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The gluon Sivers function in heavy flavor dijet production at the EIC

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Jets for 3D imaging at the EIC workshop
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Gluon Sivers function (GSF)

- **Gauge link dependent gluon TMDs**

$$\Gamma^{[U,U']\mu\nu}(x, p_T; n) = \int \frac{d\xi \cdot P d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P,S| F^{n\mu}(0)\, U_{[0,\xi]} F^{n\nu}(\xi) U'_{[\xi,0]} |P,S\rangle \Big|_{\mathrm{LF}}$$

- **GSF: T-odd object; two gauge links; process dependence more involved**
- **For any process GSF can be expressed in terms of two functions:**

(Buffing, Mukherjee, Mulders'13)

$$f_{1T}^{\perp g[U]}\left(x, \mathbf{k}_\perp^2\right) = \sum_{c=1}^{2} C_{G,c}^{[U]} f_{1T}^{\perp g(Ac)}\left(x, \mathbf{k}_\perp^2\right)$$

calculable for each channel

- $f_{1T}^{\perp g(f)}$ **f-type, C-even**
- $f_{1T}^{\perp g(d)}$ **d-type, C-odd**

$$f_{1T}^{\perp g[e\, p^\uparrow \to e' Q\bar{Q} X]}(x, p_T^2) = - f_{1T}^{\perp g[p^\uparrow p \to \gamma\gamma X]}(x, p_T^2)$$

Gluon Sivers function (GSF)

- **Theory constrain from Burkardt's sum rule: sum of the transverse momenta of quarks and gluons in a transversely polarized nucleon is zero**
- **Various pp scattering processes suggested to probe GSF**

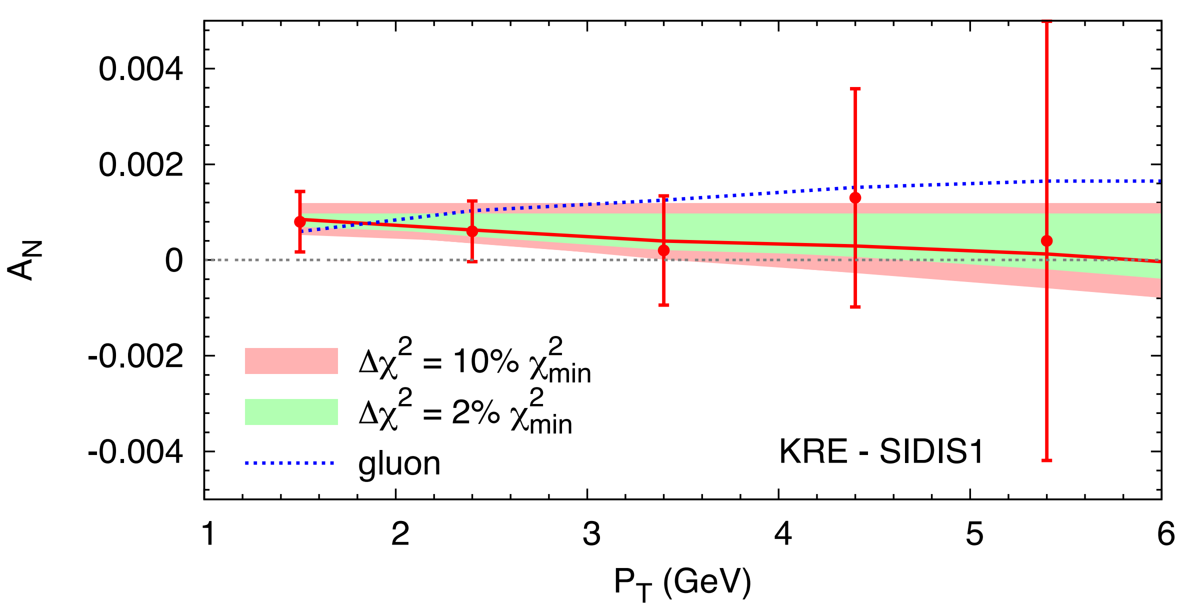
see a review (Boer, Lorce, Pisano, Zhou '15)

$$p^\uparrow p \to \text{jet jet}X \quad p^\uparrow p \to DX \quad p^\uparrow p \to \gamma X \quad p^\uparrow p \to \gamma\text{jet}X \quad p^\uparrow p \to \text{jet}X \quad p^\uparrow p \to \pi\text{jet}X$$

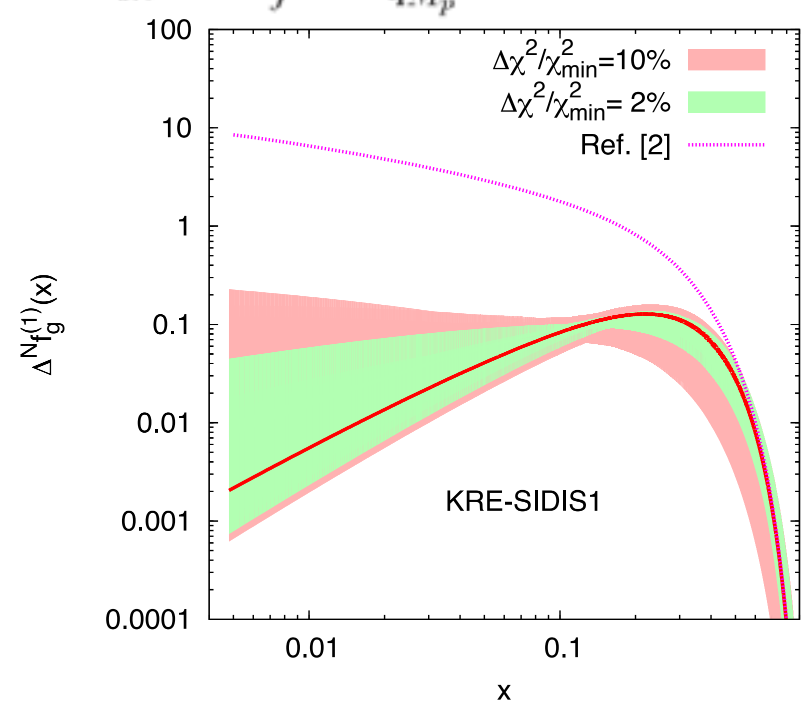
$$p^\uparrow p \to \eta_{c/b}X \quad p^\uparrow p \to Q\overline{Q}X \quad p^\uparrow p \to D^0\overline{D}^0X \quad p^\uparrow p \to J/\psi\gamma X \quad p^\uparrow p \to J/\psi J/\psi X$$

- **twist-3 collinear factorization, indirect constrain on GSF**
- **TMD factorization violation** (recent progress see Yuan's talk)
- **Within generalized parton model, first estimate of the GSF** D'Alesio, Murgia, Pisano '15

$$A_N(p^\uparrow p \to \pi X)$$



$$\Delta^N f^{(1)}_{g/p^\uparrow}(x) = \int d^2k_\perp \frac{k_\perp}{4M_p} \Delta^N f_{g/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)g}(x)$$

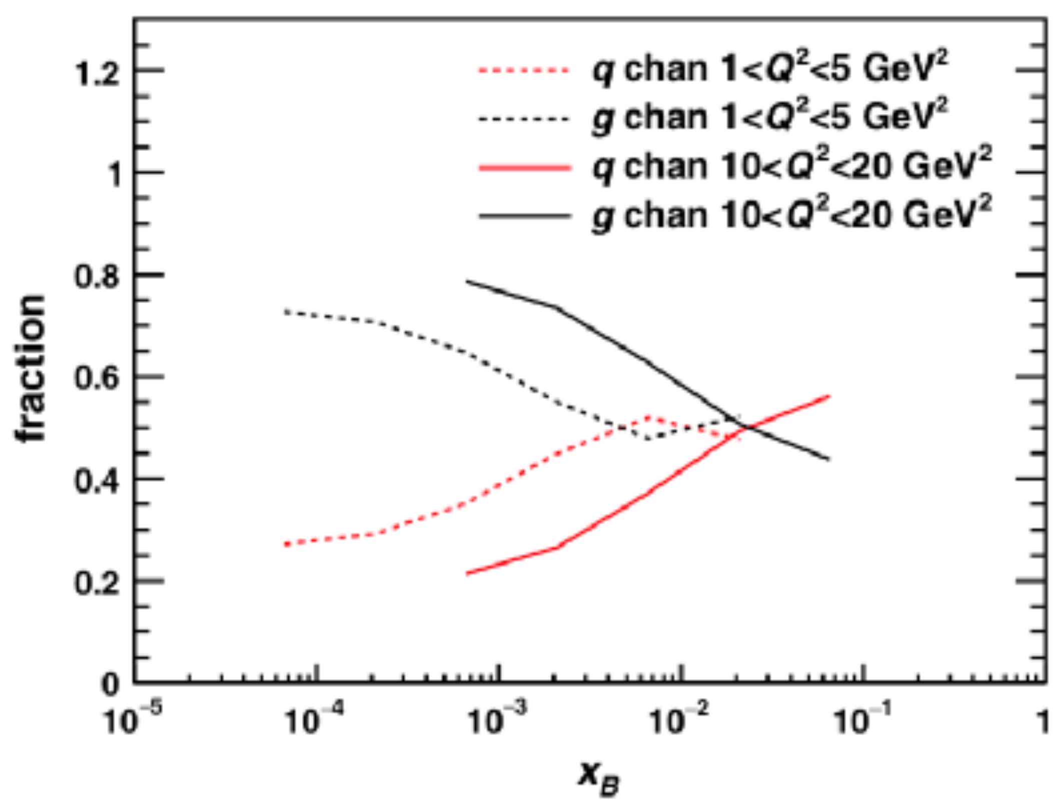


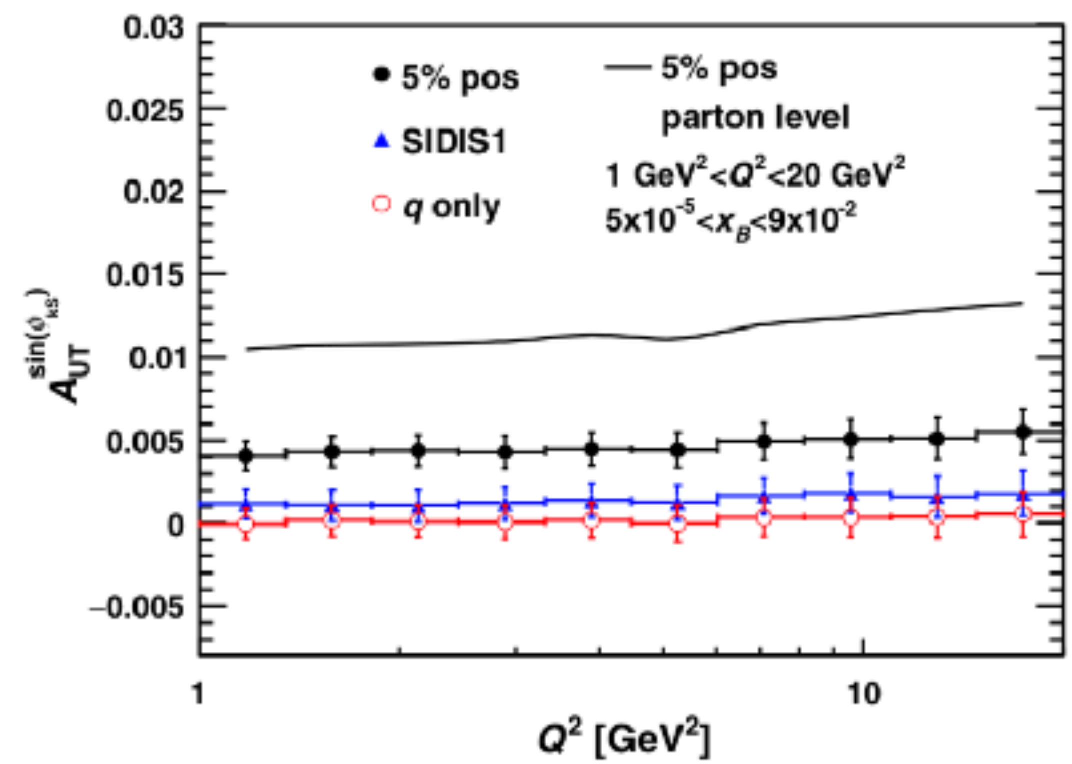
GSF and spin asymmetry in di-jet at the EIC

At the EIC , accessing of GSF via high-p_T dihadron, open di-charm, di-D-meson and dijet has been investigated using PYTHIA and reweighing methods in Zheng, Aschenauer, Lee, Xiao, Yin '18

- **They find that dijet process is the most promising channel**

At the LO di-jet production in DIS involves two processes: $\gamma^* q \to qg \qquad \gamma^* g \to q\bar{q}$

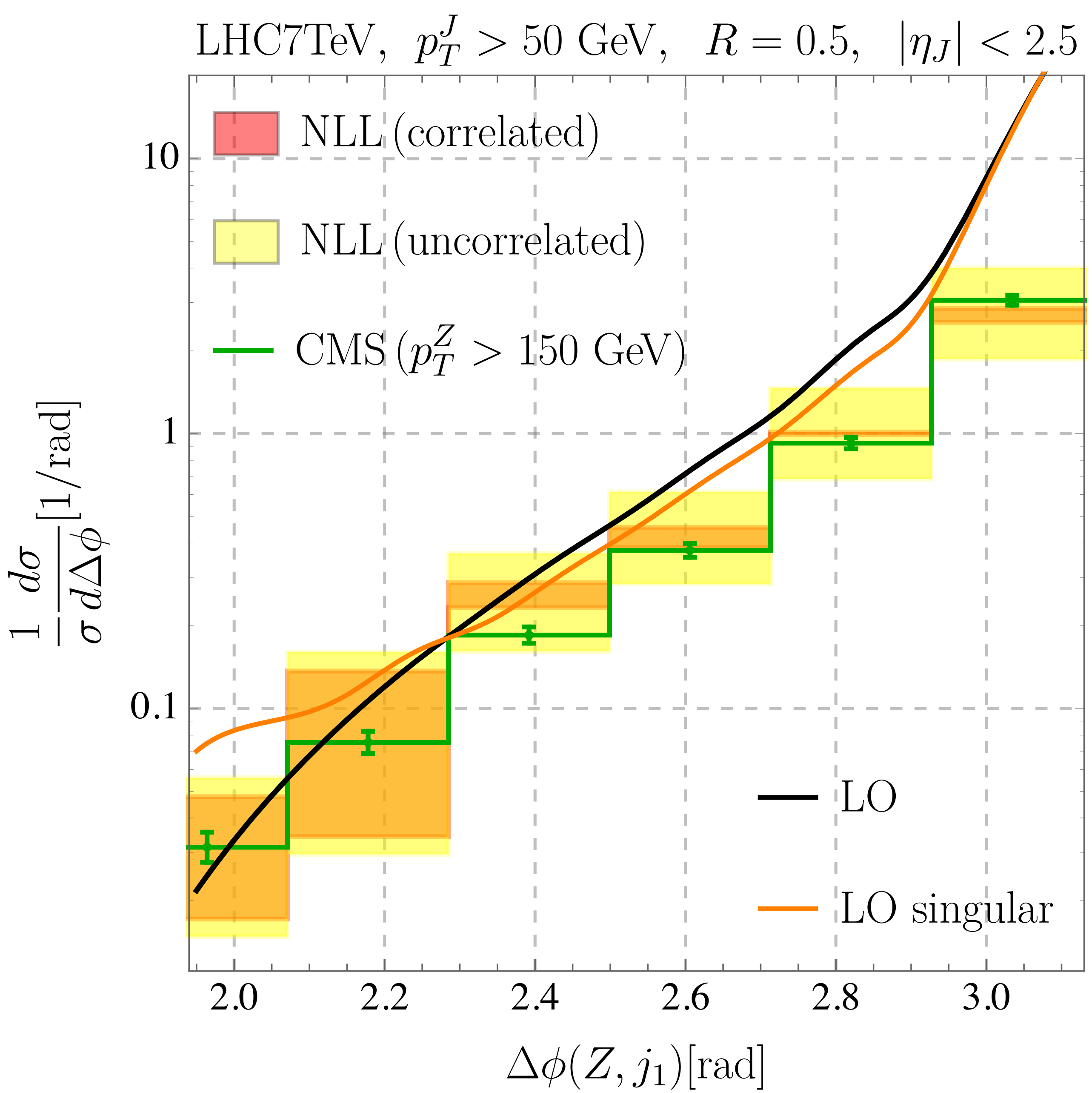




- **to distinguish different TMDs**
  - **Jet substructure (e.g. jet charge "different quark TMDs"** Kang, Liu, Mantry, DYS '20 PRL, see Liu' talk**)**
  - **Heavy-flavor (HF) dijet processes, where q-channel starts to contribute beyond the LO** (Kang, Reiten, DYS, Terry 2011.XXXXX)

Jet TMD studies at the LHC

Chien, DYS & Wu ’19



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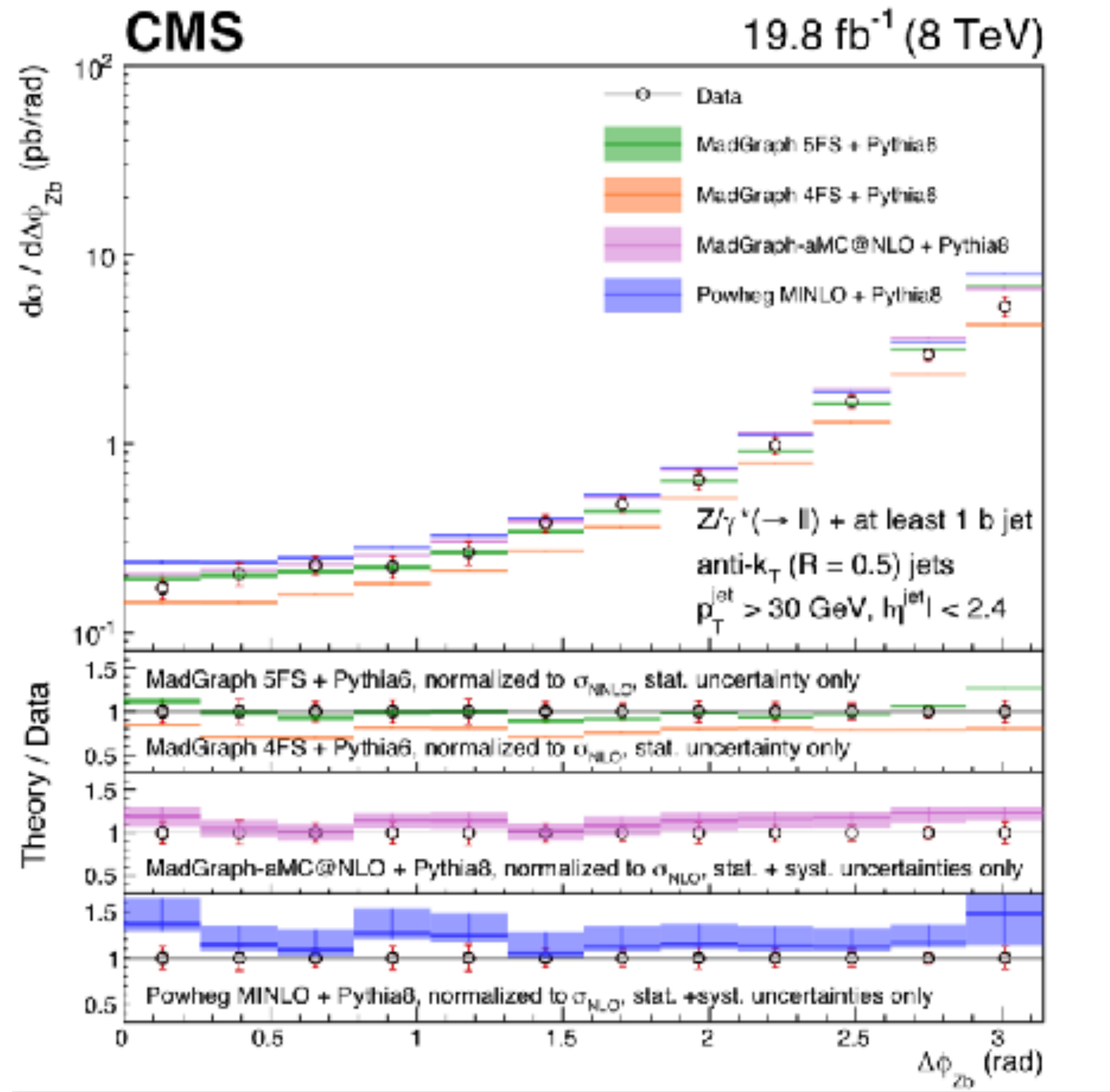
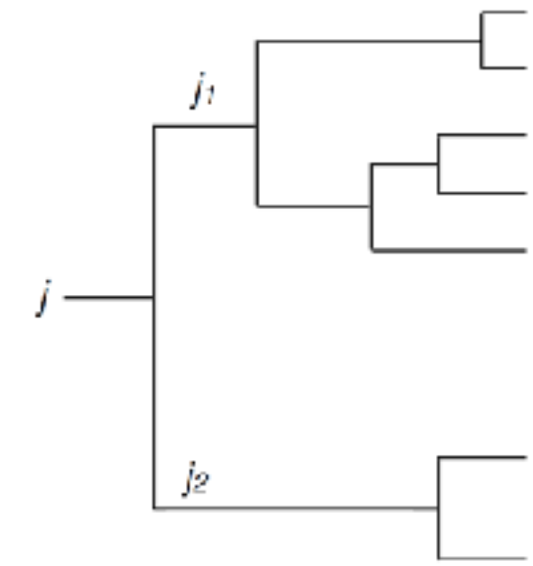


(see also Sun,Yuan,Yuan ’15; Buffing,Kang,Lee,Liu ’18,…)

All-order resummation results are consistent with the LHC data

TMD factorization for QCD jets



Jet definition

Which particles get put together?

Jet algorithm

(A new algorithm at the EIC see Arratia, Makris,Neill,Ringer '20)

How to combine their momenta?

Recombination scheme

TMD factorization for QCD jets

Recombination schemes in jet definitions:

E-scheme: add the four vectors $p_r^\mu = p_i^\mu + p_j^\mu$ (see Yuan's talks)

Non-global in jet TMD resummation **(Banfi, Dasgupta & Delenda '08)**

$$q_T = \left| \sum_{i \notin \text{ jets}} \vec{k}_{T,i} \right| + \mathcal{O}\left(k_T^2\right)$$

sum over all "soft" partons not combined with hard jets

deviation from q_T=0 are only caused by particle flow outside the jet regions

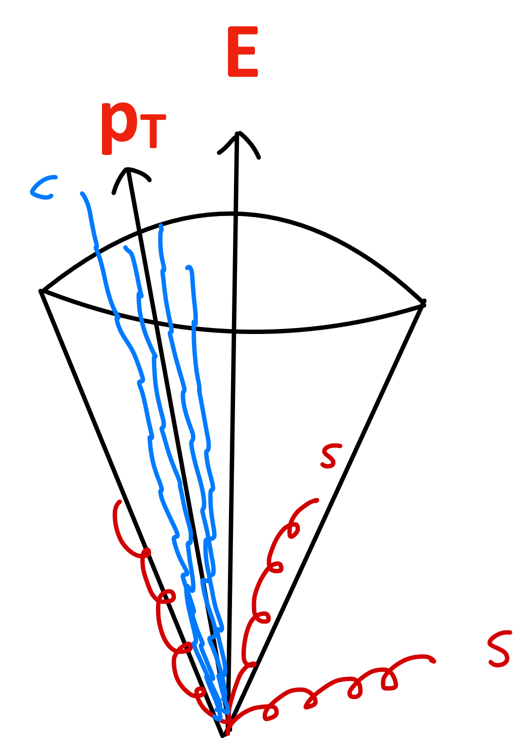
p_T-scheme:

(Ellis, Soper '93)

$$p_{t,r} = p_{t,i} + p_{t,j}\,,$$
$$\phi_r = (w_i\phi_i + w_j\phi_j)/(w_i + w_j)$$
$$y_r = (w_i y_i + w_j y_j)/(w_i + w_j)$$

$$w_i = p_t^n$$

(see Waalewijin's talk)

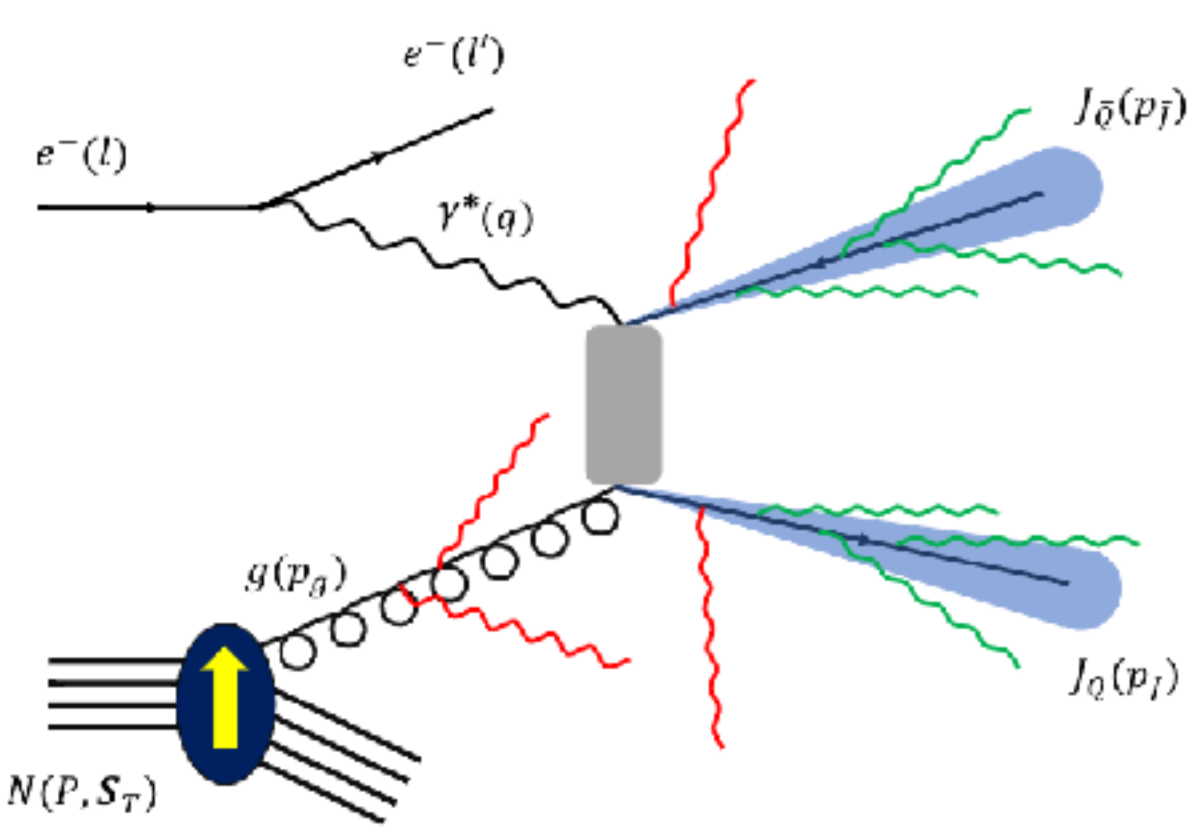


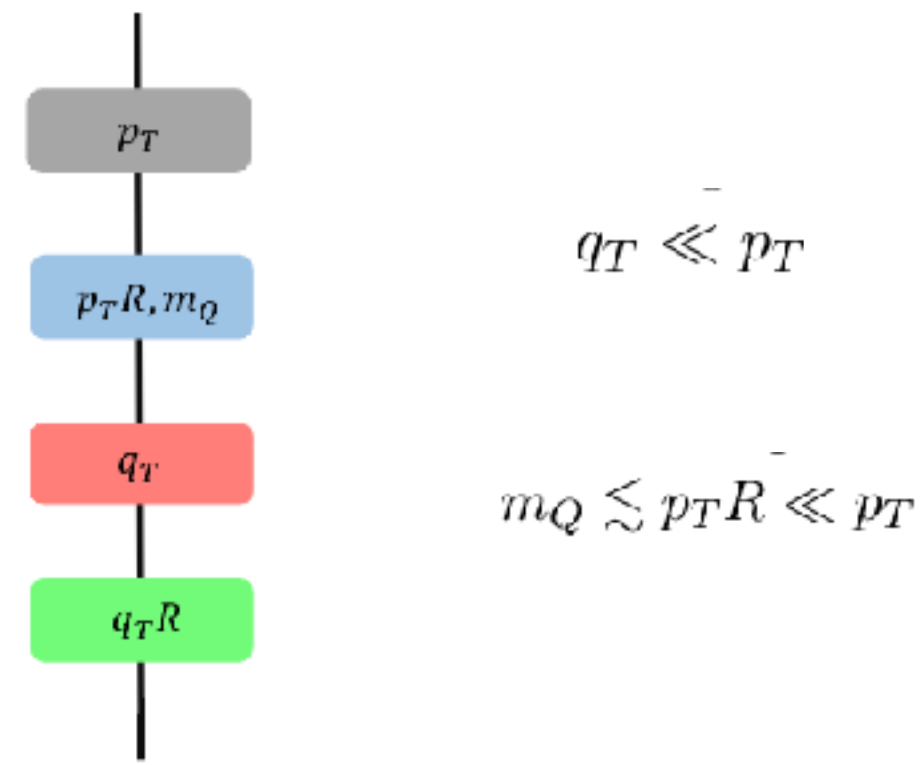
$n \to \infty$ **(Winner-take-all scheme)**

(Bertolini, Chan, Thaler '13)

TMD factorization for heavy-flavor dijet production in DIS

(Kang, Reiten, DYS, Terry 2011.XXXXX)





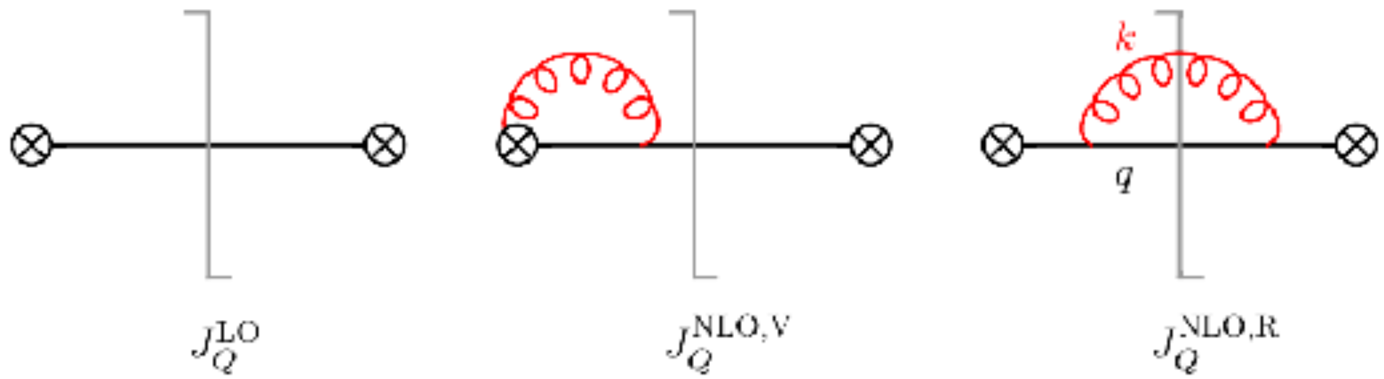
the factorized form of the spin-independent cross section (ignore non-global structures)

$$d\sigma^{UU} \sim H(Q, p_T) J_Q(p_T R, m_Q) J_{\bar{Q}}(p_T R, m_Q) S(\boldsymbol{\lambda}_T) f_g(\boldsymbol{k}_T) S^c_Q(\boldsymbol{l}_{QT}) S^c_{\bar{Q}}(\boldsymbol{l}_{\bar{Q}T}) \delta^{(2)}(\boldsymbol{k}_T + \boldsymbol{\lambda}_T + \boldsymbol{l}_{QT} + \boldsymbol{l}_{\bar{Q}T} - \boldsymbol{q}_T)$$

- **Hard and soft functions are the same as light-jet cases, since $p_T >> m_Q$**
- **Jet and collinear-soft functions are new, which receive finite quark mass correction**

Heavy quark jet function

Collinear radiation only inside the jet cone:



virtual correction: $J_Q^{\rm NLO,V}(p_T R, m_Q, \epsilon) = \frac{\alpha_s}{4\pi} C_F \left[\frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left(1 + 2\ln\frac{\mu^2}{m_Q^2}\right) + \left(1 + \ln\frac{\mu^2}{m_Q^2}\right)\ln\frac{\mu^2}{m_Q^2} + 4 + \frac{\pi^2}{6}\right]$

real correction: $J_Q^{\rm NLO,R}(p_T R, m_Q, \epsilon) = \frac{\alpha_s}{4\pi} C_F \left[-2\ln\left(\frac{m_Q^2 + p_T^2 R^2}{m_Q^2}\right) + 2 - \frac{2m_Q^2}{m_Q^2 + p_T^2 R^2}\right]\frac{1}{\epsilon} + J_Q^{R,\rm fin}$

After combining the real and virtual contributions, the logarithmic dependence of quark mass cancels

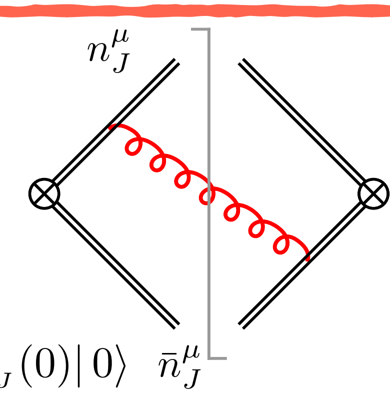
$$Z^{J_Q} = 1 + \frac{\alpha_s}{4\pi} C_F \left[\frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left(2\ln\frac{\mu^2}{m_Q^2 + p_T^2 R^2} + 3 - \frac{2m_Q^2}{m_Q^2 + p_T^2 R^2}\right)\right]$$

(see also Kim '20)

TMD collinear-soft function

collinear-soft function for the LF jet:

(Buffing,Kang,Lee,Liu '18; Chien, DYS, Wu '19)



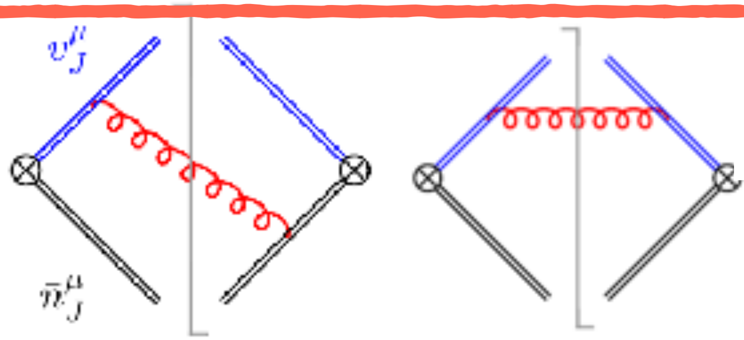
$$n_J^\mu = (1,0,0,1), \quad \bar{n}_J = (1,0,0,-1)$$

$$n_J^2 = \bar{n}_J^2 = 0$$

$$p_{cs}^\mu \sim q_T (R^2, 1, R)_{n_J \bar{n}_J}$$

$$S_q^c(R, \boldsymbol{b}, \epsilon) = \sum_{X_t} e^{\frac{i}{2} p_{cs}^{\rm out} \cdot \bar{n}_J n_{JT} \cdot b} \langle 0 | \boldsymbol{U}_{\bar{n}_J}^\dagger(0) \boldsymbol{U}_{n_J}^\dagger(0) | X_{cs} \rangle \langle X_{cs} | \boldsymbol{U}_{\bar{n}_J}(0) \boldsymbol{U}_{n_J}(0) | 0 \rangle$$

collinear-soft function for the HF jet:



$$v_J^\mu = \frac{\omega_J}{m_Q} \frac{n_J^\mu}{2} + \frac{m_Q}{\omega_J} \frac{\bar{n}_J^\mu}{2}$$

$$v_J^2 = 1, \quad \bar{n}_J^2 = 0$$

HF jet momentum: $p_{J_Q}^\mu = m_Q v_J^\mu + k_{cs}^\mu$

$$S_Q^c(R, \boldsymbol{b}, m_Q, \epsilon) = \sum_{X_t} e^{\frac{i}{2} p_{cs}^{\rm out} \cdot \bar{n}_J n_{JT} \cdot b} \langle 0 | \boldsymbol{U}_{\bar{n}_J}^\dagger(0) \boldsymbol{U}_{v_J}^\dagger(0) | X_{cs} \rangle \langle X_{cs} | \boldsymbol{U}_{\bar{n}_J}(0) \boldsymbol{U}_{v_J}(0) | 0 \rangle$$

At one-loop order:

$$S_{Q,\rm NLO}^c(b, \epsilon) = 2 C_F \, w_{\bar{n}_J v_J} - C_F w_{v_J v_J} \qquad w_{\alpha\beta} = \frac{\alpha_s \mu^{2\epsilon} \pi^\epsilon e^{\epsilon \gamma_E}}{2\pi^2} \int d^d k \, \delta^+(k^2) e^{-i \bar{n}_J \cdot k n_J \cdot b/2} \frac{\alpha \cdot \beta}{\alpha \cdot k \, k \cdot \beta} \, \theta \left[\frac{n_J \cdot k}{\bar{n}_J \cdot k} - \left(\frac{R}{2 \cosh y_J} \right)^2 \right]$$

$$w_{\bar{n}_J v_J} = \frac{\alpha_s}{4\pi} \left[-\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \left(\ln \frac{\mu^2}{\mu_b^2} + 2 \ln \frac{-2i \cos(\phi_b - \phi_J)}{R} - \ln \frac{m_Q^2 + p_T^2 R^2}{p_T^2 R^2} \right) \right] + w_{\bar{n}_J v_J}^{\rm fin},$$

$$w_{v_J v_J} = \frac{\alpha_s}{4\pi} \left[-\frac{1}{\epsilon} \left(\frac{2 m_Q^2}{m_Q^2 + p_T^2 R^2} \right) \right] + w_{v_J v_J}^{\rm fin}$$

RG consistence at one loop

Anomalous dimension for the HF quark jet function:

$$\Gamma^{j_Q}(\alpha_s) = -C_F \gamma^{\text{cusp}}(\alpha_s) \ln \frac{m_Q^2 + p_T^2 R^2}{\mu^2} + \gamma^{j_Q}(\alpha_s) \qquad \gamma_0^{j_Q} = 2C_F \left(3 - \frac{2m_Q^2}{m_Q^2 + p_T^2 R^2}\right)$$

Anomalous dimension for the HF collinear-soft function

$$\Gamma^{cs_Q}(\alpha_s) = C_F \gamma^{\text{cusp}}(\alpha_s) \ln \frac{R^2 \mu_b^2}{\mu^2} + \gamma^{cs_Q}(\alpha_s) \qquad \gamma_0^{cs_Q} = -4C_F \left[2 \ln\left[-2i \cos(\phi_b - \phi_J)\right] - \frac{m_Q^2}{m_Q^2 + p_T^2 R^2} - \ln \frac{m_Q^2 + p_T^2 R^2}{p_T^2 R^2}\right]$$

Heavy-quark mass dependence cancels out in

$$\Gamma^{j_Q} + \Gamma^{cs_Q} = \Gamma^{j_q} + \Gamma^{cs_q}$$

Then one can easily verify RG consistence at one-loop order

$$\Gamma^h + \Gamma^s + \Gamma^{f_g} + 2\left(\Gamma^{j_q} + \Gamma^{cs_q}\right) = 0 \longrightarrow \Gamma^h + \Gamma^s + \Gamma^{f_g} + 2\left(\Gamma^{j_Q} + \Gamma^{cs_Q}\right) = 0$$

Castil, Echevarria, Makris, Scimemi '20

Heavy quark mass will contribute the RG evolution between jet and collinear-sot function different from the case for the inclusive HF quark jet production Dai, Kim, Leibovich '18**.**

RG evolution and resummation

- **Resummation formula:**

$$\frac{d\sigma^{UU}}{dQ^2 dy d^2\boldsymbol{q}_T dy_J d^2\boldsymbol{p}_T} = H(Q, p_T, y_J, \mu_h) \int_0^\infty \frac{bdb}{2\pi} J_0(b\, q_T) f_{g/N}(x_g, \mu_{b*})$$
$$\times \exp\left[-\int_{\mu_{b*}}^{\mu_h} \frac{d\mu}{\mu} \Gamma^h(\alpha_s) - 2\int_{\mu_{b*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{jQ}(\alpha_s) - 2\int_{\mu_{b_*}}^{\mu_{cs}} \frac{d\mu}{\mu} \Gamma^{csQ}(\alpha_s) \right]$$
$$\times \exp\left[-S_{\rm NP}(b, Q_0, n\cdot p_g)\right]$$

- **b*-prescription to avoid Landau pole** $b_* = b/\sqrt{1+b^2/b_{\rm max}^2}$ $\mu_{b_*} = 2e^{-\gamma_E}/b_*$

- **Non-perturbative model:** $S_{\rm NP}(b, Q_0, n\cdot p_g) = g_1^f b^2 + \frac{g_2}{2}\frac{C_A}{C_F} \ln\frac{n\cdot p_g}{Q_0} \ln\frac{b}{b_*}$

Sun, Isaacson,Yuan,Yuan ‘14

- **Typical scales:** $\mu_h \sim p_T, \quad \mu_j \sim R p_T, \quad \mu_{cs} \sim R\mu_{b*}$

RG evolution and resummation

- **QCD evolution between soft and collinear-soft scale**

$$\Gamma^{csQ}(\alpha_s) = C_F \gamma^{\text{cusp}}(\alpha_s) \ln \frac{R^2 \mu_b^2}{\mu^2} + \gamma^{csQ}(\alpha_s) \qquad \gamma_0^{csQ} = -4C_F \left[2 \ln\left[-2i \cos(\phi_b - \phi_J)\right] - \frac{m_Q^2}{m_Q^2 + p_T^2 R^2} - \ln \frac{m_Q^2 + p_T^2 R^2}{p_T^2 R^2} \right]$$

$$\exp\left[-\int_{\mu_b}^{\mu_{cs}} \frac{d\mu}{\mu} \Gamma^{csQ}(\alpha_s) \right] \longrightarrow \left| \cos\left(\phi_b - \phi_J\right) \right|^{\frac{4C_F}{\beta_0} \log \frac{\alpha_s(\mu_b)}{\alpha_s(\mu_{cs})}}$$

- $\Delta\phi_{bJ}$ **integral is convergent only if**

$$-1 < p\left(\mu_b, \mu_{cs}\right) \equiv \frac{4C_F}{\beta_0} \log \frac{\alpha_s\left(\mu_b\right)}{\alpha_s\left(\mu_{cs}\right)}$$

- **One encounters such a divergence when the collinear-soft scale approaches to the non-perturbative region**
- **different schemes to avoid such problem:**
  - **turn off the QCD evolution between soft and collinear-soft** Liu,Ringer,Vogelsang,Yuan '19
  - $\Delta\phi_{bJ}$ **averaging** Buffing,Kang,Lee,Liu '18, Kang, Kyle, DYS, Terry '20
  - $\Delta\phi_{bJ}$ **dependent collinear-soft scale choice** Chien, DYS, Wu '19

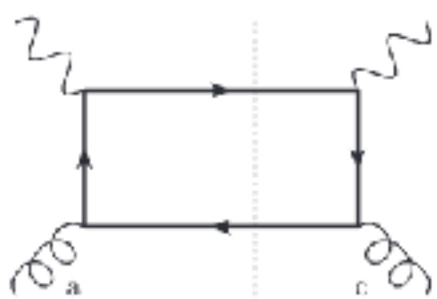
Spin dependent cross section

- **Resummation formula:**

$$\frac{d\sigma^{UT}(\boldsymbol{S}_T)}{dQ^2 dy d^2\boldsymbol{q}_T dy_J d^2\boldsymbol{p}_T} = \sin(\phi_q - \phi_s)\, H(Q, p_T, y_J, \mu_h) \int_0^\infty \frac{b^2 db}{4\pi} J_1(b\, q_T) f_{1T,g/p}^{\perp}(x_g, \mu_{b*})$$
$$\times \exp\left[-\int_{\mu_{b*}}^{\mu_h} \frac{d\mu}{\mu}\Gamma^h(\alpha_s) - 2\int_{\mu_{b*}}^{\mu_j} \frac{d\mu}{\mu}\Gamma^j(\alpha_s) - 2\int_{\mu_{b_*}}^{\mu_{cs}} \frac{d\mu}{\mu}\Gamma^{cs}(\alpha_s)\right]$$
$$\times \exp\left[-S_{\rm NP}^{\perp}(b, Q_0, n\cdot p_g)\right]$$

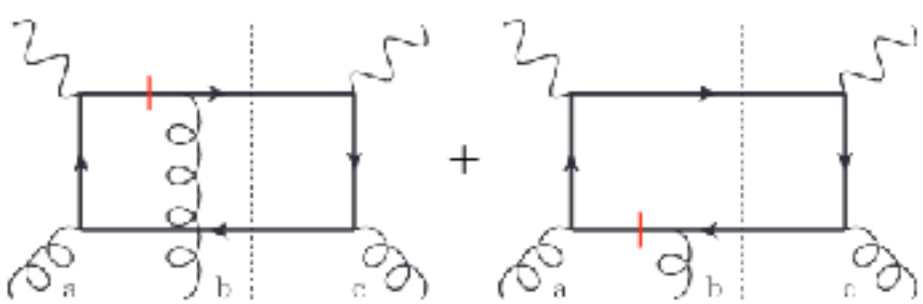
- **Polarized hard function: For the polarized process, we must consider the attachment of an additional gluon from gauge link in GSF**

unpolarized:



$= C_u\, \delta^{ac}\, h^{(0)}(Q, p_T, y_J, \mu)$

polarized:



$= (C_1 + C_2)\,(-i f^{abc})\, h^{(0)}(Q, p_T, y_J, \mu)$

polarized and unpolarized hard functions are the same $C_1 + C_2 = C_u$

f-type gluon Sivers function

Numerical results

Anti-k$_T$, R=0.6

c-jets: $5\,\mathrm{GeV} < p_T < 10\,\mathrm{GeV}, \quad |\eta_J| < 4.5,$

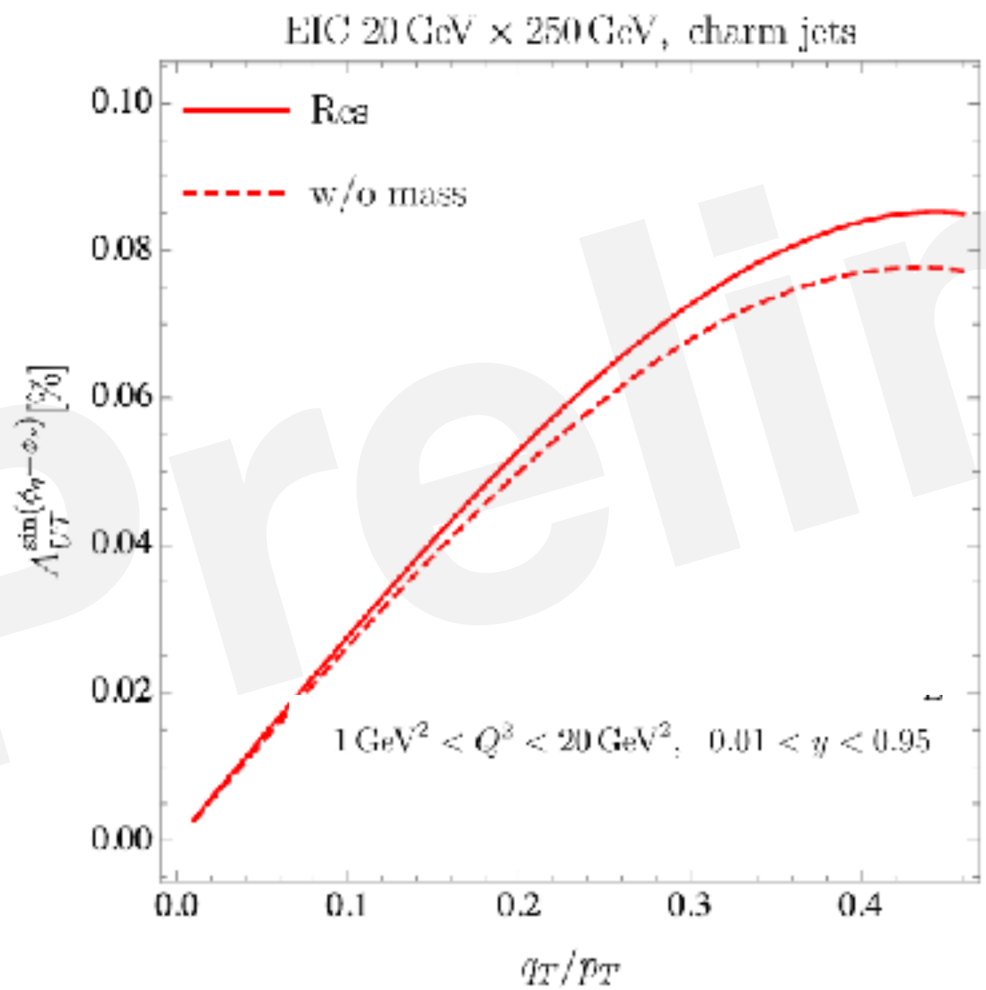
b-jets: $10\,\mathrm{GeV} < p_T < 15\,\mathrm{GeV}, \quad |\eta_J| < 4.5,$

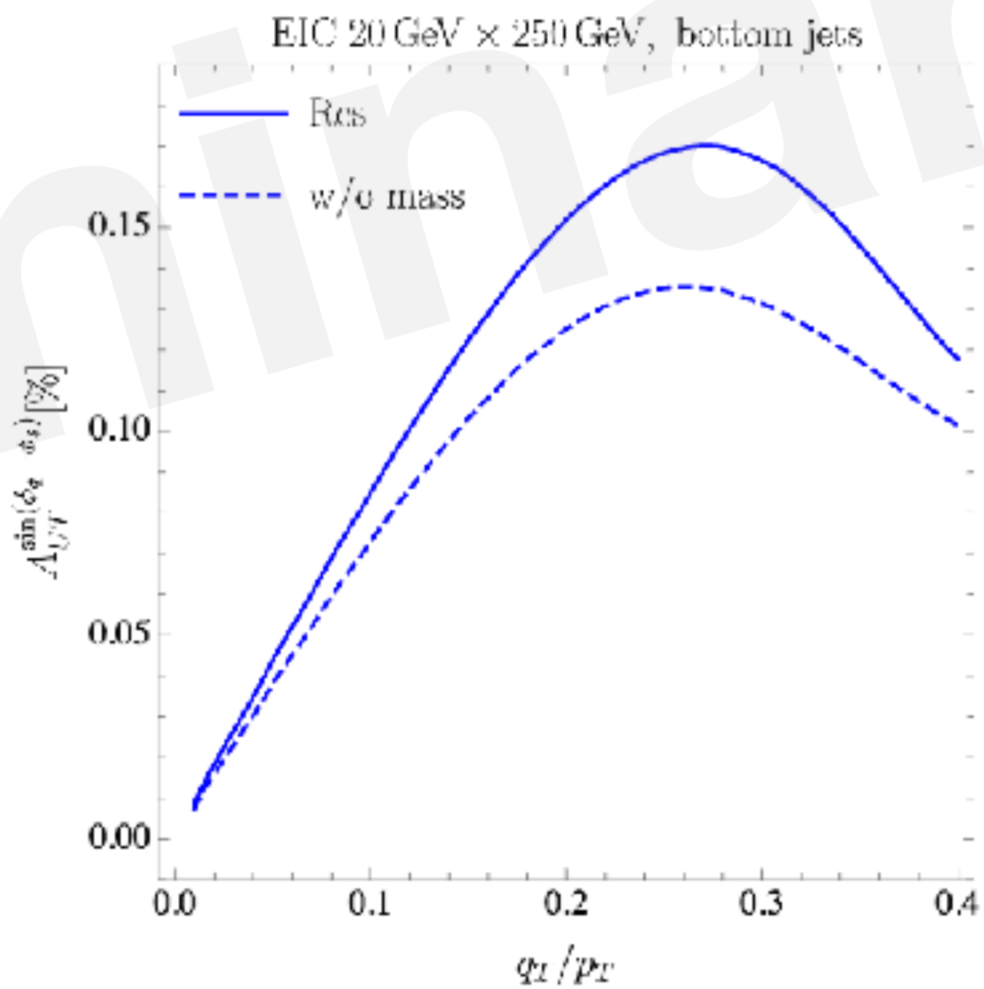
$$d\sigma(\boldsymbol{S}_T) = d\sigma^{UU} + \sin(\phi_q - \phi_s) d\sigma^{UT}$$

$$A_{UT}^{\sin(\phi_q - \phi_s)} = \frac{d\sigma^{UT}}{d\sigma^{UU}}$$

GSF: SIDIS1 set

D'Alesio, Murgia, Pisano '15





Heavy quark mass give sizable corrections to the predicted asymmetry

Conclusion

- **We develop the TMD factorization formalism for heavy flavor dijet production in electron polarized proton collisions.**

- **We consider heavy flavor mass correction in the collinear-soft and jet functions, as well as the associated evolution equations.**

- **We generate a prediction for the gluon-Sivers asymmetry for charm and bottom dijet production at the future Electron-Ion Collider.**

- **After comparing our theoretical prediction with and without considering the heavy-flavor mass effects, we find that these effects give sizable corrections to the predicted asymmetry.**

Thank you