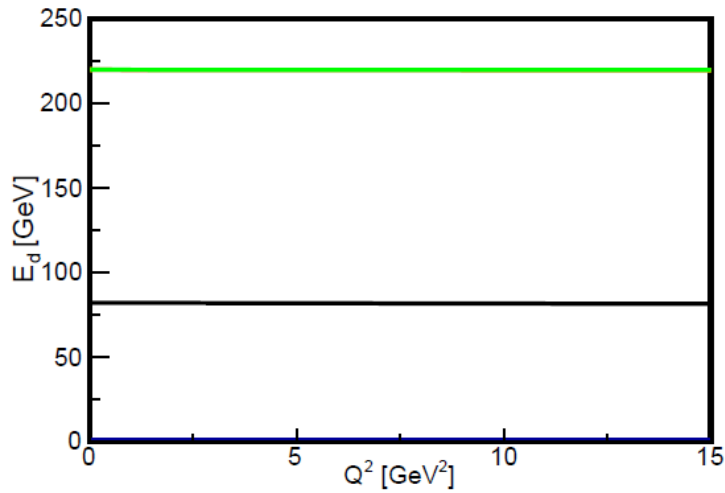
$$\cos \theta_e = \frac{\cancel{x_d}y_dE_d - (1 - y_d)E_e}{\cancel{x_d}y_dE_d + (1 - y_d)E_e}$$

$$\cos \theta_d = \frac{-y_dE_e + (1 - y_d)\cancel{x_d}E_d}{y_dE_e + (1 - y_d)\cancel{x_d}E_d}$$

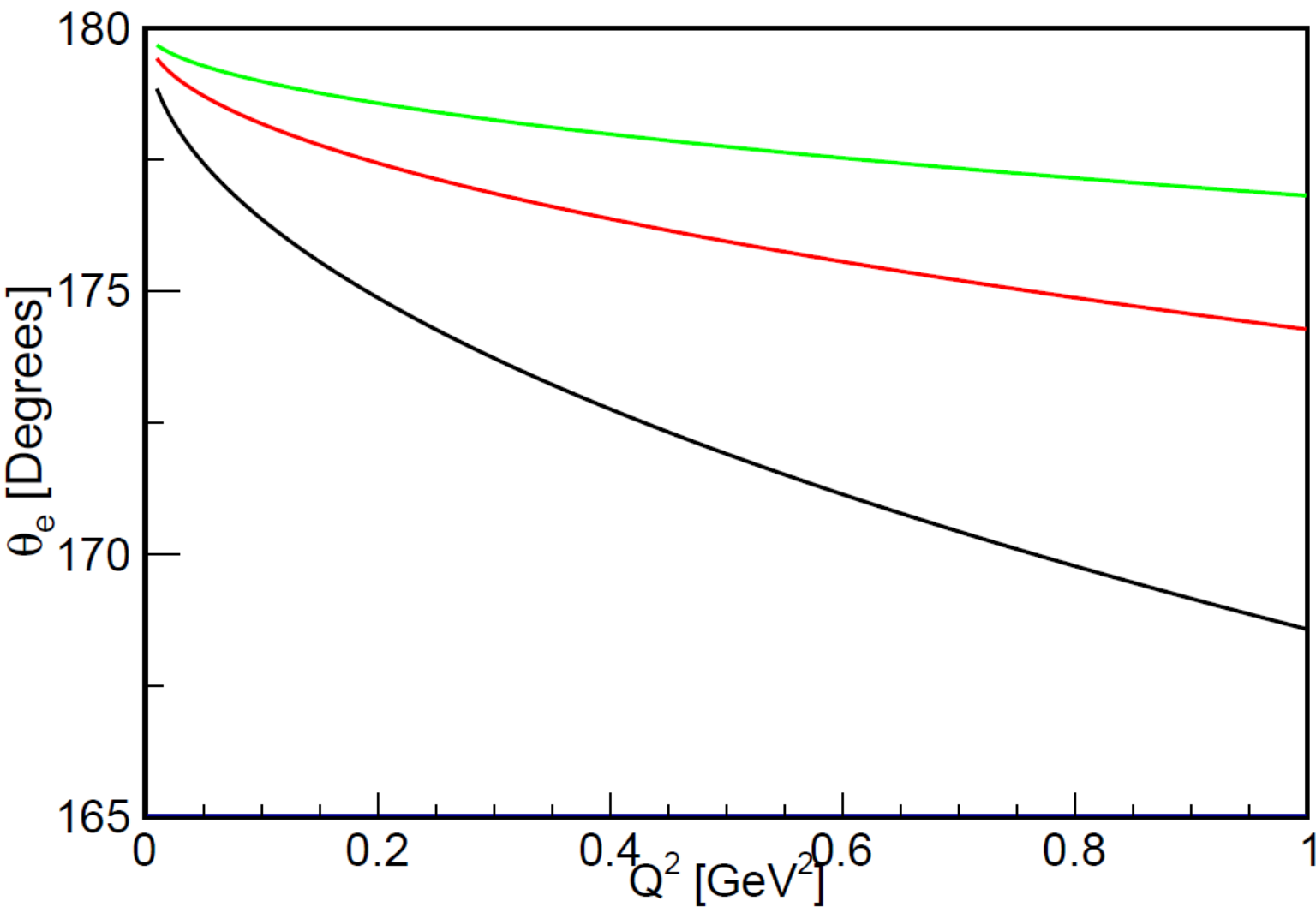
Kinematics for Elastic Electron-Deuteron Scattering at the EIC



- $E_e = 5$ GeV, $E_d = 41$ GeV/nucleon -- $\sqrt{s_{ed}} = 40.5$ GeV
- $E_e = 10$ GeV, $E_d = 110$ GeV/nucleon -- $\sqrt{s_{ed}} = 93.8$ GeV
- $E_e = 18$ GeV, $E_d = 110$ GeV/nucleon -- $\sqrt{s_{ed}} = 125.9$ GeV

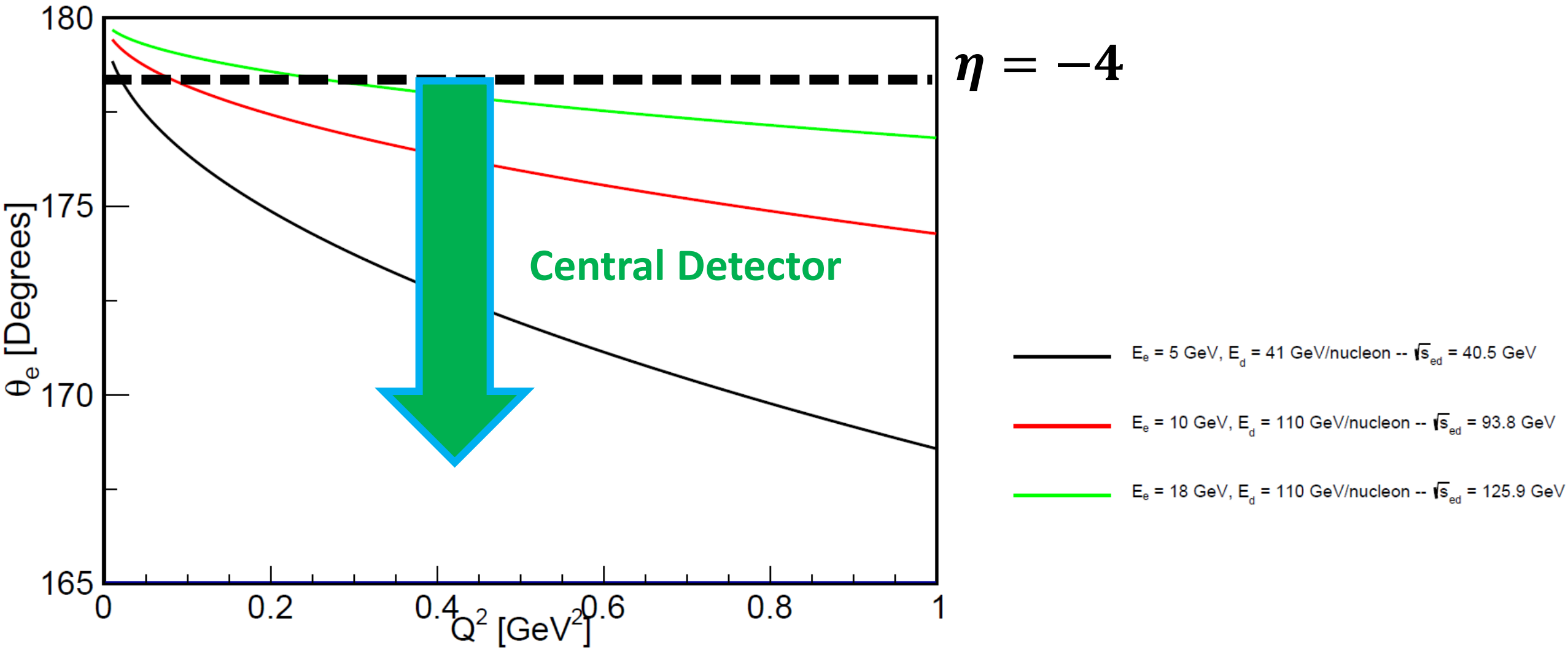


Kinematics: Low Q^2 Electron Scattering Angle

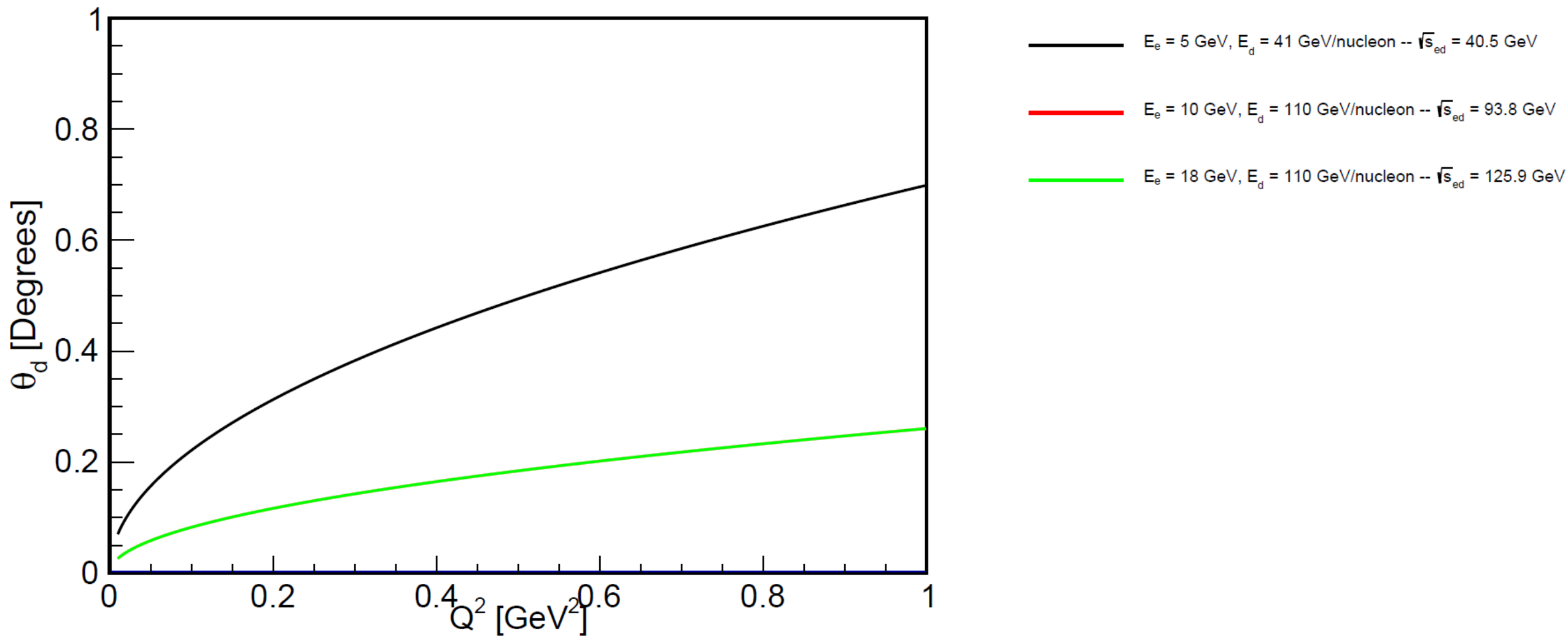


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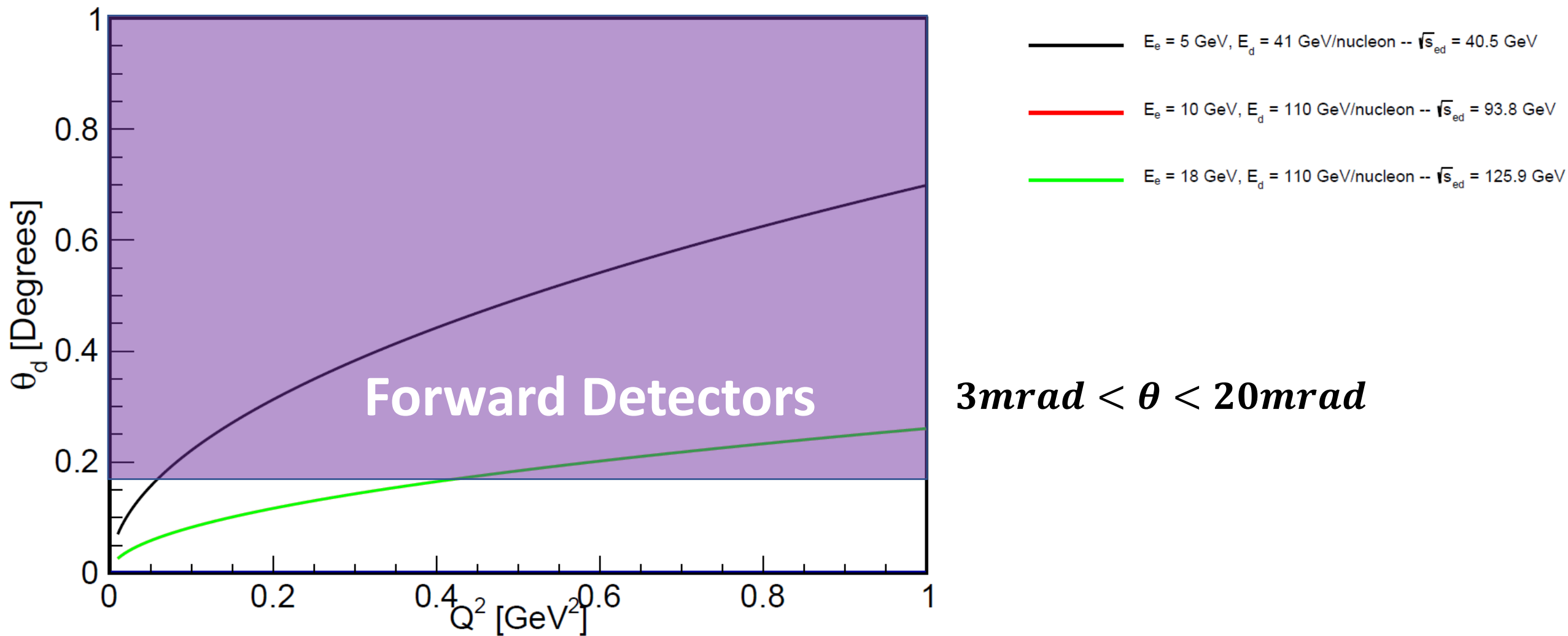
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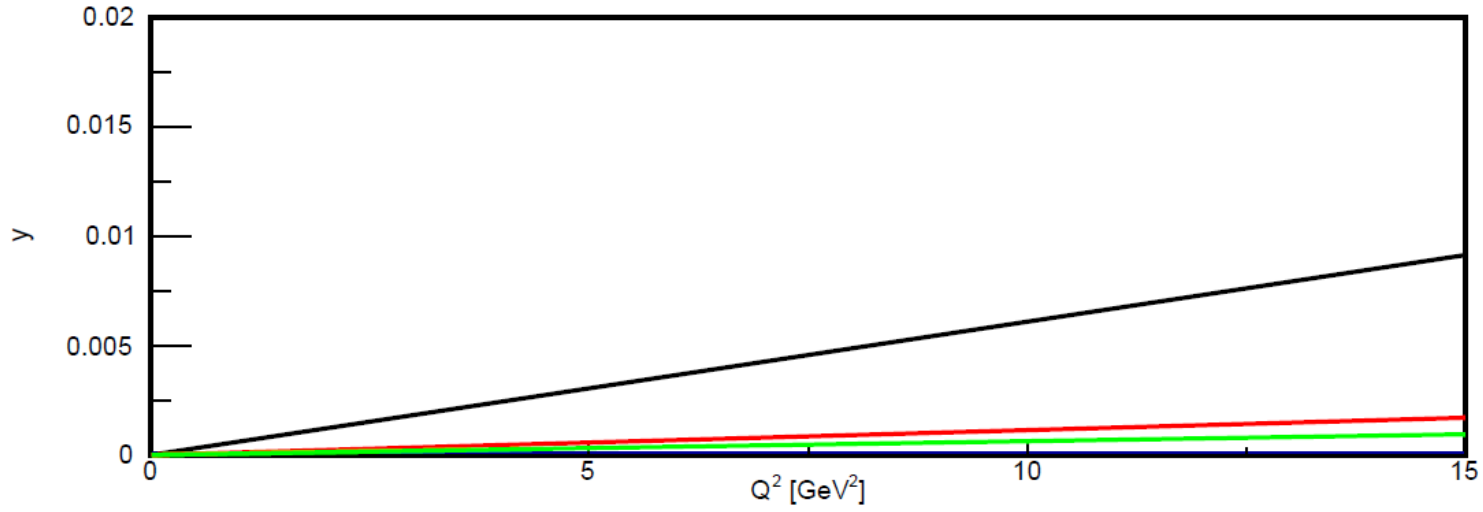
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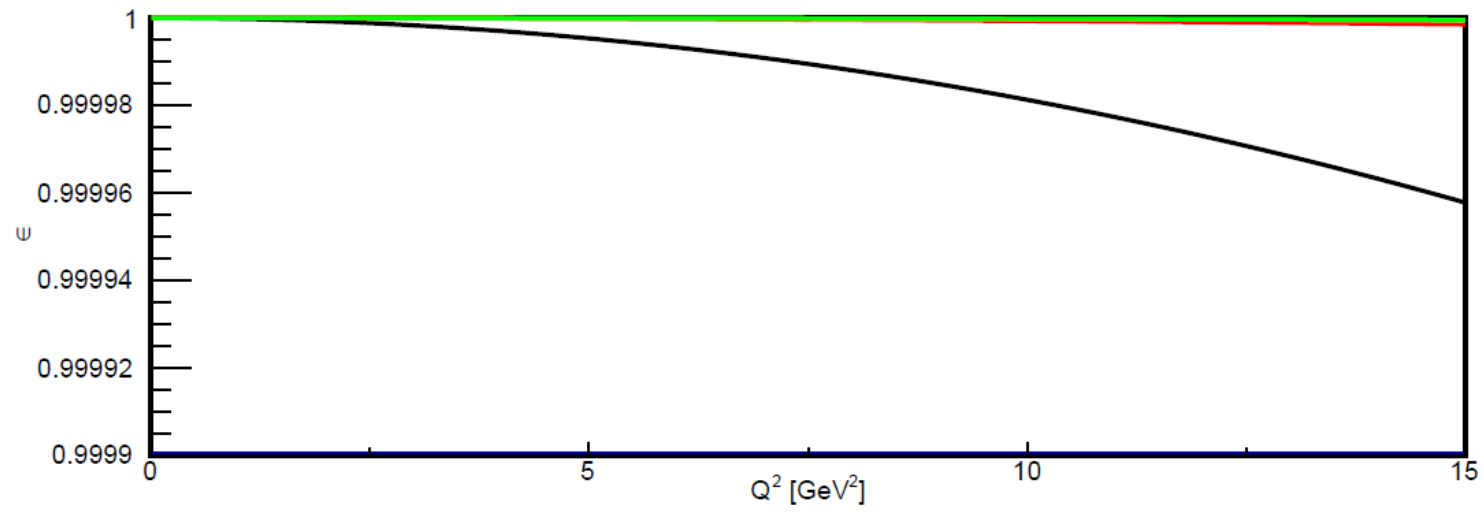
Kinematics: Low Q^2 Deuteron Scattering Angle



Kinematics: Data will be at very low y_d



- $E_e = 5 \text{ GeV}, E_d = 41 \text{ GeV/nucleon} \text{ -- } \sqrt{s_{ed}} = 40.5 \text{ GeV}$
- $E_e = 10 \text{ GeV}, E_d = 110 \text{ GeV/nucleon} \text{ -- } \sqrt{s_{ed}} = 93.8 \text{ GeV}$
- $E_e = 18 \text{ GeV}, E_d = 110 \text{ GeV/nucleon} \text{ -- } \sqrt{s_{ed}} = 125.9 \text{ GeV}$



$$\epsilon \approx \frac{1 - y_d}{1 - y_d + y_d^2/2}$$

Deuteron Form Factor Parameterization – *Abbott I* (Log Scale)

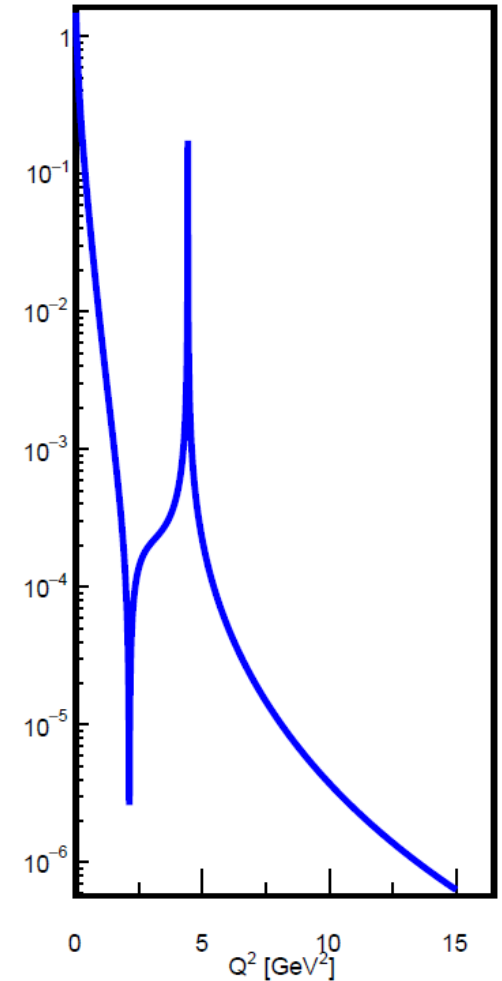
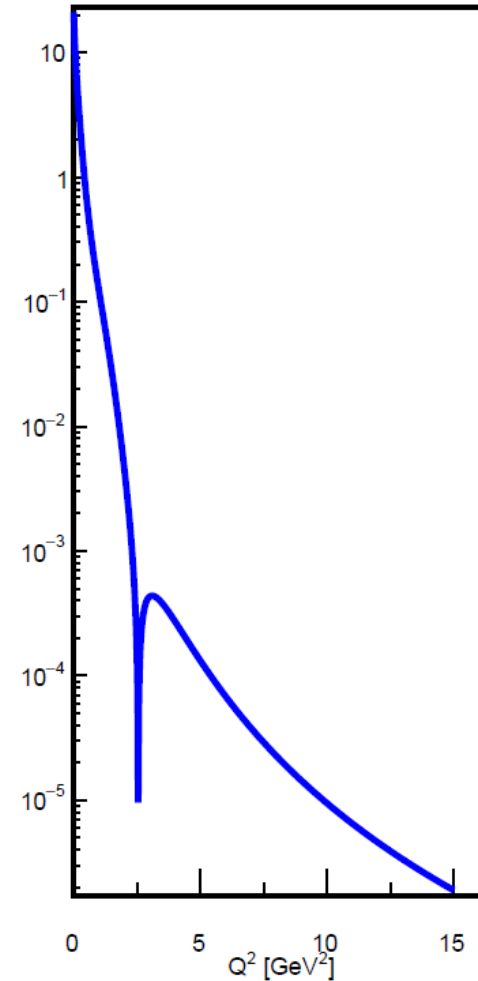
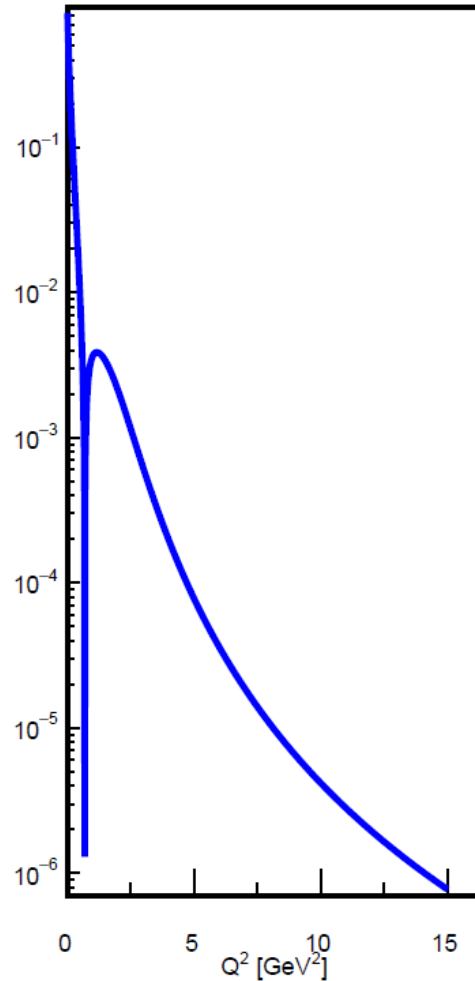
$|G_c|$ – Abbott I

$|G_q|$ – Abbott I

$|G_M|$ – Abbott I

Eur. Phys. J. A **7**, 421-427 (2000)

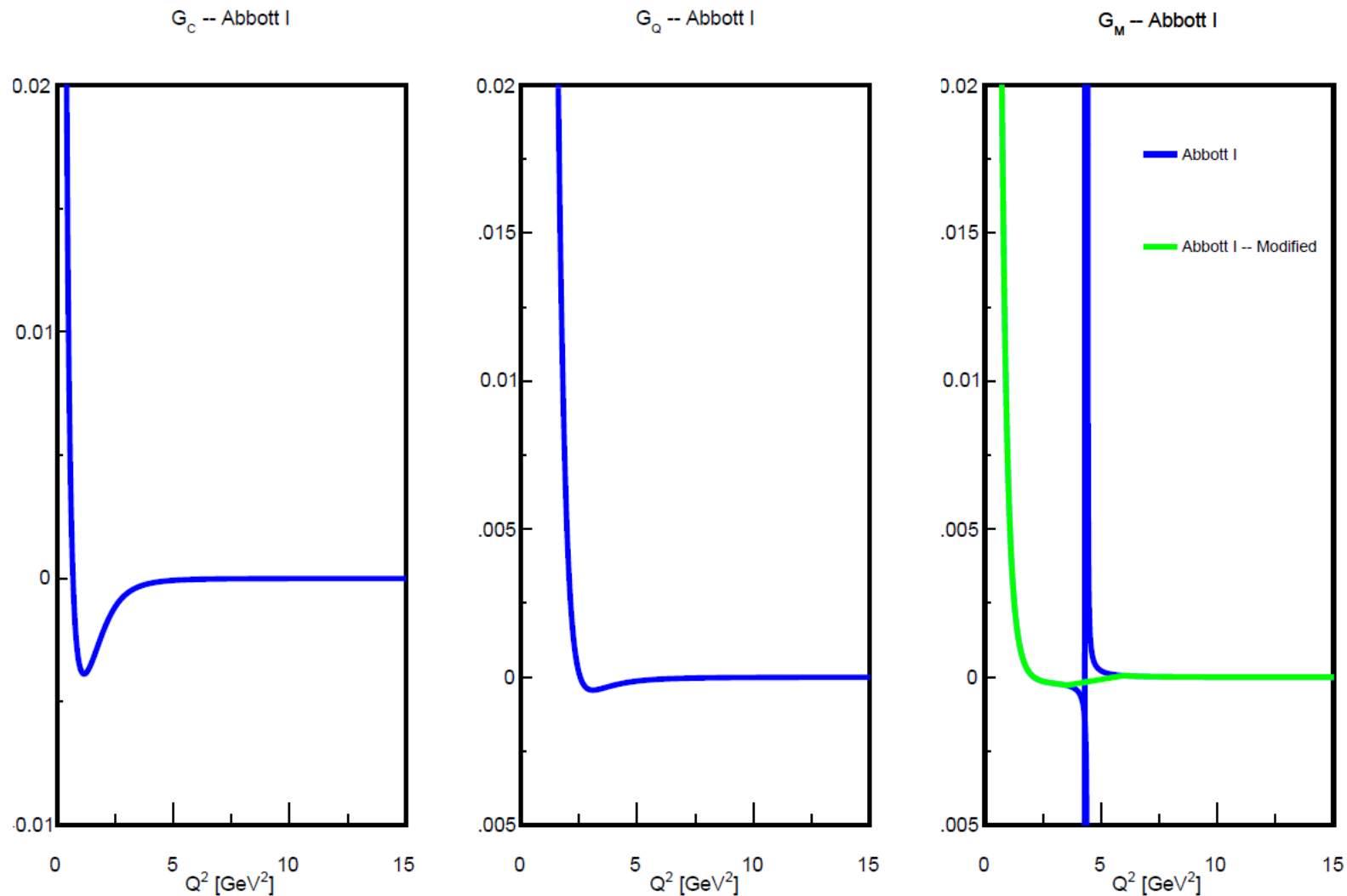
<http://irfu.cea.fr/dphn/T20/Parametrisations/>



Deuteron Form Factor Parameterization – *Abbott I* (Linear Scale)

Eur. Phys. J. A **7**, 421-427 (2000)

<http://irfu.cea.fr/dphn/T20/Parametrisations/>

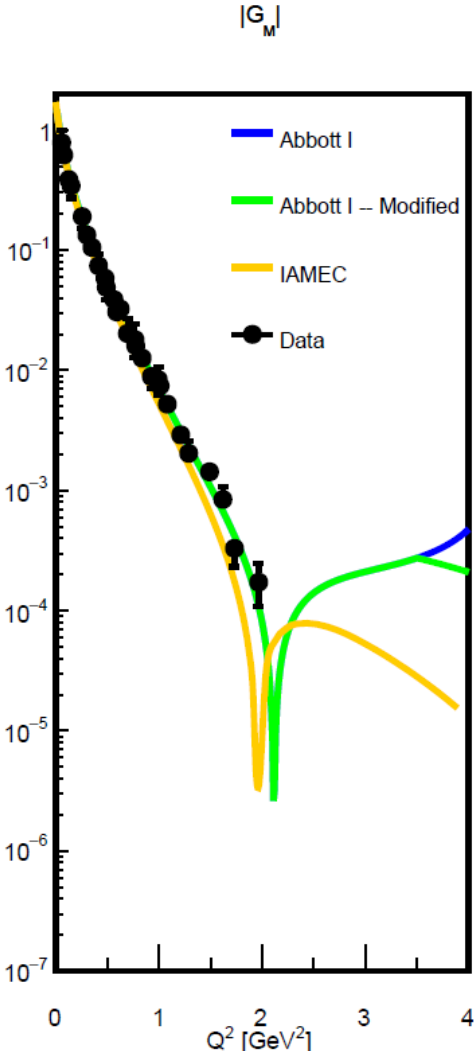
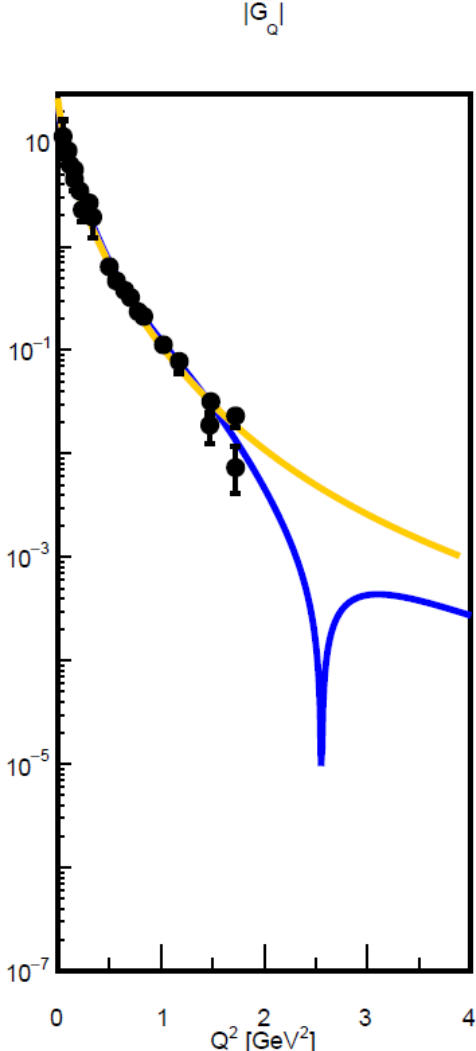
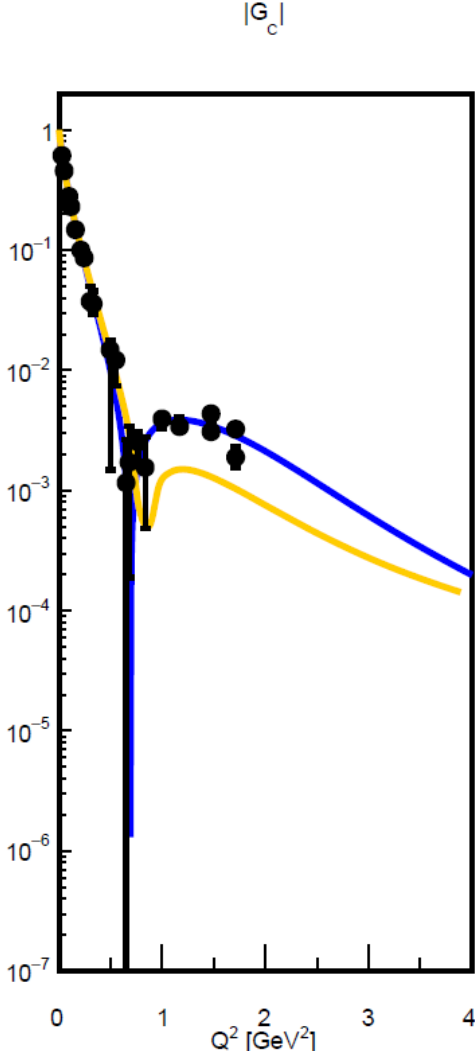


Deuteron Form Factors – Comparison to Theory and Data

Eur. Phys. J. A **7**, 421-427 (2000)

<http://irfu.cea.fr/dphn/T20/Parametrisations/>

Phys. Rev. C **49**, 21 (1994)



Deuteron Form Factors – A and B

A

B

Eur. Phys. J. A **7**, 421-427 (2000)

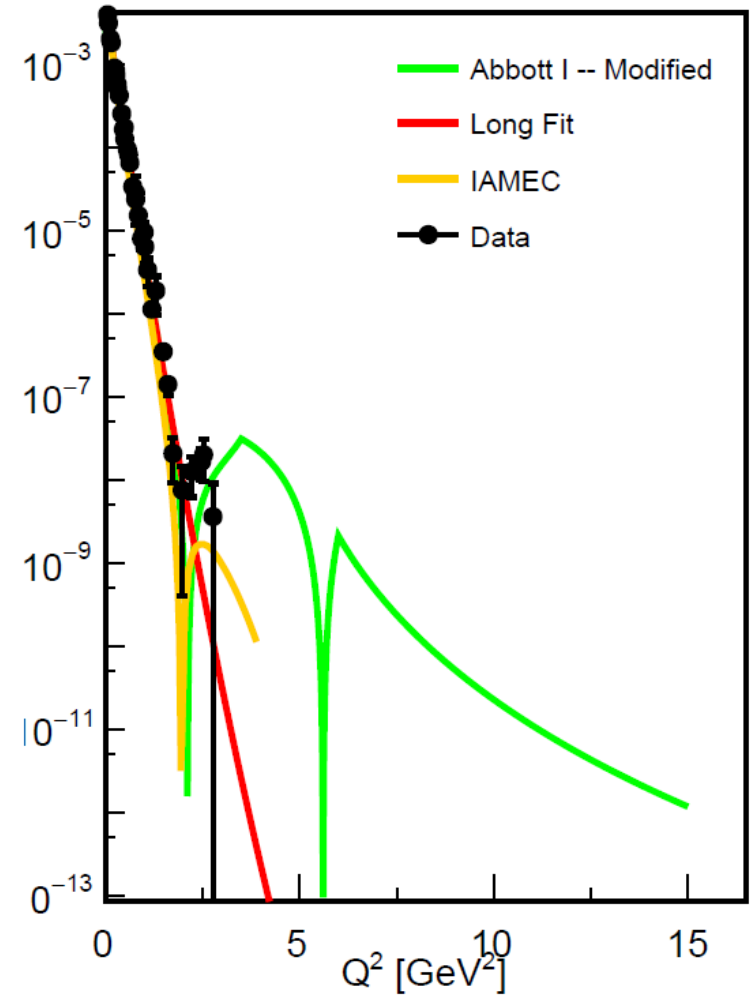
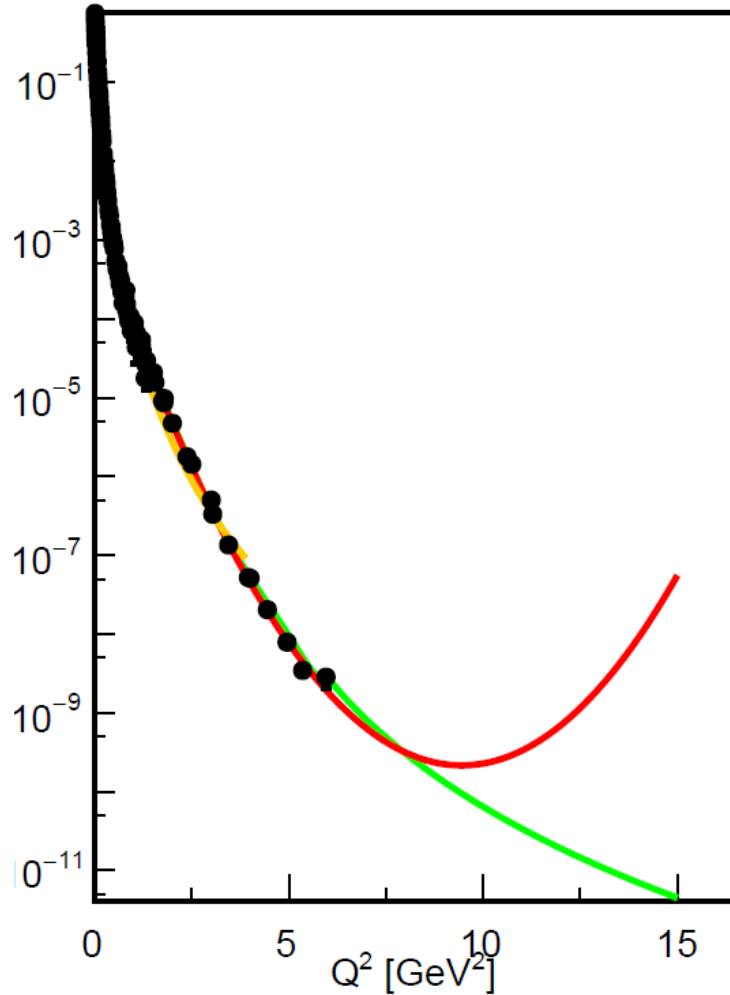
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Phys. Rev. C **49**, 21 (1994)

<https://hallcweb.jlab.org/wiki/index.php/Elong-15-02-26>

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2)$$

$$B(Q^2) = \frac{4}{3}\eta(1 + \eta)G_M^2(Q^2)$$



Electron-Deuteron Elastic Lorentz-Invariant Cross Section

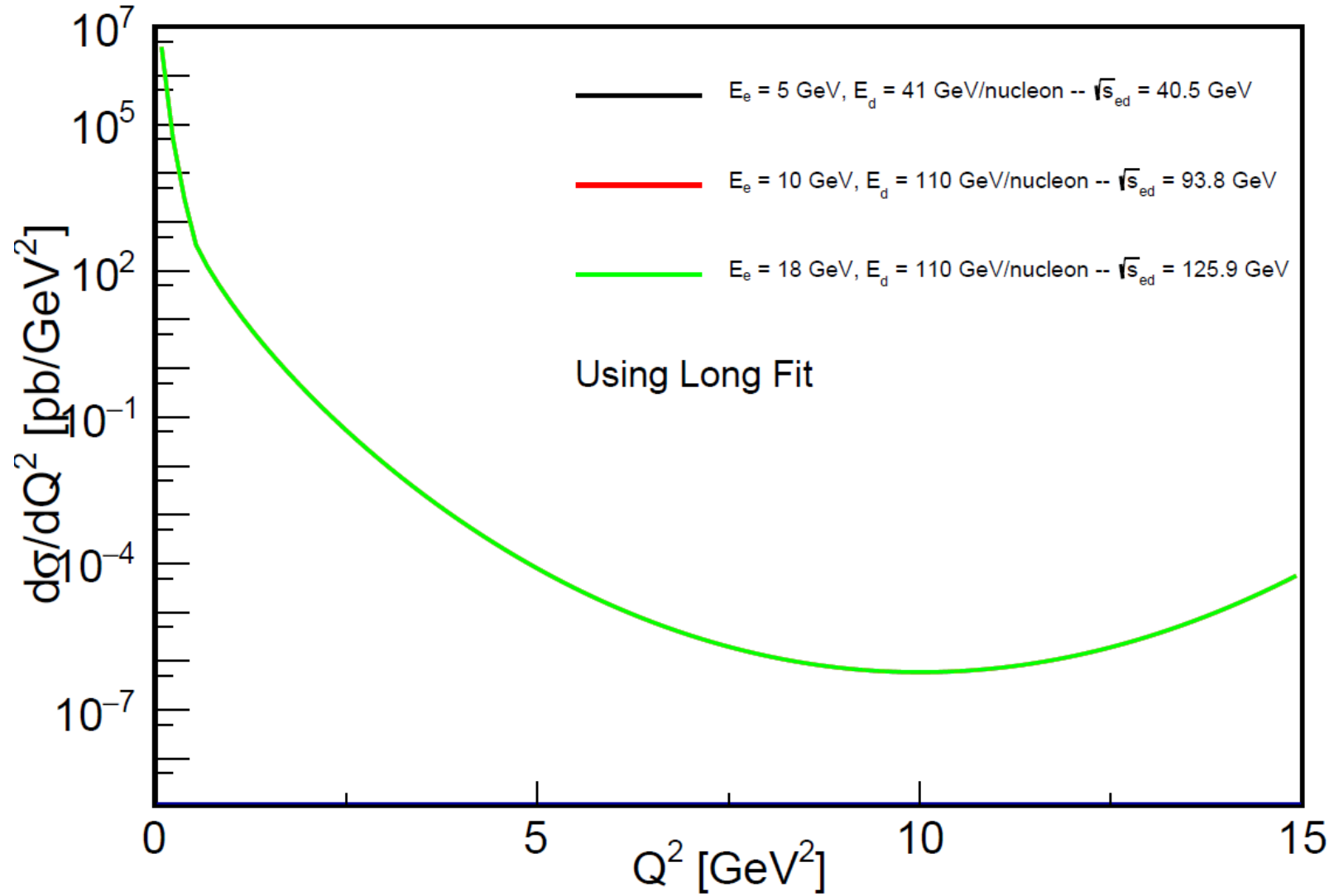
The elastic e-D cross section can be written as

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[A \left(1 - y_d - \frac{M_d^2 y_d^2}{Q^2} \right) + B \left(\frac{1}{4\eta} y_d^2 \right) \right]$$

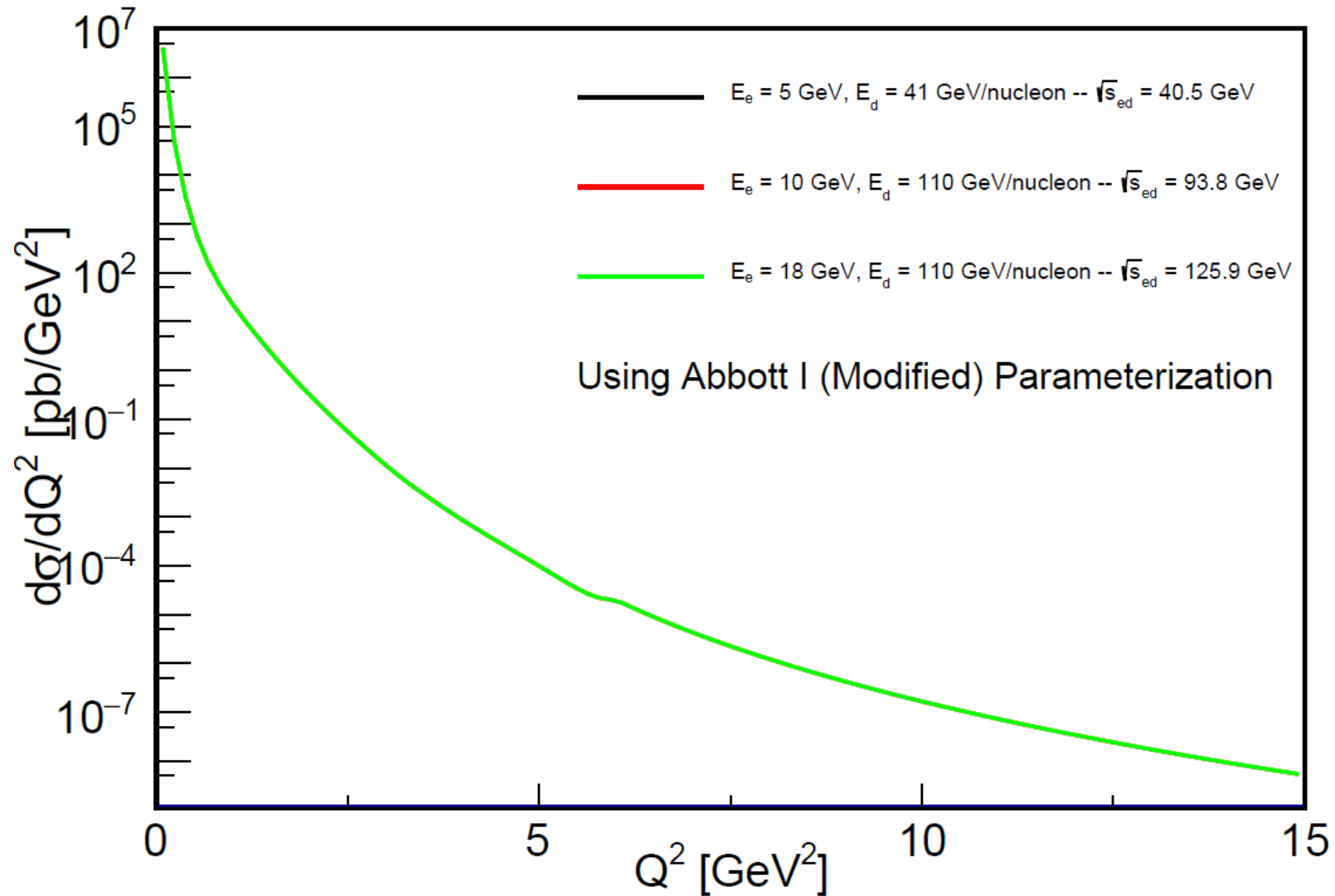
At the EIC, this is approximately

$$\frac{d\sigma}{dQ^2} \approx \frac{4\pi\alpha^2}{Q^4} A$$

Lorentz-Invariant Cross Section



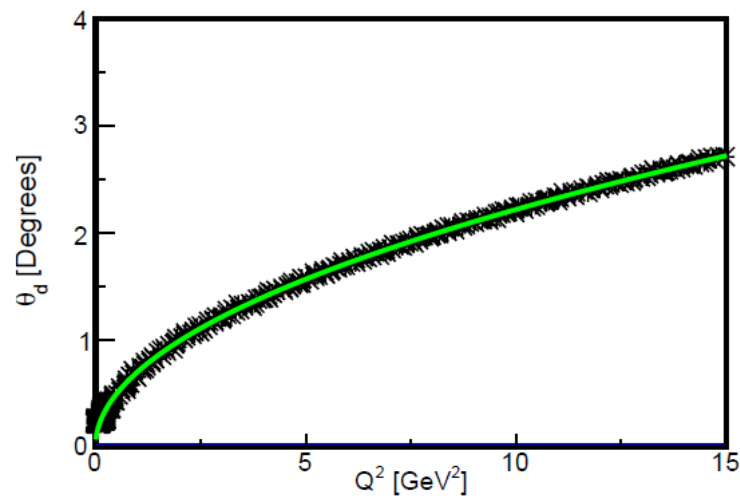
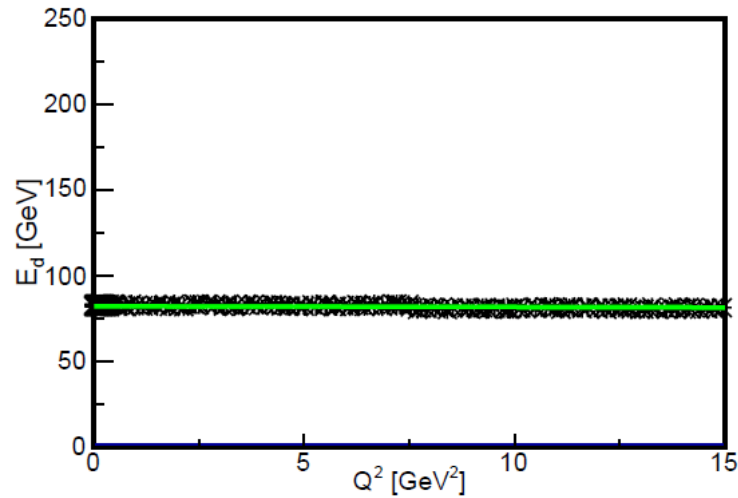
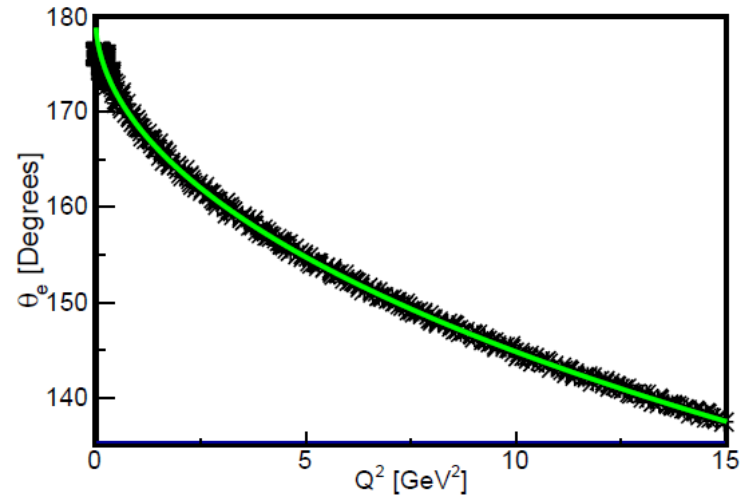
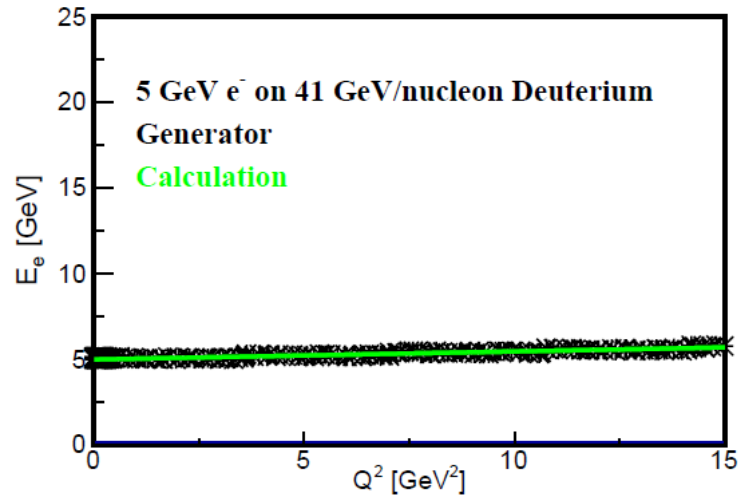
Lorentz-Invariant Cross Section



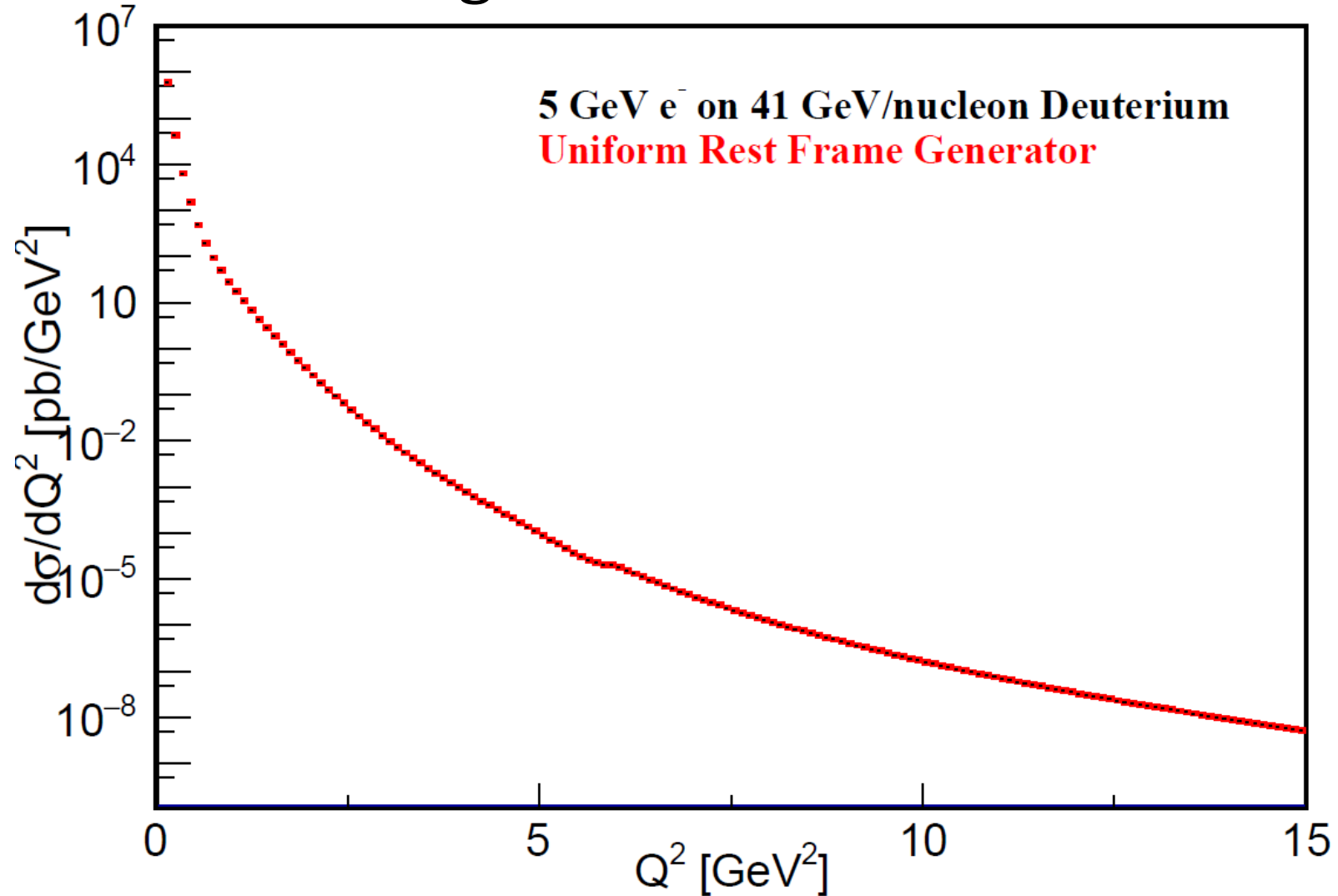
Description of rest-frame e-D Elastic generator with anti-parallel beams

1. Boost from lab frame to deuteron's rest frame
2. Generate events according to deuteron rest frame Born cross section. We use the modified *Abbott I* parametrization discussed above for the deuteron form factors.
3. Include an option to generate events according to the tensor-polarized elastic e-D cross section, as discussed below
4. We choose to generate elastic simulated data corresponding to an electron-nucleon integrated luminosity 100 fb^{-1} . This corresponds to an electron-deuteron integrated luminosity of 50 fb^{-1} (i.e. $\mathcal{L}_{ed} = \frac{1}{2} \times \mathcal{L}_{eN}$).

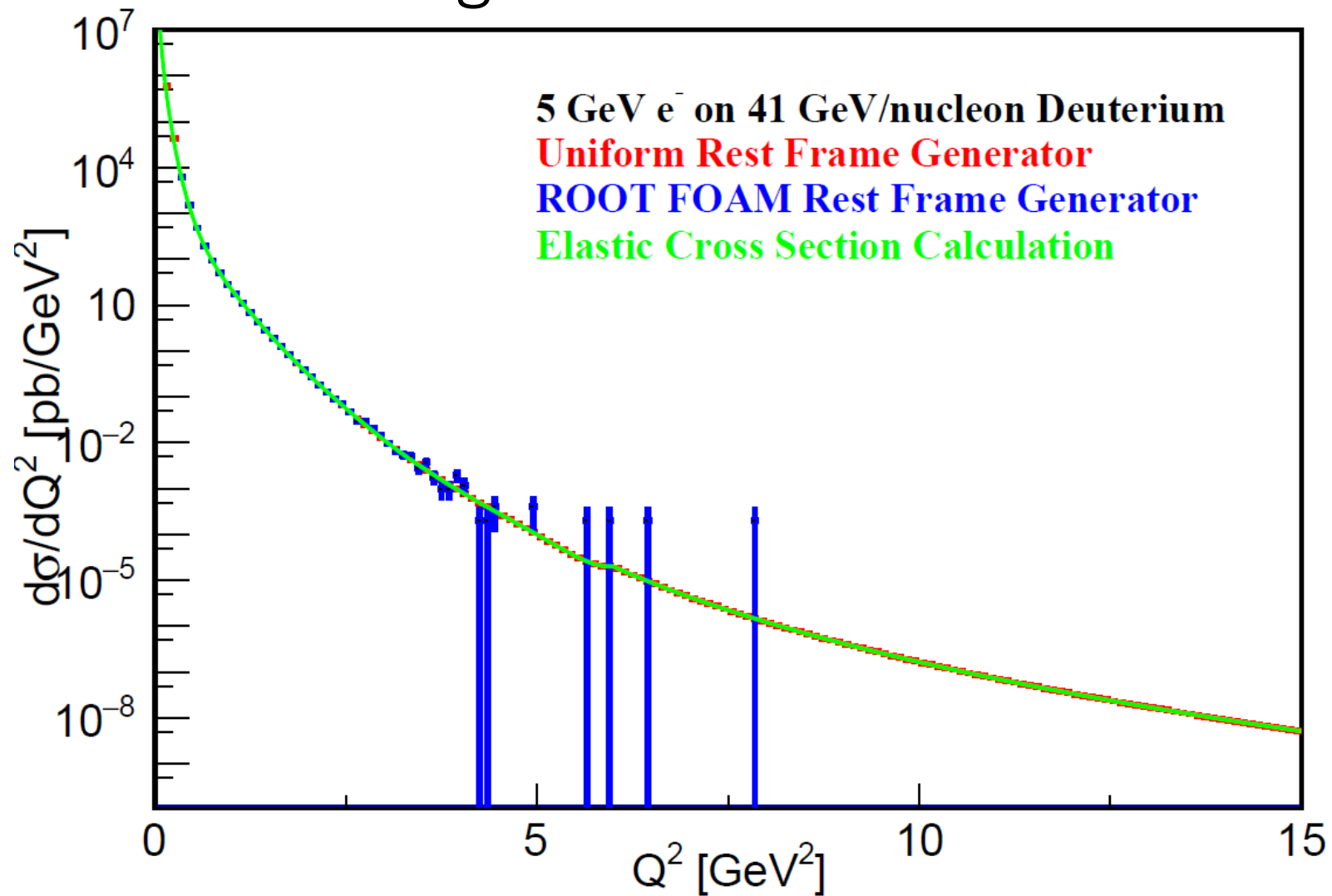
Generator results agree with calculations: Kinematics



Generator results agree with calculations: Cross Section

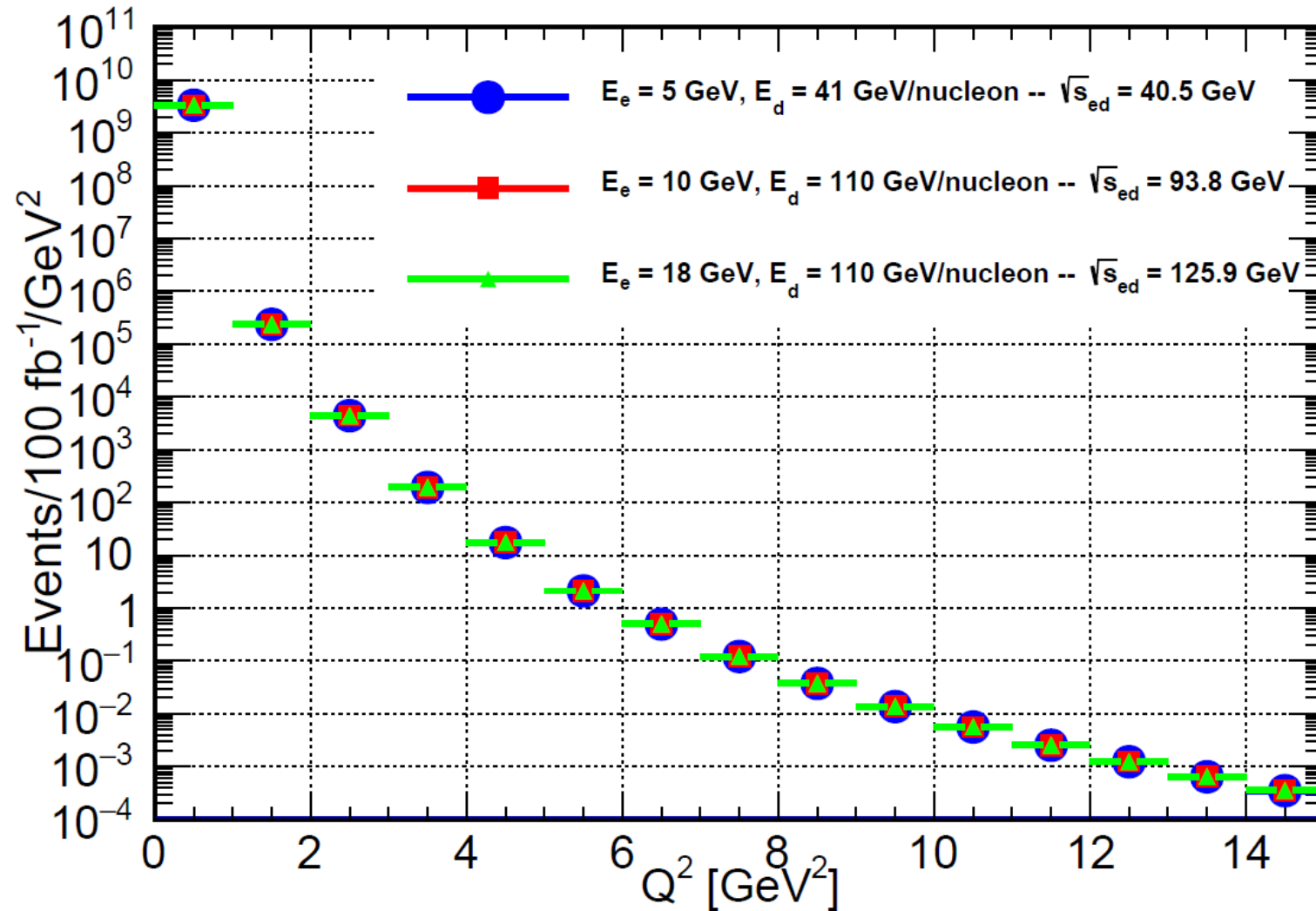


Generator results agree with calculations: Cross Section

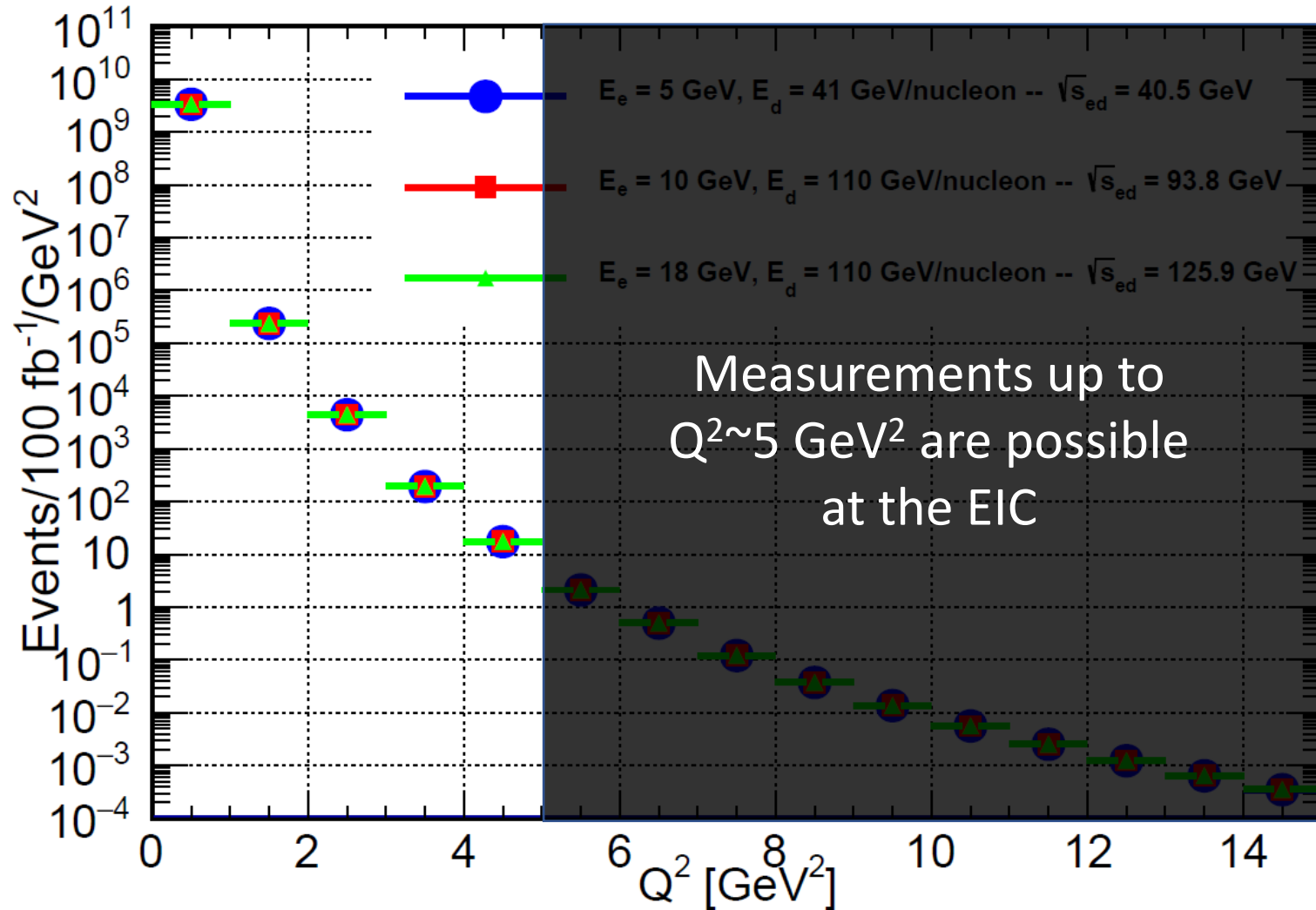


$$\frac{d\sigma}{dQ^2} \approx \frac{4\pi\alpha^2}{Q^4} A$$

e-D Elastic Scattering Expected EIC Unpolarized Yields



e-D Elastic Scattering Expected EIC Unpolarized Yields



Electron-Deuteron Cross Section with polarized deuteron...and unpolarized electron

$$\frac{\sigma}{\sigma_0} = 1 + (P_{zz}/\sqrt{2}) \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

Electron-Deuteron Cross Section with polarized deuteron...and unpolarized electron

$$\boxed{\frac{\sigma}{\sigma_0}} = 1 + (P_{zz}/\sqrt{2}) \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

**Ratio of polarized
to unpolarized
cross section**

Electron-Deuteron Cross Section with polarized deuteron...and unpolarized electron

$$\frac{\sigma}{\sigma_0} = 1 + \boxed{(P_{zz}/\sqrt{2})} \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

**Deuteron tensor
polarization**

Electron-Deuteron Cross Section with polarized deuteron...and unpolarized electron

$$\frac{\sigma}{\sigma_0} = 1 + (P_{zz}/\sqrt{2}) \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

**Tensor Analyzing Powers
(i.e. Polarization
Observables)**

$$T_{20} = -(\sqrt{2}\eta/3S) \left[4G_C G_Q + \frac{4\eta}{3} G_Q^2 + \left(\frac{1}{2} + \varepsilon \right) G_M^2 \right]$$

$$T_{21} = \frac{2}{S} \sqrt{\frac{\eta^3(1+\varepsilon)}{3}} G_Q G_M \quad T_{22} = [\eta/(2\sqrt{3}S)] G_M^2$$

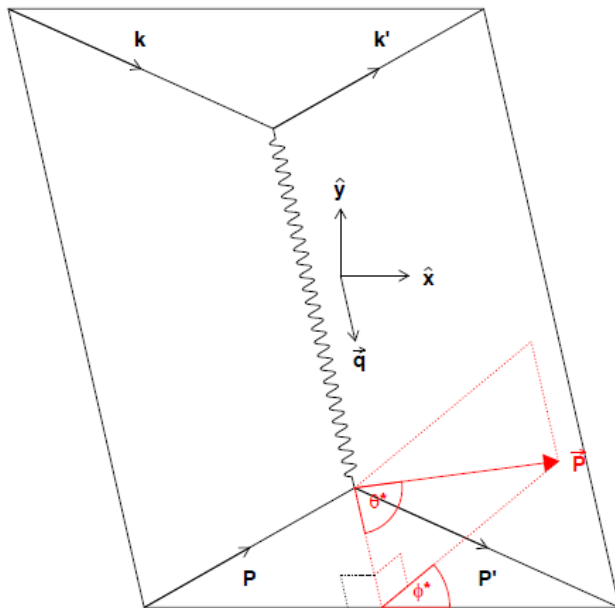
$$\eta = \frac{Q^2}{4M_d^2}$$

$$\varepsilon = (1 + \eta) \tan^2(\theta_e/2)$$

$$S \equiv A + \tan^2(\theta_e/2) B$$

Electron-Deuteron Cross Section with polarized deuteron...and unpolarized electron

$$\frac{\sigma}{\sigma_0} = 1 + (P_{zz}/\sqrt{2}) \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

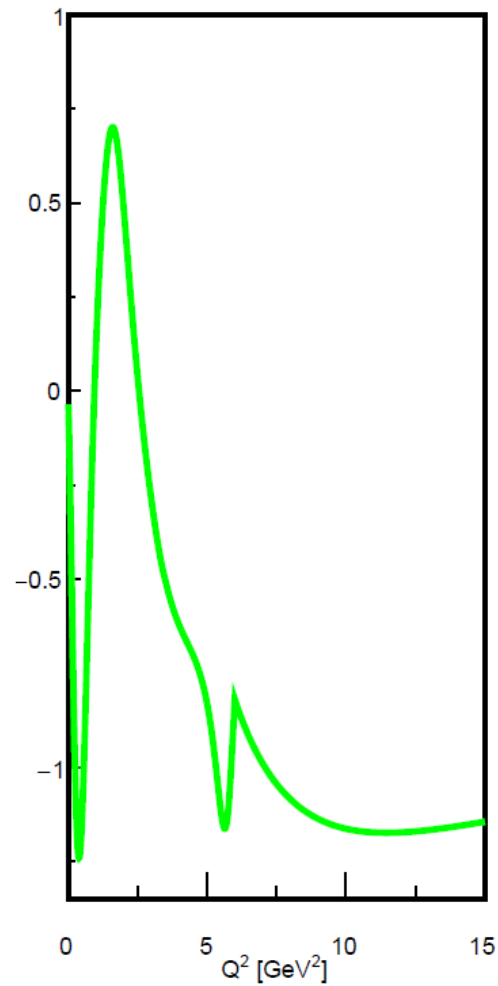


$\vec{P} \equiv$ Target polarization

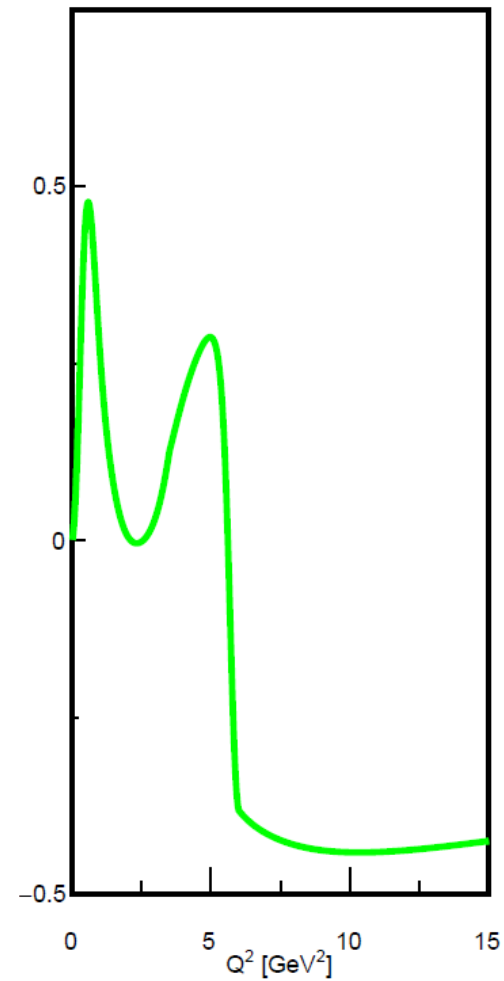
θ^* and ϕ^* give the polarization orientation with respect to the momentum transfer in the Breit (or incoming deuteron rest) frame. The terms here are proportional to the real part of the corresponding spherical harmonic.

Parameterizations of T_{20} , T_{21} and T_{22}

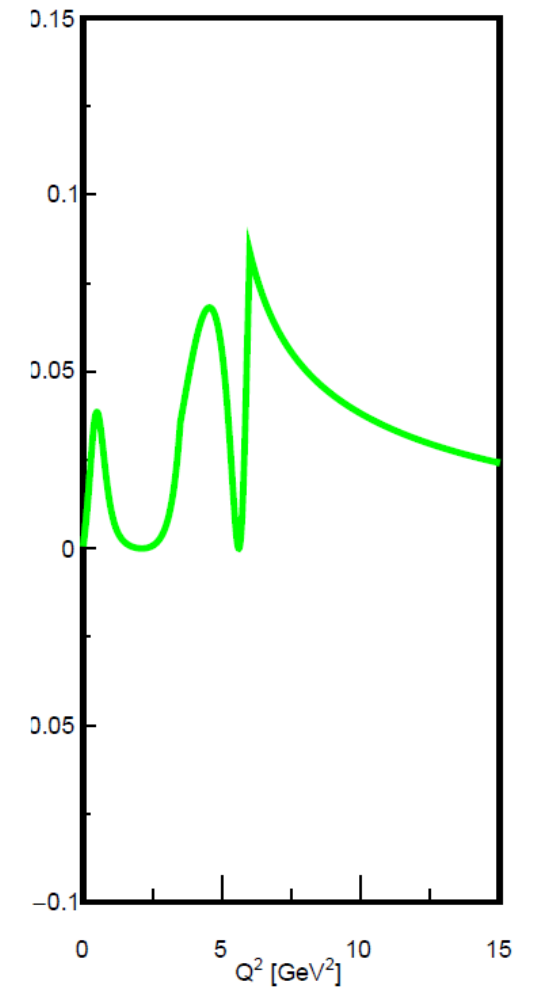
$T_{20}(70^\circ)$ -- Abbott I (Modified)



$T_{21}(70^\circ)$ -- Abbott I (Modified)



$T_{22}(70^\circ)$ -- Abbott I (Modified)



Eur. Phys. J. A **7**, 421-427 (2000)

<http://irfu.cea.fr/dphn/T20/Parametrisations/>

Comparison to Theory and Data

$T_{20}(70^\circ)$

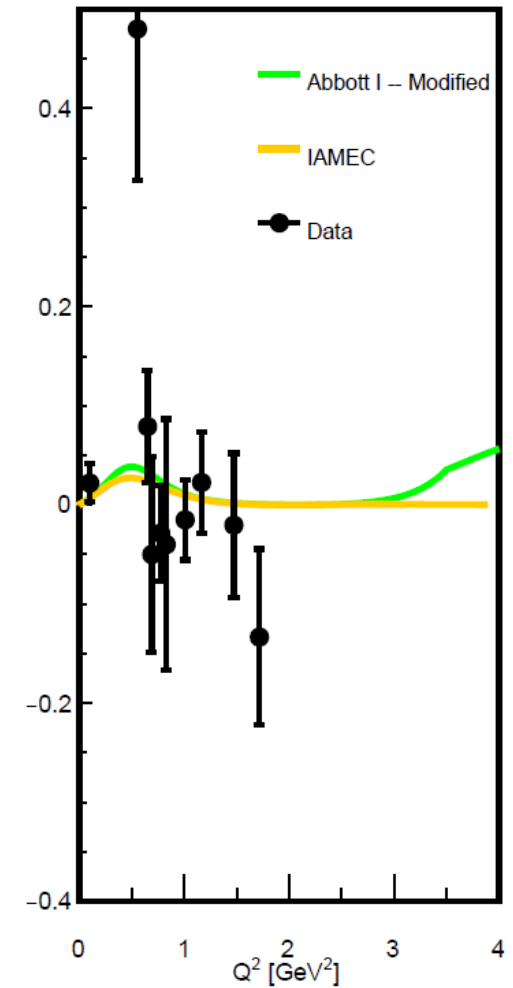
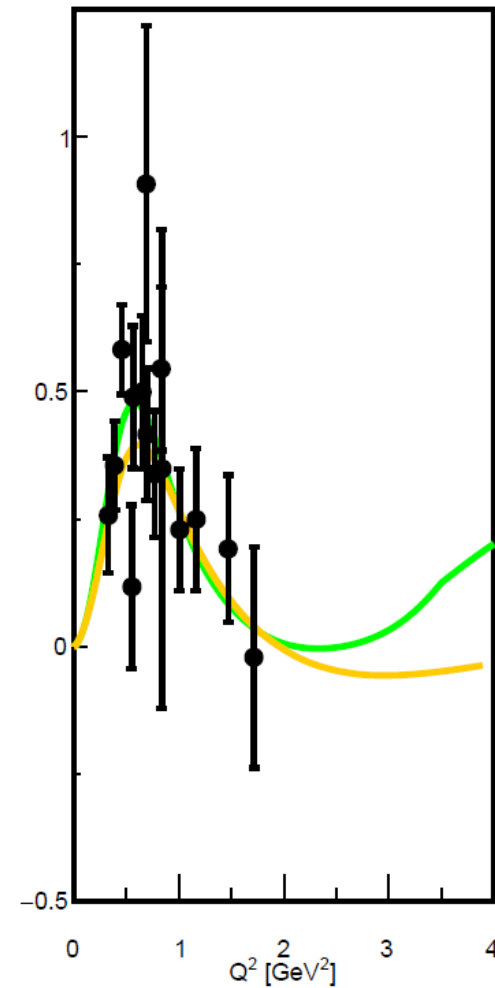
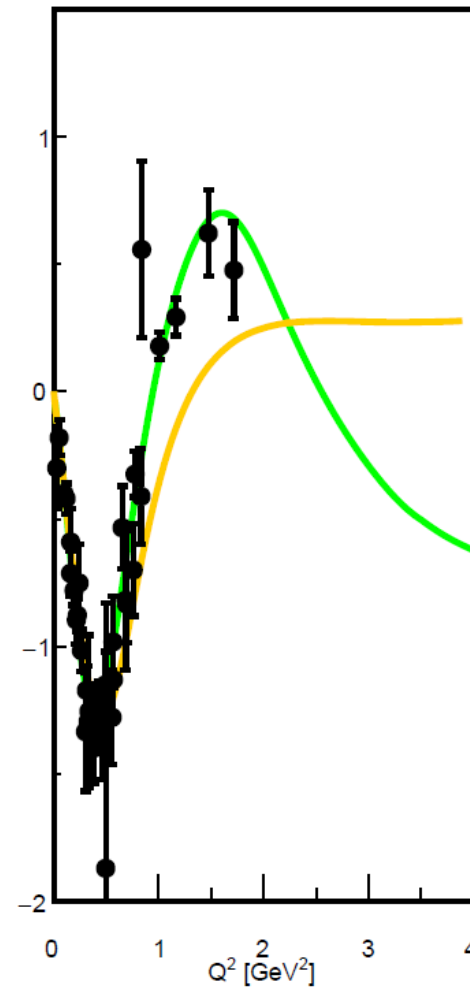
$T_{21}(70^\circ)$

$T_{22}(70^\circ)$

Eur. Phys. J. A **7**, 421-427 (2000)

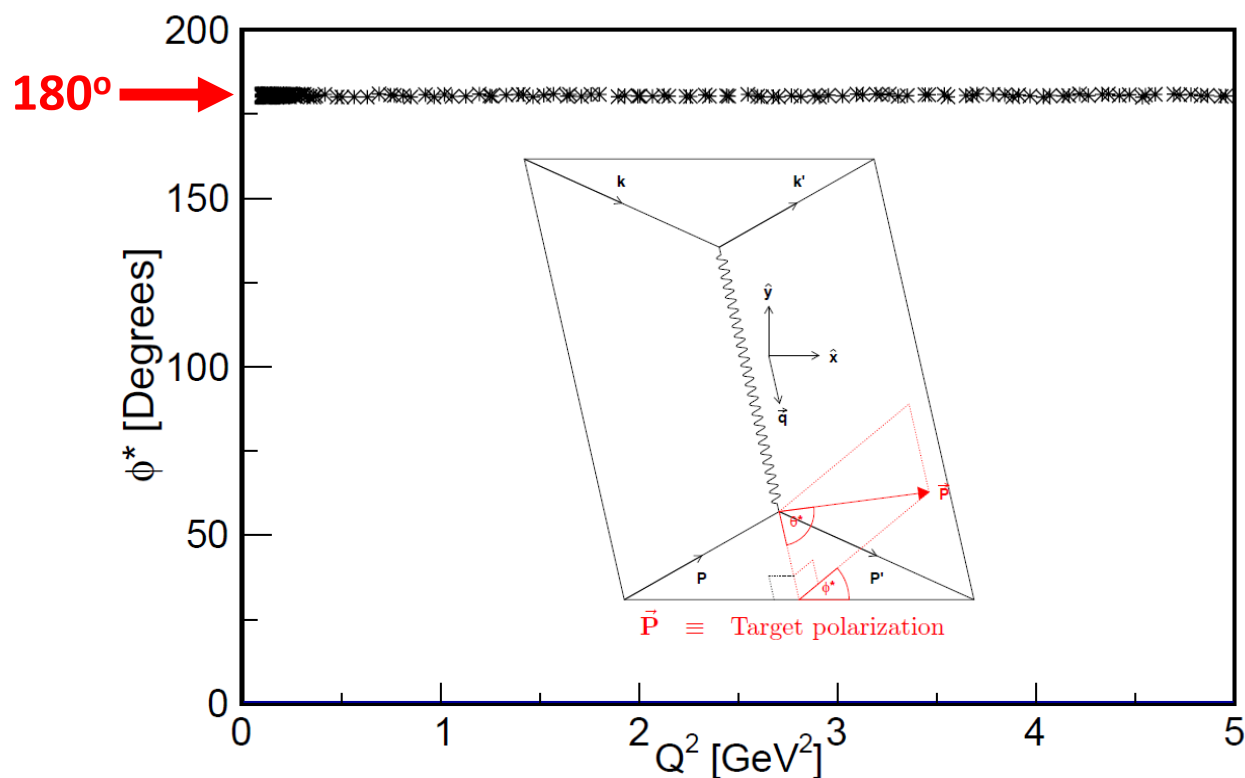
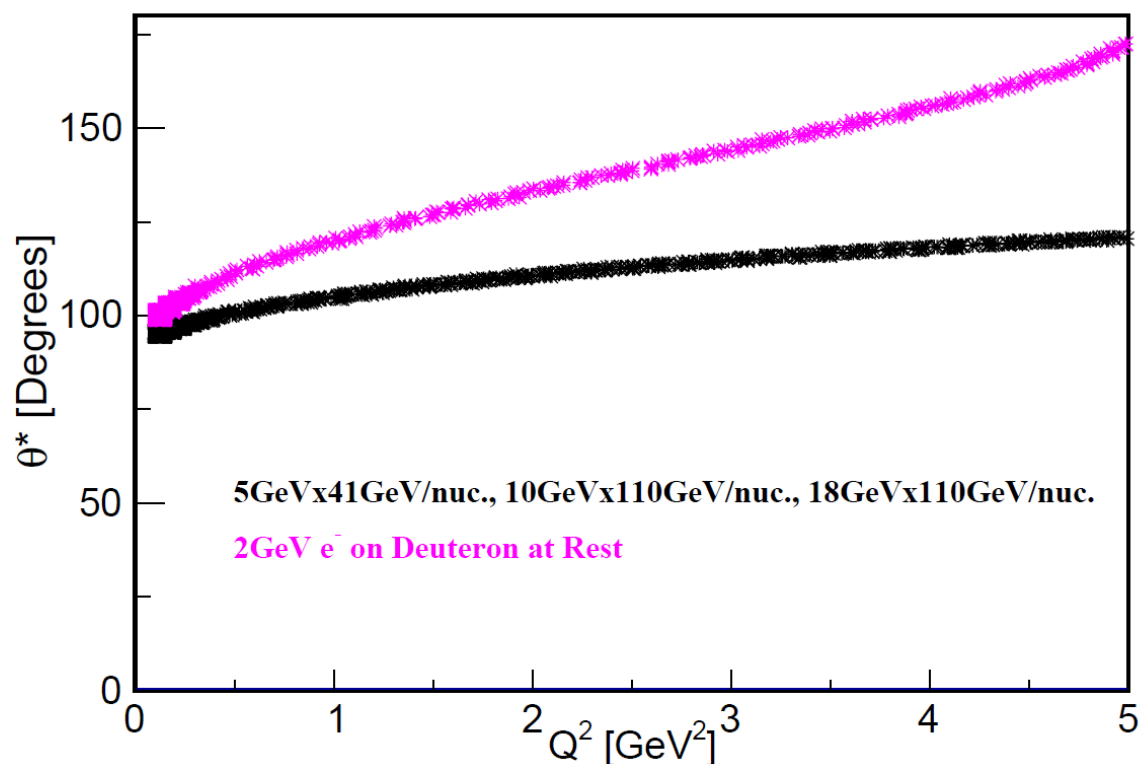
<http://irfu.cea.fr/dphn/T20/Parametrisations/>

Phys. Rev. C **49**, 21 (1994)

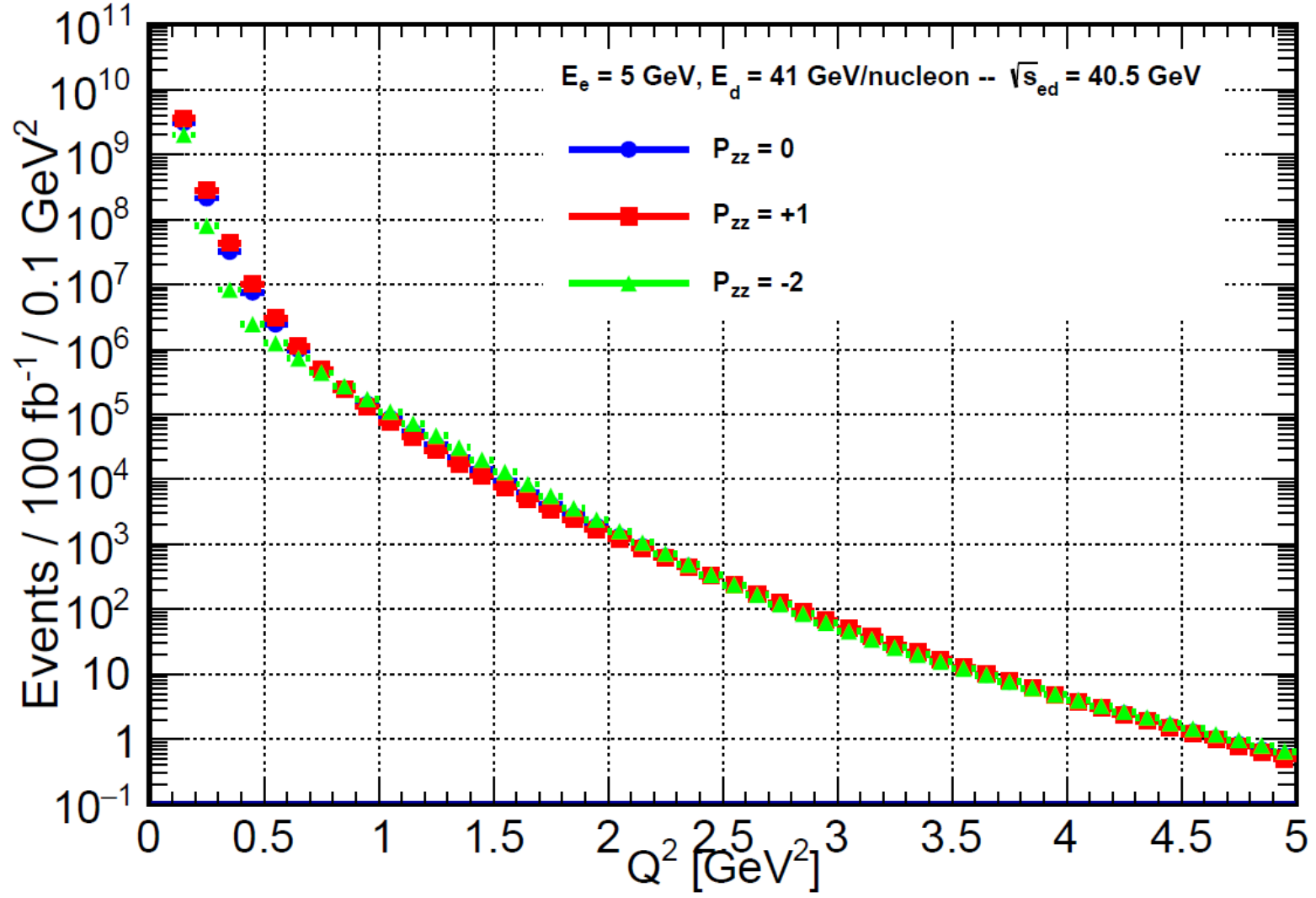


Deuteron Polarization Orientation: $\mathbf{P} = (0,0,1)$

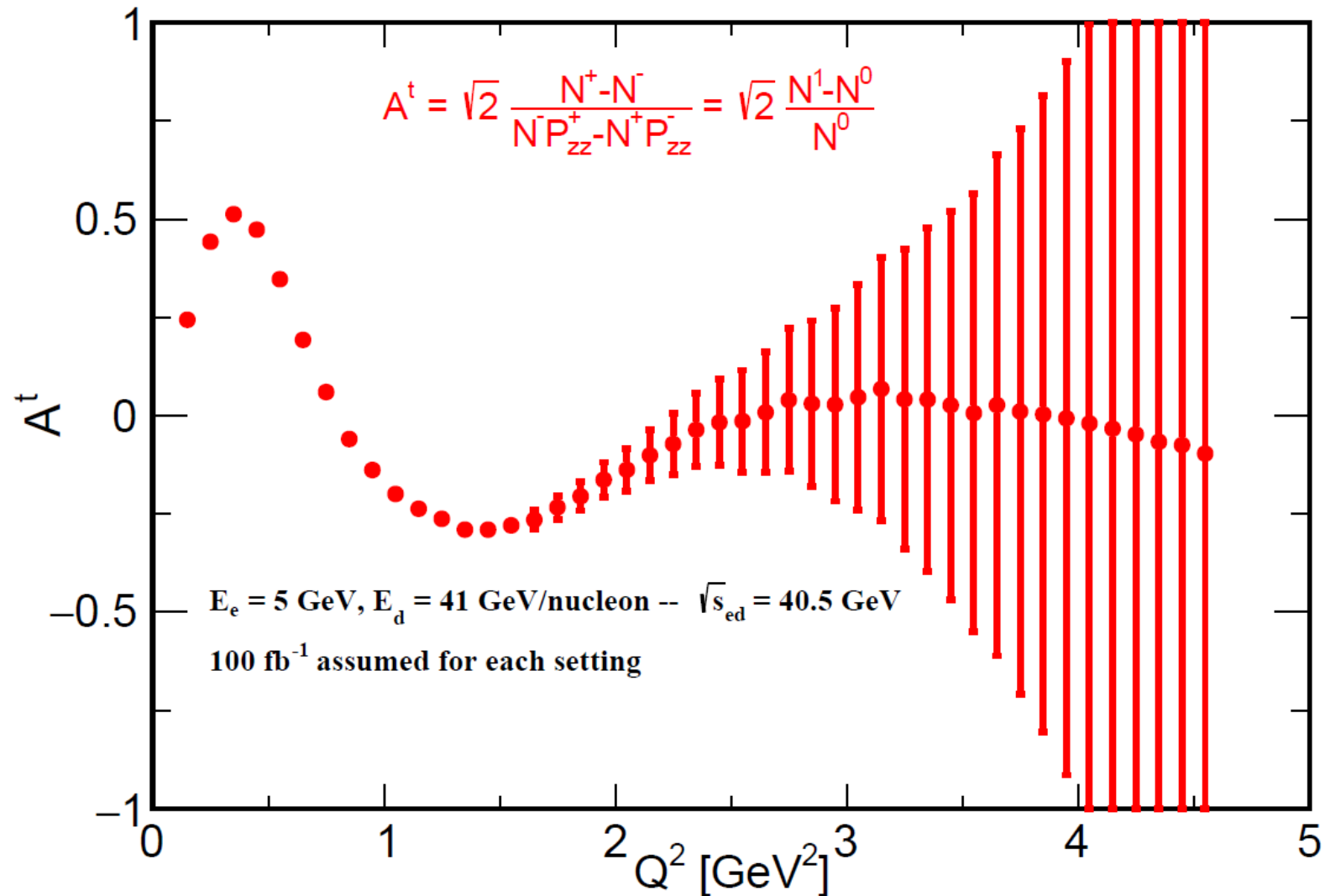
$$\frac{\sigma}{\sigma_0} = 1 + (P_{zz}/\sqrt{2}) \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$



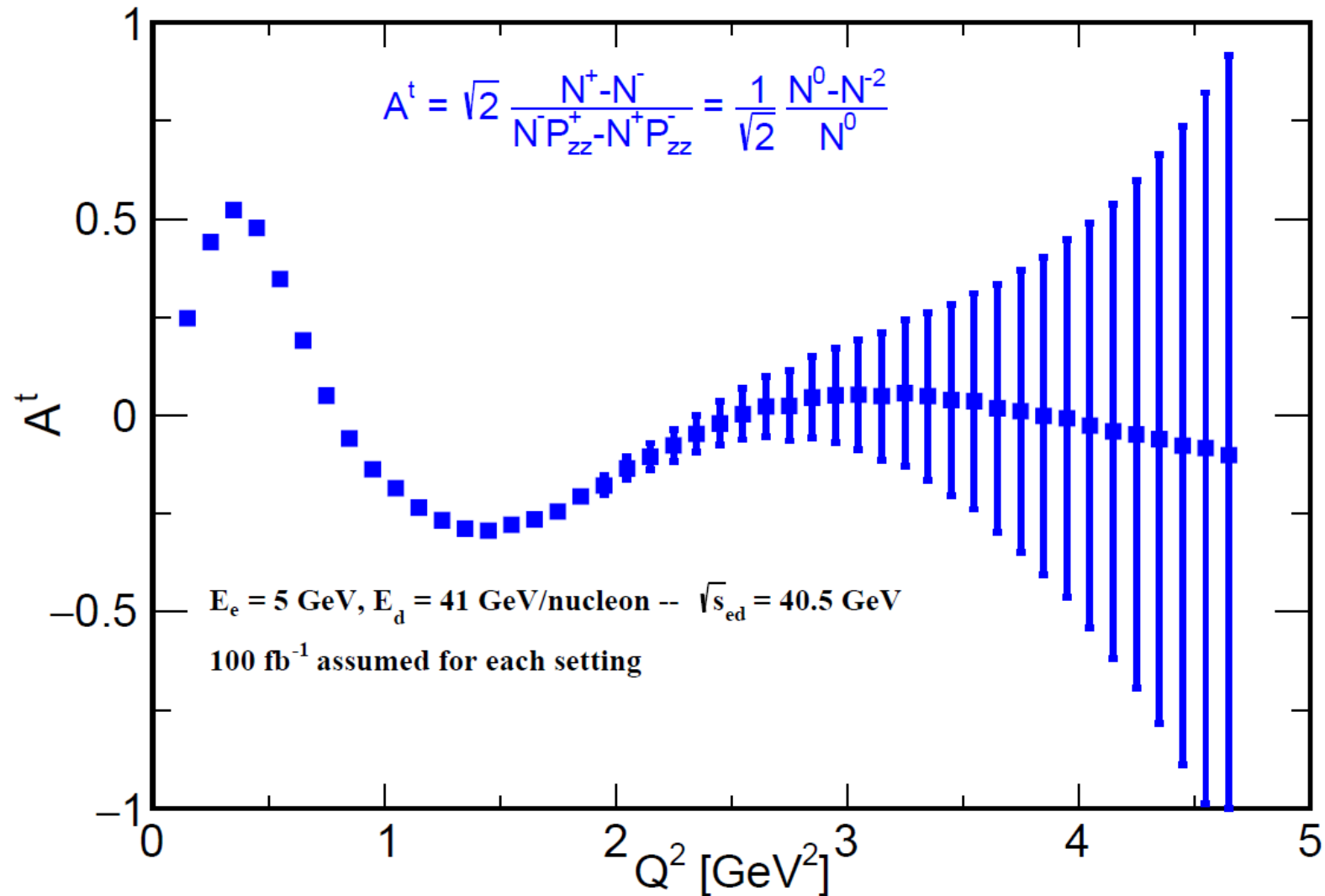
e-D Elastic Scattering Expected EIC Tensor-Polarized Yields



e-D Elastic Scattering Expected EIC Tensor-Polarized Asymmetries

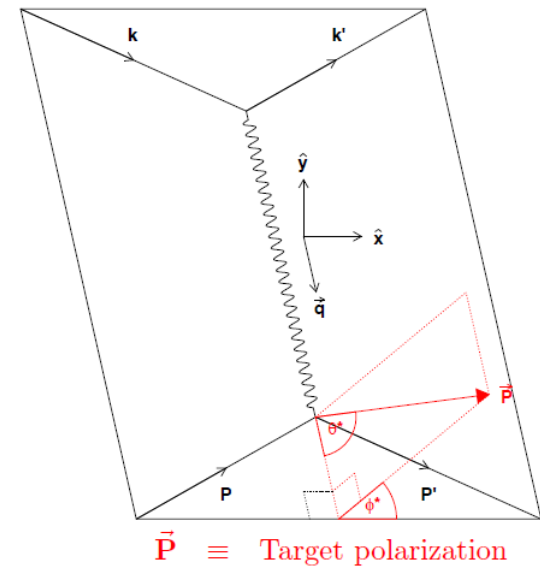
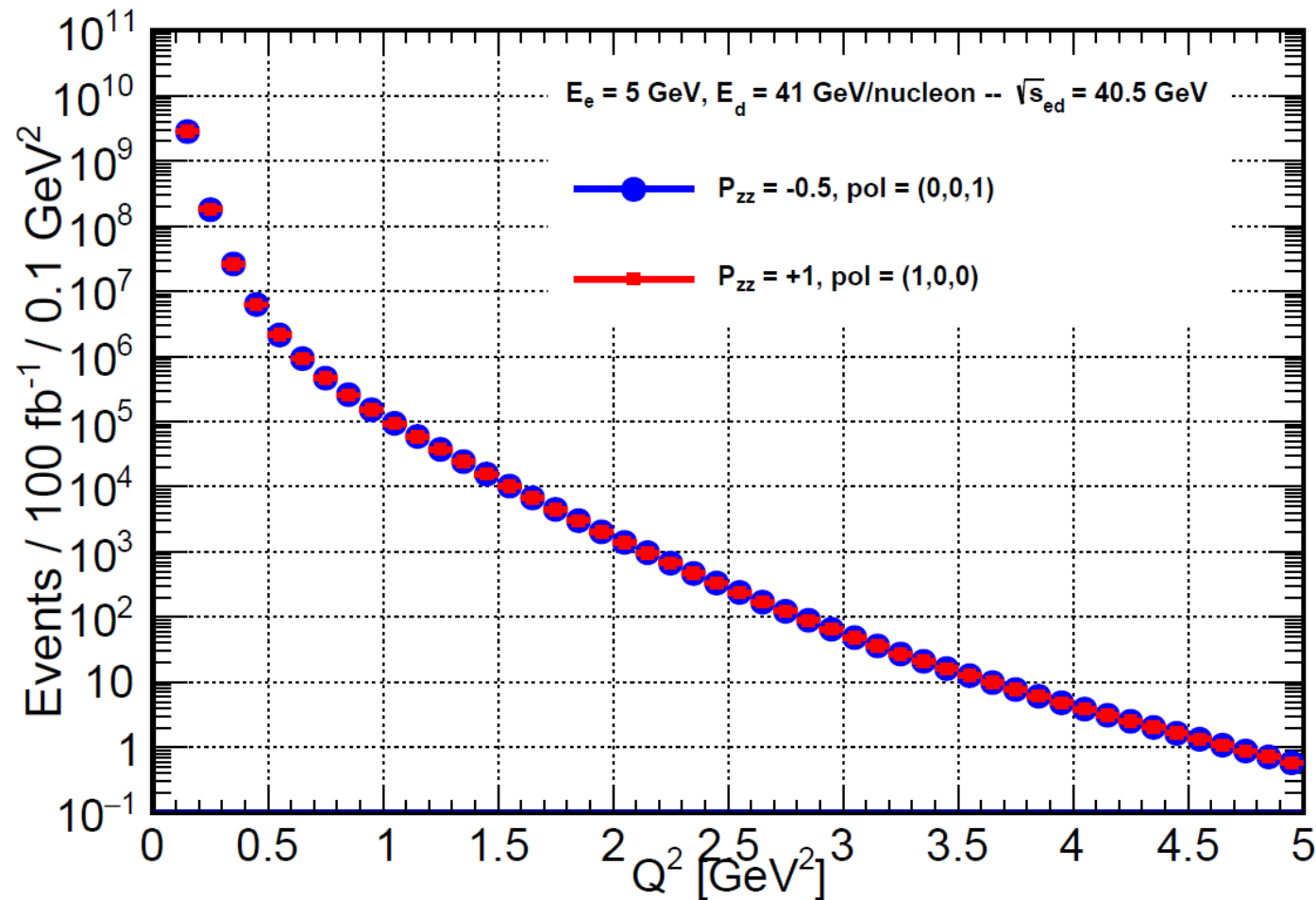


e-D Elastic Scattering Expected EIC Tensor-Polarized Asymmetries



Transverse $P_{zz} = +1$ is equivalent to Longitudinal $P_{zz} = -\frac{1}{2}$

$$\langle +1 \rangle_x = \frac{1}{2} \langle +1 \rangle_Z + \frac{1}{\sqrt{2}} \langle 0 \rangle_Z + \frac{1}{2} \langle -1 \rangle_Z \Rightarrow P_Z = 0 \ \& \ P_{ZZ} = -\frac{1}{2}$$



Possible procedure to extract the Deuteron tensor polarization at the EIC

$$A^t = \sqrt{2} \frac{(N^+ - N^-)}{(N^- P_{zz}^+ - N^+ P_{zz}^-)}$$
$$= \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

Possible procedure to extract the Deuteron tensor polarization at the EIC

$$A^t = \sqrt{2} \frac{(N^+ - N^-)}{(N^- P_{zz}^+ - N^+ P_{zz}^-)}$$
$$= \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

The procedure can be as follows:

1. Bin data in Q^2
2. For each bin, use the form factor parameterizations to calculate A^t . Note that if the polarization axis is parallel to the initial deuteron momentum direction, $\phi^*=180^\circ$ and θ^* is a function of Q^2 .
3. For each bin, calculate the charge normalized yields (i.e. N^+ & N^-) which correspond to the tensor polarization orientations (i.e. P_{zz}^+ & P_{zz}^-)
4. Using the information from steps 2 and 3, extract P_{zz}^+ & P_{zz}^-

Possible procedure to extract the Deuteron tensor polarization at the EIC

$$A^t = \sqrt{2} \frac{(N^+ - N^-)}{(N^- P_{zz}^+ - N^+ P_{zz}^-)}$$
$$= \left(\frac{3 \cos^2 \theta^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta^* \cos \phi^* T_{21} + \sqrt{\frac{3}{2}} \sin^2 \theta^* \cos 2\phi^* T_{22} \right)$$

Other option: Use high precision measurements of T_{20} at a single (or a few) Q^2 values. (Phys. Rev. Lett. **77**, 2630) No parameterization of form factors would have to be assumed here – but we need to make sure θ^* is such that T_{20} dominates the asymmetry. We would also need to take data with $P_{zz} = 0$ if we want both P_{zz}^+ & P_{zz}^- .

Conclusions

- We have conducted simulation studies at the generator level for unpolarized and tensor-polarized elastic electron-deuteron scattering at the EIC.
- We can make measurements of the unpolarized cross section up to $Q^2 \sim 5 \text{ GeV}^2$.
- For the tensor-polarized case, we can potentially use previous measurements at low Q^2 to determine tensor polarization of the deuteron beam. We can also make tensor asymmetry measurements up to $Q^2 \sim 2.5 \text{ GeV}^2$ for new measurements of the tensor polarization observables.
- We are beginning to conduct detector simulations to study acceptance/resolution requirements, as well as study background suppression.
- We have also conducted studies for elastic electron-proton scattering. We find that we can make unpolarized cross section measurements up to $Q^2 \sim 40 \text{ GeV}^2$ at the EIC— these will be the highest values ever measured. See the backup slides for details on this.

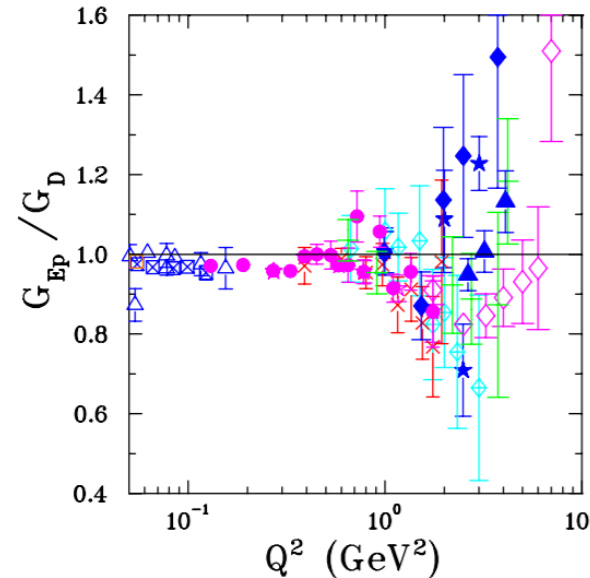
BACKUP:

Elastic Electron-Proton Scattering at the EIC

Elastic Electron-Proton Scattering at the EIC

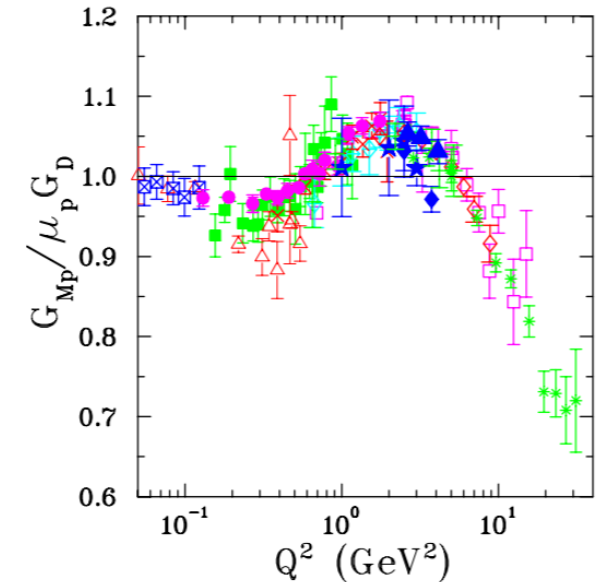
Elastic electron-proton scattering at high Q^2 can be interesting in itself:

- Precision G_M required to study approach of QCD scaling in Dirac F_1 Form Factor
- Constraints on GPDs at high- x & high- t via sum rules
- Possible increased sensitivity to hard two-photon exchange effects



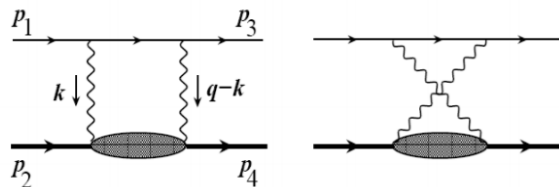
△ Han63
◆ Lit70
● Pri71
× Ber71
◇ Bar73
☆ Han73

⊠ Bor75
□ Sim80
◇ And94
★ Wal94
+ Chr04
▲ Qat05



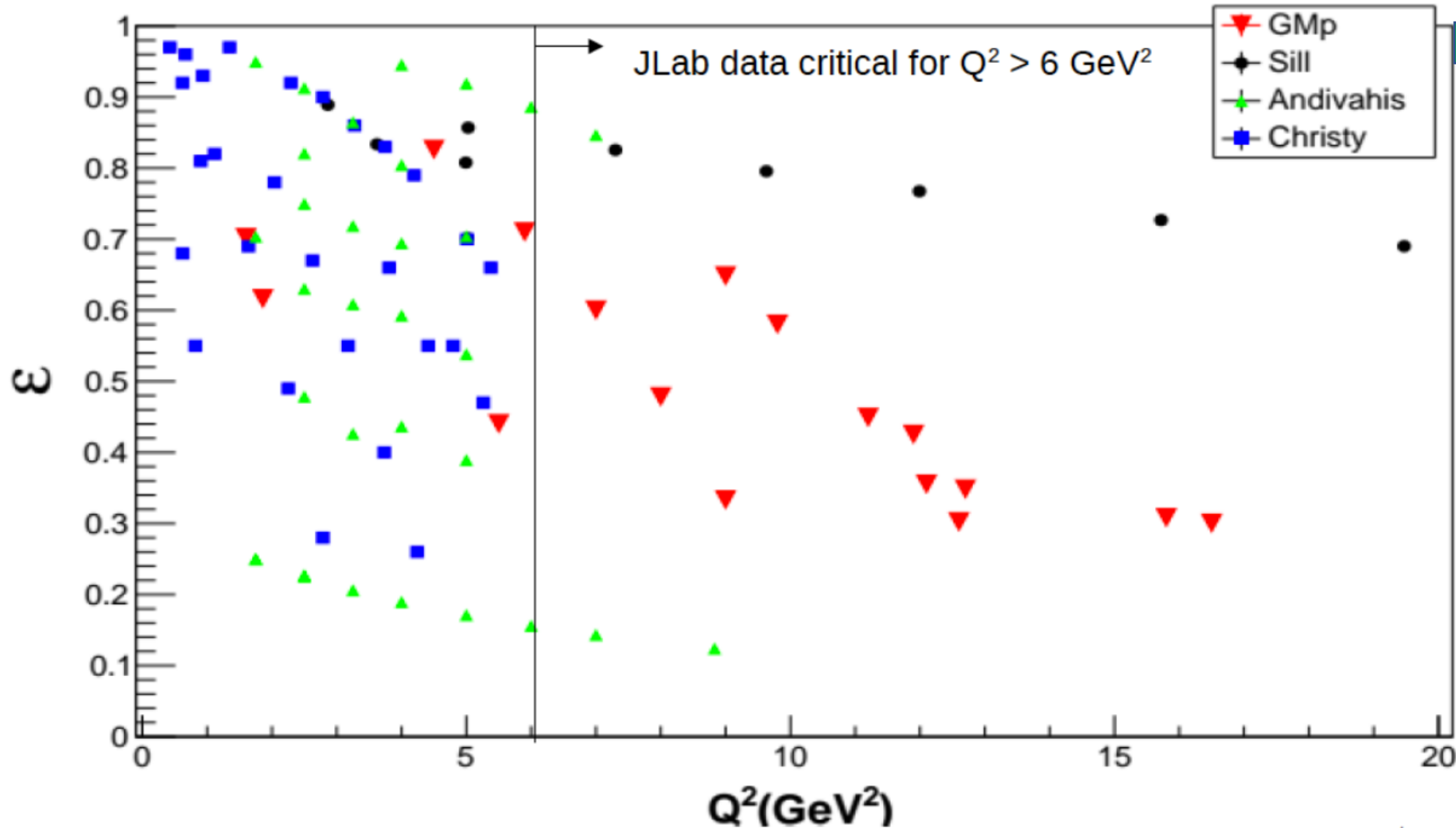
△ Han63
■ Jan66
□ Cow68
◆ Lit70
● Pri71
× Ber71
☆ Han73

◇ Bar73
⊠ Bor75
* Sil93
◇ And94
★ Wal94
+ Chr04
▲ Qat05



C.F Perdrisat, V. Punjabi, M. Vanderhaeghen, *Progress in Particle and Nuclear Physics* 59 (2007) 694–764

For ep elastic scattering, the *EIC* will allow us to probe the highest-ever values of Q^2



Up to $Q^2 \sim 40 \text{ GeV}^2$
or higher at the
EIC – all at high ε

Description of rest-frame Elastic generator with anti-parallel beams

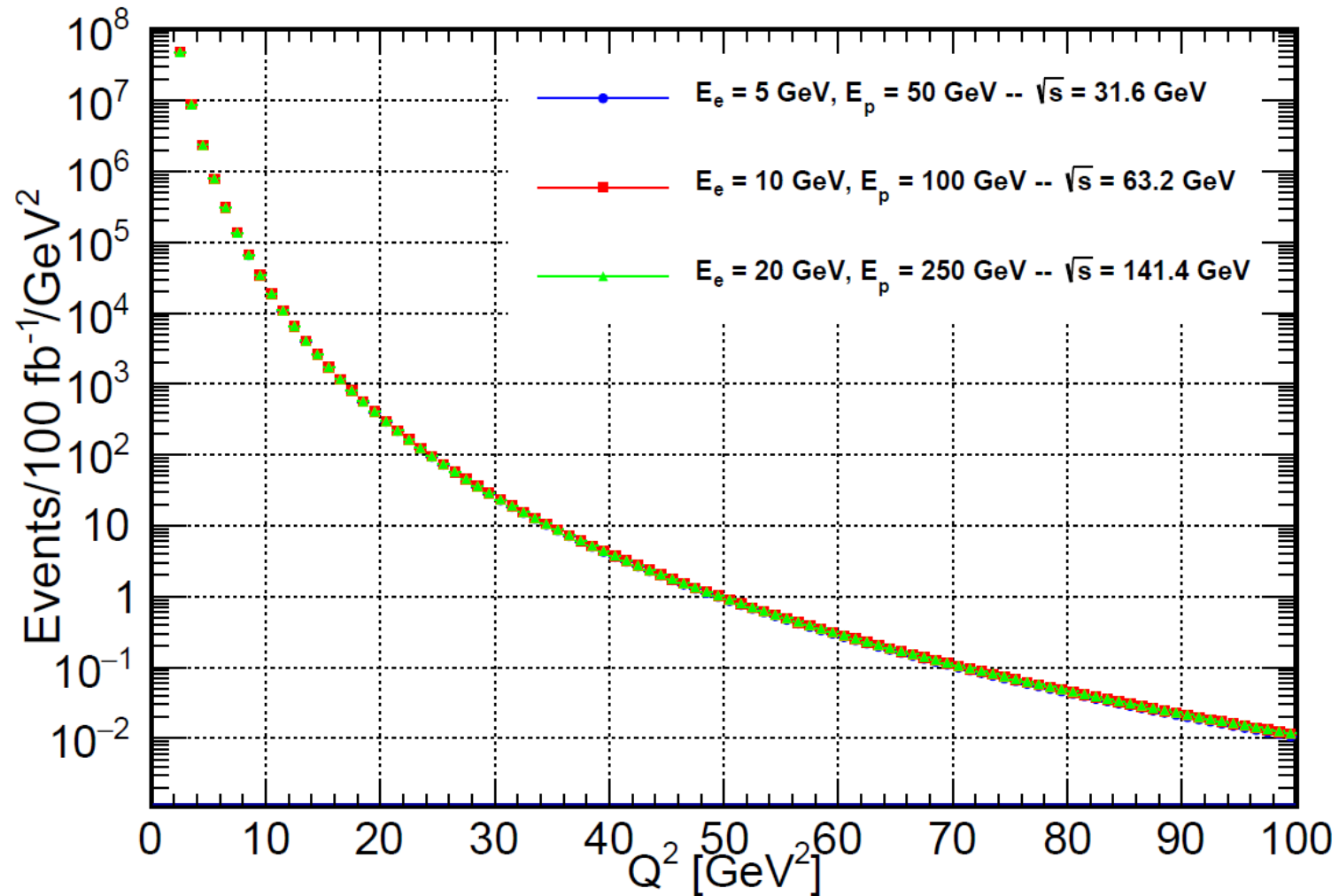
1. Boost from lab frame to proton's rest frame
2. Generate events according to Born cross section below
3. Form factor parameterization comes from *Kelly* (PHYSICAL REVIEW C **70**, 068202 2004, Phys. Rev. C **96**, 055203).
4. We choose to generate 100 fb^{-1} worth of simulation data

$$\frac{d\sigma}{d\Omega_e} = \left(\frac{d\sigma}{d\Omega_e} \right)_{Mott} \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1 + \tau)} \quad \tau \equiv \frac{Q^2}{4M_p^2}$$

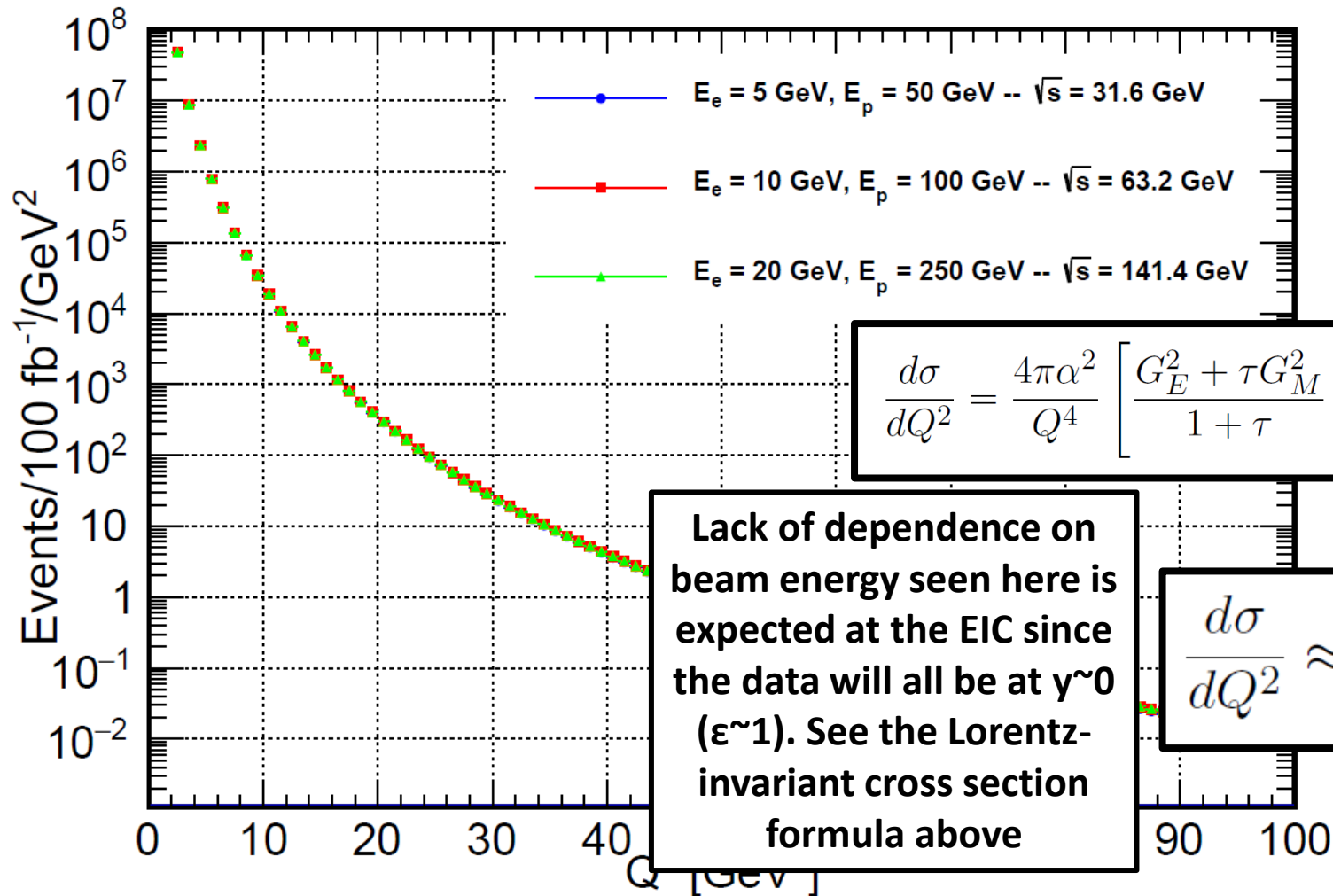
$$\left(\frac{d\sigma}{d\Omega_e} \right)_{Mott} = \frac{\alpha^2 \cos^2 \left(\frac{\theta_e}{2} \right) E'_e}{4E_e^2 \sin^4 \left(\frac{\theta_e}{2} \right) E_e} \quad \epsilon \equiv \left[1 + 2(1 + \tau) \tan^2 \left(\frac{\theta_e}{2} \right) \right]^{-1}$$

$$\sigma_R = \epsilon G_E^2 + \tau G_M^2 \quad \frac{E'_e}{E_e} = \frac{M_p}{M_p + E_e(1 - \cos \theta_e)}$$

Electron-Proton Elastic scattering expected yields



Electron-Proton Elastic scattering expected yields

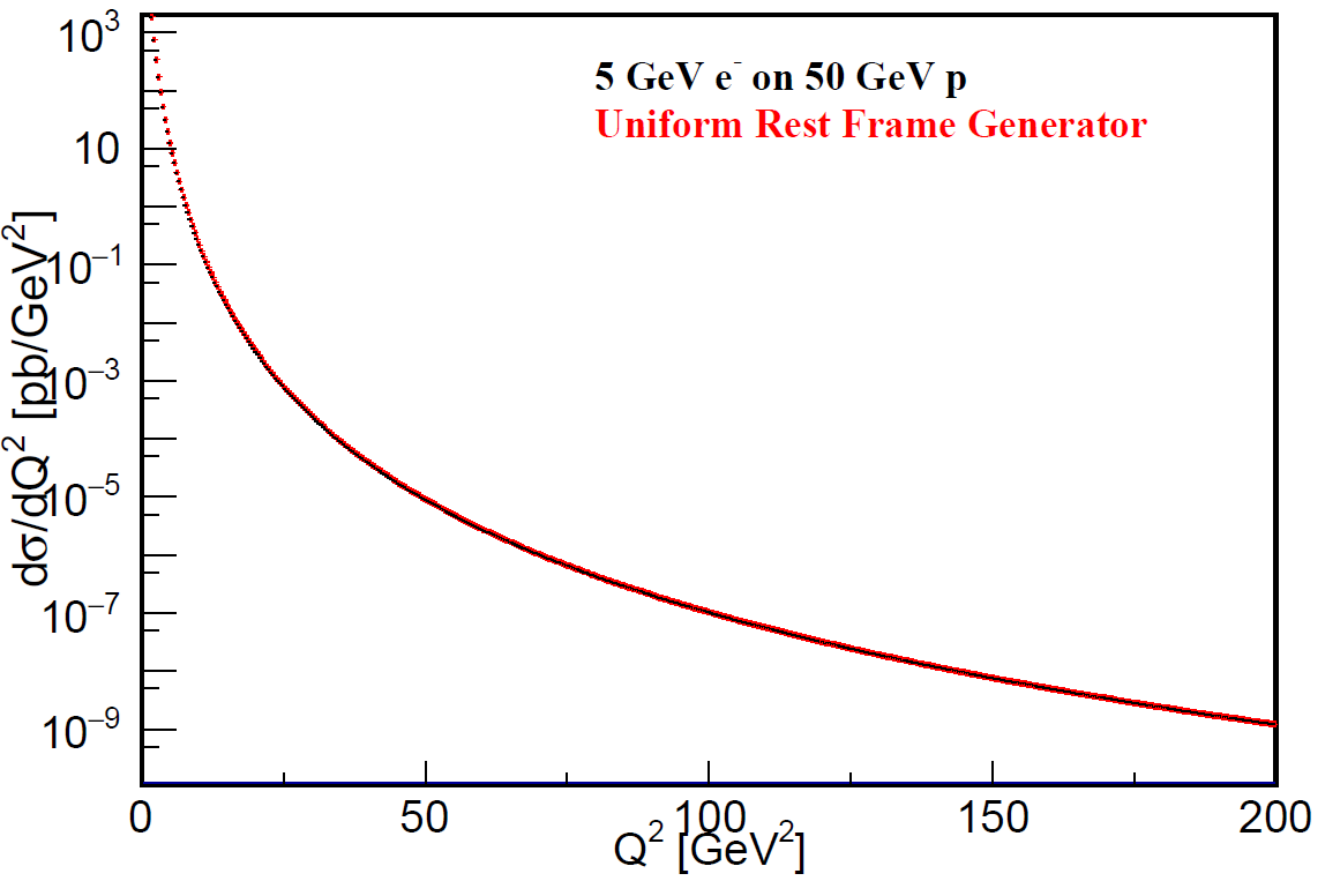


$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \left(1 - y - \frac{M_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

Lack of dependence on beam energy seen here is expected at the EIC since the data will all be at $y \sim 0$ ($\epsilon \sim 1$). See the Lorentz-invariant cross section formula above

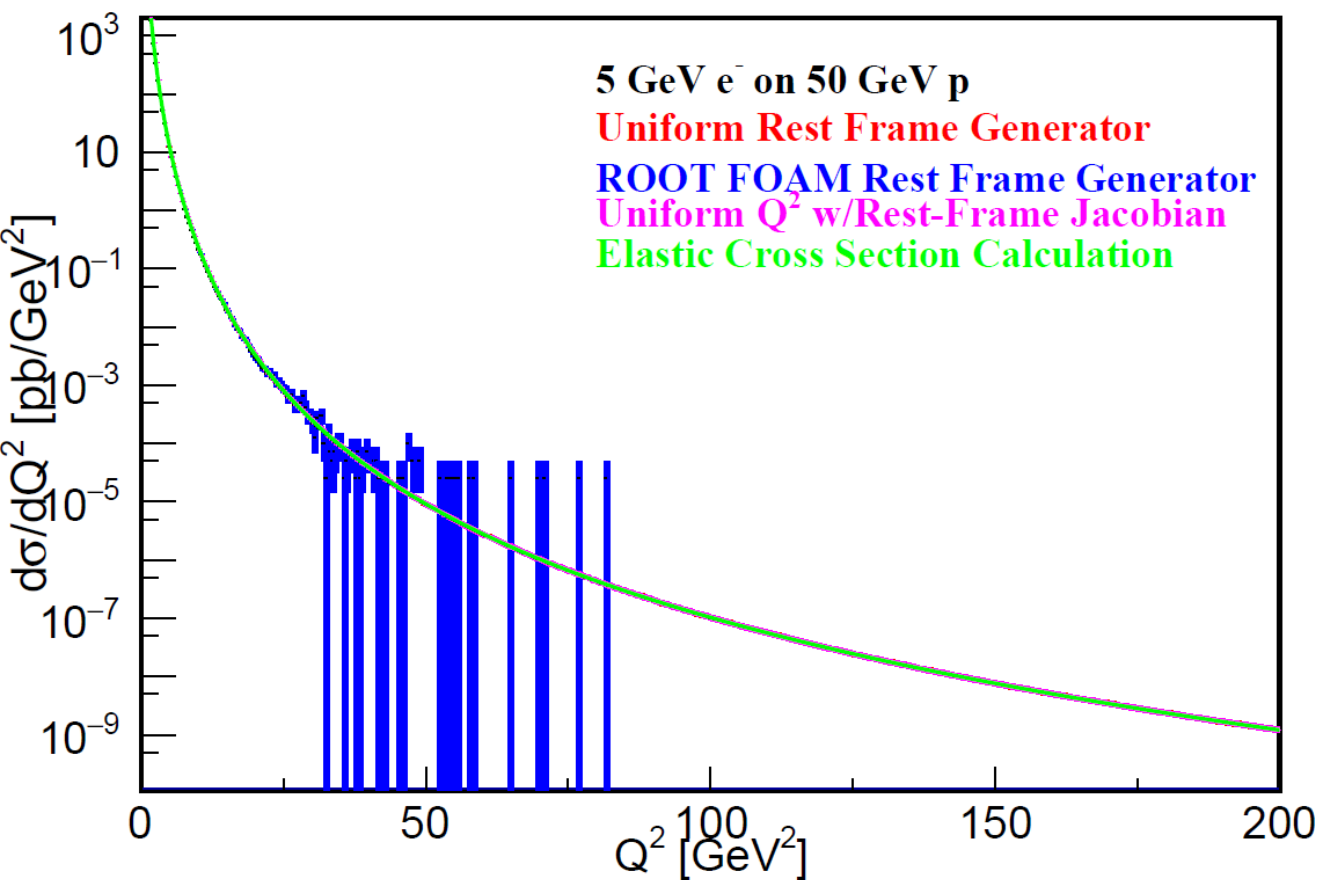
$$\frac{d\sigma}{dQ^2} \approx \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \right]$$

Generator agrees with cross section calculation



$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \left(1 - y - \frac{M_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

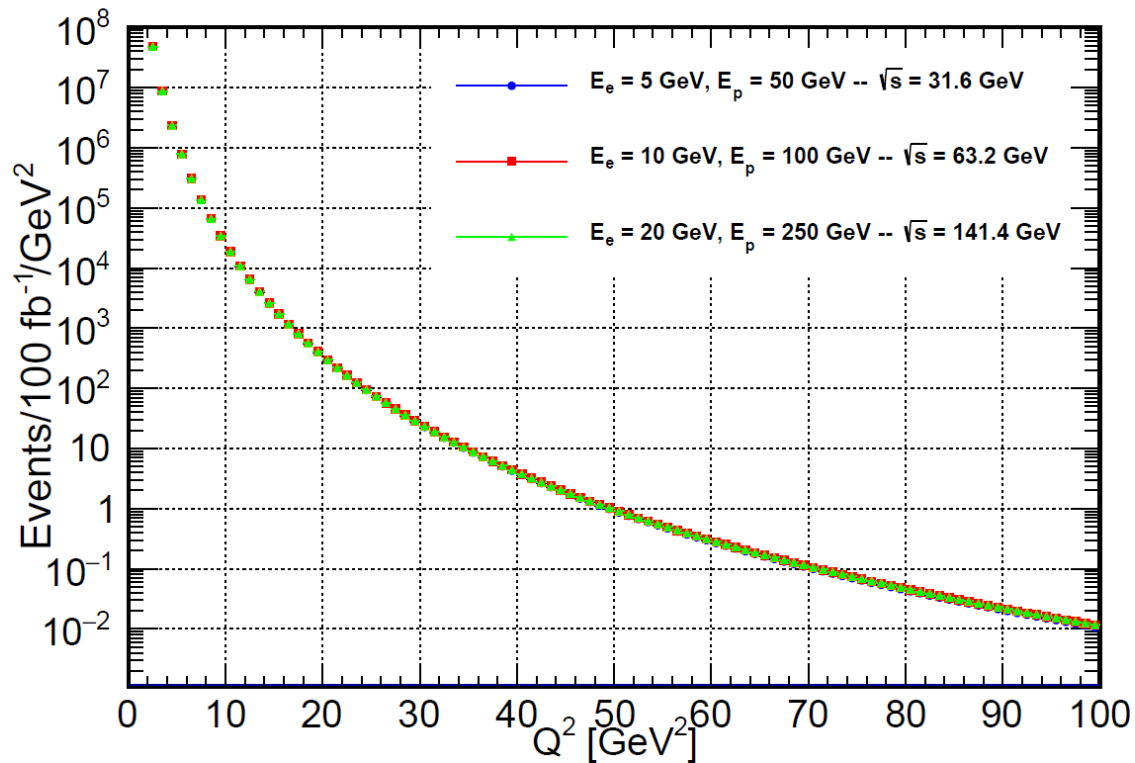
Generator agrees with cross section calculation



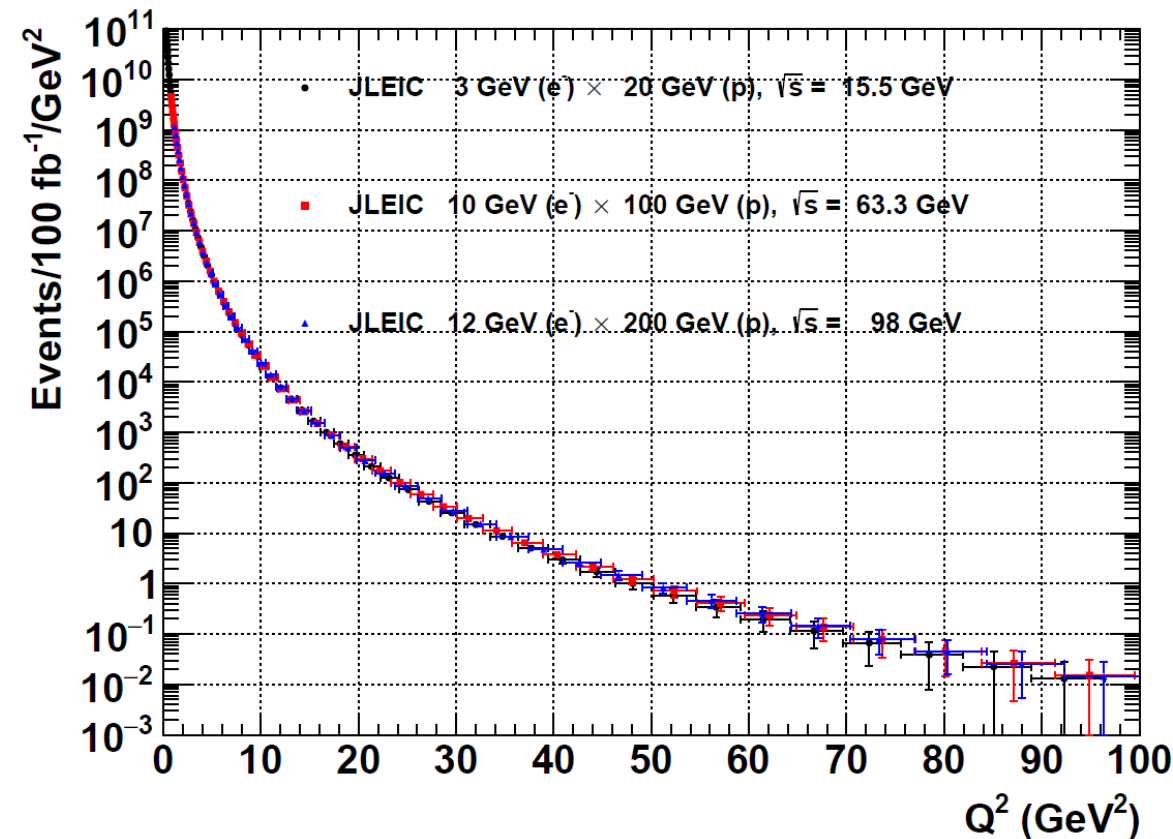
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \left(1 - y - \frac{M_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

Generating in the proton rest frame and the lab frame also gives consistent results

Rest Frame Generator



Lab Frame Generator



Kinematics for Electron-Proton Scattering at the EIC

Ignoring the masses of the electron and proton, in the collider frame we have:

$$Q^2 = xsy \qquad s = 4E_e E_p \qquad \epsilon \approx \frac{1-y}{1-y+y^2/2}$$

$$E'_e = (1-y)E_e + xyE_p$$

$$E'_p = yE_e + x(1-y)E_p$$

$$\cos \theta_e = \frac{xyE_p - (1-y)E_e}{xyE_p + (1-y)E_e}$$

$$\cos \theta_p = \frac{-yE_e + (1-y)xE_p}{yE_e + (1-y)xE_p}$$

Kinematics for Electron-Proton Scattering at the EIC

Ignoring the masses of the electron and proton, in the collider frame we have:

$$Q^2 = \cancel{c}sy$$

$$s = 4E_e E_p$$

$$\epsilon \approx \frac{1-y}{1-y+y^2/2}$$

$$E'_e = (1-y)E_e + \cancel{xy}E_p$$

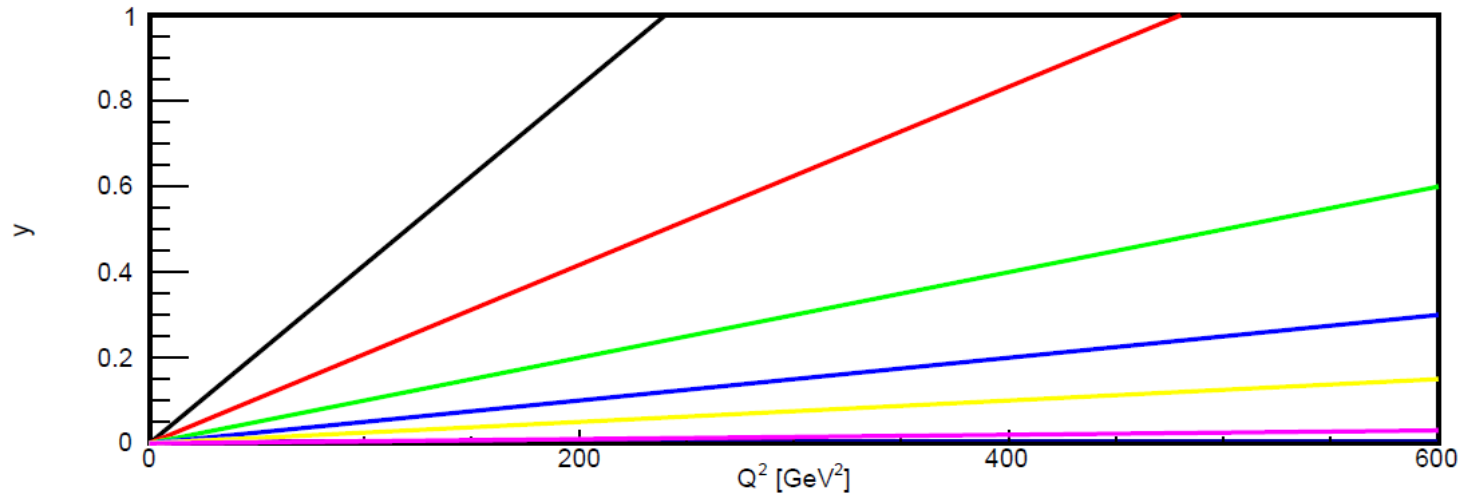
$$E'_p = yE_e + \cancel{x}(1-y)E_p$$

for e-p elastic

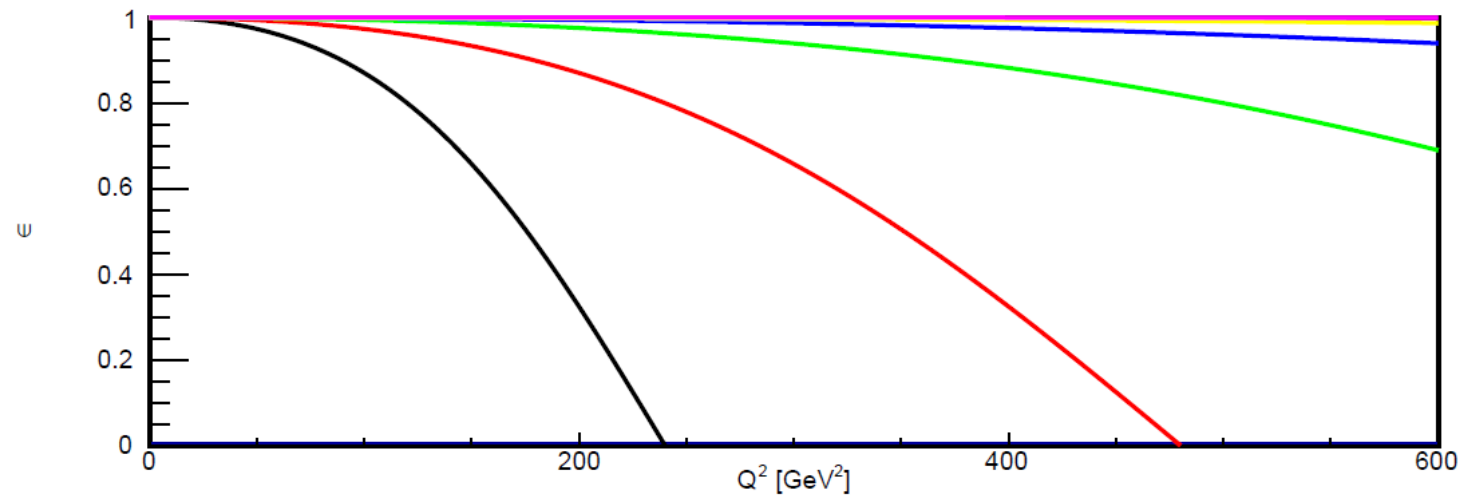
$$\cos \theta_e = \frac{\cancel{xy}E_p - (1-y)E_e}{\cancel{xy}E_p + (1-y)E_e}$$

$$\cos \theta_p = \frac{-yE_e + (1-y)\cancel{x}E_p}{yE_e + (1-y)\cancel{x}E_p}$$

Kinematics – Elastic Data will be at very low y

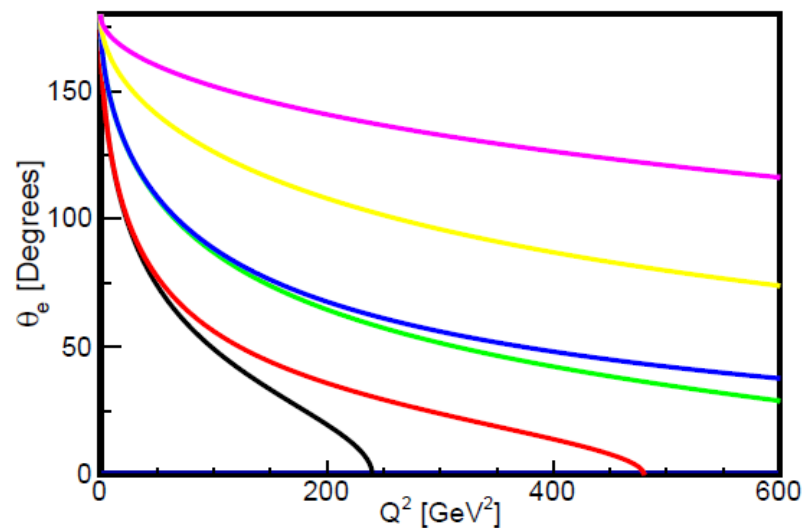
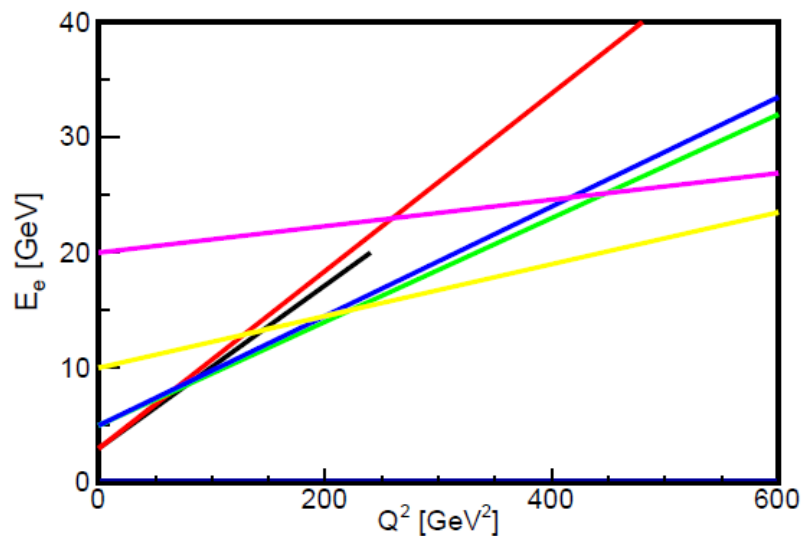


- $E_e = 3 \text{ GeV}, E_p = 20 \text{ GeV} \rightarrow \sqrt{s} = 15.5 \text{ GeV}$
- $E_e = 3 \text{ GeV}, E_p = 40 \text{ GeV} \rightarrow \sqrt{s} = 21.9 \text{ GeV}$
- $E_e = 5 \text{ GeV}, E_p = 50 \text{ GeV} \rightarrow \sqrt{s} = 31.6 \text{ GeV}$
- $E_e = 5 \text{ GeV}, E_p = 100 \text{ GeV} \rightarrow \sqrt{s} = 44.7 \text{ GeV}$
- $E_e = 10 \text{ GeV}, E_p = 100 \text{ GeV} \rightarrow \sqrt{s} = 63.2 \text{ GeV}$
- $E_e = 20 \text{ GeV}, E_p = 250 \text{ GeV} \rightarrow \sqrt{s} = 141.4 \text{ GeV}$

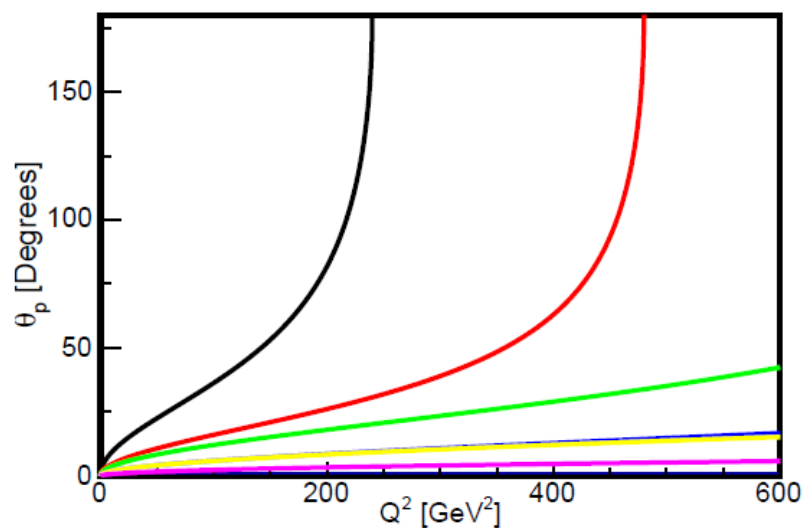
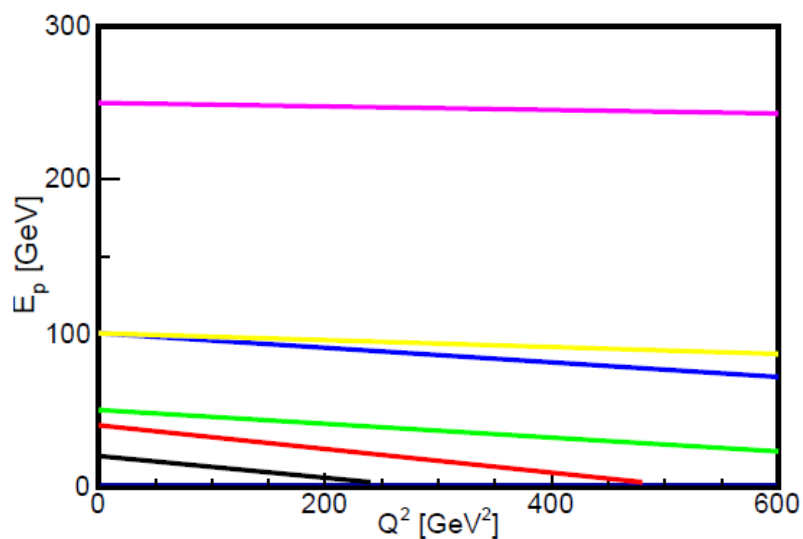


$$\epsilon \approx \frac{1 - y}{1 - y + y^2/2}$$

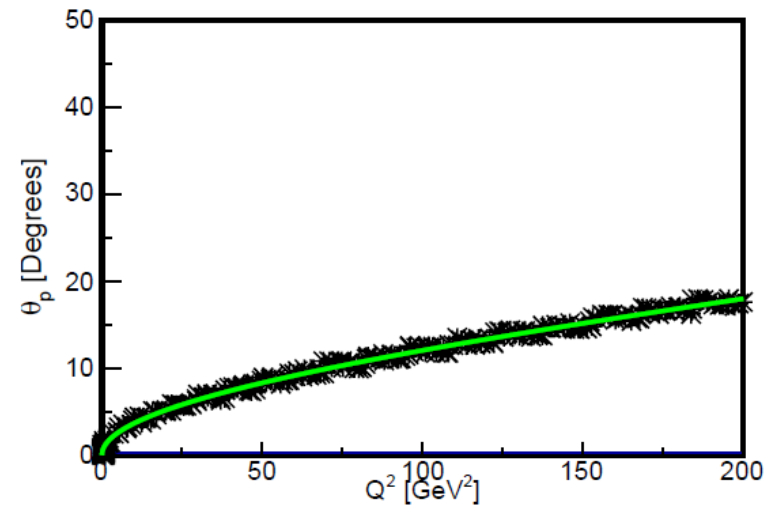
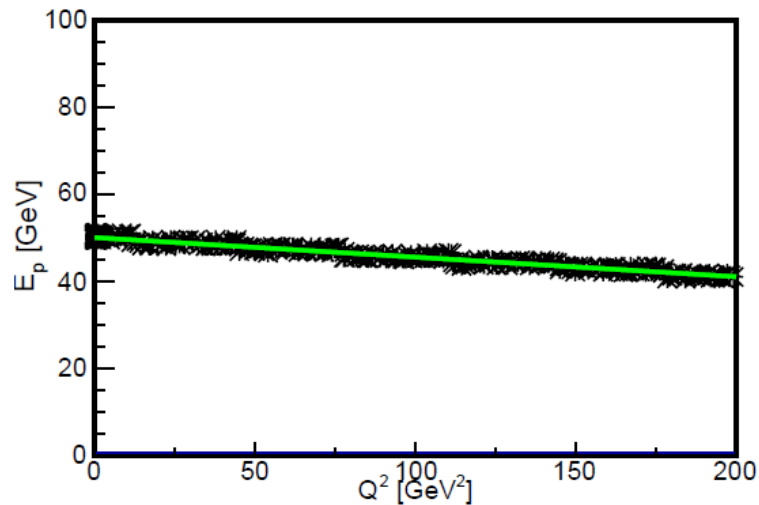
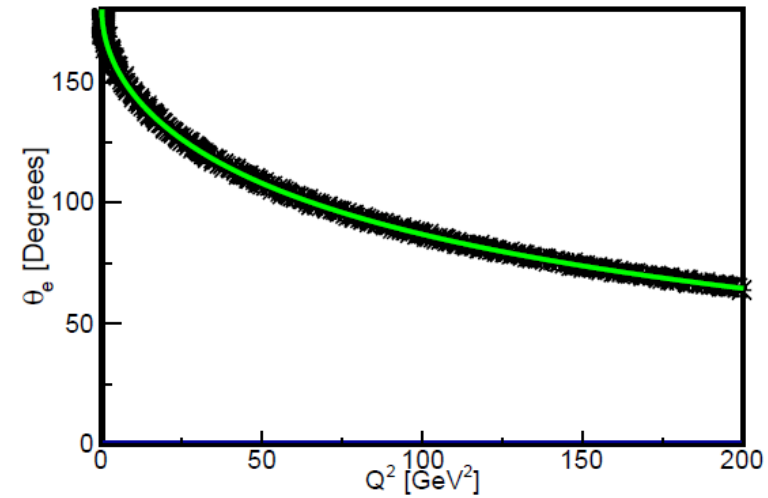
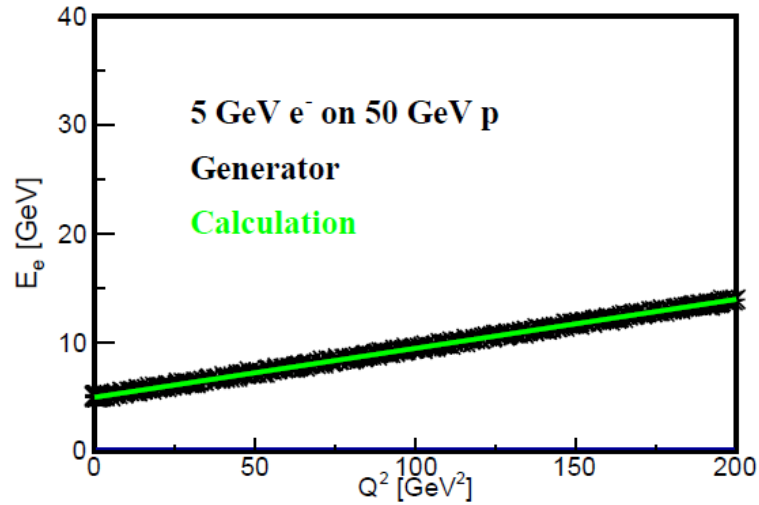
Kinematics – Scattering Angles and Energies



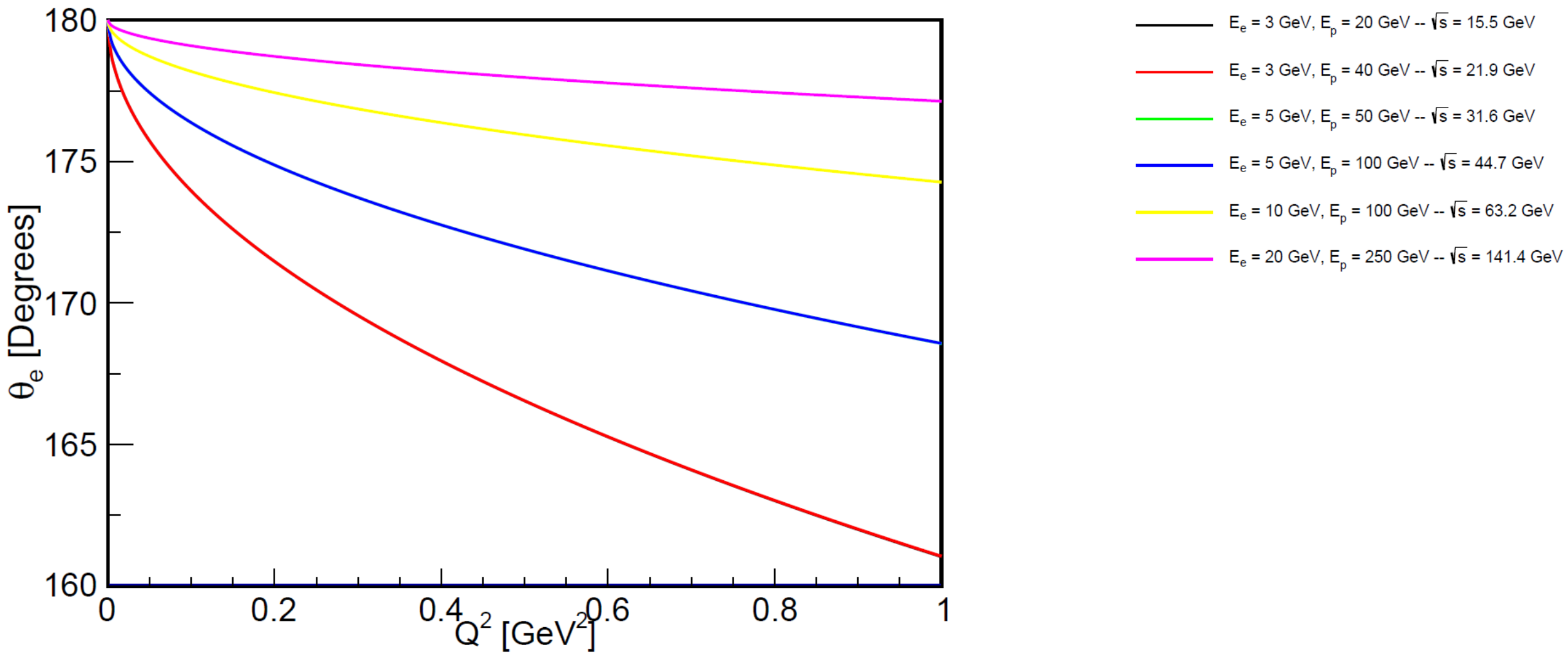
- $E_e = 3$ GeV, $E_p = 20$ GeV -- $\sqrt{s} = 15.5$ GeV
- $E_e = 3$ GeV, $E_p = 40$ GeV -- $\sqrt{s} = 21.9$ GeV
- $E_e = 5$ GeV, $E_p = 50$ GeV -- $\sqrt{s} = 31.6$ GeV
- $E_e = 5$ GeV, $E_p = 100$ GeV -- $\sqrt{s} = 44.7$ GeV
- $E_e = 10$ GeV, $E_p = 100$ GeV -- $\sqrt{s} = 63.2$ GeV
- $E_e = 20$ GeV, $E_p = 250$ GeV -- $\sqrt{s} = 141.4$ GeV



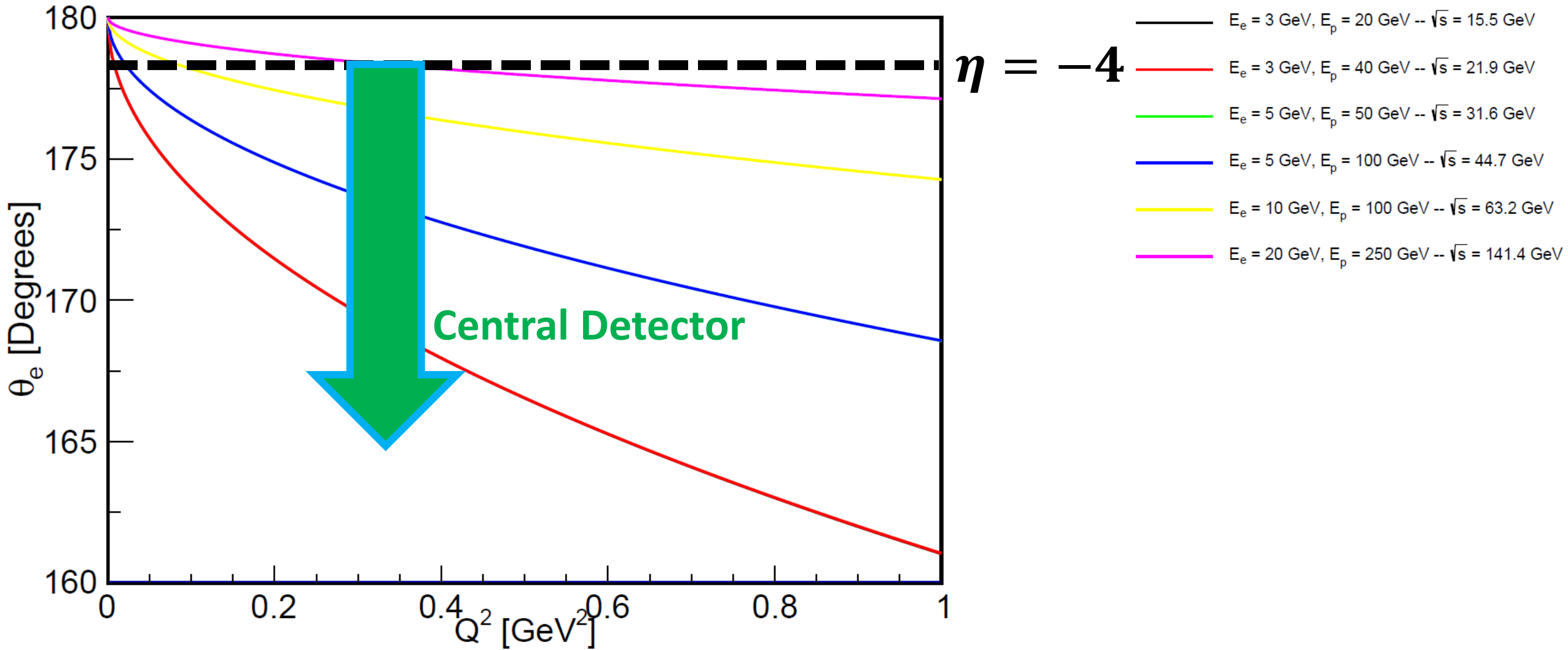
Generator results agree with Kinematics calculations



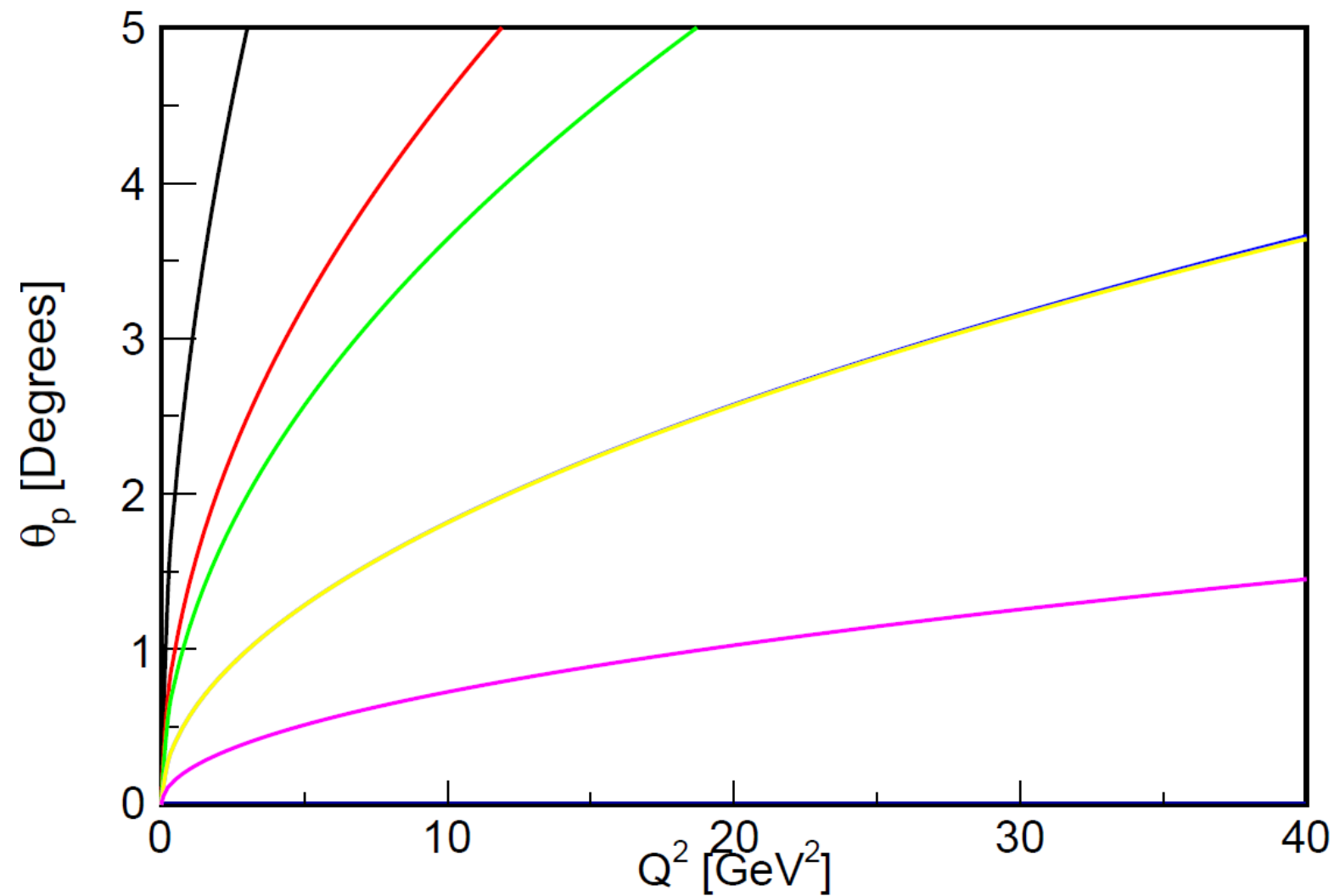
Kinematics: Low Q^2 Electron Angle



Kinematics: Low Q^2 Electron Angle

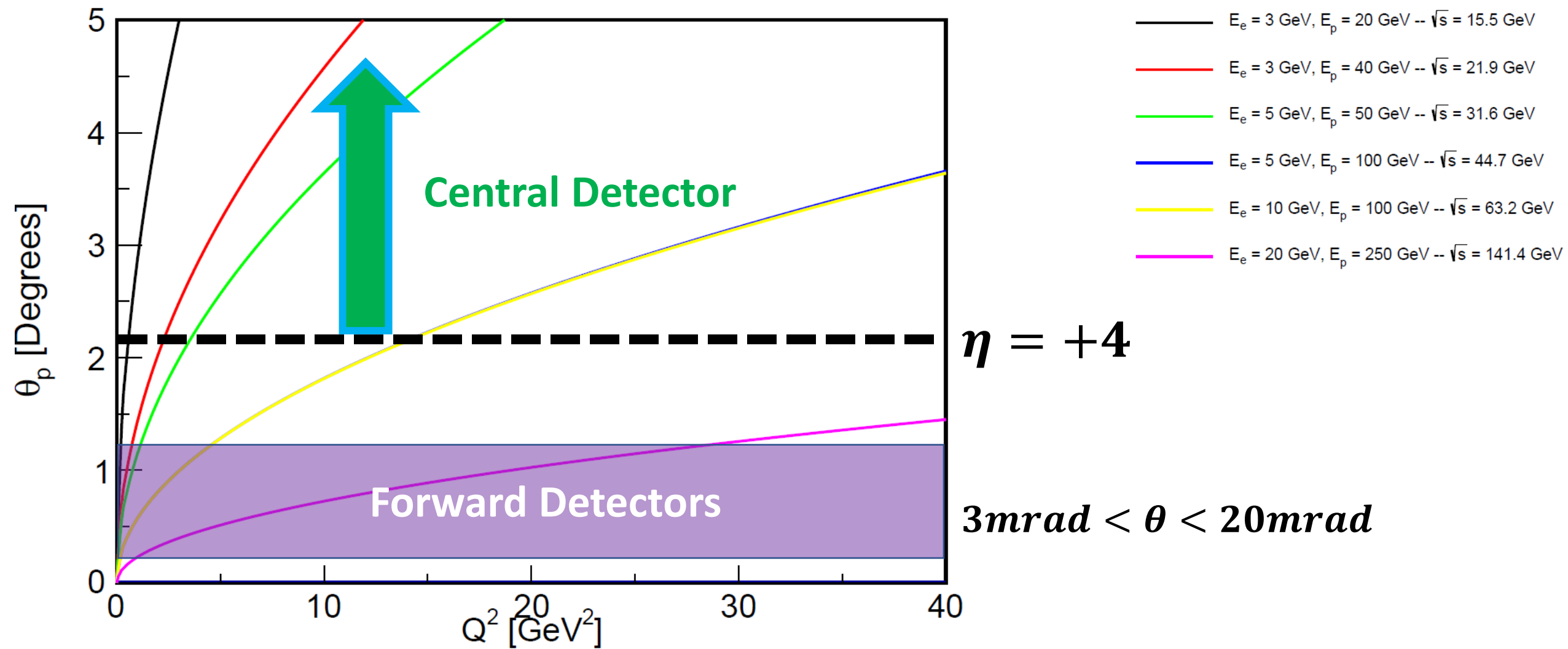


Kinematics: Proton Angle



- $E_e = 3$ GeV, $E_p = 20$ GeV -- $\sqrt{s} = 15.5$ GeV
- $E_e = 3$ GeV, $E_p = 40$ GeV -- $\sqrt{s} = 21.9$ GeV
- $E_e = 5$ GeV, $E_p = 50$ GeV -- $\sqrt{s} = 31.6$ GeV
- $E_e = 5$ GeV, $E_p = 100$ GeV -- $\sqrt{s} = 44.7$ GeV
- $E_e = 10$ GeV, $E_p = 100$ GeV -- $\sqrt{s} = 63.2$ GeV
- $E_e = 20$ GeV, $E_p = 250$ GeV -- $\sqrt{s} = 141.4$ GeV

Kinematics: Proton Angle



Polarized Electron-Proton Elastic Scattering Asymmetry Measurements

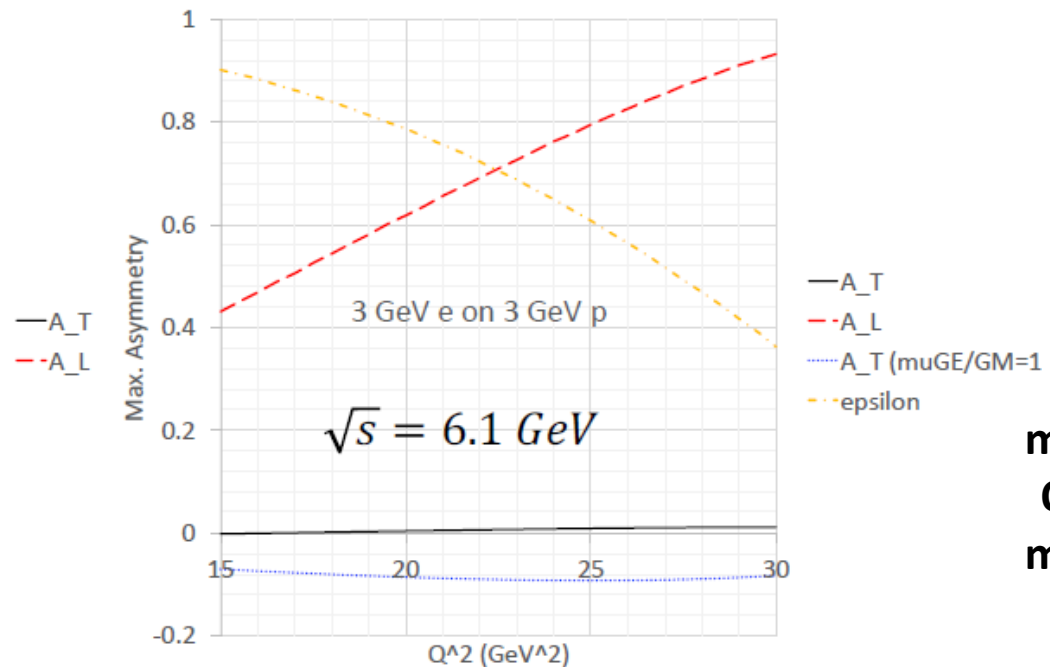
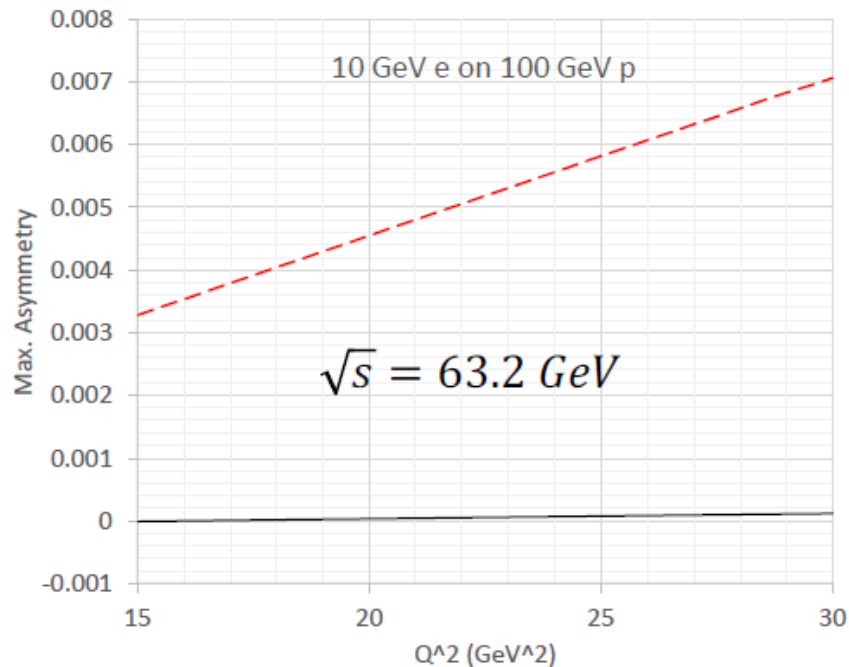
$$\begin{aligned}
 A_{eN} &= -\frac{P_{beam}P_{target}}{1 + \frac{\epsilon}{\tau}r^2} \left[\left(\sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}} \sin \theta^* \cos \phi^* \right) r + \sqrt{1-\epsilon^2} \cos \theta^* \right] \\
 &\equiv P_{target} [A_t \sin \theta^* \cos \phi^* + A_\ell \cos \theta^*]
 \end{aligned}
 \quad \Bigg| \quad r \equiv \frac{G_E}{G_M}$$

Polarized Electron-Proton Elastic Scattering Asymmetry Measurements

$$A_{eN} = -\frac{P_{beam}P_{target}}{1 + \frac{\epsilon}{\tau}r^2} \left[\left(\sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}} \sin \theta^* \cos \phi^* \right) r + \sqrt{1-\epsilon^2} \cos \theta^* \right]$$

$$\equiv P_{target} [A_t \sin \theta^* \cos \phi^* + A_\ell \cos \theta^*]$$

$r \equiv \frac{G_E}{G_M}$



To make reasonable measurements in this higher Q² range, we would need a much lower energy than will be provided by the EIC