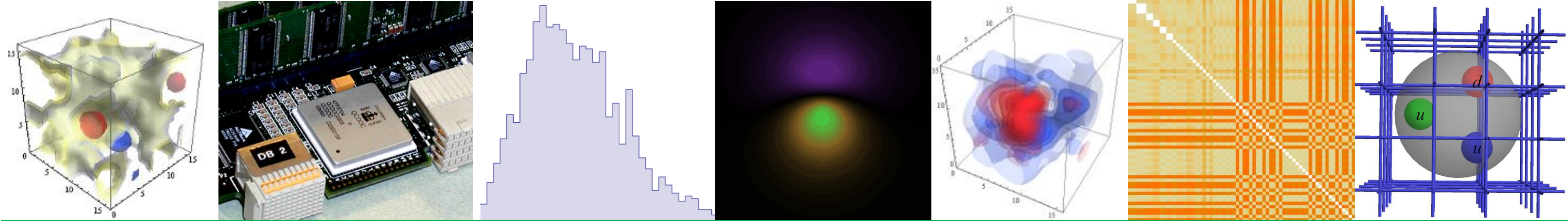

Nucleon Tomography and GPDs at Physical Pion Mass from Lattice QCD

Huey-Wen Lin

This work of HL is supported by the NSF under grant PHY 1653405
and the Research Corporation for Science Advancement through the Cottrell Scholar Award

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RESEARCH CORPORATION *for* SCIENCE ADVANCEMENT

Outline

§ Consumer's Guide to Lattice Structure Calculations

- **Nucleon** structure with controlled systematics in the physical limit ($m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty$)
- PDF Moments

§ *x*-dependent Nucleon Structure

- Recent Lattice PDFs Progress
- Applications to Generalized Parton Distributions

What is Lattice QCD?

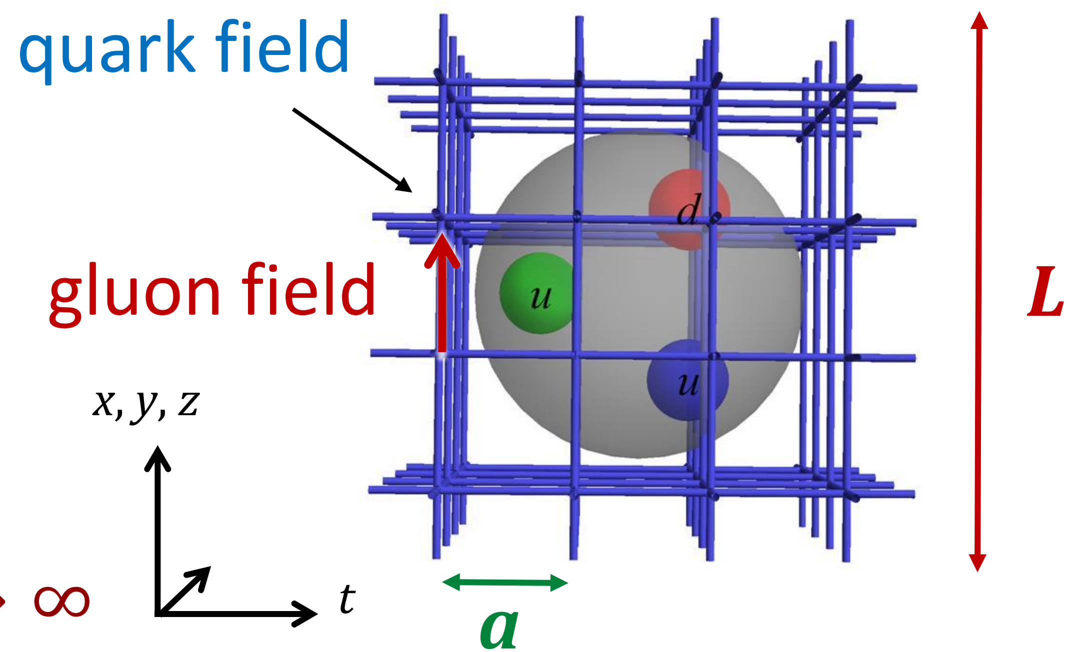
§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\left(\bar{\psi},\psi,A\right)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



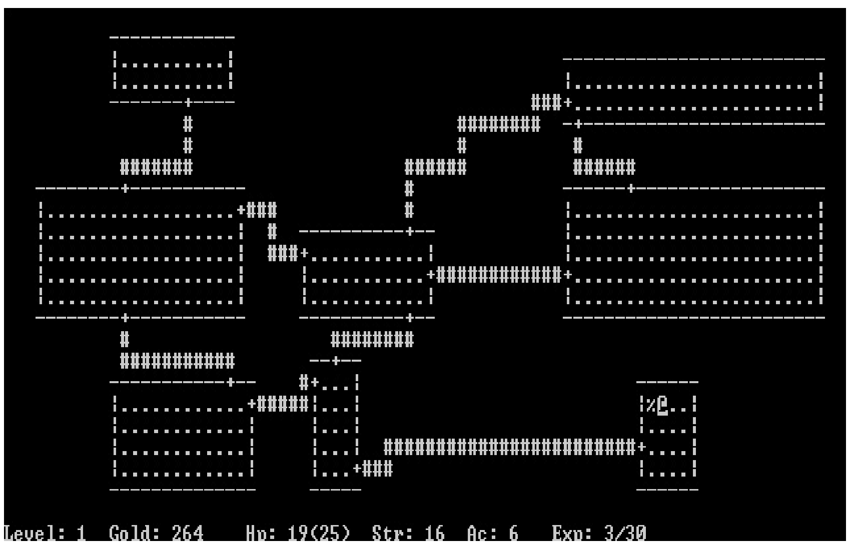
§ Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ \boldsymbol{a} \to \mathbf{0},\ \boldsymbol{L} \to \infty$$

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s



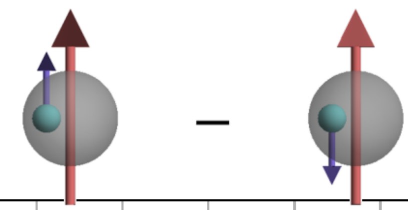
Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

Moments of PDFs

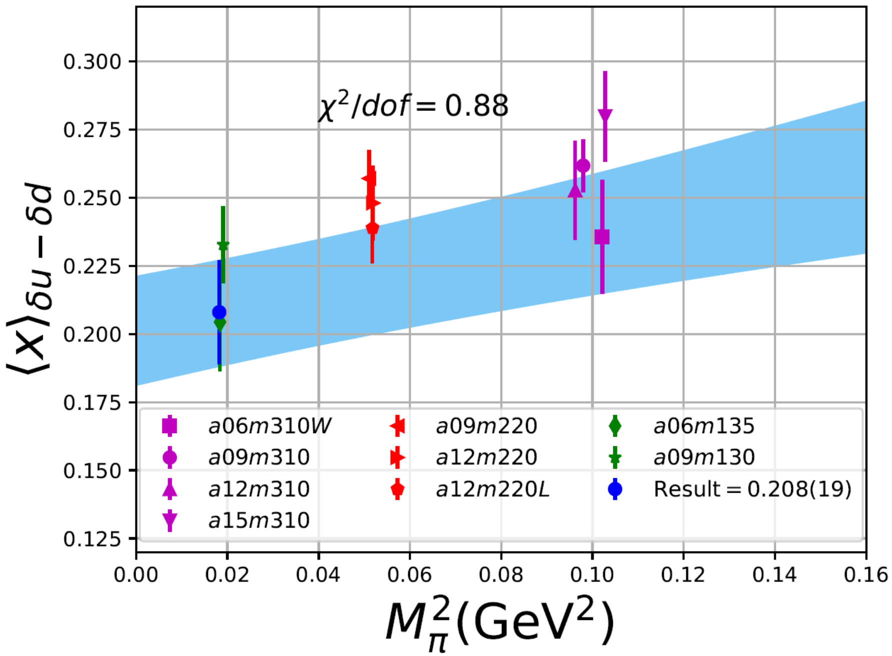
- Only lowest few moments
- State-of-the art example

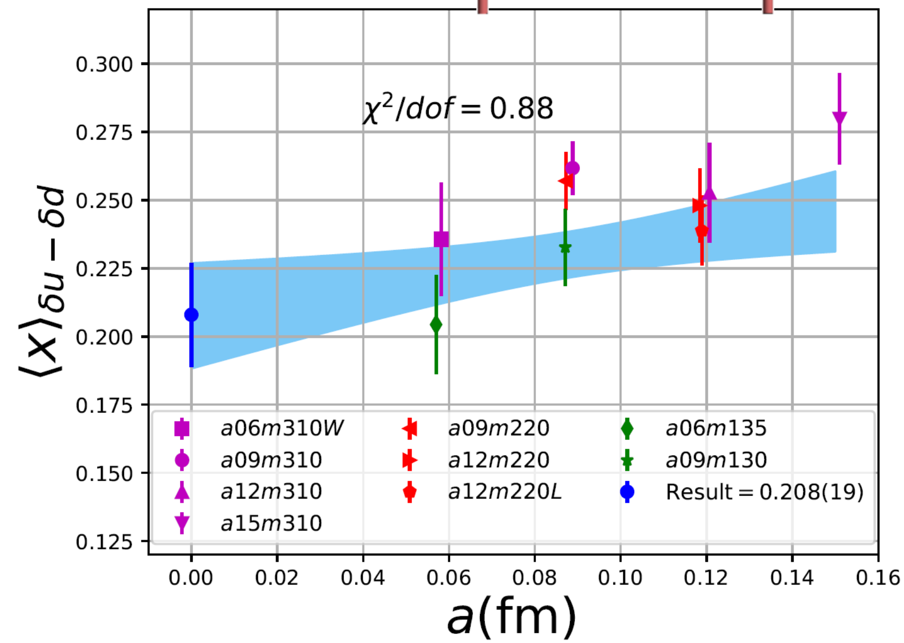
$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



- Extrapolate to the physical limit

Santanu Mondal et al (PNDME collaboration), 2005.13779





- Usually more than one LQCD calculation
  - Sometimes LQCD numbers do not even agree with each other…

Moments of PDFs

- § PDG-like rating system or average
- § LatticePDF Workshop
  - Lattice representatives came together and devised a rating system
- § Lattice QCD/global fit status

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\delta q(x)$$

LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1\rangle_{\delta u^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1\rangle_{\delta d^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1\rangle_{\delta s^-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

Moments of PDFs

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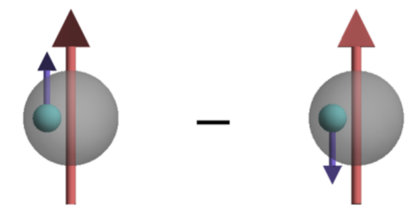
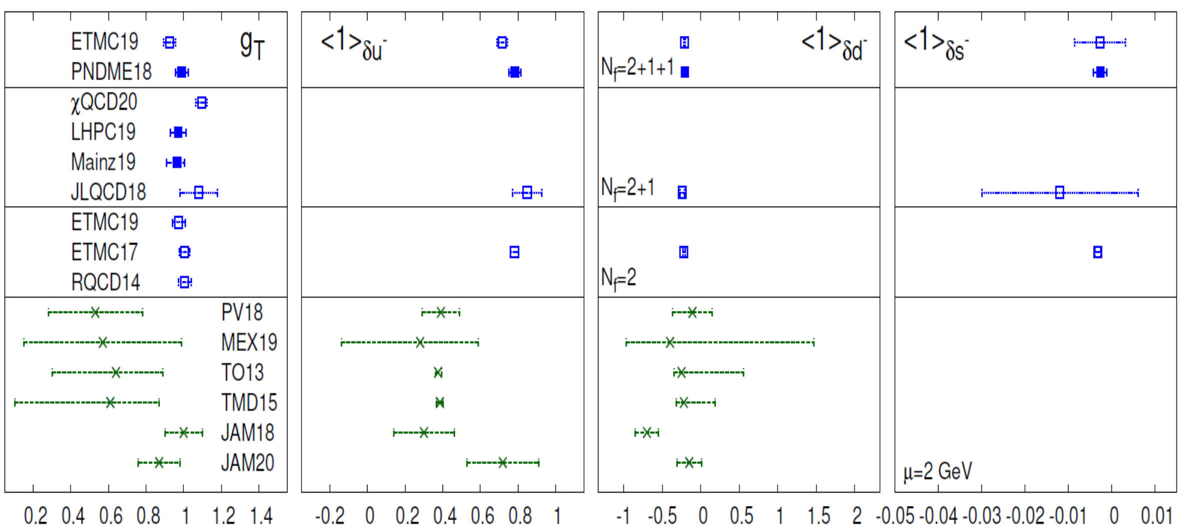
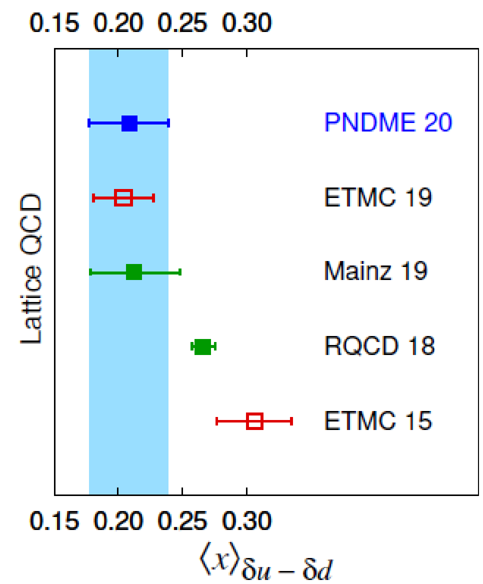
$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1}\,\delta q(x)$$



LatticePDF Report, 1711.07916, 2006.08636



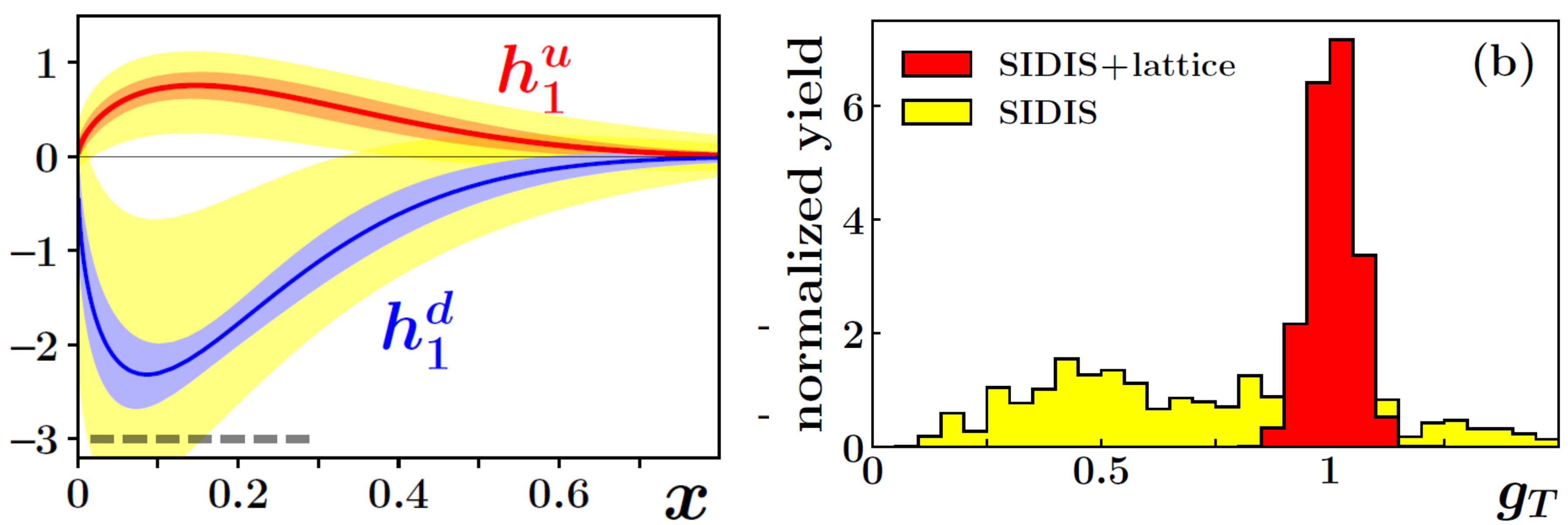


S. Mondal et al (PNDME), 2005.13779

From Charges to PDFs

§ Improved **transversity** distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets



Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

Bjorken-x Dependent Nucleon Structure

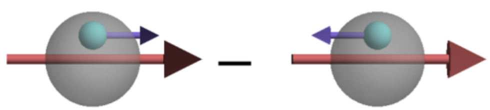
Structure on the Lattice

- Traditional lattice calculations rely on operator product expansion, only provide moments

most well known

spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent
transversely polarized

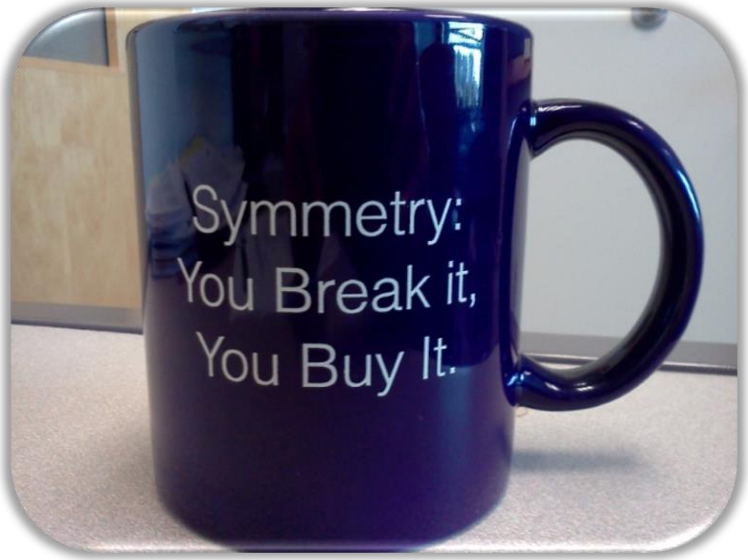
$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

- True distribution can only be recovered with all moments

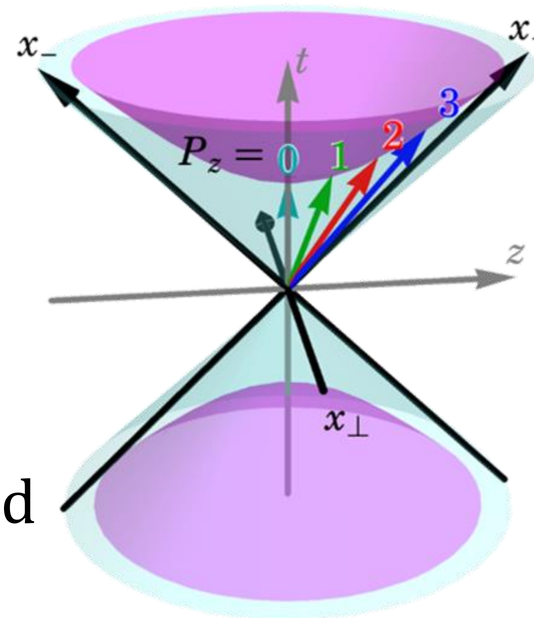
PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea



PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - No practical proposal yet to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Cannot separate valence contrib. from sea
- New Strategy: Xiangdong Ji, PRL 111, 039103 (2013);
  - Adopt lightcone description for PDFs
  - Calculate finite-boost quark distribution
    - In $P_z \to \infty$ limit, parton distribution recovered
    - For finite P_z, corrections are applied through effective theory
  - Feasible with today's resources!



Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

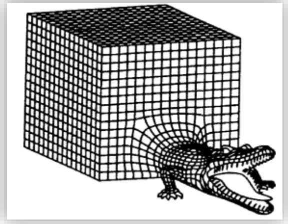
§ Longstanding obstacle to lattice calculations!

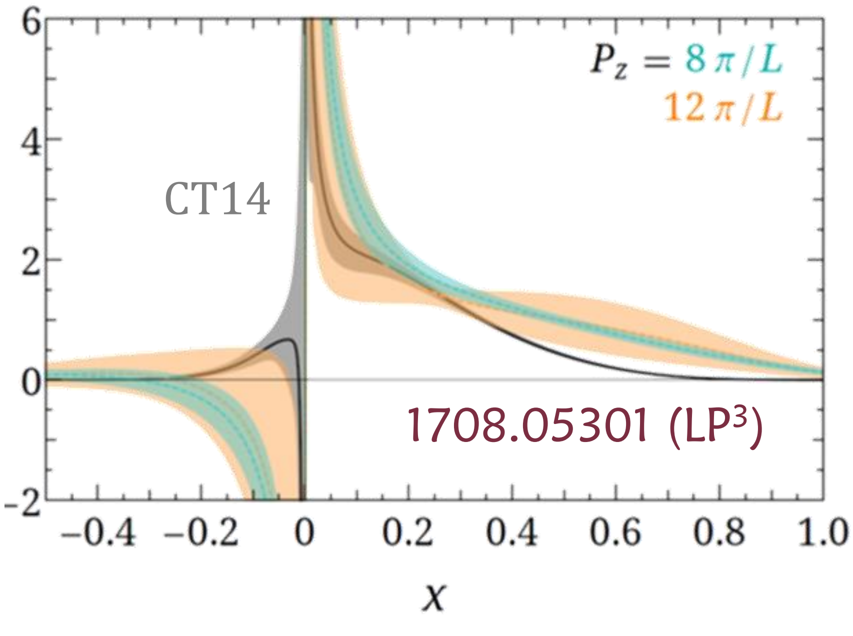
- Kernel is a complicated object;
  mostly only calculated up to one-loop level
- **Inverse problem to extract the wanted distribution**
  - Slightly different approaches from each group
  - Systematics vary
- Large momentum is needed in the lattice calculations in all methods to reach small-*x* region
  - Current projects focus on mid- to large-*x*

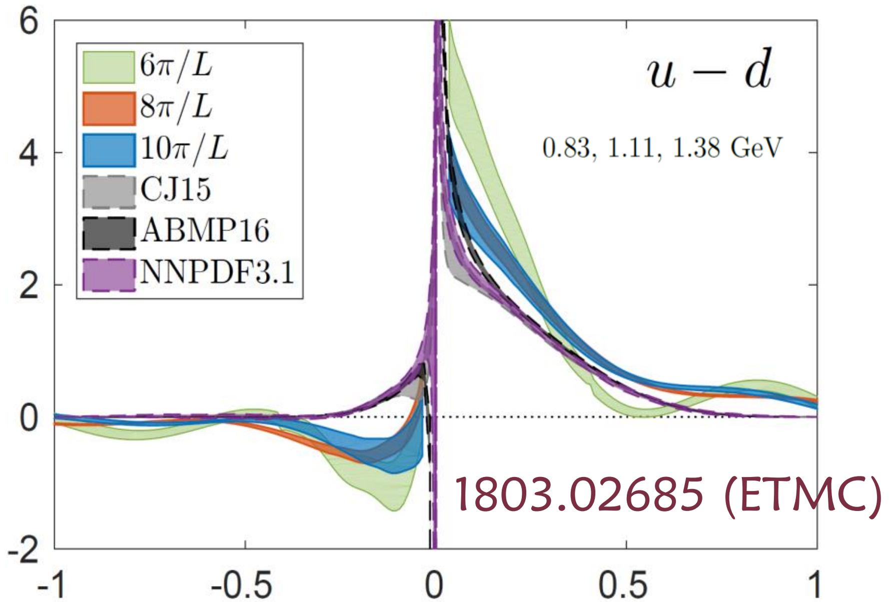
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





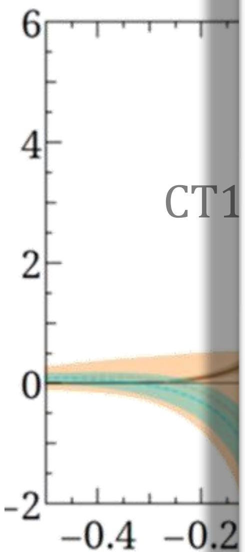


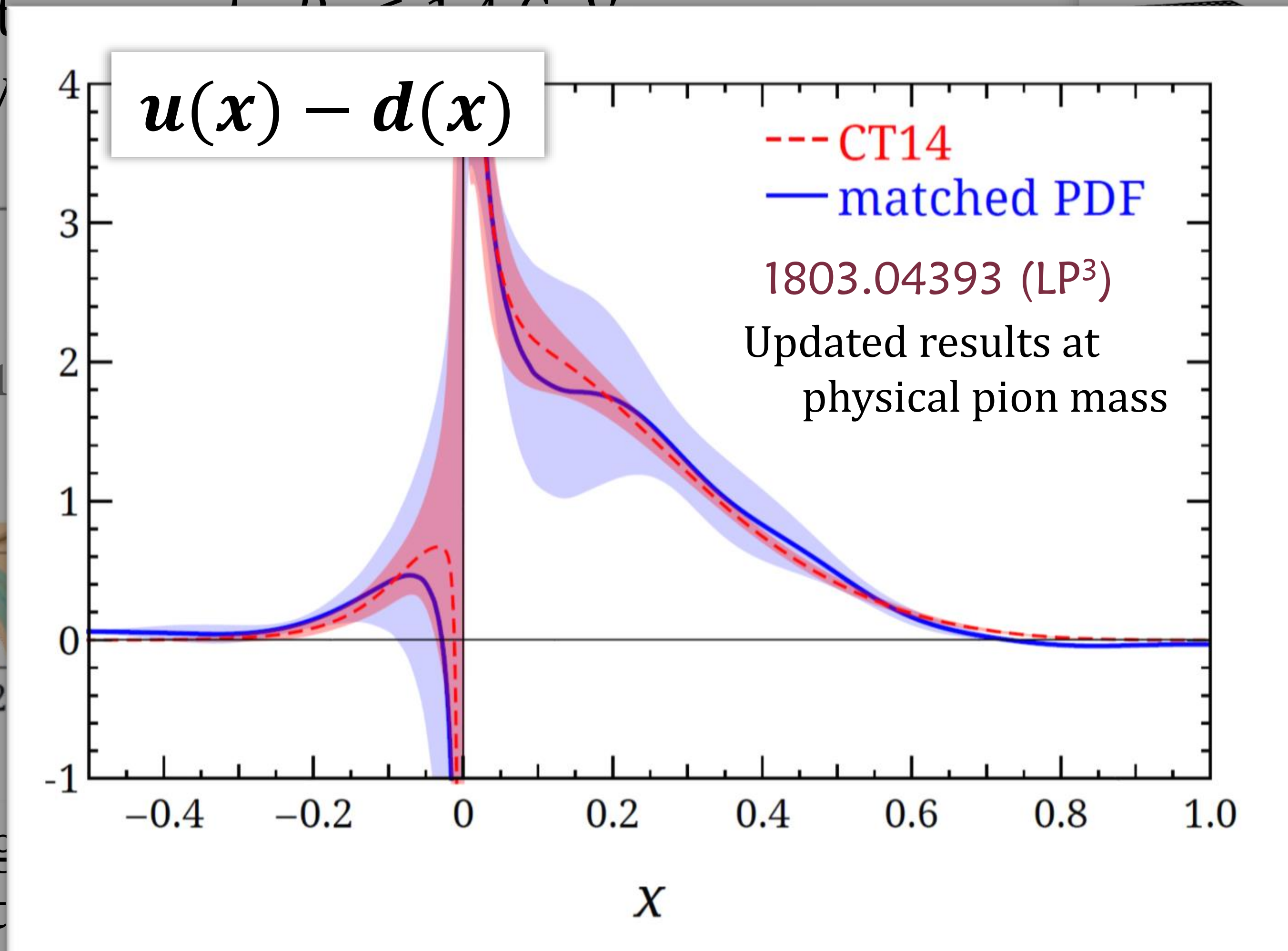
Not using parametrization (e.g. $xf(x, \mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

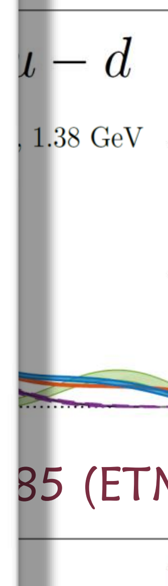
Physical Pion Mass Results

§ Quasi-PDF: two collaborations' results at physical pion mass

- Boost
- Study





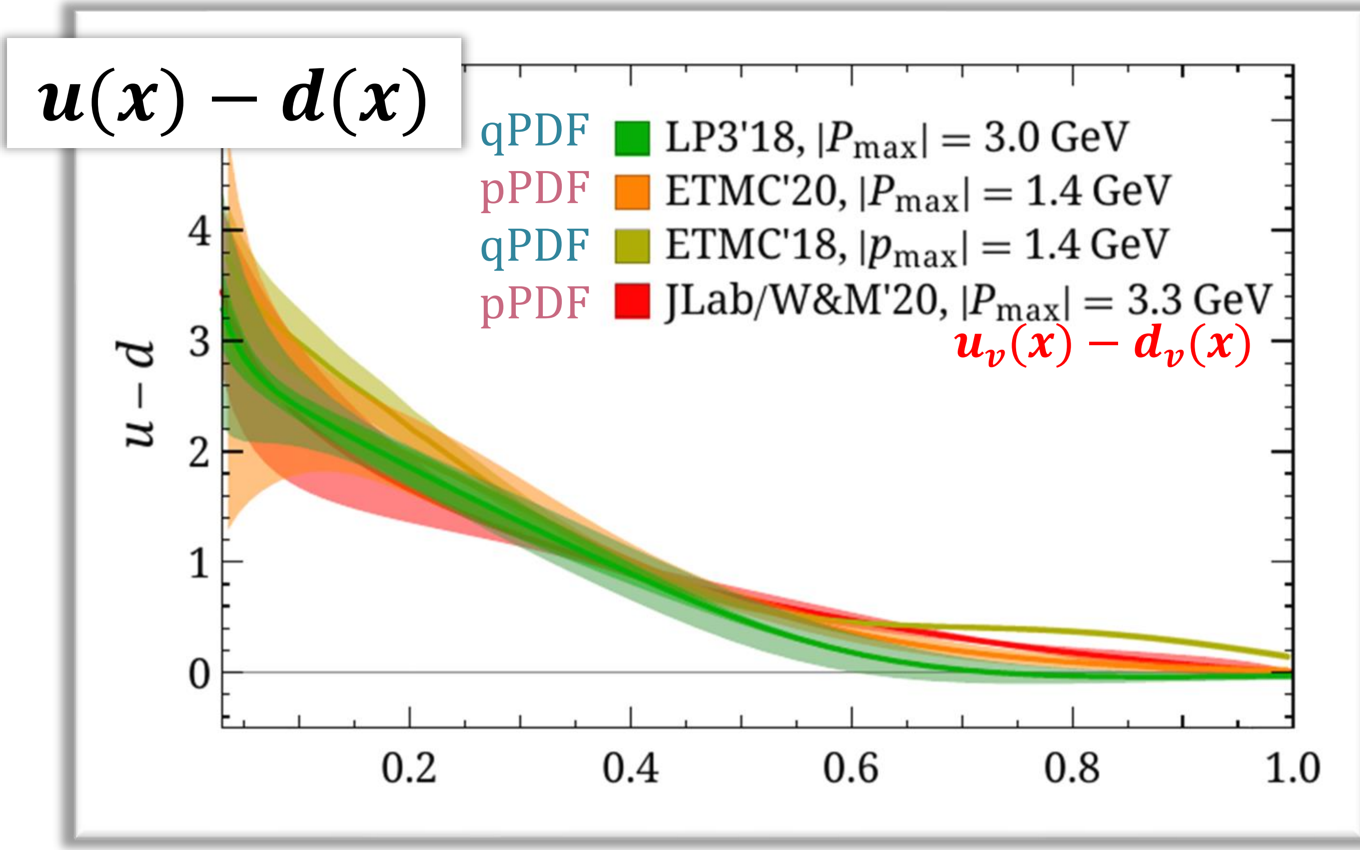


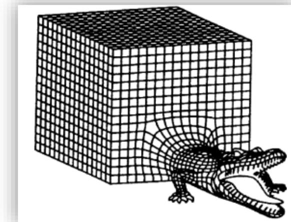
Not using)
Less pret
less likely to exactly coincide with global fits.

Physical Pion Mass Results

§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV





Finite volume,
Discretization,
…

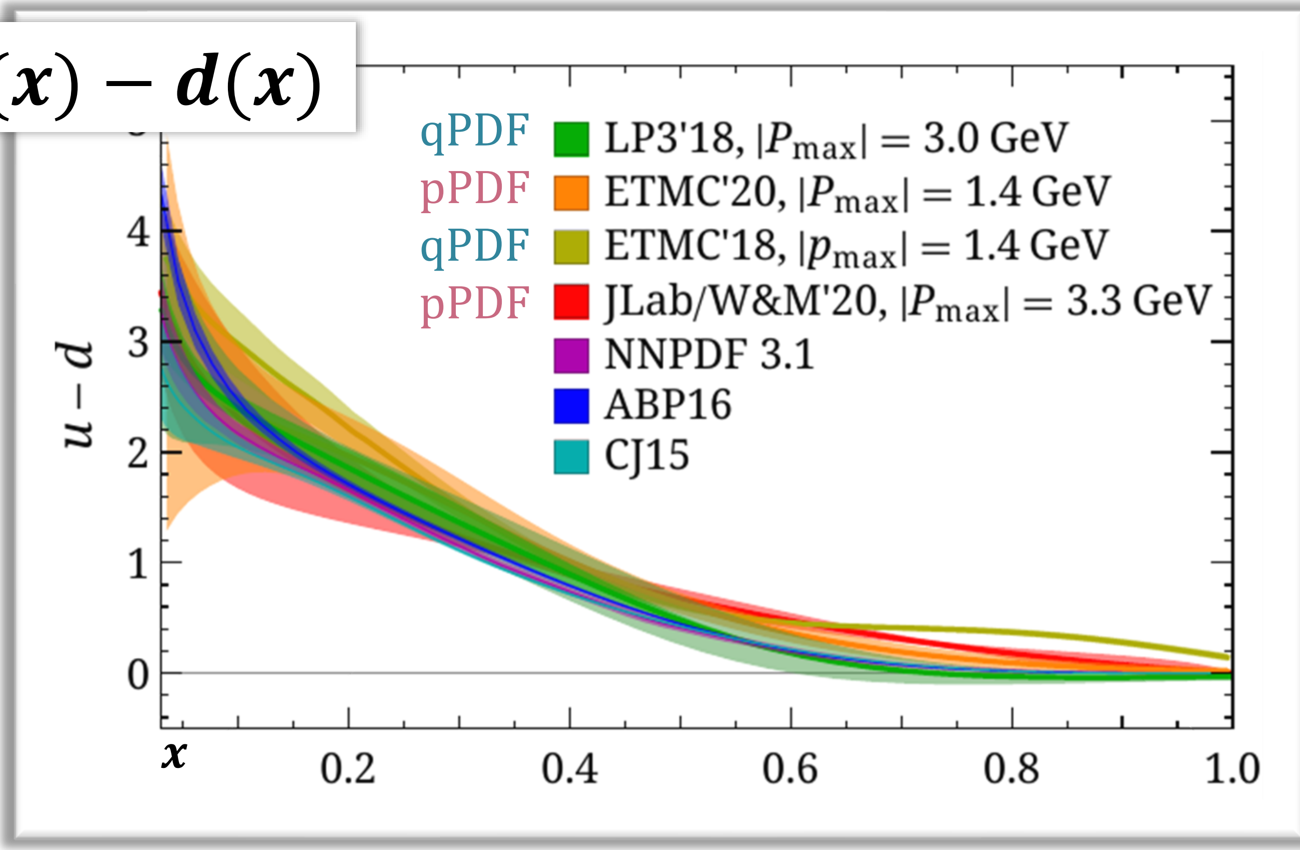
2006.08636, PDFLattice2019 report

Physical Pion Mass Results

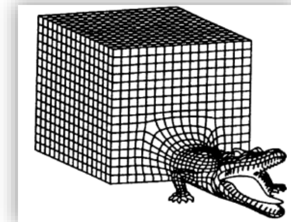
§ Summary of physical pion mass results

- Recent study increase boost momenta $P_z > 3$ GeV

$u(x) - d(x)$



2006.08636, PDFLattice2019 report



Finite volume,
Discretization,
…

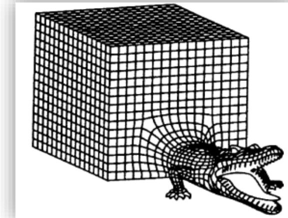
Transversity

§ Summary of physical pion mass results

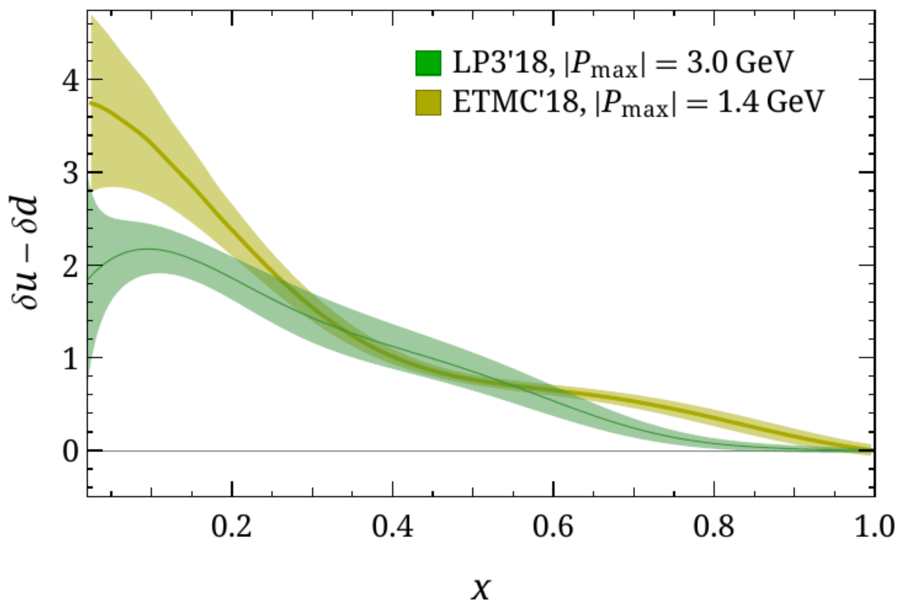
- Quasi-PDF method only

$\boldsymbol{\delta u(x) - \delta d(x)}$

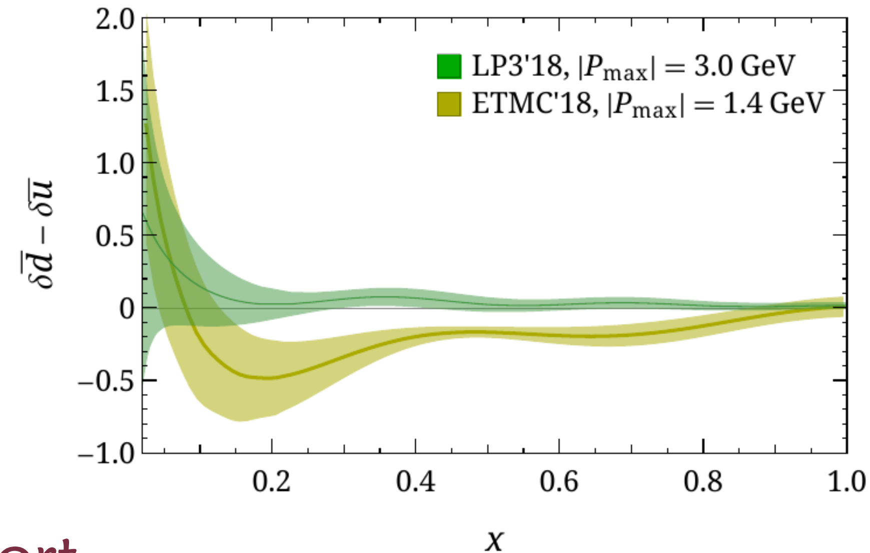
Transversity



Finite volume,
Discretization,
…



$\boldsymbol{\delta \bar{d}(x) - \delta \bar{u}(x)}$



2006.08636, PDFLattice2019 report

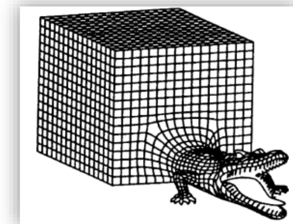
Transversity

§ Summary of physical pion mass results

- Quasi-PDF method only

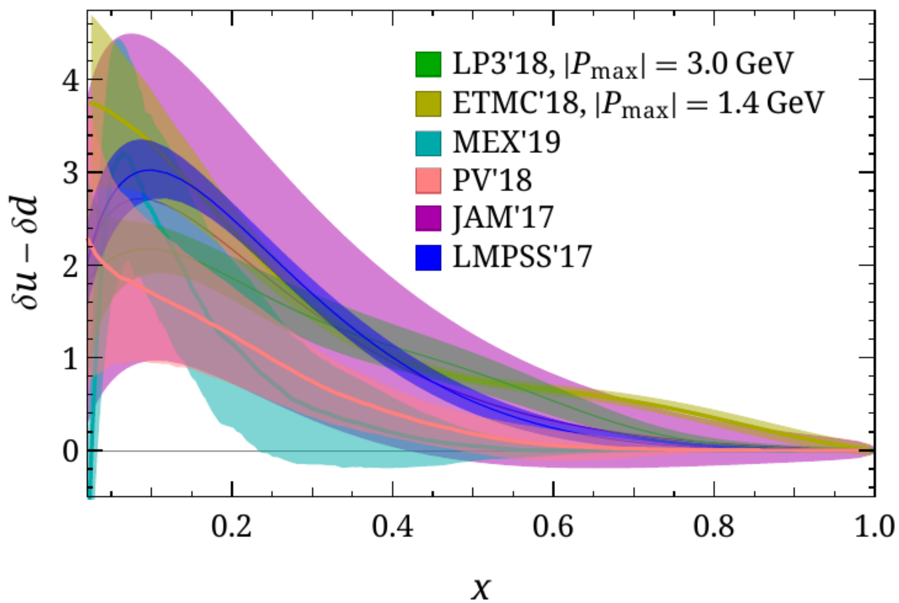
$$\delta u(x) - \delta d(x)$$

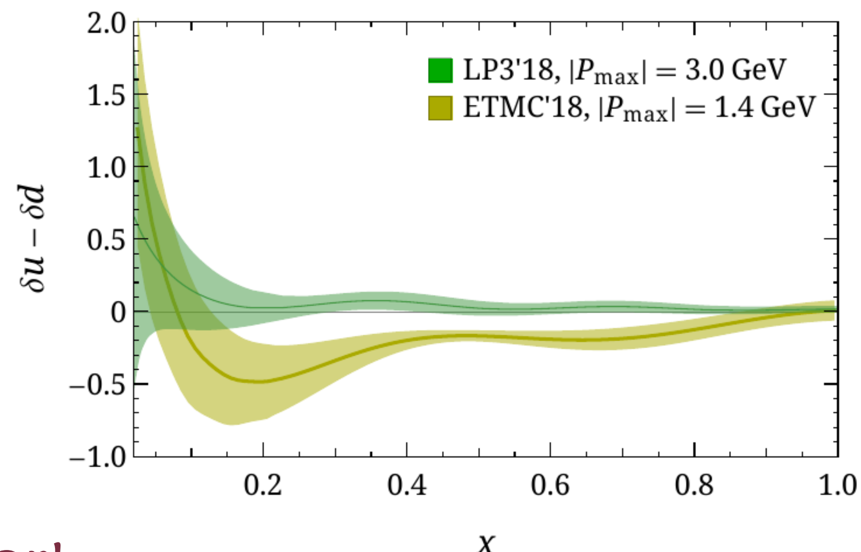
Transversity



Finite volume,
Discretization,
...

$$\delta\,\bar{d}(x) - \delta\,\bar{u}(x)$$

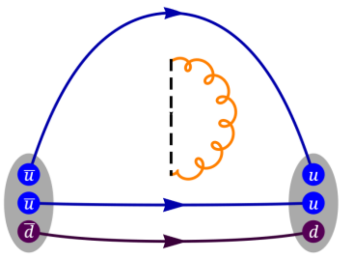




2006.08636, PDFLattice2019 report

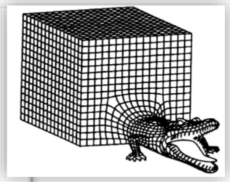
Gluon PDF in Nucleon

§ Gluon PDF using pseudo-PDF

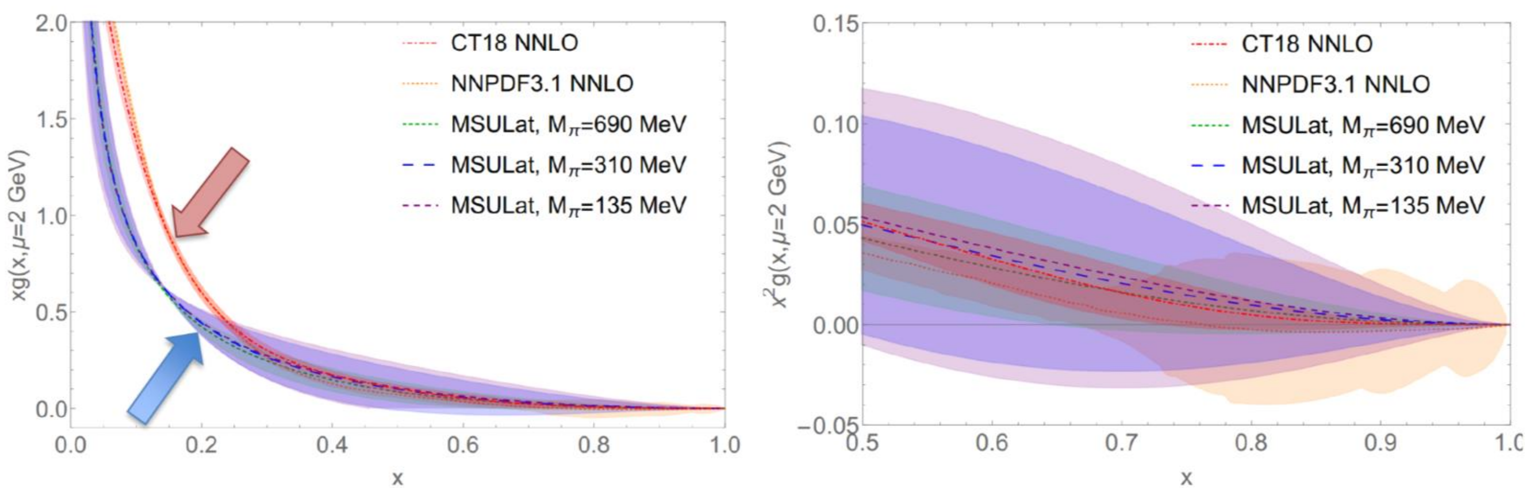


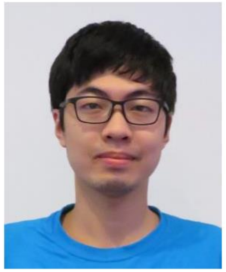
- Lattice details: clover/2+1+1 HISQ 0.12 fm, 310-MeV sea pion
- Study strange/light-quark

Z. Fan. et al (MSULat), 2007.16113



The comparison of the reconstructed unpolarized gluon PDF from the function form with CT18 NNLO and NNPDF3.1 NNLO gluon unpolarized PDF at $\mu = 2\ GeV$ in the $\overline{\mathrm{MS}}$ scheme.





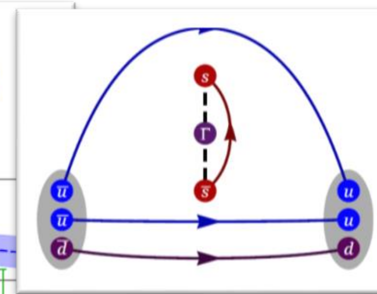
Zhouyou Fan (MSU)

Slide by Zhouyou Fan: See slides on Oct. 30th (Friday) Sec. E 10:42AM CDT

First Lattice Strange PDF

§ Large uncertainties in global PDFs

$$h^R(z,\mu^R,p_z^R,P_z)=\int_{-\infty}^{\infty}dxe^{ixzPz}\int_{-1}^{1}\frac{dy}{|y|}C(\frac{x}{y},\frac{\mu R}{\mu},\frac{\mu}{yP^z},\frac{p_z^R}{yP^z})q(y,\mu=2\text{ GeV})$$

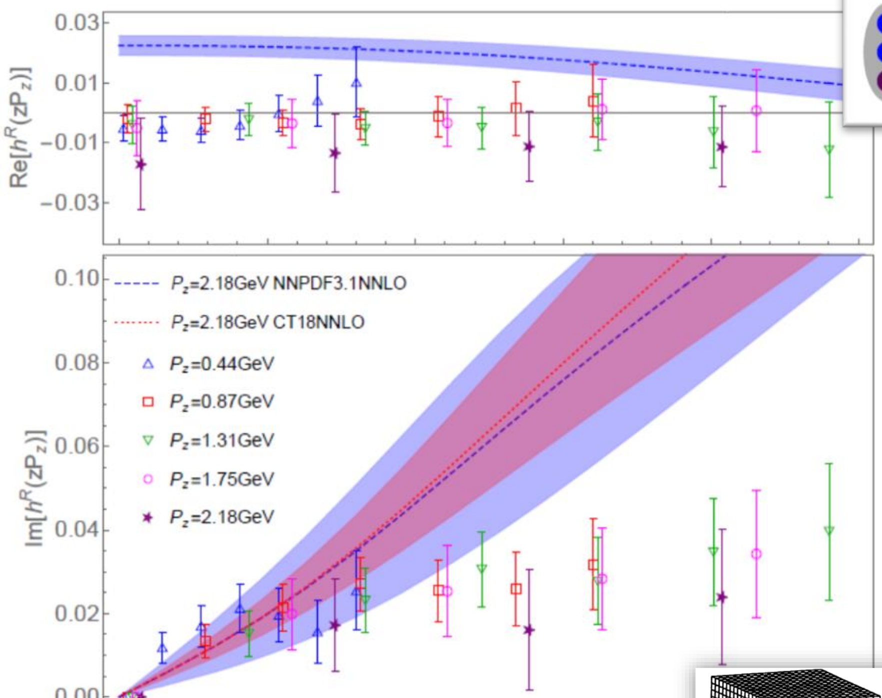


$\mathrm{Re}[h(z)] \propto$

$$\int dx\,(s(x)-\bar{s}(x))\cos(xzP_z)$$

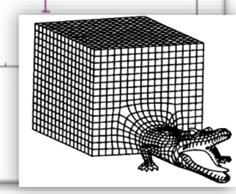
$\mathrm{Im}[h(z)] \propto$

$$\int dx\,(s(x)+\bar{s}(x))\sin(xzP_z)$$



- symmetric $s-\bar{s}$ distribution.
- smaller momentum fraction.

Rui Zhang (MSU)



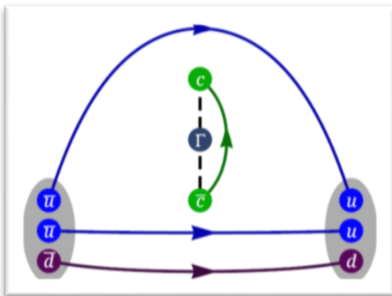
Slide by Rui Zhang: See slides on Oct. 30 (Friday) Sec. E 10:30AM CDT

First Lattice Charm PDF

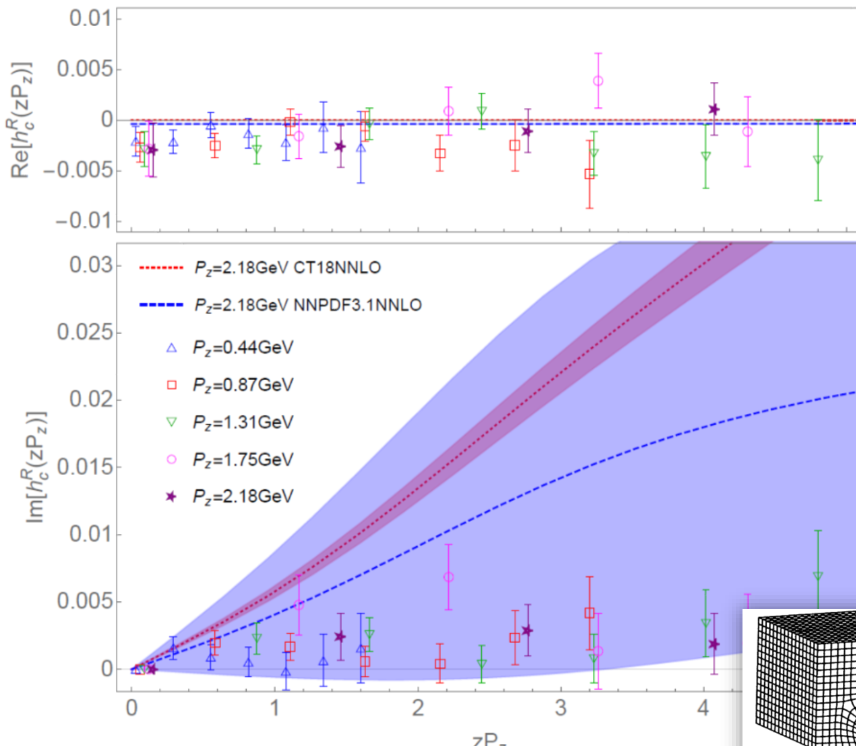
§ Large uncertainties in global PDFs

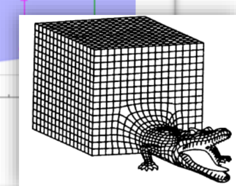
§ Results by MSULat/quasi-PDF method

2005.12015, R. Zhang et al (MSULat)



- suggest a symmetric $c-\bar{c}$ distribution
- much smaller than strange PDF





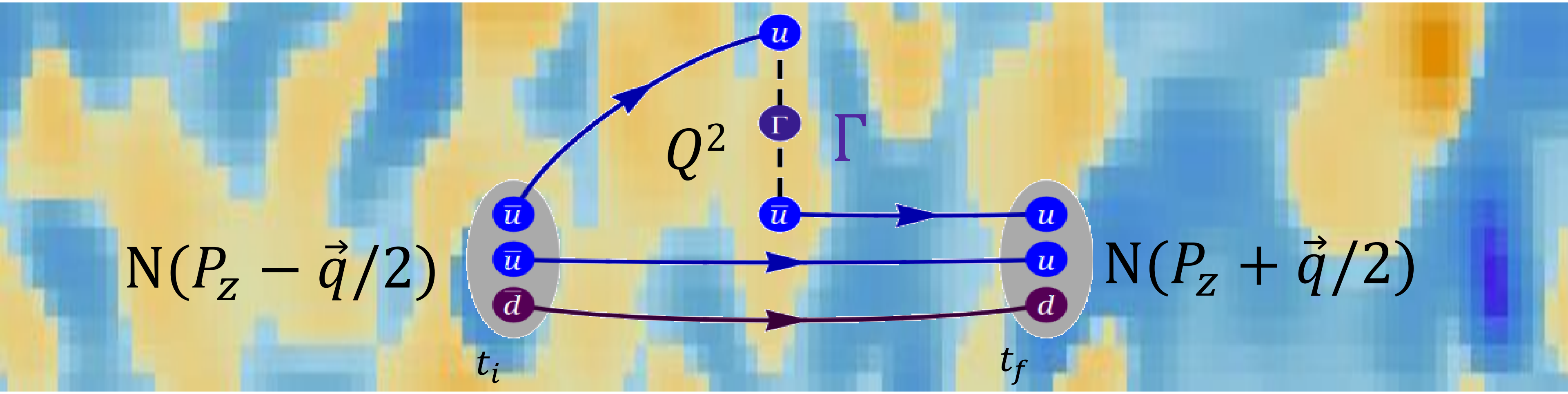
Rui Zhang (MSU)

Slide by Rui Zhang: See slides on Oct. 30th (Friday) Sec. E 10:30AM CDT

Bjorken-x Dependent GPDs

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)



$$\tilde{F}\left(x, \tilde{\xi}, t, \bar{P}_{\mathrm{Z}}\right)$$

$$= \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_Z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(\tilde{H}\left(x, \tilde{\xi}, t, \bar{P}_{\mathrm{Z}}\right) \gamma^0 + \tilde{E}\left(x, \tilde{\xi}, t, \bar{P}_Z\right) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

❧ **Inverse problem to extract the wanted distribution**

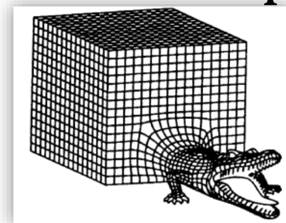
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

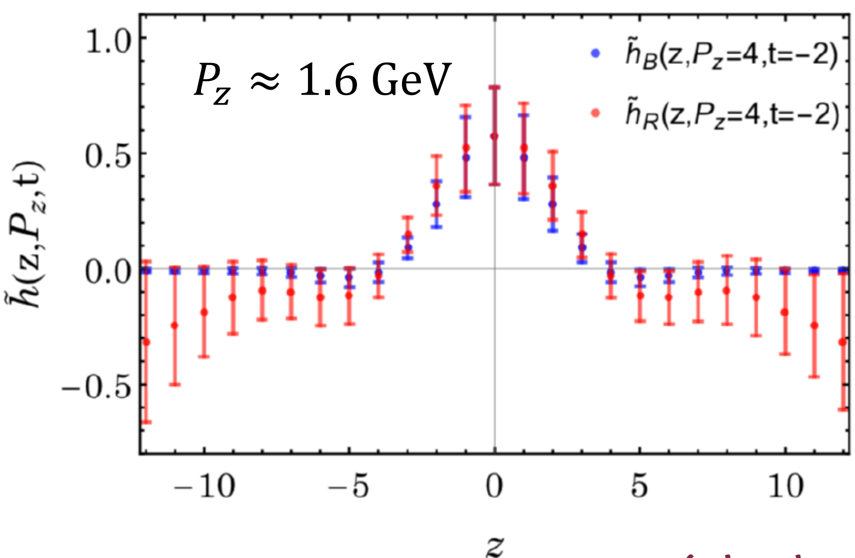
- Lattice details: clover/HISQ, 0.12fm, **310-MeV** pion mass

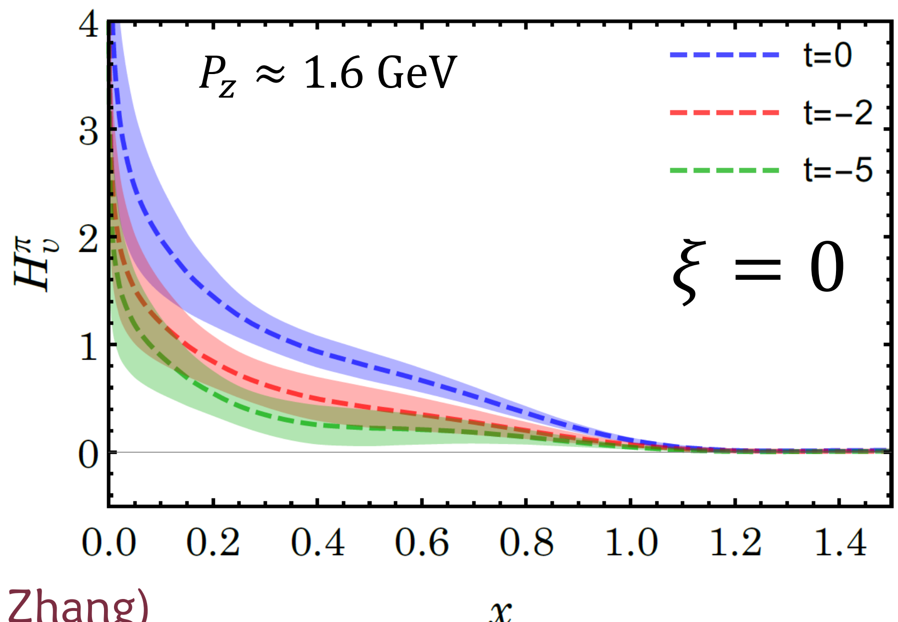
  $P_z \approx 1.3, 1.6$ GeV

  J. Chen, HL, J. Zhang, 1904.12376



$$H_q^{\pi}(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$





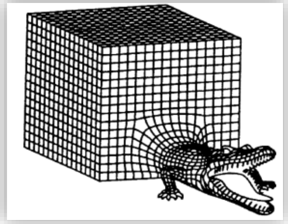
(plot by J. Zhang)

Isovector Nucleon GPDs

§ Pioneering first glimpse into nucleon GPD using quasi-PDFs

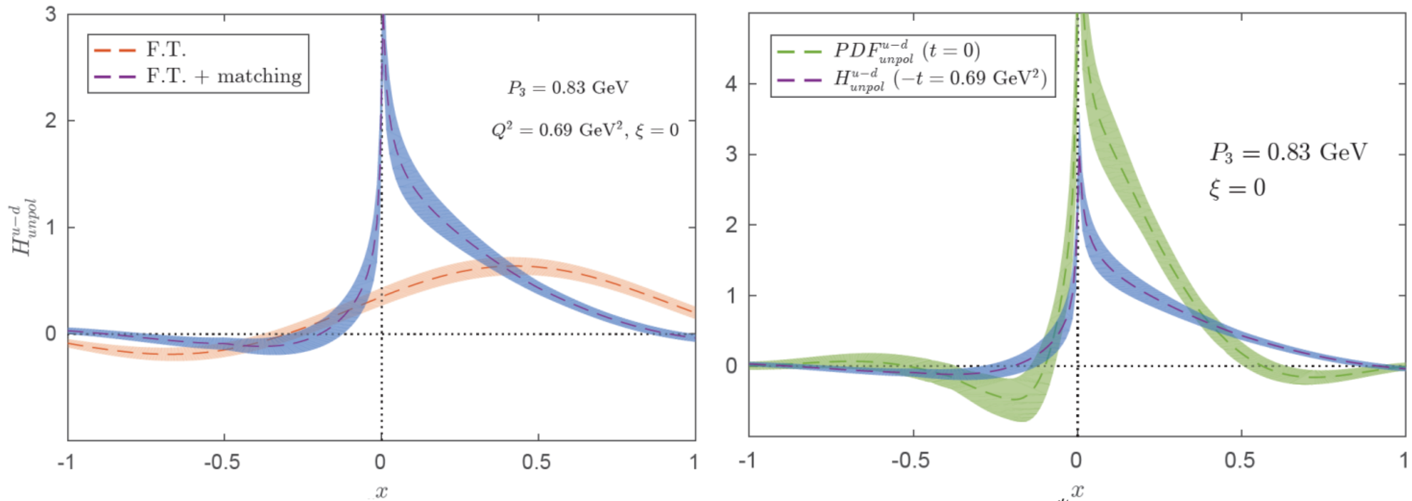
- Lattice details: twisted-mass fermions, 0.09fm, **270-MeV** pion mass, $P_z \approx 0.83$ GeV

$$F(x,\xi,t) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P' | O_{\gamma^+}(\zeta^-) | P \rangle = \frac{1}{2\bar{P}^+} \bar{u}(P') \left\{ H(x,\xi,t)\gamma^+ + E(x,\xi,t) \frac{i\sigma^{+\mu}\Delta_\mu}{2M} \right\} u(P)$$



nucleon $\xi = 0$ isovector results

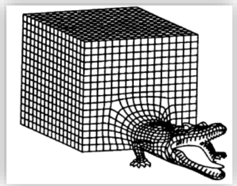
C. Alexandrou, (ETMC), 1910.13229 (Lattice 2019 Proceeding)



Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

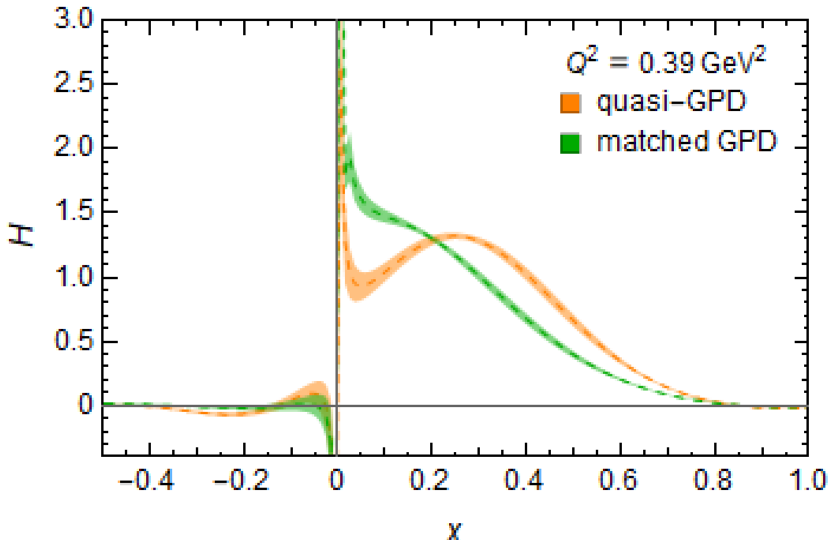
- Lattice details: clover/2+1+1 HISQ
  0.09fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

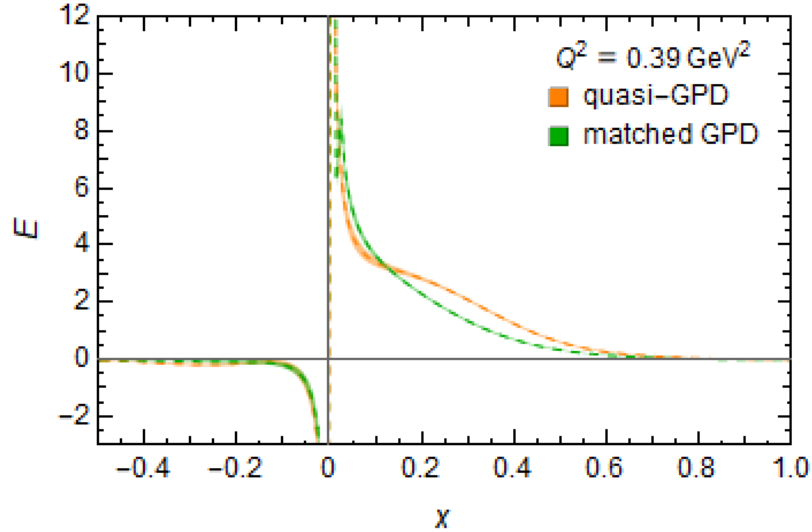


finite-volume,
discretization,
...

$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\tilde{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \tilde{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$



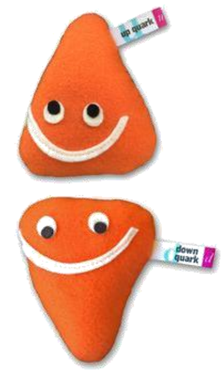


Isovector Nucleon GPDs

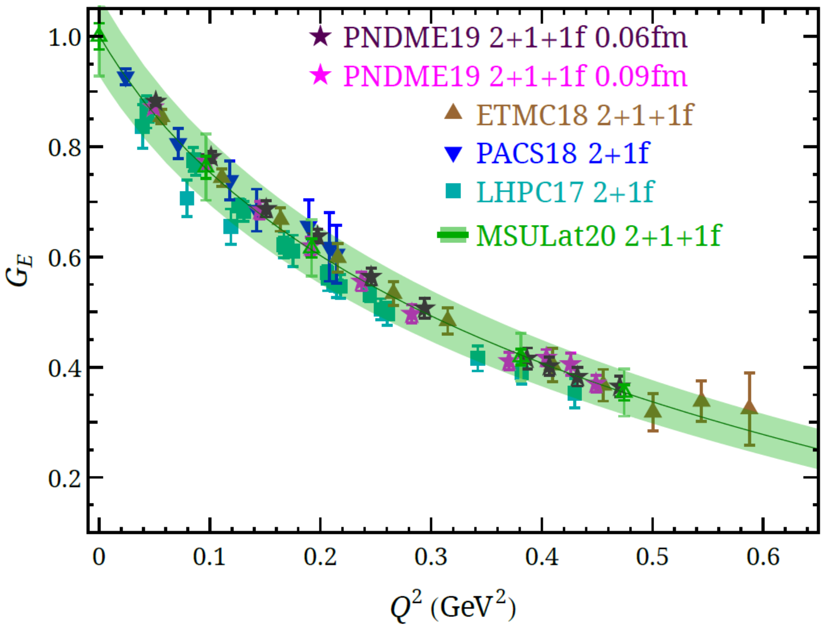
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} H^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t) \Big|_{n\text{ even}}$$



$n = 1$

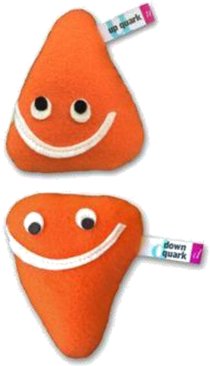


Nucleon GPDs

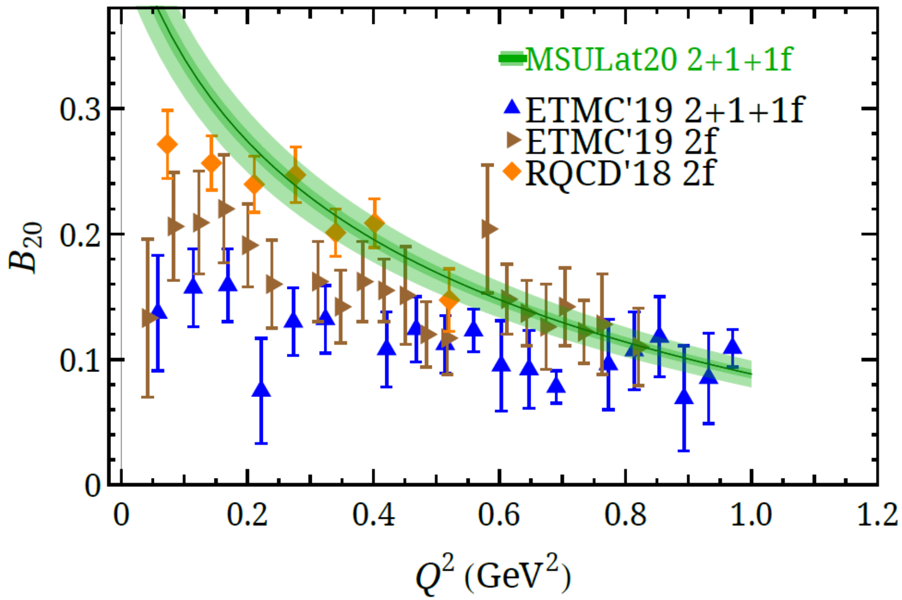
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$\int_{-1}^{+1} dx\, x^{n-1} E^q(x,\xi,t) = \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B^q_{ni}(t) - (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 2$

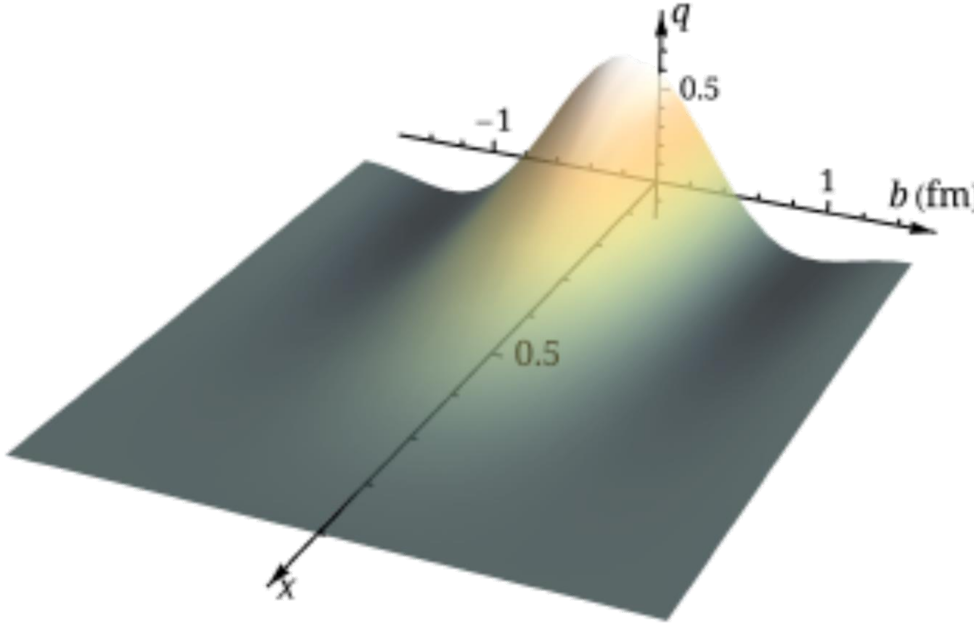


Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

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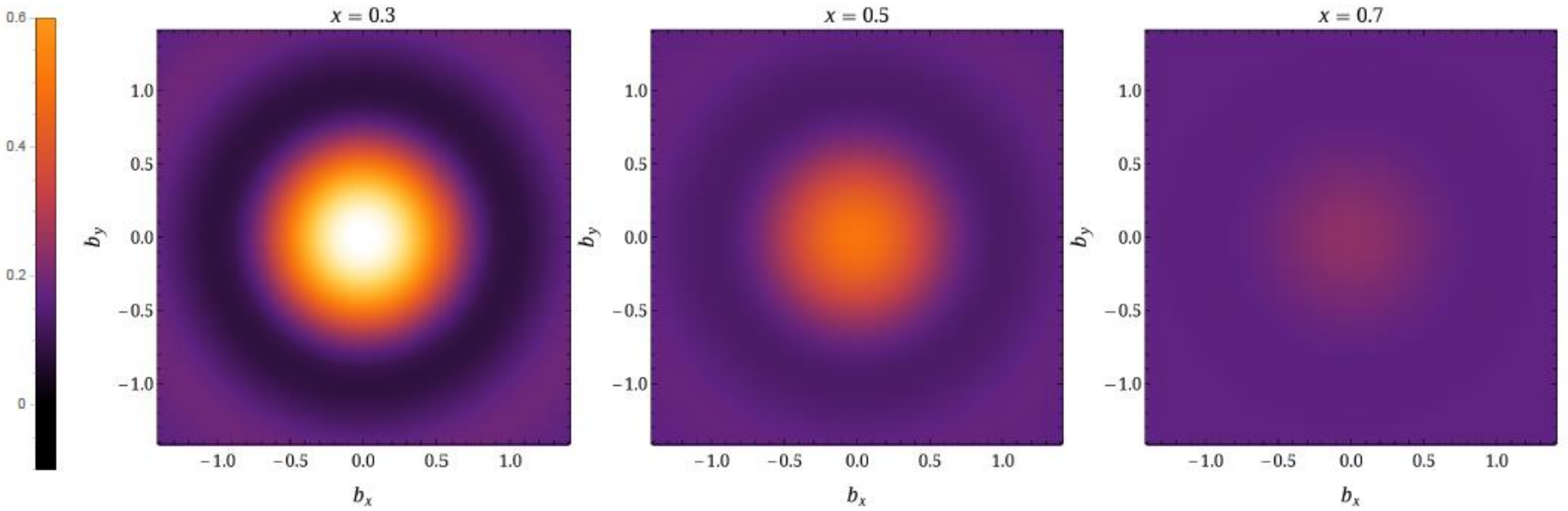

Summary

- Exciting era using LQCD to study nucleon structure
  - More nucleon matrix elements with physical pion masses
  - Well-studied systematics → precision structures
- Overcoming longstanding limitations of moment method
  - Bjorken-x dependence of parton distribution functions are widely studied with LaMET and its variants
  - Pioneer GPDs x-dependent structure in pion and nucleon
    - First nucleon tomography at physical pion mass results with $\xi = 0$
  - More study of systematics planned in the near future
- Stay tuned for many more exciting results from LQCD

Summit
@ORNL
IC@LANL

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