Unruh effect and Quantum Computing

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discussing with

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Synthetic Unruh effect in cold atoms

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We propose to simulate a Dirac field near an event horizon using ultracold atoms in an optical lattice. Such a quantum simulator allows for the observation of the celebrated Unruh effect. Our proposal involves three stages: (1) preparation of the ground state of a massless 2D Dirac field in Minkowski spacetime; (2) quench of the optical lattice setup to simulate how an accelerated observer would view that state; (3) measurement of the local quantum fluctuation spectra by one-particle excitation spectroscopy in order to simulate a De Witt detector. According to Unruh's prediction, fluctuations measured in such a way must be thermal. Moreover, following Takagi's inversion theorem, they will obey the *Bose-Einstein* distribution, which will smoothly transform into the *Fermi-Dirac* as one of the dimensions of the lattice is reduced.

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The Unruh effect and its applications

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It has been thirty years since the discovery of the Unruh effect. It has played a crucial role in our understanding that the particle content of a field theory is observer dependent. This effect is important in its own right and as a way to understand the phenomenon of particle emission from black holes and cosmological horizons. Here, we review the Unruh effect with particular emphasis to its applications. We also comment on a number of recent developments and discuss some controversies. Effort is also made to clarify what seems to be common misconceptions.

Unruh effect

- An accelerated observer through the flat Minkowski spacetime vacuum will observe a thermal bath, a : constant acceleration

$$k_B T_U = \frac{\hbar a}{2\pi}.$$

- Similar to Hawking radiation
- Event Horizon (Black wall vs Schwarzschild radius) prevents communication between different region of space time, which plays a role of heat bath
- Thermal effect without underlying stochasticity

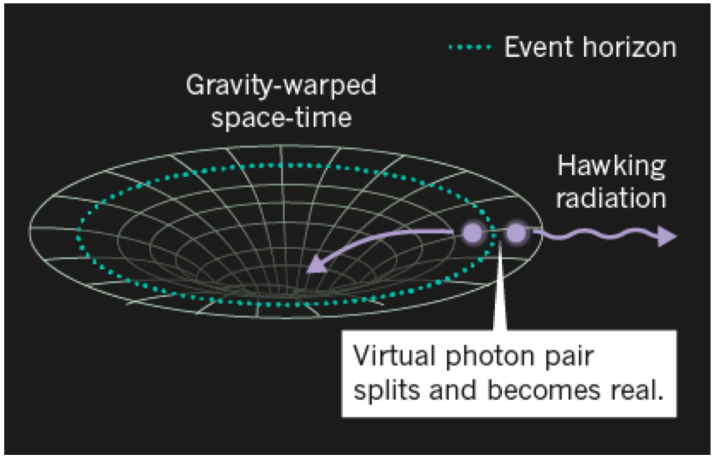
$\longrightarrow$ Thermal mixed state from pure quantum state

Hawking radiation

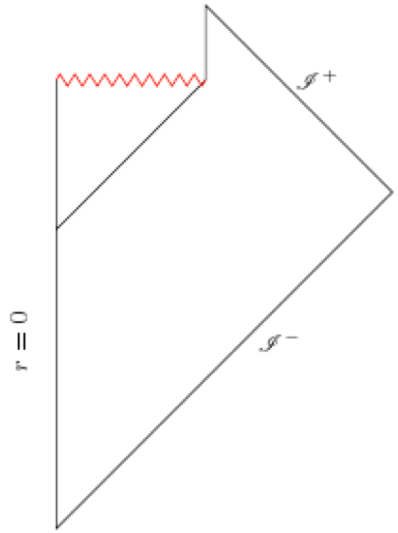
- Black Hole of mass M
- Vacuum fluctuation, particle-antiparticle pair creation/annihilation, near Schwarzschild radius
- one of pair escape from BH, and the other fall into BH
- Emitted particle spectrum turns out to be blackbody radiation with temperature

$$T = \frac{\hbar c^3}{8\pi k G M}$$

- Blackhole lose mass (energy conservation), eventually evaporate, information paradox



[Nature]



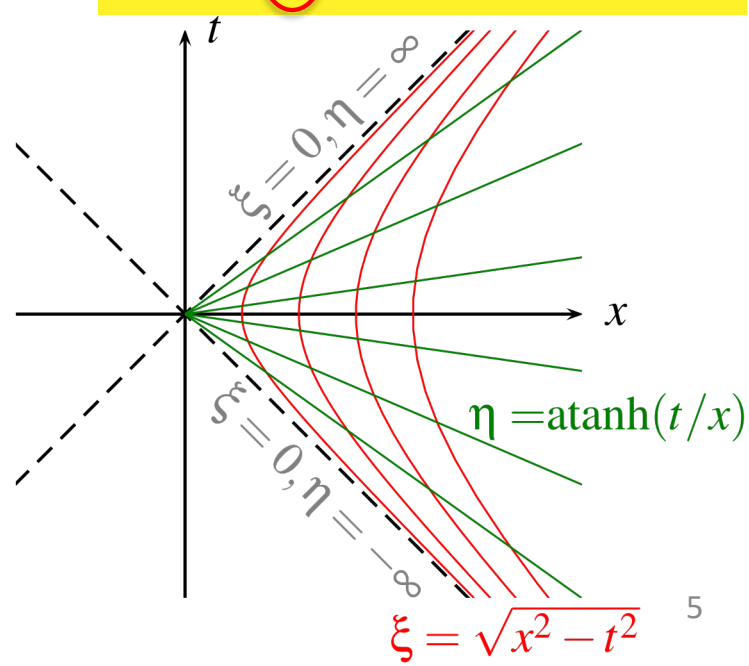
[Wikipedia]

Rindler spacetime

- Flat Minkowski spacetime
- Consider an observer with constant acceleration a=1 in x-axis. Observer is at rest at t=0 and x=1
- Natural coordinate for the observer is Rindler coordinate (co-moving coordinate) (t,x) -> (ξ, η)

$$\begin{cases} t = \xi \sinh \eta \\ x = \xi \cosh \eta \end{cases}$$

- The observer's trajectory is ξ=1, and all η
- The Minkowski metric in Rindler coordinate $ds^2 = -\xi^2 d\eta^2 + d\xi^2 + dy^2 + dz^2$
- spacetime is separate into two parts, left and right Rindler wedges, doesn't communicate each other
- From observer's point of view (ξ=1), light moves at ξ<1 is slower than 1.
- At ξ=0, light stops (Black wall).



Thermalization theorem

- Spacetime with time-like Killing vector (metric is invariant along with the vector) has well defined energy
- Thermalization theorem (Tolman-Eherenfest):

For any thermal equilibrium field, local temperature times time-like Killing vector is constant: $T \cdot g_{00}^{1/2} = \mathrm{const}$

c.f. redshift frequency

$$\nu(P') = \nu(P) \cdot \sqrt{g_{00}(P)/g_{00}(P')}$$

- Vacuum of Massless Dirac field (Fermi sea)

$$|0_M\rangle = \prod_{\omega_k^M < 0} b_k^\dagger \, |\Omega\rangle$$

- Vacuum of accelerated observable, Hamiltonian H_R

$$|0_R\rangle = \prod_{\omega_q^R < 0} d_q^\dagger \, |\Omega\rangle$$

- Minkowski vacuum is not vacuum (either eigenstate) of H_R
- For the observer, field in left Rindler Wedge (LRW) is not causal, state of LRW dof is traced out (loss of information)

$$\rho_R = \mathrm{Tr}_L \left|0_M\right\rangle \left\langle 0_M\right|$$

- Density matrix can always formally written as a thermal state using entanglement Hamiltonian H_E $\rho_R = \exp(-H_E)$
- H_E and H_R commute according to boost invariance of Minkowski vacuum $0 = \dot{\rho}_R = -\frac{i}{\hbar}[\rho_R, H_R]$
- It can be explicitly proved H_E is proportional to H_R

$$\rho_R = \exp\left(-\frac{H_R}{k_B T_U}\right) \qquad k_B T_U = \frac{\hbar a}{2\pi}.$$

- Minkowski vacuum turns out stationary to an accelerated observer

Unruh effect

- Two different mode expansions of the same fermion field with definite wavefunctions
- Bogoliubov transformation $d_q^\dagger = \sum_k U_{qk} b_k^\dagger$
- Occupation of Rindler mode on the Minkowski vacuum (IR cutoff by radius ε)

$$n_{q,\mathbf{r}} \equiv \int_{D_{\mathbf{r},\epsilon}} \langle 0_M | \, d_q^\dagger \, | \mathbf{r}' \rangle \, \langle \mathbf{r}' | \, d_q \, | 0_M \rangle = \sum_{\omega_k^M < 0} |\tilde{U}_{qk}(\mathbf{r})|^2,$$

$$n_{q,\mathbf{r}} = \frac{1}{\exp(\hbar\omega_q^R / k_B T_U(\mathbf{r})) + 1}$$

$$k_B T_U(\mathbf{r}) = \frac{\hbar}{2\pi x}$$

- De Witt detector (minimally coupled to fermion field at r), whose absorption/emission is written by Wightman function

$$G(t) \equiv \langle 0_M | \; c_{x(t)}^\dagger(t) \; c_{x(0)}(0) \; | 0_M \rangle$$

x(t) : trajectory of the observer, c_x is the annihilation operator at x

- detector response function G(ω) , Fourier transformation of G(t)

Unruh effect (contd.)

- Minkowski operator b_k in terms of coordinate space operator c_x (M_{kx} : wave function = plane wave)

$$b_k^\dagger = \sum_x M_{kx} c_x^\dagger$$

- Rindler operator d_q in terms of coordinate space operator c_x (R_{qx} : wave functions)

$$d_q^\dagger = \sum_x R_{qx} c_x^\dagger$$

- c.f. Bogoliubov transformation

$$d_q^\dagger = \sum_k U_{qk} b_k^\dagger \qquad U_{qk} = \sum_x R_{qx} \bar{M}_{kx}$$

- Detector response function

$$G_{x_0}(\omega) \equiv \int \mathrm{d}t\, \mathrm{e}^{-i\omega t} \langle 0_M | c_{x_0}^\dagger(t) c_{x_0}(0) | 0_M \rangle$$
$$= \sum_{q,q'} \delta(\omega - \omega_q^R) \bar{R}_{qx_0} R_{q'x_0} \sum_{\omega_k^M < 0} \bar{U}_{qk} U_{q'k} = \frac{1}{\exp(\hbar\omega_q^R / k_B T_U(\mathbf{r})) + 1}$$

$$k_B T_U(\mathbf{r}) = \frac{\hbar}{2\pi x}$$

is the Fermi-Dirac thermal distribution (in 1+1 dim and 3+1 dim)

is Bose-Eistein distribution (in 2+1 dim) for Fermion, and opposite for Boson (Takagi's inverse theorem)

$$G(\omega) = \frac{1}{\exp(\omega / T(x)) - 1}, \qquad T(x) = \frac{1}{2\pi x}$$

Dirac fermion in Rindler Lattice

- Minkowski Dirac Hamiltonian

$$i\partial_0\psi = \mathcal{H}\psi = -i\gamma_0\gamma^j\partial_j\psi \qquad i\partial_t\psi = -i\left(\partial_x\sigma_x + \partial_y\sigma_y\right)\psi$$

- curved spacetime Hamiltonian (w$_\mu$: spin connection)

$$\partial_\mu\psi \to D_\mu\psi \equiv \left(\partial_\mu + \frac{1}{4}w_\mu^{ab}\gamma_{ab}\right)\psi \qquad \gamma_{ab} \equiv \frac{1}{2}[\gamma_a, \gamma_b].$$

$$i\partial_t\psi = -i\gamma_t\left(\gamma^j\partial_j + \frac{1}{4}\gamma^j w_j^{ab}\gamma_{ab} + \frac{1}{4}\gamma^t w_t^{ab}\gamma_{ab}\right)\psi$$

- Rindler coordinate $ds^2 = -x^2dt^2 + dx^2 + dy^2$
- spin connection (others are zero) $w_t^{01} = x/|x|$
- Dirac equation and Hamiltonian density

$$i\partial_t\psi = -i\left(\left(|x|\partial_x + \frac{1}{2}\frac{x}{|x|}\right)\sigma_x + |x|\partial_y\sigma_y\right)\psi \qquad \mathcal{H}_R = -i\left(\left(|x|\partial_x + \frac{1}{2}\frac{x}{|x|}\right)\sigma_x + |x|\partial_y\sigma_y\right)$$

$$\mathcal{H}_R = \sqrt{x}\not{p}\sqrt{x}.$$

- Second quantized Hamiltonian $H_R = \int \mathrm{d}x\mathrm{d}y\,\bar{\psi}^\dagger\mathcal{H}_R\psi$
- x=0 is the boundary (any boundary condition is allowed at |x|=0)

Eigenstate of Rindler Hamiltonian in 1+1 dim

- 1-particle Hamiltonian (density) $H_{R(1D)} = \sqrt{x}p\sqrt{x}$
- variable transformation u = log(x). (boundary x=0 is u→ - ∞)

$$H_{R(1D)} = -i(\partial_u + 1/2)$$

- eigenvalue equation

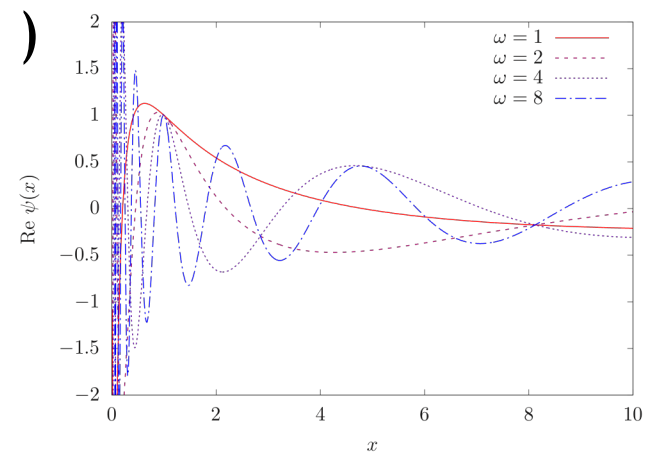
$$-i\,(x\partial_x + 1/2)\,\psi(x) = -i\,(\partial_u + 1/2)\,\psi(u) = \omega\psi(u)$$

- eigen functions (plane waves in u = log(x))

$$\psi(u) = A\ \exp\left[\left(i\omega - \frac{1}{2}\right)u\right] = A\ x^{i\omega - 1/2}$$

orthonormal

$$\int_0^\infty \mathrm{d}x \exp\left((-i\omega - 1/2)u\right) \exp\left((i\omega' - 1/2)u\right) =$$
$$\int_{-\infty}^\infty \mathrm{d}u \exp(-i\omega u) \exp(i\omega' u) = \delta(\omega - \omega').$$



- Spinor structure

$$-i\,(x\partial_x + 1/2)\,\sigma_x \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \omega \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} \qquad \begin{pmatrix}\psi_1(x,t)\\ \psi_2(x,t)\end{pmatrix} = A \begin{pmatrix}1\\ \pm 1\end{pmatrix} x^{i\omega - 1/2} e^{-i\omega t}$$

Lattice Discretization

- 1+1 dim $$H_{R(1D)} = \sqrt{x}p\sqrt{x}$$
- x_m = m d, d : lattice spacing, m = -(L-1) / 2, …, (L-1)/2, even L
- symmetric derivative for $p = -i\partial_x$

$$(p\psi)_m = -i\left(\psi_{m+1} - \psi_{m-1}\right)/(2d)$$

- 1 particle Hamiltonian $$H_{R(1D)} = \sqrt{x}p\sqrt{x}$$

$$R_{m,m+1} = -\frac{i}{2}\sqrt{m(m+1)}$$

- for 2+1 dim

$$H_R = -\sum_{m,n} t_0' \Big((m+\tfrac{1}{2})\, e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m+1,n} + m\, e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m,n+1} \Big) c_{m,n} + \text{H.c.},$$

Simulating Unruh effect

- Detector response function G(ω), Fourier transformation of

$$G(t) \equiv \langle 0_M | \; c^\dagger_{x(t)}(t) \; c_{x(0)}(0) \; | 0_M \rangle$$

- Measure overlap between the states of one-hole excitation at t=0 and t
- In Right Rindler coordinate, observer's trajectory is x(0) = x(t) = x_0 (frame independence, equivalence principle)

- Strategy : quench from Minkowski Hamiltonian to Rindler spacetime

1. Prepare Mikowski vacuum
2. Quench : Switch Hamiltonian from Minkowski H_M to Rindler H_R
   quench introduces event horizon, disconnecting the L/R Wedges
3. Measure G(t) in Rindler frame at $x=x_0$

Numerical simulation

- 2+1 dim and 1+1 dim
- space size Lx, Ly
- time step t0 for Minkowski
- time step t0' for Rindler
  chose t0' = 2 t0 / Lx
- Lattice spacing d = t0

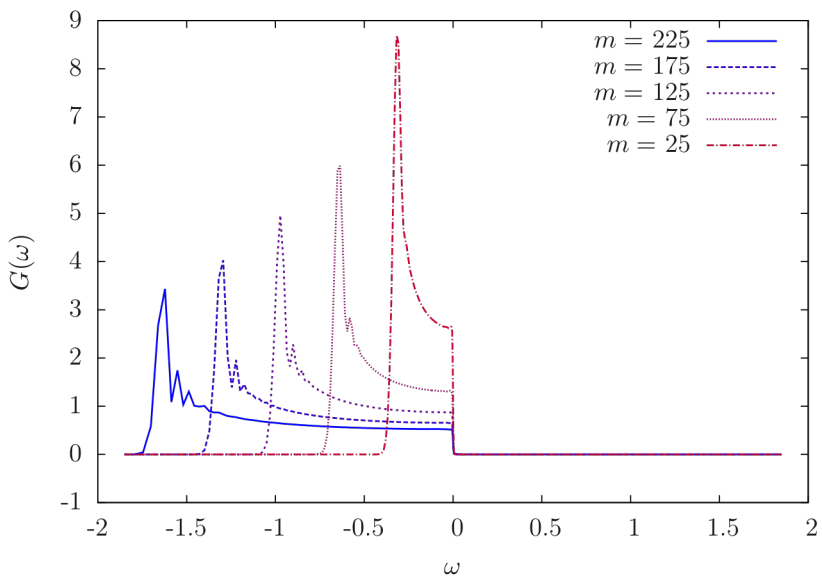
$$H_M = -\sum_{m,n} t_0 \left(e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m+1,n} + e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m,n+1} \right) c_{m,n} + \text{H.c.}$$

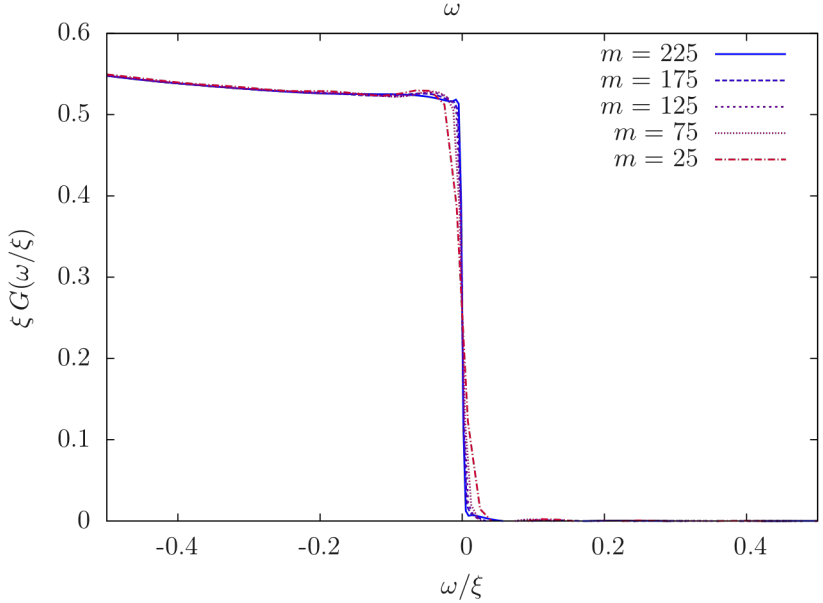
$$H_R = -\sum_{m,n} t'_0 \left((m+\tfrac{1}{2})\, e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m+1,n} + m\, e^{i\frac{\pi}{2}(m-n)} c^\dagger_{m,n+1} \right) c_{m,n} + \text{H.c.},$$

expectation $G(\omega) = \dfrac{1}{\exp\left(\omega/T(x)\right)+1}, \qquad T(x) = \dfrac{1}{2\pi x}$

$$\xi G(\omega/\xi) = \xi \frac{1}{\exp\left(\pi L_x \omega\right)+1}, \qquad \xi = 2x/L_x$$

- Lx = 500, 1+1 dim
- x = md (smaller x is hotter temperature)
- $|\omega / t_0| > 1$ is time-step discretization

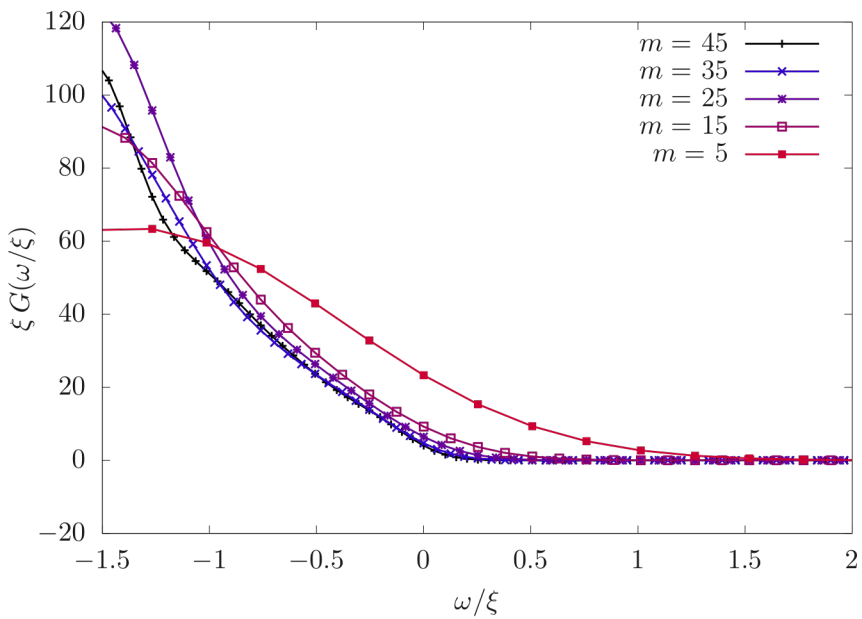




$$G(\omega) = \frac{1}{\exp\left(\omega/T(x)\right) + 1}, \qquad T(x) = \frac{1}{2\pi x}$$

$$\xi G(\omega/\xi) = \xi \frac{1}{\exp\left(\pi L_x \omega\right) + 1}, \qquad \xi = 2x/L_x$$

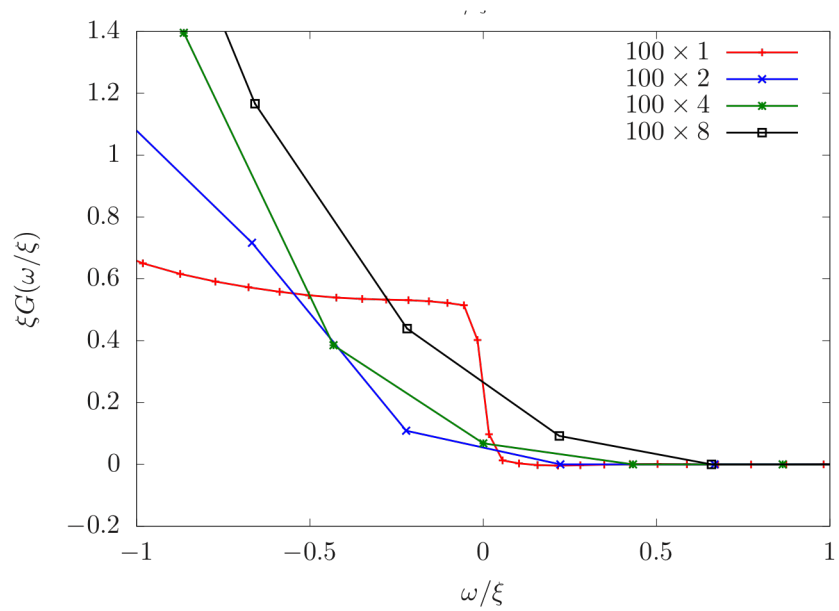
- 2+1 dim Lx = Ly = 100
- Bose Einstein distribution



$$G(\omega) = \frac{1}{\exp\left(\omega/T(x)\right) - 1}, \qquad T(x) = \frac{1}{2\pi x}$$

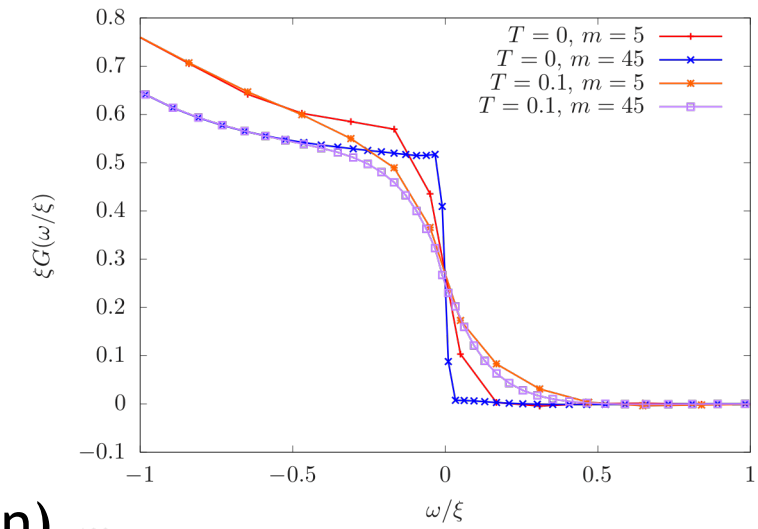
$$\xi G(\omega/\xi) = \xi \frac{1}{\exp\left(\pi L_x \omega\right) - 1}, \qquad \xi = 2x/L_x$$

- transition from 1+1 dim to 2+1 dim

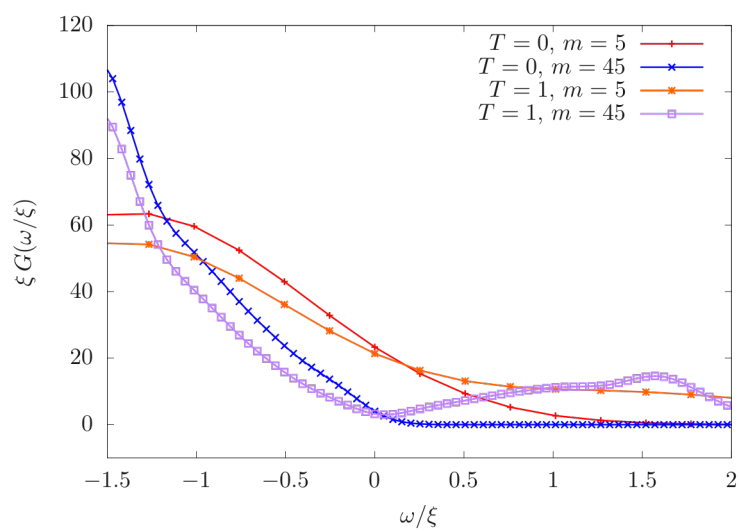


Discretization error

- 1+1 dimension (Fermi-Dirac)



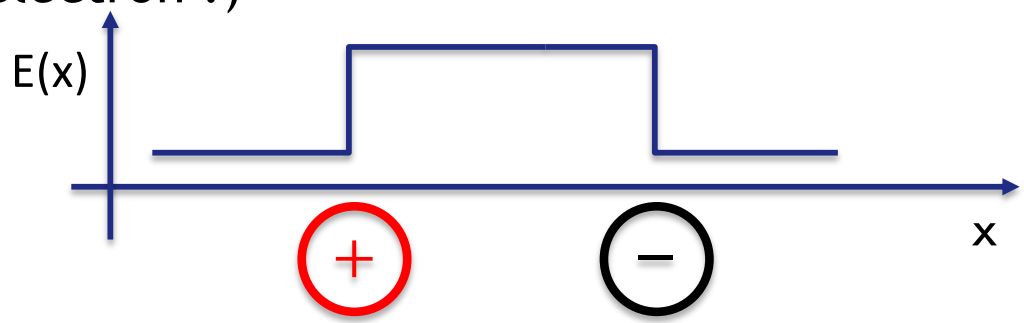
- 2+1 dim (Bose-Einstein)



Discussions and Prospects

- Authors proposed optical lattice experiment
- Apply this in digital quantum computation?
  Number of Qbits too demanding ?

- Unruh effect for interacting theory such as Schwinger model?
  (radiation for photon and electron ?)



- Pure quantum state, namely Minkowski vacuum, turns into thermal mixed state preparation, which is exponentially large operation otherwise.
- Minkowski vacuum could be prepared via adiabatic deformation from solvable Hamiltonian (Gumaro, Yuuta et al are now trying ultra-efficient preparation)
- Could we engineer different coordinate system so that the temperature is constant everywhere ?