Running α vs g-2
&
Precision Electroweak Measurements
or
How I Spent My Summer Vacation

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September 11, 2020
HET Lunch Discussion

BROOKHAVEN NATIONAL LABORATORY

Based on:

Muon g−2 and Δα connection

A. Keshavarzi, WJM, M. Passera, A. Sirlin

Phys.Rev.D 102 (2020) 3, 033002

Earlier Work

Muon g-2 and bounds on the Higgs mass

M. Passera, WJM, A. Sirlin

Phys.Rev.D 78 (2008) 013009

result

114.4GeV< m_H < 130GeV. (95% C.L.)

For details see: "The anomalous magnetic moment of the muon in the Standard Model" White Paper

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to $a_\mu^{\rm SM}$. After the experimental number from E821, the first block gives the main results for the hadronic

A Beautiful Relationship

$SU(2)_L xU(1)_Y$ + Higgs Doublet + Renormalizability

- *$\sin^2\theta^0_W = 1-(m^0_W/m^0_Z)^2 = (e^0/g^0)^2$ Natural Bare Relation*

Radiative Corrections to renormalized relations

Finite & Calculable!

Demonstrated by Bollini, Giambiagi & Sirlin (1972)

WJM(1974) Thesis: Finite Parts Calculated

but model incomplete: Charm, Color, 3rd Generation?

time not quite right for full EW Radiative Corrections

Main effect: Vacuum polarization

$\alpha=1/137 \rightarrow \alpha(m_Z)\sim 1/128$ to $1/129$ (definition)

Large 6-7% Effect

Vacuum Polarization

$$\alpha^{-1}(m_Z^2) = \alpha^{-1}(0) - \Sigma Q_f^2/3\pi[\ln(m_Z^2/m_f^2) - 5/3]$$

$$\alpha^{-1}(0) = 137$$

$$\alpha^{-1}(m_Z^2) = 128.5 \text{ (1979 estimate)}$$

Implications

1) 3% increase in m_W & m_Z predictions

2) Factor 10 decrease in SU(5) GUT scale ~10^{14}GeV

Proton lifetime prediction went from exp. Unobservable to IMB ruling out minimal SU(5) Georgi-Glashow Model

3) Weak Mixing Angle Studies Initiate an era of precision Radiative Corrections.

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Vacuum Polarization Running

$$\alpha(q^2) = \alpha / \left(1 - \Delta\alpha(q^2)\right), \text{ where } \Delta\alpha(q^2) = \Delta\alpha_{\text{lep}}(q^2) + \Delta\alpha^{(5)}_{\text{had}}(q^2)$$

leptonic loops 314.979×10^{-4} ~3.15%

Hadronic Part via Dispersion Relation

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{M_Z^2}{4\alpha\pi^2} \, \mathrm{P} \int_{s_{th}}^{\infty} \mathrm{d}s \frac{\sigma_{\text{had}}(s)}{M_Z^2 - s}$$

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)[\text{KNT19}] = (276.09 \pm 1.12) \times 10^{-4}$$

No Comparable Lattice calculation of $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$

Seems relatively easy (high precision).

What is the real alpha?

$\alpha^{-1}(0)=137$ Mystical Number

$\alpha^{-1}(m_Z^2)=129$ On Shell Value

$\alpha^{-1}(m_Z^2)_{MS}=128$ MSbar Definition (GUTS)

M. Goldhaber & WJM E_6 $\alpha(M_{GUT})=3/8$

Dirac 80th Birthday Celebration

$\alpha^{-1}(0)=137.035999149(33)$ g_e-2

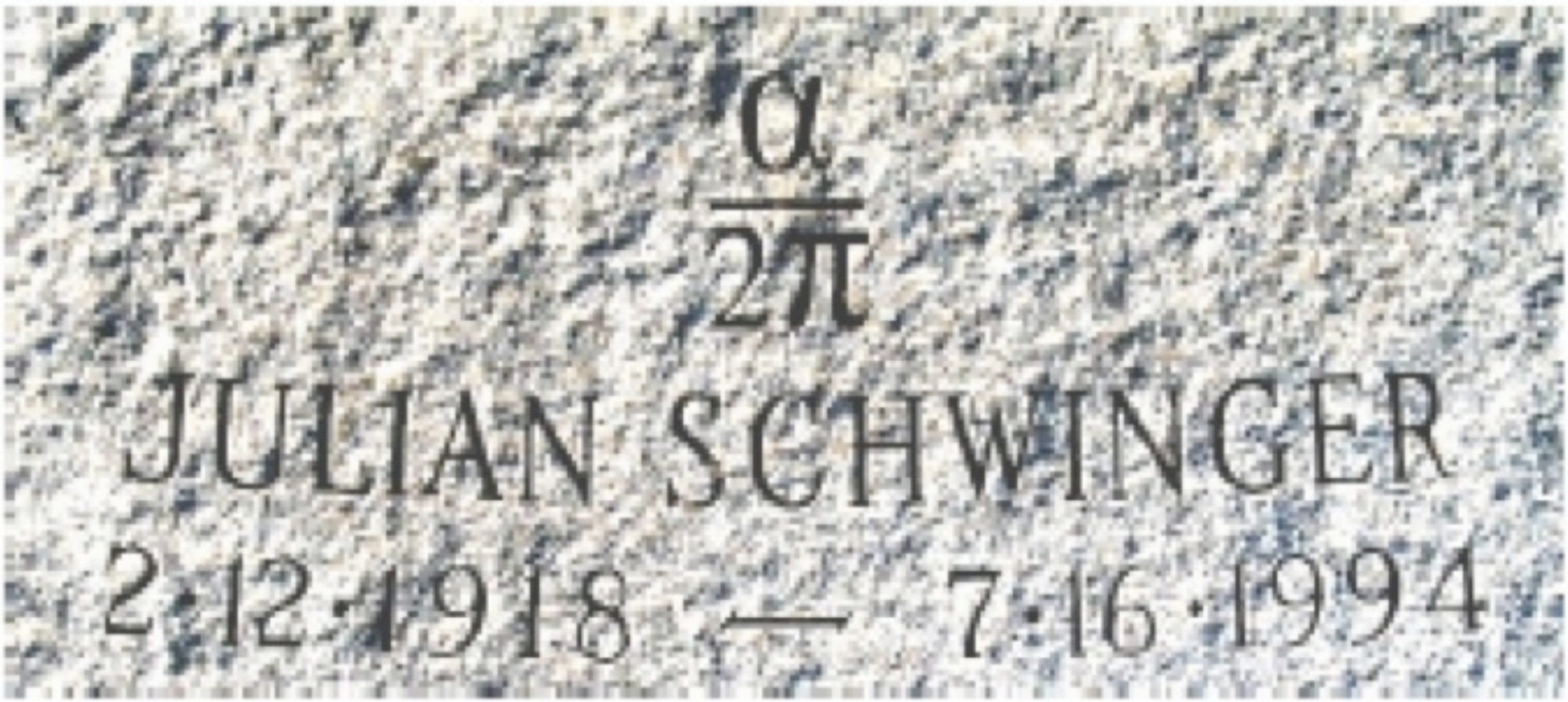
$\alpha^{-1}(0)=137.035999046(27)$ Cs Mass, Ry

2.4 sigma Tension

First α /g-2 Connection

Mount Auburn Cemetery

Courtesy Lee Roberts



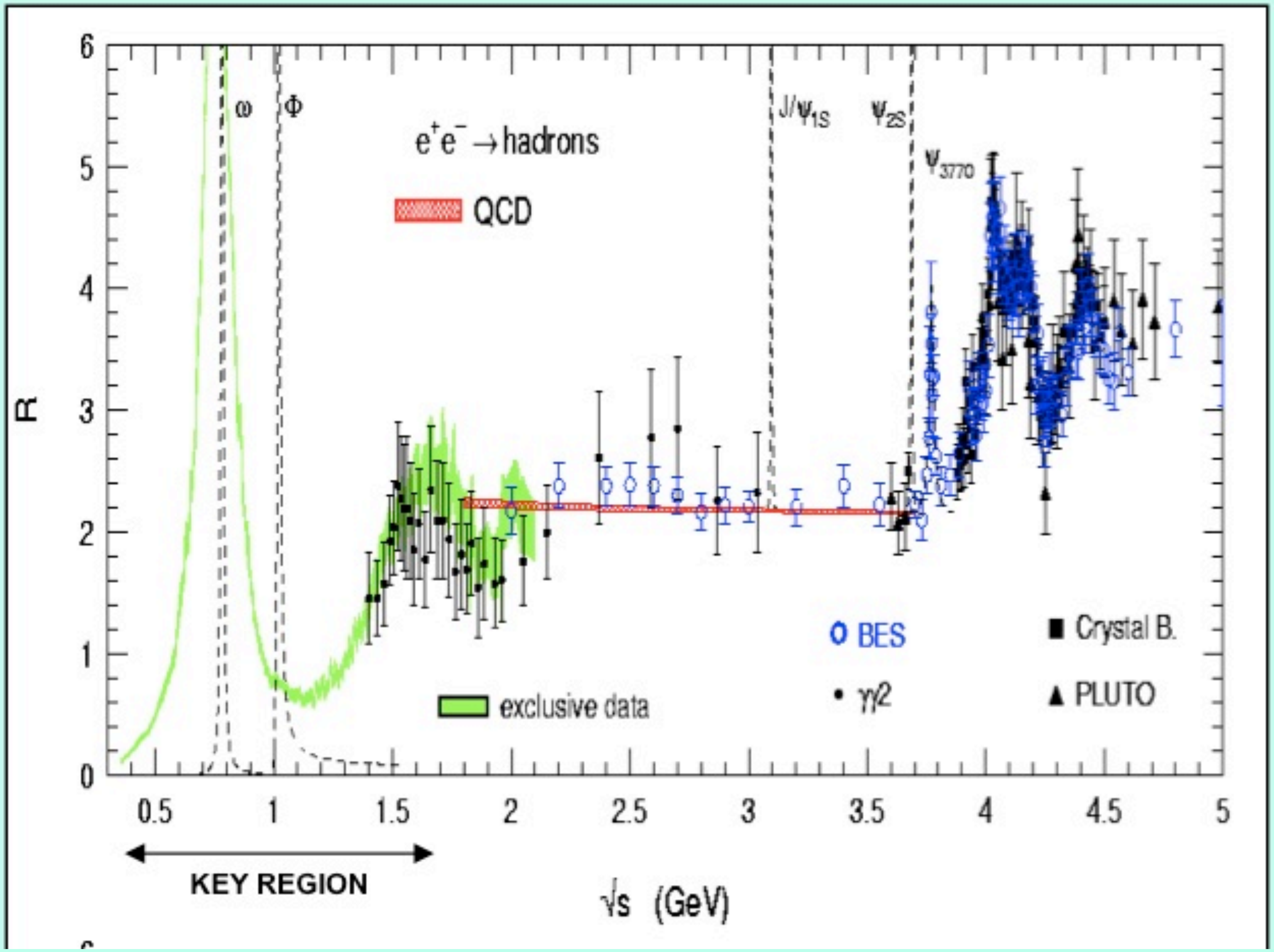
The Hadronic Vacuum Polarization Connection
same data used for α **and g-2**

$$\sigma_{\rm had}(s) \equiv \sigma^0\left(e^+e^- \to \gamma^* \to {\rm hadrons} + (\gamma)\right)$$

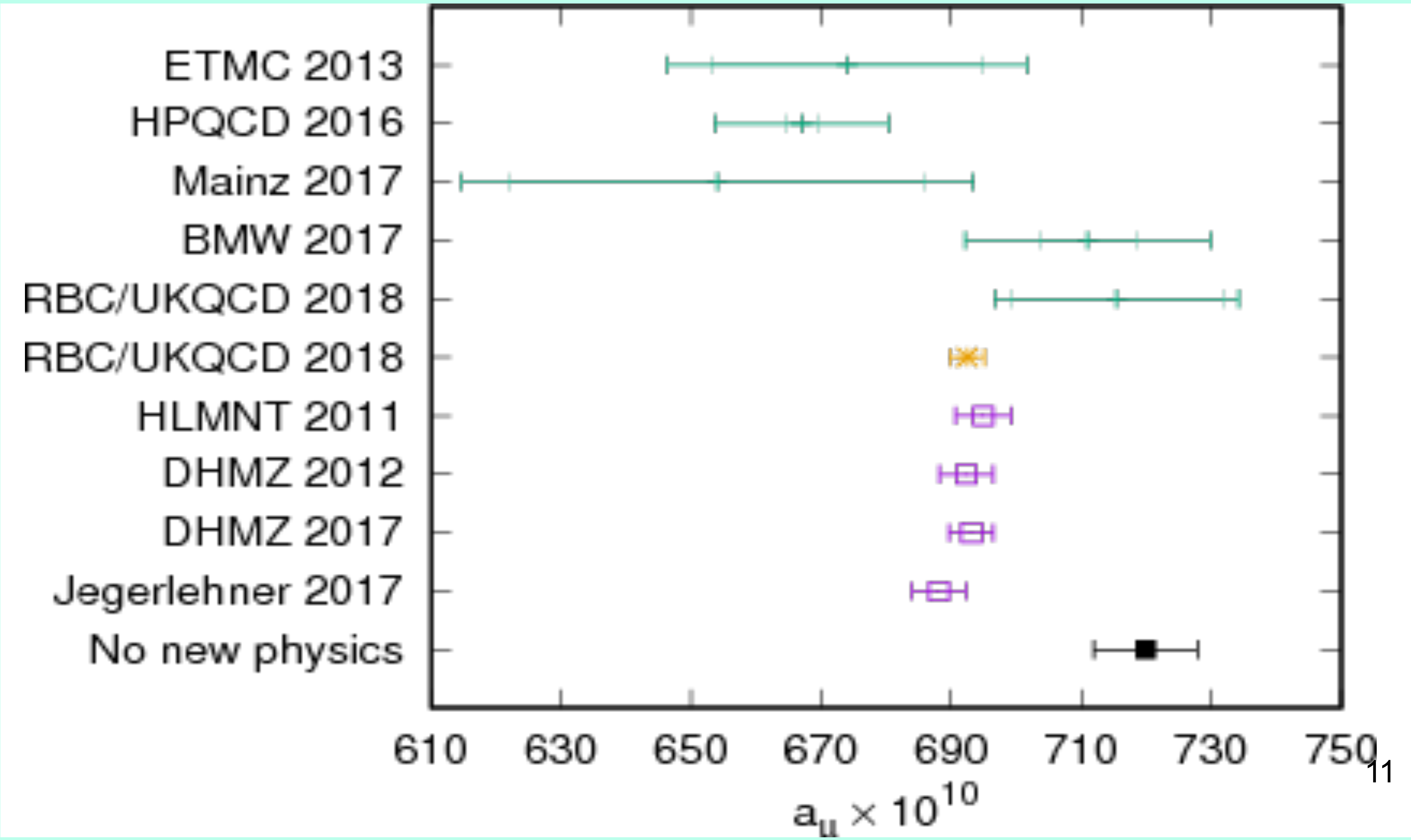
$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = \frac{M_Z^2}{4\alpha\pi^2}\,{\rm P}\int_{s_{th}}^{\infty} {\rm d}s\,\frac{\sigma_{\rm had}(s)}{M_Z^2 - s}$$

$$a_\mu^{\rm had,\,LO\,VP} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} {\rm d}s\,K(s)\,\sigma_{\rm had}(s)$$

$$K(s) = \int_0^1 {\rm d}x\,\frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)}$$


e⁺e⁻ → hadrons
QCD
exclusive data
BES
γγ2
Crystal B.
PLUTO
ω
Φ
J/ψ1S
ψ2S
ψ3770
R
√s (GeV)
KEY REGION

From Blum et al. PRL 2018
Green = Lattice QCD
Purple= e^+e^- data
Gold =Mixed LQCD & e^+e^- data



Dispersive results vs BMW lattice

	KNT	DHMZ	BMW	
$a_\mu^{\text{had, LO VP}}$	6928(24)	6940(40)	7087(53)	$\times 10^{-11}$
$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	279(76)	282(85)	120(90)	$\times 10^{-11}$
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	0.02761(11)	0.02760(10)	-	

BMW challenges "new physics" interpretation of g-2

Naively expect somewhat larger hadronic contribution to $\alpha(\mathbf{m_Z^2})$.

Other tests of Hadronic contribution?

Weighting of DR Integrands

$\alpha_{had}(m_Z^2)$=**0.02761(11). 12.5% rho**

a_μ **$^{had.LO}$= 6928(24)x10$^{-11.}$ 73% rho**

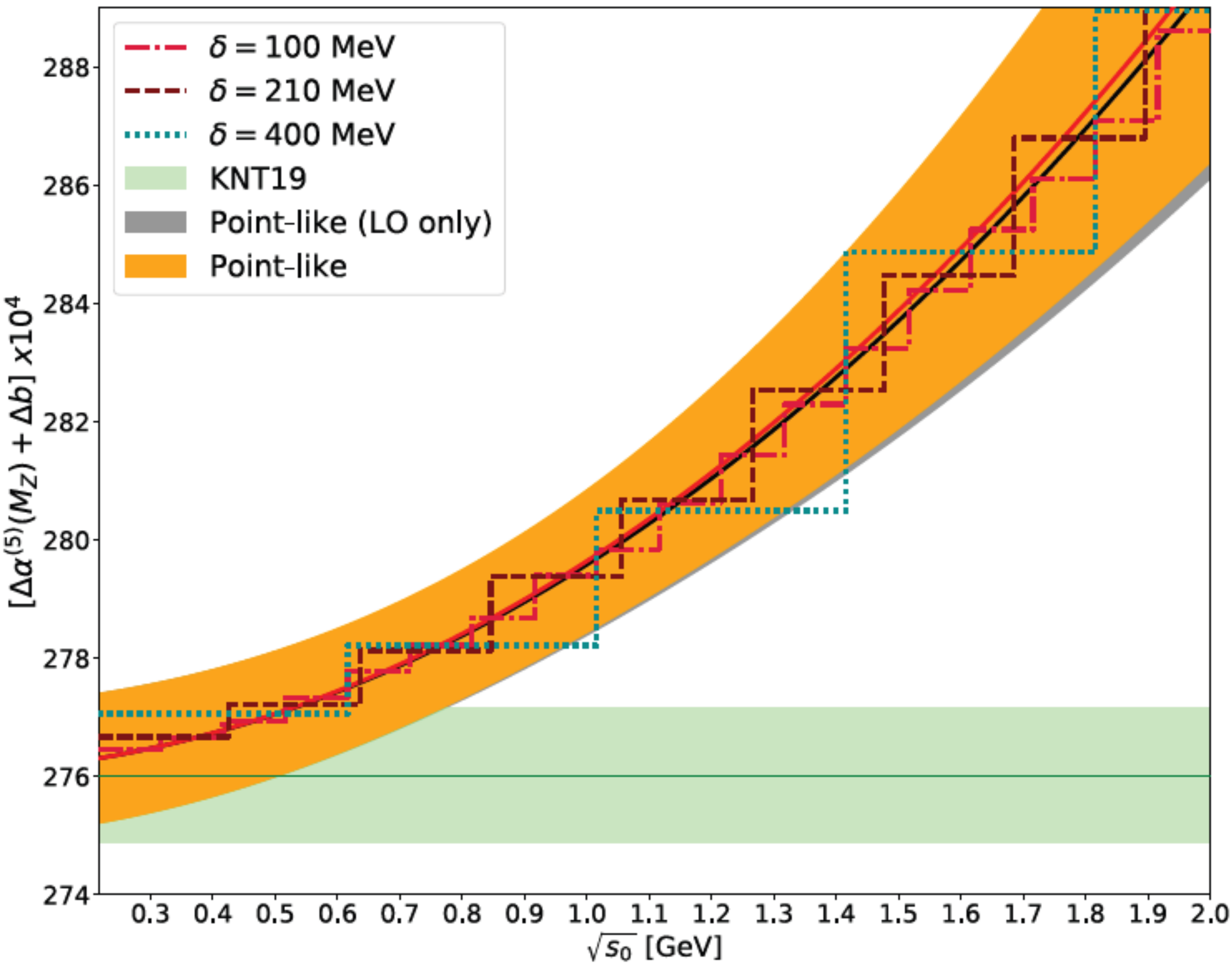
If BMW equivalent to 1.7% uniform shift. Expect 4 sigma increase in $\alpha_{had}(m_Z^2)$. If all from rho, expect 1.5 sigma etc

EW parameter	Value used in [7] (2008)	Value used in this work (2020)
M_W [GeV]	80.398(25) [76, 100, 101]	80.379(12) [5]
m_t [GeV]	172.6(1.4) [102]	172.9(4) [5, 105–107]
$\alpha_s(M_Z^2)$	0.118(2) [103]	0.1179(10) [5]
$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$	0.02768(22) [65]	0.02761(11) [23]
M_H [GeV]	–	125.10(14) [5, 97–99]
$\sin^2\theta_{\rm eff}^{\rm lep}$	0.23153(16) [77]	0.23151(14) [104]

2: Comparison of the values of the EW precision observables used in [7] and in this work.

Global fit to EW Dat

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W [GeV]	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H [GeV]	125.10(14)	[5]	125.10(14)	$94^{+20}_{-18}{}^{+6}_{-6}$
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t [GeV]	172.9(4)	[5]	173.0(4)	–
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	–
M_Z [GeV]	91.1876(21)	[5].	91.1883(20)	–
Γ_Z [GeV]	2.4952(23)	[5]	2.4940(4)	–
Γ_W [GeV]	2.085(42)	[5]	2.0903(4)	–
$\sigma^0_{\rm had}$ [nb]	41.541(37)	[112]	41.490(4)	–
R^0_l	20.767(25)	[112]	20.732(4)	–
R^0_c	0.1721(30)	[112]	0.17222(8)	–
R^0_b	0.21629(66)	[112]	0.21581(8)	–
$\bar{m}_c$ [GeV]	1.27(2)	[5]	1.27(2)	–
$\bar{m}_b$ [GeV]	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	–
$A^{0,l}_{\rm FB}$	0.0171(10)	[112]	0.01622(7)	–
$A^{0,c}_{\rm FB}$	0.0707(35)	[112]	0.0737(2)	–
$A^{0,b}_{\rm FB}$	0.0992(16)	[112]	0.1031(2)	–
A_ℓ	0.1499(18)	[75, 112]	0.1471(3)	–
A_c	0.670(27)	[112]	0.6679(2)	–
A_b	0.923(20)	[112]	0.93462(7)	–



δ = 100 MeV
δ = 210 MeV
δ = 400 MeV
KNT19
Point-like (LO only)
Point-like
[Δα^(5)(M_Z) + Δb] x10^4
√s_0 [GeV]
274
276
278
280
282
284
286
288
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0

Standard Model Predictions Through 2 loops Assuming No New Physics

$\sin^2\theta_W(m_Z)_{MS} = \pi\alpha/\sqrt{2}m_W^2 G_\mu(1-\Delta r(m_Z)_{MS})$

$\Delta r(m_Z)_{MS} = 0.0693(2)$ $\Rightarrow$ $\sin^2\theta_W(m_Z)_{MS} = \underline{0.23110(9)}$

$\sin^2 2\theta_W(m_Z)_{MS} = 2\sqrt{2}\pi\alpha/m_Z^2 G_\mu(1-\Delta r'(m_t,m_H))$

$\Delta r'(m_t,m_H) = 0.0598(2)$ $\Rightarrow$ $\sin^2\theta_W(m_Z)_{MS} = \underline{0.23124(6)}$ $\pm 0.03\%$

Error Expected to be reduced (improved m_t) to ~ $\pm 0.01\%$

Corresponds to $m_W = 80.362(6)$

Any significant difference with other precise $\sin^2\theta_W$ measurement Implies "New Physics"

Currently $\sin^2\theta_W(m_Z)_{ave} = 0.23119(16)$ Excellent Agreement

Best Off Z Resonance Measurements of $\sin^2\theta_W$

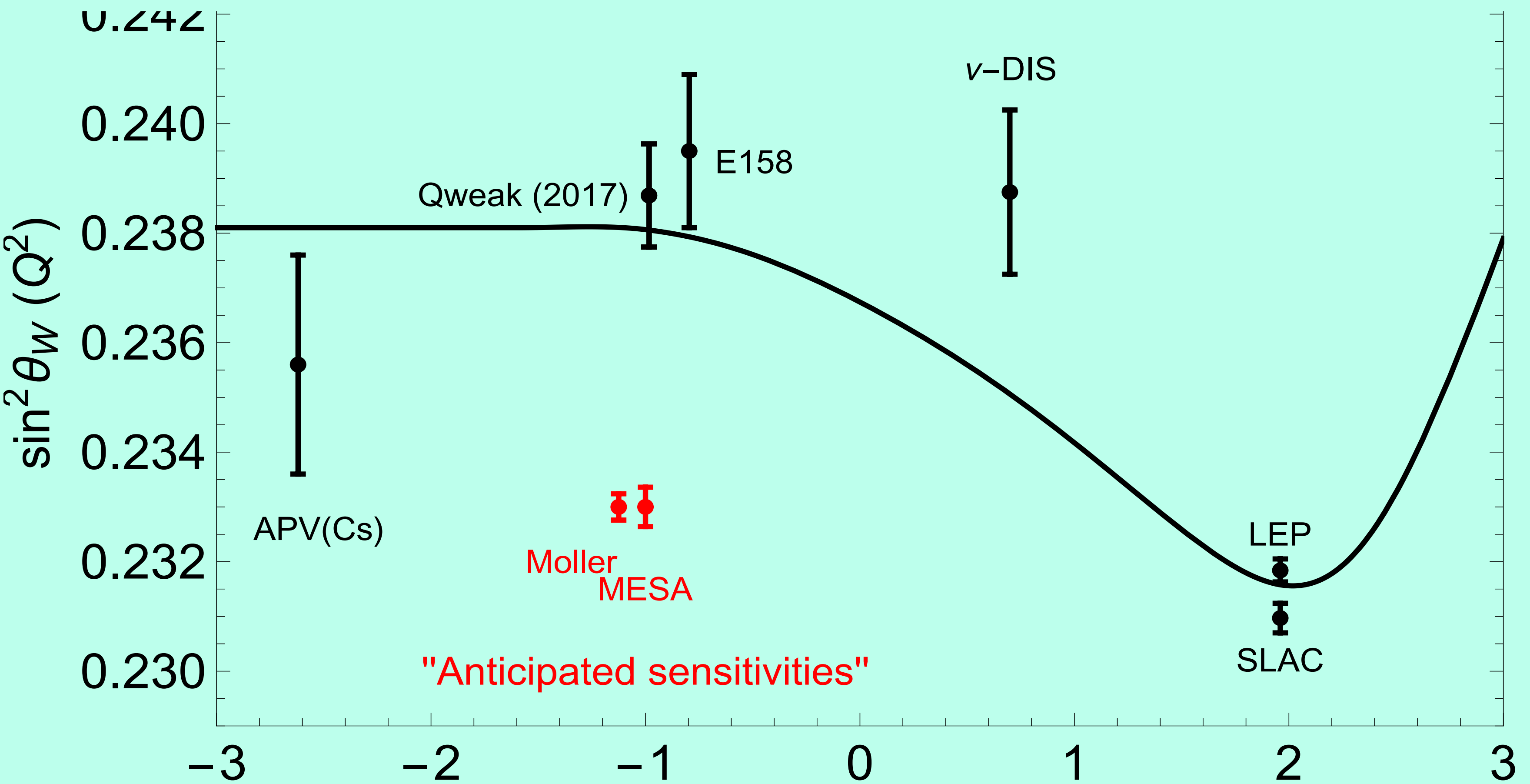
(Not Competitive with Z Pole)

Reaction	$\sin^2\theta_W(m_Z)_{MS}$	<Q>
Cs APV	0.2283(20)	2.5MeV
E158 ee	0.2329(13)	160MeV
Q_{weak} ep	0.2320(9)	160MeV
	0.2310(11)	
6GeV Dis eD	0.2299(43)	1.5GeV
NuTeV $\nu_\mu N$	0.2356(16)	3-4GeV

Average Low Q^2 Determination

$\sin^2\theta_W(m_Z)_{MS}$=0.23216(64) vs 0.23125(16) Z Pole

Recent Qweak Result & Future Sensitive Proposals



Comparison of various

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)$$

$$a_\mu^{\mathrm{had,\,LO\,VP}}[\mathrm{BMW}] = 708.7(5.3) \times 10^{-10}.$$

$$a_\mu^{\mathrm{had,\,LO\,VP}}[\mathrm{KNT19}] = (692.78 \pm 2.42) \times 10^{-10},$$

$$\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2)[\mathrm{KNT19}] = (276.09 \pm 1.12) \times 10^{-4}$$

comparisons

Dispersion rel.	0.02761(11)
All EW Fit	0.02758(11)
Independent Fit.	0.02722(34)
$\sin^2\theta_W(m_Z)_{MS}$=0.23119(14).	**0.02761(66)**
M_W=80.379(12)GEV.	**0.02700(80)**
$\sin^2\theta_W(0)_{MS}$=0.23866(79)	**0.02790(340)**

Conclusions

1) SM muon g-2 prediction pretty solid

DR approach reached amazing precision

~3.5 sigma deviation (still some tension)

passed $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)[\rm KNT19] = (276.09 \pm 1.12) \times 10^{-4}$

stress test

2) Lattice approach has come a long way since

I lent Tom Blum my QED Book (Kinoshita)

Should do α^{-1}**(m_Z^2) and $\sin^2\theta_W(0)$**

muon g-2 has been good to us. (so far)

Acts to Follow

3) Anxiously awaiting Fermilab result

Best and Worst Outcomes?

4) Electron g-2 confrontation may be even

more exciting! No Hadronic Unc.

Clash of two experimental giants

LBL (Parker) vs Northwestern (Gabrielse)

Some Important Precision EW Parameters Tied Together by Natural Relationships

Quantity	2008 Value		2020 Value	Comment
α^{-1}	137.035999084(51)		***137.035999046(27)***	$\alpha^{-1}(a_e)$ vs $\alpha^{-1}(Cs)$
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	0.02768(22)		**0.02761(11)**	
G_μ	$1.16637(1)\times10^{-5}\mathrm{GeV}^{-2}$		$1.1663787(6)\times10^{-5}\mathrm{GeV}^{-2}$	$\tau_{\mu+}$ PSI
m_Z	91 .1875(21)GeV		91.1876(21)GeV	-
*m_t	171.4(2.1)GeV	→	172.6(0.8)GeV	FNAL/LHC
*m_H	>114GeV	→	**125.10(0.24)GeV**	
m_W	80.410(32)GeV	→	80.379(12)GeV	LEP2/FNAL/LHC
$\sin^2\theta_W(m_Z)_{\rm ave}$	0.23125(16)		0.23119(14)	Z Pole Ave.

Best individual Z pole determinations:

$\sin^2\theta_W(m_Z)$	0.23070(26)	0.23070(26)	SLAC A_{LR}
$\sin^2\theta_W(m_Z)$	0.23193(29)	0.23193(29)	CERN $A_{FB}(bb)$

(3 sigma difference?)

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