Operator Basis for Effective Theories

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Outline

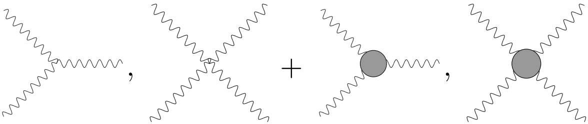
Motivation

- We want to check NP in a model independent way

$\Rightarrow$ Using bottom-up effective field theory

- Need NP operators to extend the minimal model (Standard Model)

$$\mathcal{L} = \mathcal{L}_{min} + \sum_{d \geq 4} \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}_i^d$$



Requirements for Operator Set

- Complete

$\rightarrow$ systematic ordering using building blocks

$\rightarrow$ gauge invariant under $SU(2)_L \times U(1)_Y$

- Minimal

$\rightarrow$ eliminating linearly dependent operators

- Suited

$\rightarrow$ for which purpose

$+$ dependent on minimal model

Minimal Models

No Higgs Heavy Higgs	Non-linear Representation		Weinberg: Phys. Rev. 166 (1967)
	$\downarrow$ Adding scalar resonance		
Light Higgs	Non-linear model with σ	$\rightarrow$	Linear σ-model Standard Model

Model names originate from hadron physics

Non-linear Model

Chiral Lagrangian

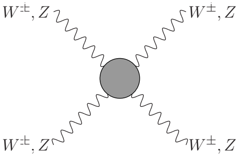
$$\mathcal{L} = \frac{1}{2}\,\mathrm{tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right] + \frac{1}{2}\,\mathrm{tr}\left[\mathbf{B}^{\mu\nu}\mathbf{B}_{\mu\nu}\right] + \frac{1}{4}v^2\,\mathrm{tr}\left[\mathbf{V}^{\mu}\mathbf{V}_{\mu}\right]$$

Building Blocks (BB)

BB	dimension	description
$\mathbf{W}^{\mu\nu}, \mathbf{B}^{\mu\nu}$	2	Field strength tensors $SU(2)_L$ and $U(1)_Y$ $\equiv$ transversal modes of VB
$\mathbf{D}^{\mu}$	1	Covariant derivative
$U(w^a/v)$	0	Non-linear representation (unitary) of goldstone bosons; 2×2-matrix
$\mathbf{V}^{\mu}$	1	$U\left(\mathbf{D}_{\mu}U\right)^{\dagger} \equiv$ longitudinal modes of VB
$\mathbf{T}^{\mu}$	0	$U\tau_3 U^{\dagger}$

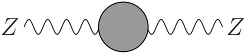
Operator Examples

$$\mathcal{L}_{\alpha_4} = \alpha_4 \,\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right] \mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right]$$



- Effect all possible combination of 4 longitudinal weak gauge bosons
- Anomalous Quartic Gauge Coupling (AQGC)

$$\mathcal{L}_{\beta'} = \beta' \frac{v^2}{4} \mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]$$



- **T** project out only Z boson

$\rightarrow$ Contribution to Z-boson-mass

CP-conserving Operators in NLM

Appelquist/Bernard: Phys. Rev. D22 Issue 1 (1980) Longhitano: Phys. Rev. D22 Issue 5 (1980)

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} + \mathcal{L}_{\beta'} + \sum_i \mathcal{L}_i$$

$$\mathcal{L}_{\alpha_1} = \alpha_1 g g' \,\text{tr}\left[U \mathbf{B}_{\mu\nu} U^\dagger \mathbf{W}^{\mu\nu}\right]$$

$$\mathcal{L}_{\alpha_2} = i\alpha_2 g' \,\text{tr}\left[U \mathbf{B}_{\mu\nu} U^\dagger [\mathbf{V}^\mu, \mathbf{V}^\nu]\right]$$

$$\mathcal{L}_{\alpha_3} = \alpha_3 i \,\text{tr}\left[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]\right]$$

$$\mathcal{L}_{\alpha_4} = \alpha_4 \,\text{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right] \text{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right]$$

$$\mathcal{L}_{\alpha_5} = \alpha_5 \,\text{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \text{tr}\left[\mathbf{V}_\nu \mathbf{V}^\nu\right]$$

$$\mathcal{L}_{\alpha_{11}} = \alpha_{11} g \varepsilon^{\mu\nu\rho\lambda} \,\text{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \text{tr}\left[\mathbf{V}_\nu \mathbf{W}_{\rho\lambda}\right]$$

$$\mathcal{L}_{\alpha_6} = \alpha_6 \,\text{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right] \text{tr}\left[\mathbf{T}\mathbf{V}^\mu\right] \text{tr}\left[\mathbf{T}\mathbf{V}^\nu\right]$$

$$\mathcal{L}_{\alpha_7} = \alpha_7 \,\text{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \text{tr}\left[\mathbf{T}\mathbf{V}_\nu\right] \text{tr}\left[\mathbf{T}\mathbf{V}^\nu\right]$$

$$\mathcal{L}_{\alpha_8} = \tfrac{1}{4}\alpha_8 \,\text{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right] \text{tr}\left[\mathbf{T}\mathbf{W}^{\mu\nu}\right]$$

$$\mathcal{L}_{\alpha_9} = \tfrac{i}{2}\alpha_9 \,\text{tr}\left[\mathbf{T}\mathbf{W}_{\mu\nu}\right] \text{tr}\left[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]\right]$$

$$\mathcal{L}_{\alpha_{10}} = \alpha_{10} \tfrac{1}{2} \left(\text{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \text{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right)^2$$

$$\mathcal{L}_{\beta'} = \beta' \frac{v^2}{4} \,\text{tr}\left[\mathbf{T}\mathbf{V}_\mu\right] \text{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]$$

- All operators (beside $\mathcal{L}_{\beta'}$) have dimension 4
- $SU(2)_C$-breaking operators: **T** and **B**

Notation

$$\mathbf{V}_\mu = U(\mathbf{D}_\mu U)^\dagger$$
$$T = U\tau^3 U^\dagger$$

Custodial Symmetry

$$\beta' \frac{v^2}{8} \operatorname{tr}[T\mathbf{V}_\mu] \operatorname{tr}[T\mathbf{V}^\mu]$$

- Free parameter $\beta' = \beta'(\rho_*)$
- Experimental data constrains ρ_*:

$\rightarrow$ $\beta'(\rho_* \equiv 1) = 0$

- Impose approximate symmetry to forbid above term

$\Rightarrow$ $SU(2)_L \times U(1)_Y \rightarrow SU(2)_L \times SU(2)_R$

Fermionic sector

Very strong violation due large top mass

Bosonic sector

- Broken by coupling $B\tau_3 U \propto s_w^2$

$\Rightarrow$ Only small violation of $M_W = M_Z$

- Higgs mechanism: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$

Higgs Mechanism

- Higgs boson has mass of order of gauge bosons
- Additional degree of freedom from fluctuation of composite field $\mathrm{tr}\left[U^\dagger U\right]$

$\rightarrow$ U is not unitary

- Adding invariant potential term

$$\mathcal{L}_U = -\frac{\mu^2 v^2}{4}\,\mathrm{tr}\left[U^\dagger U\right] - \frac{\lambda v^4}{16}\,\mathrm{tr}\left[U^\dagger U\right]^2$$

- Normalized ground state

$$\langle \frac{1}{2}\,\mathrm{tr}\left[U^\dagger(x)U(x)\right]\rangle = 1$$

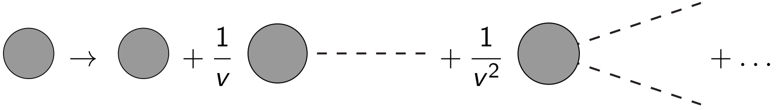
$\Rightarrow$ $\mathcal{L}_U$ breaks spontaneously $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$

$\Rightarrow$ Associated to scalar Higgs boson

Adding generic Higgs

Adding scalar higgs resonance through substitution

$$U \to U\left(1 + \left(\frac{h}{v}\right)^n\right)$$



Building blocks

$$F_U\left(\frac{h}{v}\right) = \sum_{n=1}^{\infty} f_{U,n}\left(\frac{h}{v}\right)^n \quad (\text{dim} = 0)$$

$$\frac{\partial^\mu h}{v} : \quad (\text{dim} = 1)$$

Additional Higgs term

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{4}v^2 \,\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] F_U\left(\frac{h}{v}\right) + V\left(\frac{h}{v}\right)$$

Examples for NLM with Higgs

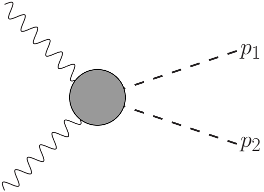
Buchalla/Cata/Krause: arxiv/1307.5017

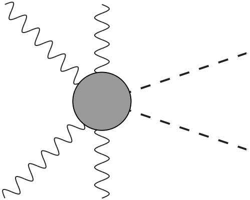
$$\mathcal{O}_{D2} = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}_\nu]\,\mathrm{tr}\,[\mathbf{V}^\mu \mathbf{V}^\nu] \times F_{D2}\left(\frac{h}{v}\right)$$

$$\mathcal{O}_{D5} = \mathrm{tr}\,[\mathbf{T}\mathbf{V}^\mu]\,\mathrm{tr}\,[\mathbf{T}\mathbf{V}^\nu]\,\mathrm{tr}\,[\mathbf{V}^\mu \mathbf{V}^\nu] \times F_{D5}\left(\frac{h}{v}\right)$$

$$\mathcal{O}_{XU1} = gg' B_{\mu\nu}\,\mathrm{tr}\,[\mathbf{W}^{\mu\nu}\mathbf{T}] \times \left(1 + F_{XU1}\left(\frac{h}{v}\right)\right)$$

$$\mathcal{O}_{D8} = \mathrm{tr}\,[\mathbf{V}_\mu \mathbf{V}_\nu]\,\frac{\partial^\mu h \partial^\nu h}{v^2} \times F_{D8}\left(\frac{h}{v}\right)$$





Notation

$$\mathbf{V}_\mu = U(\mathbf{D}_\mu U)^\dagger$$
$$T = U\tau^3 U^\dagger$$

Summary: Non-linear model

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- Most generic
- Higgs is formally separated from GB sector
- GB and Higgs fields "are" dimensionless
- NP Operators starting at dim 4
- $SU(2)_C$ breaking is explicit
- Suited for composite or heavy Higgs models
- Includes linear representation as special case

Higgs: Linear Form

- SM Higgs sector consists of three Goldstone bosons $w^{\pm}$, z and a scalar Higgs boson h
- Associating U with linear representation of Higgs H

$$U \to \frac{1}{v} H = \frac{1}{v} \begin{pmatrix} h + v - \mathrm{i} w^3 & -\mathrm{i}\sqrt{2} w^+ \\ -\mathrm{i}\sqrt{2} w^- & h + v + \mathrm{i} w^3 \end{pmatrix}$$

$\Rightarrow$ SM Model Lagrangian (important terms for VBS)

$$\begin{aligned} \mathcal{L}_{min} = & -\frac{1}{2} \operatorname{tr}\left[\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}\right] - \frac{1}{2} \operatorname{tr}\left[\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}\right] \\ & - \frac{1}{2} \operatorname{tr}\left[\left(\mathbf{D}_\mu H\right)^\dagger \mathbf{D}^\mu H\right] - \frac{\mu^2}{4} \operatorname{tr}\left[H^\dagger H\right] + \frac{\lambda}{16} \left(\operatorname{tr}\left[H^\dagger H\right]\right)^2 \end{aligned}$$

- Continuous interpolation between non-linear and linear

Kilian, Riesselmann: hep-ph/9801265 (1998)

Custodial Symmetry

- Lowest $SU(2)_C$ violating term has dimension 6

$$\mathcal{L}_\beta = \frac{f_\beta}{8\Lambda^2} \operatorname{tr}\left[H^\dagger \tau^3 \mathbf{D}_\mu H\right] \operatorname{tr}\left[H^\dagger \tau^3 \mathbf{D}^\mu H\right]$$

$\Rightarrow$ Suppressed by energy scale $\frac{1}{\Lambda^2} \rightarrow \beta'$ is small $\rightarrow \rho_* \approx 1$

- $SU(2)_C$ violating AC is more suppressed than corresponding $SU(2)_C$ conserving AC

Anomalous Quartic Gauge Coupling

$$\mathcal{L}_{S0} = \frac{f_{S0}}{\Lambda^4} \operatorname{tr}\left[(\mathbf{D}_\mu H)^\dagger \mathbf{D}_\nu H\right] \operatorname{tr}\left[(\mathbf{D}^\mu H)^\dagger \mathbf{D}^\nu H\right]$$

$$\mathcal{L}_{\alpha_6} = \frac{f_{\alpha_6}}{\Lambda^6} \operatorname{tr}\left[(\mathbf{D}_\mu H)^\dagger \mathbf{D}_\nu H\right] \operatorname{tr}\left[H^\dagger \tau^3 \mathbf{D}^\mu H\right] \operatorname{tr}\left[H^\dagger \tau^3 \mathbf{D}^\nu H\right]$$

Higgs Doublet Representation

Buchmüller, Wyler : Nucl. Phys B286 (1986) Hagiwara, Ishihara, Szalpaski, Zeppenfeld Phys Rev D48 (1993)

Relation to matrix realization:

$$\phi = H \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \tilde{\phi} = H \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

Example:

$$\begin{aligned} \mathcal{L}_{S,0} &= \frac{f_{S,0}}{\Lambda^4} \left[(\mathbf{D}_\mu \Phi)^\dagger \mathbf{D}_\nu \Phi \right] \left[(\mathbf{D}^\mu \Phi)^\dagger \mathbf{D}^\nu \Phi \right] \\ &= \frac{f_{S,0}}{\Lambda^4} \operatorname{tr} \left[(\mathbf{D}_\mu H)^\dagger (1 + \tau_3) \mathbf{D}_\nu H \right] \operatorname{tr} \left[(\mathbf{D}^\mu H)^\dagger (1 + \tau_3) \mathbf{D}^\nu H \right] \end{aligned}$$

Higgs doublet realization will break down custodial symmetry $SU(2)_C$

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- GB and Higgs form Higgs sector
- NP from operators with dimension > 4

$\rightarrow$ Dimension of operators are more natural

- Special case of non-linear representation

$\rightarrow$ Fewer operators than non-linear model

- Higgs doublet realization will break $SU(2)_C$

Basis shortcuts

- HISZ — Hagiwara, Ishihara, Szalpaski, Zeppenfeld Phys Rev D48 (1993)
- SILH (Strongly Interacting Light Higgs) — Giudice, Grojean, Pomarol, Ratazzi: hep-ph/0703164 (2007)
- CDZ — Chen, Dawson, Zhang ariv/1311.3107

Reduction of Basis

Methods

- Bianchi Identities
- Jacobi Identities
- $[\mathbf{D}^\mu, \mathbf{D}^\nu] \propto F^{\mu\nu}$
- Integration by parts
- Equation of Motions (EOM)

Literature

- NLM ($\sim$ 82 operators) :

  Buchalla/Cata/Krause: arxiv/1307.5017

- Higgs doublet (dim 6):

  Grzadkowski, Iskrzynski, Misiak, Rosiek : arxiv/1008.4884 (2010)

$\rightarrow$ # of operators 80 $\rightarrow$ 59 (baryon number conserving)

- 8dim TGC

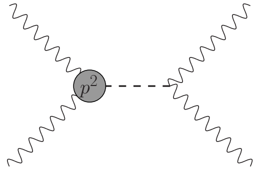
  Degrande: arxiv/1308.6323

Example EOM

$$\mathcal{L}_h = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{4} v^2 \operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] \left(1 + \frac{h}{v}\right) + \frac{1}{2} m_h^2 h^2$$

Operator reduction via EOM

$$\begin{aligned}
\partial^2 h &= \frac{1}{4} v \operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] && + m_h^2 h \\
\Rightarrow \frac{1}{\Lambda} \operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] \partial^2 h &\sim \frac{1}{4} \frac{v}{\Lambda} \operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right]^2 && + \frac{m_h^2}{\Lambda} \operatorname{tr}\left[\mathbf{V}^\mu \mathbf{V}_\mu\right] h \\
\frac{p^2}{p^2 - m_h^2} &= 1 && + \frac{m_h^2}{p^2 - m_h^2}
\end{aligned}$$



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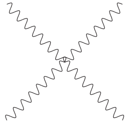

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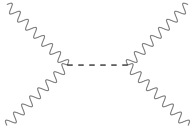

More about EOM

- S-matrix is invariant under
  - EOM transformation   Georgi: Nuclear Phys B361 (1991)
  - Field redefinition   Bergere, Lam: Phys Rev D 13 (1976)
- To reduce an operator of given dimension,
  all operators with lower dimension have to be taken into account
- Reduction by EOM $\equiv$ Reduction by field redefinition

(Re)Normalization

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Contribution of Dim 6 Operators to minimal Lagrangian

$$\mathcal{O}_{D\phi} = \phi^\dagger \phi \left(D_\mu \phi\right)^\dagger D^\mu \phi \supset v^2 \partial_\mu h \partial^\mu h$$

Effects kinetic term of higgs $\rightarrow$ A redefinition for canonical normalization is necessary

$$h \rightarrow h \left(1 - \frac{c_{D\phi} v^2}{\Lambda^2}\right)$$

Alternative solution mixing of operators

Boos, Bunichev, Dubinin, Kurihara: arxiv/1309.5410

$$\mathcal{O}_{D\phi} = \left(\phi^\dagger \phi - \frac{v^2}{2}\right) \left(D_\mu \phi\right)^\dagger D^\mu \phi$$

Further discussion of operators

- Contribution from resonance or loops ($\frac{1}{16\pi^2}$ suppression)
- Distinguish between strong ($\Lambda \sim 4\pi v$) and weak coupled NP
- Canonical power counting ($\frac{1}{\Lambda^n}$ suppression)
  Naive dimensional analysis   Manohar, Georgi: Nucl.Phys. B234 189
  Counting of chiral dimensions   Buchalla, Cata, Krause: arxiv/1312.5624
- NLO treatment
  - Renormalization
  - UV behavior   Passarino arxiv/1209.5538
  - Renormalization group evolution   Jenkins, Manohar, Trott: arxiv/1308.2627

Summary

- NP operators extend minimal model (U, h, H, ϕ)
- Linear representation is special case of non-linear model
- Explicit $SU(2)_L \times SU(2)_R$ symmetry besides Higgs doublet realization
- Using building blocks to create complete operator set systematically

$\rightarrow$ Reduction to minimal set of operators

Backup Slides

Power Counting/Rescaling of Operators

Rescaling of Naive Dimensional Analysis

$$\mathcal{L} \sim v^2\Lambda^2 \left(\frac{\psi}{v\sqrt{\Lambda}}\right)^a \left(\frac{H}{v}\right)^b \left(\frac{yH}{\Lambda}\right)^c \left(\frac{D}{\Lambda}\right)^d \left(\frac{gX}{\Lambda^2}\right)^e$$

$$\frac{v^2}{\Lambda^4} g^3 X^3,\ \frac{\Lambda^2}{v^4} H^6,\ \frac{1}{v^2} H^2 D^2,\ \frac{1}{\Lambda^2} g^2 X^2 H^2,$$

$$\frac{1}{v^2} y\psi^2 H^3,\ \frac{1}{\Lambda^2} y\psi^2 yXH,\ \frac{1}{v^2}\psi^2 H^2 D,\ \frac{1}{v^2}\psi^4$$

Chiral Dimension

$$[\partial_\mu]_C = 1,\ [\phi]_C = [h]_C = 0,\ [X_{\mu\nu}]_C = 2,$$

$$[\psi]_C = \frac{1}{2},\ [g]_C = [y]_C = 1$$

Continuous Interpolation/Transition

Kilian, Riesselmann: hep-ph/9801265 (1998)

$$U = \frac{1}{\sqrt{2}}\left[\left(\frac{v}{\eta} + H\right)\exp\left(\mathrm{i}\frac{\eta}{v}w^a\tau^a\right) + \left(1 - \frac{1}{\eta}\right)v\right]$$

- $\eta \to 1$
  - Non-linear representation
  - Purely non-decoupling scenario
  - Additional operators needed to get renormalizable theory

  $\Rightarrow$ Naive or chiral power counting needed
- $\eta \to 0$
  - Linear representation
  - Decoupling limit
  - Renormalizable

  $\Rightarrow$ Canonical power-counting possible