# Implementation of the Goloskokov-Kroll model pseudoscalar meson production in PARTONS

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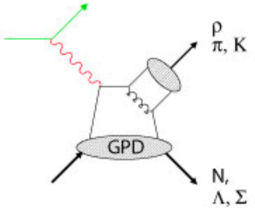
# Chiral-odd GPDs

- Four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p,\lambda).$$

  where $i = 1, 2$ is the transversity index [Diehl '03]

- Accessible through exclusive meson production processes



# Chiral-odd GPDs

- Factorization for electroproduction of mesons, only for longitidunally polarized photons, has been proven [Collins-Frankfurt-Strikman '97]
- For transversely polarized photons, cross section is power suppressed by $1/Q$ [Collins-Frankfurt-Strikman '97]
- But in some kinematics, it is apparent that transversely polarized photons contributes substantially [HERMES Collaboration, arXiv:0907.2596[hep-ex]]

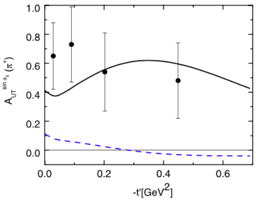



**Fig. 2** (Color online) The $\sin\phi_s$ moment for a transversely polarized target at $Q^2 \simeq 2.45$ GeV$^2$ and $W = 3.99$ GeV. The prediction from our handbag approach is shown as a *solid line*. The *dashed line* is obtained disregarding the twist-3 contribution. Data are taken from [10]

Figure: [Goloskokov-Kroll '10]

# Goloskokov-Kroll Model

- Goloskokov-Kroll(GK) model for pseudoscalar meson production considers the region of small $\xi$ and small $-t$, but large $Q^2$ and $W$.
  [Goloskokov-Kroll '10]
- Contributions from transversely polarized photons can be computed as a twist-3 effect in the handbag mechanism
- In pseudoscalar meson production, the following amplitudes are relevant

$$\mathcal{M}_{0+,0+} = \sqrt{1-\xi^2}\frac{e}{Q}[\langle\tilde{H}\rangle - \frac{\xi^2}{1-\xi^2}\langle\tilde{E}\rangle]$$

$$\mathcal{M}_{0-,0+} = \frac{e}{Q}\frac{-t'}{2m}[\xi\langle\tilde{E}\rangle]]$$

$$\mathcal{M}_{0-,++} = \sqrt{1-\xi^2}e\,\langle H_T\rangle$$

$$\mathcal{M}_{0+,\mu+} = -\frac{e}{4m}\sqrt{-t'}\langle\bar{E}_T\rangle$$

# Goloskokov-Kroll Model

- Generically, $\langle F \rangle$ represents a convolution of a GPD $F$ with an appropriate subprocess amplitude

$$\langle F \rangle = \sum_{\lambda} \int_{-1}^{1} dx\, \mathcal{H}_{\mu'\lambda,\mu\lambda}(x, \xi, Q^2)\, F(x, \xi, t)$$

  where $\lambda$ denotes unobserved helicities of the partons.
- Subprocesses are calculated in the so-called modified perturbative approach: Transverse momenta of the quark and the anti-quark in the meson are kept and gluon radiations are taken into account through Sudakov factor
- GPDs appear in the following combination

$$F^0(x, \xi, t) = \frac{1}{\sqrt{2}}\Big(e_u\, F^u(x, \xi, t) - e_d\, F^d(x, \xi, t)\Big)$$

$$F^+(x, \xi, t) = F^u(x, \xi, t) - F^d(x, \xi, t)$$

# Goloskokov-Kroll Model

- In impact space

$$\mathcal{H}_\pi = \int d\tau d^2\vec{b}\, \hat{\Psi}_\pi(\tau, -\vec{b})\hat{\mathcal{F}}^i_\pi(\bar{x}, \xi, \tau, Q^2, \vec{b})\alpha_s(\mu_R)\; exp\big(-S(\tau, \vec{b}, Q^2)\big)$$

- Hard scattering kernels has the following forms in momentum space

$$\mathcal{F}^q_{\pi^0} = \frac{N_c^2-1}{2N_c}\sqrt{\frac{2}{N_c}}\frac{Q}{\xi}\Big[\frac{1}{k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi) - i\epsilon} - \frac{1}{k_\perp^2 - \bar{\tau}(\bar{x}-\xi)Q^2/(2\xi) - i\epsilon}\Big]$$

$$\mathcal{F}^q_{\pi^+} = \frac{N_c^2-1}{2N_c}\sqrt{\frac{2}{N_c}}\frac{Q}{\xi}\Big[\frac{e_d}{k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi) - i\epsilon} - \frac{e_u}{k_\perp^2 - \bar{\tau}(\bar{x}-\xi)Q^2/(2\xi) - i\epsilon}\Big]$$

- A Gaussian meson wave function is used at twist-2

$$\Psi_\pi(\tau, \vec{b}) \sim \tau(1-\tau)exp\Big[\frac{\tau(\tau-1)}{4}\frac{\vec{b}^2}{a_\pi^2}\Big]$$

# Goloskokov-Kroll Model

- Sudakov factor has the form

$$S(\tau, b, Q) = s(\tau, b, Q) + s(\bar{\tau}, b, Q) - \frac{4}{\beta_0}\, ln\, \frac{ln(\mu_R/\Lambda_{QCD})}{\hat{b}}$$

where

$$s(\tau, b, Q) = \frac{8}{3\beta_0}\left(\hat{q}\; ln\left(\frac{\hat{q}}{\hat{b}}\right) - \hat{q} + \hat{b}\right) + NLL$$
$$\hat{b} = -ln(b\,\Lambda_{QCD})$$
$$\hat{q} = ln\left(\tau Q/(\sqrt{2}\Lambda_{QCD})\right)$$

- Twist-3 meson wave function

$$\Psi_\pi(\tau, \vec{b}) \sim exp\left[-\frac{\vec{b}^2}{8a_\pi^2}\right] I_0\left(\frac{\vec{b}^2}{8a_\pi^2}\right)$$

# Goloskokov-Kroll Model GPDs

- GPDs are constructed from double distribution ansatz

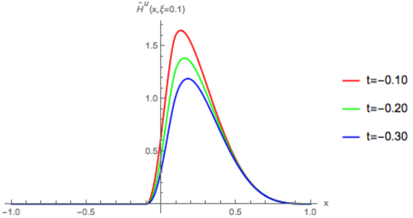
$$F_i^a(\bar{x}, \xi, t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta \, \delta(\rho + \xi\eta - \bar{x}) f_i^a(\rho, \eta, t)$$

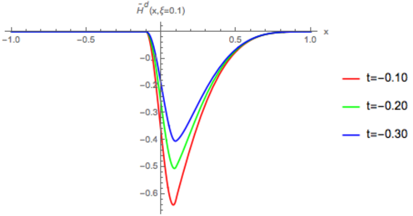
where for valence-quark GPDs;

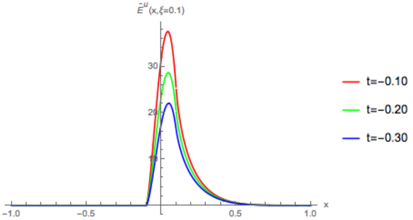
$$f_i(\rho, \eta, t) = exp[(b_i - \alpha_i' \, ln \, \rho)t] \, F_i^a(\rho, \xi = t = 0) \, \frac{3}{4} \frac{(1-\rho)^2 - \eta^2}{(1-\rho)^3} \Theta(\rho)$$

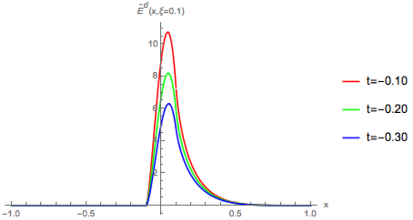
- Parameters of the forward limits
  - $\tilde{H}$ : DSSV, Phys. Rev. D **80**, 034030 (2009)
  - $H_T$: ABM, Phys. Rev. D **86**, 054009 (2012) and DSSV, Phys. Rev. D **80**, 034030 (2009)
  - $\tilde{E}$ : LHPC Collaboration, Phys. Rev. D **77**, 094502 (2008)
  - $\bar{E}_T$ : QCDSF and UKQCD Collaborations, Phys. Rev. Lett. **98**, 222001 (2007)

# Goloskokov-Kroll Model GPDs



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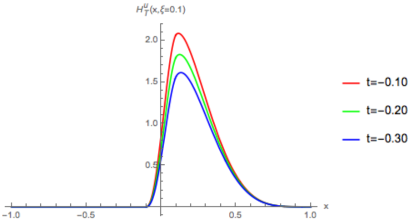
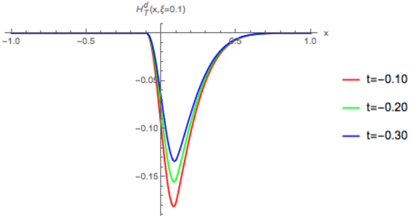
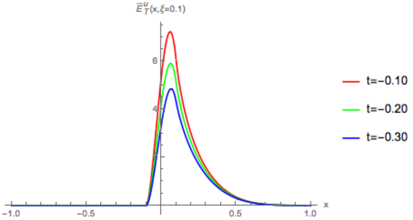
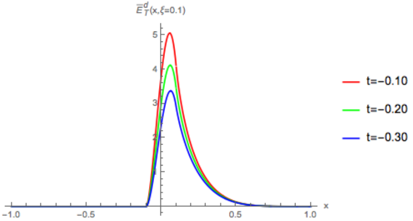


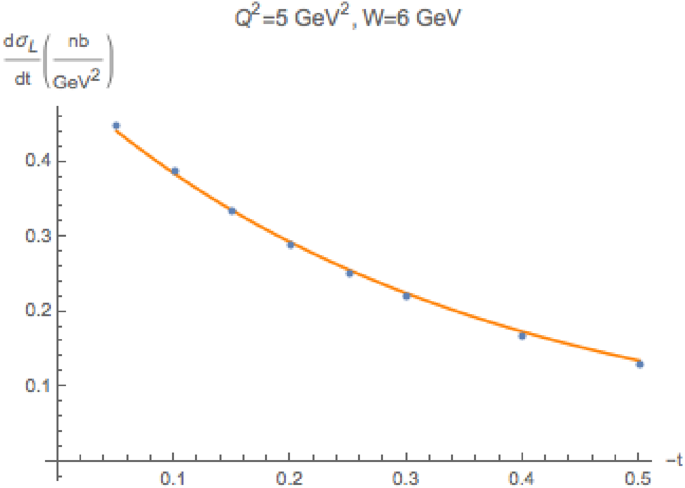



# Goloskokov-Kroll Model

- So altogether, to compute the longitudinal cross section, we need: Twist-2 meson wave function, kernel, running coupling, Sudakov factor and GPDs $\tilde{H}$ and $\tilde{E}$.
- To compute the transverse cross section, we need: Twist-3 meson wave function, kernel, running coupling, Sudakov factor and GPDs $H_T$ and $\bar{E}_T$.
- 3 dimensional integrals, over $\bar{x}, \tau$ and $b$, are performed in impact space
- In many different processes, meson wavefunction has the same structure.
- Sudakov factor has also the same structure
- $\pi^+$ electroproduction also receives a pion pole contribution, besides the handbag contribution

# Hepgen vs. Mathmematica

- Comparision between the Hepgen and Mathmematica codes



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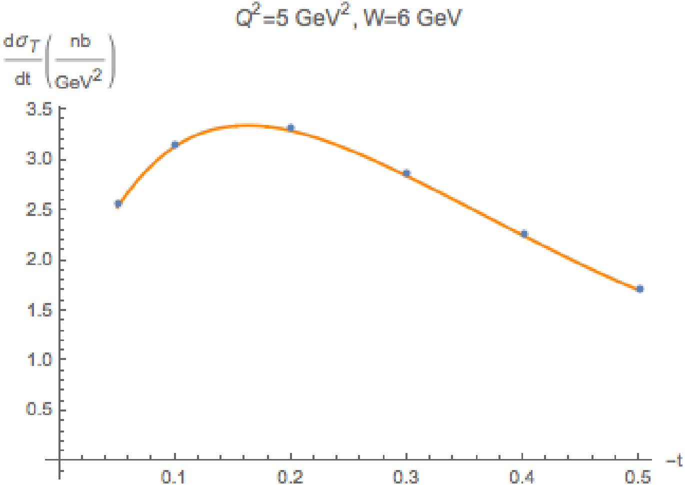

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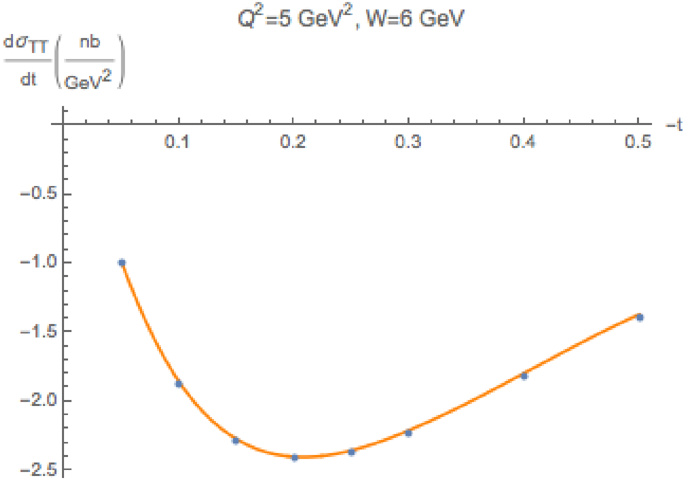

# Beam Spin Asymmetry

- Beam spin asymmetry for $\pi^+$ [Goloskokov-Kroll '10]

$$A_{LU}\sigma_0 = \sqrt{\epsilon(1-\epsilon)}\, Im\Big[(\mathcal{M}^*_{0+,++} - \mathcal{M}^*_{0+,-+})\mathcal{M}_{0+,0+} + (\mathcal{M}^*_{0-,++} - \mathcal{M}^*_{0-,-+})\mathcal{M}_{0-,0+}\Big]$$

where

$$\sigma_0 = \frac{1}{2}\Big[|\mathcal{M}_{0+,++}|^2 + |\mathcal{M}_{0-,-+}|^2 + |\mathcal{M}_{0-,++}|^2 + |\mathcal{M}_{0+,-+}|^2\Big] + \epsilon\Big[|\mathcal{M}_{0+,0+}|^2 + |\mathcal{M}_{0-,0+}|^2\Big]$$

# Goloskokov-Kroll Model

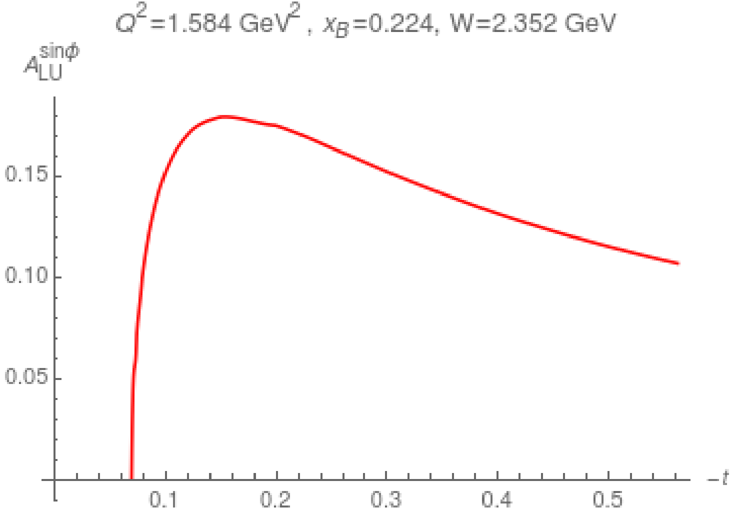


Figure: BSA results for $\pi^+$ *production*