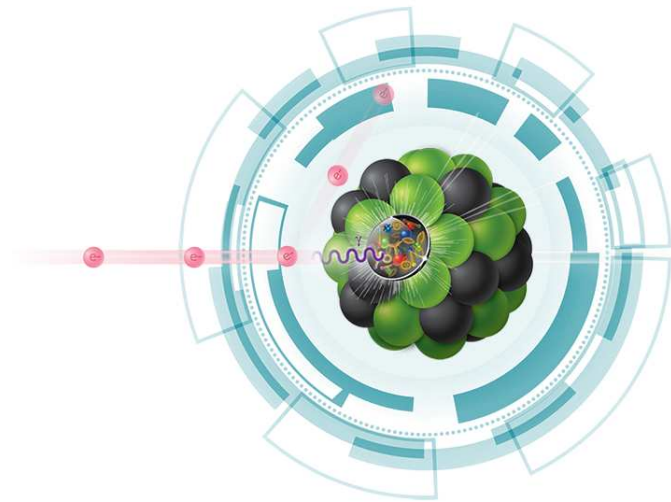


Weak mixing angle measurements at different Q^2

A. Freitas

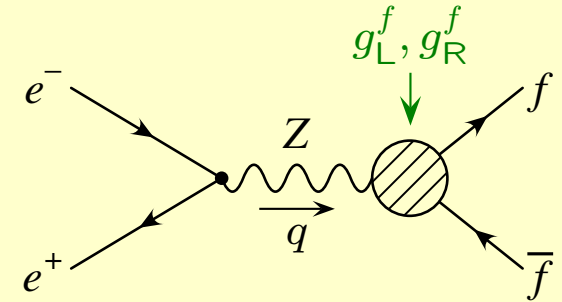
University of Pittsburgh



- Theory of $\sin^2 \theta_W$
- Measurement at Z pole
- Measurement at low energies
- Measurement at EIC

■ **On-shell** $s_w^2 \equiv M_W^2/M_Z^2$

■ **Effective** $\sin^2 \theta_{\text{eff}}^f \equiv \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)} \quad (q^2 = M_Z^2)$



■ **Off-shell effective** $\sin^2 \theta_{\text{eff}}^f$ for $q^2 \neq M_Z^2$
 → Not gauge invariant

■ **MS** $\sin^2 \bar{\theta}(\mu) \equiv \frac{\bar{g}_R^f(\mu)}{2|Q_f|(\bar{g}_R^f(\mu) - \bar{g}_L^f(\mu))}$

$$\sin^2 \theta_{\text{eff}}^f = s_w^2(1 + \Delta\kappa)$$

$$\sin^2 \theta_{\text{eff}}^f = \sin^2 \bar{\theta}(\mu)(1 + \Delta\bar{\kappa})$$

$$\Delta\kappa, \Delta\bar{\kappa} = \text{rad. corr.}, \quad \Delta\kappa(M_Z) \ll \Delta\bar{\kappa}$$

s_w^2	0.22337
$\sin^2 \theta_{\text{eff}}^f$	0.23153
$\sin^2 \bar{\theta}(M_Z)$	0.23121
$\sin^2 \bar{\theta}(0)$	0.23857

Effective field theory: $\mathcal{L}_{\text{BSM}} = \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \quad (\Lambda \gg M_Z)$

$$\sin^2 \theta_{\text{eff}}^f = s_W^2 \left(1 + \Delta \kappa_{\text{SM}} \underbrace{-\frac{g^2/4}{c_W^2 - s_W^2} \bar{c}_{\text{BW}}}_{\propto S \text{ parameter}} \underbrace{-\frac{c_W^2}{c_W^2 - s_W^2} \bar{c}_{\text{T}}}_{\propto T \text{ parameter}} - \frac{s_W^2}{2g_R^f} \bar{c}_L^f - \frac{s_W^2 g_L^f}{2(g_R^f)^2} \bar{c}_R^f \right)$$

$$\sin^2 \theta_{\text{eff}}^f = \sin^2 \bar{\theta}(M_Z) \left(1 + \Delta \bar{\kappa}_{\text{SM}} + \underbrace{-\frac{s_W^2}{2g_R^f} \bar{c}_L^f - \frac{s_W^2 g_L^f}{2(g_R^f)^2} \bar{c}_R^f}_{\propto S \text{ parameter}} \right)$$

$$\bar{c}_i = v^2 / \Lambda^2 c_i$$

$$\mathcal{O}_{\text{T}} = \frac{1}{2} (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi)$$

$$\mathcal{O}_{\text{BW}} = \Phi^\dagger B_{\mu\nu} W^{\mu\nu} \Phi$$

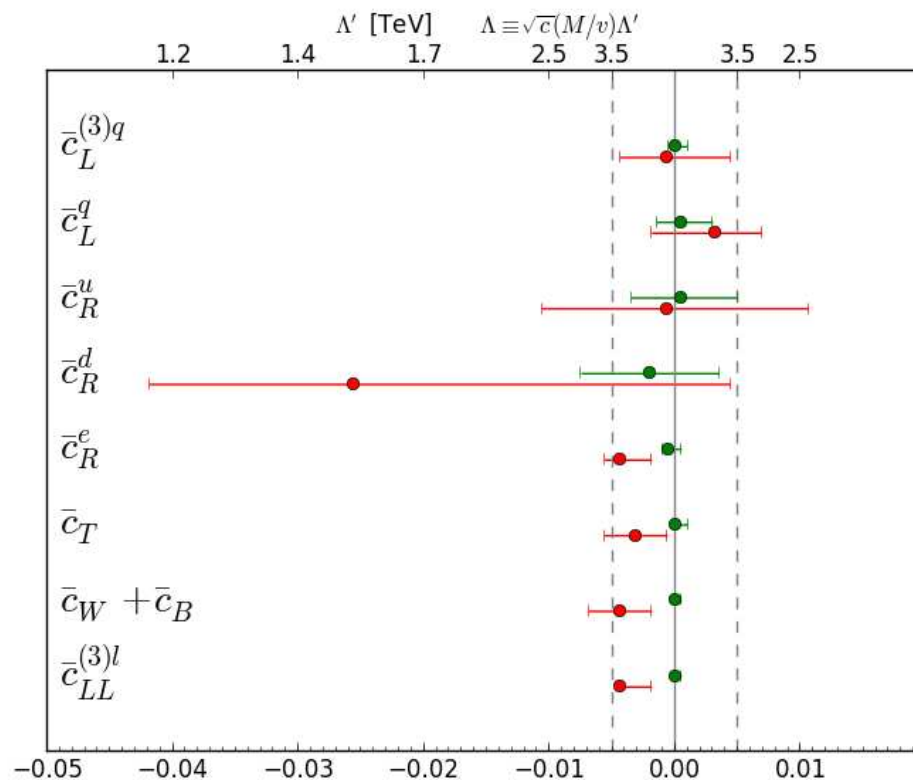
$$\mathcal{O}_{\text{R}}^f = i (\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{f}_{\text{R}} \gamma^\mu f_{\text{R}})$$

$$f = e, \mu, \tau, b, l, q$$

$$\mathcal{O}_{\text{L}}^F = i (\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) (\bar{F}_{\text{L}} \gamma^\mu F_{\text{L}})$$

$$F = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} u, c \\ d, s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

Assuming flavor universality:



$$\delta \sin^2 \theta_{\text{eff}}^f \sim \pm 10^{-4}$$

$$\Rightarrow \Lambda \sim 10 \text{ TeV}$$

Significant correlation/
degeneracy between
different operators

Pomaral, Riva '13
Ellis, Sanz, You '14

Forward-backward asymmetry:

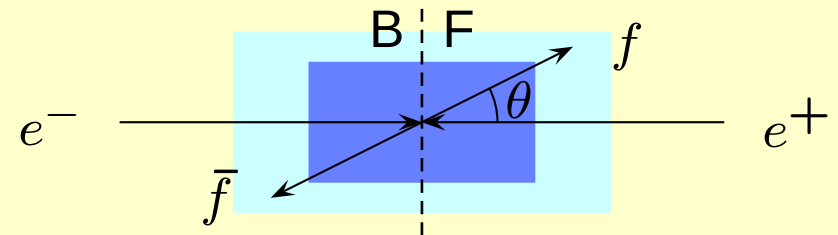
$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2 \theta_{\text{eff}}^f)}{1 + (1 - 4\sin^2 \theta_{\text{eff}}^f)^2}$$

$$\sin^2 \theta_{\text{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Main systematic uncertainties:

- For $f = b$: charge tagging, jet clustering
- For $f = \mu$: calibration of $\sqrt{s}$, muon angle



Left-right asymmetry:

Electron beam polarized with degree P_{e^-} :

$$A_{LR} = \frac{1}{P_{e^-}} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\mathcal{A}_\tau$$

Forward-backward asymmetry:

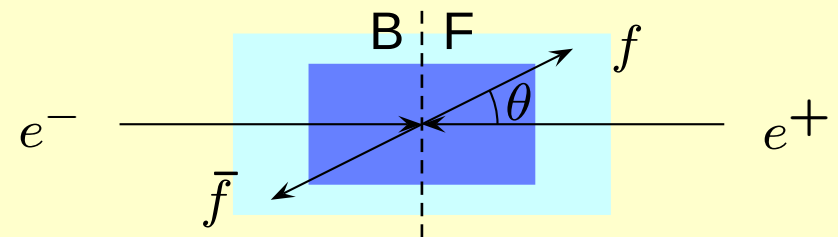
$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2 \theta_{\text{eff}}^f)}{1 + (1 - 4\sin^2 \theta_{\text{eff}}^f)^2}$$

$$\sin^2 \theta_{\text{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Main systematic uncertainties:

- For $f = b$: charge tagging, jet clustering
- For $f = \mu$: calibration of $\sqrt{s}$, muon angle



← most robust for future colliders

Left-right asymmetry:

Electron beam polarized with degree P_{e^-} :

$$A_{LR} = \frac{1}{P_{e^-}} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = -\mathcal{A}_\tau$$

■ Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85

Berends, Burgers, v. Neerven '88

Kniehl, Krawczyk, Kühn, Stuart '88

Beenakker, Berends, v. Neerven '89

Skrzypek '92

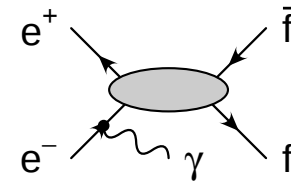
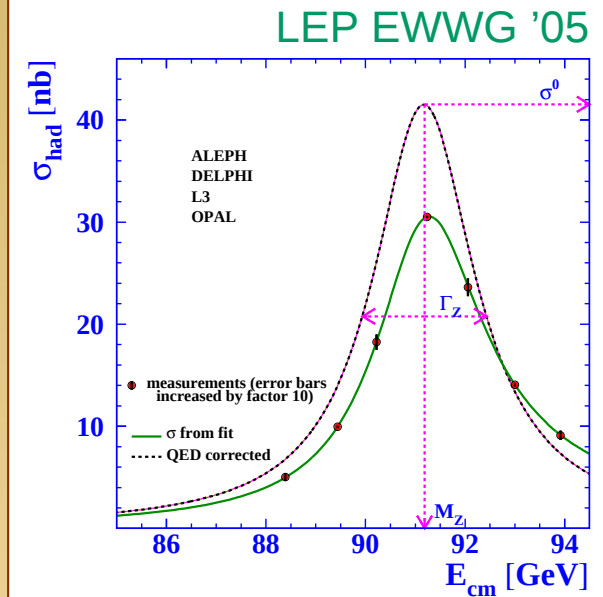
Montagna, Nicrosini, Piccinini '97

Soft photons (resummed) + collinear photons

$$\mathcal{R}_{\text{ini}} = \frac{\zeta(1 - s'/s)^{\zeta-1}}{\Gamma(1 - \zeta)} e^{-\gamma_E \zeta + 3\alpha L/2\pi} - \frac{\alpha}{\pi} L \left(1 + \frac{s'}{s}\right) + \alpha^2 L^2 \dots + \alpha^3 L^3 \dots$$

$$\zeta = \frac{2\alpha}{\pi} (L - 1)$$

$$L = \log \frac{s}{m_e^2}$$



- Deconvolution of initial-state QED radiation:

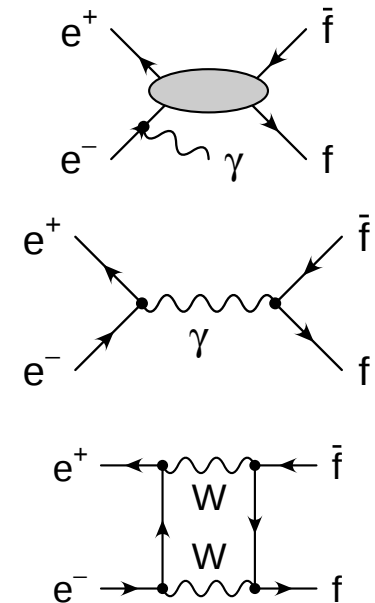
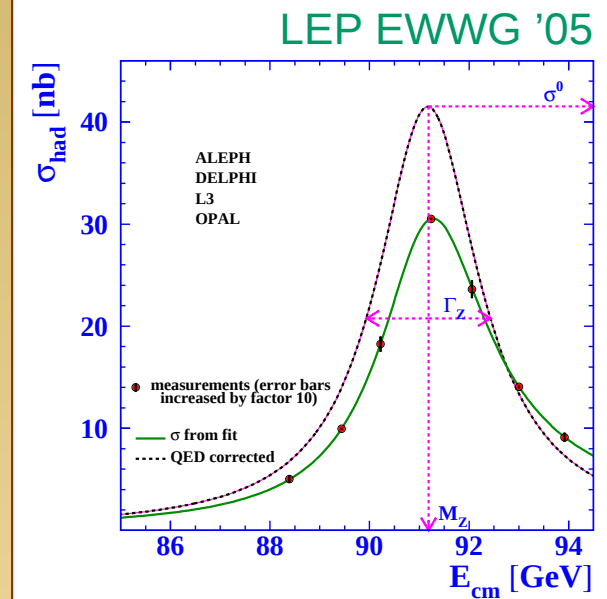
$$\sigma[e^+e^- \rightarrow f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ -exchange, γ -Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$

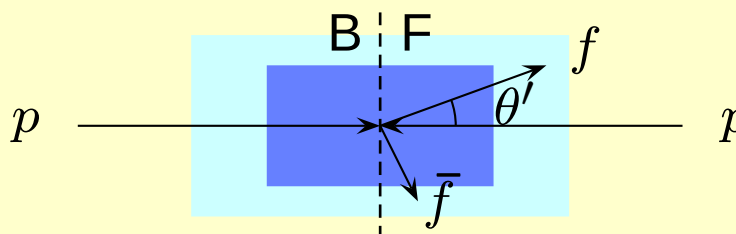
- Z-pole contribution:

$$\sigma_Z = \frac{R}{(s - \overline{M}_Z^2)^2 + \overline{M}_Z^2 \overline{\Gamma}_Z^2} + \sigma_{\text{non-res}}$$

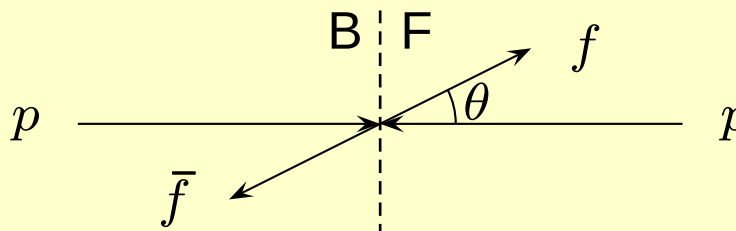


Forward-backward asymmetry: “forward” defined through overall boost
(valence quark typically has higher momentum than sea anti-quark)

lab frame:



center-of-mass frame:

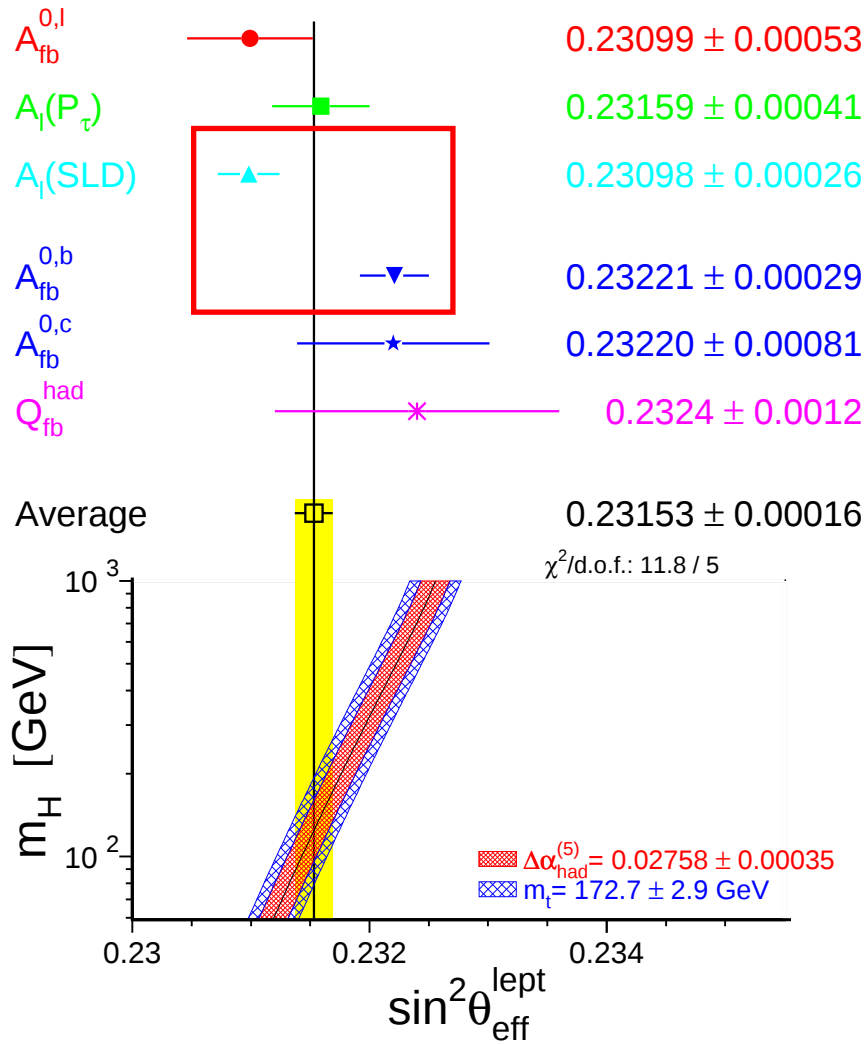


Main systematic uncertainties:

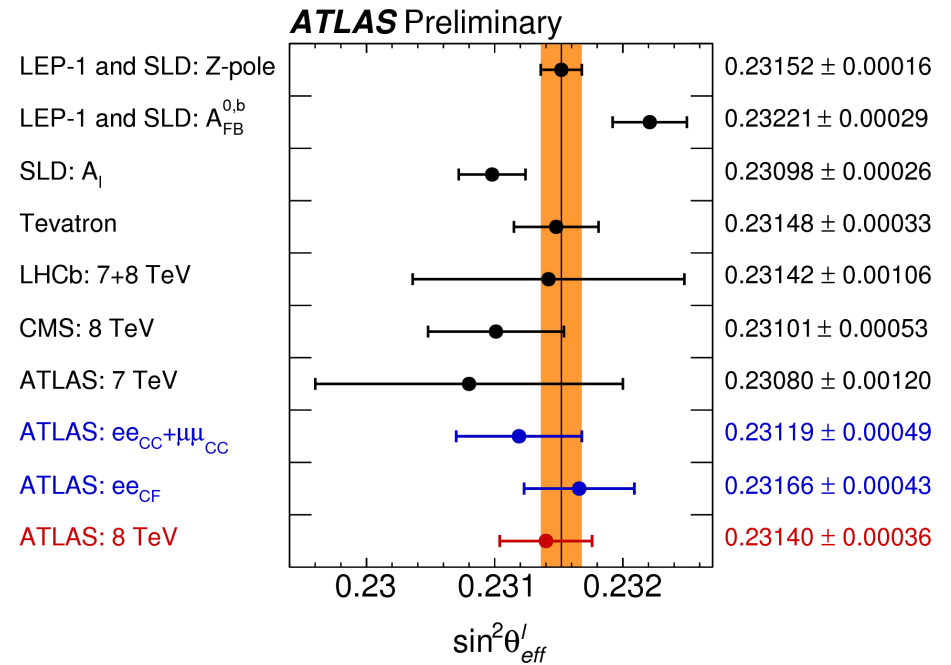
- PDFs
- QCD (QCD $\times$ EW) corrections

Current status of $\sin^2 \theta_{\text{eff}}^l$

9/23



LEP EWWG '05



ATLAS '18

	Current exp.	CEPC	FCC-ee
M_W [MeV]	15	1	1
Γ_Z [MeV]	2.3	0.5	0.1
$R_\ell = \Gamma_Z^{\text{had}}/\Gamma_Z^\ell$ [10^{-3}]	25	2	1
$R_b = \Gamma_Z^b/\Gamma_Z^{\text{had}}$ [10^{-5}]	66	4.3	6
$\sin^2 \theta_{\text{eff}}^\ell$ [10^{-5}]	13*	<1	0.5

* naive combination of LEP/SLC/TeV/LHC

- Improved measurements of several EWPOs necessary to improve global fit
- Will need 3-loop and partial 4-loop SM corrections

- Polarized ee , ep , ed scattering
($Q_W(e)$, $Q_W(p)$, eDIS)

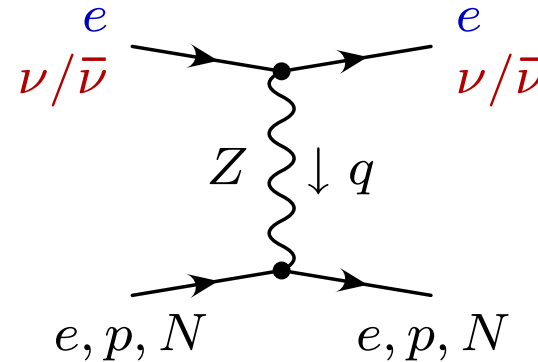
E158 '05; Qweak '17;
JLab Hall A '13

- $\nu N/\bar{\nu}N$ scattering NuTeV '02

- Atomic parity violation

($Q_W(^{133}\text{Cs})$) Wood et al. '97
Guéna, Lintz, Bouchiat '05

→ Test of running $\overline{\text{MS}}$ weak
mixing angle $\sin^2 \bar{\theta}(\mu)$,
 $\mu^2 \sim |q^2|$



$$g_{AV}^{ef} [\bar{e}\gamma^\mu\gamma_5 e] [\bar{f}\gamma_\mu f]$$

$$g_{VA}^{ef} [\bar{e}\gamma^\mu e] [\bar{f}\gamma_\mu\gamma_5 f]$$

$$g_{AV}^{ef} = \frac{1}{2} - 2|Q_f|\sin^2 \bar{\theta}(\mu)$$

$$g_{VA}^{ef} = \frac{1}{2} - 2\sin^2 \bar{\theta}(\mu)$$

- Polarized ee , ep , ed scattering
($Q_W(e)$, $Q_W(p)$, eDIS)

E158 '05; Qweak '17;
JLab Hall A '13

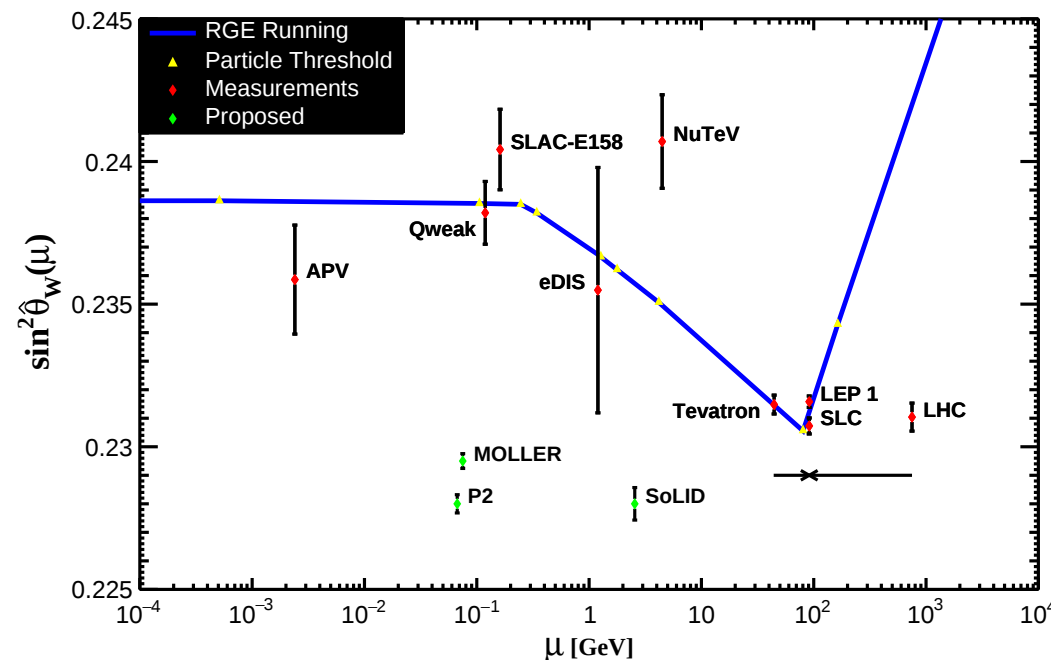
- $\nu N/\bar{\nu}N$ scattering NuTeV '02

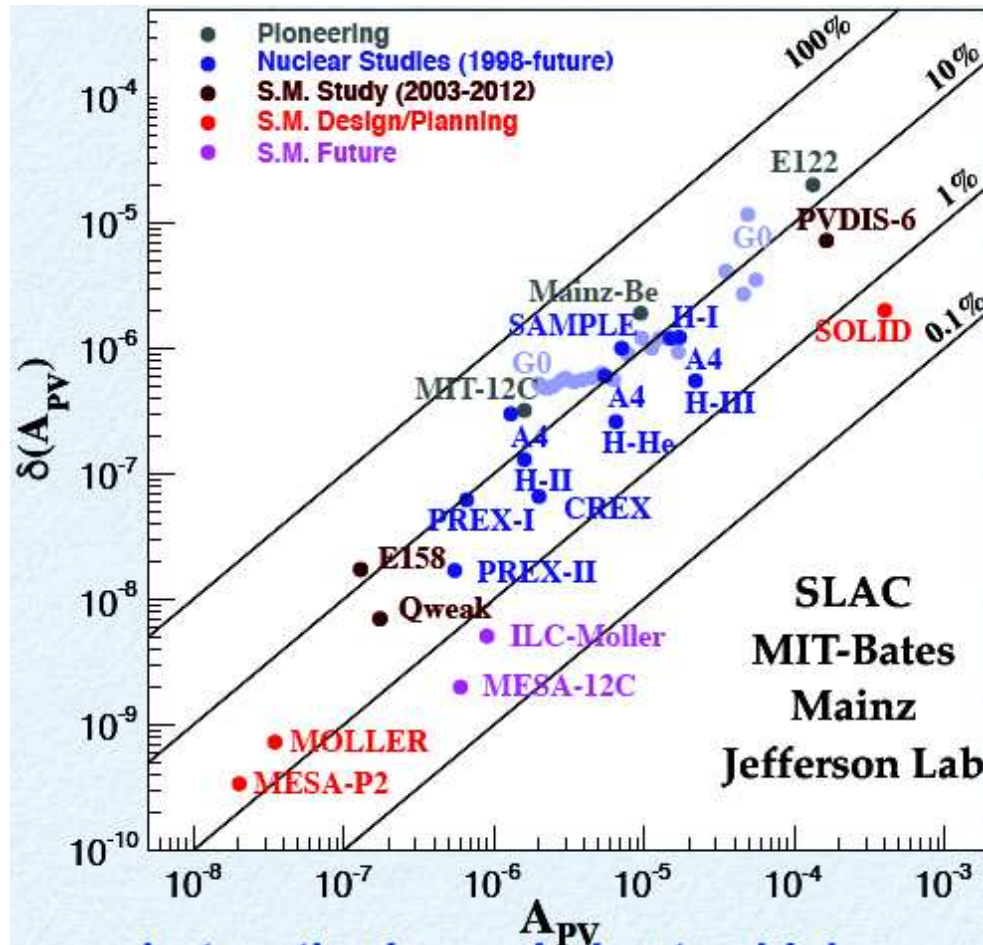
- Atomic parity violation

($Q_W(^{133}\text{Cs})$) Wood et al. '97
Guéna, Lintz, Bouchiat '05

→ Test of running $\overline{\text{MS}}$ weak
mixing angle $\sin^2 \bar{\theta}(\mu)$,
 $\mu^2 \sim |q^2|$

Erler, Ferro-Hernández '17





MOLLER experiment at JLab (ee):

$$\delta_{\text{exp}} A_{LR} = 0.73 \times 10^{-9} \text{ (2.4\%)}$$

$$\delta_{\text{exp}} \sin^2 \theta_W \sim 0.1\%$$

Current best ee (SLAC E158):

$$\delta_{\text{exp}} A_{LR} = 14\%$$

P2 experiment at MESA (ep):

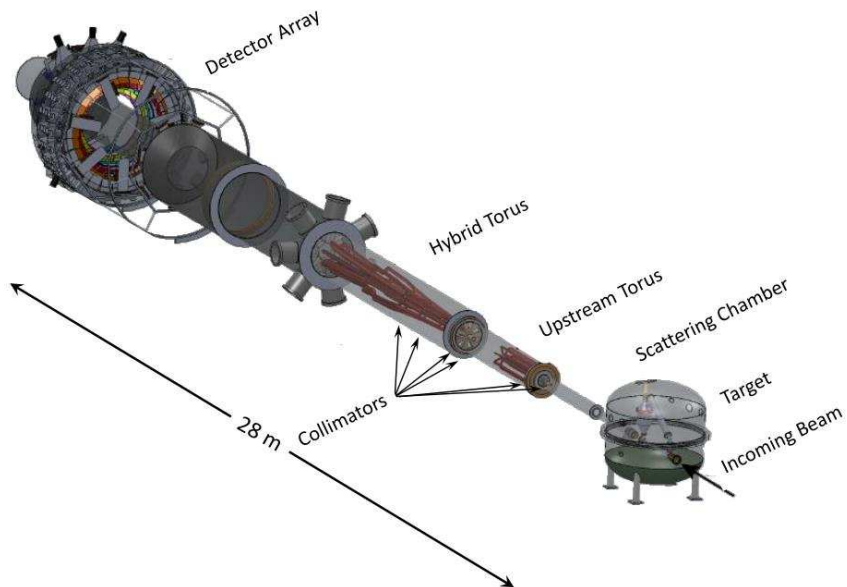
$$\delta_{\text{exp}} A_{LR} = 0.56 \times 10^{-9} \text{ (1.4\%)}$$

$$\delta_{\text{exp}} \sin^2 \theta_W \sim 0.1\%$$

Current best ep (Qweak):

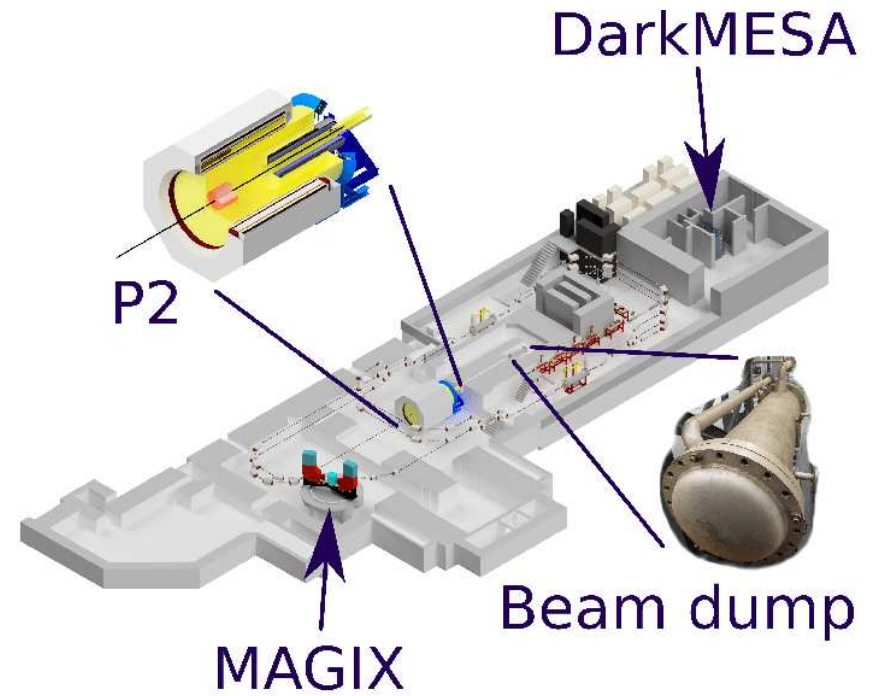
$$\delta_{\text{exp}} A_{LR} = 8\%$$

The MOLLER Experiment



10/2/2013

MENU 2013



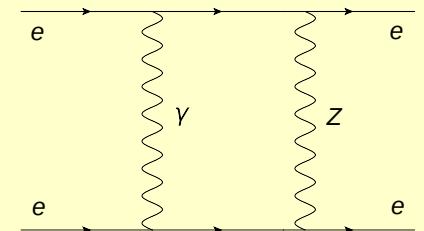
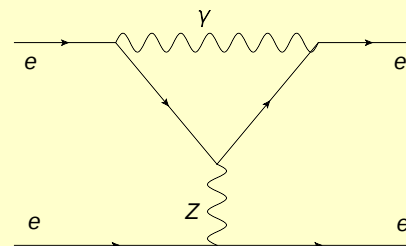
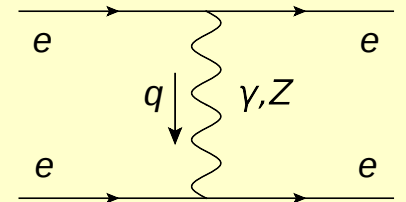
- Polarized e^- on $e^-/p/N$ target
- LR asymmetry for point-like target:

$$A_{LR}^{ee} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{G_\mu(-q^2)}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} Q_W(e)$$

$$A_{LR}^{ep} \approx \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{G_\mu(-q^2)}{4\sqrt{2}\pi\alpha} Q_W(p)$$

$$y \approx \frac{1}{2}(1 - \cos\theta_{CM})$$

- $Q_W(e) = Q_W(p) = 1 - 4\sin^2\theta_W$
- For low $Q^2 = -q^2$ proton is approx. point-like, but form factor corrections needed ($Q^2 \sim 0.005 \text{ GeV}^2$ at P2)
- Radiative corrections must be included:
 $1 - 4\sin^2\theta_W \rightarrow [1 - 4\kappa(\mu)\sin^2\bar{\theta}(\mu)] + \Delta Q(\mu)$



$$A_{\text{LR}} = \frac{\rho G_\mu q^2}{\sqrt{\pi} \alpha} \frac{-1 + y}{1 + y^4 + (1 - y)^4} \left[1 - 4\kappa(\mu) \sin^2 \bar{\theta}(\mu) + \Delta Q(\mu) \right]$$

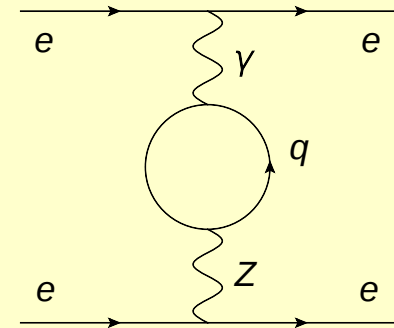
- From γ - Z self-energy:

$$\kappa(M_Z) = 1 - \frac{\alpha}{6\pi s^2} \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln \frac{m_f^2}{M_Z^2}$$

- Sensitivity to m_q : non-perturbative hadron physics

- $\kappa(0)$ is free of $\ln m_f^2$ terms

→ Absorbed into running of $\sin^2 \bar{\theta}(\mu)$



Determination of $\Delta_{\gamma Z} = \sum_f (I_{3f} Q_f - 2s^2 Q_f^2) \ln m_f^2 / M_Z^2$:

- a) Directly from e^+e^- data using reweighting of different flavors
[SU(3)_{u,d,s} symmetry,
pQCD for u, d, s at c, b thrsh.]

Wetzel '81; Marciano, Sirlin '84
Jegerlehner '86,17

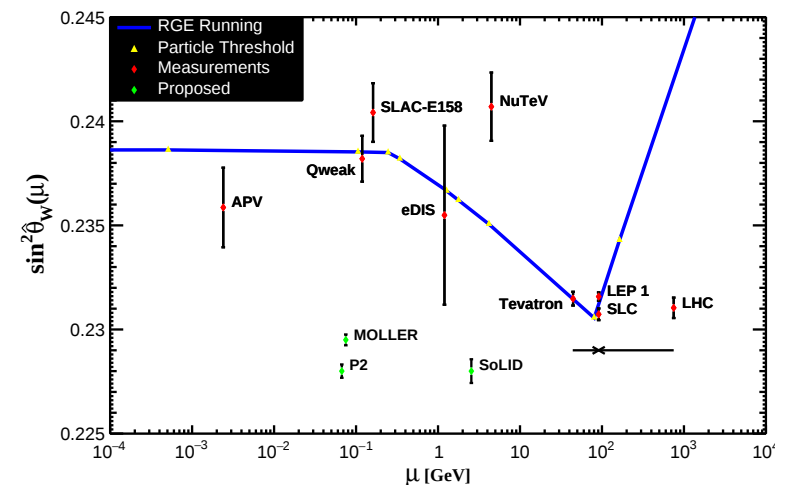
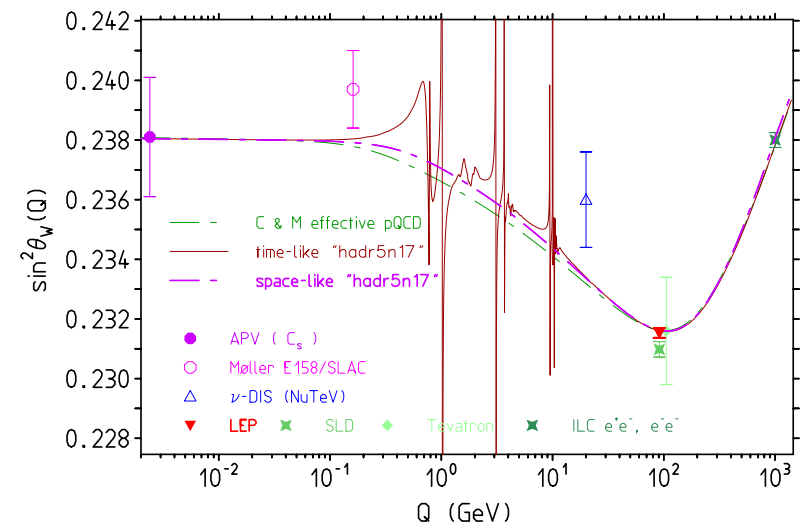
- b) Determine “threshold masses”

$\bar{m}_{u,d,s,c,b}$ from $\Delta\alpha(q^2)$;
pQCD RG running btw. thresholds

Erler, Ramsey-Musolf '04
Erler, Ferro-Hernández '17

- c) Lattice QCD

Otnad 'PVES 2018



- Polarized e^- on p for $Q^2 \gg \Lambda_{\text{QCD}}$

- LR asymmetry:

$$A_{\text{LR}}^{ep} \approx \frac{\sigma_{\text{L}} - \sigma_{\text{R}}}{\sigma_{\text{L}} + \sigma_{\text{R}}} = \frac{G_{\mu}(-q^2)}{4\sqrt{2}\pi\alpha} \left[\frac{F_1^{\gamma Z}}{F_1^{\gamma}} + (1 - 4\sin^2 \theta_{\text{W}}) \frac{y(1-y)}{1 + (1-y)^2} \frac{F_3^{\gamma Z}}{F_1^{\gamma}} \right]$$

$$y = 1 - E'_e/E_e$$

- DIS regime (rather than point-like as at P2):

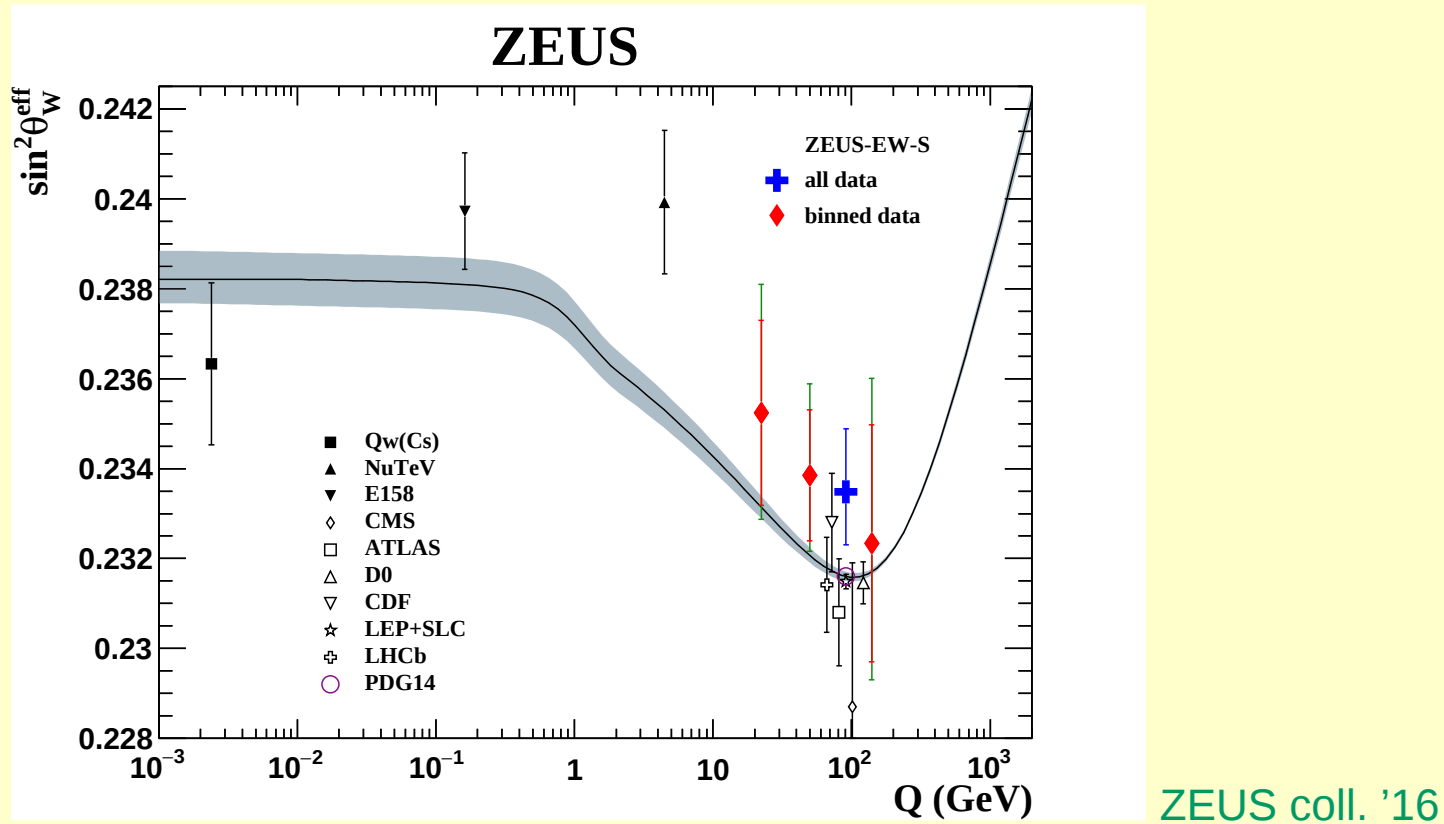
$$F_1^{\gamma} = \sum_q q q (f_q + f_{\bar{q}})$$

$$F_1^{\gamma Z} = \sum_q q q g_V^q (f_q + f_{\bar{q}})$$

$$F_1^{\gamma Z} = 2 \sum_q q q g_A^q (f_q + f_{\bar{q}})$$

→ Need precise knowledge of PDFs for $100 \text{ GeV}^2 \lesssim Q^2 \lesssim 5000 \text{ GeV}^2$

- Polarized e^- on p for $Q^2 \gg \Lambda_{\text{QCD}}$
- Result from HERA (0.3 fb $^{-1}$):



- Polarized e^- on d for $Q^2 \gg \Lambda_{\text{QCD}}$
- d is iso-singlet $\rightarrow$ PDF dependence approximately cancels in LR asymmetry:
- Assuming valence quark dominance and charge symmetry:

$$f_u \approx f_d,$$

$$f_{\bar{u}} \approx f_{\bar{d}} \approx f_{s,c,b} \approx f_{\bar{s},\bar{c},\bar{b}} \approx 0$$

$$A_{\text{LR}}^{ep} \approx \frac{G_\mu(-q^2)}{4\sqrt{2}\pi\alpha} \left[\frac{9}{5} - \sin^2 \theta_W + \frac{9}{5}(1 - 4\sin^2 \theta_W) \frac{y(1-y)}{1 + (1-y)^2} \right]$$

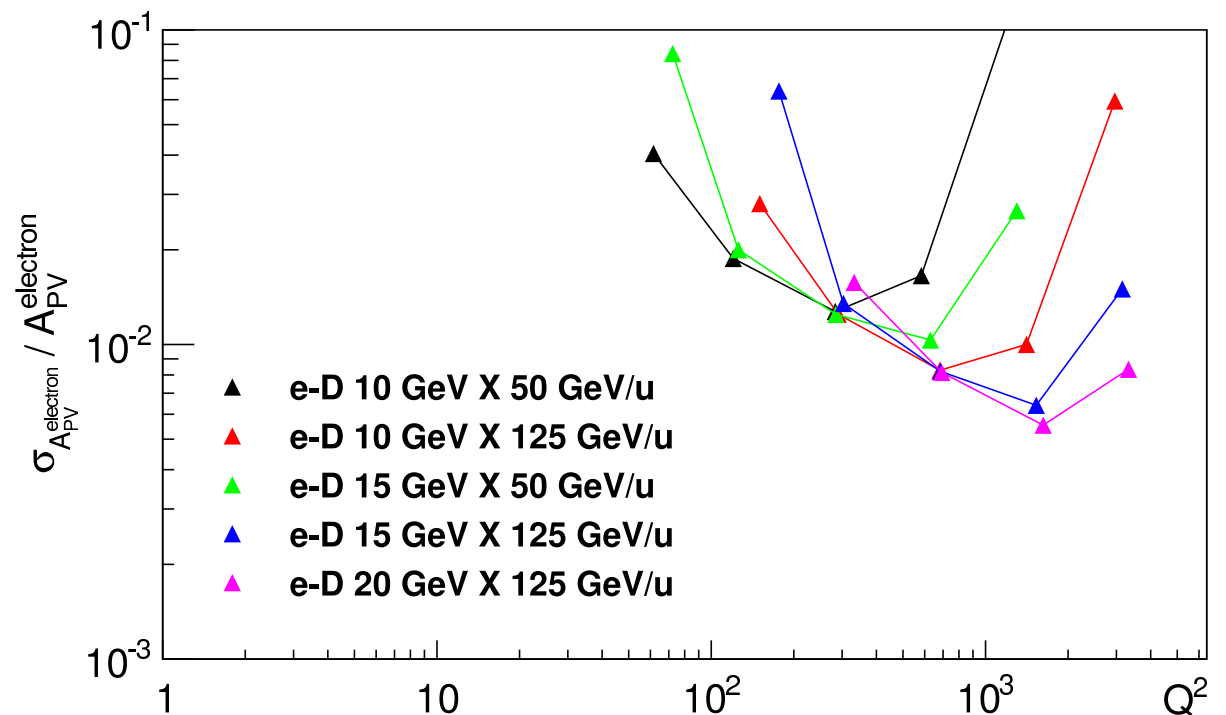
$\rightarrow$ Reduced need for precision PDF input

Simulation with QED and QCD radiative effects (DJANGO), CTEQ6.1M PDFs, and detector smearing
Zhao, Deshpande, Huang, Kumar, Riordan '16

Assumptions:

- Neglect higher-twist effects and PDF evolution in Q^2 range
- Differential luminosity uncertainty $< 10^{-4}$
- Polarization uncertainty $< 1\%$

$$\mathcal{L}_{\text{int}} = 267 \text{ fb}^{-1}$$



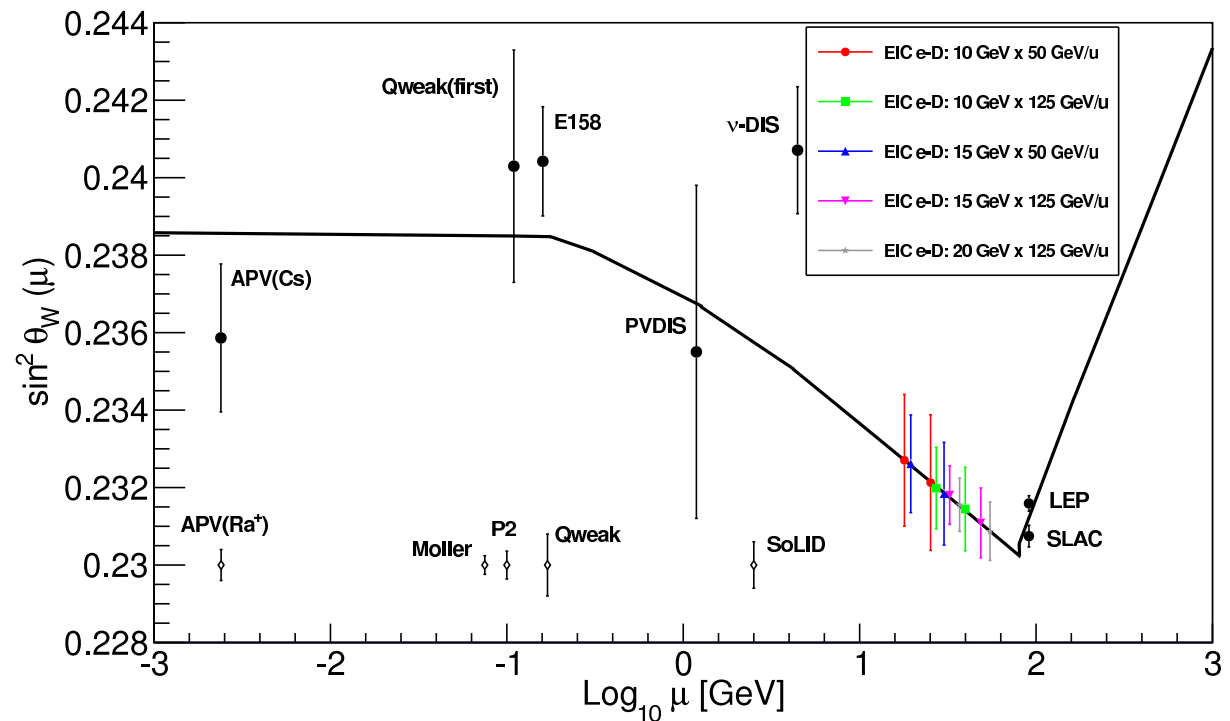
Simulation with QED and QCD radiative effects (DJANGO), CTEQ6.1M PDFs, and detector smearing
 Zhao, Deshpande, Huang, Kumar, Riordan '16

Assumptions:

- Neglect higher-twist effects and PDF evolution in Q^2 range
- Differential luminosity uncertainty $< 10^{-4}$
- Polarization uncertainty $< 1\%$

$$\mathcal{L}_{\text{int}} = 267 \text{ fb}^{-1}$$

(2 Q^2 bins)



- ⊕ EIC can determine $\sin^2 \bar{\theta}(\mu)$ in poorly explored range
 $10 \text{ GeV} \lesssim \mu \lesssim 70 \text{ GeV}$
 - Experimentally probe QCD running
 - May help to resolve tension between A_{FB}^b and A_{LR}^e
- ⊖ Precision is lower than at LEP, SLC, LHC, MOLLER, P2
 - No improved sensitivity to high-scale new physics

Open questions:

- PDF uncertainties (use EIC ep data as an input)
- Treatment of radiative corrections, non-perturbative corrections?

Backup slides

Z-pole asymmetries

Left-right asymmetry: (using polarization e^- beams)

$$A_{LR} \equiv \frac{1}{P_{e^-}} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \mathcal{A}_e + \Delta A_{\gamma Z} + \Delta A_{\gamma}$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2 \theta_{\text{eff}}^f)}{1 + (1 - 4\sin^2 \theta_{\text{eff}}^f)^2} \quad \sin^2 \theta_{\text{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Limited by systematic uncertainty of P_{e^-}
0.5% at SLD, 0.1% possible in future

Karl, List '17

Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available)

Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{LR} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$$\delta A_{LR} \approx 10^{-4} \quad \Rightarrow \quad \delta \sin^2 \theta_{\text{eff}}^{\ell} \approx 1.3 \times 10^{-5}$$

Mönig, Hawkings '99

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	$80.379 \pm 0.012 \text{ MeV}$	4 MeV	$\alpha^3, \alpha^2\alpha_s$
Γ_Z	$2495.2 \pm 2.3 \text{ MeV}$	0.4 MeV	$\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2$
R_ℓ	20.767 ± 0.025	0.005	$\alpha^3, \alpha^2\alpha_s$
R_b	0.21629 ± 0.00066	0.0001	$\alpha^3, \alpha^2\alpha_s$
$\sin^2 \theta_{\text{eff}}^\ell$	0.23153 ± 0.00016	4.5×10^{-5}	$\alpha^3, \alpha^2\alpha_s$

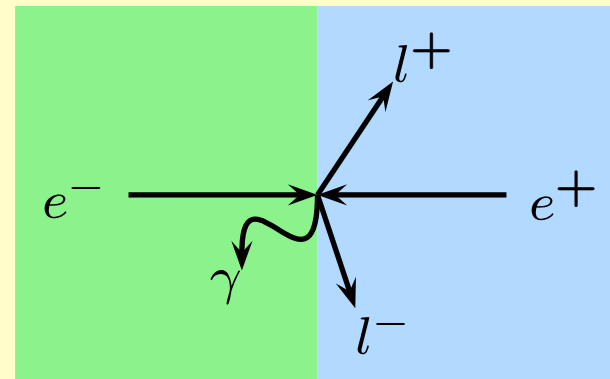
- Theory error estimate is not well defined, ideally $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
 - Count prefactors ($\alpha, N_c, N_f, \dots$)
 - Extrapolation of perturbative series
 - Renormalization scale dependence
 - Renormalization scheme dependence

QED radiation in Z asymmetries

QED radiation in principle cancels in asymmetries, e.g. $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$

Some effects from detector acceptance and cuts

Typical influence $< 10^{-3}$



Implementation of QED effects:

a) Analytical formulae, e.g. ZFITTER

Arbuzov, Bardin, Christova, Kalinovskaya, Riemann, Riemann, ...

→ exact $\mathcal{O}(\alpha^2)$ ISR/FSR corrections

b) Monte Carlo event generator, e.g. KORALZ, KKMC

Jadach, Ward, ...

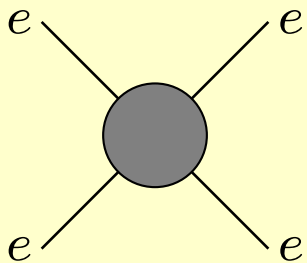
→ only $\mathcal{O}(\alpha^2 L)$ accuracy, but more flexible

$$L = \log(s/m_e^2)$$

Sensitivity to new physics scales

4-lepton operator

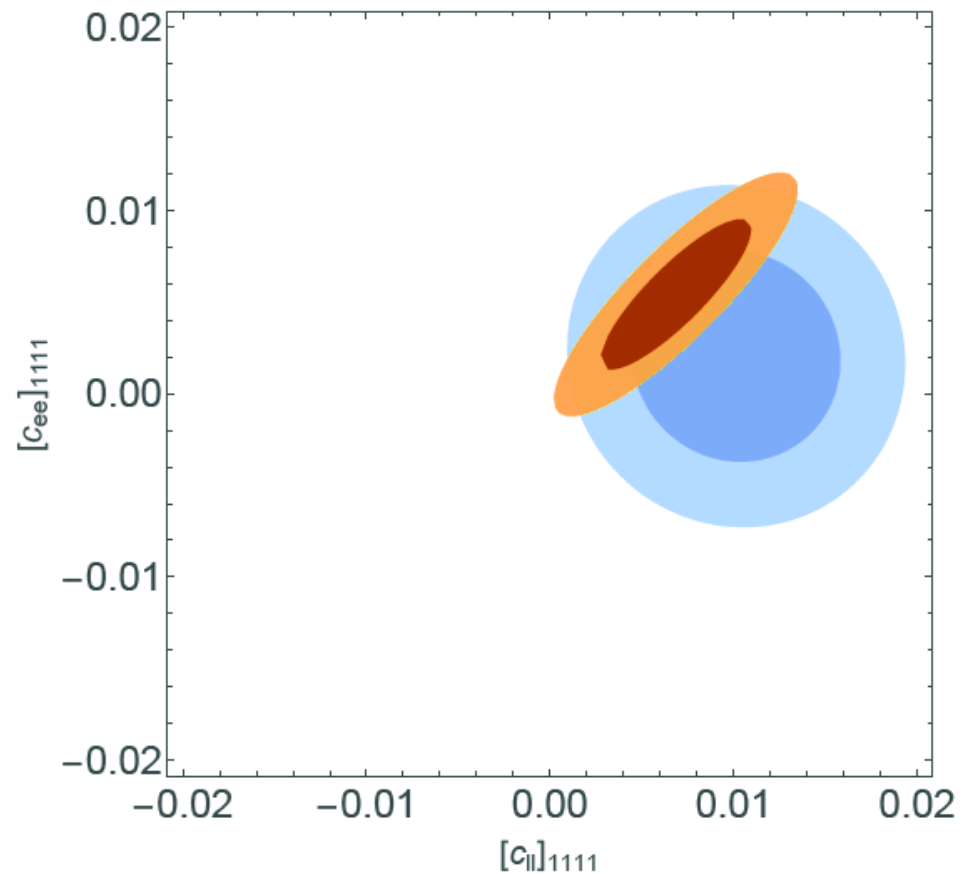
$$\frac{4\pi}{\Lambda^2} [\bar{e}\gamma^\mu\gamma_5 e] [\bar{e}\gamma_\mu e]$$



E158: $\Lambda \lesssim 17 \text{ TeV}$

MOLLER: $\Lambda \lesssim 39 \text{ TeV}$

Erler, Horowitz, Mantry, Souder '14



Falkowski, Gonzalez-Alonso, Mimouni '17

Falkowski et al. '18

$$\frac{c_{ee}}{v^2} [\bar{e}\gamma^\mu P_R e] [\bar{e}\gamma_\mu P_R e]$$

$$\frac{c_{\ell\ell}}{v^2} [\bar{e}\gamma^\mu P_L e] [\bar{e}\gamma_\mu P_L e]$$