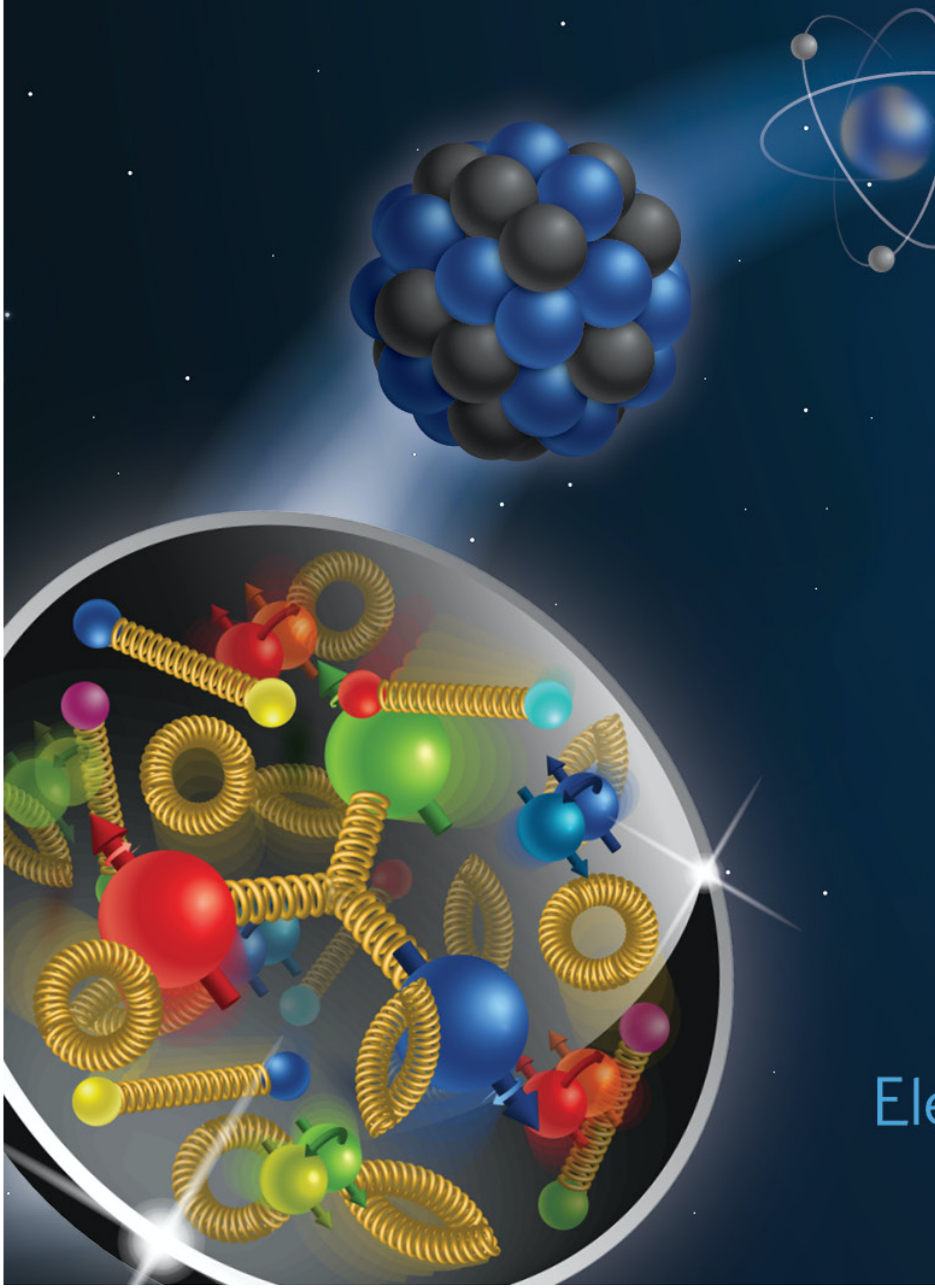

Spin rotator design and spin matching

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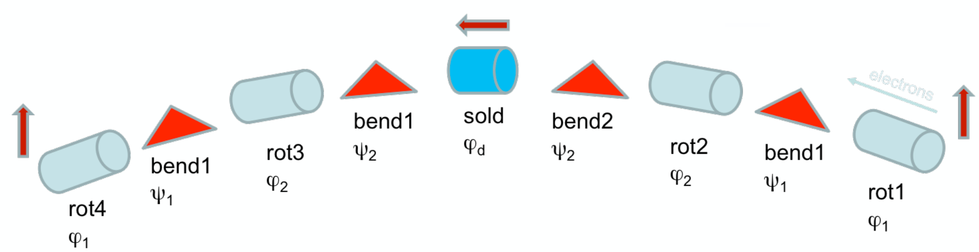
EIC electron spin rotators

- Electron energy range: 5-18 GeV
- A HERA-type rotator (based on sequence of vertical and horizontal bend) creates meter scale orbit excursion at lower energies.
- The rotator design capable to operate in all energy range is based on the combination of solenoidal and horizontal bending magnets.

eRHIC spin rotator

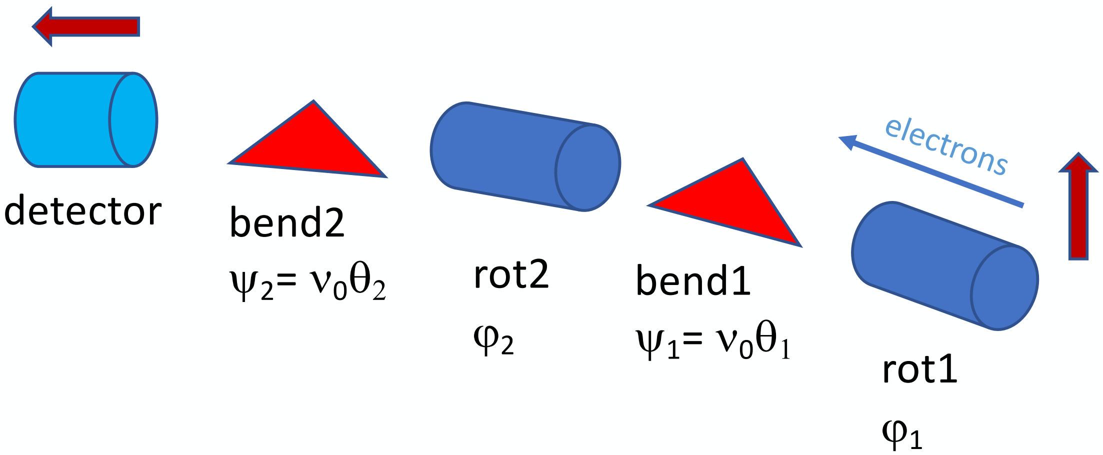
C-type bending configuration



φ_j – spin rotation angle in solenoids

ψ_i – spin rotation angle in bends

Spin rotator: spin rotation angle definitions



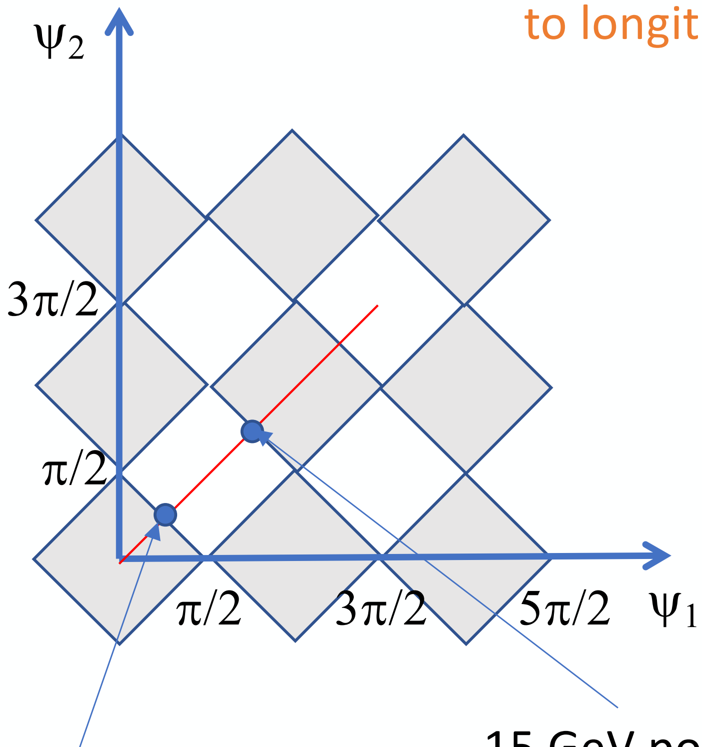
$\nu_0 = \gamma a$
$\theta_{1,2}$ – bending angles

Relations between solenoidal rotation angles (φ) and horizontal bend spin rotation angles (ψ) to convert vertical spin to longitudinal:

$$\tan\varphi_1 = \pm\frac{\cos\psi_2}{\sqrt{-\cos(\psi_1+\psi_2)\cos(\psi_1-\psi_2)}}$$

$$\cos\varphi_2 = \cot\psi_1 \ \cot\psi_2$$

Selecting rotator parameters



15 GeV point

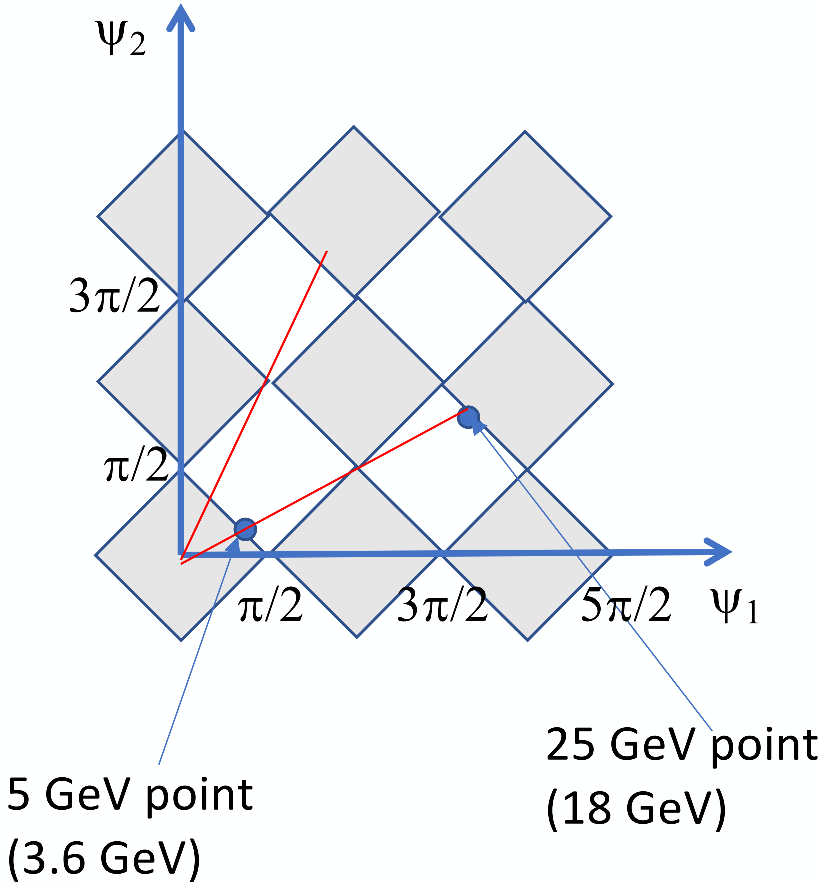
5 GeV point
(required $\theta_2 + \theta_1$ = 138.4 mrad)

In gray areas of (ψ_1, ψ_2) plane the spin transformation from vertical to longitudinal can not be realized

- Spin rotation angles (ψ_1, ψ_2) are related with bend angle values:
  $\psi_1 = \gamma a\, \theta_1$, $\psi_2 = \gamma a\, \theta_2$
- Thus, any straight line passing through the origin point corresponds to varying electron energy at fixed dipole bending angles.
- For instance, the red line on the plot corresponds to varying energy at fixed and equal θ_1 and θ_2

With this rotator design example the longitudinal polarization at 18 GeV energy **can not be realized**, since energy range is limited (5-15 GeV)

Achieving largest continuous energy range



- Red lines $\theta_1=2\theta_2$ and $\theta_2=2\theta_1$ span over longest continuous good parameter intervals
- Thus, these lines (and bend angle relations) provides largest possible continuous energy range:
  Maximum-to-minimum energy ratio = 5
- $\boldsymbol{\theta_1=2\theta_2}$ line leads to smaller values of required solenoid fields, than $\theta_2=2\theta_1$
  Thus this is a preferable choice for EIC rotator

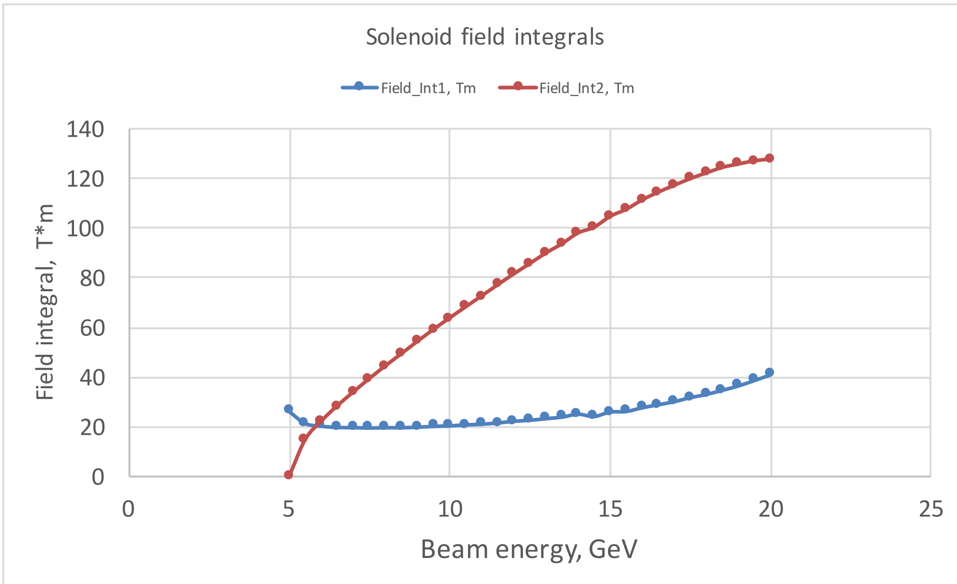
Two examples of continuous ranges covering all EIC energies (using $\boldsymbol{\theta_1=2\theta_2}$ line)

5-25 GeV: θ_1= 92.27 mrad
θ_2= 46.14 mrad

3.6-18 GeV: θ_1= 128.15 mrad
θ_2= 64.08 mrad

Spin rotator: required solenoid fields

Provides longitudinal polarization in IP throughout the whole energy range.



Calls for SC solenoids with up to 7T field

	18 GeV	5 GeV	Magnet length
1st rotator solenoid field integral , T*m	33.2	26.1	5.4 m
2nd rotator solenoid field integral , T*m	121.9	0	18 m
1st bend angle, mrad	46.1		
2nd bend angle, mrad	92.3		

Polarization evolution in electron ring

Synchrotron radiation determines the polarization evolution through Sokolov-Ternov spin-flip emission and spin diffusion caused by quantum emission of S photons. Both processes combined define the equilibrium polarization P_{eq} and polarization relaxation time t.

$$P(t) = (P_0 - P_{eq})\, e^{-t/\tau} + P_{eq}$$

Derbenev-Kondratenko: (1973)

$$P_{eq} = -\frac{8}{5\sqrt{3}}\frac{\alpha_-}{\alpha_+}$$

$$\tau^{-1} = \frac{5\sqrt{3}}{8}\frac{\hbar\, r_0}{m}\gamma^5\,\alpha_+$$

$$\alpha_- = \left\langle \oint \frac{d\theta}{|\rho|^3}\,\hat{\boldsymbol{b}}(\boldsymbol{n}-\boldsymbol{d}) \right\rangle$$

$$\alpha_+ = \left\langle \oint \frac{d\theta}{|\rho|^3}\left[1 - \frac{2}{9}(\boldsymbol{n}\hat{\boldsymbol{v}})^2 + \frac{11}{18}|\boldsymbol{d}|^2\right] \right\rangle$$

Depolarization caused by spin diffusion is defined by a derivative of invariant spin field over $\delta = \frac{\Delta E}{E} = \frac{\Delta\gamma}{\gamma}$:

$$\boldsymbol{d} = \gamma\frac{\partial \mathbf{n}}{\partial \gamma}$$ (taken at const x, x', y, y')

$$\left(\frac{\partial \mathbf{n}}{\partial \delta}\right)_{A_x, A_y} \qquad \frac{\partial \mathbf{n}}{\partial A_x}$$

First-order spin perturbation consideration

The magnetic fields on design orbit define the periodical spin solution $\mathbf{n_0}$ and two others spin eigenvectors $\mathbf{k_0} = \mathbf{l_0} + i\mathbf{m_0}$, $\mathbf{k_0}^*$.

$\mathbf{l_0}, \mathbf{m_0}, \mathbf{n_0}$ form normalized triad convenient for considering spin motion perturbations.

In the first order spin perturbation α_0 by momentum deviation or betatron motion is described by the following equation:

$$\frac{d\alpha_0}{ds} = -i\mathbf{w}\cdot\mathbf{k}_0$$

where components perturbation precession vector $\mathbf{w}$ (neglecting terms of order of anomalous magnetic moment a) are:

$$w_x = \left(1+\gamma_0 a\right)y'' + K_s x'; \quad w_s = K_y' y - K_s\frac{\Delta E}{E_0} - \gamma_0 a K_y y'; \quad w_y = -\left(1+\gamma_0 a\right)x'' + \gamma_0 a K_y \frac{\Delta E}{E_0} + K_s y'$$

$$K_y = \frac{B_y}{B\rho} \quad ; \qquad K_s = \frac{Bs}{B\rho}$$

With proper periodical conditions the solution of this equation gives the invariant spin field in first order.

Exploring spin matching conditions

The goal: eliminate (minimize) dependence of the spin invariant field (α_0) on horizontal betatron amplitude A_x and energy deviation δ outside the rotator system.

$$\longrightarrow \partial \mathrm{n}/\partial\delta = 0 \quad \partial \mathrm{n}/\partial A_x = 0$$

Thus avoiding any spin dynamics distortion by synchrotron radiation in the arc bends.

The following integral over the whole spin rotator system must be made 0 for terms proportional to A_x and δ:

$$\int_{s_{in}}^{s_{out}} \left(w_x k_{0x} + w_s k_{0s} + w_y k_{0y} \right) ds = 0$$

The orbital motion is considered in a standard form through components of betatron motion eigen-vectors f_I *and* f_{II} and dispersion functions D_x, D_y:

$$x = f_{Ix} A_x + f_{Ix}^* A_x^* + f_{IIx} A_y + f_{IIx}^* A_y^* + D_x \delta$$

$$y = f_{Iy} A_x + f_{Ix}^* A_x^* + f_{IIy} A_y + f_{IIy}^* A_y^* + D_y \delta$$

Spin matching conditions for solenoidal rotators

We assumed following reasonable optics conditions:

-betatron coupling is fully compensated individually for each of four solenoidal insertions by **dividing each solenoid in two halves** and using set of quadrupoles/skew quadrupoles between and around them

-the vertical dispersion function D_y does not leak into the horizontal bends

Then, using integration by parts one gets following set of spin matching conditions (neglecting terms of order a):

$$\sum_{rot:j=1,4} H_j\left(f_I\right) = 0\,; \quad \sum_{rot:j=1,4} H_j\left(f_I^*\right) = 0\,;$$

Betatron motion conditions

$$a\gamma \sum_{rot:j=1,4} H_j\left(D\right) + \sum_{rot:j=1,4} \varphi_j k_{sj} - \sum_{bends:i=1,4} \psi_j k_{yi} = 0$$

Direct energy effect condition

where:

$$H_j\left(F\right) = \frac{\varphi_j}{2}\left[\left(k_x\left(F_x' + \frac{K_s}{2}F_y\right) + k_y\left(F_y' - \frac{K_s}{2}F_x\right)\right)_{j,entrance} + \left(k_x\left(F_x' + \frac{K_s}{2}F_y\right) + k_y\left(F_y' - \frac{K_s}{2}F_x\right)\right)_{j,exit}\right]$$

F is either f_I or D

entrance of first part of solenoid

exit of second half of solenoid

Direct energy effect spin matching

Contribution from solenoids

Contribution from IR an rotator dipoles

$$a\gamma \sum_{rot:j=1,4} H_j\left(D\right) + \sum_{rot:j=1,4} \varphi_j k_{sj} - \sum_{bends:i=1,4} \psi_j k_{yi} = 0$$

This term can be nullified either by not allowing dispersion function inside solenoids or by proper optics

This combination is completely defined by the choice of bending angles of the dipoles and solenoidal fields.

For S-type bending configuration around IP and spin-up to spin-up transformation through the whole rotator system: automatically zero .

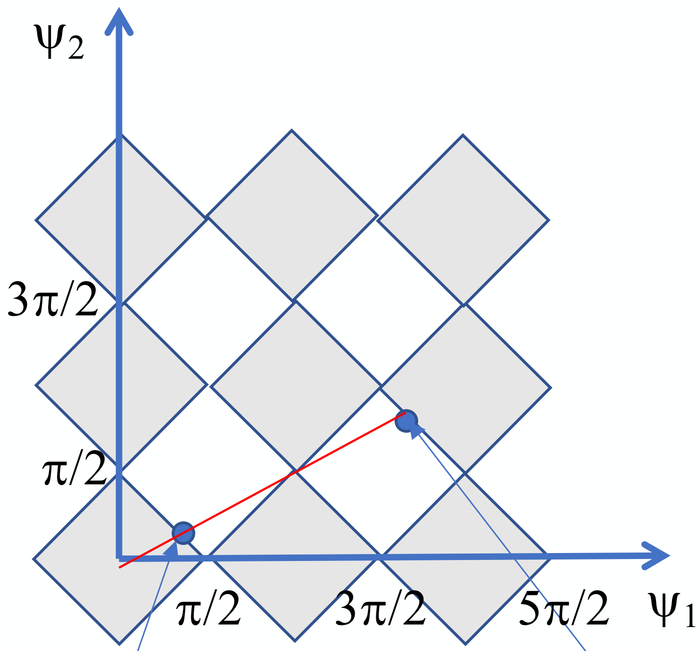
For C-type bending configuration around IR, used in EIC, can be nullified at **a particular energy** with following choice of rotator parameters:

$\varphi_1 = \varphi_4 = 0.524$ rad, $\varphi_2 = \varphi_3 = 2.094$ rad

$\psi_1 = \psi_4 = \pi$ rad, $\psi_2 = \psi_3 = \pi/2$ rad

It makes sense to consider fully longitudinal matching the rotators at EIC highest energy, **18 GeV**

The rotator solution corresponding to full direct energy spin match at 18 GeV



30 GeV point

6 GeV point
(required $\theta_2 + \theta_1$ = 115.34 mrad)

Solenoid strength:
φ_1 = 0.524 rad, φ_2 = 2.094 rad

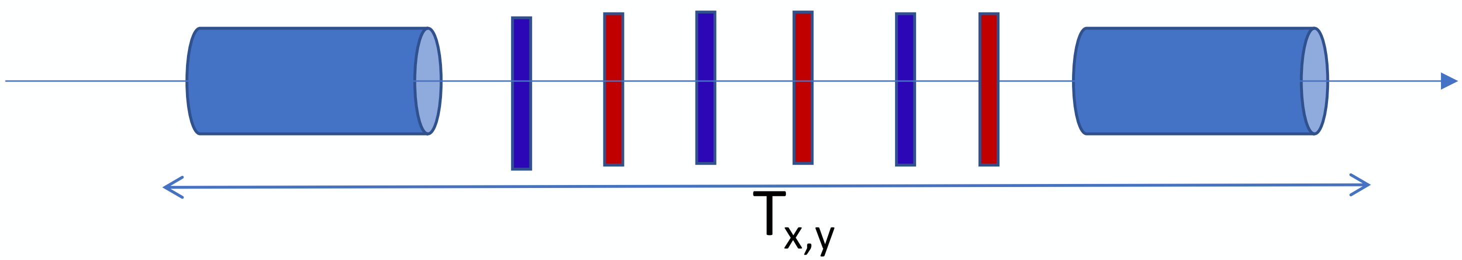
Dipole bend angles:
θ_1= 76.89 mrad, θ_2= 38.45 mrad

But with this rotator design the longitudinal polarization only realized down to 6 GeV energy.

Solenoidal insertion with betatron spin matching

Spin matching conditions related with betatron motion can be satisfied for each individual solenoidal insertion, using two solenoid halves and (at least 6) quadrupoles between them.

That is for each j : $H_j(f_I) = 0 \quad \text{and} \quad H_j(f_I^*) = 0$



For a betatron spin-matched and fully decoupled solenoidal insertion the horizontal and vertical transport matrices must have following forms:

$$\mathrm{T_X} = \begin{pmatrix} -\cos(\varphi) & -\frac{2}{K_s}\sin(\varphi) \\ \frac{K_s}{2}\sin(\varphi) & -\cos(\varphi) \end{pmatrix}; \quad \mathrm{T_Y} = -\mathrm{T_X} = \begin{pmatrix} \cos(\varphi) & \frac{2}{K_s}\sin(\varphi) \\ -\frac{K_s}{2}\sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

$$K_s = \frac{B_s}{B\rho} \qquad \varphi = (1+a)K_s L$$

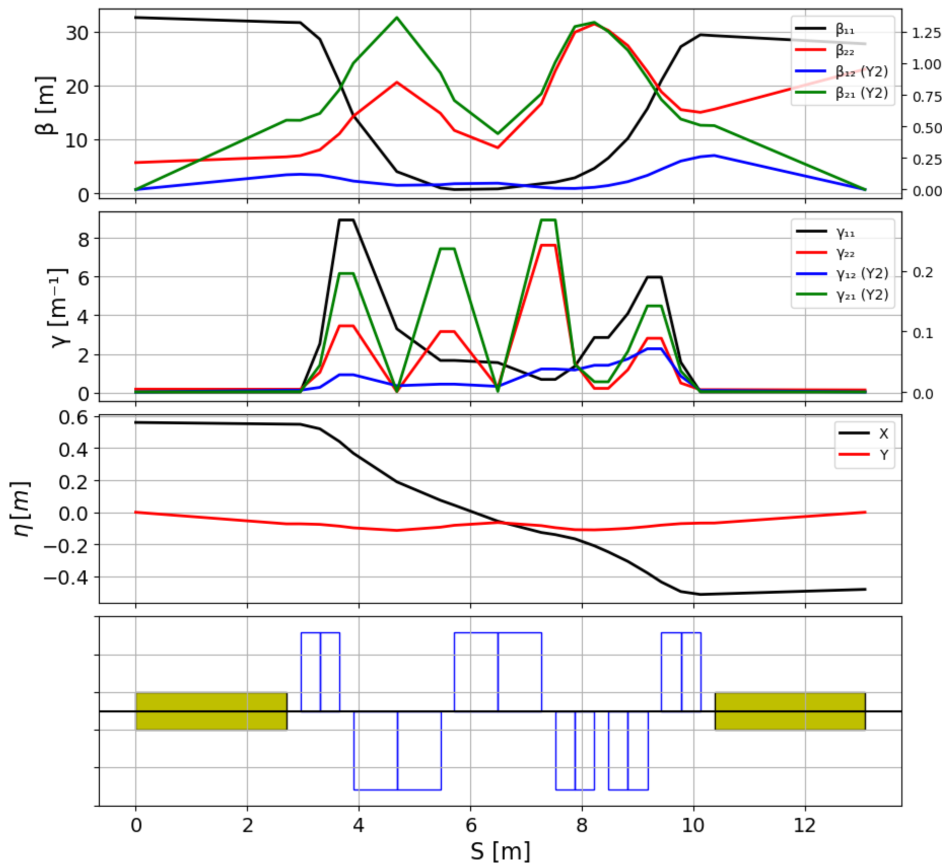
Realization of short solenoid insertion

V.5.3 lattice

Quad	Length [m]	Gradient [T/m]
QB15	0.70	48.3334
QB16	1.56	-32.3936
QB17	1.56	53.8891
QB18	0.70	-50.9092
QB19	0.70	-20.4009
QB20	0.70	41.6940

$\mu_x = 0.5556672$ $\mu_y = 0.1790836$

$Length = 13.07\ m$



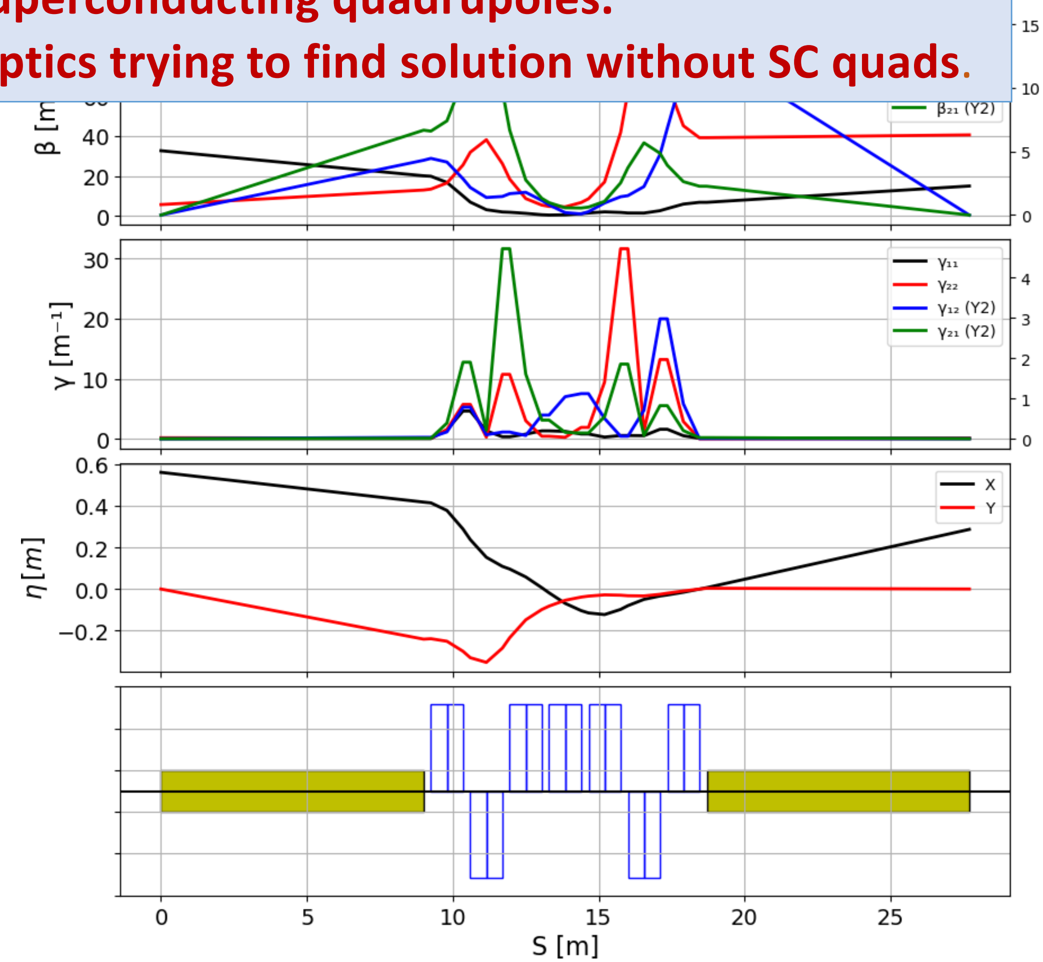
Realization of long solenoid insertion

However, present solution requires superconducting quadrupoles.
Thus, we are continuing to work on optics trying to find solution without SC quads.

Quad	Length [m]	Gradient $[T/m]$
QA15	1.1	27.9878
QA16	1.1	-52.3538
QA17	1.1	47.5649
QA18	1.1	45.4466
QA19	1.1	53.9845
QA20	1.1	-54.1593
QA21	1.1	28.7995

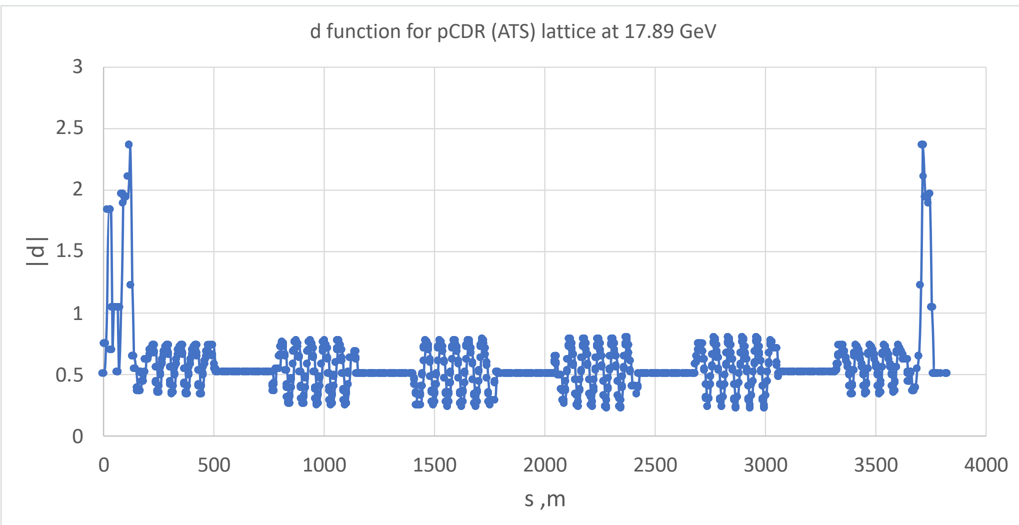
$\mu_x = 0.8823279 \ \mu_y = 0.2800422$

$Length = 27.7\ m$



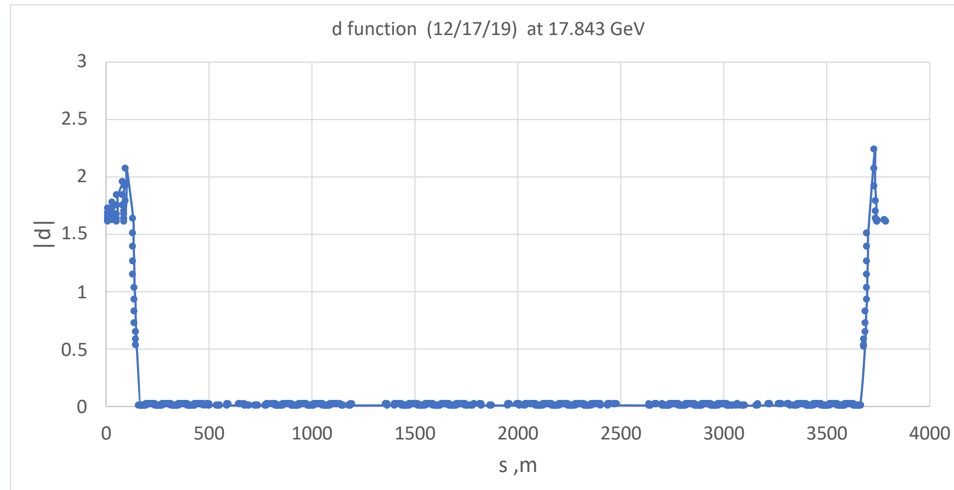
Example of d-function around the ring for two lattices

pCDR'18 lattice



Spin matching was not very good

v5.2 lattice



Optimization of rotator layout for 18 GeV resulted in good spin match into the arc in v5.2.

However, sufficiently large d-function is still present throughout IR.

Three rotator design options for comparison

- Option 1: Minimized depolarization at 18 GeV. But limited energy range (to 6 GeV)

  θ_1= 76.89 mrad
  θ_2= 38.45 mrad

- Option 2: covers 5-18 GeV energy range, but stronger depolarization is expected

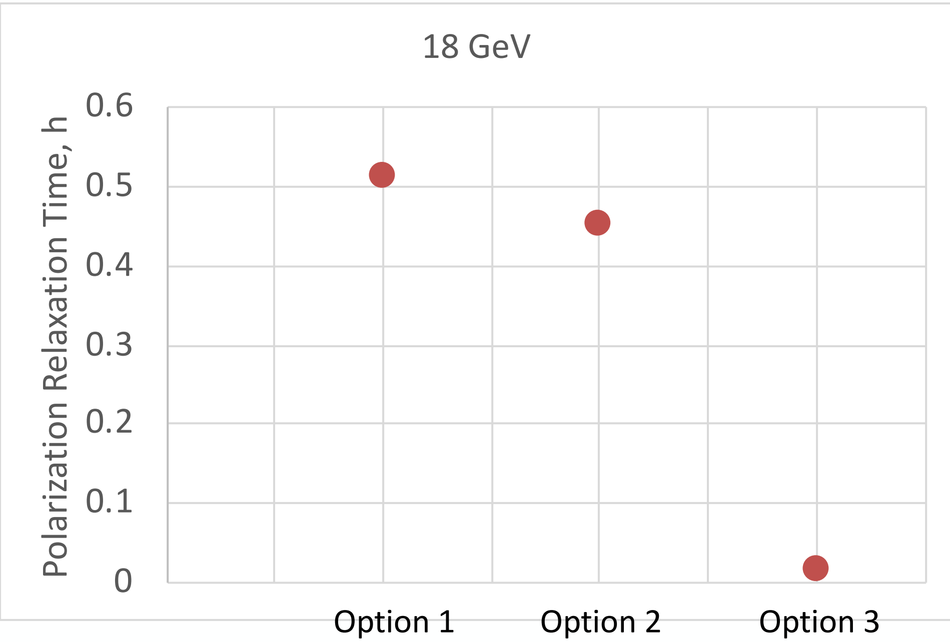
  θ_1= 92.27 mrad
  θ_2= 46.14 mrad

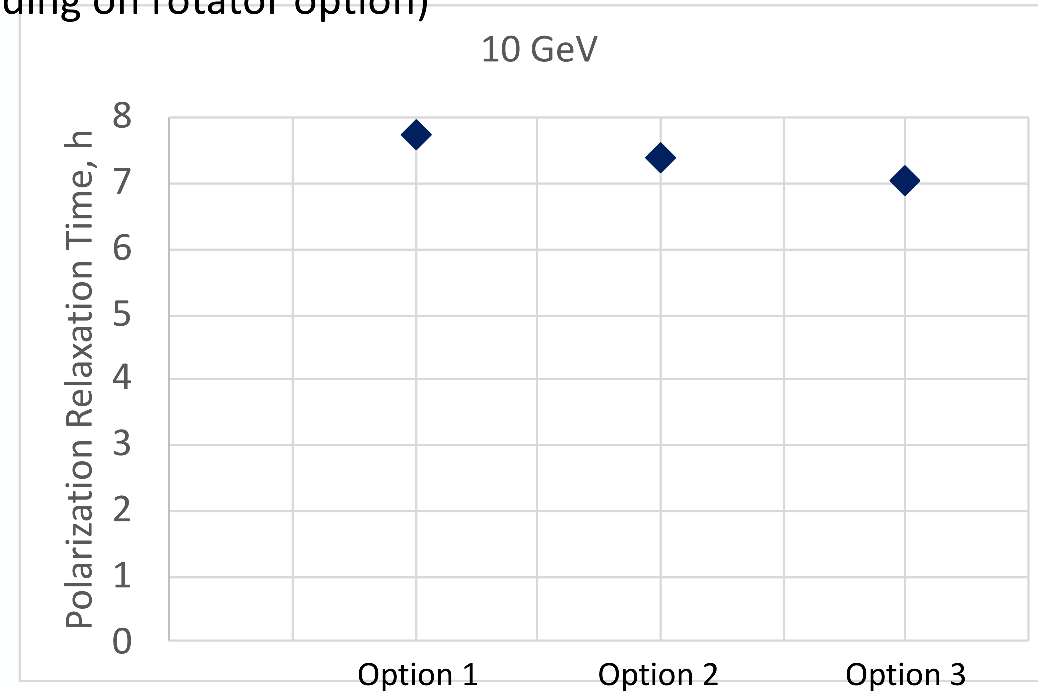
- Option 3: covers 5-18 GeV energy range, but stronger depolarization is expected

  θ_1= 128.15 mrad
  θ_2= 64.08 mrad

Polarization Relaxation Time Comparison (first-order calculation)

Calculations are done at energies corresponding to 0.5 fractional spin tune (17.84-17.89 GeV and 10.25-10.34 GeV depending on rotator option)





Conclusion:
Although the Option 1 demonstrates longer polarization relaxation times, the times in the Option 2 is not much lower.
Both option 1 and 2 remains on the table as a possible implementation of rotator design.

Polarization formulas

$$\mathbf{P_{eq}} = -\frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \mathbf{b} \rangle}{\langle |\rho^{-3}| \rangle}$$

Sokolov-Ternov (ST)

$$\mathbf{P_{eq}} = -\frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \mathbf{b} \cdot \mathbf{n} \rangle}{\langle |\rho^{-3}| (1 - \frac{2}{9} (\mathbf{n} \cdot \mathbf{e_v})^2) \rangle}$$

Baier-Katkov-Strakhovenko (BKS)

$$\mathbf{P_{eq}} = -\frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \mathbf{b} \cdot (\mathbf{n} - \mathbf{d}) \rangle}{\langle |\rho^{-3}| (1 - \frac{2}{9} (\mathbf{n} \cdot \mathbf{e_v})^2) \rangle}$$

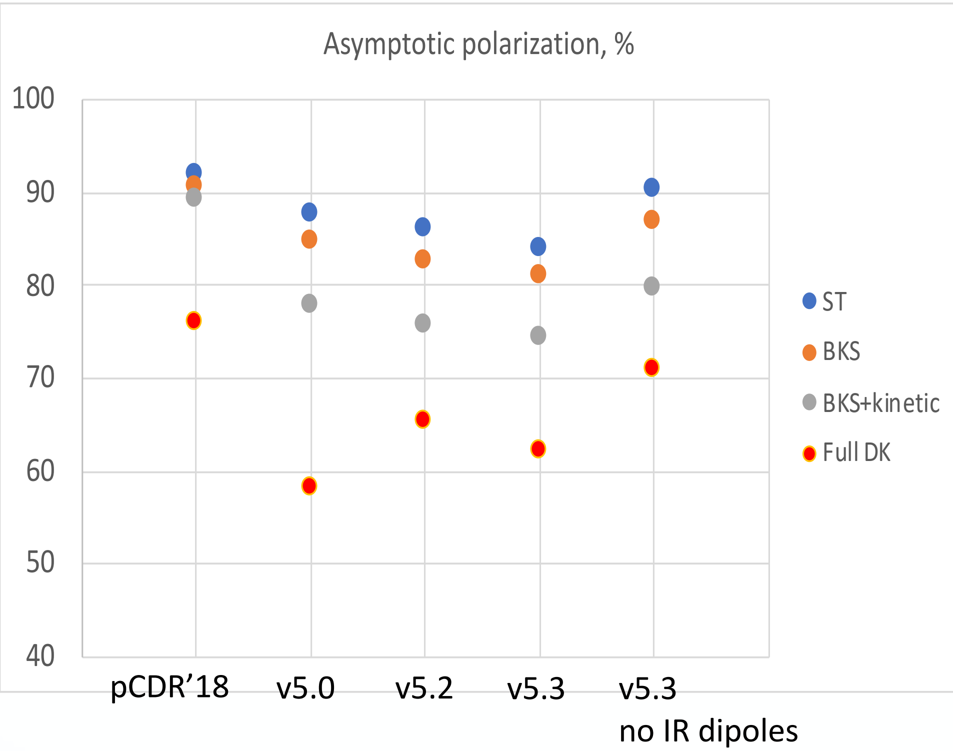
BKS + kinetic polarization mechanism

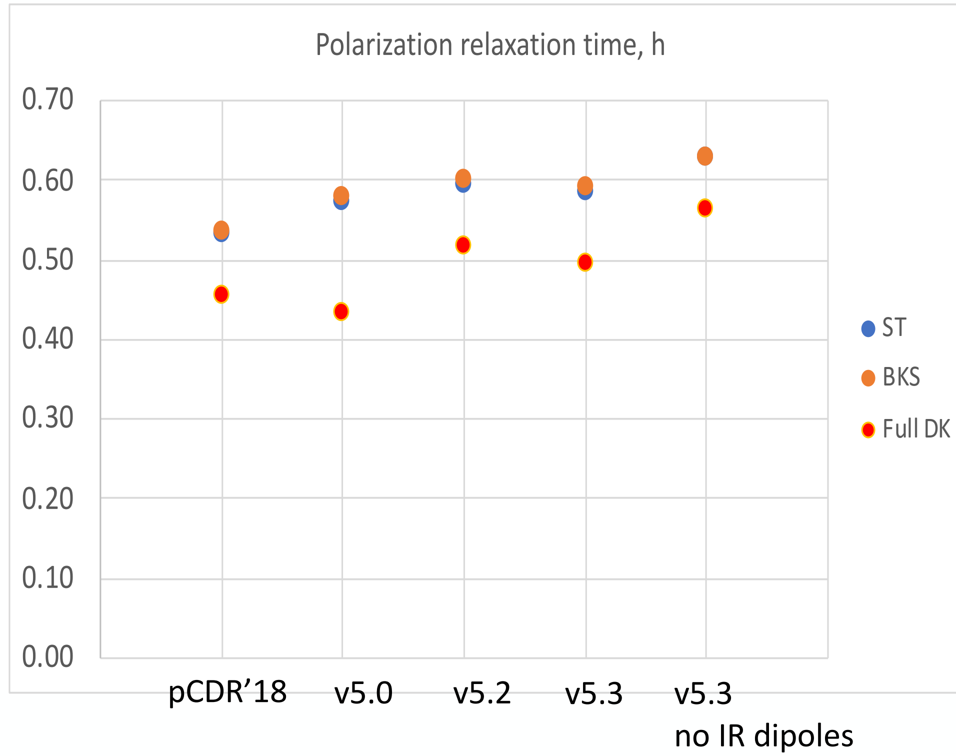
$$\mathbf{P_{eq}} = -\frac{8}{5\sqrt{3}} \frac{\langle |\rho^{-3}| \mathbf{b} \cdot (\mathbf{n} - \mathbf{d}) \rangle}{\langle |\rho^{-3}| (1 - \frac{2}{9} (\mathbf{n} \cdot \mathbf{e_v})^2 + \frac{11}{18} |\mathbf{d^2}|) \rangle}$$

Derbenev-Kondratenko (Full DK)
(with stochastic depolarization)

18 GeV lattices: disentangling different contributions in the polarization

Energy used for calculation: 17.84 GeV (corresponding to ~.5 fractional spin tune)





pCDR’18: eSR not yet fitted in the tunnel; Option 2 rotator

v5.0: eSR partially fitted in the tunnel; lattice with longer bends; Option 1 rotator

v5.2: eSR fully fitted in the tunnel, Option 1 rotator

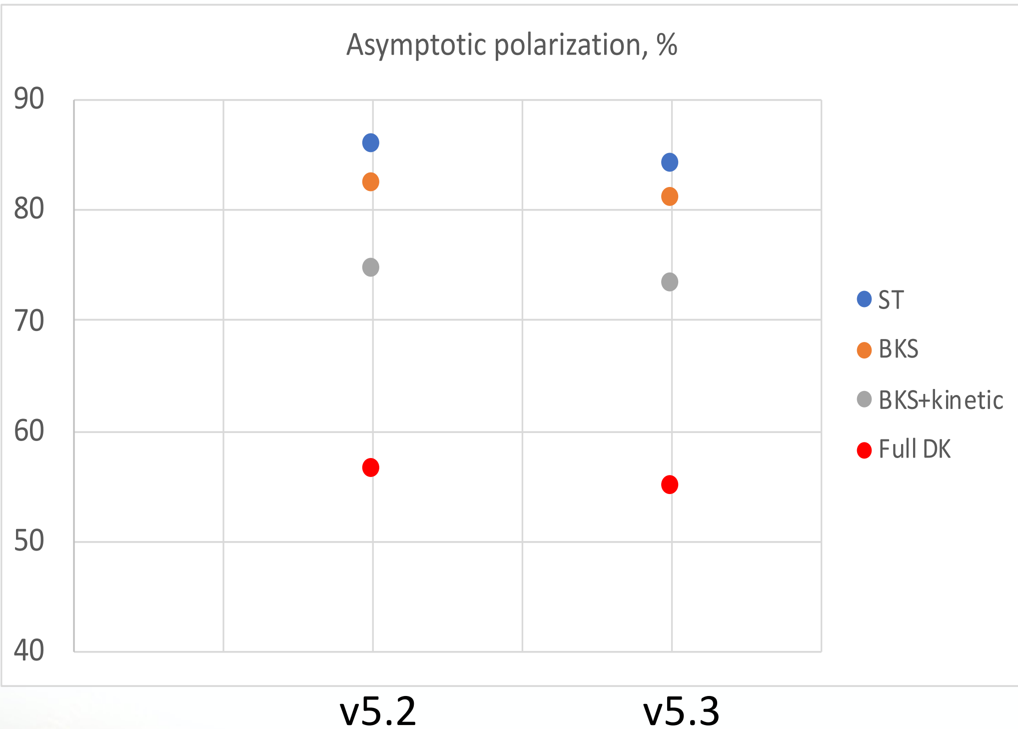
v5.3: IR design layout adjustments, Option 1 rotator

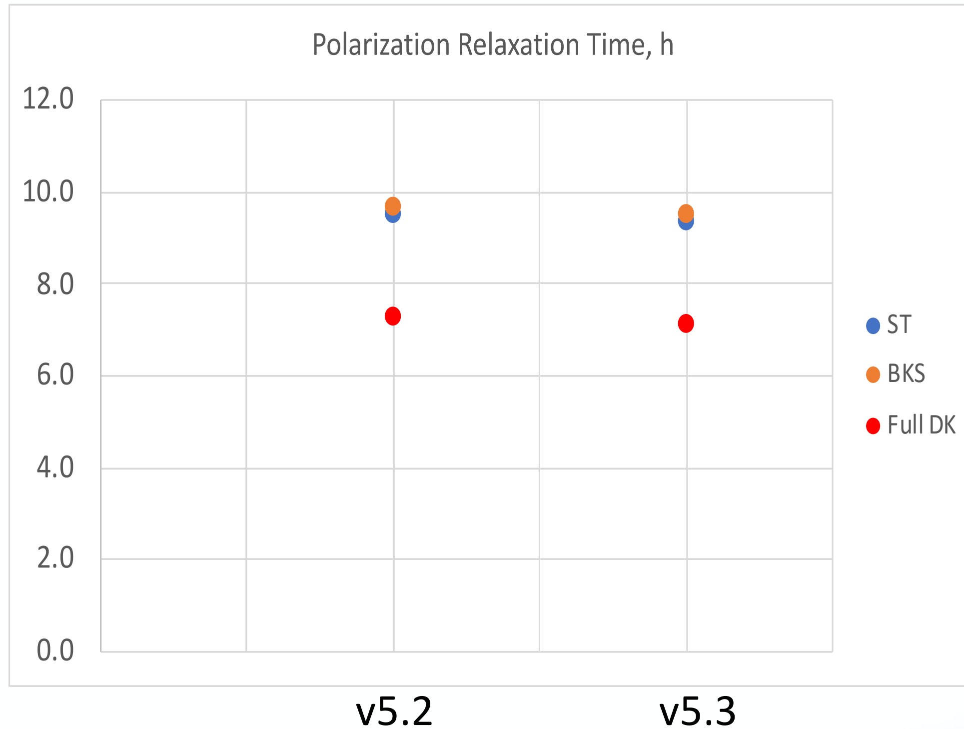
v5.3 no IR dipoles: excluded (only for the analysis purposes) 0.44-0.46 T dipoles in IR6 and IR8

10 GeV lattices: disentangling different contributions in the polarization

Kinetic polarization effect is notable for both 10 GeV and 18 GeV lattices

Energy used for calculation: 10.25 GeV





Spin matching at 10 GeV is worse than at 18 GeV resulting in less asymptotic polarization. But, it is not so important since the polarization relaxation time is very long at 10 GeV.

Summary

- EIC spin rotator design has been developed based on combination of solenoidal and dipole magnets covering wide energy range required by EIC
- The design optimization continues:
  - reducing compensation quad strength
  - optimal selection of rotator configuration (two options)
- The conditions for spin matching have been derived from spin-orbital integrals and implemented in the rotator optics
- Betatron related spin-matching can be done by using a special transport matrix of solenoidal insertions
- There is a rotator configuration that can provide full spin matching at 18 GeV, but its minimum energy is limited to 6 GeV
- Alternative version of spin rotator is presently being evaluated, with potential of reducing space taking by rotator system (and saving cost). Please, see next talk by Fanglei.