Unitarity Issues and Simplified Models for High-Energy Electroweak Interactions

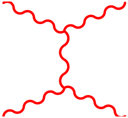
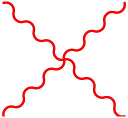
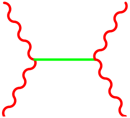
Wolfgang Kilian

University of Siegen

Multi-Boson Interactions Workshop
October 2014
Brookhaven National Laboratory

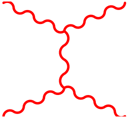
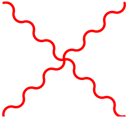
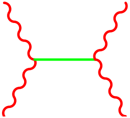
A. Alboteanu, WK, J. Reuter, JHEP 0811 (2008) 010
WK, T. Ohl, J. Reuter, M. Sekulla, arXiv:1408.6207

Higgs and Vector-Boson Scattering

$O(E^4)$  $+ O(E^4)$  $+ O(E^2)$  $= O(1)$

Higgs exchange cancels the E^2 rise exactly (in the SM):
the Minimal SM Higgs Sector.

Higgs and Vector-Boson Scattering

$O(E^4)$  $+ O(E^4)$  $+ O(E^2)$  $= O(1)$

Higgs exchange cancels the E^2 rise exactly (in the SM):
the Minimal SM Higgs Sector.

Discoveries

1. Higgs production in WW fusion: the Higgs boson exists.
2. SM confirmed in VBS: the Higgs mechanism works as expected.

Future Expectation for VBS

If SM is true,

Future Expectation for VBS

If SM is true, VBS amplitude is bounded and small: m_H^2/v^2.

LHC:
Production cross section falls of with increasing effective energy, i.e., invariant mass of the WW pair system.

NLO: some logarithmic corrections.

Future Expectation for VBS

If SM is true, VBS amplitude is bounded and small: m_H^2/v^2.

LHC:

Production cross section falls of with increasing effective energy, i.e., invariant mass of the *WW* pair system.

NLO: some logarithmic corrections.

No problem with unitarity, of course.

And What If Not?

Two classes of modifications to the SM (or mixture):

1. New weakly interacting particles, direct production. Example: 2HDM

And What If Not?

Two classes of modifications to the SM (or mixture):

1. New weakly interacting particles, direct production. Example: 2HDM
2. Small deviations from the SM prediction (linear Higgs rep.)

And What If Not?

Two classes of modifications to the SM (or mixture):

1. New weakly interacting particles, direct production. Example: 2HDM
2. Small deviations from the SM prediction (linear Higgs rep.)

Formalism:

Effective Field Theory

- Add higher-dimensional operators to the SM Lagrangian.
- Use only SM fields, respect SM gauge invariance
- Operator of dimension n carries prefactor $1/\Lambda^{n-4}$

And What If Not?

Two classes of modifications to the SM (or mixture):

1. New weakly interacting particles, direct production. Example: 2HDM
2. Small deviations from the SM prediction (linear Higgs rep.)

Formalism:

Effective Field Theory

- Add higher-dimensional operators to the SM Lagrangian.
- Use only SM fields, respect SM gauge invariance
- Operator of dimension n carries prefactor $1/\Lambda^{n-4}$

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \sum_{d=5}^{\infty} \frac{1}{\Lambda^{n-4}} \mathcal{O}_n$$

Concrete Examples:

Anomalous Interactions

$$\mathcal{L}_{HD} = F_{HD}\ \mathrm{tr}\left[\mathbf{H}^\dagger\mathbf{H} - \frac{v^2}{4}\right] \cdot \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger(\mathbf{D}^\mu\mathbf{H})\right] \qquad HVV \quad D = 6$$

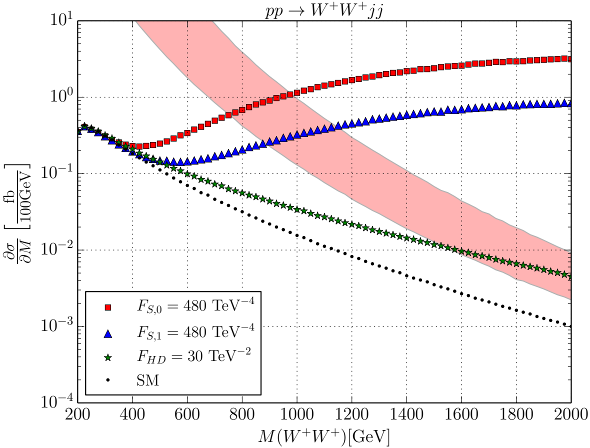
$$\mathcal{L}_{S,0} = F_{S,0}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\,\mathbf{D}_\nu\mathbf{H}\right] \cdot \mathrm{tr}\left[(\mathbf{D}^\mu\mathbf{H})^\dagger\,\mathbf{D}^\nu\mathbf{H}\right] \qquad VVVV \quad D = 8$$

$$\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[(\mathbf{D}_\mu\mathbf{H})^\dagger\,\mathbf{D}^\mu\mathbf{H}\right] \cdot \mathrm{tr}\left[(\mathbf{D}_\nu\mathbf{H})^\dagger\,\mathbf{D}^\nu\mathbf{H}\right] \qquad VVVV \quad D = 8$$

Linear Higgs/Goldstone Field Representation:

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h - \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix} \quad . \tag{1}$$

Nice, but. . .



Calculation: WHIZARD

What happened?

Gauge invariance + Higgs exchange
remove two orders of the Taylor expansion.

$\Rightarrow$ Effect of anomalous couplings rapidly rises with energy. ($D = 8$ operators!) cancels the PDF suppression

What happened?

Gauge invariance + Higgs exchange
remove two orders of the Taylor expansion.

- $\Rightarrow$ Effect of anomalous couplings rapidly rises with energy. ($D = 8$ operators!) cancels the PDF suppression
- $\Rightarrow$ Window (in energy) where effective theory is useful for describing deviations at the LHC: absent.

Basically, forget about (perturbative) quantum field theory?

This is not the same situation as in VB pair production.

What happened?

Gauge invariance + Higgs exchange
remove two orders of the Taylor expansion.

- ⇒ Effect of anomalous couplings rapidly rises with energy. ($D = 8$ operators!) cancels the PDF suppression
- ⇒ Window (in energy) where effective theory is useful for describing deviations at the LHC: absent.

Basically, forget about (perturbative) quantum field theory?

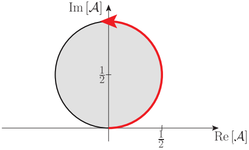
This is not the same situation as in VB pair production.

[There are perturbative models, e.g, the 2HDM. But they access only a small fraction of the conceivable Model Space.]

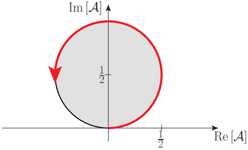
Unitarity

The scattering of w, z is a (quasi-) elastic process. Properly diagonalized (isospin I, spin J) and normalized, the partial-wave amplitudes must lie on the Argand Circle.

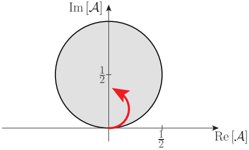
Possibilities



A: Saturation



B: Resonance



C: Inelastic

Unitarization

Unitarization

There are zillions of papers that investigate this problem.

- Heavy Higgs as Unitarization
- K-Matrix Unitarization
- Padé Unitarization
- Inverse Amplitude Method
- O(N) Model Unitarization
- N/D Method
- ...

Unitarization

There are zillions of papers that investigate this problem.

- Heavy Higgs as Unitarization
- K-Matrix Unitarization
- Padé Unitarization
- Inverse Amplitude Method
- O(N) Model Unitarization
- N/D Method
- ...

Small caveat: 99 % of those papers don't have a light Higgs.

Unitarization

There are zillions of papers that investigate this problem.

- Heavy Higgs as Unitarization
- K-Matrix Unitarization
- Padé Unitarization
- Inverse Amplitude Method
- O(N) Model Unitarization
- N/D Method
- ...

Small caveat: 99 % of those papers don't have a light Higgs.

Which makes a difference.

Unitarization after 2012

Repeat the game with light Higgs?

Unitarization methods are tailored for the quasi-elastic WW system, not for arbitrary processes.

Unitarization after 2012

Repeat the game with light Higgs?

Unitarization methods are tailored for the quasi-elastic WW system, not for arbitrary processes.

- measure low-energy parameters
- extrapolate, using analytic properties and assumptions
- get a prediction.

Unitarization after 2012

Repeat the game with light Higgs?

Unitarization methods are tailored for the quasi-elastic WW system, not for arbitrary processes.

- measure low-energy parameters
- extrapolate, using analytic properties and assumptions
- get a prediction.

Do we want a prediction with assumptions?

Unitarization after 2012

Repeat the game with light Higgs?

Unitarization methods are tailored for the quasi-elastic WW system, not for arbitrary processes.

- measure low-energy parameters
- extrapolate, using analytic properties and assumptions
- get a prediction.

Do we want a prediction with assumptions? We want a framework.

For the experimentalist:

A class of models that

- is in accordance with SM, EFT, and unitarity
- exhausts the possibilities as far as they are experimentally accessible
- let us quote a result in the form of a few parameter values

For the phenomenologist:

A class of models that

- can be implemented in a Monte Carlo that computes the full process, not just some Goldstone-Boson idealization
- can be systematically improved
- works for any process (in principle)

For the model builder:

A class of models that

- can accomodate any scenario for high-energy interactions
- in a unitary version
- makes use of all information that is put in
- but not more
- doesn't modify a model that is already unitary
- is not limited to perturbation theory

$\Rightarrow$ no traditional scheme fits the description.

K Matrix

(Heitler 1941, for QED): Cayley Transform

$$S = \frac{\mathbb{1} + \mathrm{i}K/2}{\mathbb{1} - \mathrm{i}K/2}\,, \qquad \text{where} \quad K = K^\dagger \qquad \text{and} \quad S = \mathbb{1} + \mathrm{i}\,T$$

The K Matrix, exactly:

$$K = \frac{T}{\mathbb{1} + \mathrm{i}\,T/2}\,.$$

K Matrix

(Heitler 1941, for QED): Cayley Transform

$$S = \frac{\mathbb{1} + \mathrm{i}K/2}{\mathbb{1} - \mathrm{i}K/2}\,, \qquad \text{where} \quad K = K^\dagger \qquad \text{and} \quad S = \mathbb{1} + \mathrm{i}\,T$$

The K Matrix, exactly:

$$K = \frac{T}{\mathbb{1} + \mathrm{i}\,T/2}\,.$$

The K Matrix, in Perturbation Theory:

$$K = T - \frac{\mathrm{i}}{2}T^2 \pm \ldots$$

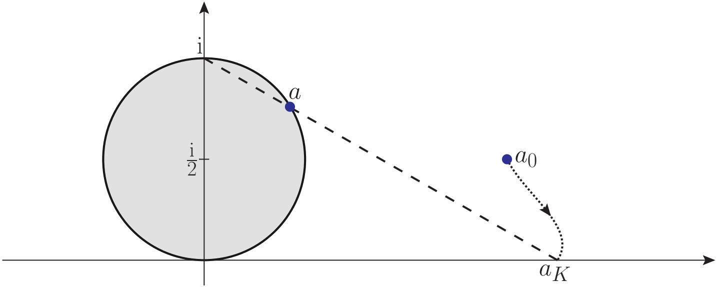
Original K Matrix algorithm (Gupta, for QCD/EW):

- Compute T matrix perturbatively
- Reconstruct K matrix order by order
- Insert into S matrix formula, without expanding again

This is elegant, but relies on perturbation theory.

Graphical Visualization: K Matrix

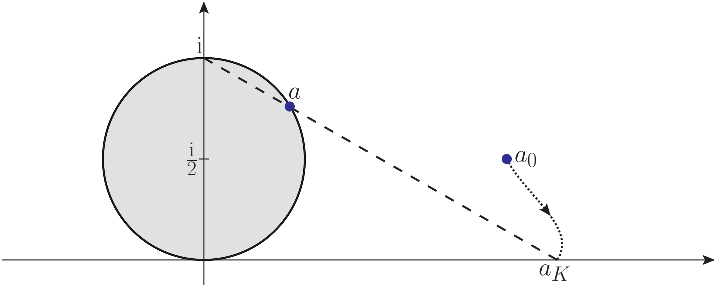
Start from arbitrary amplitude a_0 in perturbative expansion:



First reconstruct a_K, then compute a

Graphical Visualization: K Matrix

Start from arbitrary amplitude a_0 in perturbative expansion:

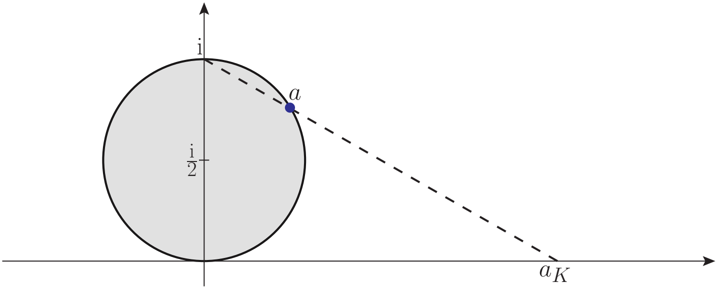


First reconstruct a_K, then compute a

Our suggestion: compute unitarized T matrix directly, without detour

Graphical Visualization: Direct T Matrix Unitarization

Start from real amplitude $a_0 = a_K$: Inverse stereographic projection

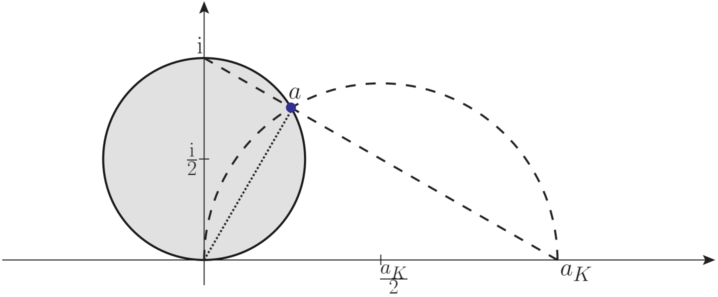


⇒ No reference to perturbative expansion

⇒ Unitary amplitude a_0 left invariant

Graphical Visualization: Direct T Matrix Unitarization

Start from real amplitude $a_0 = a_K$: Thales circle projection

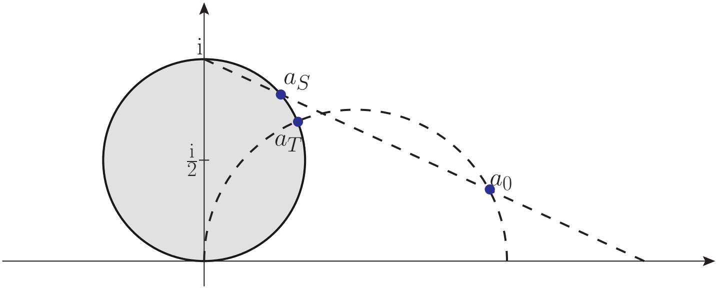


⇒ No reference to perturbative expansion

⇒ Unitary amplitude a_0 left invariant

Graphical Visualization: Direct T Matrix Unitarization

Start from complex amplitude a_0:



⇒ No reference to perturbative expansion

⇒ Unitary amplitude a_0 left invariant

⇒ But scheme dependence for complex a_0

Linear Construction "Stereographic"

$$T = \frac{\operatorname{Re} T_0}{\mathbb{1} - \frac{\mathrm{i}}{2} T_0^\dagger}.$$

for normal matrices ($T^\dagger T = TT^\dagger$), otherwise need operator ordering

- well behaved near $T = 0$
- weird behavior for eigenvalues above $T = \mathrm{i}$

Circular Construction "Thales"

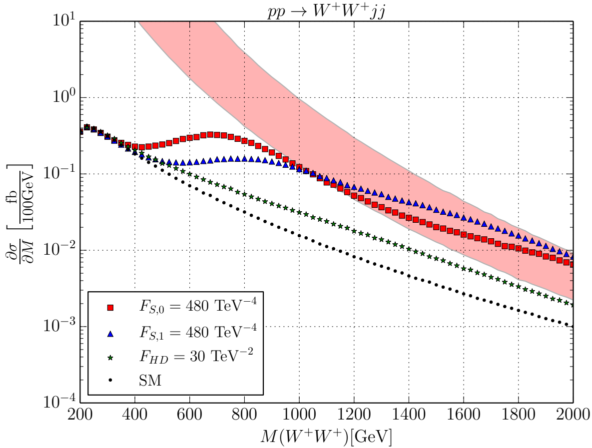
$$T = \frac{1}{\mathrm{Re}\left(\frac{1}{T_0}\right) - \frac{\mathrm{i}}{2}\mathbb{1}} \qquad .$$

- singular at $T = 0$ (but harmless)
- well behaved above $T = \mathrm{i}$

Algorithm

1. Start with input model
2. Extract strong-interaction part in Goldstone limit
3. Unitarize via T Matrix projection
4. Re-insert correction as form factor into Feynman rules
5. Extrapolate off-shell
6. Use in Monte Carlo simulation

Result: Unitarized Cross Section



Calculation: WHIZARD

And Beyond?

- Padé & Co. yield predictions: resonances
- work in QCD (vector dominance) . . . ?
- restricted to quasi-elastic scattering?

⇒ Add any additional information in T Matrix framework

Resonances and Anomalous Couplings

A resonance is a pole in the elastic scattering matrix:

$$A(s) = \frac{g^2}{s - \hat{m}^2} + \hat{A}_{\text{nonres}}(s)$$

The parameters g^2 and $\hat{m}^2$ are well defined: pole location and residue.

Resonances and Anomalous Couplings

A resonance is a pole in the elastic scattering matrix:

$$A(s) = \frac{g^2}{s - \hat{m}^2} + \hat{A}_{\text{nonres}}(s)$$

The parameters g^2 and $\hat{m}^2$ are well defined: pole location and residue.
Applying T-matrix unitarization, we get a Breit-Wigner resonance

$$A(s) = \frac{g^2}{s - m^2 + im\Gamma} + A_{\text{nonres}}(s)$$

Resonances and Anomalous Couplings

A resonance is a pole in the elastic scattering matrix:

$$A(s) = \frac{g^2}{s - \hat{m}^2} + \hat{A}_{\text{nonres}}(s)$$

The parameters g^2 and $\hat{m}^2$ are well defined: pole location and residue.
Applying T-matrix unitarization, we get a Breit-Wigner resonance

$$A(s) = \frac{g^2}{s - m^2 + im\Gamma} + A_{\text{nonres}}(s)$$

At low energy, the resonant amplitude has a Taylor expansion

$$A(s) = -\frac{g^2}{m^2} + \frac{g^2}{m^4}\,s + \ldots$$

The second term corresponds to an anomalous coupling (matching).

Guideline for Simplified Models

- The rise of an amplitude (anomalous coupling) may be the Taylor expansion of a resonance.
- We have no idea which resonances exist and where they come from.
- Including a resonance in the model, there still may be further sources for anomalous couplings (further resonances, $A_{\text{nonres}}(s)$, deviation from the Breit-Wigner shape, etc.)
- Beyond the resonance, the amplitude may eventually rise and need unitarization again.

Consequence:

We allow for resonances in all accessible spin/isospin channels.
We also include extra anomalous couplings.

Simplified Models: Generic Resonances

I	0	1	2
$J=0$	σ^0	.	$\phi^{--}, \phi^{-}, \phi^0, \phi^{+}, \phi^{++}$
1	.	$\rho^{-}, \rho^0, \rho^{+}$	.
2	f^0	.	$t^{--}, t^{-}, t^0, t^{+}, t^{++}$
...	...	...	...

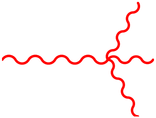
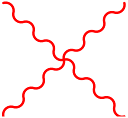
- $I = 0$: resonant in W^+W^- and ZZ scattering
- $I = 1$: resonant in W^+Z and W^-Z scattering
- $I = 2$: resonant in W^+W^+ and W^-W^- scattering

Model Parameters

VBS, total (isospin preserved, CP, higher spin ignored):

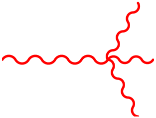
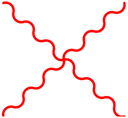
- 5 resonances with 3 parameters each (M, g_L, g_T)
- quartic anomalous couplings of longitudinal VB
- quartic anomalous couplings of transversal VB
- quartic anomalous couplings mixing T and L

Other Processes? Such as

relate  to  ??

Same Feynman graphs (in a complete model), but. . .

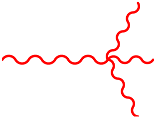
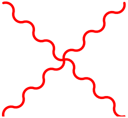
Other Processes? Such as

relate  to  ??

Same Feynman graphs (in a complete model), but. . .

one external $W/Z/\gamma$ is always far off-shell.

Other Processes? Such as

relate  to  ??

Same Feynman graphs (in a complete model), but. . .

one external $W/Z/\gamma$ is always far off-shell.

Unitary & Simplified Models: Next project (not yet done)

Summary

- Effective theory: good for TGC, limited applicability for QGC.
- Unitarization schemes tend to introduce theoretical prejudice

$\Rightarrow$ We propose a framework how to reconcile EFT with unitarity without losing its benefits

$\Rightarrow$ Direct T-Matrix unitarization as catch-all scheme for new models

Summary

- Effective theory: good for TGC, limited applicability for QGC.
- Unitarization schemes tend to introduce theoretical prejudice

⇒ We propose a framework how to reconcile EFT with unitarity without losing its benefits

⇒ Direct T-Matrix unitarization as catch-all scheme for new models

- Possible Realization: generic resonances = simplified model.

Summary

- Effective theory: good for TGC, limited applicability for QGC.
- Unitarization schemes tend to introduce theoretical prejudice

⇒ We propose a framework how to reconcile EFT with unitarity without losing its benefits

⇒ Direct T-Matrix unitarization as catch-all scheme for new models

- Possible Realization: generic resonances = simplified model.

- Extended Framework for quantitative tests of the SM version of electroweak interactions