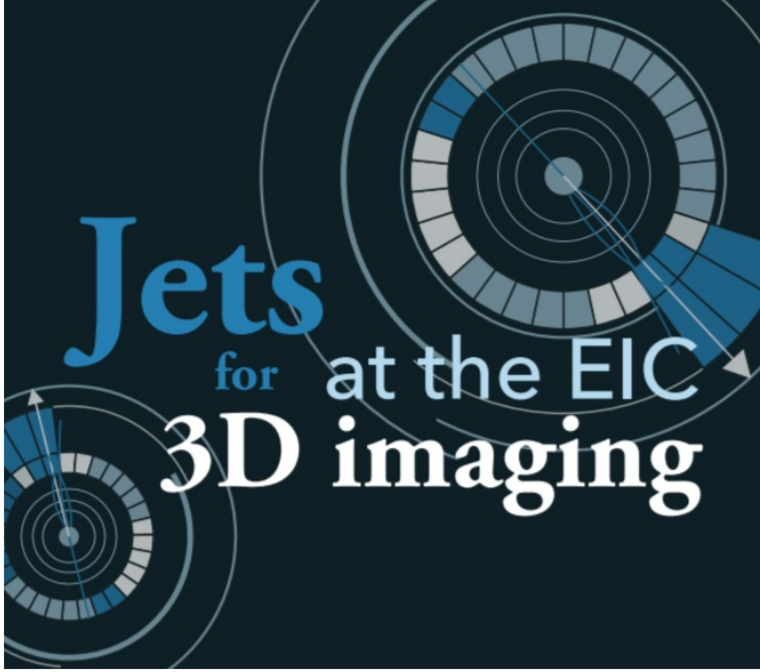

Andrea Simonelli

In collaboration with M. Boglione

Factorization of $e^+e^- \to H\,X$ cross section, differential in z_h, P_T and T in the 2-jet limit

Table of Content

- Provide a scheme that allows to extend TMDs *beyond* the standard processes

M. Boglione and A. Simonelli
arXiv:2007.13674v2

$P\,P \to \ell^+\ell^-$	$\ell^- P \to H\,X$	$e^+e^- \to H_1\,H_2\,X$
Drell-Yan	SIDIS	e^+e^- annihilation

- Description of BELLE data: $e^+e^- \to H\,X$

M. Boglione and A. Simonelli
arXiv:2011.07366

Incredibly good agreement with data!

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TWO hadrons → **2-h class**

- Description of BELLE data: $e^+ e^- \to H\, X$

M. Boglione and A. Simonelli
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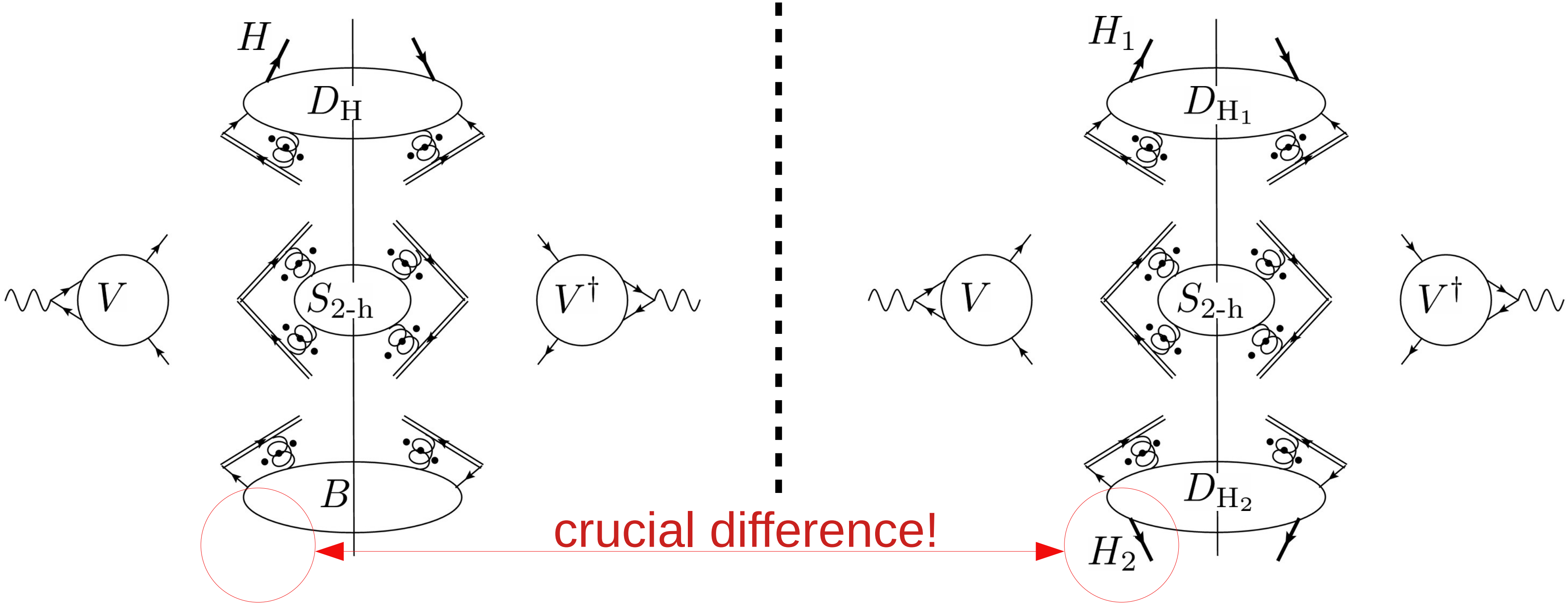
ONE hadron → **1-h class**

Incredibly good agreement with data!

1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$

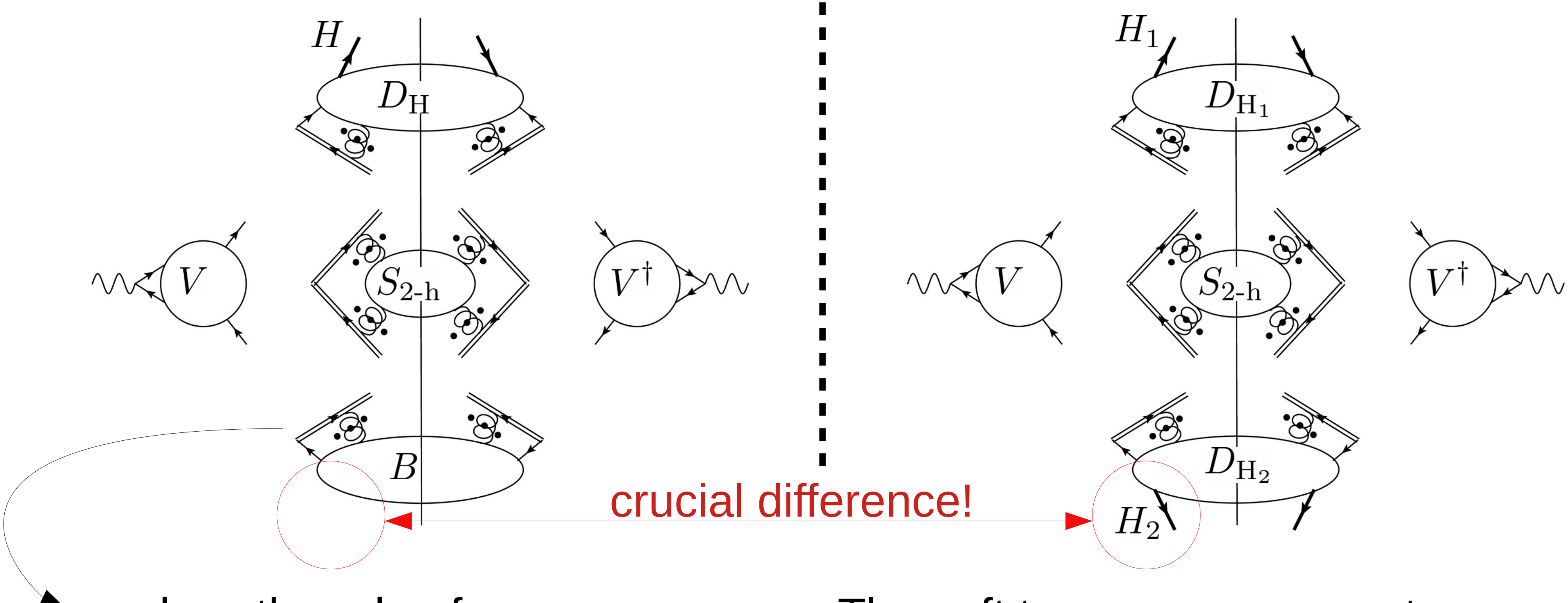
$$e^+e^- \to H_1\,H_2\,X$$



1-h class vs 2-h class

$e^+e^- \to H\,X \quad (T \sim 1)$

$e^+e^- \to H_1\,H_2\,X$

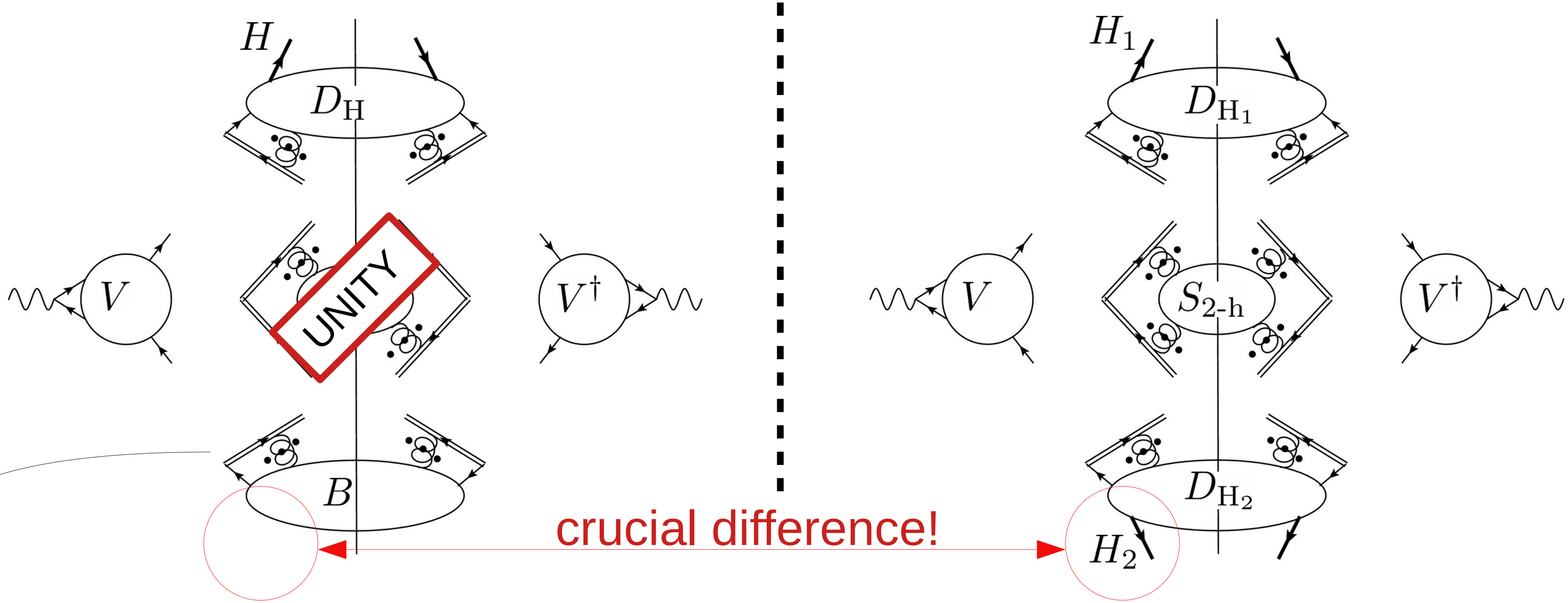


$\bar{q}$ plays the role of a hard real emission → The soft transverse momentum is washed out in integration

J. Collins, Foundations of perturbative QCD. Cambridge University Press, 2011.

1-h class vs 2-h class

$e^+e^- \to H\,X \quad (T \sim 1)$ $\qquad\qquad$ $e^+e^- \to H_1\,H_2\,X$



$\bar{q}$ plays the role of a hard real emission

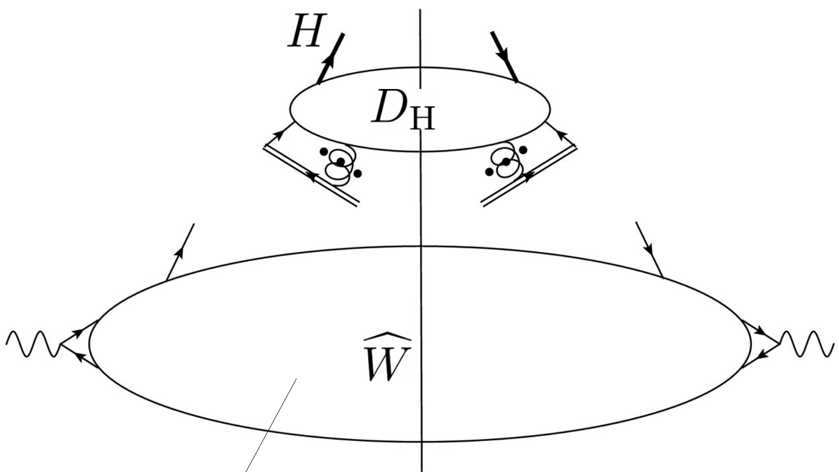
The soft transverse momentum is washed out in integration

The CSS 2-h soft factor is unity

J. Collins, Foundations of perturbative QCD. Cambridge University Press, 2011.

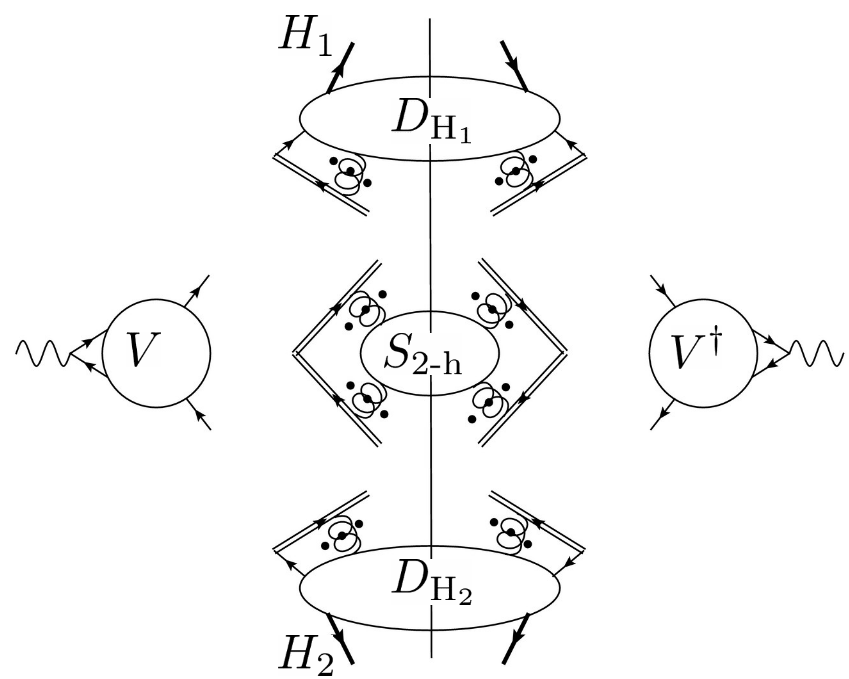
1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$



Partonic version of the final state tensor

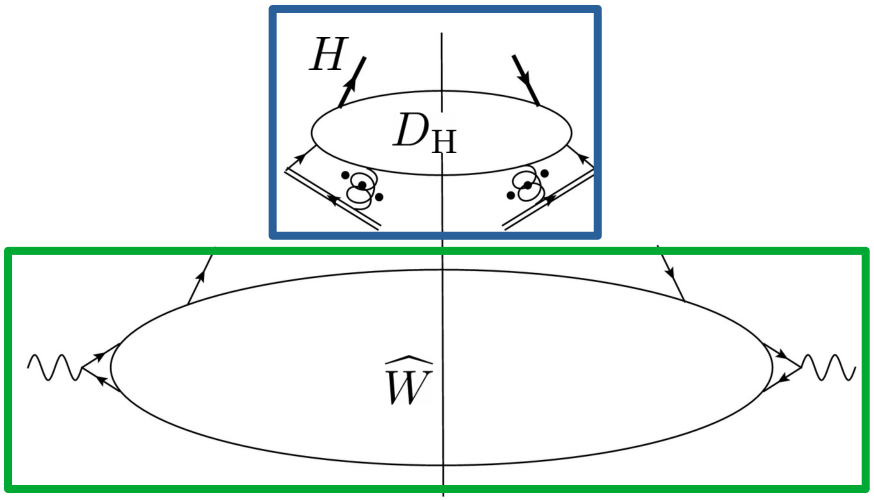
$$e^+e^- \to H_1\,H_2\,X$$



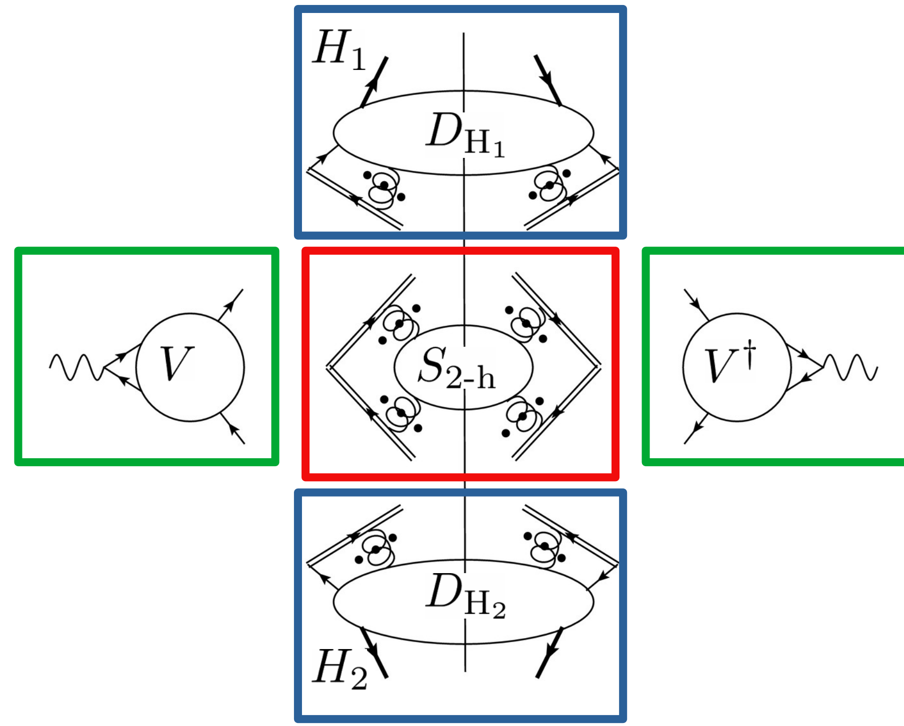
It includes SOFT and BACKWARD radiation, but their contributions are totally predicted by perturbative QCD

1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$



$$e^+e^- \to H_1\,H_2\,X$$



= PERT.

= NON PERT. , DIRECT PHENO

= NON PERT. , INDIRECT PHENO

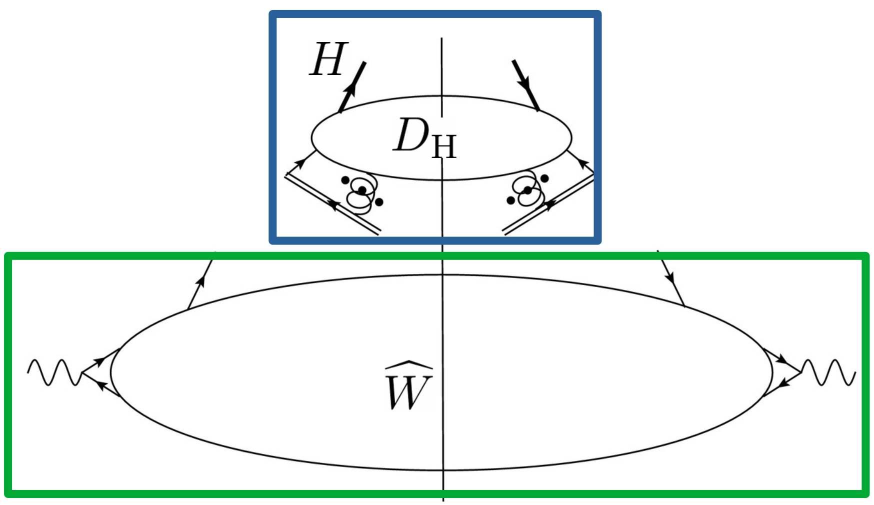
$$W \sim \widehat{W}\; D_H$$

?

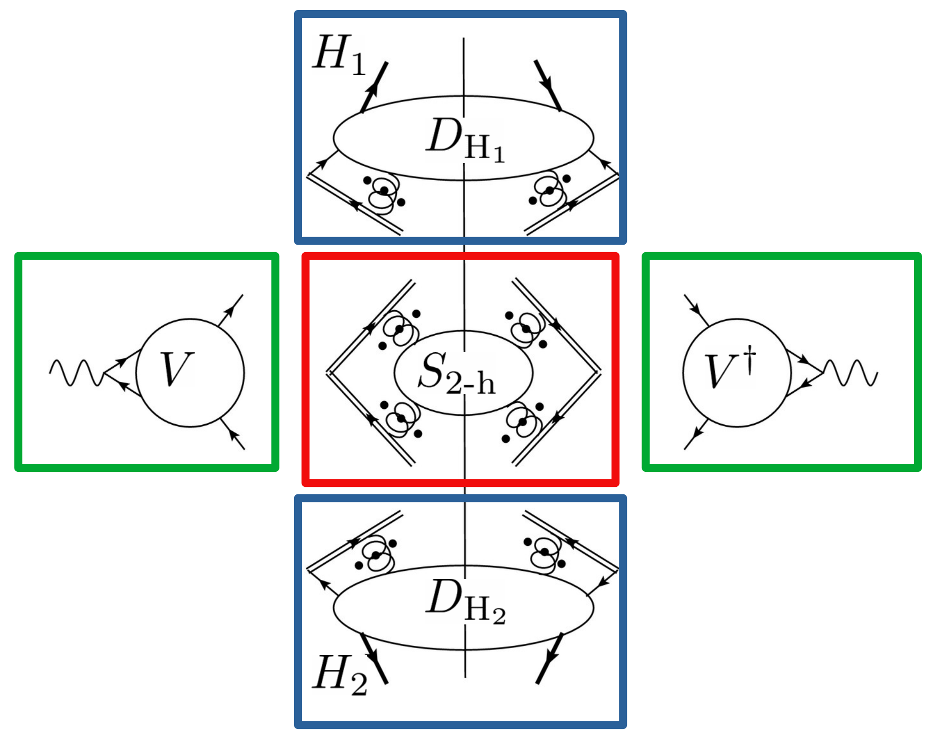
$$W \sim V\,V^\dagger\,D_{H_1}\boxed{S_{2\text{-h}}}D_{H_2}$$

1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$



$$e^+e^- \to H_1\,H_2\,X$$



- = PERT.
- = NON PERT. , DIRECT PHENO
- = NON PERT. , INDIRECT PHENO

A brilliant solution!

$$W \sim \widehat{W}\, D_H$$

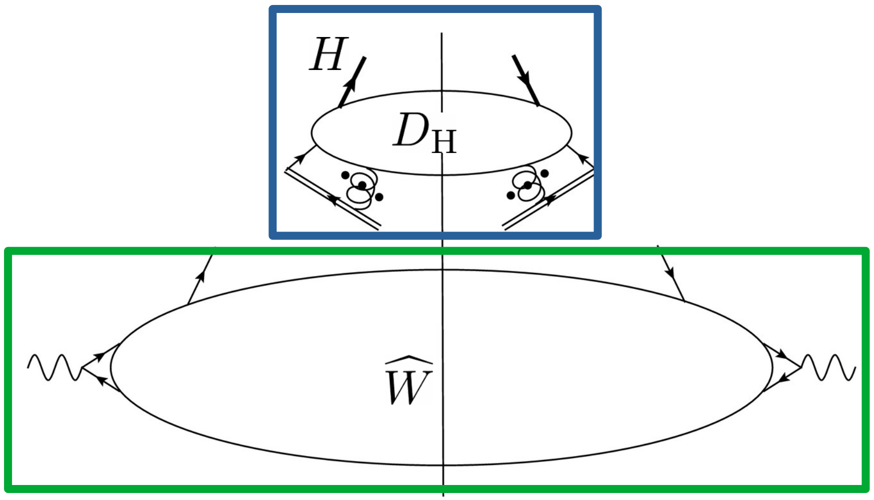
$$\sqrt{S_{2\text{-h}}} \qquad \sqrt{S_{2\text{-h}}}$$

$$W \sim V\,V^\dagger\,D_{H_1}\,S_{2\text{-h}}\,D_{H_2}$$

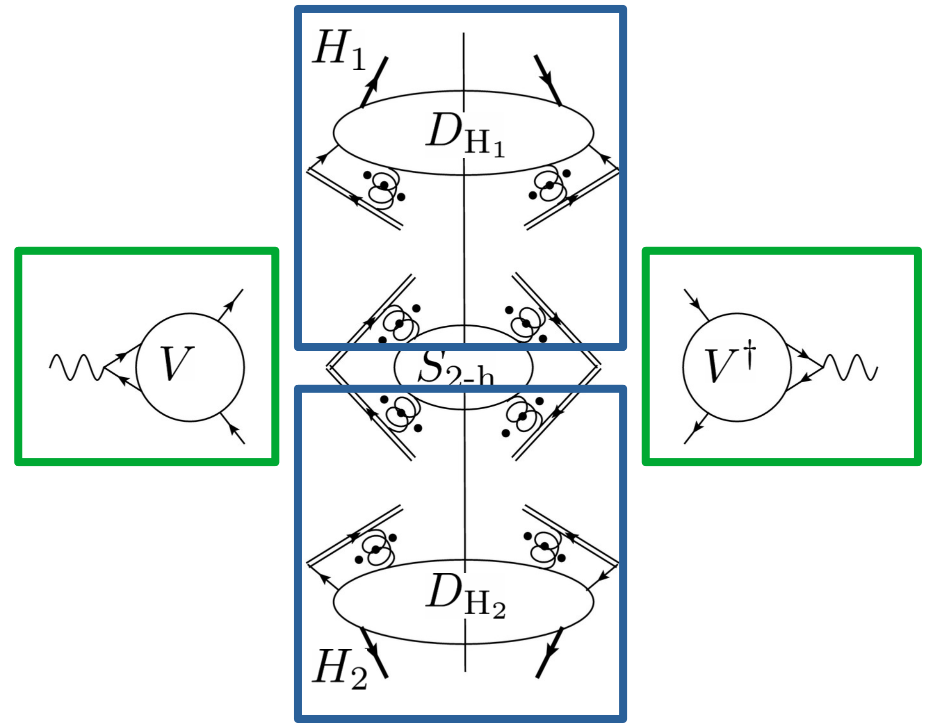
J. Collins, Foundations of perturbative QCD. Cambridge University Press, 2011.

1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$



$$e^+e^- \to H_1\,H_2\,X$$



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A brilliant solution!

$$W \sim \widehat{W}\, D_H$$

$$W \sim V\, V^\dagger\, D_{H_1}^{\mathrm{sqrt}}\, D_{H_2}^{\mathrm{sqrt}}$$

J. Collins, Foundations of perturbative QCD. Cambridge University Press, 2011.

1-h class vs 2-h class

$$e^+e^- \to H\,X \quad (T \sim 1)$$

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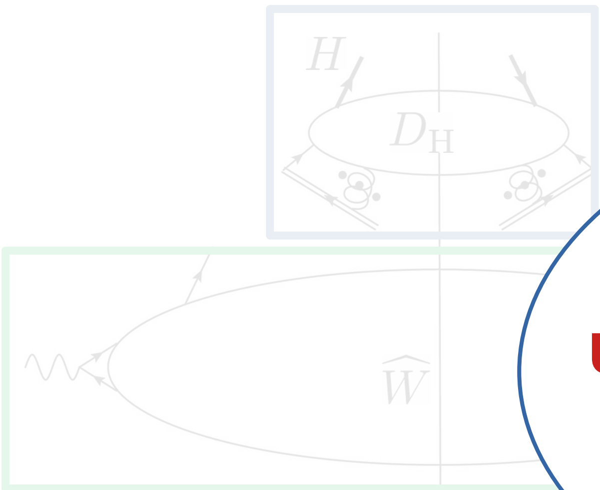
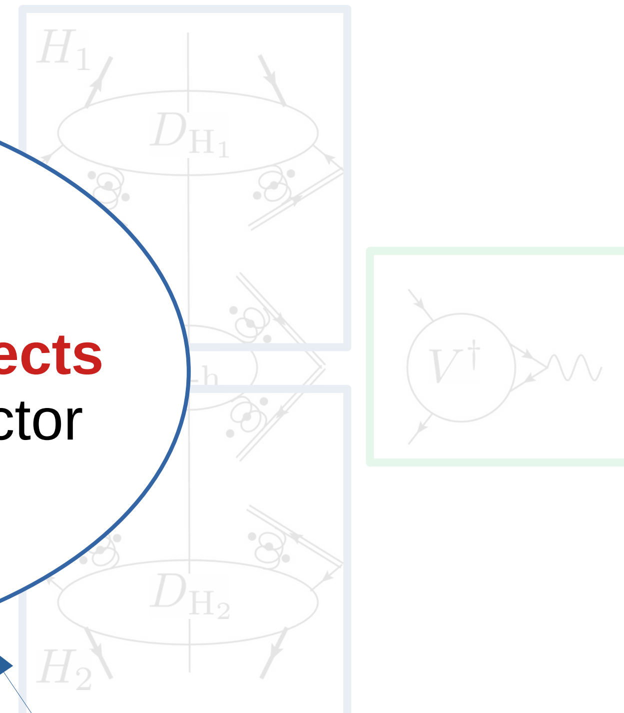




Different objects!

Universality breaking effects
induced by the 2-h soft factor

M. Boglione and A. Simonelli
arXiv:2007.13674v2

= PERT.

= NON PERT. , DIRECT PHENO

= NON PERT. , INDIRECT PHENO

A brilliant solution!

$$W \sim \widehat{W}\, D_H$$

$$W \sim V\, V^\dagger\, D^{\mathrm{sqrt}}_{H_1}\, D^{\mathrm{sqrt}}_{H_2}$$

J. Collins, Foundations of perturbative QCD.
Cambridge University Press, 2011.

Two different definitions for TMDs

1-h class TMD FFs

2-h class
(usual) TMD FFs

$$\widetilde{D}^{\text{sqrt}}_{H_1/f} = \widetilde{D}_{1,\,H/f}\,\sqrt{M_S}$$

NEW ingredient!

Function describing the long-distance behavior of the 2-h Soft Factor

- Non-perturbative

All quantities predicted by pQCD
(Wilson Coefficients, perturbative Sudakov)
are the same in both definitions!

- It encodes information about soft radiation typical of the 2-h class

The square root def. is optimal
for the 2-h class but it lowers
the degree of universality of the TMDs

M. Boglione and A. Simonelli
arXiv:2007.13674v2

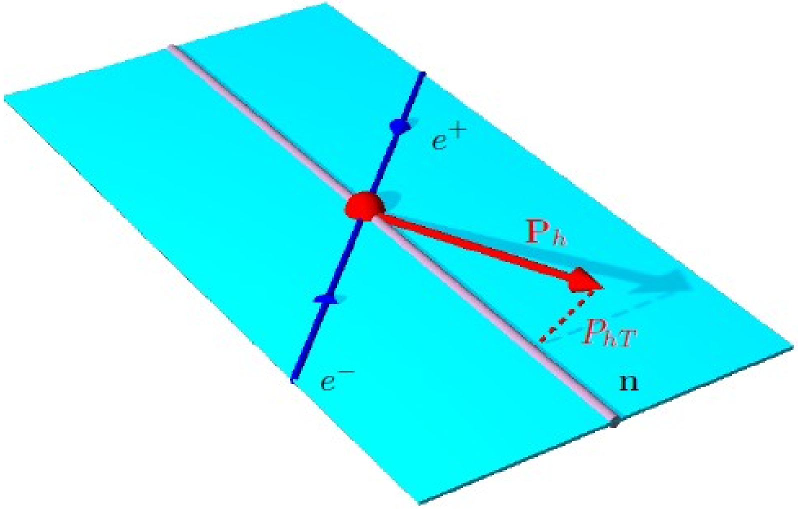
The BELLE Cross Section

Cross section of $e^+e^- \to H\,X$ (1-h class), differential in:

z_h	T	P_T
Fractional energy of the detected hadron: $z_h = \frac{2P\cdot q}{q^2} = \frac{2E_H}{Q}$ $Q = 10.58\ \mathrm{GeV}$	Topology of the final state: $T = \mathrm{Max}\frac{\sum_i \lvert\vec{p}_i\cdot\vec{n}\rvert}{\sum_i \lvert\vec{p}_i\rvert}$ $0.5 \le T \le 1$	Transverse Momentum of the detected hadron with respect to the thrust axis

Assumptions:

- Thrust axis = jet axis
- 2-jet limit $T \sim 1$
- Spinless hadron (charged pions $\pi^\pm$)



BelleCollaboration, R. Seidl et al., Phys. Rev.D99(2019), no. 11 112006

The BELLE Cross Section

Naive structure (1-h class): $\sigma \sim \hat{\sigma} \otimes D_1$

Actual structure, resulting from CSS factorization procedure:

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_j \int_{z_h}^{1} \frac{dz}{z} \frac{d\hat{\sigma}_j}{dz_h/z\, dT} D_{1,\,\pi^\pm/j}(z,\, P_T) \left[1 + \begin{matrix}\text{power}\\ \text{suppressed}\\ \text{terms}\end{matrix} \right]$$

Partonic Cross Section,
totally predicted
by pQCD (**NLO**)

Unpolarized TMD FF, includes:

- Perturbative contributions (**NLL**)
- Non-Perturbative contributions (**pheno**menological models)

Suppressed corrections of order P_T^2/Q^2 and M_π^2/Q^2

Still not the final version:

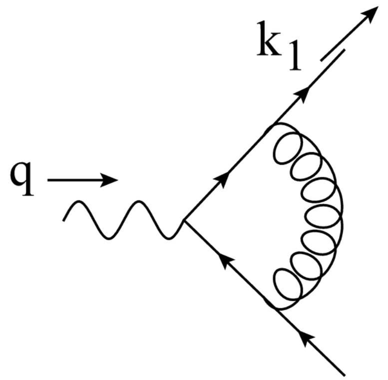
- Subtraction and Renormalization mechanism (Partonic Cross Section)
- Rapidity cut-offs

Partonic Cross Section at NLO

M.Boglione and A. Simonelli
arXiv:2011.07366

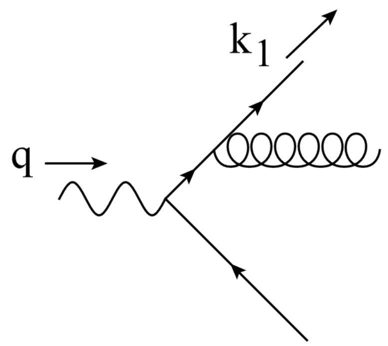
Lets consider a fragmenting quark of flavor f

- Virtual Emission



$$\longrightarrow \text{LO} \times V^{[1]}(\epsilon)\,\delta(1-T)\,\delta(1-z)$$

- Real Emission



$\longrightarrow$ 2-jet topology

Gluon collinear to fragmenting quark $\propto J^{[1]}_{q/q}(\epsilon;\,1-T,\,z)$

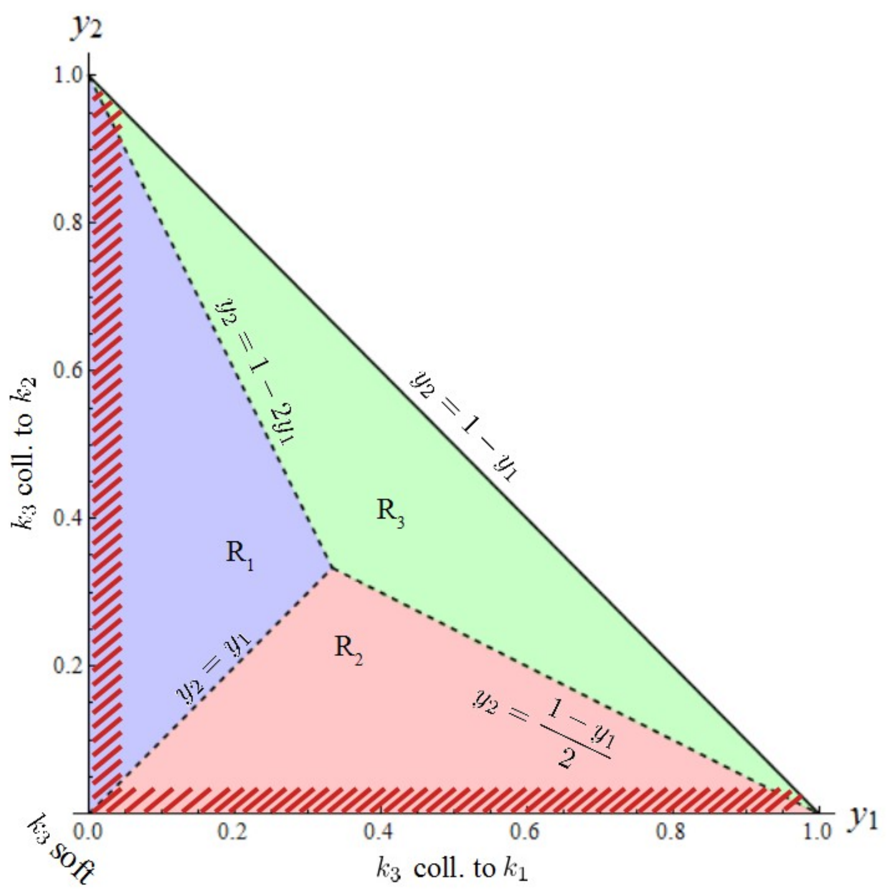
Soft Gluon $\propto S^{[1]}(\epsilon;\,1-T)\,\delta(1-z)$

Gluon collinear to antiquark (backward) $\propto J^{[1]}_{B}(\epsilon;\,1-T)\,\delta(1-z)$

Partonic Cross Section at NLO

Topology cut-off $\tau = 1 - T \leq \tau_{\text{MAX}}$

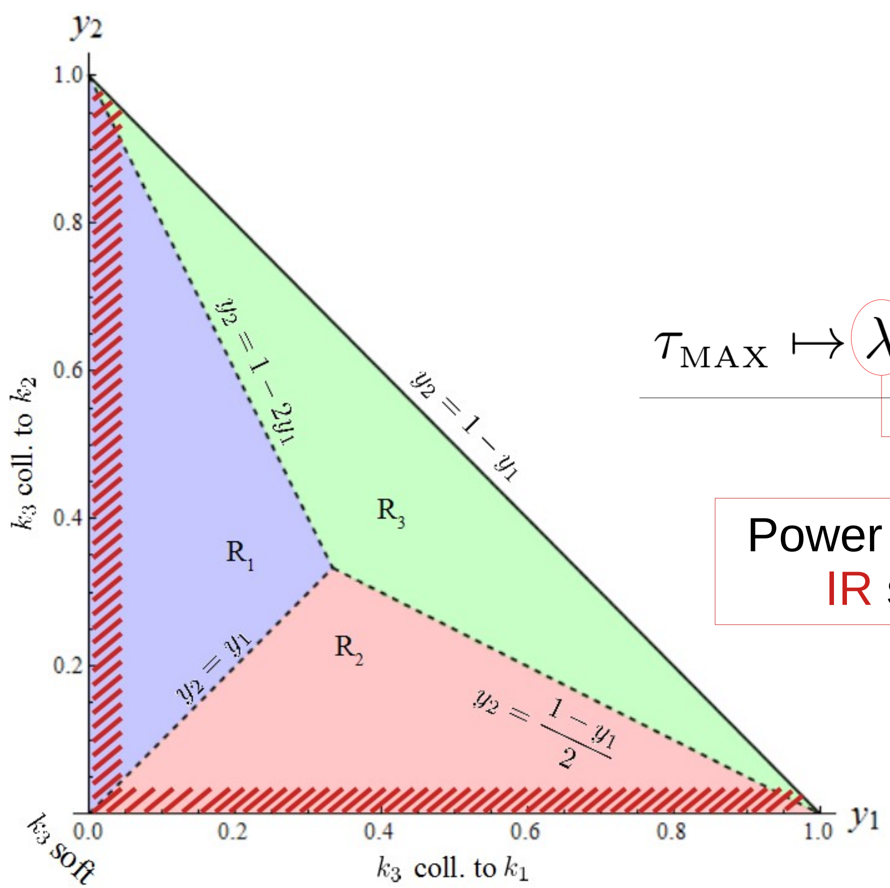
2-jet limit $\longleftrightarrow$ $\tau_{\text{MAX}} \to 0$



Partonic Cross Section at NLO

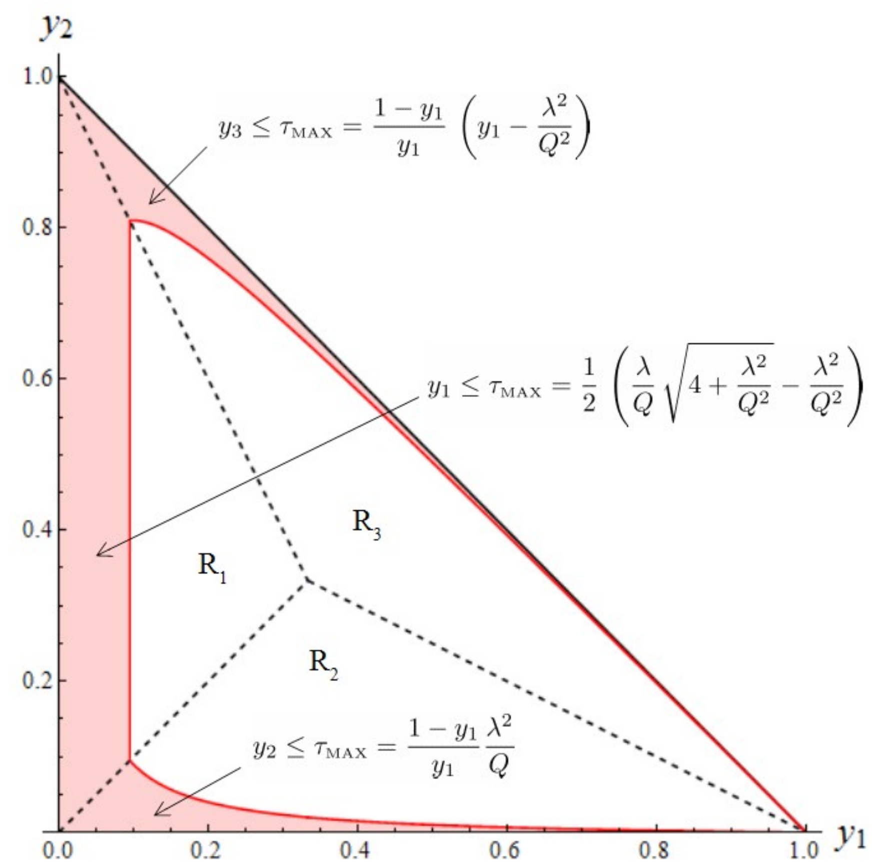
Topology cut-off $\tau = 1 - T \leq \tau_{\mathrm{MAX}}$

2-jet limit $\longleftrightarrow$ $\tau_{\mathrm{MAX}} \to 0$ $\longleftrightarrow$ $\lambda \to 0$



$\tau_{\mathrm{MAX}} \mapsto \lambda \geq k_T$

Power counting
IR scale



Partonic Cross Section at NLO

$$\frac{d\widehat{\sigma}_f^{[1]}}{dz\,dT} = \sigma_B\, z\, N_C\, e_f^2 \left[\delta(1-z)\left(\delta(\tau)\, V^{[1]}(\epsilon) + S^{[1]}(\epsilon;\,\tau) + J_B^{[1]}(\epsilon;\,\tau)\right) + \right.$$
$$\left. + J_{q/q}^{[1],\,(\lambda)}(\epsilon;\,\tau,\,z)\right]$$

Born cross section:

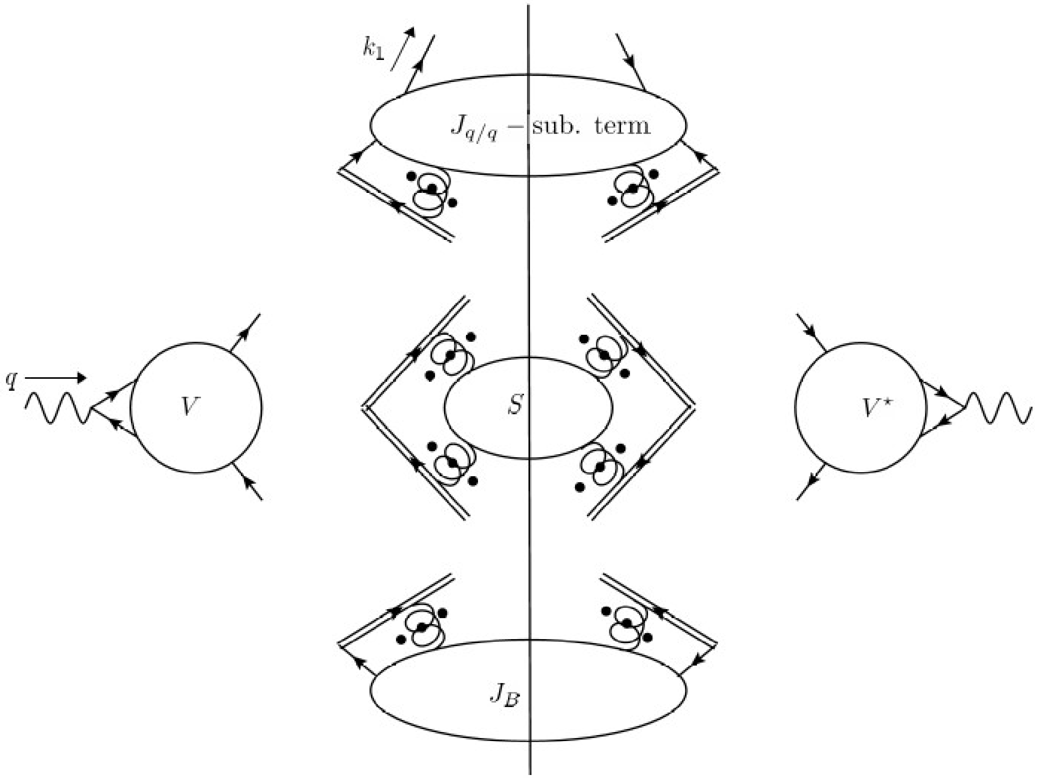
$$\sigma_B = \frac{4\pi\alpha^2}{3Q^2}$$

Partonic Cross Section at NLO

$$\frac{d\widehat{\sigma}_f^{[1]}}{dz\,dT} = \sigma_B\, z\, N_C\, e_f^2 \left[\delta(1-z)\left(\delta(\tau)\, V^{[1]}(\epsilon) + S^{[1]}(\epsilon;\,\tau) + J_B^{[1]}(\epsilon;\,\tau)\right) + \right.$$
$$\left. + J_{q/q}^{[1],\,(\lambda)}(\epsilon;\,\tau,\,z) - z\,\widetilde{D}_{q/q}^{[1],\,(\lambda)}(\epsilon;\,z,\,\zeta)\,\delta(\tau)\right]$$

Born cross section:

$$\sigma_B = \frac{4\pi\alpha^2}{3Q^2}$$



Subtraction Term:

- Cancellation of all poles (collinear, UV)
- Renormalization
- Cancellation of double counting (overlapping between hard and collinear momentum region)

Evolution and Resummation

Four energy scales: Q, μ, ζ and λ

$$
\left\{
\begin{aligned}
&\frac{\partial}{\partial \log \mu} \log \frac{d\widehat{\sigma}_f(\mu,\, \lambda,\, \zeta)}{dz\, dT} = -\gamma_D\left(\alpha_S(\mu),\, \zeta/\mu^2\right)\\
&\frac{\partial}{\partial \log \sqrt{\zeta}} \log \frac{d\widehat{\sigma}_f(\mu,\, \lambda,\, \zeta)}{dz\, dT} = \frac{1}{2}\, \widehat{K}\left(\alpha_S(\mu),\, \mu^2/\lambda^2\right)\\
&\frac{\partial}{\partial \log \lambda} \log \frac{d\widehat{\sigma}_f(\mu,\, \lambda)}{dz\, dT} = G\left(\alpha_S(\mu),\, \mu^2/Q^2,\, \zeta/\mu^2,\, \mu^2/\lambda^2\right)
\end{aligned}
\right.
$$

$$
\frac{d\widehat{\sigma}_f(\mu,\, \lambda,\, \zeta)}{dz\, dT} = \left.\frac{d\widehat{\sigma}_f}{dz\, dT}\right|_{\text{ref.}} \exp\left\{ \int_\mu^Q \frac{d\mu'}{\mu'}\, \gamma_D\left(\alpha_S(\mu'),\, \zeta/(\mu')^2\right) \right\} \times
$$

$$
\times \exp\left\{ \frac{1}{4}\, \widehat{K}\left(\alpha_S(Q),\, 1\right) \log \frac{\zeta}{Q^2} - \int_\lambda^Q \frac{d\lambda'}{\lambda'}\, G\left(\alpha_S(Q),\, 1,\, \zeta/Q^2,\, Q^2/(\lambda')^2\right) \right\}
$$

Evolution and Resummation

Four energy scales: $Q, \cancel{\mu}, \zeta$ and λ

$$\begin{cases} \dfrac{\partial}{\partial \log \mu} \log \dfrac{d\widehat{\sigma}_f(\mu,\, \lambda,\, \zeta)}{dz\, dT} = -\gamma_D \left(\alpha_S(\mu),\, \zeta/\mu^2\right) \longrightarrow \\[2ex] \dfrac{\partial}{\partial \log \sqrt{\zeta}} \log \dfrac{d\widehat{\sigma}_f(\mu,\, \lambda,\, \zeta)}{dz\, dT} = \dfrac{1}{2}\, \widehat{K} \left(\alpha_S(\mu),\, \mu^2/\lambda^2\right) \\[2ex] \dfrac{\partial}{\partial \log \lambda} \log \dfrac{d\widehat{\sigma}_f(\mu,\, \lambda)}{dz\, dT} = G \left(\alpha_S(\mu),\, \mu^2/Q^2,\, \zeta/\mu^2,\, \mu^2/\lambda^2\right) \end{cases}$$

RG-invariance of the whole cross section

$$\frac{d\widehat{\sigma}_f(\cancel{\mu},\, \lambda,\, \zeta)}{dz\, dT} = \left.\frac{d\widehat{\sigma}_f}{dz\, dT}\right|_{\text{ref.}} \cancel{\exp\left\{ \int_\mu^Q \frac{d\mu'}{\mu'} \gamma_D \left(\alpha_S(\mu'),\, \zeta/(\mu')^2\right) \right\}} \times$$

$$\times \exp\left\{ \frac{1}{4}\, \widehat{K} \left(\alpha_S(Q),\, 1\right) \log \frac{\zeta}{Q^2} - \int_\lambda^Q \frac{d\lambda'}{\lambda'} G \left(\alpha_S(Q),\, 1,\, \zeta/Q^2,\, Q^2/(\lambda')^2\right) \right\}$$

Evolution and Resummation

Four energy scales: $Q, \cancel{\mu}, \zeta$ and λ

In the partonic cross section, the contribution associated to the radiation collinear to the fragmenting quark is given by:

$J_{q/q}^{[1],\,(\lambda)}(\epsilon;\,\tau,\,z)$	$-\,z\,\widetilde{D}_{q/q}^{[1],\,(\lambda)}(\epsilon;\,z,\,\zeta)\,\delta(\tau)$	
$0 \leq k_T \leq \lambda$	$0 \leq k_T \leq \lambda$	Matched
$\frac{1}{2}\log\frac{2(k^+)^2}{\lambda^2} \leq y \leq +\infty$	$\frac{1}{2}\log\frac{2(k^+)^2}{\zeta} \leq y \leq +\infty$	

$$\frac{d\widehat{\sigma}_f(\cancel{\mu},\,\lambda,\,\zeta)}{dz\,dT} = \left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}} \cancel{\exp\left\{\int_{\mu}^{Q}\frac{d\mu'}{\mu'}\gamma_D\left(\alpha_S(\mu'),\,\zeta/(\mu')^2\right)\right\}}\times$$

$$\times\exp\left\{\frac{1}{4}\widehat{K}\left(\alpha_S(Q),\,1\right)\log\frac{\zeta}{Q^2} - \int_{\lambda}^{Q}\frac{d\lambda'}{\lambda'}G\left(\alpha_S(Q),\,1,\,\zeta/Q^2,\,Q^2/(\lambda')^2\right)\right\}$$

Evolution and Resummation

Four energy scales: $Q, \cancel{\mu}, \cancel{\zeta}$ and λ

In the partonic cross section, the contribution associated to the radiation collinear to the fragmenting quark is given by:

$J_{q/q}^{[1],\,(\lambda)}(\epsilon;\ \tau,\ z)$	$-\, z\, \widetilde{D}_{q/q}^{[1],\,(\lambda)}(\epsilon;\ z,\ \zeta)\, \delta(\tau)$	
$0 \leq k_T \leq \lambda$	$0 \leq k_T \leq \lambda$	Matched
$\frac{1}{2} \log \frac{2(k^+)^2}{\lambda^2} \leq y \leq +\infty$	$\frac{1}{2} \log \frac{2(k^+)^2}{\zeta} \leq y \leq +\infty$	Matched $(\tau = 0)$ $\longleftrightarrow$ $\zeta = \lambda^2$

$$\frac{d\widehat{\sigma}_f(\cancel{\mu},\ \lambda,\ \cancel{\zeta})}{dz\, dT} = \left.\frac{d\widehat{\sigma}_f}{dz\, dT}\right|_{\text{ref.}} \cancel{\exp\left\{ \int_\mu^Q \frac{d\mu'}{\mu'} \gamma_D\left(\alpha_S(\mu'),\ \zeta/(\mu')^2\right) \right\}} \times$$

$$\times \exp\left\{ \frac{1}{4}\, \widehat{K}\left(\alpha_S(Q),\ 1\right) \log \frac{\lambda^2}{Q^2} - \int_\lambda^Q \frac{d\lambda'}{\lambda'} G\left(\alpha_S(Q),\ 1,\ \lambda^2/Q^2,\ Q^2/(\lambda')^2\right) \right\}$$

Partonic Cross Section at NLO

$$\frac{d\widehat{\sigma}_f}{dz\,dT} = \underbrace{\left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}}}_{\text{NLO}} \underbrace{\exp\left\{\frac{1}{4}\,\widehat{K}\left(\alpha_S(Q)\right)\,\log\frac{\lambda^2}{Q^2} - \int_\lambda^Q \frac{d\lambda'}{\lambda'}\,G\left(\alpha_S(Q),\,Q^2/(\lambda')^2\right)\right\}}_{\text{1-loop}}$$

$$= \exp\left\{-\frac{\alpha_S(Q)}{4\pi}\,3C_F\,\log^2\frac{\lambda^2}{Q^2} + \mathcal{O}\left(\alpha_S(Q)^2\right)\right\}$$

- Suppression as $\lambda \to 0$
- Not a rigorous resummation

$$= \sigma_B\,e_f^2\,N_C\,\Bigg(\delta(1-z)\,\delta(\tau) +$$

$$+\frac{\alpha_S(Q)}{4\pi}\,2\,C_F\left\{\delta(1-z)\left[\delta(\tau)\left(-\frac{9}{2}+\frac{\pi^2}{3}\right)-\frac{3}{2}\left(\frac{1}{\tau}\right)_+ - 4\left(\frac{\log\tau}{\tau}\right)_+\right]+\right.$$

$$\left.+\,2\left[-\frac{z}{1-z}\,\log z - \log(1-z) + \left(\frac{\log(1-z)}{1-z}\right)_+\right]\delta(\tau)\right\} + \mathcal{O}\left(\alpha_S(Q)^2\right)\Bigg)$$

Partonic Cross Section at NLO

$$\left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}} \overset{\text{NLO}}{=} \sigma_B\, e_f^2\, N_C \Bigg(\delta(1-z)\,\delta(\tau) +$$
$$+\frac{\alpha_S(Q)}{4\pi}\, 2\, C_F \Bigg\{ \delta(1-z) \left[\delta(\tau)\left(-\frac{9}{2}+\frac{\pi^2}{3}\right) - \frac{3}{2}\left(\frac{1}{\tau}\right)_+ - 4\left(\frac{\log\tau}{\tau}\right)_+\right] +$$
$$+ 2\left[-\frac{z}{1-z}\,\log z - \log(1-z) + \left(\frac{\log(1-z)}{1-z}\right)_+\right]\delta(\tau)\Bigg\} + \mathcal{O}\left(\alpha_S(Q)^2\right)\Bigg)$$

Written in terms of τ-distributions ⟶ **Pheno** requires functions

Solution: **RESUMMATION** in both z and τ

Partonic Cross Section at NLO

$$
\left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}} \overset{\text{NLO}}{=} \sigma_B\, e_f^2\, N_C \Bigg(\delta(1-z)\,\delta(\tau) +
$$

$$
+ \frac{\alpha_S(Q)}{4\pi}\, 2\, C_F \Bigg\{ \delta(1-z) \left[\delta(\tau) \left(-\frac{9}{2} + \frac{\pi^2}{3} \right) - \frac{3}{2} \left(\frac{1}{\tau} \right)_+ - 4 \left(\frac{\log \tau}{\tau} \right)_+ \right] +
$$

$$
+ 2 \left[-\frac{z}{1-z} \log z - \log(1-z) + \left(\frac{\log(1-z)}{1-z} \right)_+ \right] \delta(\tau) \Bigg\} + \mathcal{O}\left(\alpha_S(Q)^2\right) \Bigg)
$$

Written in terms of τ-distributions ⟶ **Pheno** requires functions

Solution: **RESUMM[illegible]N** in both z and τ

DIFFICULT TASK

Partonic Cross Section at NLO

$$\left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}} \overset{\text{NLO}}{=} \sigma_B\, e_f^2\, N_C \Bigg(\delta(1-z)\,\delta(\tau) +$$

$$+ \frac{\alpha_S(Q)}{4\pi}\, 2\, C_F \left\{ \delta(1-z) \left[\delta(\tau) \left(-\frac{9}{2} + \frac{\pi^2}{3} \right) - \frac{3}{2} \left(\frac{1}{\tau} \right)_+ - 4 \left(\frac{\log \tau}{\tau} \right)_+ \right] + \right.$$

$$\left. + 2 \left[-\frac{z}{1-z} \log z - \log(1-z) + \left(\frac{\log(1-z)}{1-z} \right)_+ \right] \delta(\tau) \right\} + \mathcal{O}\left(\alpha_S(Q)^2\right) \Bigg)$$

Written in terms of τ-distributions $\longrightarrow$ **Pheno** requires functions

Solution: **RESUMMATION** in both z and τ

DIFFICULT TASK

Easy (and rough) shortcut: neglect $\tau = 0$

Partonic Cross Section at NLO

$$\left.\frac{d\widehat{\sigma}_f}{dz\,dT}\right|_{\text{ref.}} \overset{\text{NLO}}{=} -\sigma_B\, e_f^2\, N_C\, \frac{\alpha_S(Q)}{4\pi}\, C_F\, \delta(1-z)\, \left[\frac{3+8\log\tau}{\tau}\right] + \mathcal{O}\left(\alpha_S(Q)^2\right)$$

- The limit $T=1$ cannot be reached $\longrightarrow$ Pheno in the range $0.7 \leq T \leq 0.9$
- The z-dependence is compromised, especially at large T

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Almost finished...we still have to fix λ! Remember: $k_T \leq \lambda$

But k_T is naturally constrained by kinematics: $k_T \leq \sqrt{\tau} Q$

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Now we are ready for phenomenology!

$\lambda = \sqrt{\tau}Q$

But k_T is naturally constrained by kinematics: $k_T \leq \sqrt{\tau}Q$

Final Results

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = \pi \sum_{f} \int_{z_h}^{1} \frac{dz}{z} \overbrace{\underbrace{\frac{d\widehat{\sigma}_f}{dz_h/z\, dT}}}^{\textbf{NLO}} \overbrace{D_{1,\,\pi^\pm/f}(z,\; P_T,\, \boxed{Q,}\; \boxed{(1-T)\,Q^2})}^{\textbf{NLL}}$$

Only fermions,
the fragmenting gluon
is suppressed by $\mathcal{O}(1-T)$

RG-invariance

The TMD FF acquires
a dependence on
thrust through its
rapidity cut-off

Computed at NLO:

$$\stackrel{\text{NLO}}{=} -\sigma_B\, e_f^2\, N_C\, \frac{\alpha_S(Q)}{4\pi}\, C_F\, \delta\left(1 - \frac{z_h}{z}\right) \left[\frac{3 + 8\log(1-T)}{1-T}\right] \times$$

$$\times \exp\left\{-\frac{\alpha_S(Q)}{4\pi}\, 3C_F \log^2(1-T)\right\}$$

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Collinear FFs, **NNFF10NLO**

NNPDFCollaboration, V. Bertone, S. Carrazza, N. P. Hartland, E. R. Nocera, and J. Rojo,A,Eur. Phys. J. C77(2017), no. 8 516

Fourier transform of:

$$\widetilde{D}_{1,\pi^\pm/f}(z,\, b_T;\, Q,\, \tau Q^2) = \frac{1}{z^2} \sum_k \left[\boxed{d_{\pi^\pm/k}} \otimes \overbrace{\mathcal{C}_{k/f}}^{\textbf{NLO}}\right](\mu_b) \times$$

$$\left.\times \exp\left\{ \frac{1}{4}\, \widetilde{K}\, \log\frac{\tau Q^2}{\mu_b^2} + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_D - \frac{1}{4}\, \gamma_K\, \log\frac{\tau Q^2}{\mu'^2}\right] \right\} \times \right\} \textbf{NLL}$$

$$\times \boxed{(M_D)_{f,\,\pi^\pm}\,(z,\, b_T)}\; \exp\left\{ -\frac{1}{4} \boxed{g_K(b_T)}\, \log\left(\tau \frac{z_h^2\, Q^2}{M_H^2}\right) \right\}$$

Non-Perturbative functions **(pheno)**

Final Results

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NNPDFCollaboration, V. Bertone, S. Carrazza, N. P. Hartland, E. R. Nocera, and ...), no. 8 516

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The very soul of the specific TMD FF

Universal, independent of the TMD definition used

Non-Perturbative functions (**pheno**)

Final Results

Non-perturbative functions:

$g_K(b_T) = a\, b_T^2 \longrightarrow$ **Quadratic behavior** (common choice)

In general $0.01\ \mathrm{GeV}^2 \leq a \leq 0.1\ \mathrm{GeV}^2 \longrightarrow$ Our choice: $a = 0.05\ \mathrm{GeV}^2$

$$\left(M_D\right)_{f,\,\pi^\pm}(z,\, b_T) \equiv M_D(b_T) = \underbrace{\frac{2^{2-p}}{\Gamma(p-1)} (b_T\, m)^{p-1} K_{p-1}(b_T\, m)}_{\textbf{Power-law model}}$$

Simplest choice:
no dependence on the z
fragmenting quark flavor,
detected hadron,

$$\mathcal{FT}\{M_D\} = \frac{\Gamma(p)}{\pi\,\Gamma(p-1)} \frac{m^{2(p-1)}}{(k_T^2 + m^2)^p}$$

Common sense:

$$\left\{ \begin{array}{ll} m = 1\ \mathrm{GeV} & \text{generic hadronic mass} \\ p = 2 & \text{propagator squared} \end{array} \right.$$

Final Results

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$g_K(b_T) = a\, b_T^2$ → **Quadratic behavior** (common choice)

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Only three parameters,
here fixed to sensible values

aw model

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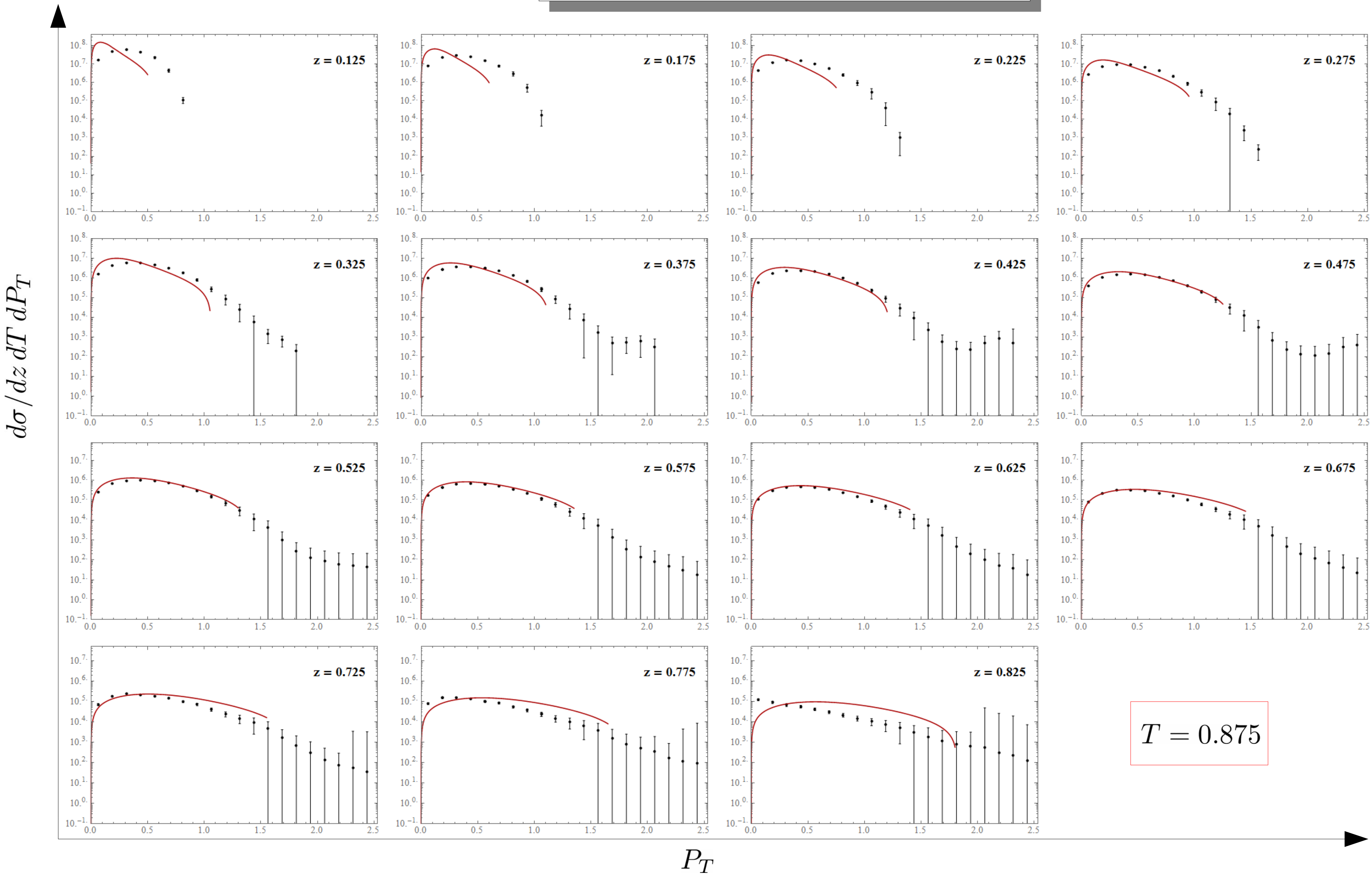
Final Results

M. Boglione and A. Simonelli
arXiv:2011.07366

NO FIT, this is a **PREDICTION**



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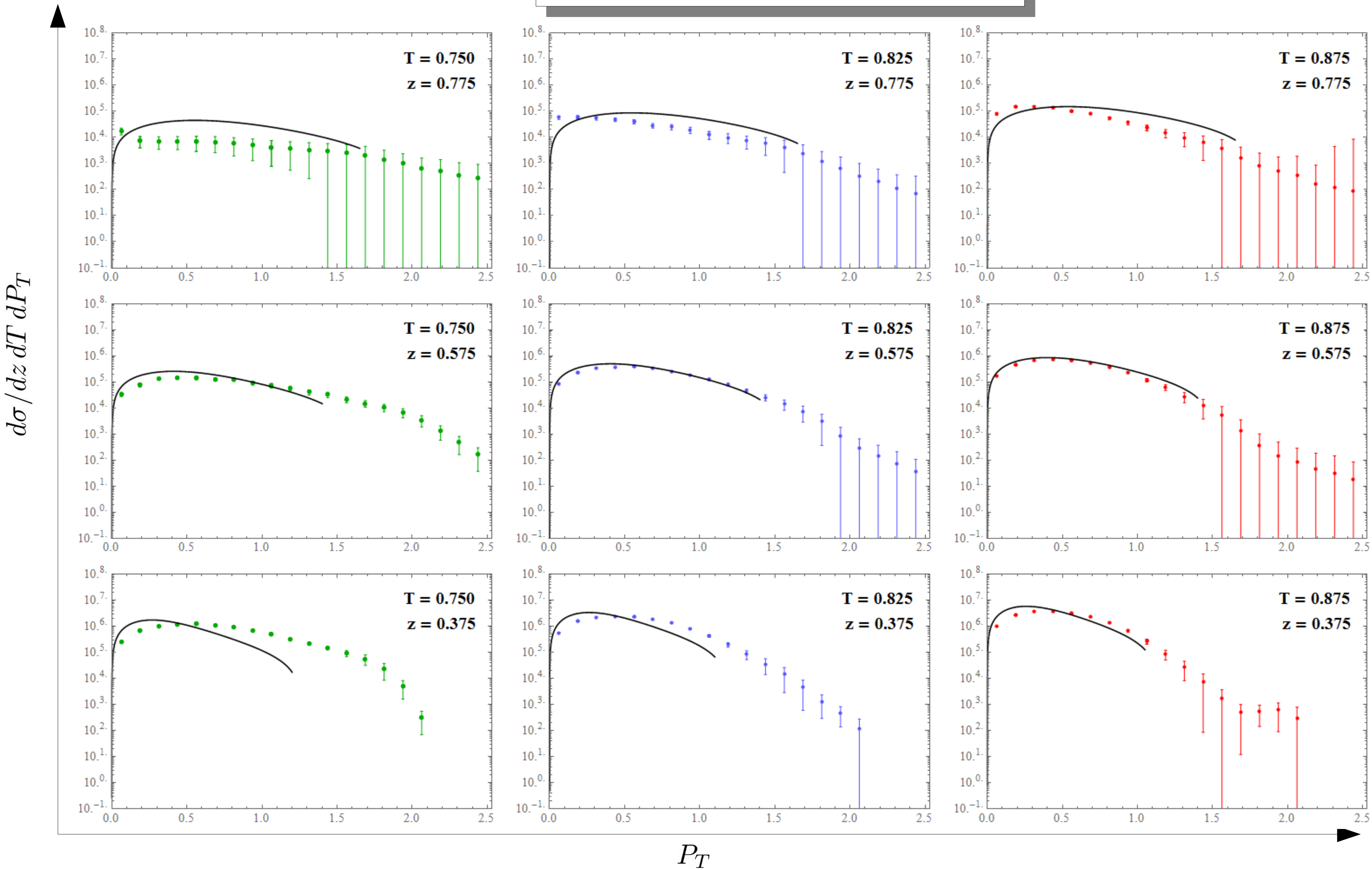

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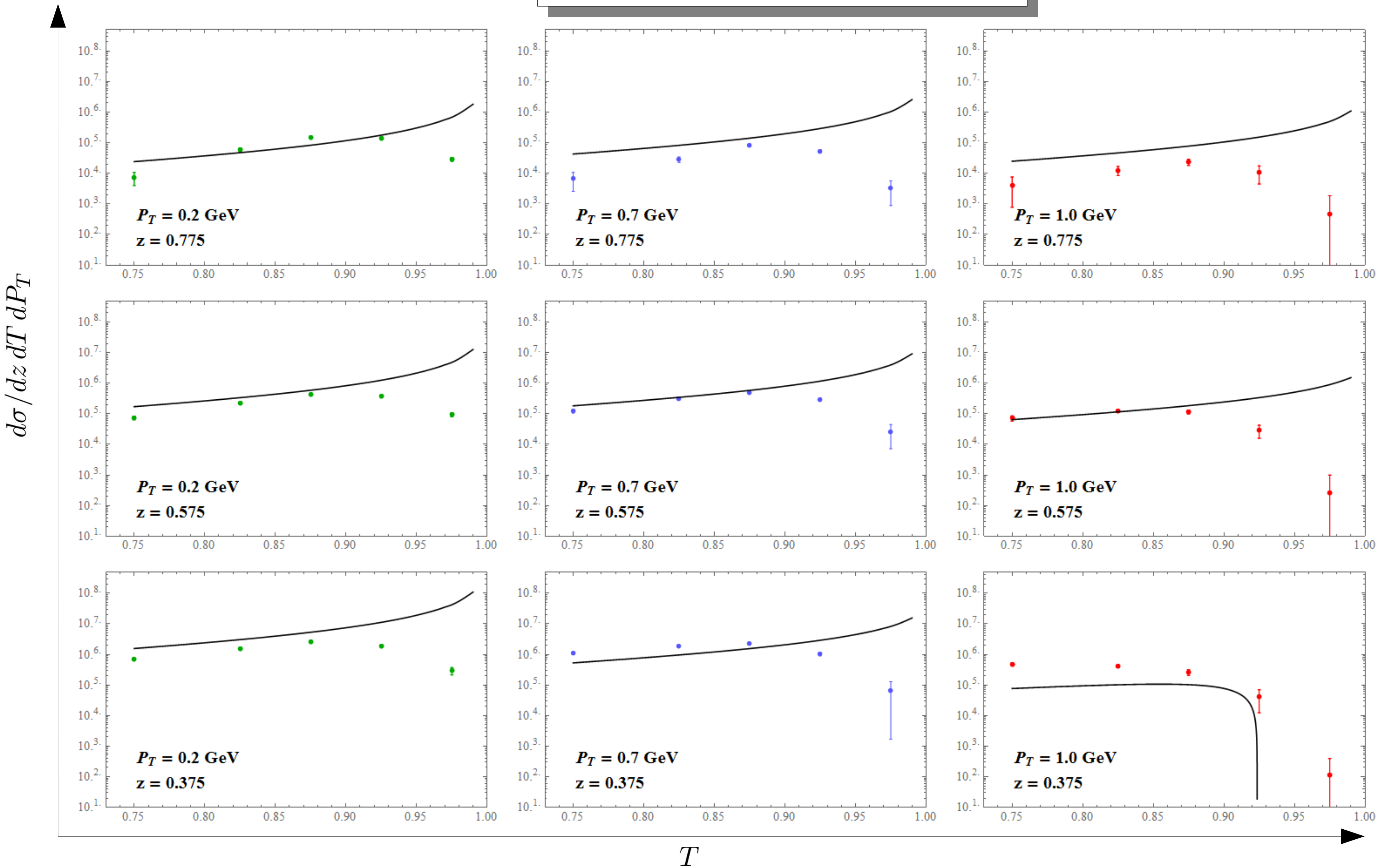


Conclusions and Future Remarks

- We have factorized (CSS) the cross section of $e^+e^- \to H\,X$, differential in z_h, P_T and T.
- Incredibly good agreement with BELLE data (only three parameters, fixed to sensible values – no fit).
- Different definition of TMD FF:

$$\boxed{\widetilde{D}^{\text{sqrt}}_{H_1/f}} = \boxed{\widetilde{D}_{1,\,H/f}}\;\boxed{\sqrt{M_S}}$$

Unknown

All the pheno of the past 20 years (2-h class)

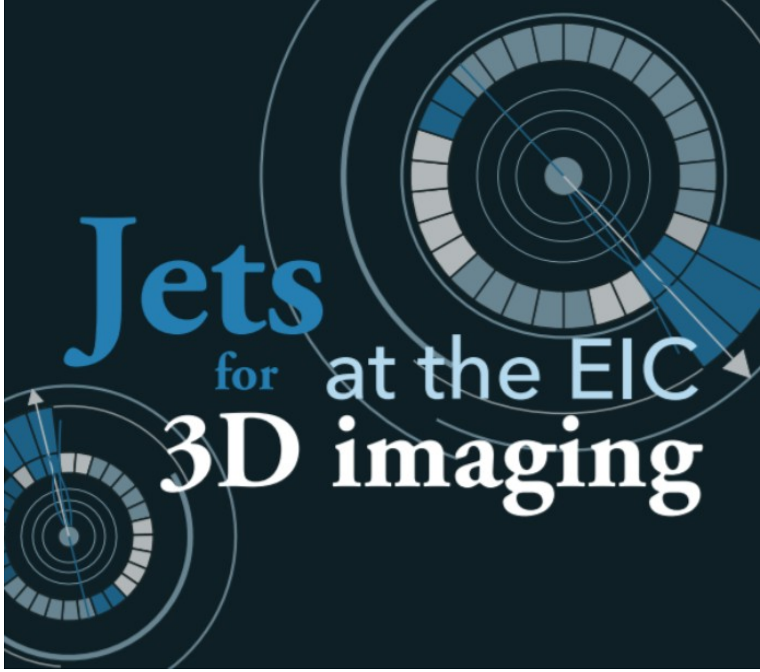
BELLE data pheno (1-h class)

- Promising applications in SIDIS, $e^+e^- \to H_1\,H_2\,X$ with back-to-back hadrons.

THANK YOU FOR YOUR ATTENTION!

Andrea Simonelli

In collaboration with M. Boglione



BACKUP SLIDES

Definition of TMDs: Building Blocks

- Unsubtracted TMD FF:

$$\widetilde{D}^{(0),\,\mathrm{unsub}}_{1,\,H/f}(z,\,\boxed{b_T};\,\mu,\,y_P,\,-\infty) =$$

$$= \frac{1}{z}\sum_X \langle P\,(H),\,X;\,\mathrm{out}|\overline{\psi}_f(-x/2)\,W_q(-x/2,\infty;\,n_1(y_1)\,)^\dagger|0\rangle$$

$$\langle 0|W_q(x/2,\infty;\,w_-\,)\,\psi_f(x/2)|P\,(H),\,X;\,\mathrm{out}\rangle\,|_{\mathrm{NO\ S.I.}}$$

Fourier conjugate space to transverse momentum

- 2-h Soft Factor

$$\widetilde{\mathbb{S}}^{(0)}_{\text{2-h}}(\boxed{b_T};\,\mu,\,y_1-y_2) =$$

$$= \frac{\mathrm{Tr}_C}{N_C}\,\langle 0|W(-\vec{b}_T/2,\,\infty;\,n_1(y_1)\,)^\dagger\,W(\vec{b}_T/2,\,\infty;\,n_1(y_1)\,)$$

$$W(\vec{b}_T/2,\,\infty;\,n_2(y_2)\,)^\dagger\,W(-\vec{b}_T/2,\,\infty;\,n_2(y_2)\,)|0\rangle\,|_{\mathrm{NO\ S.I.}}$$

Definition of TMDs: Building Blocks

- Unsubtracted TMD FF:

Rapidity Range: $-\infty \leq y \leq y_P \sim +\infty$

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- 2-h Soft Factor

Rapidity Range: $y_2 \leq y \leq y_1$

$$\widetilde{\mathbb{S}}^{(0)}_{\text{2-h}}(b_T;\,\mu,\,\boxed{y_1 - y_2}) =$$

$$= \frac{\mathrm{Tr}_C}{N_C}\,\langle 0|W(-\vec{b}_T/2,\,\infty;\,n_1(y_1)\,)^\dagger\,W(\vec{b}_T/2,\,\infty;\,n_1(y_1)\,)$$

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Need for UV renormalization

Two different definitions for TMDs

- $e^+e^- \to H\,X \quad (T \sim 1) \longrightarrow W \sim \widehat{W}\, D_H$

$$\widetilde{D}_{1,\,H/f}(z,\,b_T;\,\mu,\,y_P - y_1) =$$

$$= Z_j(\mu,\,y_P - y_1) Z_2\left(\alpha_S(\mu)\right) \lim_{y_{u_2} \to -\infty} \frac{\widetilde{D}^{(0),\,\text{unsub}}_{1,\,H/f}(z,\,b_T;\,\mu,\,y_P - y_{u_2})}{\mathbb{S}^{(0)}_{\text{2-h}}(b_T;\,\mu,\,y_1 - y_{u_2})}$$

- $e^+e^- \to H_1\,H_2\,X \longrightarrow W \sim V\,V^\dagger\, D^{\text{sqrt}}_{H_1}\, D^{\text{sqrt}}_{H_2}$

$$\widetilde{D}^{\text{sqrt}}_{H_1/f}(z,\,b_T;\,\mu,\,y_P - y_1) =$$

$$= Z_j(\mu,\,y_P - y_1) Z_2\left(\alpha_S(\mu)\right) \lim_{\substack{y_{u_1} \to +\infty \\ y_{u_2} \to -\infty}} \widetilde{D}^{(0),\,\text{unsub}}_{1,\,H/f}(z,\,b_T;\,\mu,\,y_P - y_{u_2}) \times$$

$$\times \sqrt{\frac{\widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu,\,y_{u_1} - y_1)}{\widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu,\,y_{u_1} - y_{u_2})\;\widetilde{\mathbb{S}}_{\text{2-h}}(b_T;\,\mu,\,y_1 - y_{u_2})}}$$

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- $e^+e^- \to H_1\,H_2\,X \longrightarrow W \sim V\,V^\dagger\, D^{\text{sqrt}}_{H_1}\, D^{\text{sqrt}}_{H_2}$

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$$\widetilde{D}^{\text{sqrt}}_{H_1/f}(z,\, b_T;\, \mu,\, \boxed{y_P - y_1}) =$$

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$$\times \sqrt{\frac{\widetilde{\mathbb{S}}_{\text{2-h}}\left(b_T;\, \mu,\, y_{u_1} - y_1\right)}{\widetilde{\mathbb{S}}_{\text{2-h}}\left(b_T;\, \mu,\, y_{u_1} - y_{u_2}\right)\, \widetilde{\mathbb{S}}_{\text{2-h}}\left(b_T;\, \mu,\, y_1 - y_{u_2}\right)}}$$

UV renormalization

Cross Section Structure

$$\frac{d\sigma}{dz_h\, dT\, dP_T^2} = z_h\, \frac{\alpha^2}{4Q^4} \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, L_{\mu\nu}(\theta)\, \frac{dW_H^{\mu\,\nu}(z_h,\, T,\, P_T)}{dP_T^2}$$

Leptonic Tensor (LO in QED):

$$L^{\mu\nu}(\theta) = l_1^\mu\, l_2^\nu + l_2^\mu\, l_1^\nu - g^{\mu\nu}\, l_1 \cdot l_2$$

Hadronic Tensor:

$$W_H^{\mu\,\nu}(z_h,\, T,\, P_T) = 4\pi^3 \sum_X \delta^{(4)}\, (p_X + P - q)\, \times$$

$$\times\ \langle 0|\, j^\mu(0)|\, P,\, X,\, \text{out}\,\rangle_T\, {}_T\langle P,\, X,\, \text{out}|j^\nu(0)|\, 0\rangle =$$

$$= \frac{1}{4\pi} \sum_X \int d^4z\, e^{iq\cdot z} \langle 0|\, j^\mu\, (z/2)\,|\, P,\, X,\, \text{out}\,\rangle_T\, {}_T\langle P,\, X,\, \text{out}|j^\nu\, (-z/2)\,|\, 0\rangle$$