$\Lambda^{\uparrow}$ Polarization in e^+e^- collisions & twist

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w/ Zhong-Bo Kang, Ding Yu Shao, John Terry, Fanyi Zhao

Outline

- Motivation to study on $\Lambda^{\uparrow}$ physics
- Review TSSA and "outsized" role of Lambda & PFF
- Simplest process $e^+e^- \to \Lambda^{\uparrow}X$ most interesting to process to study
- Fragmentation processes both at twist-2 and twist-3 analysis
- Twist -2 TMD description in terms of PFF $D_{1T}^{\perp}(z, p_{\perp}, Q^2)$
  - ⋆ Thrust/jet observable $\Lambda\,(\mathrm{Thrust}) + X$ $\quad e^+e^- \to \Lambda^{\uparrow}(\mathrm{Thrust})\,X$
  - ⋆ Back to back hadrons $h + \Lambda$ $\quad e^+e^- \to \Lambda^{\uparrow}\,h\,X$
- Twist -3 inclusive description in terms of $D_T(z, Q^2)$ $\quad e^+e^- \to \Lambda^{\uparrow}X$
- Test of time reversal in QCD

Dilemma

Kane, Pumplin, and Repko predict zero from QCD

VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

Transverse Quark Polarization in Large-p_T Reactions, e^+e^- Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
Physics Department, University of Michigan, Ann Arbor, Michigan 48109

and

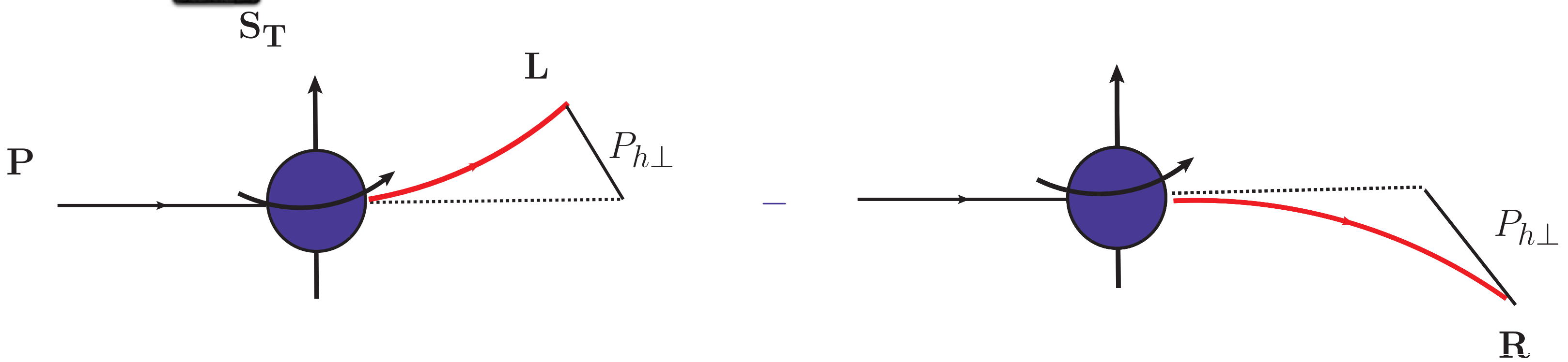
J. Pumplin and W. Repko
Physics Department, Michigan State University, East Lansing, Michigan 48823
(Received 5 July 1978)

We point out that the polarization P of a scattered or produced quark is calculable perturbatively in quantum chromodynamics for $e^+e^- \to q\bar{q}$, large-p_T hadron reactions, and large-Q^2 leptoproduction, and is infrared finite. The quantum-chromodynamics prediction is that $P=0$ in the scaling limit. Experimental tests are or will soon be possible in $pp \to \Lambda X$ [where presently $P(\Lambda) \simeq 25\%$ for $p_T > 2$ GeV/c] and in $e^+e^- \to$ quark jets.

At least for the cases when P is small, tests should be available soon in large-p_T production [where currently $P(\Lambda) = 25\%$ for $p_T \gtrsim 2$ GeV/c], and e^+e^- reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.

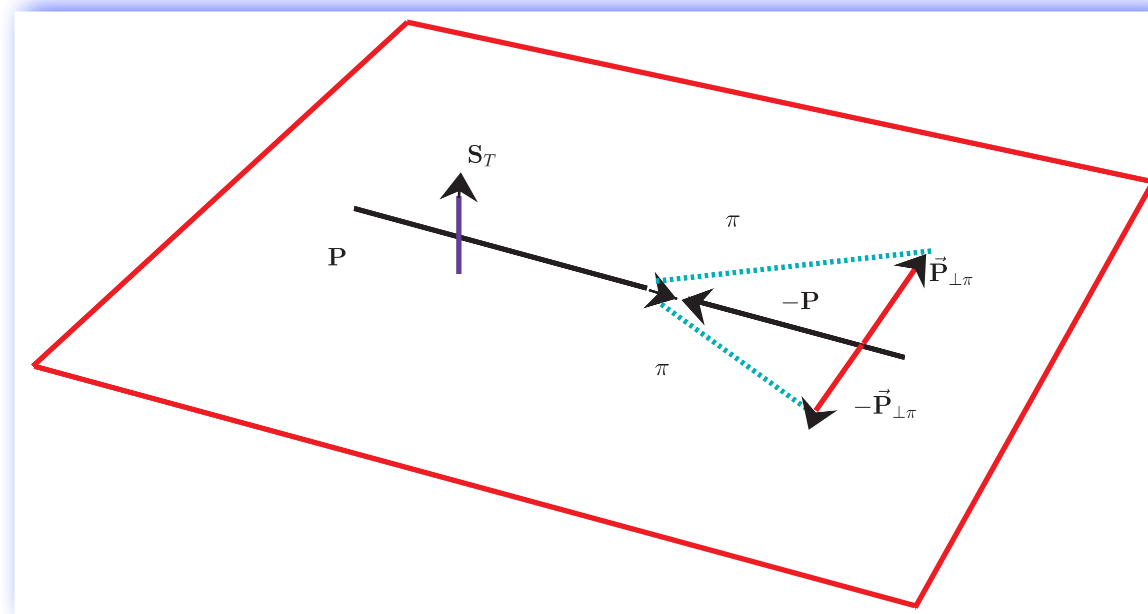
Mulders Tangerman, NPB1996
Boer, Jakob, Mulders NPB1997, 2000
Anselmino Boer, D'Alesio, Murgia. PRD 2001, 2002
Boer, Kang, Vogelsang, Yuan, PRL 2010

Essential transverse single spin asymmetry TSSAs



$$\sigma^{\downarrow}(x, P_\perp) = \sigma^{\uparrow}(x, -P_\perp)$$

$$P_\Lambda(\ldots) = \frac{\sigma^{\uparrow}(x, P_\perp) - \sigma^{\uparrow}(x, -P_\perp)}{\sigma^{\uparrow}(x, P_\perp) + \sigma^{\uparrow}(x, -P_\perp)}$$



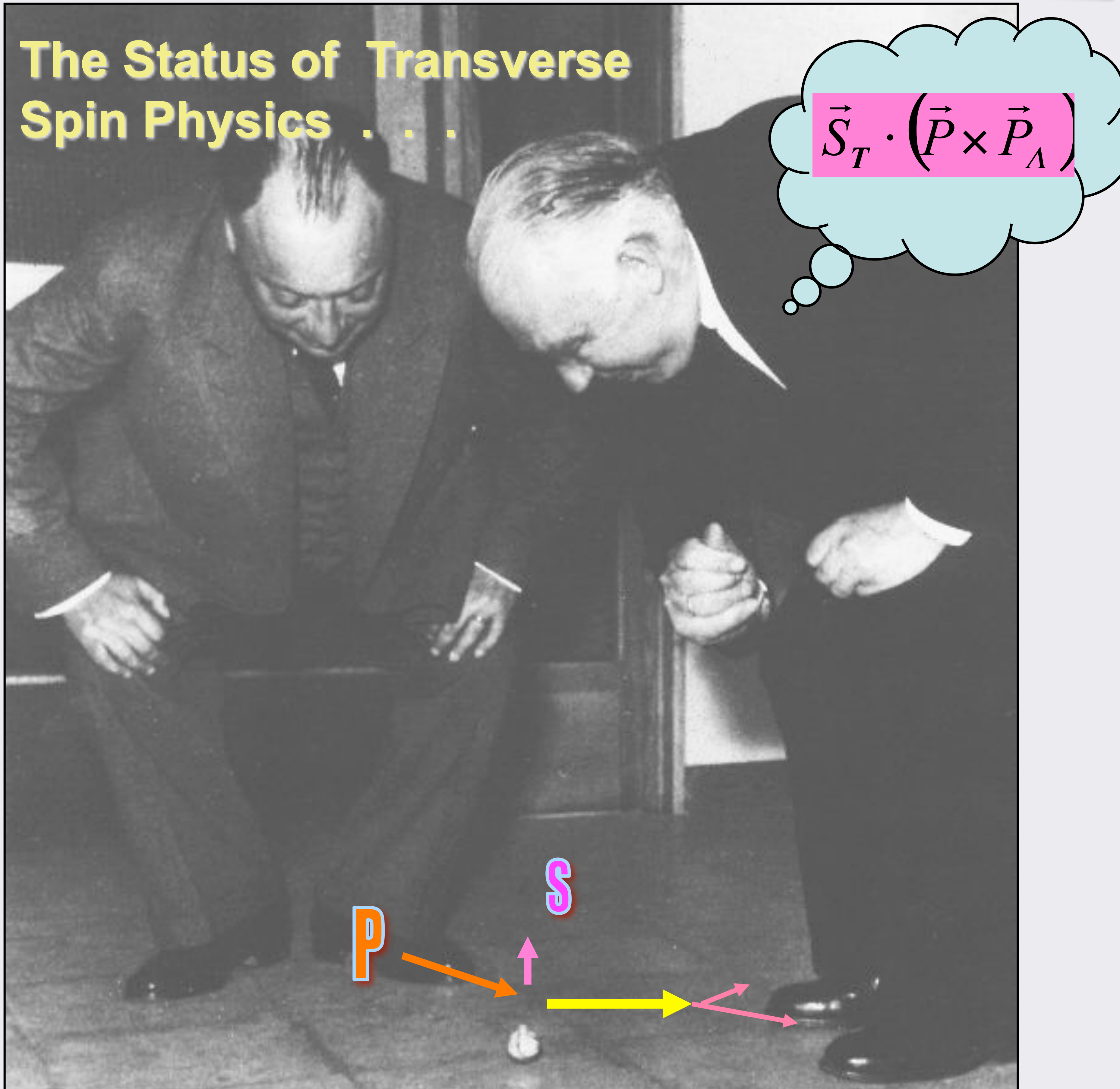
QCD is Parity Conserving   TSSAs Scattering plane transverse to spin

Naively "T-odd"

$$\Delta\sigma \sim i S_T \cdot (\mathbf{P} \times P_\perp) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

Spin orbit

Kinematics of parity conserving Transverse Spin



Measurement of Lambda-spin through weak decay $\Lambda^0 \rightarrow p\,\pi^-$

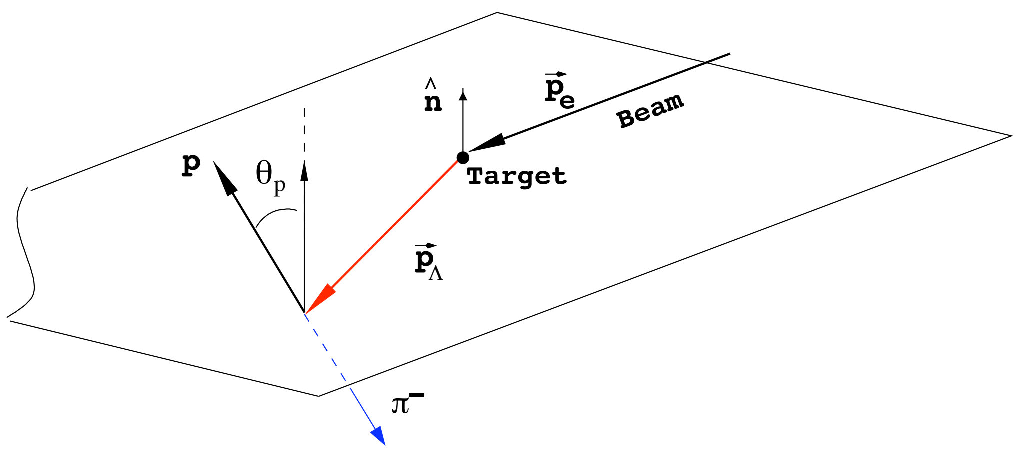


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the Λ rest frame.

- Proton preferentially emitted along Λ-spin
- In Λ rest frame: pol. decay distribution

$$\left(\frac{dN}{d\Omega_p}\right)_{\text{pol}} = \left(\frac{dN}{d\Omega_p}\right)_{\text{unpol}} \left(1 + \alpha\, P_n^{\Lambda} \cos(\theta_p)\right)$$

P^{Λ}: Transverse Lambda Polarization

QCD is Parity conserving so any final state hadron must be polarised perpendicular to the production plane

Weak decay of Lambda; proton is preferentially along the hyperon spin direction can reconstruct the Lambda spin

The Status of Transverse Spin Physics . . .

$$\vec{S}_T \cdot \left(\vec{P} \times \vec{P}_\Lambda\right)$$

What does Exp Say ...

$$pA \to \Lambda^{\uparrow} X$$

$$pp \to \Lambda^{\uparrow} X$$

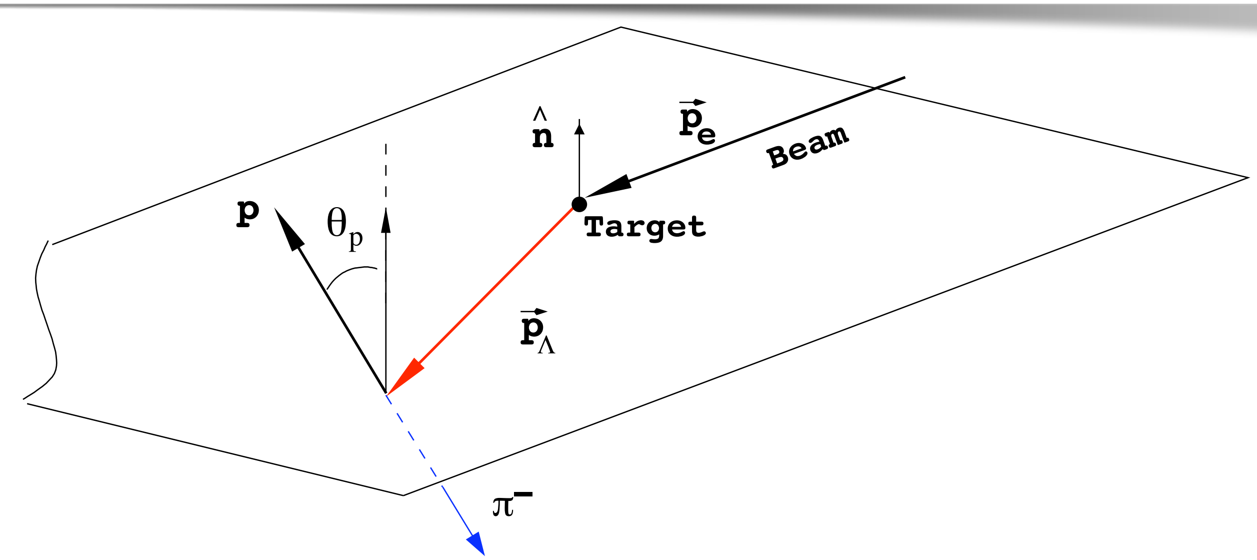


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the Λ rest frame.

$$\nu N \to \Lambda^{\uparrow} X$$

$$\gamma^* N \to \Lambda^{\uparrow} X$$

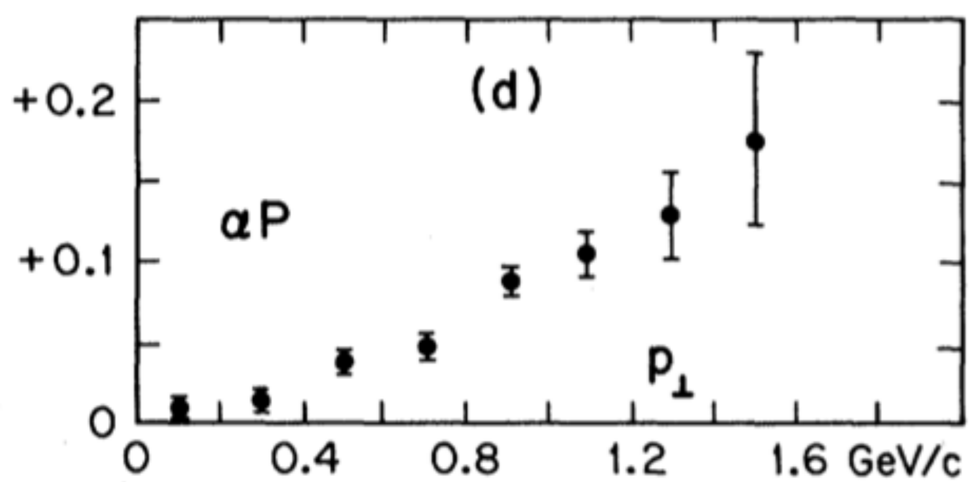
$$e^+ e^- \to \Lambda^{\uparrow} X$$

Transverse Λ polarisation a long history

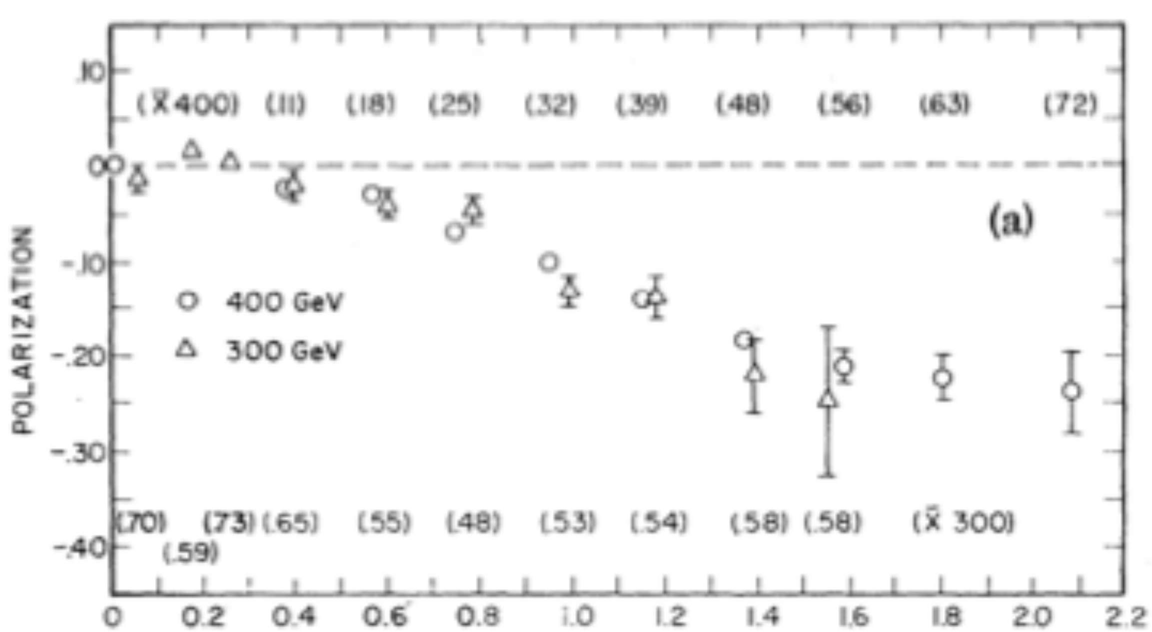
$$p + Be \rightarrow \Lambda^{\uparrow} + X$$

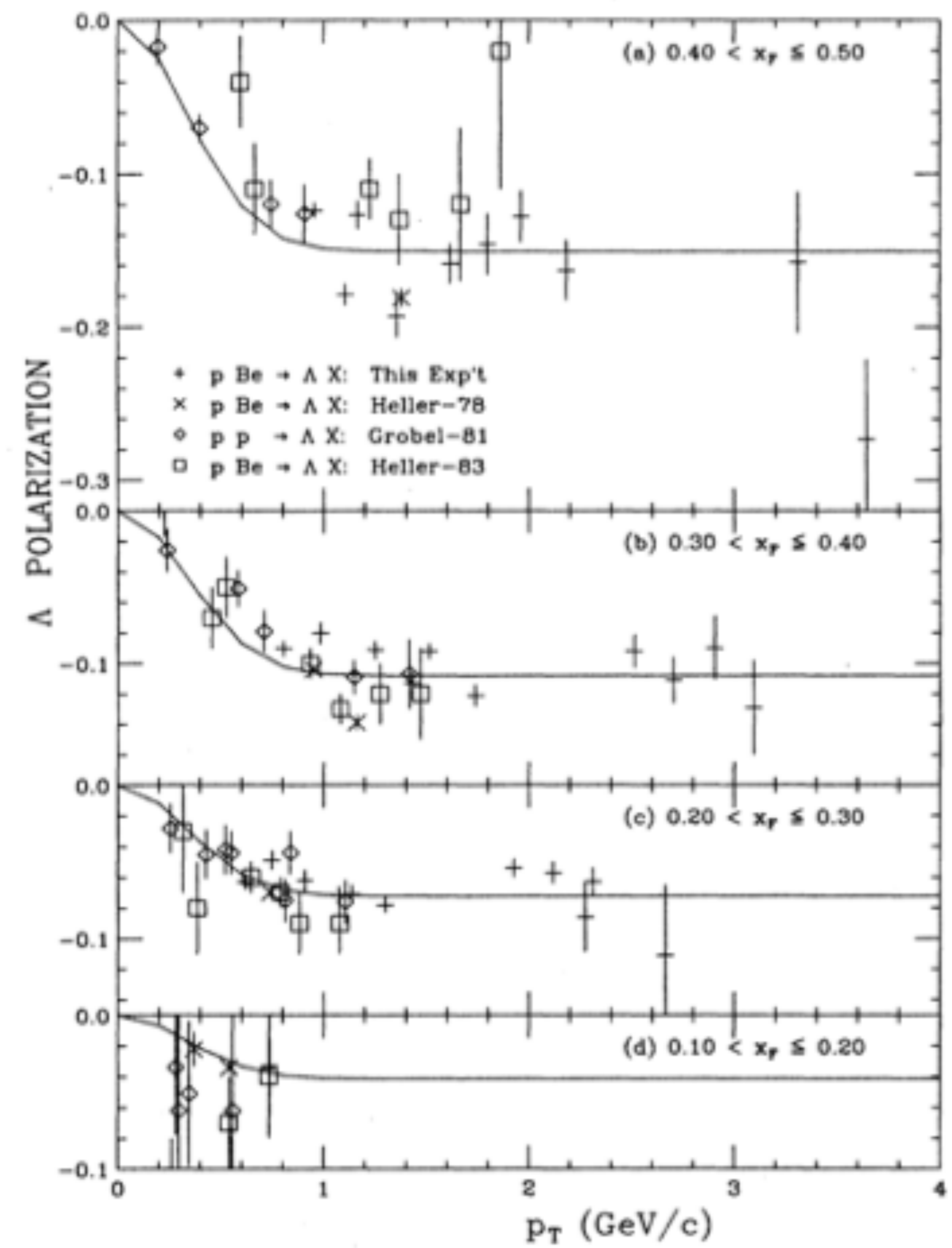
One of the first transverse spin effects at Fermilab (1976):

PRD 89 Lundberg



Bunce PRL 76
Heller PRL 78





Proton-Nuclei cont …

$$pA \rightarrow \Lambda^{\uparrow} X$$

V. Fanti et al.: NA 48 450 GeV proton energy

Eur. Phys. J. C 6, 265–269 (1999) **CERN SPS**

Lundberg et al prd40 (1989) 400 GeV

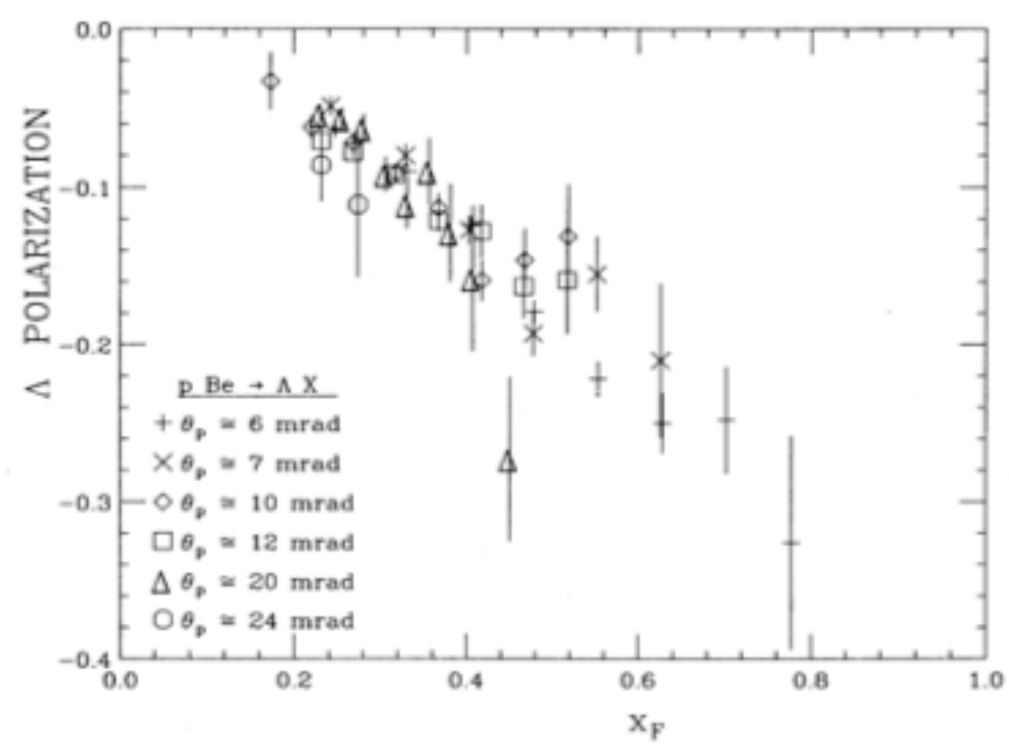
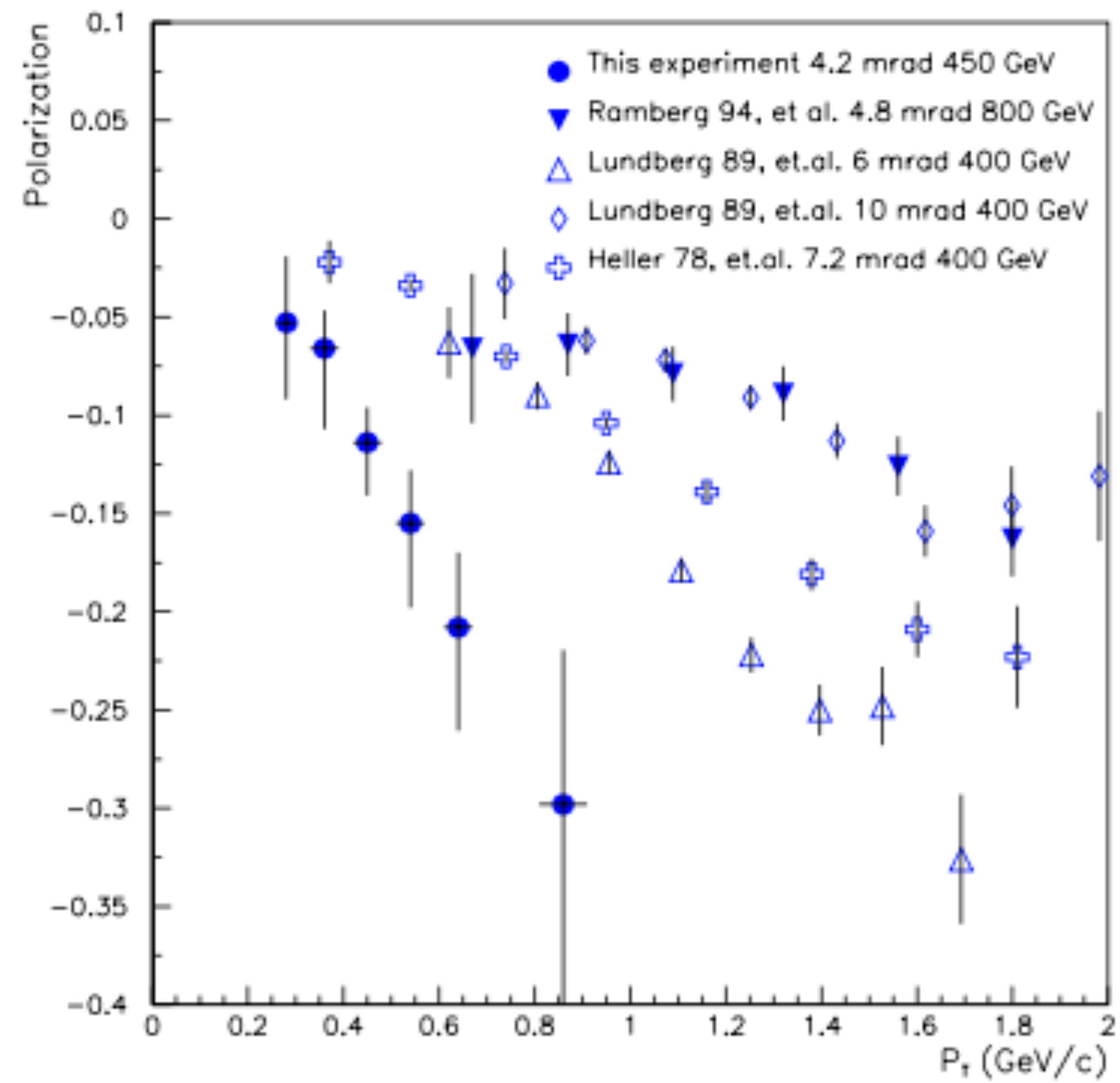


FIG. 4. The Λ polarization is shown as a function of x_F for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or p_T) independent.



Proton-proton

$$pp \to \Lambda^{\uparrow} X$$

Erhan et al. PLB 1979

Λ^0 produced in the inclusive reaction with $\sqrt{s}$ = 53 and 62 GeV at the CERN Intersecting Storage Rings (ISR) are observed to be polarized along the normal to the production plane. In the ranges of longitudinal and transverse momenta, 15-24 and 0.6-1.4 GeV/c, respectively, the mean polarization is found to be -(0.357 +/- 0.055)

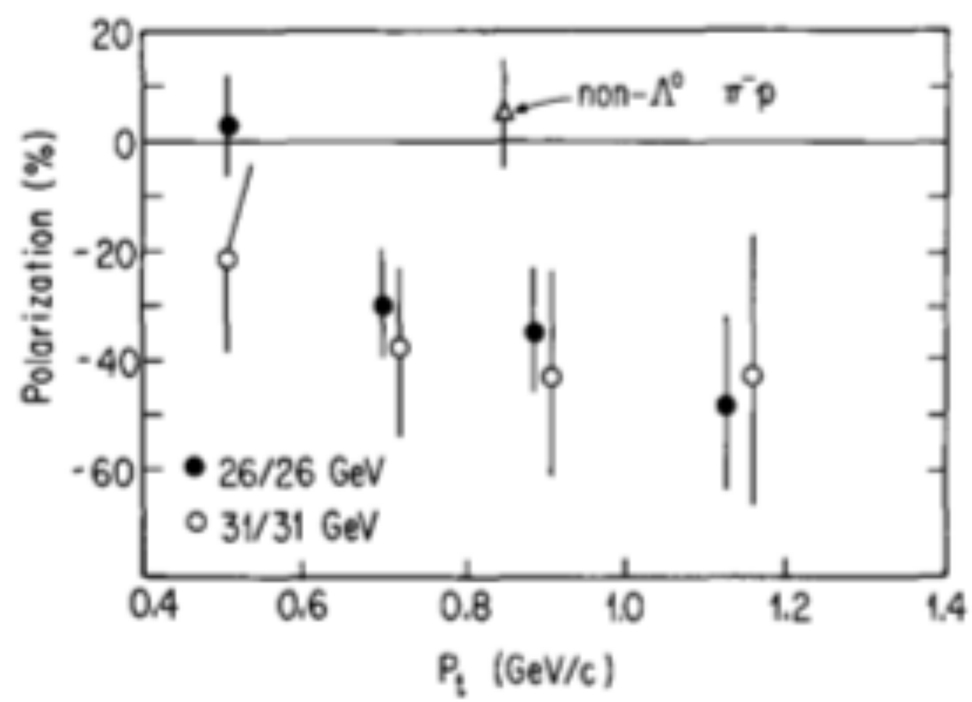


Fig. 4. Final polarization values for 26/26 and 31/31 GeV data samples as a function of P_t. The polarization measurement for $\pi^- p$ events which are not Λ^0 but have $P_t > 0.6$ GeV/c is shown as the open triangle.

Lepton-Nucleon

$$\nu N \to \Lambda^{\uparrow} X$$

Nuclear Physics B 588 (2000)

The Λ polarization in νμ charged current interactions measured in the **NOMAD** experiment. The dependence of the absolute value of *P*y on the Λ transverse momentum with respect to the hadronic jet direction is in qualitative agreement with the results from unpolarized hadron–hadron experiments.

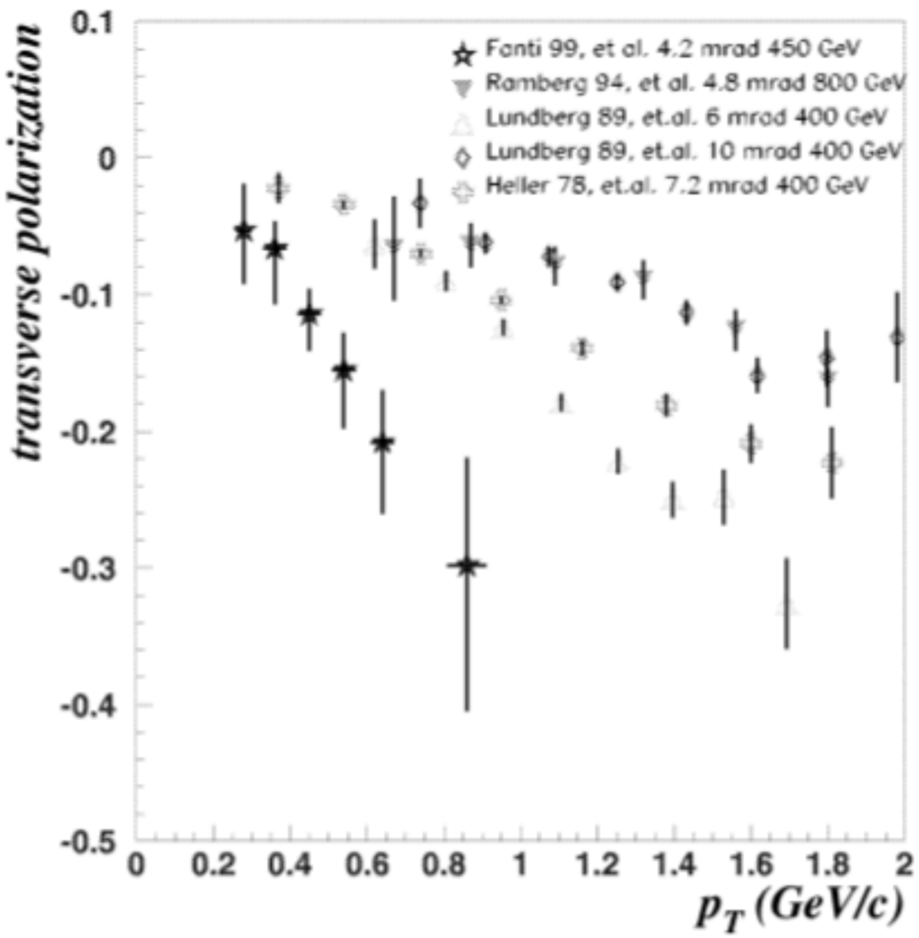


Fig. 16. The measured dependence of the transverse Λ polarization on p_T in hadron–hadron experiments.

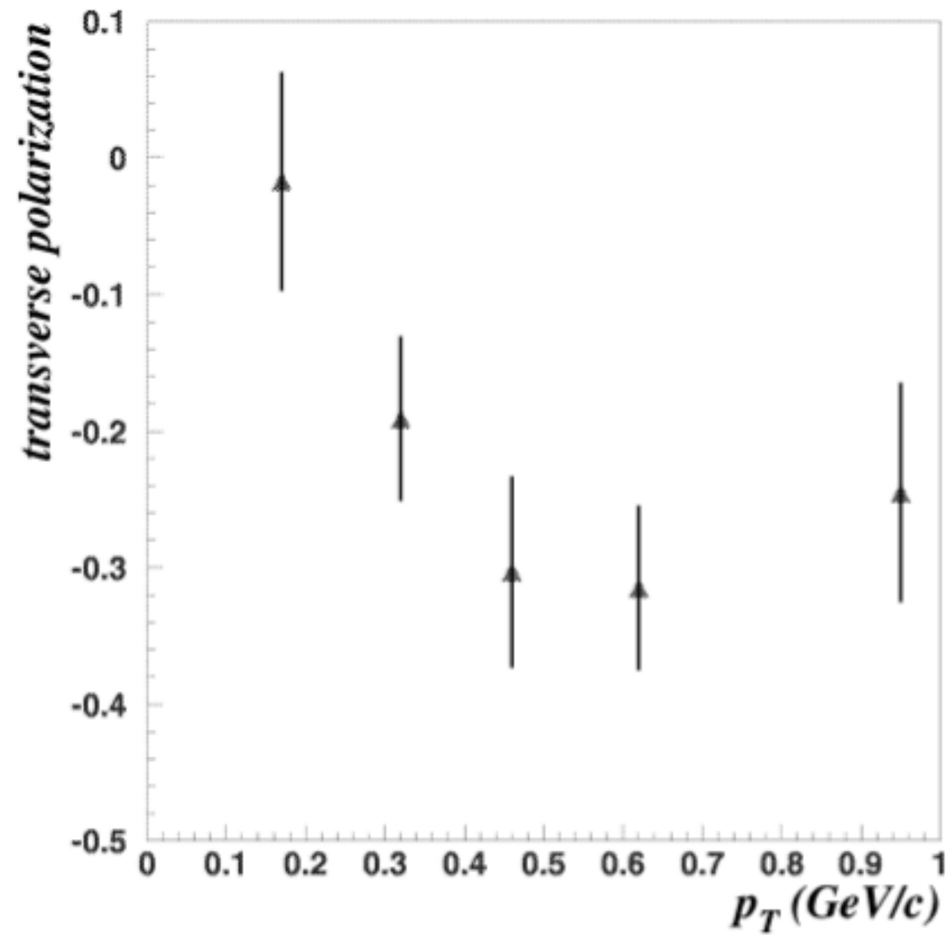


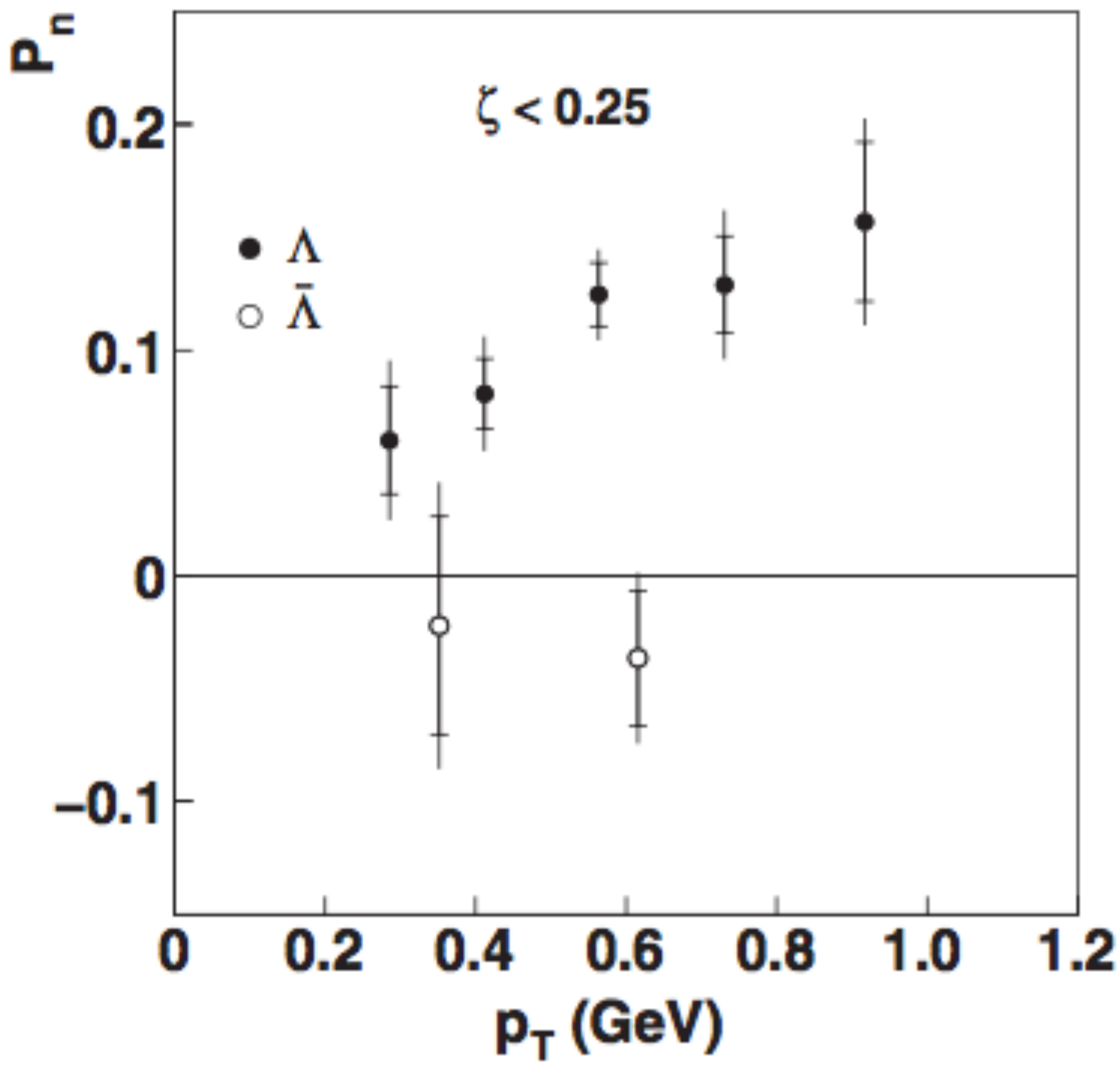
Fig. 17. The dependence of the transverse Λ polarization on p_T observed in the NOMAD experiment.

Quasi Real + Nucleon HERMES

$$\gamma^* N \to \Lambda^\uparrow X$$

The HERMES experiment has measured the transverse polarization of Λ and $\bar{\Lambda}$ hyperons produced inclusively in quasi-real photoproduction at a positron beam energy of 27.6 GeV. The transverse polarization $P_{\Lambda N}$ of the Λ hyperon is found to be positive while the observed $\bar{\Lambda}$ polarization is compatible with zero

A. AIRAPETIAN *et al.*



PRD 76, 092008 (2007)

Quasi Real + Nucleon HERMES

$$\gamma^* N \to \Lambda^{\uparrow} X$$

PRD 90, 072007 (2014)

The transverse polarization of Λ hyperons was measured in inclusive quasireal photoproduction for various target nuclei ranging from hydrogen to xenon. The polarization observed is positive for light target nuclei and is compatible with zero for krypton and xenon.

TRANSVERSE POLARIZATION OF Λ HYPERONS ...

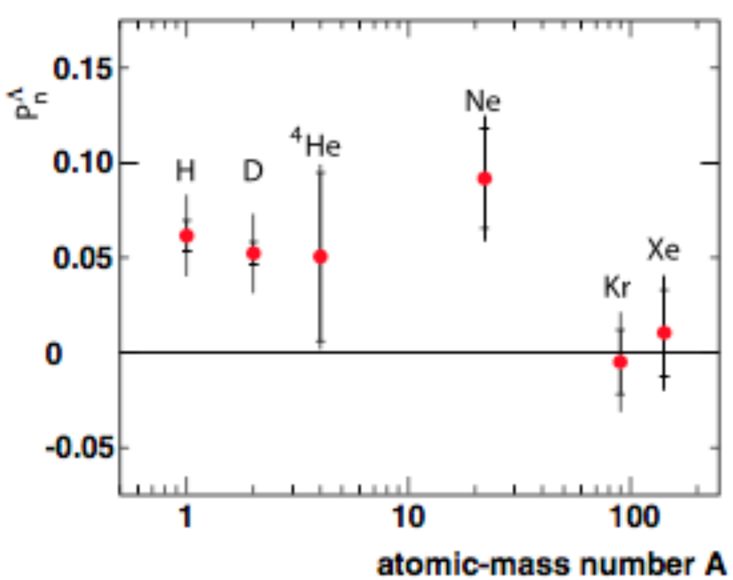
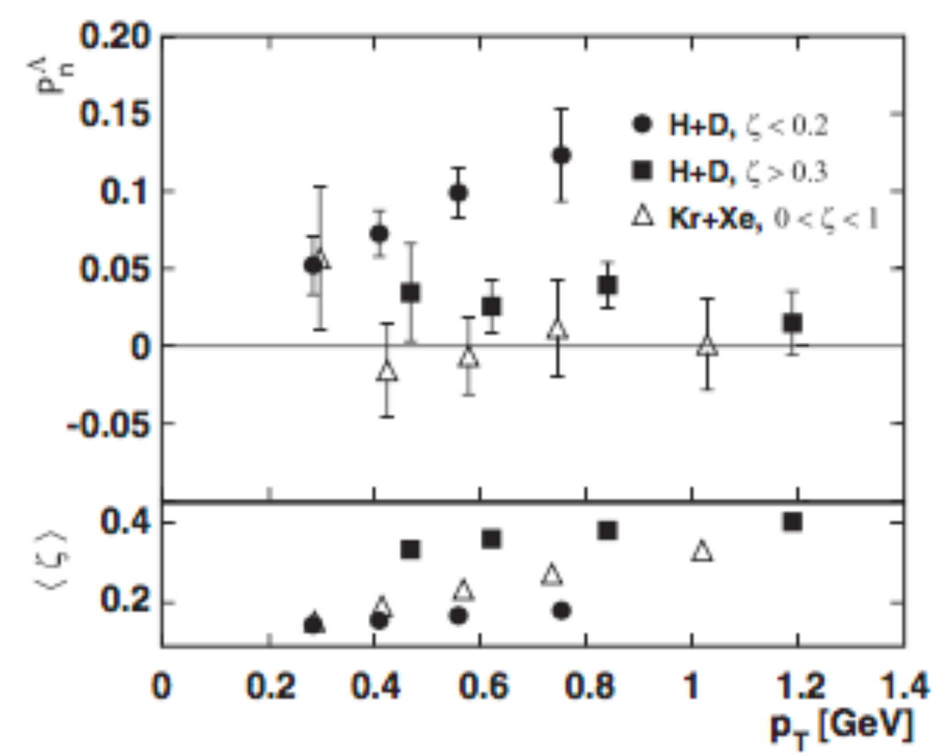
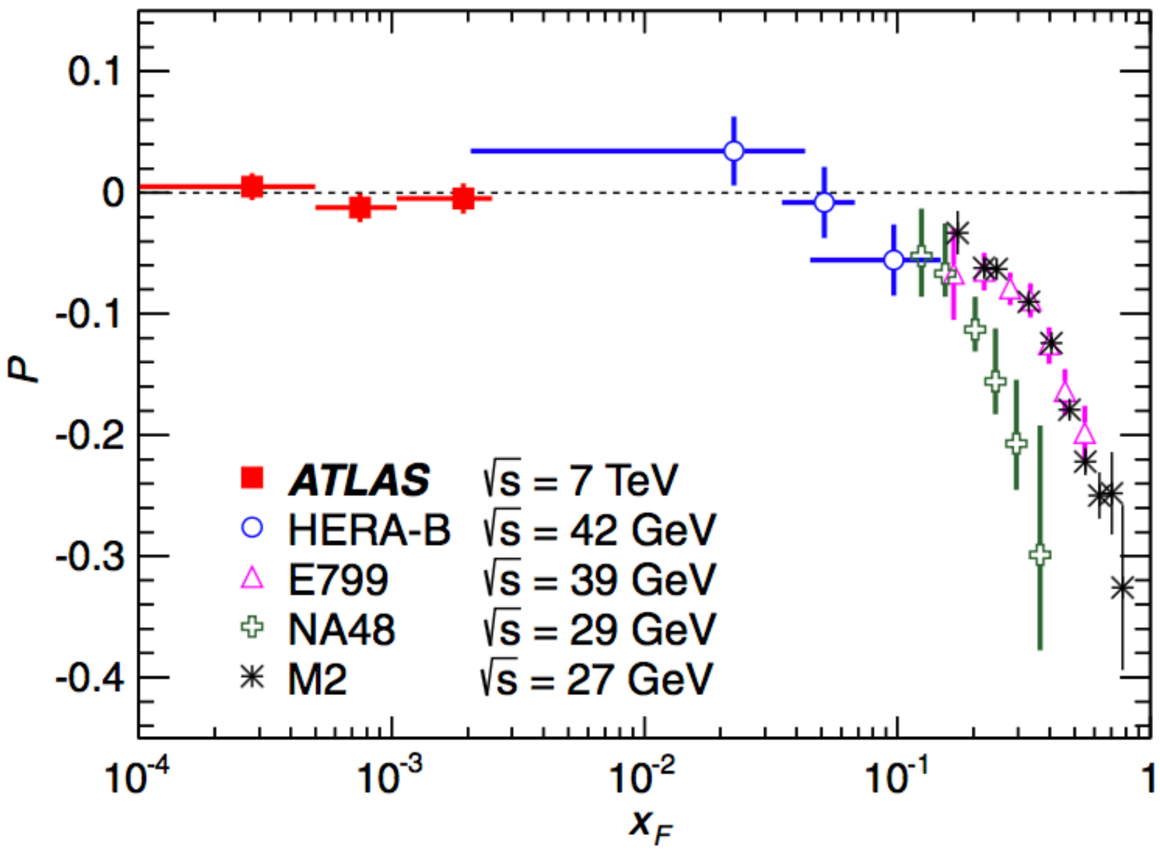


FIG. 3 (color online). Dependence of the transverse polarization P_n^Λ on the atomic-mass number A of the target nuclei. The inner error bars represent the statistical uncertainties; the full error bars represent the total uncertainties, evaluated as the sum in quadrature of statistical and systematic uncertainties.



What about LHC?

Is it feasible at a high energy collider?



Recent ATLAS measurement

at $\sqrt{S}$ = 7 TeV

PRD 91, 032004 (2015)

Polarisation at mid rapidity

What does Exp Say ...

$$pA \to \Lambda^{\uparrow} X$$

$$pp \to \Lambda^{\uparrow} X$$

$$\nu N \to \Lambda^{\uparrow} X$$

$$\gamma^* N \to \Lambda^{\uparrow} X$$

$$e^+ e^- \to \Lambda^{\uparrow} X$$

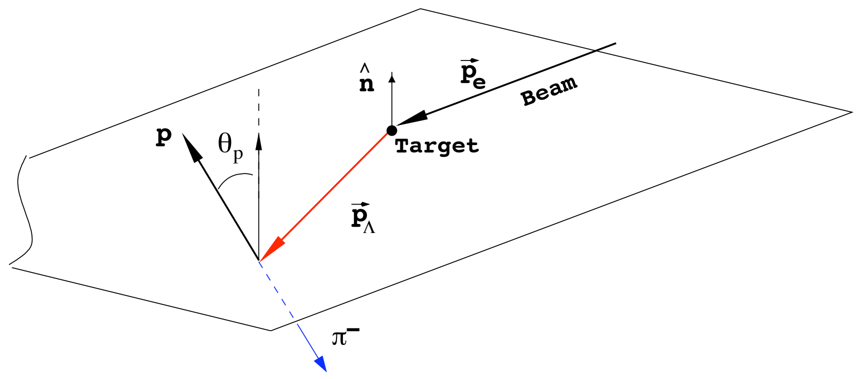


FIG. 1: Schematic diagram of inclusive Λ production and decay. The angle θ_p of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the Λ rest frame.

Simplest and cleanest process Λ polarization

$$e^+e^- \to \Lambda^\uparrow X - \mathrm{inclusive}$$

$$e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})\, X - \mathrm{semi-inclusive}$$

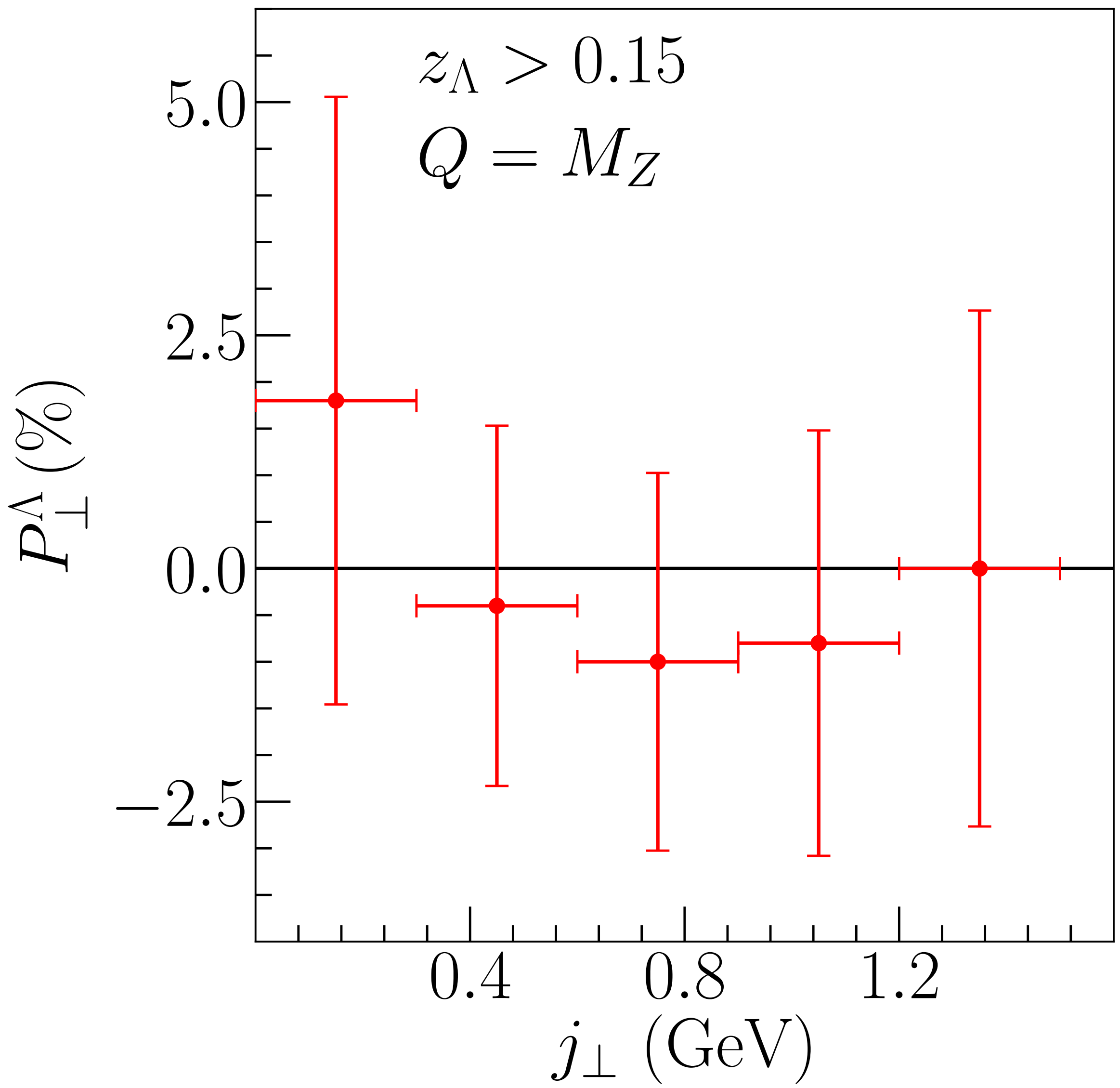
$$e^+e^- \to \Lambda^\uparrow h\, X - \mathrm{semi-inclusive}$$

Most data is inclusive !

Simplest and cleanest process : $e^+e^- \to \Lambda^\uparrow(\text{Thrust})\,X$

OPAL at LEP on Z-pole [Eur.Phys.J C2, 49 (1998)]

Longitudinal Polarization, but
no significant Transverse Polarization



Simplest and cleanest process $\Lambda^{\uparrow}$ in e^+e^-

Belle data: Transverse Polarization

Y. Guan, et al. PRL 122 (2019) → talk by Anselm here @ Jets Workshop

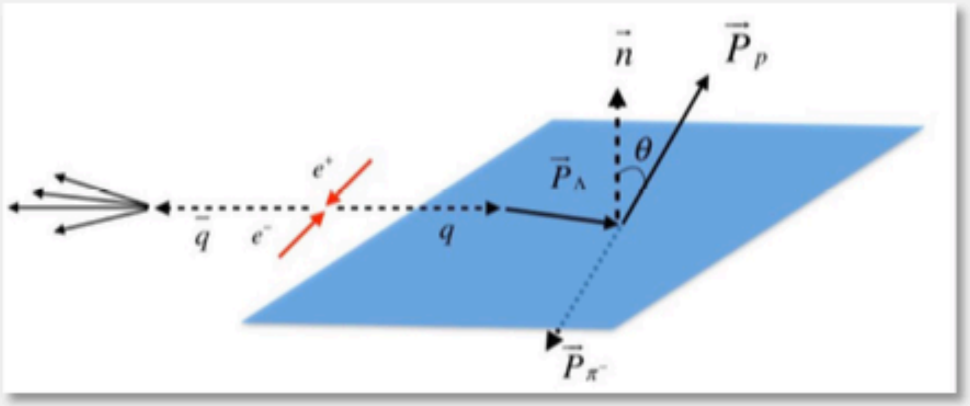
⇒ significant transverse polarization

$$e^+e^- \to \Lambda^{\uparrow}(\mathrm{Thrust})\, X$$

$$e^+e^- \to \Lambda^{\uparrow}\, h\, X,$$

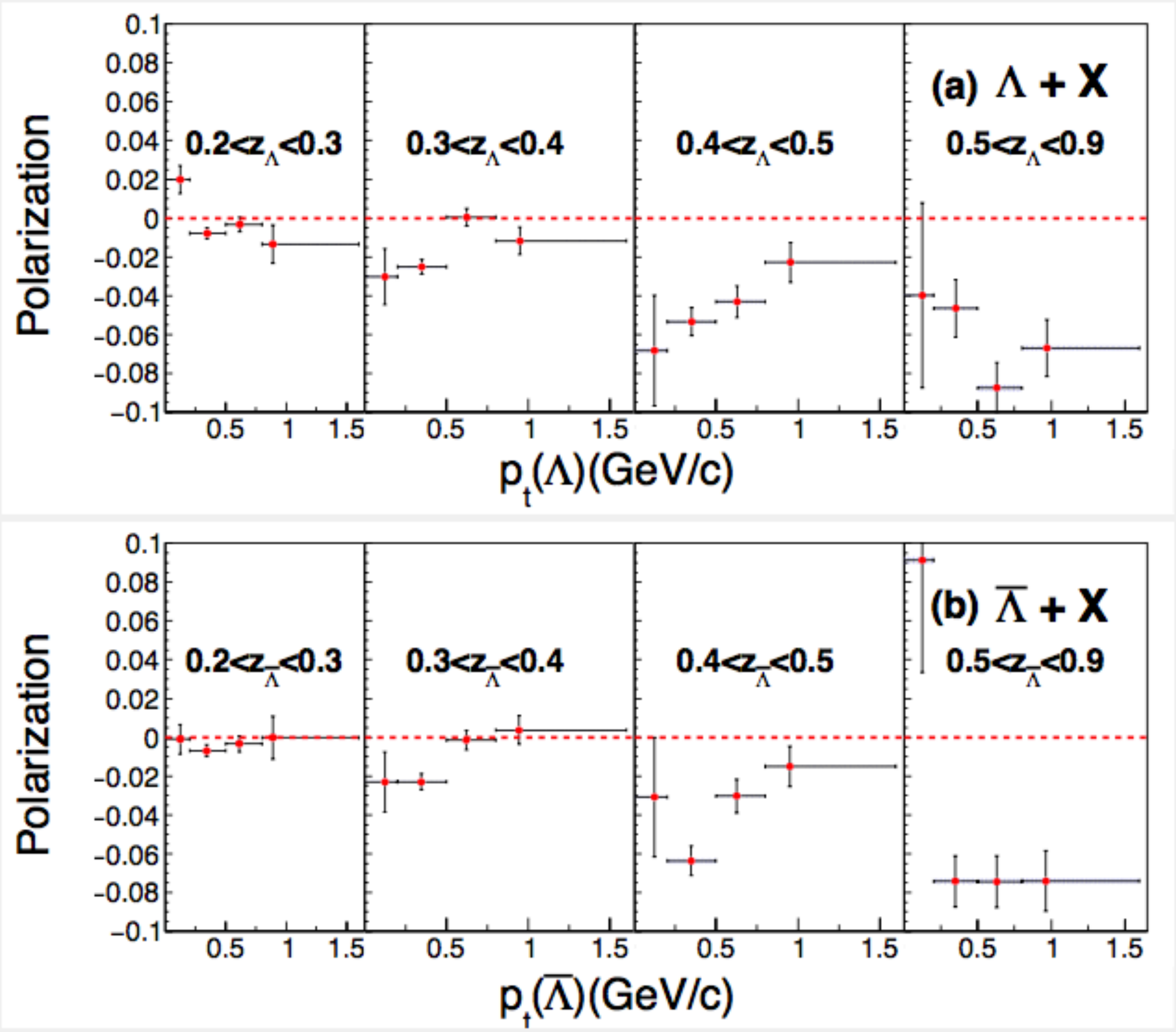
Measured w.r.t. thrust axis & back to back hadrons="bTOb"

FIRST OBSERVATION BY BELLE



PHYSICAL REVIEW LETTERS 122, 042001 (2019)

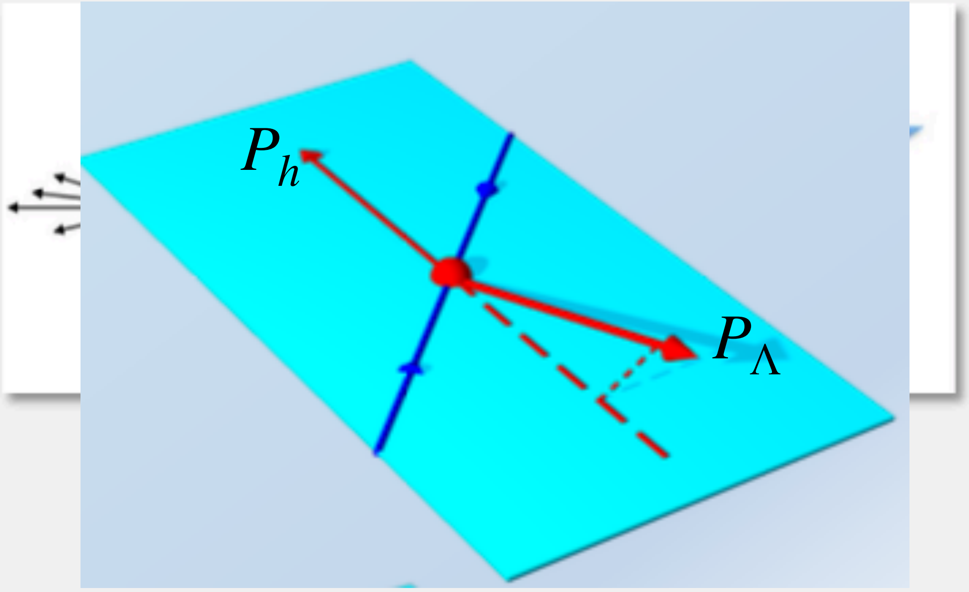
From Anselm's INT talk



The P_t is measured as the transverse momentum of Λ relative to the **thrust axis**

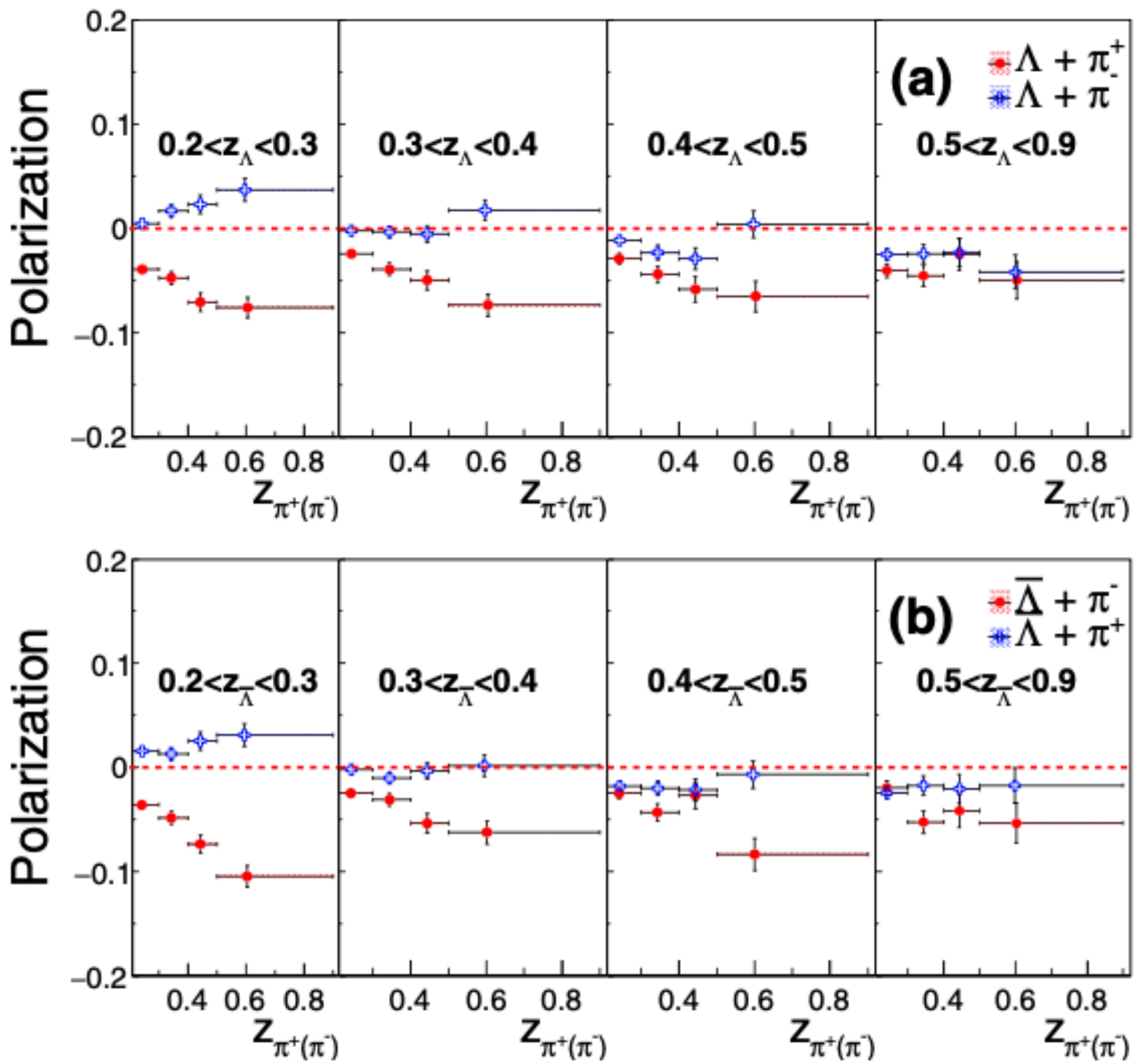
$$e^+e^- \to \Lambda^\uparrow(\text{Thrust})\, X$$

FIRST OBSERVATION BY BELLE



PHYSICAL REVIEW LETTERS 122, 042001 (2019)

From Anselm's INT talk



$$e^+e^- \to \Lambda^\uparrow h X$$

Back to back hadrons
integrated over $p_\perp$NOT SMALL

Simplest and cleanest process $\Lambda^{\uparrow}$ in e^+e^-

$\Rightarrow$ significant transverse polarization

$$e^+e^- \to \Lambda^{\uparrow}(\mathrm{Thrust})\, X$$

$$e^+e^- \to \Lambda^{\uparrow}\, h\, X,$$

Measured w.r.t. thrust axis & back to back hadrons="bTOb"

Questions/issues:

QCD prediction of Physics un-suppressed ?

- TMD factorization two scale problem
  - ✳TMD factorization formalism for thrust axis measurement
- Twist-3 factorization one hard scale-power suppressed

Simplest and cleanest process semi-inclusive

$$e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})\,X$$

$$e^+e^- \to \Lambda^\uparrow\,h\,X,$$

Questions/issues:

QCD prediction of Physics of un-suppressed ?

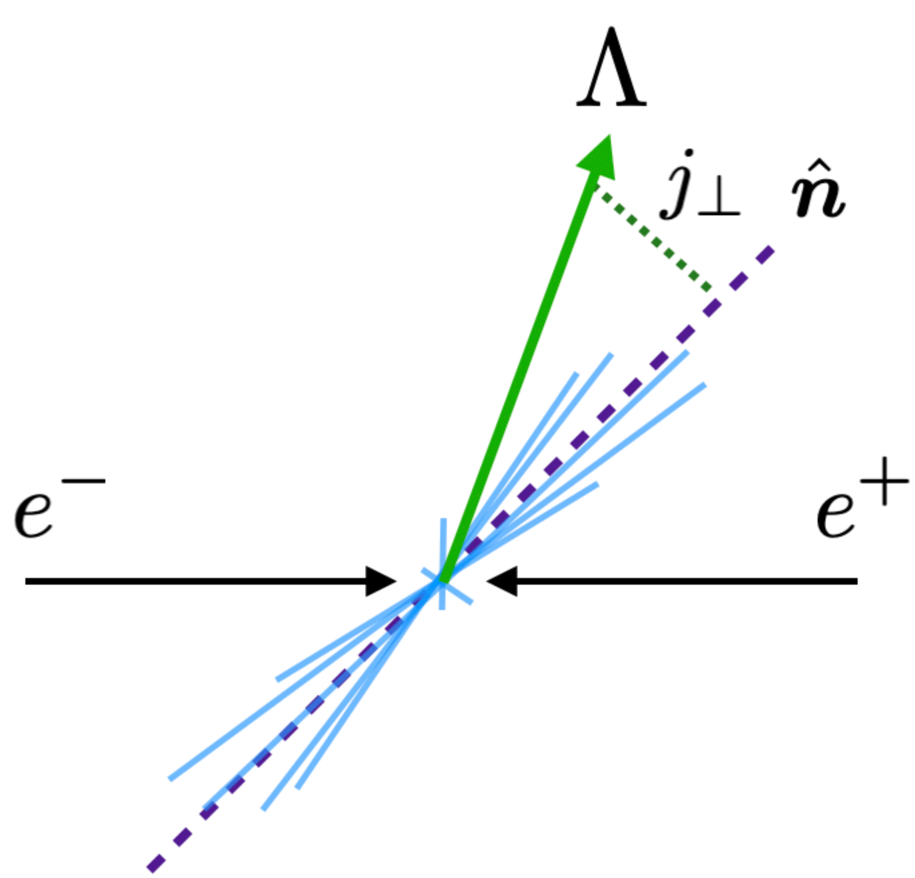
- TMD factorization two scale problem

$$\Lambda_{QCD} \lesssim p_\perp \ll Q$$

$$\Lambda_{QCD} \lesssim j_\perp \ll Q$$

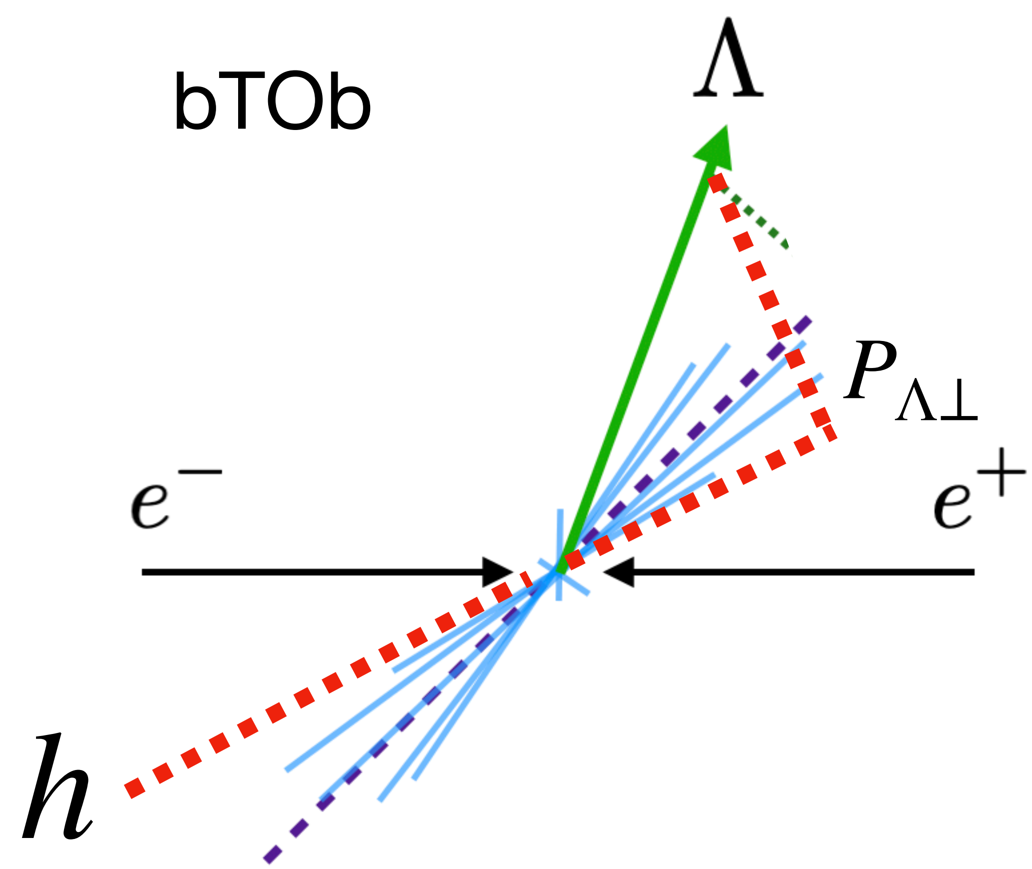
Want to test Universality Belle BeS BaBar + EIC

- Is it the same PFF function in the bTOb hadron and hadron + thrust measurements ???



thrust

bTOb



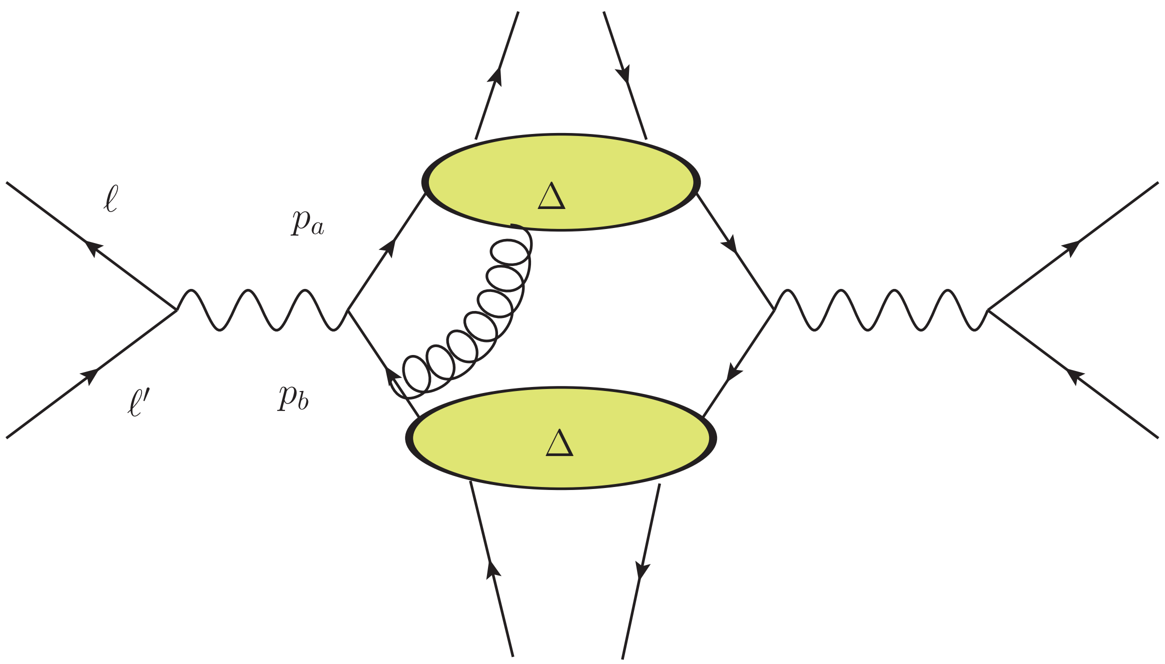
$$T = \frac{\sum_i |\boldsymbol{p}_i \cdot \hat{n}|}{\sum_i |\boldsymbol{p}_i|}$$

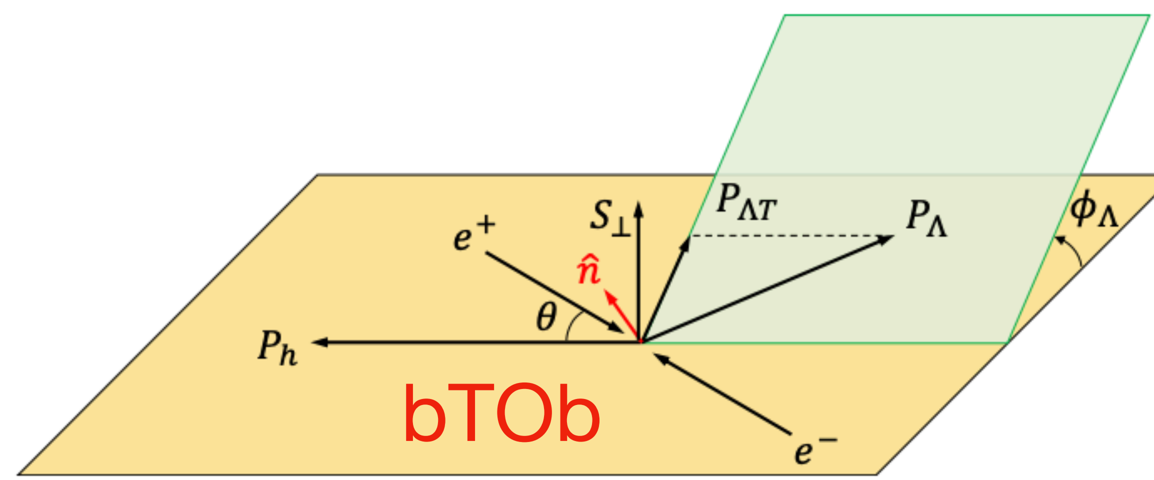
- What about “T”-odd universality can we test it with all data?

Explain via TMD fact. $e^+e^- \to \Lambda^\uparrow h X$, polarization fragmentation function PFF

In TMD factorization framework for back to back production of $\Lambda + h$ chiral even, naively T-odd fragmentation function, universal

Parton Model factorization Mulders & Tangerman 1996, Boer Jakob Mulders 1996





$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{p}_\perp, \mathbf{S}_\perp, Q) = \frac{1}{2}\Big[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) + \frac{1}{z_\Lambda M_\Lambda} D^\perp_{1T,\Lambda/q}\left(z_\Lambda, p_\perp, Q\right)\epsilon_{\perp\rho\sigma} p_\perp^\rho S_\perp^\sigma\Big]$$

Explain via TMD fact.

TMD PDFs (x, k_T)

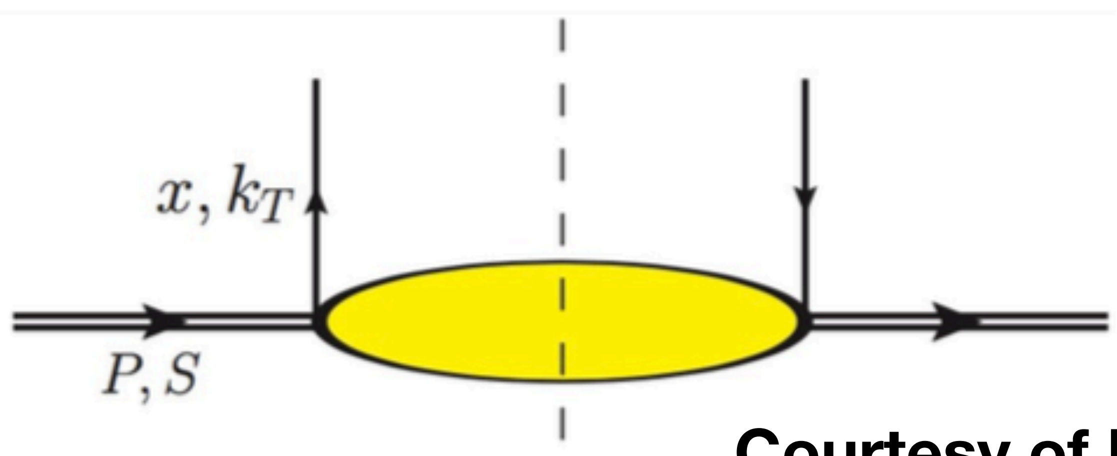
q pol. / H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

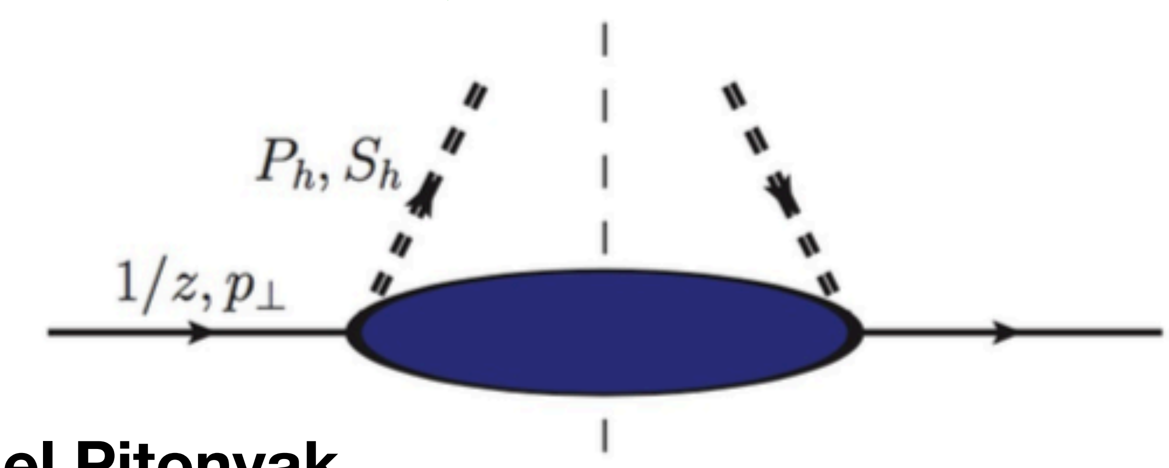
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs ($z, p_\perp$)

q pol. / H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))





Courtesy of Daniel Pitonyak

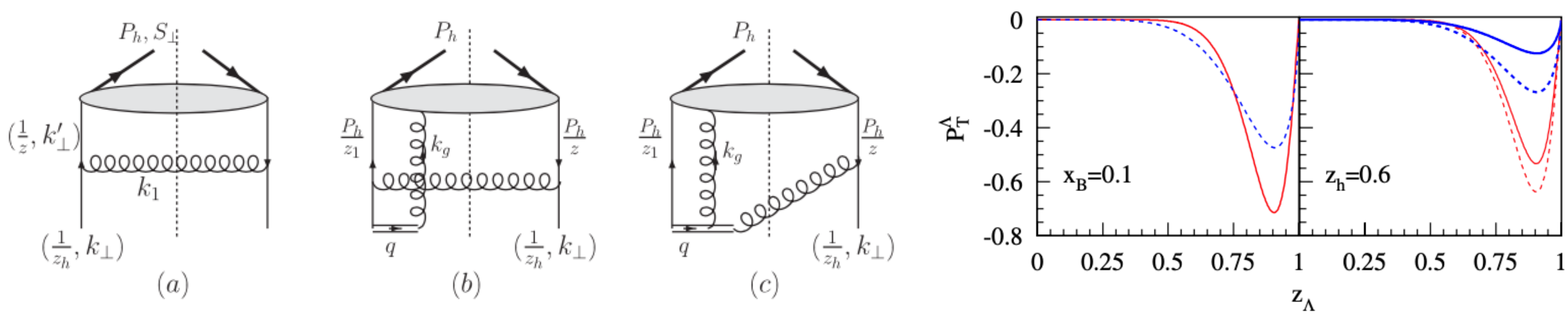
Use both data sets to study universality of T-odd fragmentation?
What is prediction of TMD Factorization

Universality of T-odd Collins function: $H_{1,\pi/q}^{\perp(1)}(z,b,Q)$

Metz PLB2002,
Boer Mulders Pijlman NPB2003
Collins Metz PRL 2004,
Gamberg, Mukerjee, Mulders PRD2007,
Meissner Metz PRL 2009,
Gamberg Mukherjee, Mulders PRD 2008

Universality of T-odd PFF prediction from pQCD - $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda,b,Q)$
phase from FSI but not gluonic/fermionic pole

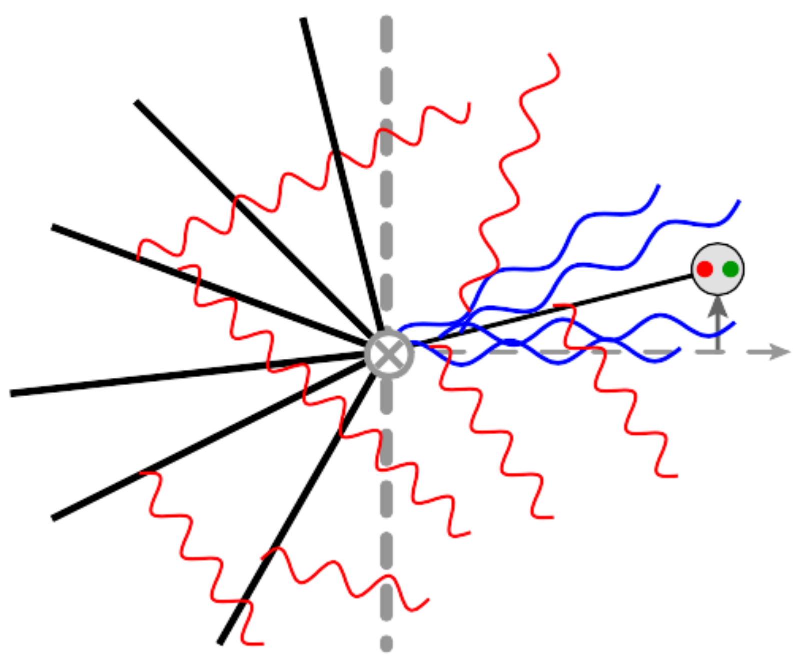
Boer, Kang, Vogelsang, Yuan PRL 2010

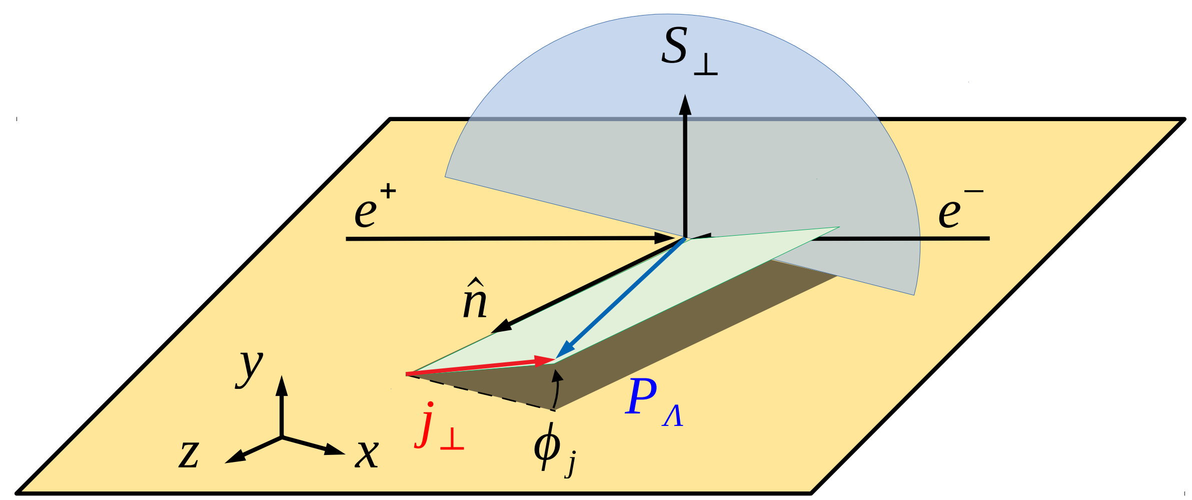


? Same PFF ? in $e^+e^- \to \Lambda^\uparrow(\text{Thrust})$

In TMD factorization framework for back to back production of Λ (Thrust) non-global observable "right hemisphere" only

? chiral even, naively T-odd fragmentation function, universal ?





- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

?

$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{p}_\perp, \mathbf{S}_\perp, Q) = \frac{1}{2}\Big[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) + \frac{1}{z_\Lambda M_\Lambda} D^\perp_{1T,\Lambda/q}\left(z_\Lambda, p_\perp, Q\right) \epsilon_{\perp\rho\sigma} p_\perp^\rho S_\perp^\sigma\Big]$$

Λ Belle data fall into 2 classes

$$e^+e^- \to \Lambda^\uparrow h\, X, \quad \& \quad \Lambda^\uparrow(\text{Thrust})\, X$$

Recent extractions address this

1) D'Alesio & Murgia PRD2020 bTOb + Thrust
2) Callos, Kang, Terry PRD2020 bTOb only

$$D^{\perp}_{1T,\Lambda/q}\left(z_\Lambda, p_\perp, Q\right)$$

Λ fragmentation TMD Factorization

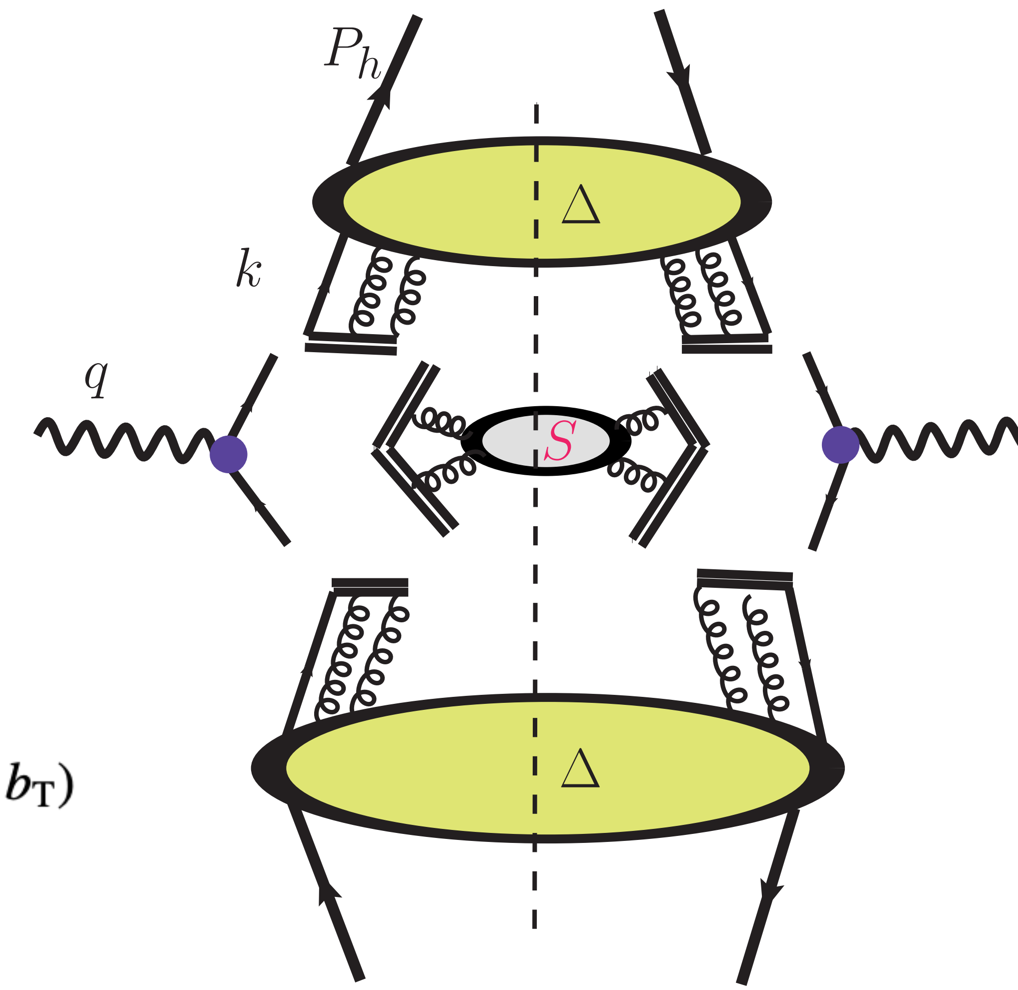
QCD factorization Collins Soper 1982 NPB,

Collins Foundations of PQCD Cambridge Press 2011

$$W^{\mu\nu} \overset{\text{prelim}}{=} \frac{8\pi^3 z_A z_B}{Q^2} \sum_f \mathrm{Tr}\, k^+_{A,\gamma}\gamma^- \, H^\nu_f(Q)\, k^-_{B,\gamma}\gamma^+ \, \overline{H}^\mu_f(Q)$$

$$\times \int \frac{d^{2-2\epsilon}\boldsymbol{b}_T}{(2\pi)^{2-2\epsilon}} e^{-i\boldsymbol{q}_{hT}\cdot\boldsymbol{b}_T}\, \tilde{S}(b_T)\tilde{D}_{1,\,H_A/f}(z_A, b_T)\, \tilde{D}_{1,\,H_B/\bar{f}}(z_B, b_T)$$

$$+ \text{polarized terms.}$$

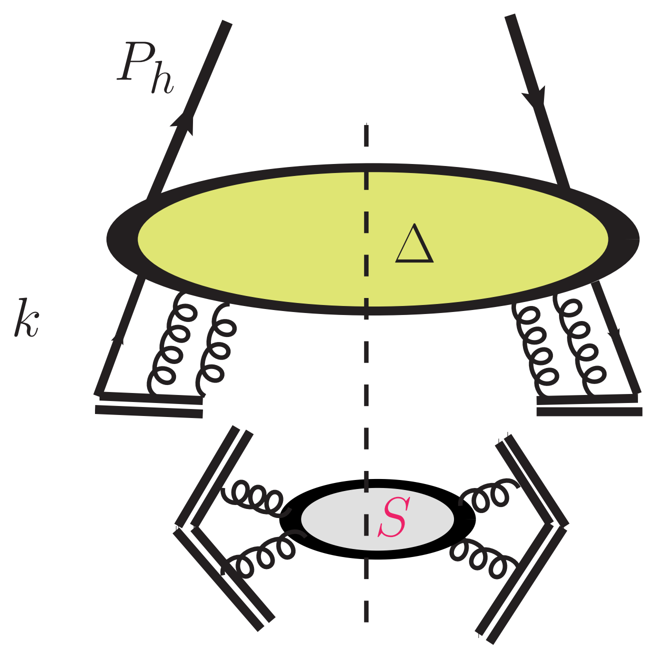


Elements TMD Evolution/Factorization QCD

TMD factorization/evolution CSS in b space region analysis & Ward Identities

- TMDs w/Gauge links: color invariant
- TMD PDFs & Soft factor have rapidity/LC divergences
- Rapidity regulator introduced to regulate these divergences

Collins Soper (81,82), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04,05,06), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun,Yuan (13), Echevarria, Idilbi, Scimemi JHEP 2012, Collins Rogers 2015 ....
SCET: Bauer, Flemming Pirjol Rothstein, Stewart PRD 2002, Chiu, Jain, Neill, Rothstein JHEP 2013, Rothstein & Stewart JHEP 2016, ...



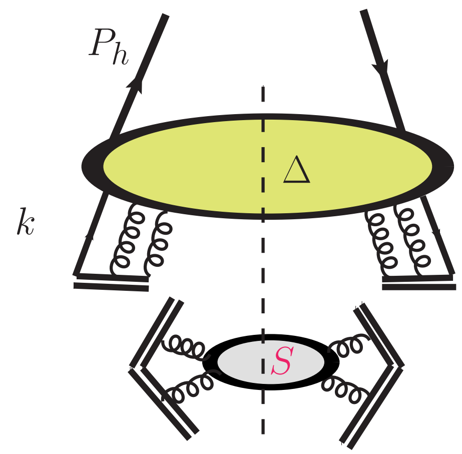
$$\tilde{D}^{\rm sub}_{H/j}(z_A, b_T; \mu, \zeta) = \lim_{\substack{y_A\to+\infty \\ y_B\to-\infty}} \underbrace{\tilde{D}^{\rm unsub}_{H/j}(z_A, b_T; \mu, y_A - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

$$\Updownarrow$$

$$\tilde{D}^{\rm unsub}_{H/j}(z_A, b_T; \mu, y_P - y_B) = \frac{1}{z_A}\int \frac{db^+}{2\pi} e^{-ik_A^- b^+} \langle 0|\gamma^- \mathcal{U}_{[0,b]}\psi(b)|X P_A\rangle\langle P_A X|\bar{\psi}(0)|0\rangle\big|_{b^-=0}$$

TMD Evolution follows from independence of rapidity scale

Collins Cambridge press 2011, Aybat & Rogers 2011 PRD

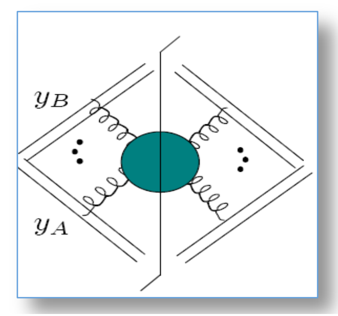


From operator definition get

$$\tilde{D}^{\rm sub}_{H/j}(z_A, b_T; \mu, \zeta) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \tilde{D}^{\rm unsub}_{H/j}(z_A, b_T; \mu, y_A - y_B) \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

Collins Soper Equation

$$\frac{\partial \ln \tilde{D}(z_A, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

$$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_k(g(\mu))$$

$$\frac{d \ln \tilde{D}(z_A, b_T; \mu, \zeta)}{d \ln \mu} = \gamma_D(g(\mu, \zeta/\mu^2)$$

RGE: get anomalous for *D* & *K*

JCC Soft factor further "repartitioned"
This is done to

1) cancel LC divergences in "unsubtracted" TMDs
2) separate "right & left" movers i.e. full factorization
3) remove double counting of momentum regions

Solve Collins Soper & RGE eqs. obtain "subtracted" TMD

$$\tilde{D}(z_A, b_T; \mu, \zeta) = \tilde{D}(z, b_T; \mu_0, Q_0^2)\, e^{\ln \frac{\sqrt{\zeta}}{Q_0} \tilde{K}(b_T, \mu_0) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left(\gamma_D(g(\mu'; 1) - \ln \frac{\sqrt{\zeta}}{\mu'} \gamma_k(g(\mu'))\right)}$$

TMD factorization & Thrust observable

Recent work

- M. Boglione & A. Simonelli, 2007.13674
- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425
- M. Boglione & A. Simonelli, 2007.13674-**very recent: & talk by Simonelli Wednesday**

Kang et al.
Derive TMD factorization for unpolarized transverse momentum distribution for the single hadron production with the thrust axis in electron-positron collision

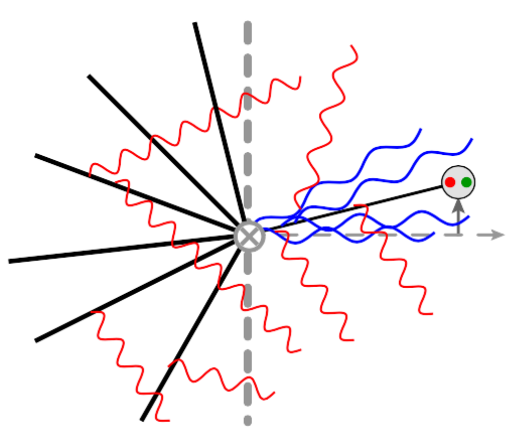
Lets Drill Down TMD factorization

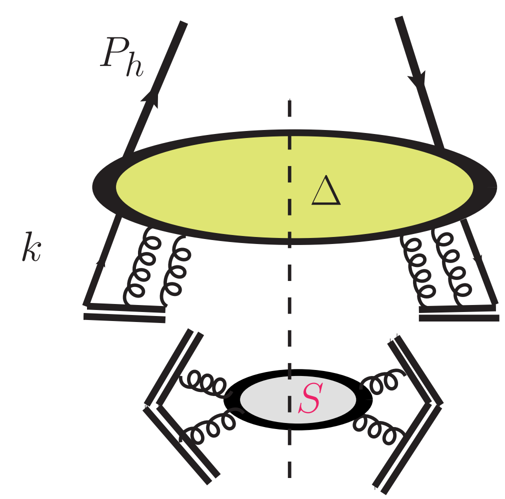
Recent work

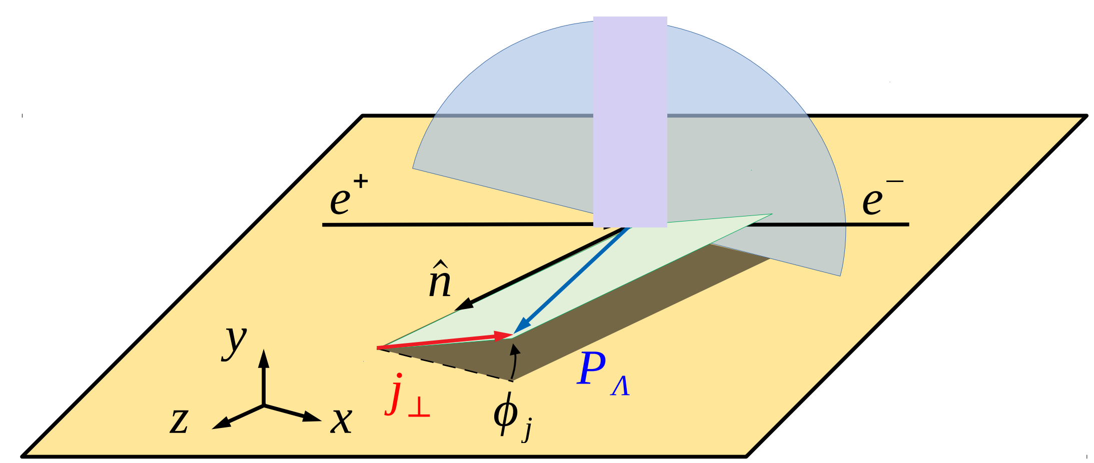
- Z.-B Kang, D.Y. Shao, F. Zhao 2007.14425

Derive TMD factorization for unpolarized TMD FF for single hadron production with the thrust axis in electron-positron collision $e^+e^- \to \Lambda^\uparrow(\text{Thrust})$ non-global observable

$$\frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \sigma_0 H(Q,\mu)\sum_q e_q^2 \int d^2p_\perp d^2\lambda_\perp \delta^{(2)}(\boldsymbol{j}_\perp - \boldsymbol{p}_\perp - z_\Lambda \boldsymbol{\lambda}_\perp) D_{\Lambda/q}(z_\Lambda, p_\perp, \mu, \zeta/\nu^2) S_{\text{hemi}}(\lambda_\perp, \mu, \nu)$$







$$\frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \sigma_0^{\text{TMD}} H(Q,\mu)\sum_q e_q^2 \int_0^\infty \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{j}_\perp/z_\Lambda} D_{\Lambda/q}(z_\Lambda, b, \mu, \zeta/\nu^2) S_{\text{hemi}}(b, \mu, \nu)$$

Lets Drill Down TMD factorization

Recent work

- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

Kang et al.

Fourier transform to b space

$$\frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \sigma_0^{\rm TMD} H(Q,\mu) \sum_q e_q^2 \int_0^\infty \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{j}_\perp/z_\Lambda} D_{\Lambda/q}(z_\Lambda, b, \mu, \zeta/\nu^2) S_{\rm hemi}(b,\mu,\nu)$$

Calculated to NLO and NLL

$$S_{\rm hemi}(b,\mu,\nu) = \sqrt{S(b,\mu,\nu)}$$

$$\frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \sigma_0^{\rm TMD} H(Q,\mu) \sum_q e_q^2 \int_0^\infty \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{j}_\perp/z_\Lambda} D_{\Lambda/q}(z_\Lambda, b, \mu, \zeta/\nu^2) \sqrt{S(b,\mu,\nu)}$$

Where UV & Rapidity subtracted TMD

$$D_{\Lambda/q}^{\rm TMD}(z_\Lambda, b, \mu, \zeta) = D_{\Lambda/q}(z_\Lambda, b, \mu, \zeta/\nu^2) \sqrt{S(b,\mu,\nu)}$$

TMD factorization non-global observable
Unpolarized

$$\frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \sigma_0^{\rm TMD} H(Q,\mu) \sum_q e_q^2 \int_0^\infty \frac{d^2b}{(2\pi)^2} e^{i\boldsymbol{b}\cdot\boldsymbol{j}_\perp/z_\Lambda} D_{\Lambda/q}^{\rm TMD}(z_\Lambda, b, \mu, \zeta)$$

Match onto collinear TMD FF

$$D_{\Lambda/q}^{\rm TMD}(z_\Lambda, b, Q, Q^2) = \frac{1}{z_\Lambda^2} D_{\Lambda/q}(z_\Lambda, \mu_{b_*}) e^{-S_{\rm pert}(\mu_{b_*}, Q) - S_{\rm NP}(b, z_\Lambda, Q_0, Q)}$$

$$b_* = b/\sqrt{1+(b/b_{\rm max})^2}$$

$$\frac{d\sigma}{dz_{la} d^2\boldsymbol{j}_\perp} = \sigma_0^{\rm TMD} \sum_q e_q^2 \int_0^\infty \frac{bdb}{(2\pi)} J_0\left(\frac{b\, j_\perp}{z_\Lambda}\right) \frac{1}{z_\Lambda^2} D_{\Lambda/q}(z_\Lambda, \mu_{b_*})\, e^{-S_{\rm NP}(b, z_\Lambda, Q_0, Q) - S_{\rm pert}(\mu_{b_*}, Q)} \underbrace{U_{\rm NG}(\mu_{b_*}, Q)}_{\text{non-global logs resummed}}$$

- Z.B Kang, D.Y. Shao, F. Zhao 2007.14425

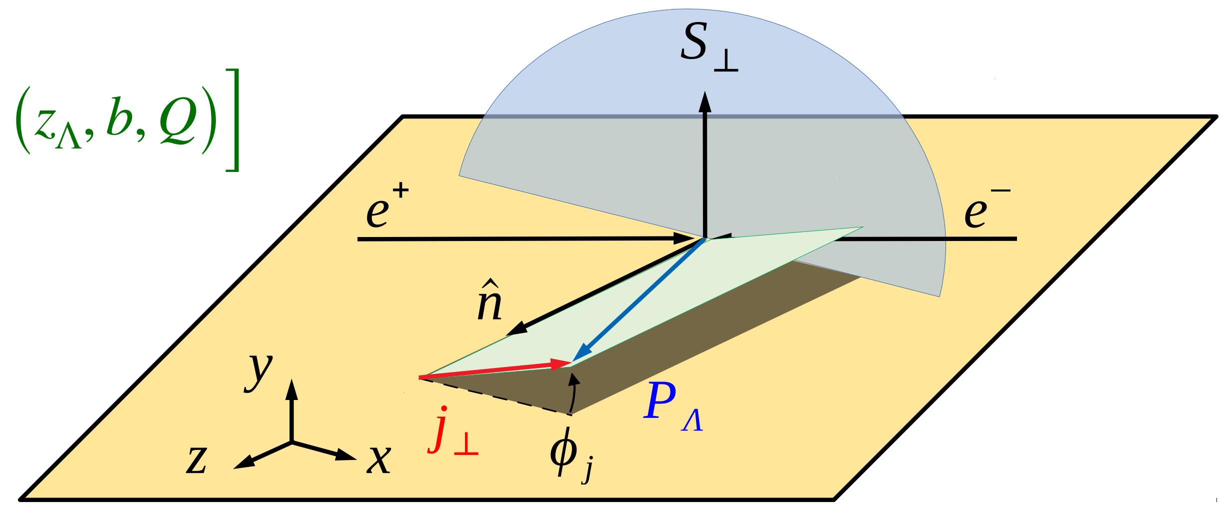
non-global
logs resummed
Factorization
Becher Rahn Shao JHEP 2017
M.Dasgupta & G.Salam PLB2001

We extend TMD factorization PFF

Gamberg, Kang, Shao, Terry, Zhao to appear

$$\hat{D}_{\Lambda/q}(z_\Lambda, \mathbf{b}, \mathbf{S}_\perp, Q) = \frac{1}{2}\Big[D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q) - i\epsilon_{\perp\rho\sigma} b^\rho S_\perp^\sigma\, M_\Lambda D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, b, Q)\Big]$$

Boer, Gamberg, Musch, Prokudin JHEP 2011



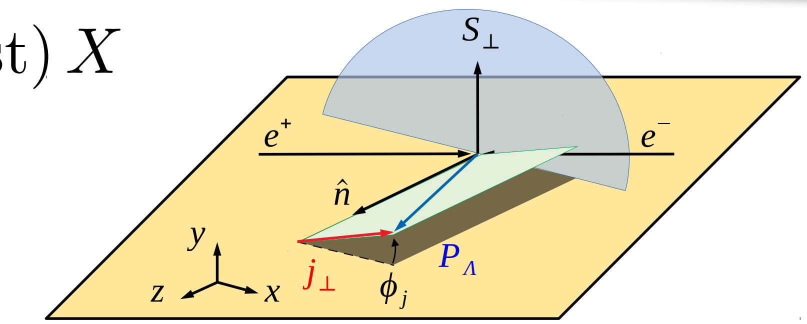
$$\frac{d\Delta\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} = \frac{d\sigma(\boldsymbol{S}_\perp)}{dz_\Lambda d^2\boldsymbol{j}_\perp} - \frac{d\sigma(-\boldsymbol{S}_\perp)}{dz_\Lambda d^2\boldsymbol{j}_\perp}$$
$$= \sigma_0^{\rm TMD} \sin(\phi_s - \phi_j) \sum_q e_q^2 \int_0^\infty \frac{b^2 db}{4\pi} J_1\left(\frac{b j_\perp}{z_\Lambda}\right)$$
$$\times \frac{M_\Lambda}{z_\Lambda^4} D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda, \mu_{b_*}) e^{-S_{\rm NP}^{\perp}(b, z_\Lambda, Q_0', Q) - S_{\rm pert}(\mu_{b_*}, Q)} U_{\rm NG}(\mu_{b_*}, Q)$$

UV & Rapidity subtracted TMD
Universal PFF

Establish factorization for $e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})$ carry out pheno

Postage stamp of input for Pheno

$$e^+e^- \to \Lambda^\uparrow(\text{Thrust})\,X$$



$$P_\perp^\Lambda(z_\Lambda, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} \bigg/ \frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp}\,.$$

$$S_{\rm NP}(b, z_\Lambda, Q_0, Q) = g_h \frac{b^2}{z_\Lambda^2} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b}{b_*}$$

Aidala Field Gamberg Rogers PRD 2014

$$g_h = 0.042\,\text{GeV}^2 \qquad g_2 = 0.84\,\text{GeV}^2$$

Implementation Issacson Sun Yuan 2014 MPA

$$U_{\rm NG}(\mu_{b_*}, Q) = \exp\left[-C_A C_F \frac{\pi^2}{3} u^2 \frac{1+(au)^2}{1+(bu)^c}\right]$$

Dasgupta Salam, PLB 2001

$$\text{with } a = 0.85 C_A,\ b = 0.86 C_A,\ c = 1.33$$

$$u \equiv \int_{\mu_{b_*}}^{Q} \frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{2\pi} = \frac{1}{\beta_0}\ln\left[\frac{\alpha_s(\mu_{b_*})}{\alpha_s(Q)}\right]$$

$$D_{1T,h/q}^\perp(z, p_\perp, Q_0') = \frac{M_\Lambda}{\langle M_D^2\rangle} D_{1T,h/q}^\perp(z, Q_0') \frac{e^{-p_\perp^2/\langle M_D^2\rangle}}{\pi\langle M_D^2\rangle}$$

$$D_{1T,h/q}^\perp(z, Q_0') = \mathcal{N}_q(z) D_{h/q}(z, Q_0') \qquad Q_0' = 10.58\text{ GeV}$$

$$\mathcal{N}_q(z) = N_q z^{\alpha_q}(1-z)^{\beta_q}\frac{(\alpha_q+\beta_q-1)^{\alpha_q+\beta_q-1}}{(\alpha_q-1)^{\alpha_q-1}\beta_q^{\beta_q}}$$

$$S_{NP}^\perp(b, z, Q_0', Q) = \frac{\langle M_D^2\rangle}{4}\frac{b^2}{z^2} + \frac{g_2}{2}\ln\frac{Q}{Q_0'}\ln\frac{b}{b_*}$$

Parameters fit from bTOb Belle data
Callos, Kang, Terry PRD2020

Belle data fit $e^+e^- \to \Lambda^\uparrow h X$

$D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda,,Q)$

Recent extractions address this
Callos, Kang, Terry PRD2020 bTOb only

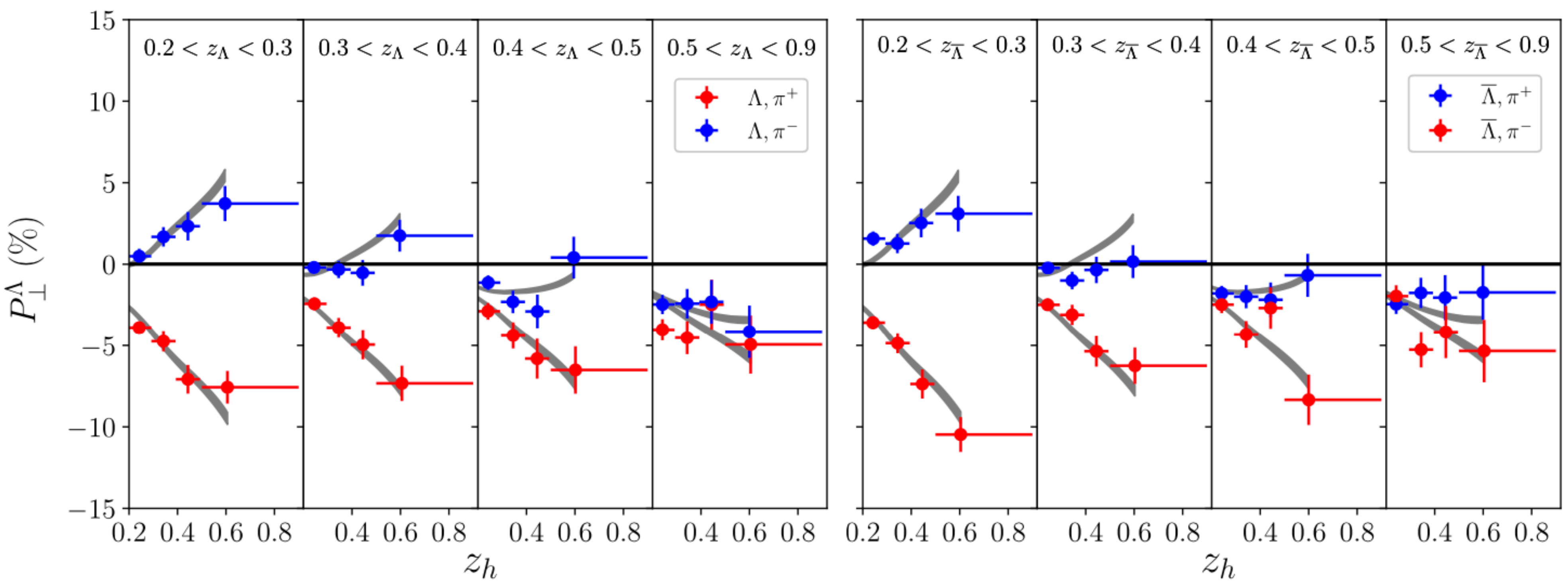


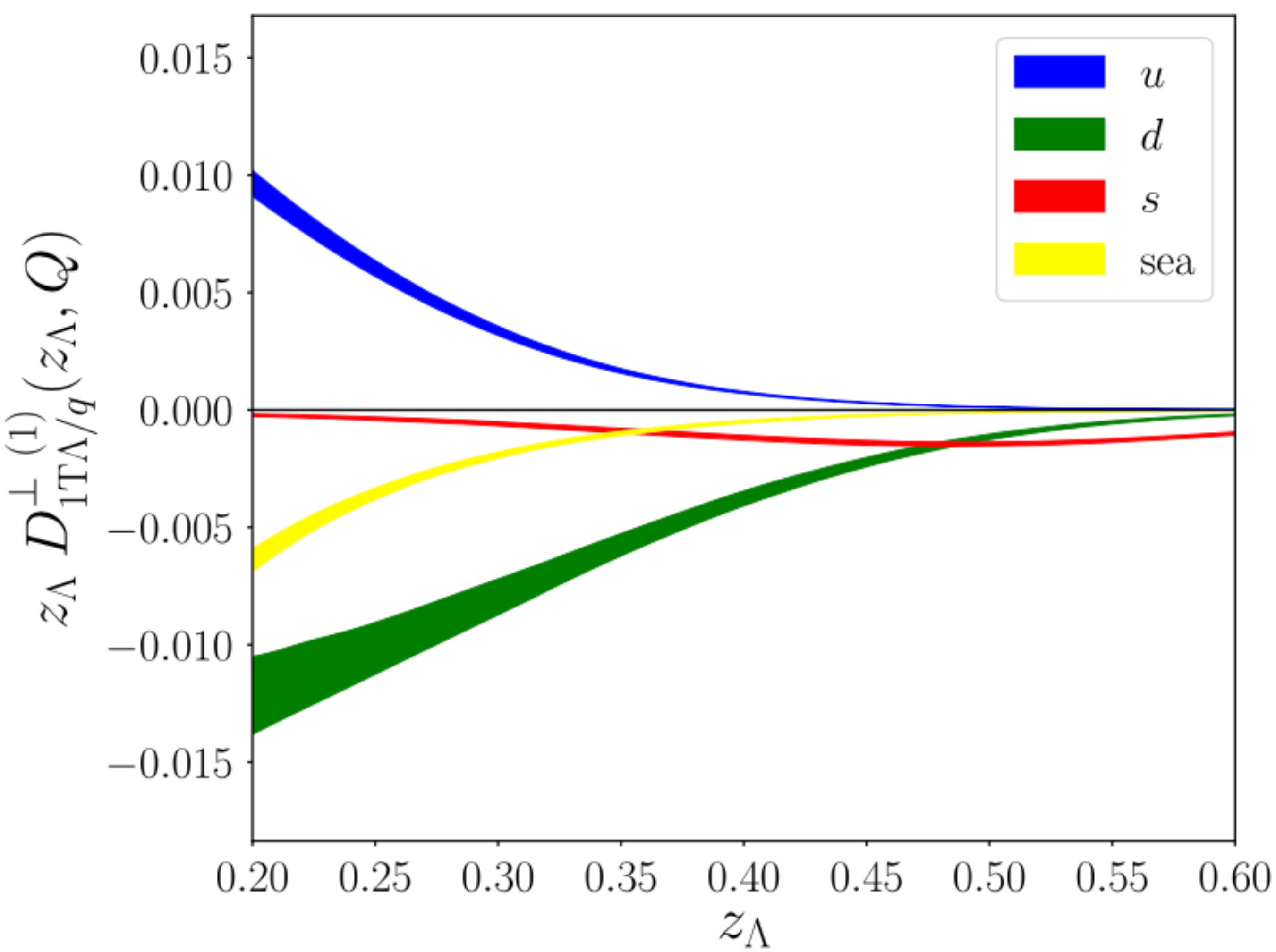
FIG. 3. The fit to the experimental data for π mesons is shown, with the gray uncertainty band displayed is generated by the replicas at 68% confidence. The left plots are for the production of $\Lambda + \pi^\pm$, while the right plots are for the production of $\bar\Lambda + \pi^\pm$.

And for kaons …

Belle data $e^+e^- \to \Lambda^\uparrow h\, X,$

$D^{\perp(1)}_{1T,\Lambda/q}(z_\Lambda,, Q)$

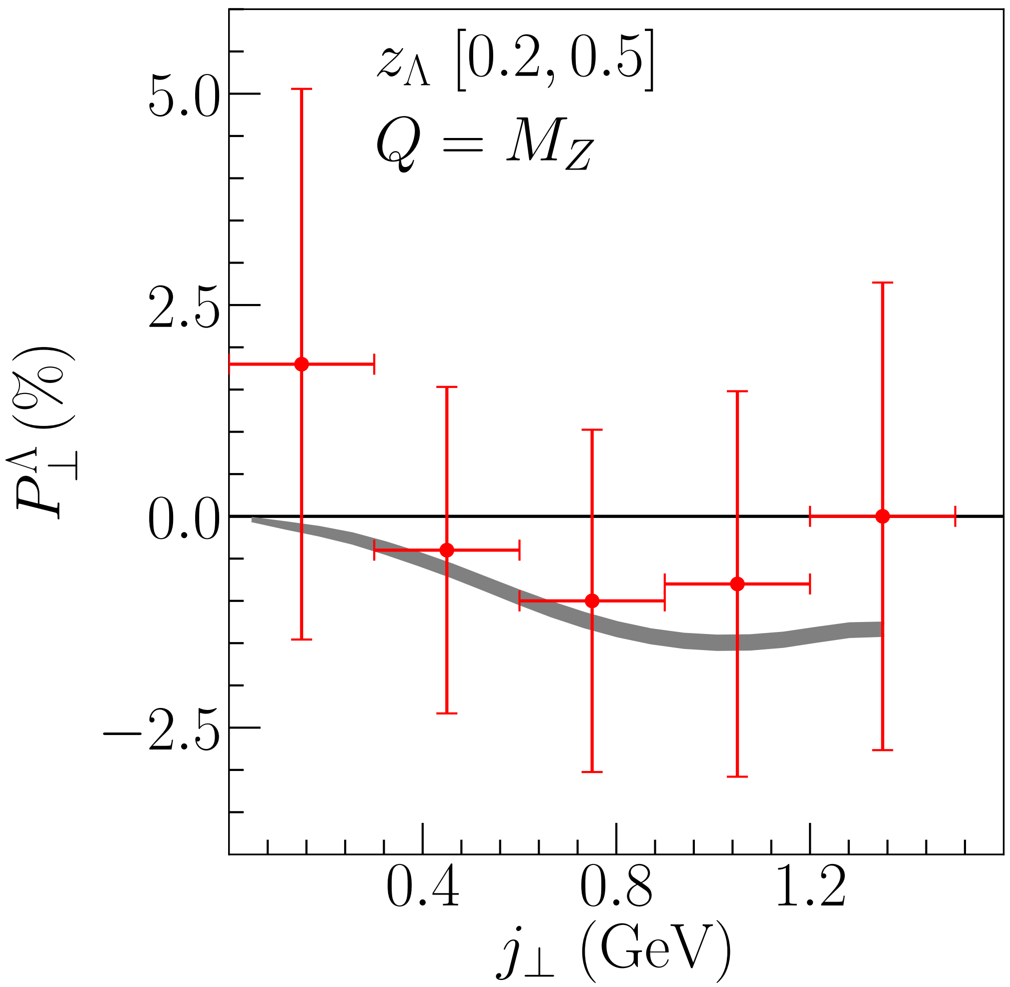
Recent extractions
Callos, Kang, Terry PRD2020 bTOb only



Exploit Universality to describe $e^+e^- \to \Lambda^\uparrow(\mathrm{Thrust})$

Compare theory predictions to OPAL & Belle data

$$e^+e^- \to \Lambda^\uparrow(\text{Thrust})$$



Gamberg, Kang, Shao,Terry, Zhao
to appear

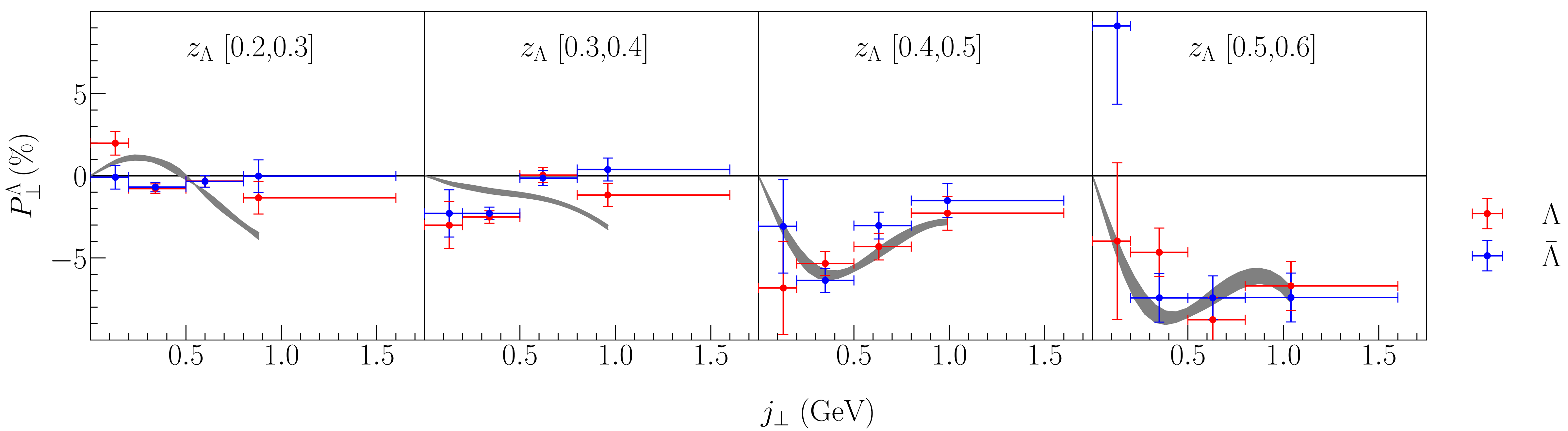
$$P_\perp^\Lambda(z_a, j_\perp) = \frac{d\Delta\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp} \Big/ \frac{d\sigma}{dz_\Lambda d^2\boldsymbol{j}_\perp}\,.$$

$P_\perp^\Lambda(z_\Lambda, j_\perp)$ for OPAL [19]. Theory curve is integrated over the region 0.2 $< z_\Lambda <$ 0.5.
total experimental uncertainty vertical error bar while the experimental uncertainty is $j_\perp$ horizontal error bar.
Theoretical uncertainty generated from the replicas for the TMD PFF
Callos, Kang, Terry PRD2020.

Compare theory predictions to OPAL & Belle

$$e^+e^- \to \Lambda^\uparrow(\text{Thrust})$$

Gamberg, Kang, Shao,Terry, Zhao to appear



- $P_\perp^\Lambda(z_\Lambda, j_\perp)$ for the Belle data [20]. left to right theory integrated from $0.2 < z_\Lambda < 0.3$, $0.3 < z_\Lambda < 0.4$, $0.4 < z_\Lambda < 0.5$, $0.5 < z_\Lambda < 0.6$
- The data in red is for Λ production while the data in blue is for $\bar{\Lambda}$ production
- Data plotted with total exp. uncertainty as vertical error bar & uncertainty on $j_\perp$ horizontal error bar
- Gray band is the theoretical uncertainty which was generated from the replicas for the TMD PFF. Callos, Kang, Terry PRD2020

Simplest and cleanest process $e^+e^- \to \Lambda^\uparrow X$

$\Rightarrow$ significant transverse polarization ?

$$e^+e^- \to \Lambda^\uparrow X$$

Measure w.r.t. COM in principle can measure at Belle ?

Questions/issues:

QCD prediction of Physics twist-3

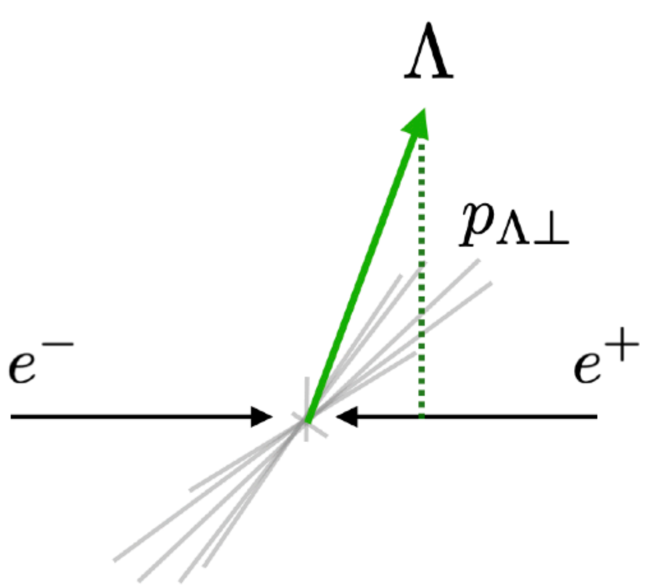
- Twist-3 factorization one hard scale

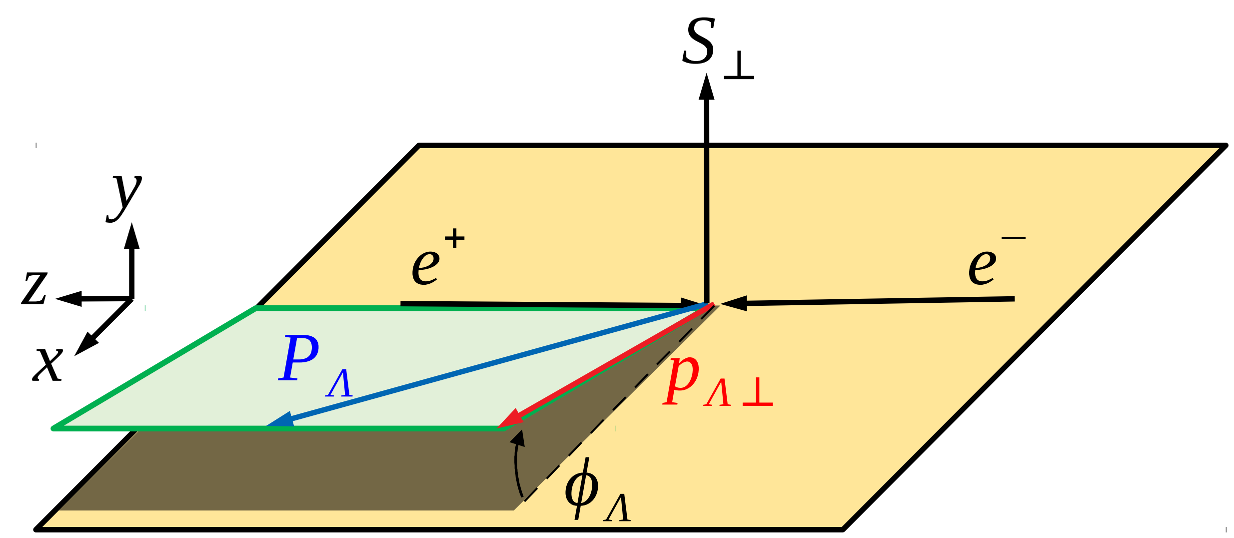
Simplest and cleanest process $e^+e^- \to \Lambda^\uparrow X$

$\Rightarrow$ significant transverse polarization ?

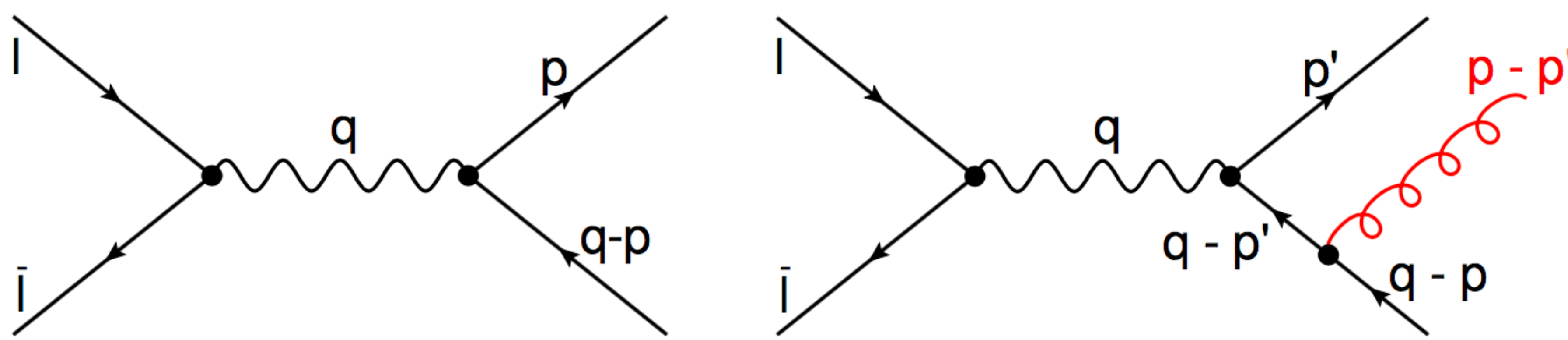
$$e^+e^- \to \Lambda^\uparrow X$$

Measured w.r.t. COM $P_{\Lambda\perp} \sim Q$ twist-3 factorization





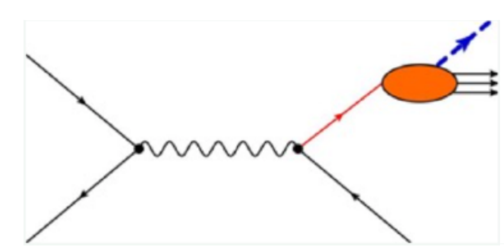
Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

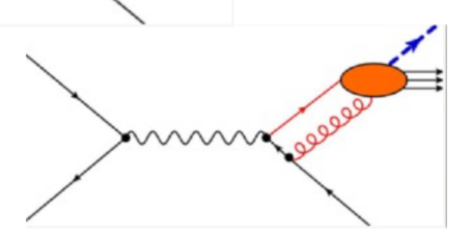


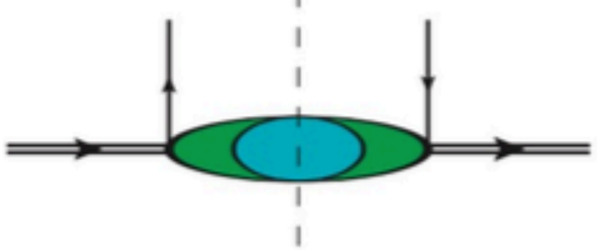
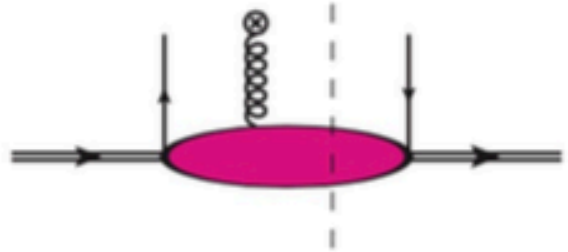
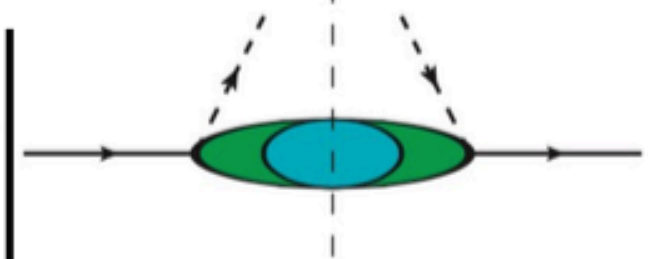
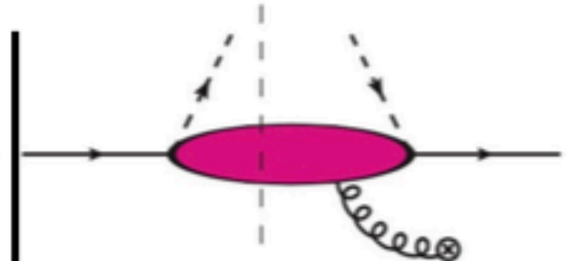
(a) LO quark fragmentation. **(b)** LO quark-gluon fragmentation.

$e^+e^- \longrightarrow \Lambda^{\uparrow}\, X$ inclusive $\Lambda^{\uparrow}$ production

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO



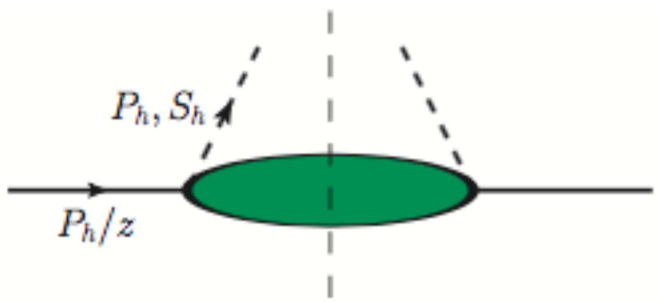


Hadron Pol.	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

Courtesy of Daniel Pitonyak

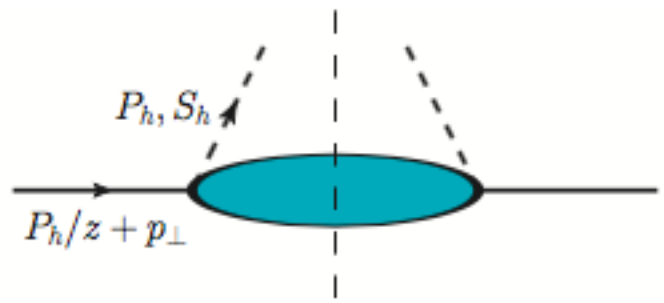
Transverse spin $\Lambda^{\uparrow}$ Collinear Tw-3 Formalism

Intrinsic



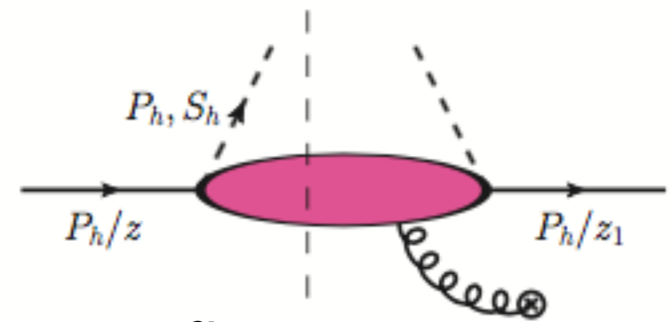
γ^{α}

Kinematic



k_T^{α}

Dynamical



A_T^{α}

See Metz,Pitonyak PLB2013
I will review for Lambdas

+

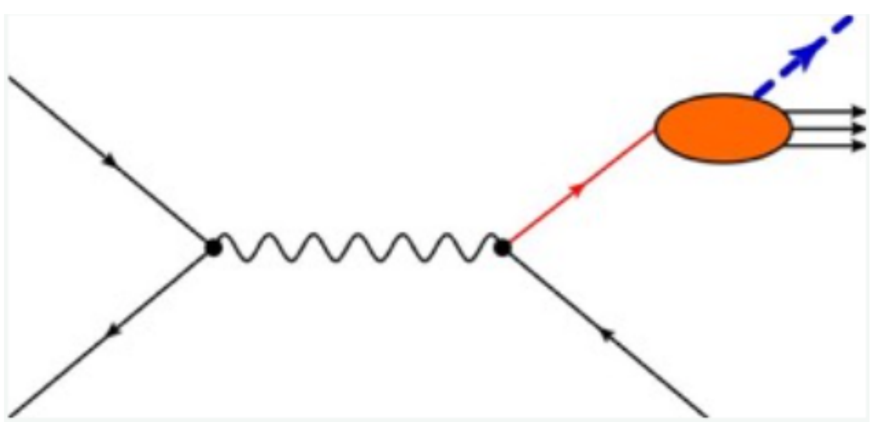
- **Gauge Invariance**
- **Equation of motions relations (EoMs)**
- **Lorentz Invariance relations (LIRs)**

Now Consider Transverse $e^+e^- \to \Lambda^\uparrow X$ polarization

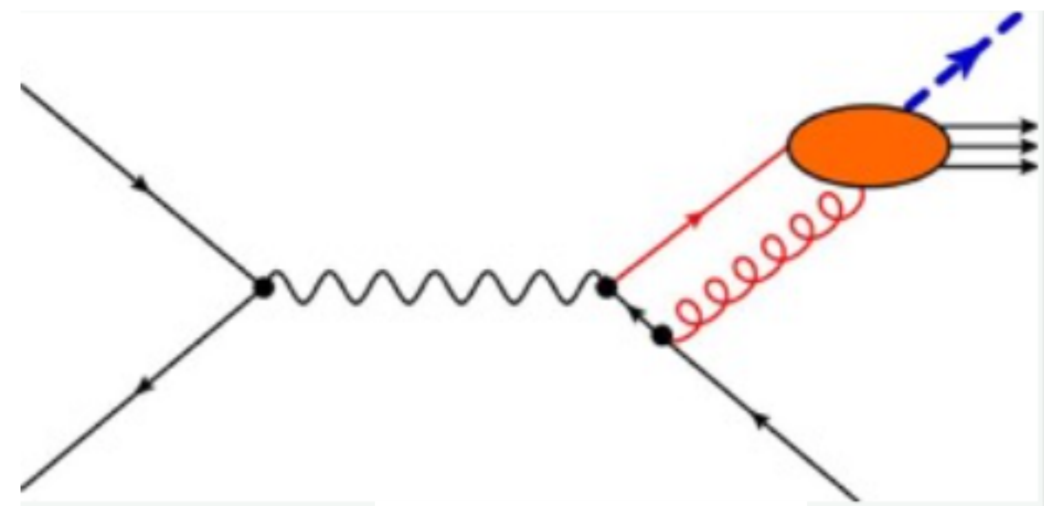
Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO

There are contributions from

’Intrinsic’ & ‘kinematical’ twist-3 FF



Dynamical’ twist-3 FF:



Intrinsic Kinematical Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\,\frac{\Im[\hat{D}_{FT}-\hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta}\right]$$

Using the EOMs and LIRs CS can be expressed soley in terms of $D_T^{\Lambda/q}(z)$

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

Boer, Jakob, Mulders NPB (1997)
in TMD framework at twist-3

Twist - 3 Pheno

$$\frac{d\Delta\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}} = -\sin(\phi_s - \phi_\Lambda)\sigma_0^{\rm Col}\left(\frac{8M_\Lambda}{Q}\right)\frac{p_{\Lambda\perp}}{Q}\frac{1}{z_\Lambda^3}\sum_q e_q^2\frac{D_{T,\Lambda/q}(z_\Lambda,Q)}{z_\Lambda}$$

To describe this process, only need a parameterization for $D_{T,\Lambda/q}(z_\Lambda, Q)$

Given our lack of knowledge of this fundamental twist-3 T-odd fragmentation function we will employ the approach outlined in
Gamberg, Metz, Pitonyak, Prokudin PLB 2017

Re-express the $D_{T,\Lambda/q}(z_\Lambda)$ in terms of our knowledge of $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$

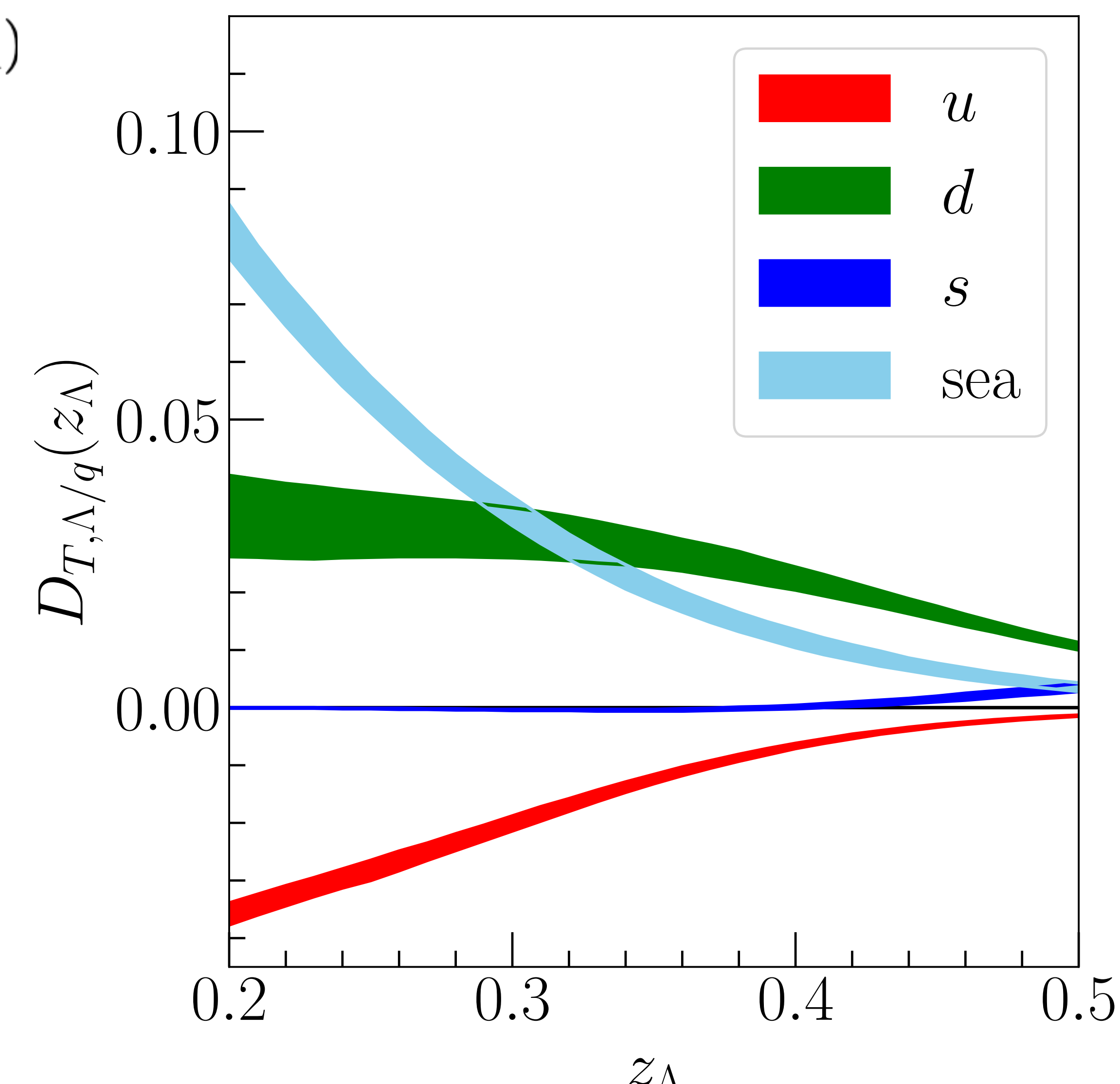
Twist - 3 Pheno

$$\frac{d\Delta\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}} = -\sin(\phi_s - \phi_\Lambda)\sigma_0^{\rm Col}\left(\frac{8M_\Lambda}{Q}\right)\frac{p_{\Lambda\perp}}{Q}\frac{1}{z_\Lambda^3}\sum_q e_q^2 \frac{D_{T,\Lambda/q}(z_\Lambda, Q)}{z_\Lambda}$$

Re-express the $D_{T,\Lambda/q}(z_\Lambda)$ **in terms of our knowledge of** $D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$

$$\frac{1}{z_\Lambda}D_{T,\Lambda/q}(z_\Lambda) = -\left(1 - z_\Lambda\frac{d}{dz_\Lambda}\right)D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda) - 2\int_0^1 d\beta\, \frac{\Im\left[\hat{D}_{FT}^{qg}(z_\Lambda,\beta)\right]}{(1-\beta)^2}$$

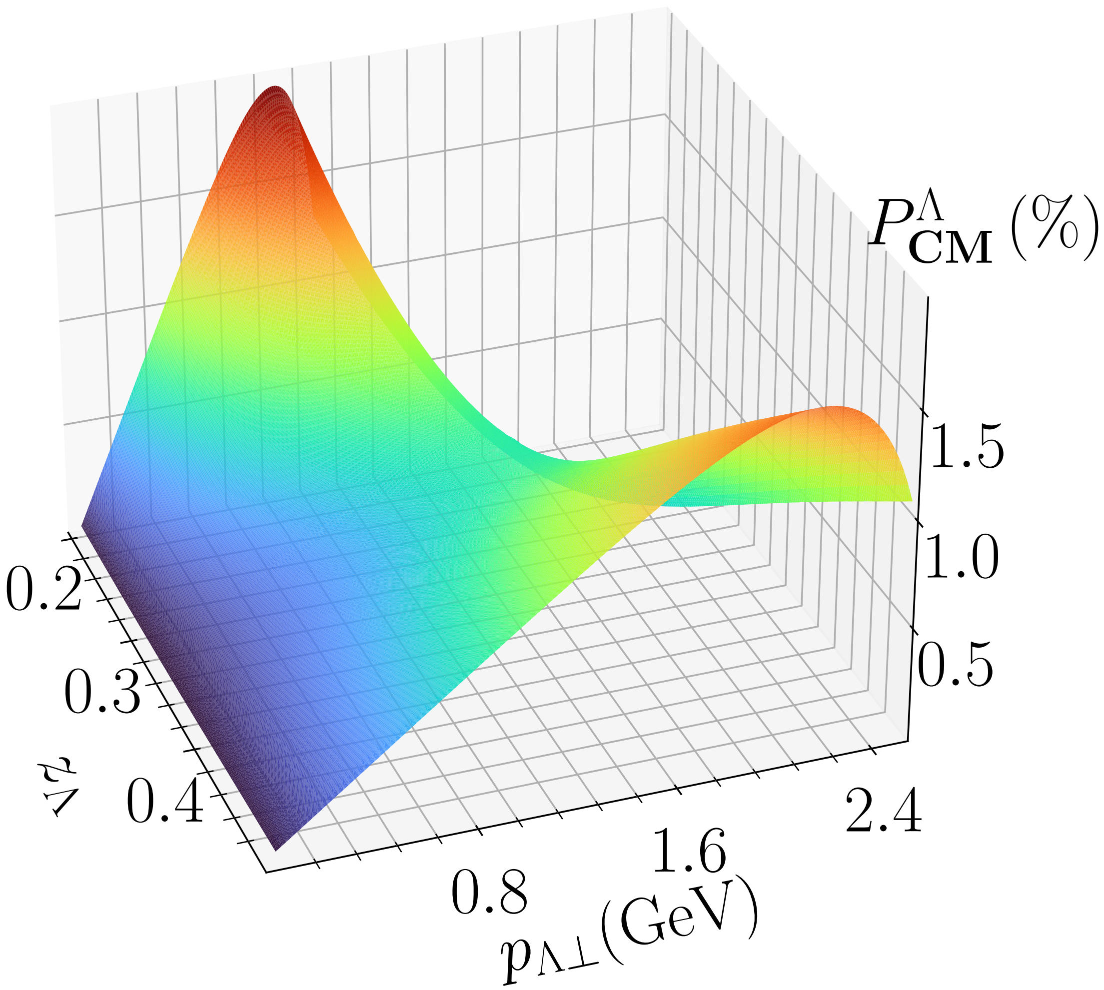
$$\frac{1}{z_\Lambda}D_{T,\Lambda/q}(z_\Lambda) \approx -\left(1 - z_\Lambda\frac{d}{dz_\Lambda}\right)D_{1T,\Lambda/q}^{\perp(1)}(z_\Lambda)$$

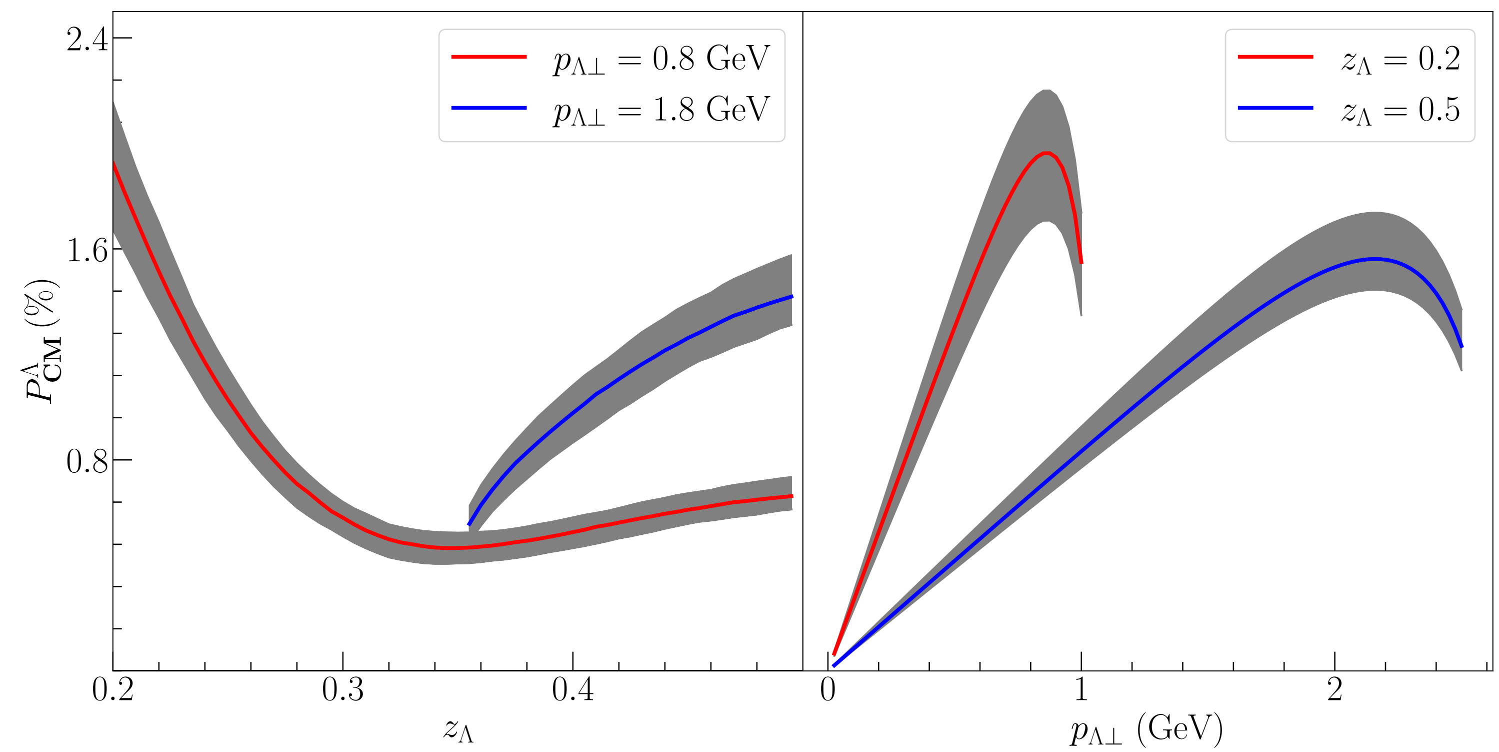


Prediction for Belle

Gamberg, Kang, Shao, Terry, Zhao to appear

$$P^{\Lambda}_{\mathbf{CM}}(z_\Lambda, p_{\Lambda\perp}) = \frac{d\Delta\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}} \Big/ \frac{d\sigma}{dz_\Lambda\, d^2p_{\Lambda\perp}}\,.$$





$P^{\Lambda}_{\mathrm{CM}}$ — 3-D plot of the polarization in z_Λ and $p_{\Lambda\perp}$

Center:Plot of the polarization as a function of only z_Λ ,

Right: Plot of the polarization as a function of $p_{\Lambda\perp}$: polarization in our scheme is ~ 1-2%

Plots are generated only using the central fit

The red and blue curves are generated using the central fit, gray band is the theoretical uncertainty

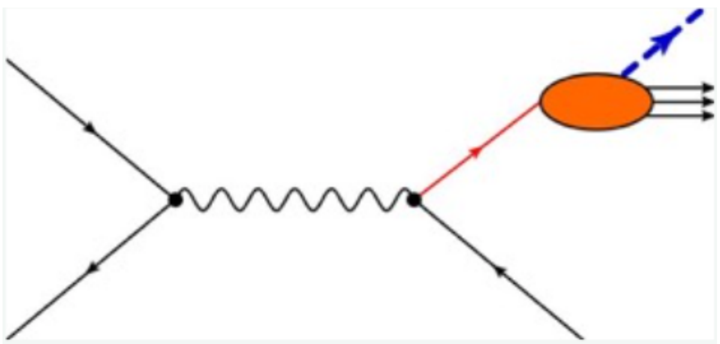
Comments ...$e^+e^- \to \Lambda(\mathrm{Thrust})\,X$ and $e^+e^- \to \Lambda\,X$

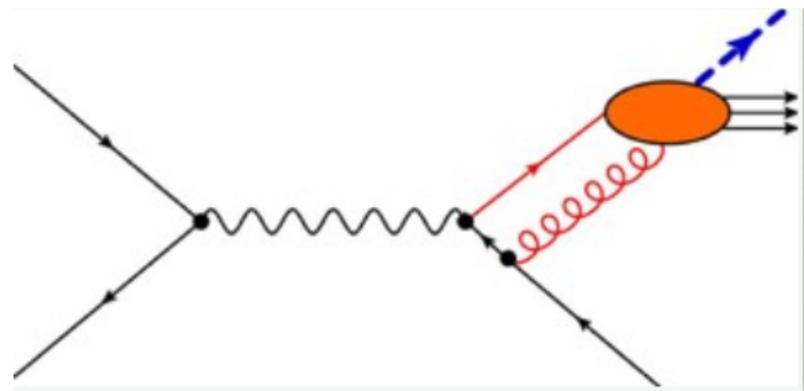
- Interesting that while these two measurements probe different distribution functions, they differ only by the definition of the measurement axis.
- That is, a measurement the polarization as a function of $j_\perp$ is a useful process for probing the properties of the PFF $D_{1T}^{\perp}$ with respect to the thrust axis
- while a measurement if polarization as a function of $p_{\Lambda\perp}$, the transverse momentum of the Λ in the lepton center-of-mass (COM) frame, is a useful process for probing the D_T function.
- Therefore the polarization in the COM frame can in principle be studied from the existing Belle data by reanalyzing the data for the inclusive $e^+e^- \to \Lambda(\mathrm{Thrust})\,X$ measurement in COM $e^+e^- \to \Lambda\,X$

Comments …

Single-Transverse $\Lambda^{\uparrow}$ spin asymmetry

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ $\rightarrow$ absent in DIS (1 γ)





Intrinsic

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

See also Boer, Jakob, Mulders NPB (1997)

n.b. **some intuition …**

Consider crossing this process to inclusive DIS for transverse polarised target

Would have the function $f_T^{q/\Lambda}(x)$, $\frac{d\sigma(S_{\Lambda\, T})}{dx d\phi} \sim \sin(\phi_S)\sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!!$

Constraints from time reversal on quark correlation function

Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960

A unique test of time reversal in QCD: Non-zero intrinsic

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ → absent in DIS (1 γ)

Single-Transverse $\Lambda^{\uparrow}$ spin asymmetry

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

$$\frac{d\sigma(S_{\Lambda\, T})}{dxd\phi} \sim \sin(\phi_S)\sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!! \qquad f_T^{q/\Lambda}(x)$$

Constraints from time reversal on quark correlation function
Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960

Simplest and cleanest process Λ polarization

$$e^+e^- \to \Lambda^\uparrow X - \text{inclusive}$$

$$e^+e^- \to \Lambda^\uparrow(\text{Thrust})\, X - \text{semi} - \text{inclusive}$$

$$e^+e^- \to \Lambda^\uparrow h\, X - \text{semi} - \text{inclusive}$$

Most data is inclusive

Belle both thrust & inclusive in principle

$$D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q), \quad D_{1T,\Lambda/q}^{\perp(1)}\left(z_\Lambda, b, Q\right)$$

Take aways

- Non-zero $e^+e^- \to \Lambda^\uparrow X$ inclusive result is an indication that there are no gluonic poles in ffs, ie time reversal is not a constraint on FFs: the simplest process is an interesting a test of time reversal in QCD, $D_T^{\Lambda/q} \neq 0$

- We are performing a test of twist-3 factorisation at NLO in $e^+e^- \to \Lambda^\uparrow X$

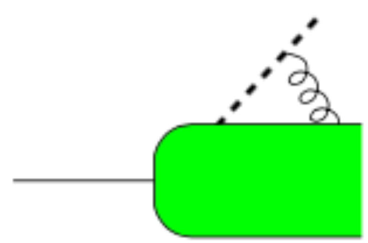
- Would be great if Belle carried out a fully inclusive measurement to directly test $D_T^{\Lambda/q} \neq 0$

Extras

Comment on time reversal constraint on ffs versus pdfs

The non-zero effect in the one-photon approximation in the annihilation process caused by the intrinsic twist-3 fragmentation function $D_T^{\Lambda/q}(z)$ can be attributed to the fact that fragmentation processes are not constrained by time-reversal

Boer, Mulders Pijlman, NPB



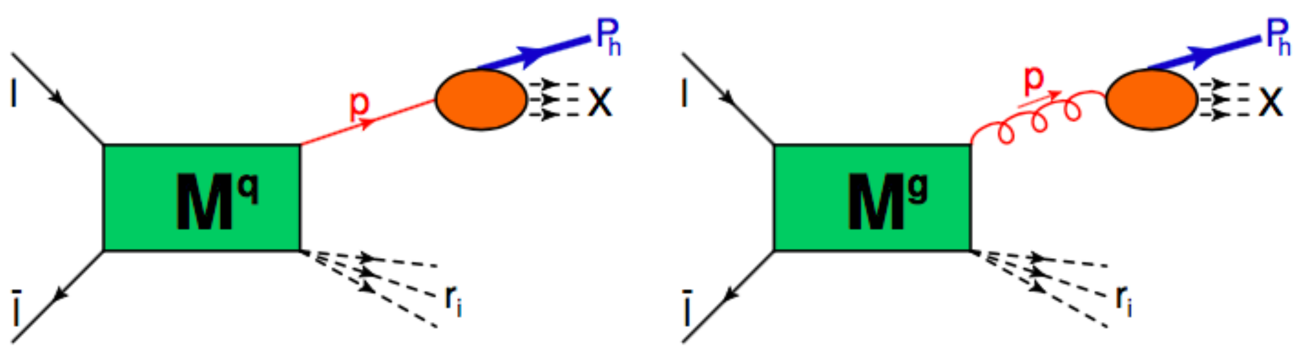
Due due to non- perturbative interactions in the in and out states in the definition of $D_T^{\Lambda/q}(z)$ On the other hand, a corresponding intrinsic twist-3 parton correlation function in the nucleon, $f_T^{q/\Lambda}(x)$ is forbidden by time-reversal.

Goeke, Metz, Schlegel plb 2006, Bacchetta et al. JHEP 2007

Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

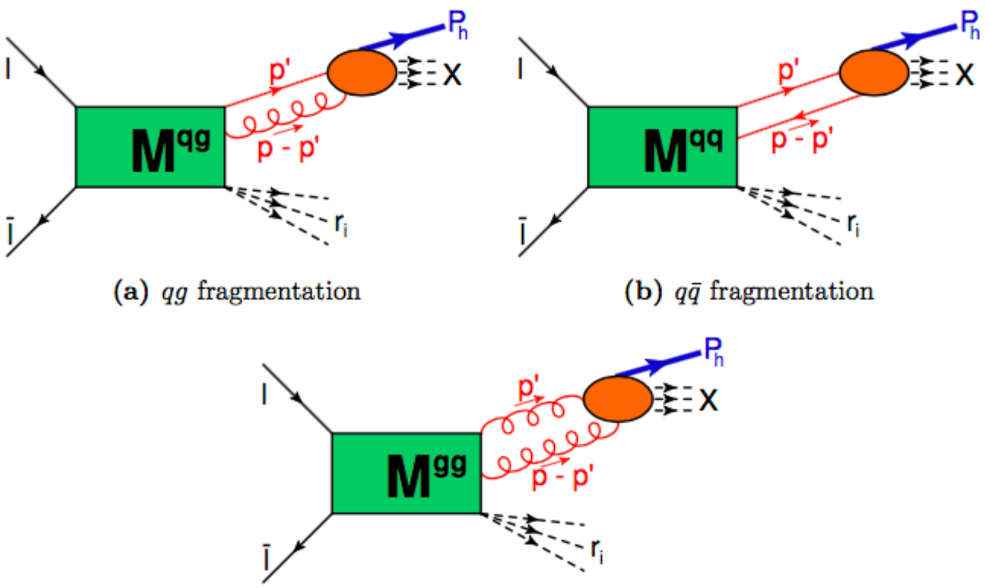
These NLO dynamics for twist-3 fragmentation are the simplest process

Calculate the contributions from all channels qq, gg, qgq, qqg,



(a) Single quark fragmentation **(b)** Single gluon fragmentation

Figure 3. Fragmentation mechanism in e^+e^- annihilation for intrinsic and kinemati contributions.



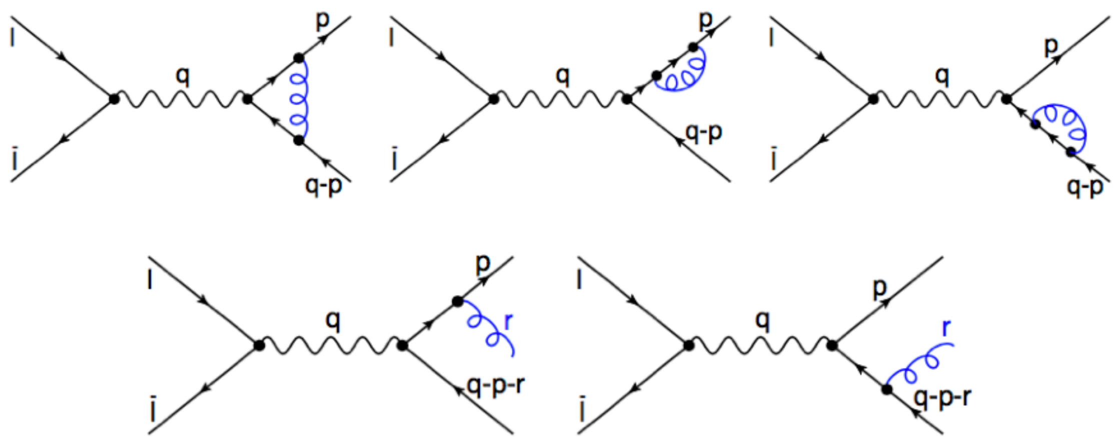
(a) qg fragmentation **(b)** $q\bar{q}$ fragmentation

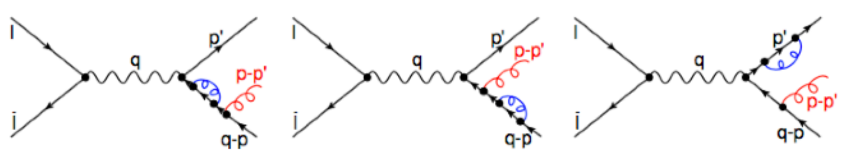
(c) gg fragmentation

Figure 4. Fragmentation mechanism in e^+e^- annihilation for dynamical contributions.

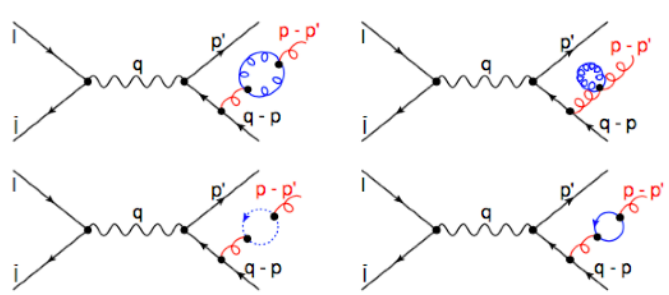
Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

Calculate intrinsic, kinematical, dynamical contributions in all channels qq, gg, qgq, qqg, ggg

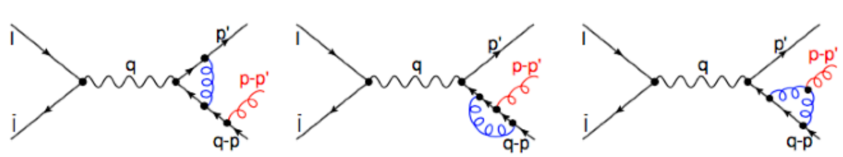




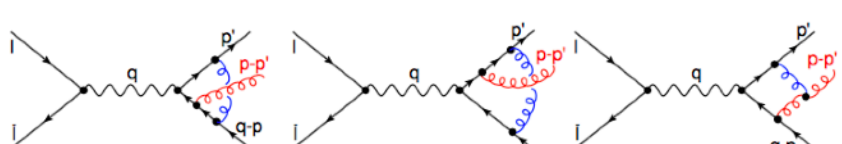
(a) Self-energy corrections to the quark lines



(b) Self-energy corrections on the gluon line



(c) Vertex corrections



(d) Box diagram

Figure 7. Virtual one-loop diagrams.

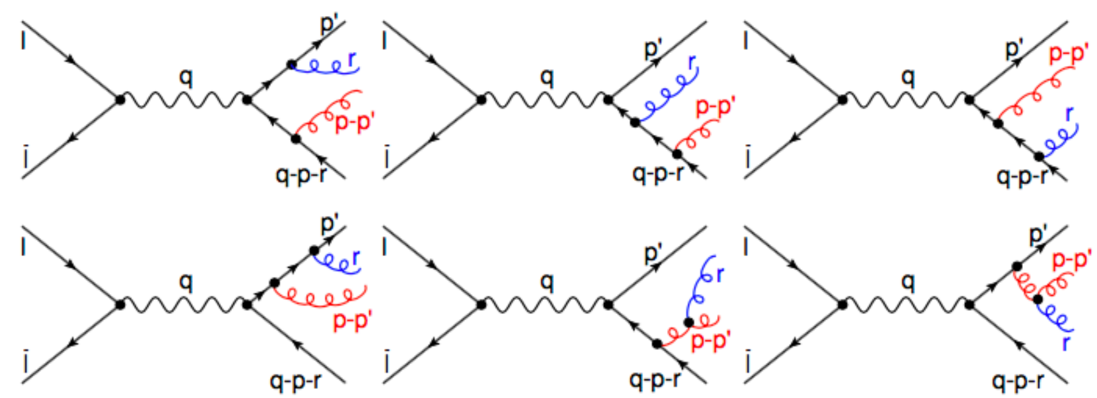


Figure 8. NLO corrections from real gluon radiation to quark-gluon fragmentation.

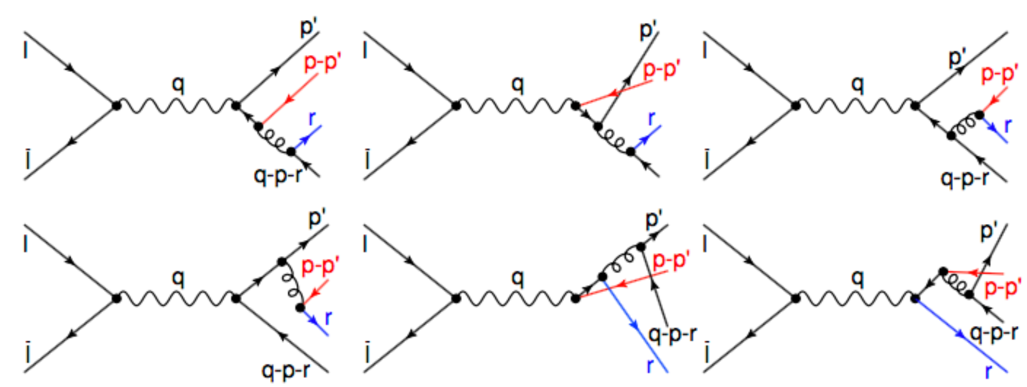


Figure 9. NLO corrections from quark radiation to quark/anti-quark fragmentation.

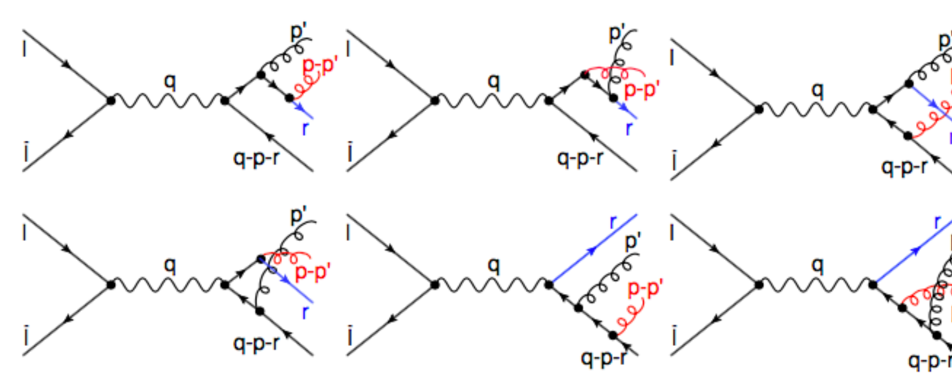


Figure 10. NLO corrections from quark radiation to gluon-gluon fragmentation.

Comments

- All partonic factors calculated in Feynman gauge & Light-cone gauge, @NLO both calculations *agree*!
- E.o.M. - relations are crucial:
- Eliminating 'intrinsic' twist-3 contributions: can show color gauge invariance at NLO! ✔
- Infrared $1/\varepsilon^2$ - poles cancel ✔
- $1/\varepsilon$ - poles of imaginary parts of loops cancel through E.o.M. ✔
- $1/\varepsilon$ collinear poles of real parts of loops through MSbar - renormalization (?????)

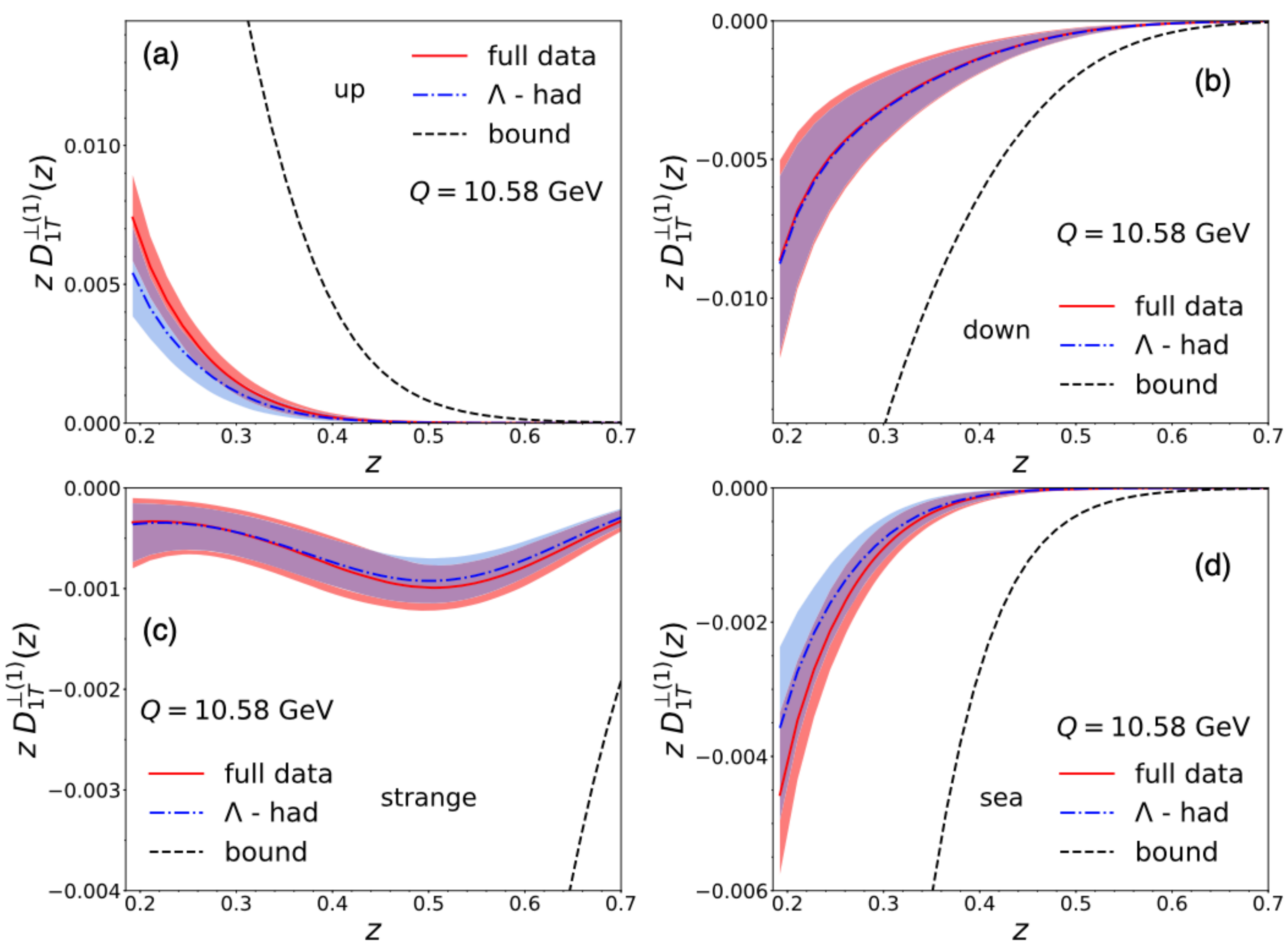
Λ Belle data fall into 2 classes

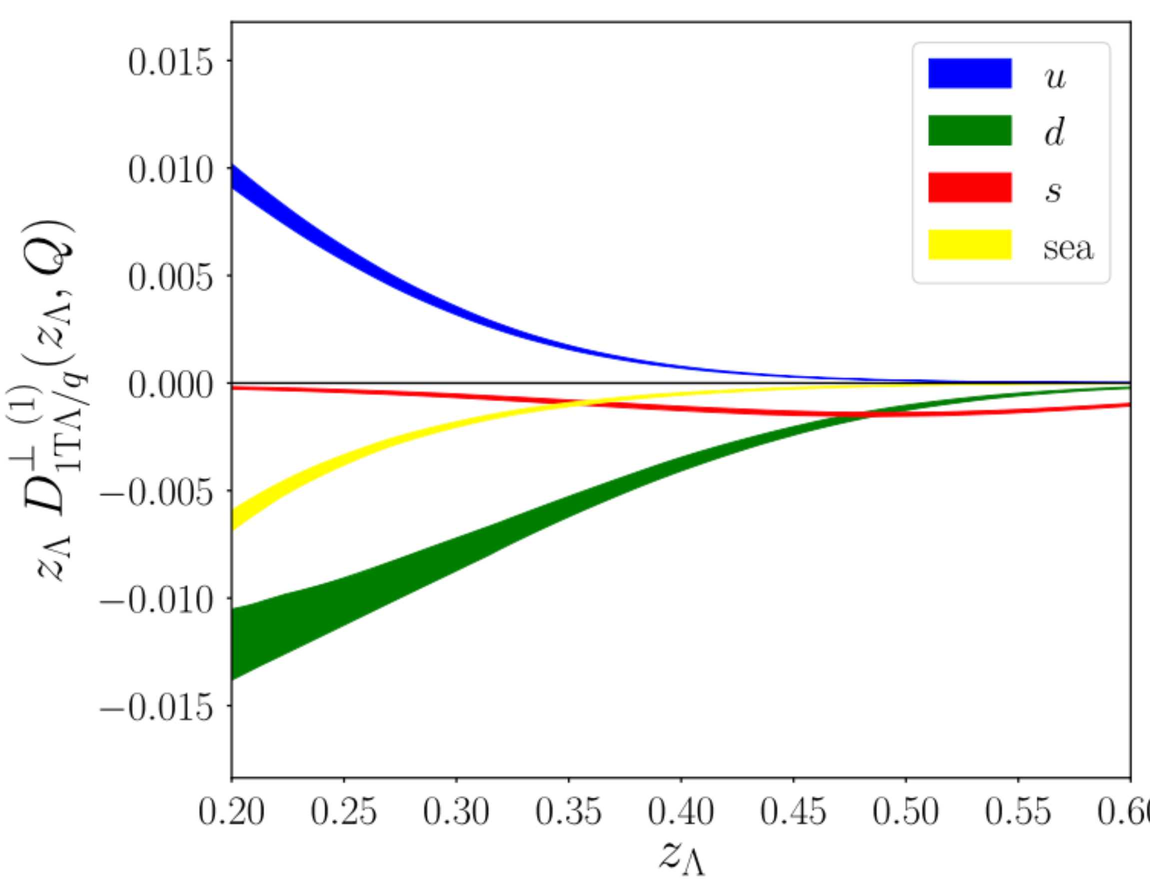
$$e^+e^- \to \Lambda^\uparrow h\, X, \quad \& \quad \Lambda^\uparrow(\text{Thrust})\, X$$

$D^\perp_{1T,\Lambda/q}(z_\Lambda, p_\perp, Q)$

Recent extractions address this

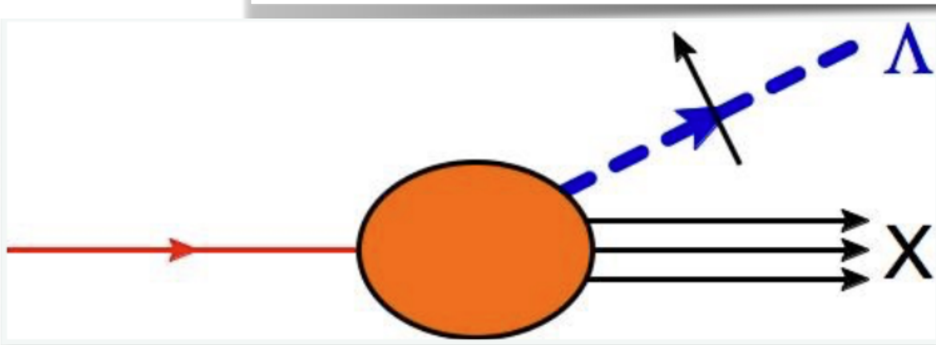
1) D'Alesio & Murgia PRD2020 bTOb + Thrust
2) Callos, Kang, Terry PRD2020 bTOb only





Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

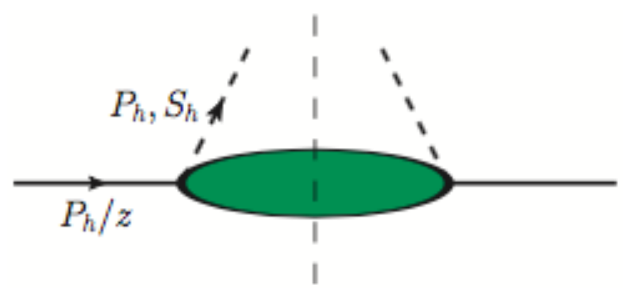


$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_{ij}(z) = \frac{1}{N_c}\sum_X \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}}\, \langle 0|\, [\infty m, 0]\, q_i(0) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}_j(\lambda m)[\lambda m, \infty m]\, |0\rangle$$

Fragmentation at Twist-3 Intrinsic twist 3 with transverse spin Λ

$$p^\mu \simeq \frac{1}{z} P_h^\mu.$$

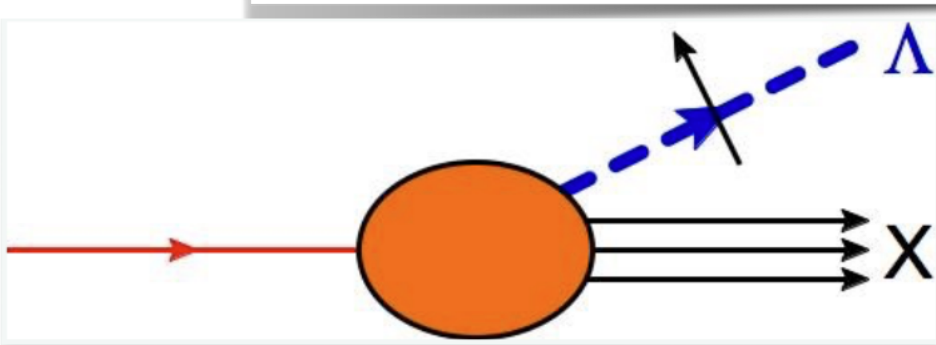


$$D_T^{\Lambda/q}(z)$$

$$G_T^{\Lambda/q}(z)$$

Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

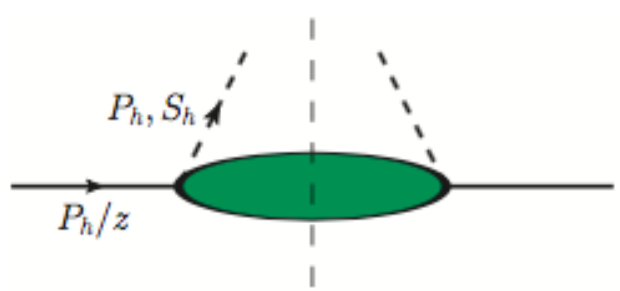


$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_{ij}(z) = \frac{1}{N_c}\sum_X\!\!\!\!\!\!\!\int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}}\, \langle 0|\, [\infty m, 0]\, q_i(0)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}_j(\lambda m)[\lambda m, \infty m]\, |0\rangle$$

Fragmentation at Twist-3 Intrinsic twist 3 with transverse spin Λ

$$p^\mu \simeq \frac{1}{z} P_h^\mu.$$



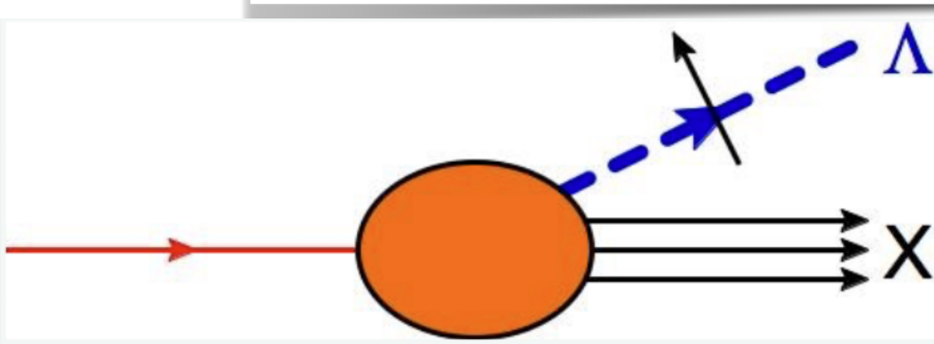
$D_T^{\Lambda/q}(z)$

$G_T^{\Lambda/q}(z)$

$$\Delta(z) = \frac{1}{z}\left(\cdots - M_h \epsilon^{P_h n \alpha S_h} \gamma_\alpha\, D_T^q(z) - M_h \not{S}_{hT} \gamma_5\, G_T^q(z) \ldots\right)$$

Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

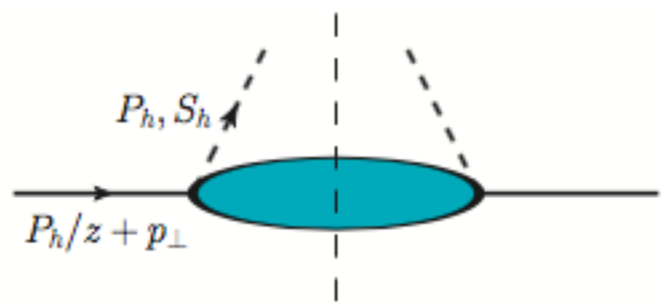


$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_\partial^\rho(z) = \int d^2 p_T\, p_T^\rho \Delta(z, z p_T)$$

Fragmentation at Twist-3 Kinematic twist 3 with transverse spin Λ

$$p^\mu \simeq \frac{1}{z} P_h^\mu + p_T^\mu \,.$$

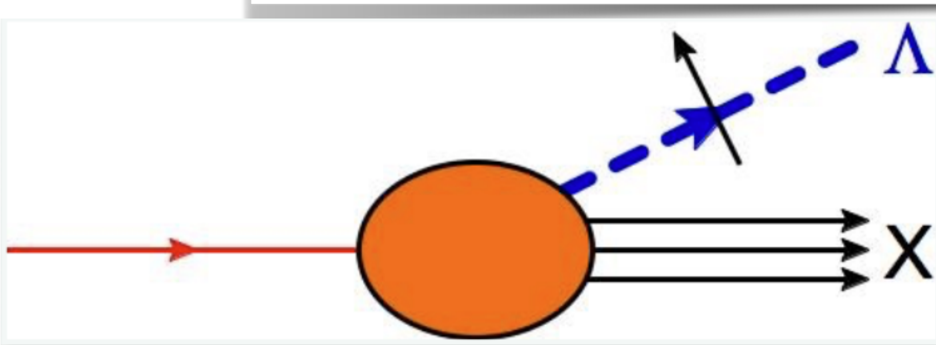


$$D_{1T}^{\perp(1),\Lambda/q}(z)$$

$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

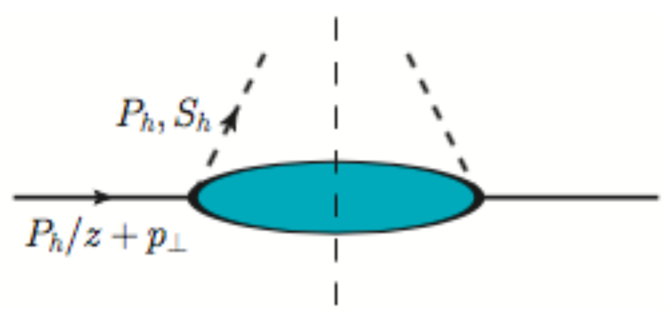


$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta^\rho_\partial(z) = \int d^2 p_T\, p_T^\rho \Delta(z, z p_T)$$

Fragmentation at Twist-3   Kinematic twist 3 with transverse spin Λ

$$p^\mu \simeq \frac{1}{z} P_h^\mu + p_T^\mu .$$



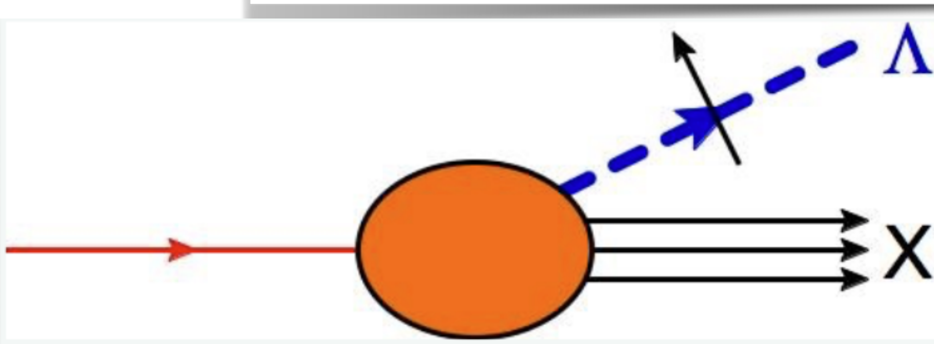
$$D_{1T}^{\perp(1),\Lambda/q}(z)$$

$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

$$\Delta^\rho_\partial(z) = \frac{1}{z} M_h \left(\epsilon^{P_h n \rho S_{hT}}\, \not{P}_h\, D_{1T}^{\perp(1),q}(z) - S_{hT}^\rho\, \not{P}_h \gamma_5\, G_{1T}^{\perp(1),q}(z) + \ldots\right)$$

Partonic picture of fragmentation,

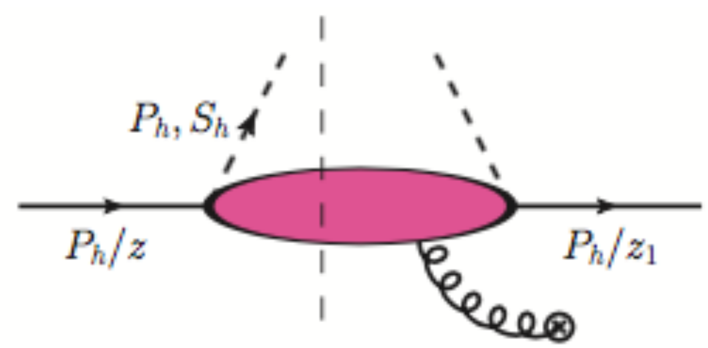
Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994



$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_F^\alpha(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

Fragmentation at Twist-3 "Dynamical twist 3 with transverse spin Λ



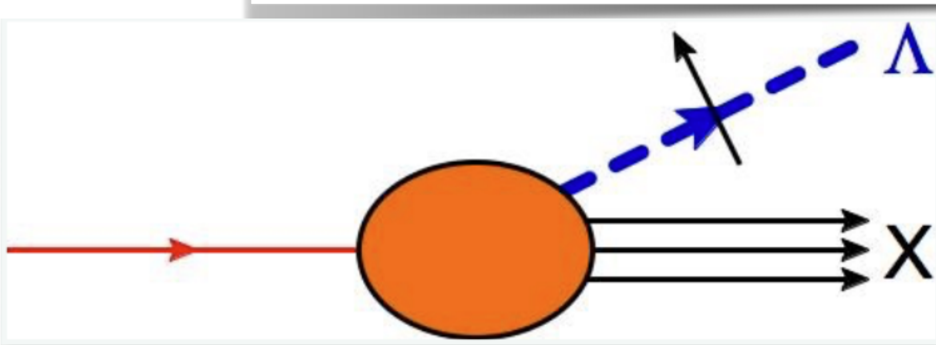
$\hat{D}_{FT}^{\Lambda/q}(z,z')$

$\hat{G}_{FT}^{\Lambda/q}(z,z')$

A_T^α

Partonic picture of fragmentation,

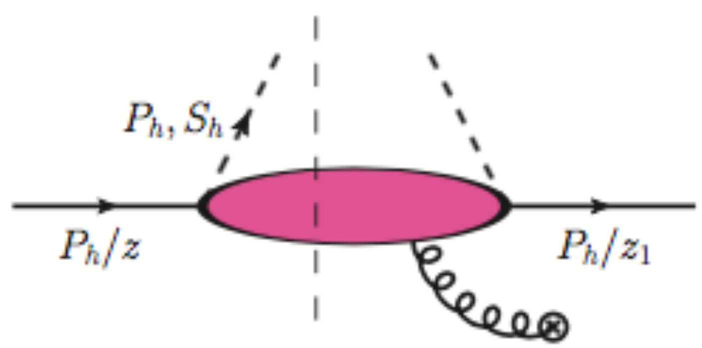
Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994



$$\langle P_\Lambda, S_\Lambda; X|\,\bar{q}(0)\,|0\rangle$$

$$\Delta_F^\alpha(z,z') \sim \langle 0|\,q(\lambda m)\,gF^{m\alpha}(\mu m)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\,\bar{q}(0)\,|0\rangle$$

Fragmentation at Twist-3 "Dynamical twist 3 with transverse spin Λ



$\hat{D}_{FT}^{\Lambda/q}(z,z')$

$\hat{G}_{FT}^{\Lambda/q}(z,z')$

A_T^α

$$\Delta^\alpha(z,z') = \frac{M_h}{z}\left(\epsilon^{P_h n \alpha S_h}\,\not{P}_h\,i\hat{D}_{FT}^{qg}(z,z') + S_{hT}^\alpha\,\not{P}_h\gamma_5\,\hat{G}_{FT}^{qg}(z,z')\ldots\right)$$

Relations: Equation of Motion & Lorentz-Invariance

See talk of Daniel Pitonyak

Kanazawa, Koike, Metz, Pitonyak, MS, PRD 93, 054024 (2016) …

EOM

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

LIR

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right) G_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

Two equations, three functions

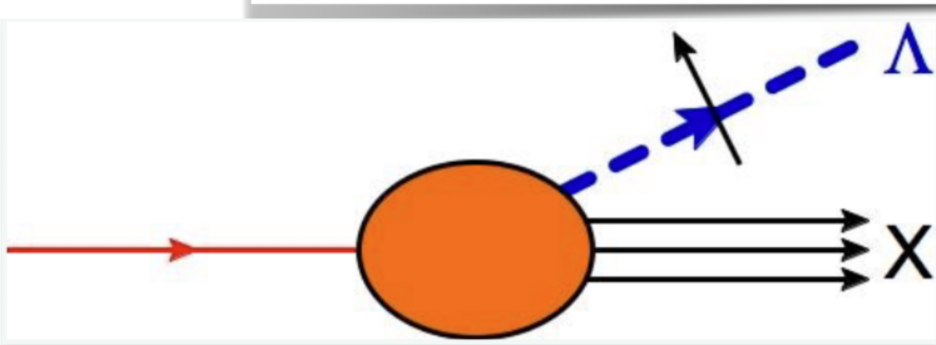
→ can eliminate

'intrinsic & kinematical twist-3'

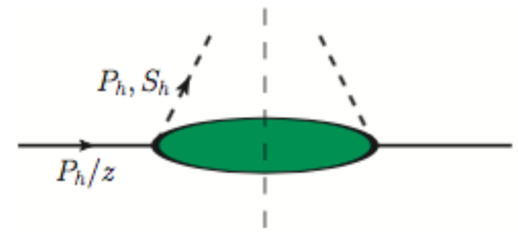
Kanazawa, Metz, Pitonyak, Schlegel, PLB **742** (2015); Kanazawa, Metz, Pitonyak, Schlegel, PLB **744** (2015); Koike, Pitonyak, Takagi, Yoshida PLB **752** (2016); Koike, Pitonyak, Yoshida PLB **759** (2016); Kanazawa, Koike, Metz, Pitonyak, Schlegel, PRD **93** (2016); Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP **1901** (2019))

Partonic picture of fragmentation,

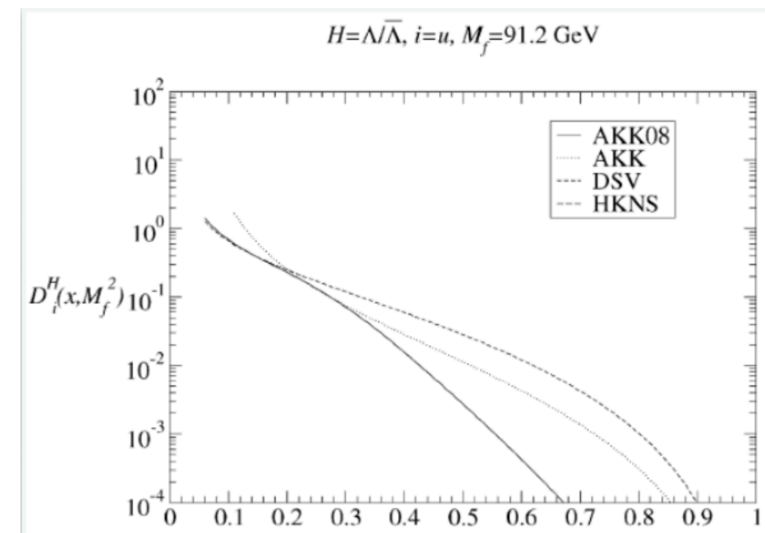
Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994



$$\langle P_\Lambda, S_\Lambda; X |\, \bar{q}(0)\, |0\rangle$$



$$\Delta_{ij}(z) = \frac{1}{N_c} \sum_X \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}} \langle 0|\, [\infty m, 0]\, q_i(0) | P_\Lambda, S_\Lambda; X\rangle \langle P_\Lambda, S_\Lambda; X |\, \bar{q}_j(\lambda m) [\lambda m, \infty m]\, |0\rangle$$



Λ Fragmentation at leading Twist

$D_1^{\Lambda/q}(z)$ FF of unpolarized q $\longrightarrow \Lambda$

fairly known [fits by AKK08, DSV, ...]

$G_1^{\Lambda/q}(z)$ FF of longitudinally pol. q $\longrightarrow \Lambda$

poorly known [attempts by DSV to fit LEP data]

$H_1^{\Lambda/q}(z)$ FF of transversely pol. q $\longrightarrow \Lambda$

unknown, chiral-odd, hard to extract from single-inclusive processes Candidate to explain large transverse Λ polarization?

K. Chen, G.R. Goldstein, R.L. Jaffe and X.-D. Ji, NPB 1995

Collinear PDFs (x)			
H pol. \ q pol.	U	L	T
U	f_1 unpolarized		
L		g_1 helicity	
T			h_1 transversity

Collinear FFs (z)			
H pol. \ q pol.	U	L	T
U	D_1		
L		G_1	
T			H_1

Courtesy of Daniel Pitonyak

$$\Delta^q_{ij}(z) = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}} \langle 0|[\infty;0]\, q_i(0)\, |P_h S_h; X\rangle\langle P_h S_h; X|\, \bar{q}_j(\lambda n)\, [\lambda;\infty]|0\rangle$$

$$= \frac{1}{z}\left(\not{P}_h\, D^q_1(z) - S_{hL}\, \not{P}_h \gamma_5\, G^q_1(z) - \tfrac{1}{2}[\not{P}_h, \not{S}_h]\gamma_5\, H^q_1(z) \right)$$

Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

$$\Delta_F^{\alpha}(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m) |P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\,|0\rangle$$

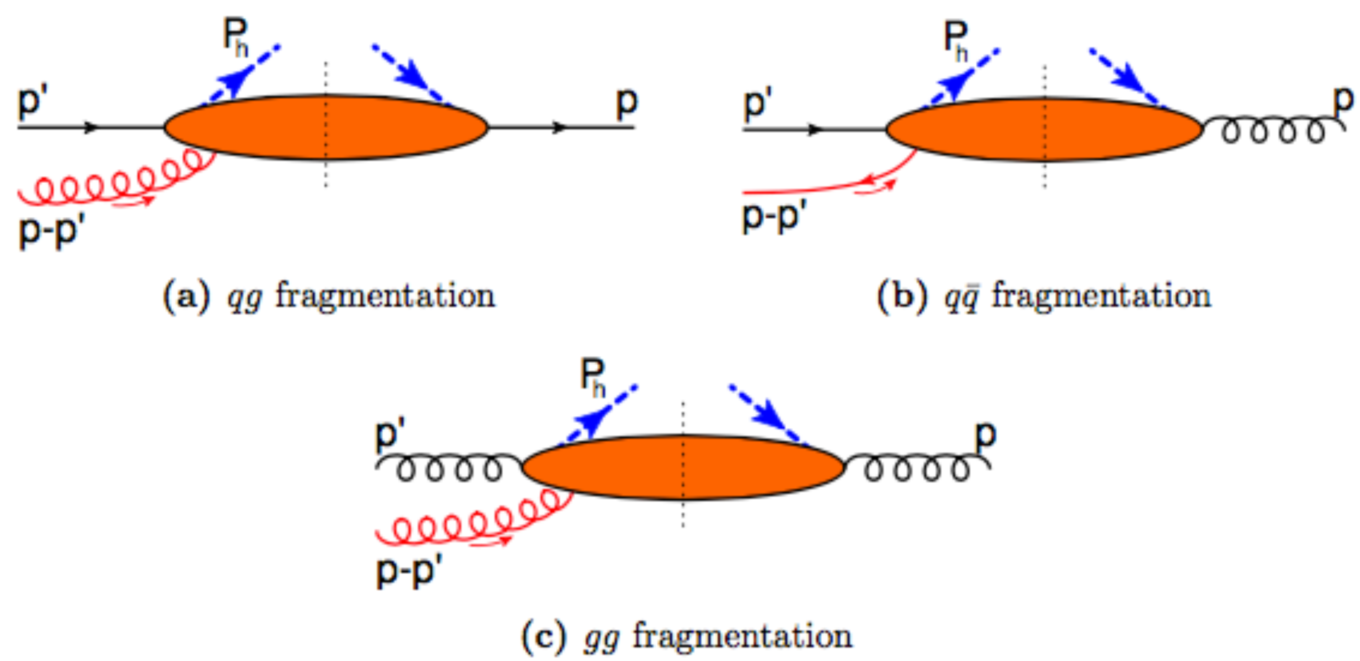


Figure 2. Diagrammatic representation of three-parton fragmentation correlations.

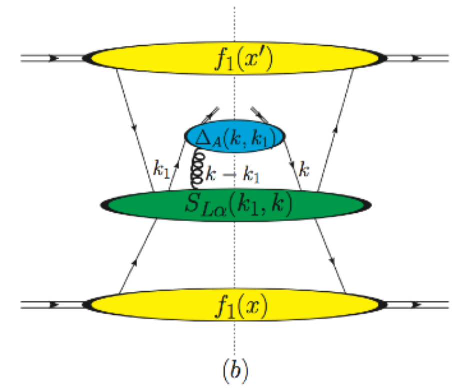
Can have interference w/ qg & q, qq & g, gg & g

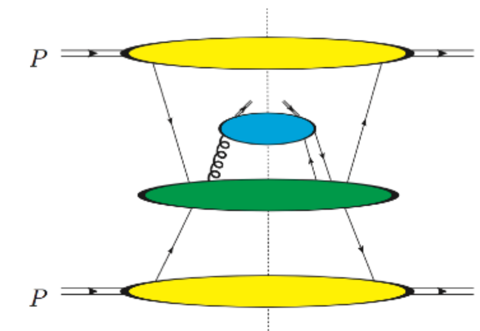
Gluonic *poles* of the dynamical FF *vanish* (Metz, 2003, Meissner & Metz (2009), Gamberg, Mukherjee, Mulders 2008, 2011, Boer, Kang, Vogelsang, Yuan Phys.Rev.Lett. 105 (2010) .

This makes the calculation of twist-3 fragmentation effects different from the calculation of soft-gluon and soft-fermion poles on the PDF side.

Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

pp - collisions $pp \rightarrow \Lambda^{\uparrow} X$



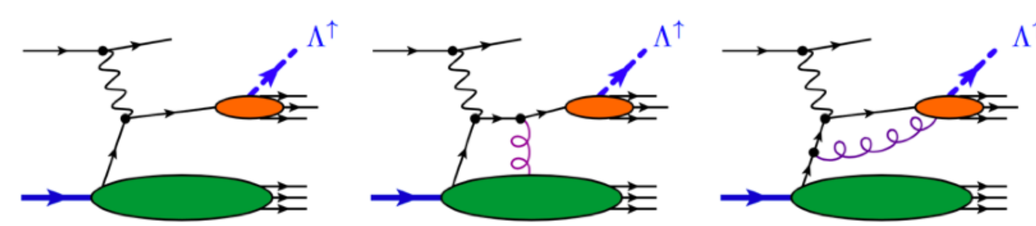


complete LO formulae not available, complicated

Koike, Metz, Pitonyak, Yabe, Yoshida, PRD 2017]

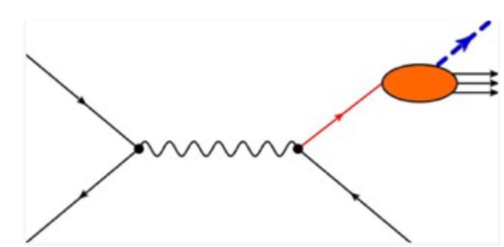
$ep \rightarrow \Lambda^{\uparrow} X$ single-inclusive $\Lambda^{\uparrow}$- production

complete LO-formula (including EoM & LIR)

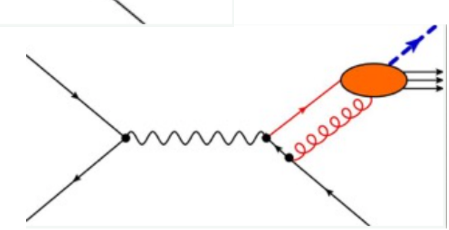


Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016

$e^+e^- \longrightarrow \Lambda^{\uparrow} X$ inclusive $\Lambda^{\uparrow}$ production

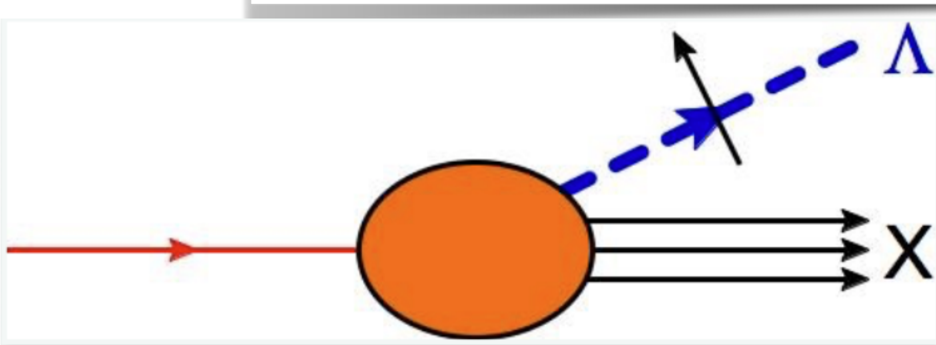


Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO



Partonic picture of collinear fragmentation

Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994



$$\langle P_\Lambda, S_\Lambda; X |\, \bar{q}(0)\, |0\rangle$$

- Partonic interpretation of the fragmentation process a matrix element that describes the hadronization of a parton into a jet of hadrons
- q stands for a generic partonic field (quark, anti-quark, or gluon) and $|X\rangle$ is an arbitrary hadronic multi-particle state which forms an (unobserved) jet
- one hadrons of the jet is detected and its four-momentum P_{h} and four-spin S_{h} are measured
- Soft fragmentation process in pQCD formula is the "square" as a cut forward transition amplitude where sum over all possible unobserved hadron states.

$$\Delta^q_{ij}(z) = \frac{1}{N_c} \sum\!\!\!\!\!\!\!\int_X \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}} \langle 0|[\infty;0]\, q_i(0)\, |P_h S_h; X\rangle\langle P_h S_h; X|\, \bar{q}_j(\lambda n)\, [\lambda;\infty]|0\rangle$$

Unpolarized and PFF evolve in same way

Obeys CS Equation, thus unpolarised and PFF evolve "similarly"

$$\frac{\partial \ln \hat{D}(z_A, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

Idilbi,Ji,Ma,Yuan PRD 2004
Aybat Rogers Collins Qiu PRD 2012
also see Kang Yuan Xiao PRL 2011

Perturbative Evolution for PFF and unpolarized the same

$$D_{\Lambda/q}(z_\Lambda, p_{\Lambda\perp}, Q), \quad D_{1T,\Lambda/q}^{\perp(1)}\left(z_\Lambda, b, Q\right)$$