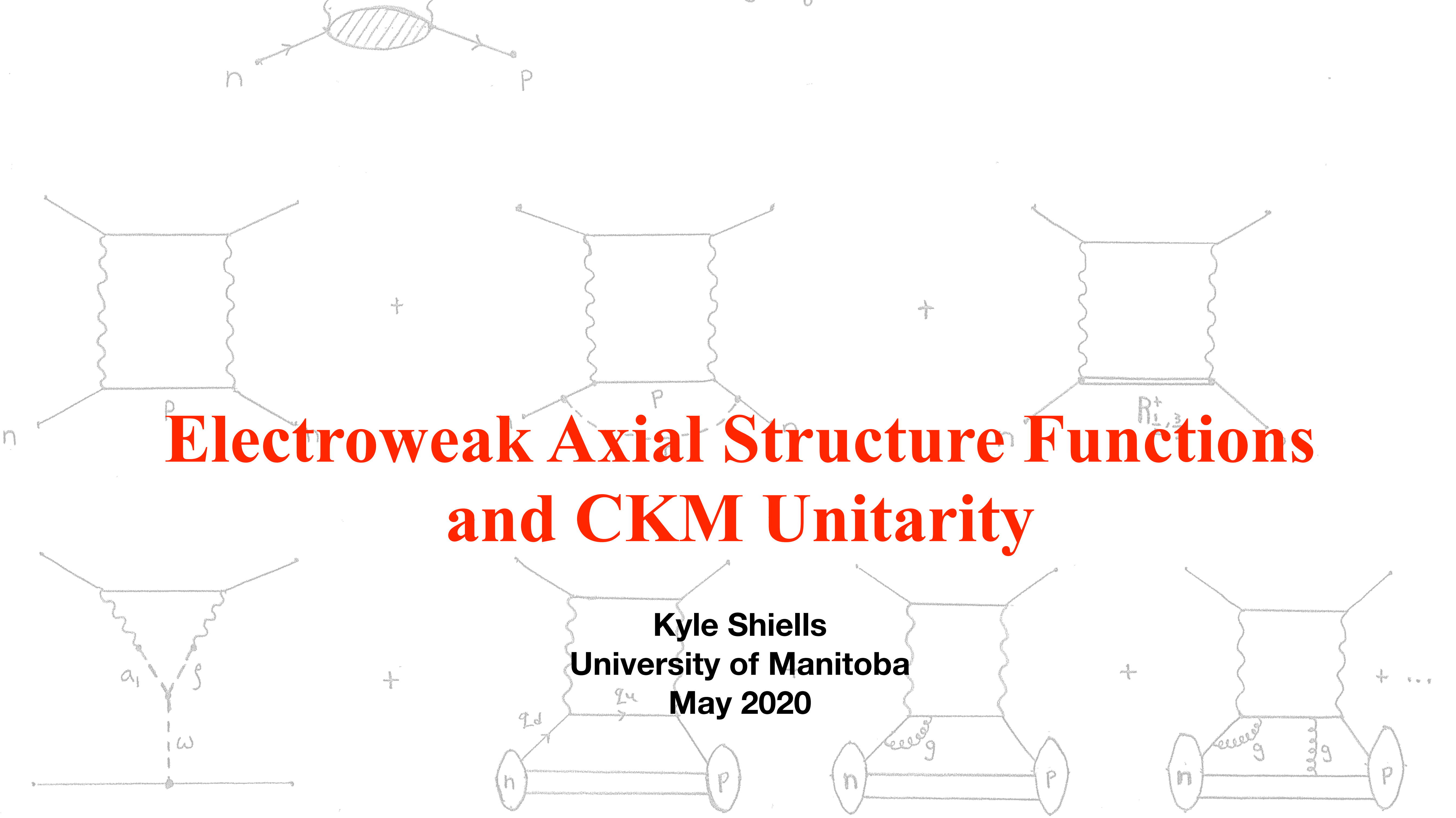

Electroweak Axial Structure Functions and CKM Unitarity

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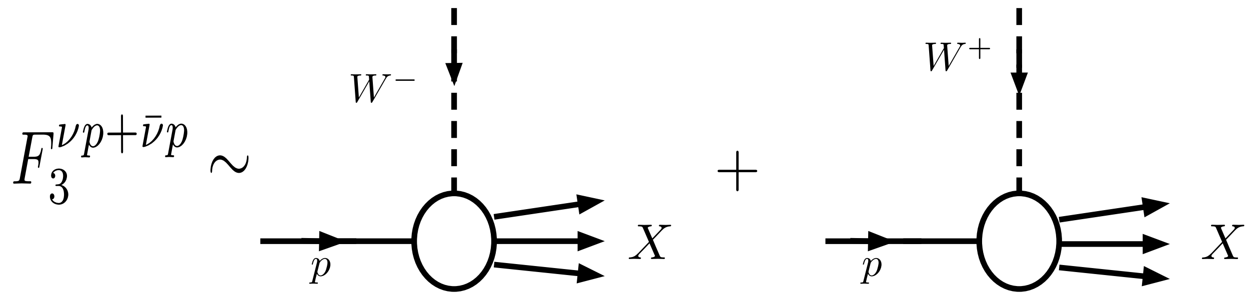
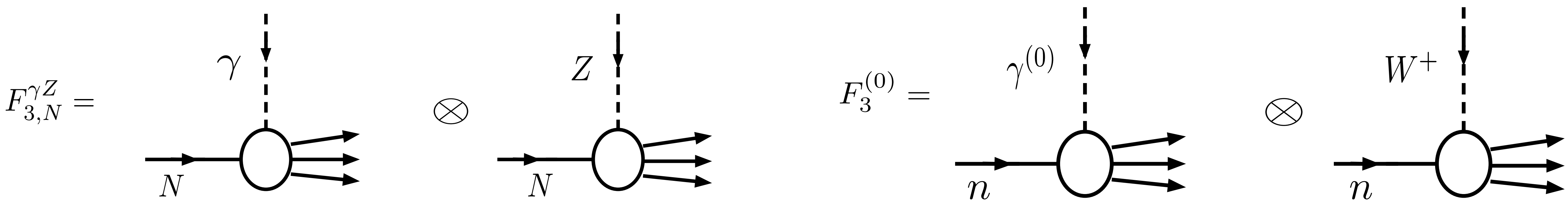
In collaboration with Peter Blunden (Manitoba) and Wally Melnitchouk (Jefferson Lab)

Axial Structure Functions





$$\boxed{\begin{aligned} F_3^{(0)} &= F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z} \\ F_3^{\nu p+\bar{\nu}p} &= F_{3,p}^{\gamma Z} + F_{3,n}^{\gamma Z} \end{aligned}}$$

$$F_3^{(0)} \equiv F_3^{\gamma^{(0)}W}$$

- 4 SF's, 2 equations ➡ **express any 2 SF's in terms of the other 2**
- all are constrained by PDFs at high Q, but only $F_3^{\nu p+\bar{\nu}p}$ has experimental constraints at low Q
- $F_{3,p}^{\gamma Z}$ has been previously modeled for the PV $\Box_A^{\gamma Z}$ used for Qweak.

CKM Unitarity

- The Yukawa couplings between the quarks and Higgs fields is allowed to mix generations.
- One can then perform a basis change of the quark generations to diagonalize those terms.
- The cost is that we complicate the charged current interaction:

$$\frac{-g}{\sqrt{2}}(\overline{u_L}, \overline{c_L}, \overline{t_L})\gamma^\mu W_\mu^+ V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \mathrm{h.c.} \qquad V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- The CKM matrix elements act like coupling constants between the W boson and two left-handed quarks of opposite isospin projection.
- Its unitarity means the sum of the squares of the top row elements is 1:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

largest term

contribution from superallowed Fermi beta decays

$$0^+ \to 0^+$$

extracted from

K_{l3} decays

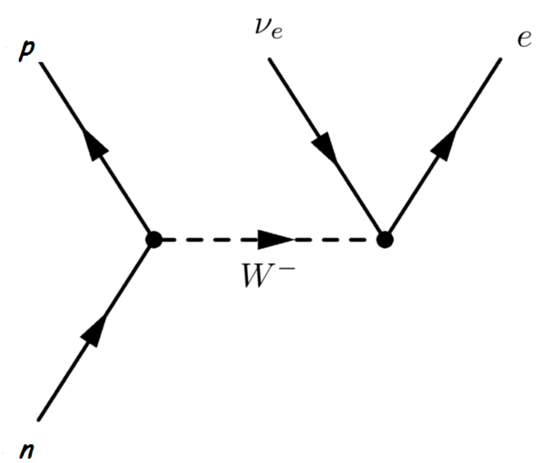
$$K \to \pi l \nu_l$$

extracted from

$$B \to \pi l \nu$$

1 Loop Effects on Superallowed Fermi Transitions:

Master formula relating V_{ud} to lifetime measurements:



$$\boxed{|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}}$$

Note: several universal RCs are common to both beta and muon decay and these cancel in

$$|V_{ud}|^2 \sim \frac{\text{beta decay}}{\text{muon decay}}$$

$\mathcal{F}t$ is a product of the statistical decay rate factor and decay lifetime and contains nuclear-dependent RCs

Nucleus-independent RCs: $\Delta_R^V = \frac{\alpha}{2\pi}\left[3\ln\frac{M_W}{M_p} - 4\ln c_W\right] + 2\Box_A^{\gamma W}$

axial current interaction

Sirlin 1978: $\Box_A^{\gamma W} = \frac{\alpha}{4\pi}\left[\ln\frac{M_W}{M_A} + 2C_{Born} + A_g\right]$

Marciano 2006: $\Box_A^{\gamma W} = \frac{\alpha}{8\pi}\int_0^\infty \frac{M_W^2}{Q^2+M_W^2}F(Q^2)dQ^2$

form factor models hadron

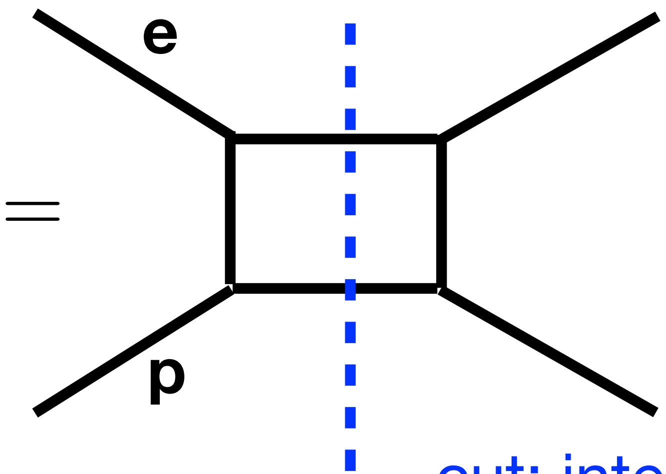
Dispersion Relations in QFT

Cutkosky Cutting Rule:

$\mathrm{Im}M(s) = -\frac{1}{2}\mathrm{Disc}M(s)$

$$M(s) = $$

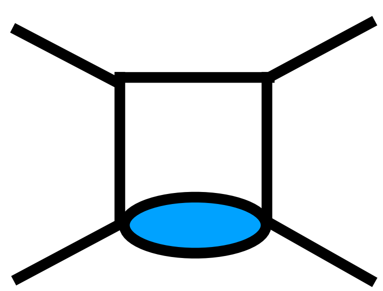
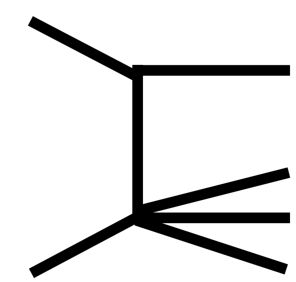
$\xrightarrow[s \geq m_p^2]{}$



cut: intermediate state energy

$(m_e + m_p)^2 \approx m_p^2$

- cutting the diagram at the intermediate state, placing the intermediate state virtual particles on their mass shell
- sum over all possible phase space of these on shell particles

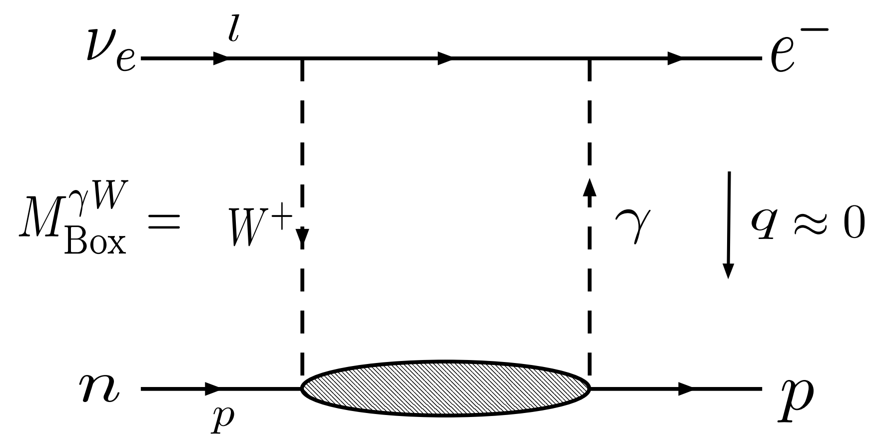
Im  $\overset{\text{forward limit}}{=}$ $\int d\Pi_{LIPS}$ $\left| \right.$  $\left. \right|^2$

Optical Theorem

Cauchy's Integral theorem: $\square(s_0) = \frac{1}{2\pi i}\oint_\Gamma \frac{\square(s)}{s-s_0}ds$

If a function has an analytical structure in the complex plane, application of the Cauchy integral theorem using an appropriate contour can yield a Dispersion Relation.

Dispersion Relation for the γW Box



$$M^{\rm Box}_{\gamma W}|_{\rm fwd} = \frac{-ig^2 e^2}{2M_W^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2(1-k^2/M_W^2)}\bar{u}_e(l)\gamma_\lambda \frac{\not{k}-\not{l}+m_e}{(l-k)^2-m_e^2}\gamma_\rho P_L u_\nu(l) T^{\lambda\rho}_{(\gamma)}(k)$$

Numerator can be written as: $L^{\gamma W}_{\mu\nu} H^{\mu\nu}_{\gamma W}$

For on-shell states, hadronic tensor involves structure functions:

$$H^{\mu\nu}_{\gamma W} = 4\pi\left[\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right)F_1^{\gamma W} + \frac{p^\mu p^\nu}{p\cdot k}F_2^{\gamma W} + \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha k_\beta}{2p\cdot k}F_3^{\gamma W}\right]$$

only need axial piece

The axial part of the gW box correction is odd with respect to the neutrino's incident energy *E*:

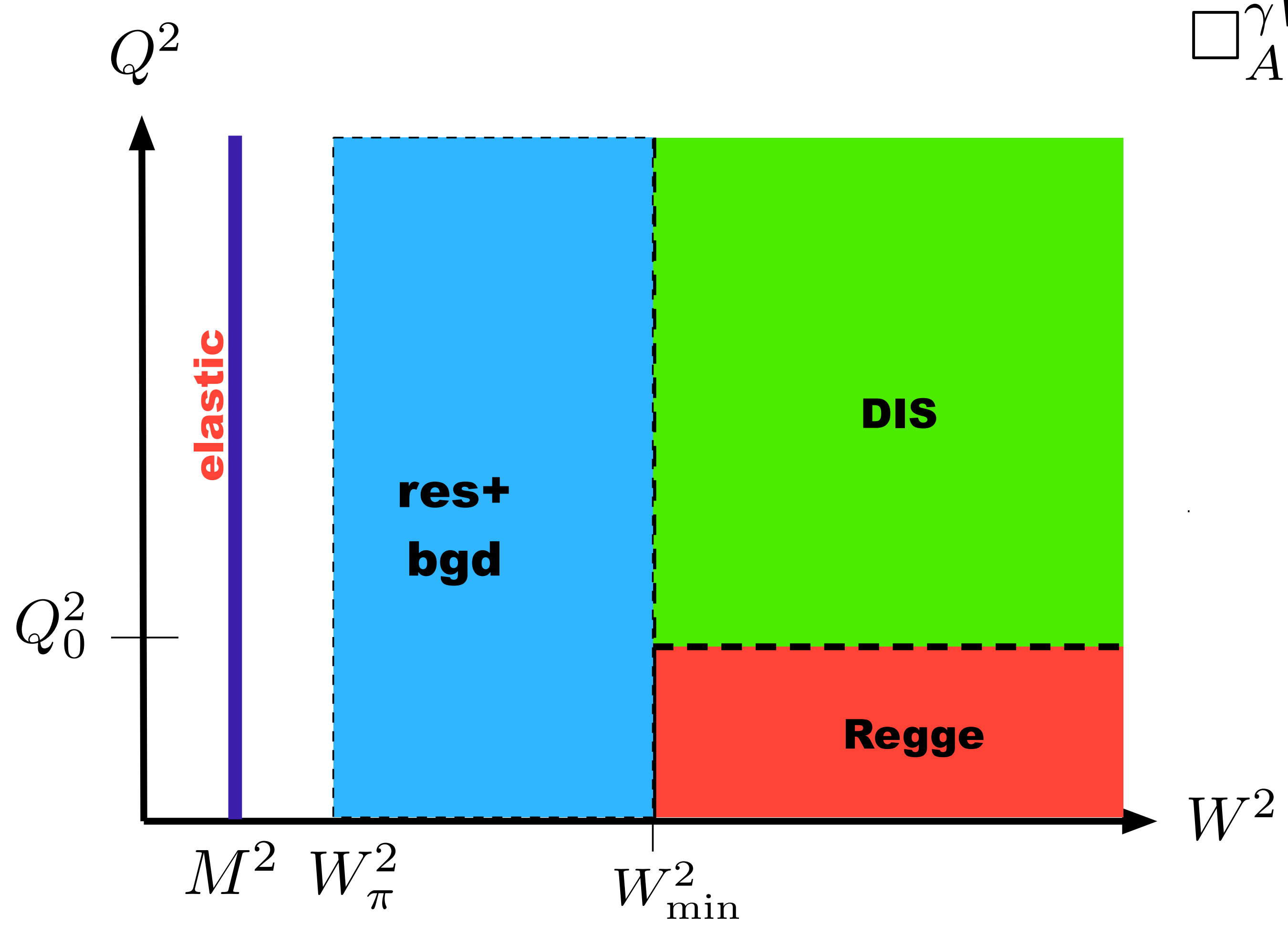
$$\Rightarrow \mathrm{Re}\Box^{(A)}_{\gamma W}(E) = \frac{2}{\pi}\int_{\nu_\pi}^{\infty} dE' \frac{E'}{E'^2-E^2}\mathrm{Im}\Box^{(A)}_{\gamma W}(E')$$

$$\boxed{\Box^{(A)}_{\gamma W} = \frac{\alpha}{2\pi}\int_{W_\pi^2}^{\infty} dW^2 \int_0^\infty dQ^2 \frac{F_3^{\gamma W}(W^2,Q^2)}{1+Q^2/M_W^2}\frac{1}{ME_{min}}\left(\frac{2}{\chi} - \frac{1}{4ME_{min}}\right)}$$

Depends on knowledge of the F_3 structure function at all W^2 and Q^2.

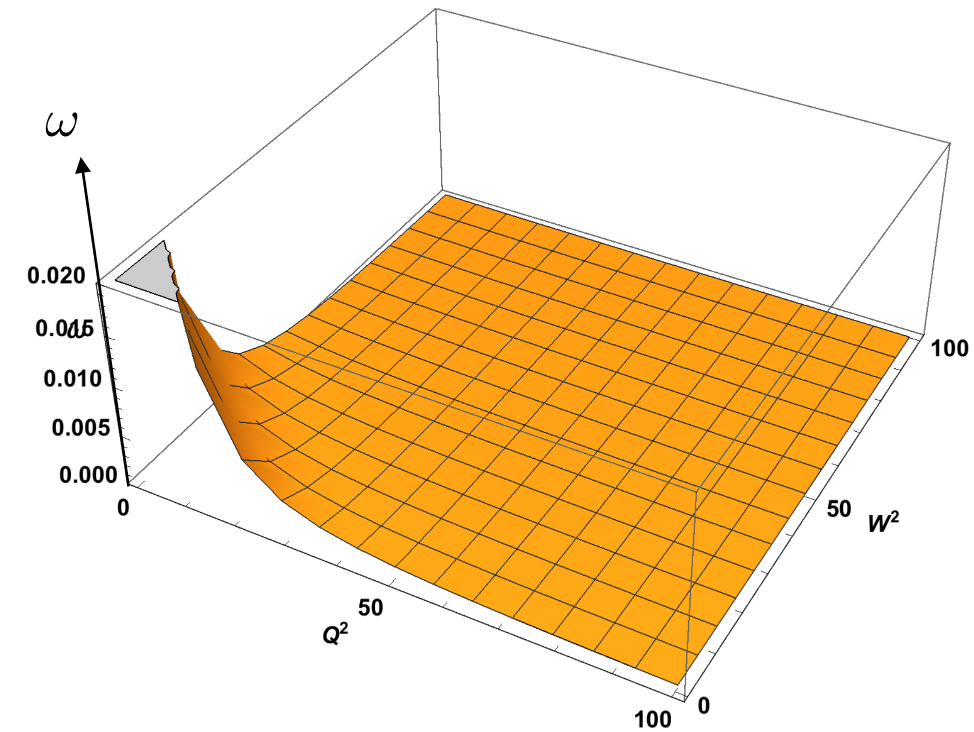
Kinematical Regions of $F_3^{\gamma W}$

- One needs different models for F_3 for different regions in the plane.
- The dispersion weight favours small W^2 and Q^2.
- The structure function should be continuous at the (moveable) boundaries, and the final box correction insensitive to their choice.

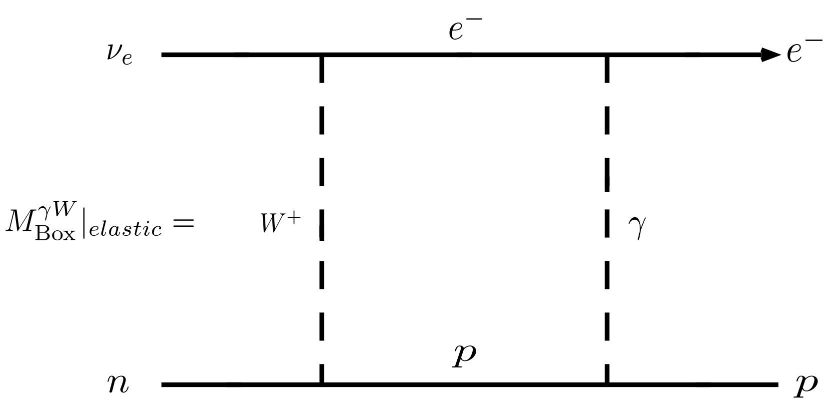


$$\Box_A^{\gamma W} \sim \int\int dW^2 dQ^2 \omega(Q^2, W^2) F_3^{\gamma W}$$

Integrand weight function



Elastic Contribution:



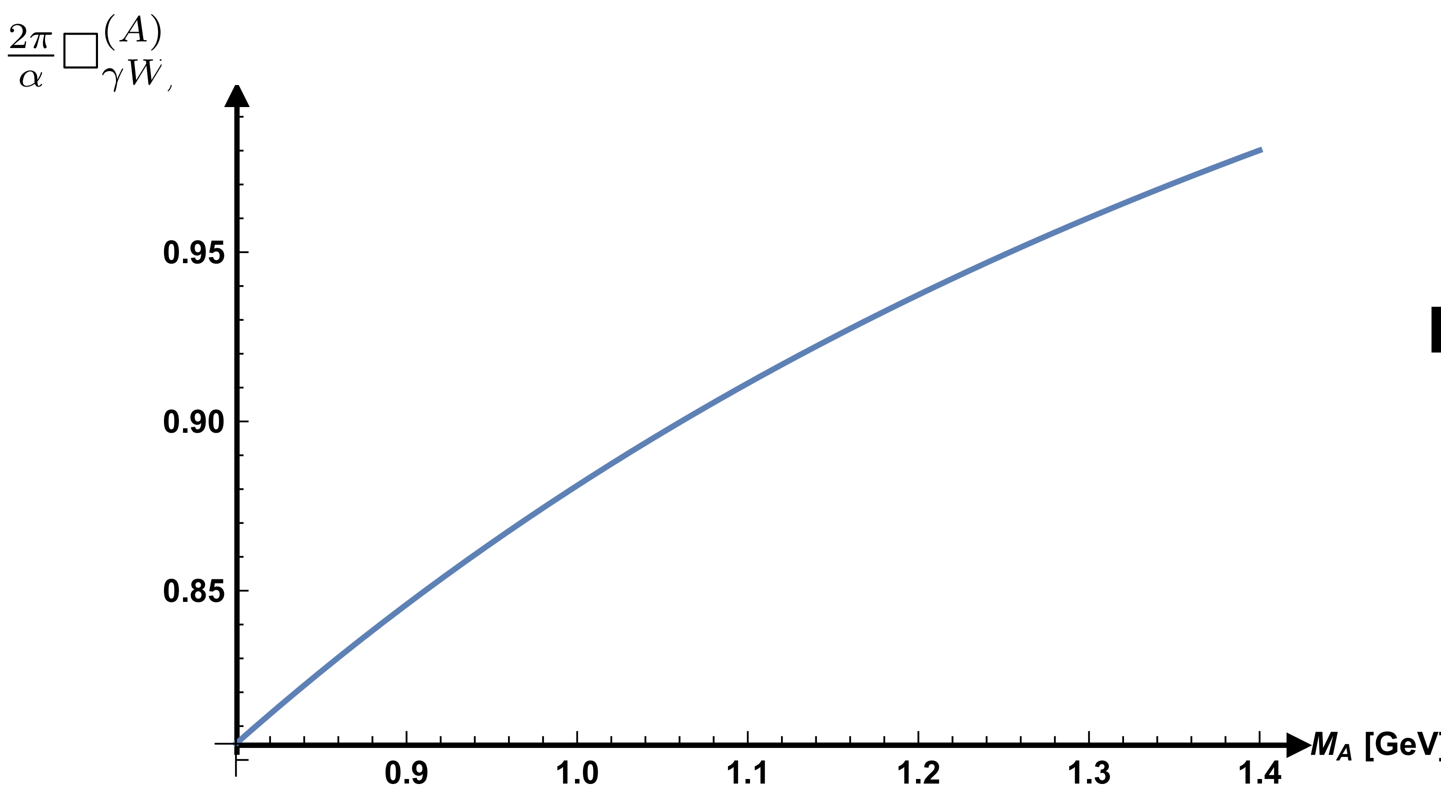
$$\Box^{\gamma W}_{A,el} = \frac{\alpha}{2\pi}\int_0^\infty dQ^2 \frac{[G_M^p(Q^2)+G_M^n(Q^2)]}{Q^2(1+Q^2/M_W^2)} \frac{g_A}{(1+Q^2/M_A^2)^2} \frac{1+2\sqrt{1+4M^2/Q^2}}{(1+\sqrt{1+4M^2/Q^2})^2}$$

$$g_A = 1.2723 \qquad\qquad M_A = 1.05(10)\ \text{GeV}$$

G_M^p, G_M^n taken from Ye, Arrington & Hill 2018 data

$$\Rightarrow \Box^{(A)}_{\gamma W,el} = (0.8967 \pm 0.0684)\frac{\alpha}{2\pi} = \boxed{1.04(6)\times 10^{-3}}$$

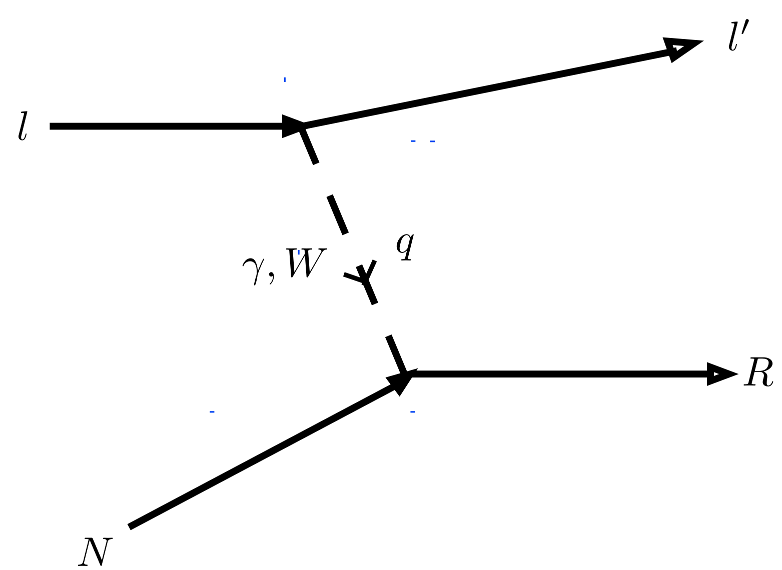
Important note: the dispersion treatment doesn't add anything new to the loop calculation of the elastic contribution



Box correction is sensitive to the axial mass parameter!

Resonance Contributions:

Origin: the first exchanged vector boson in the box diagrams can excite the neutron into an excited resonance state



the photon has an isovector (V) and an isoscalar (S) component
W boson only has isovector (V)

BOX + XBOX $\Rightarrow$ only (S) part of photon survives

$\Rightarrow$ only I=1/2 resonances are allowed

$$R = P_{11}(1440), D_{13}(1520), S_{11}(1535), ...$$

We can use the Lalakulich or MAID helicity amplitudes to find F_3 from these resonances.
example: Lalakulich D_{13}

$$F_3^{\gamma W}(D_{13}) = -\frac{4\nu}{3M}\left[-C_4^S(Q^2-\nu M)+C_5^S\nu M+C_3^S\frac{M}{M_R}(2M_R^2-2MM_R+Q^2-\nu M)\right]C_5^A\Gamma_R(W,M_R)$$

Using MAID:

Resonance	$\Box_{A,res}^{\gamma W}(\times 10^{-3})$
D_{13}	0.054
P_{11}	-0.009
S_{11}	-0.002
total	0.04

$C_i^{A,S}$ are form factors found from scattering data

DIS Contribution:

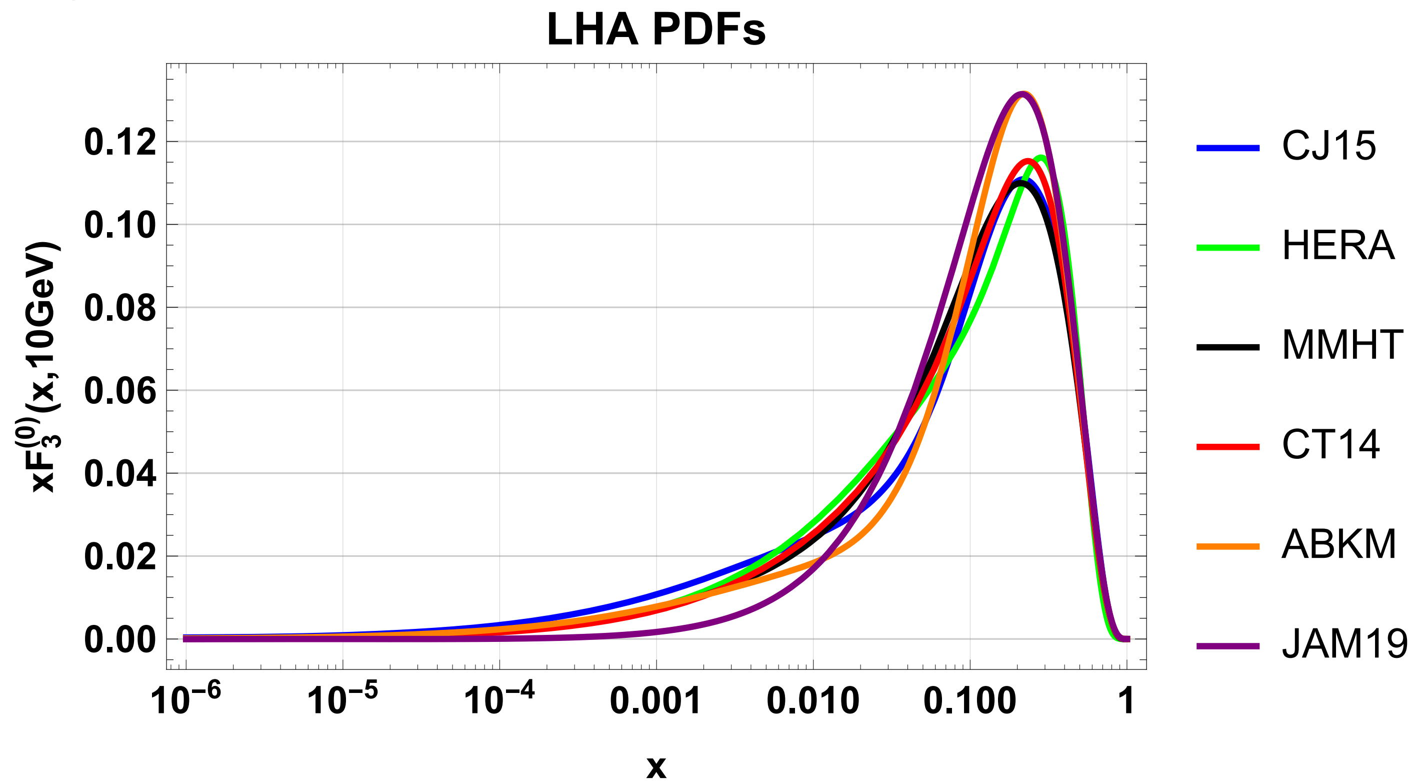
High Q^2 means the hadron looks like individual free quarks, so we use pQCD and factorization:

$$\Box^{\mathrm{A(DIS)}}_{\gamma W} = \frac{1}{2\pi}\int_{Q_0^2}^{\infty} dQ^2 \frac{\alpha_{EM}(Q^2)}{Q^2(1+Q^2/M_W^2)}\int_0^{x_{\max}} dx\ F^{(0)}_{3,DIS}(x,Q^2)\,\frac{(2r-1)}{r^2}$$

$$r \equiv 1+\sqrt{1+4M^2x^2/Q^2}$$

Perturbatively include effects of the strong interaction at NLO

$$F^{(0)}_{3,\mathrm{DIS}}(x,Q^2) = \int_x^1 \frac{dz}{z} C_3^{(1)}(z)$$
$$\times \frac{1}{3}\left(u_v(x/z,Q^2) - d_v(x/z,Q^2)\right)$$



- The effect of the NLO pQCD correction suppresses the LO prediction by $1-\frac{\alpha_S}{\pi}$
- The running of $\alpha_{EM}(Q^2)$ enhances the box correction by 4% from atomic limit

$$\boxed{\Box^{\gamma W}_{A,DIS} = 2.29(3)\times 10^{-3}} \quad \langle Q^2\rangle = 12\ \mathrm{GeV}^2$$

Regge Contribution:

At low Q^2 and high W^2, the strong interaction becomes nonperturbative

Model 1 for F$_3$: Seng, Gorchtein, Ramsey-Musolf, Phys. Rev. Lett. **121**, 241804 (2018)

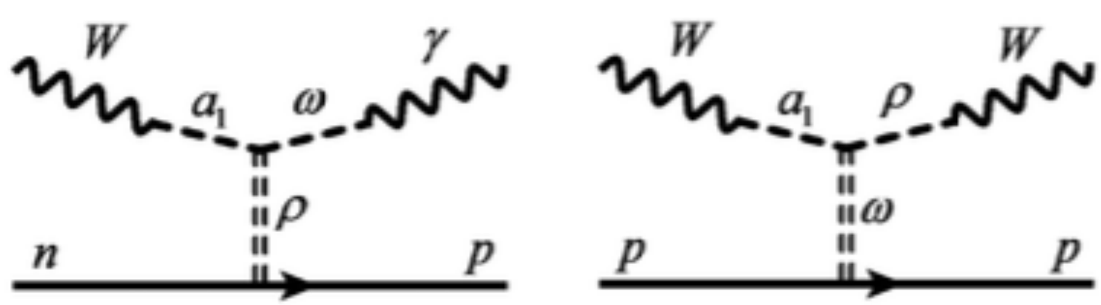
$$F^{(0)}_{3,\mathrm{Reg}}(W^2,Q^2) = \frac{f(1+gQ^2)}{(1+Q^2/m_\rho^2)(1+Q^2/m_{a_1}^2)} f_{th}(W)\left(\frac{\nu}{\nu_0}\right)^{\alpha_0}$$

$$f_{th}(W) = \Theta(W^2 - W_{th}^2)(1 - e^{(W_{th}^2 - W^2)/\Lambda_{th}^2})$$

$$W_{th} = M + m_\pi$$

The true Q^2 -dependence of this structure function is not well-determined by theory.

VMD Processes:



Diagrams: Seng et. al. Phys. Rev. Lett. **121**, 241804 (2018)

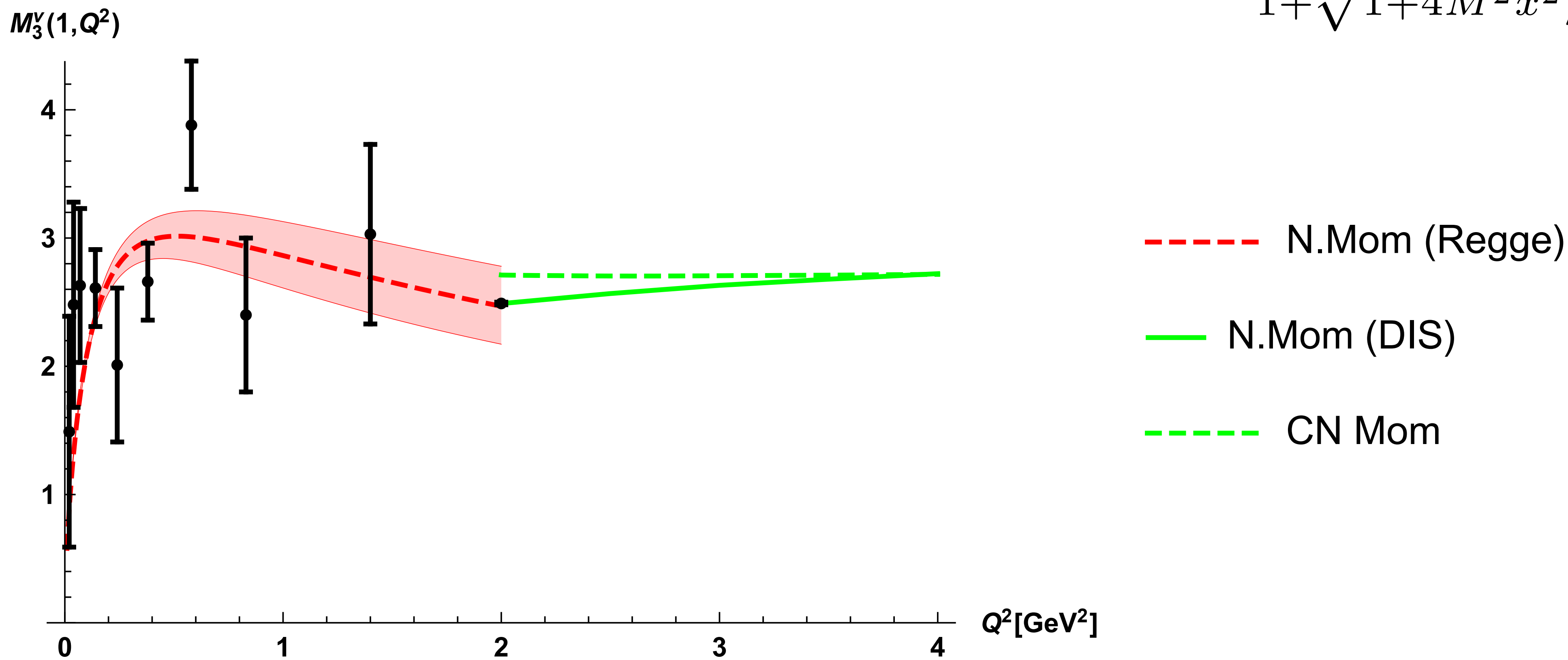
Idea: match this function to the well-known value in the DIS region around $Q^2 = 2\ \mathrm{GeV}^2$ AND constrain it from available data on $F_3^{\nu p + \bar{\nu} p}$

$$\frac{F_3^{\nu p+\bar{\nu}p}}{F_3^{(0)}} \approx 9$$ (more on this later)

Some data exists on the 1st Nachtmann moment of $F_3^{\nu p+\bar{\nu}p}$

$$M_3^{\nu p+\bar{\nu}p}(1,Q^2)\Big|_{\text{low } Q^2} = \frac{2}{3}\int_0^1 dx\,\frac{\xi}{x^2}\left(2x-\frac{\xi}{2}\right)\left[F_{3,\text{el}}^{\nu p+\bar{\nu}p} + F_{3,\text{res}}^{\nu p+\bar{\nu}p} + 9F_{3,\text{Reg}}^{(0)}\right]$$

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$



Data points: T. Bolognese et al., Phys. Rev. Lett. **50**, 224 (1983)

nonlinear fit:

$$f = 0.80(3)$$
$$g = 0.63(10)\ \text{GeV}^{-2}$$

$$\boxed{\square_{A,Reg}^{\gamma W} = .37(10)\times 10^{-3}}$$

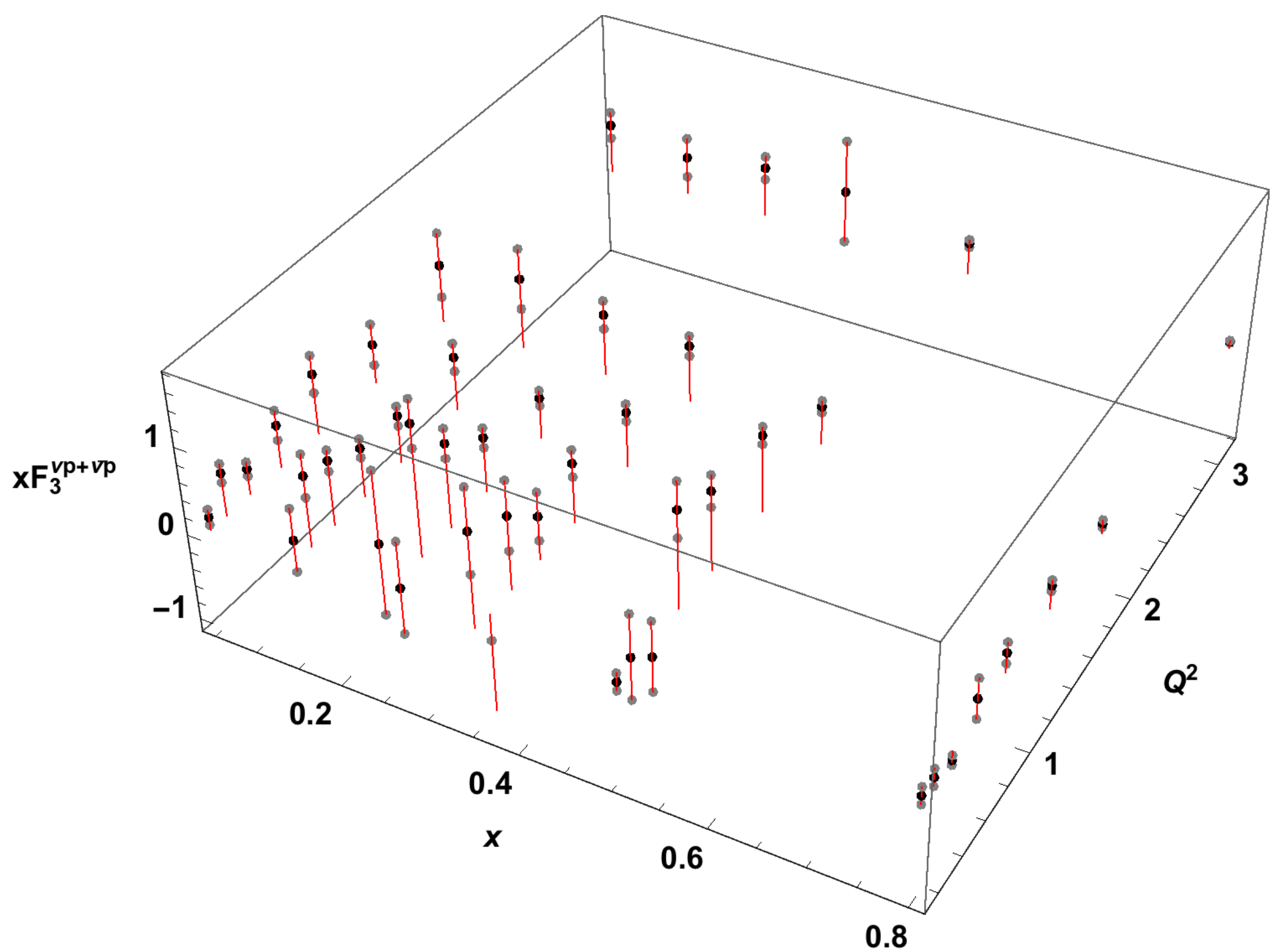
(Model 1 result)

Model 2 for F_3: A. Capella *et al.*, Phys. Lett. B **337**, 358 (1994)

$$F^{(0)}_{3,\mathrm{Reg}} = A_{p-n} x^{-\alpha_R}(1-x)^c \left(\frac{Q^2}{Q^2+\Lambda_R^2}\right)^{\alpha_R}$$

Similarly, we can model the purely axial $F_3^{\nu p + \bar{\nu} p}$ in the same way

P.C. Bosetti *et al.*, Nucl. Phys. B **203**, 362 (1982):

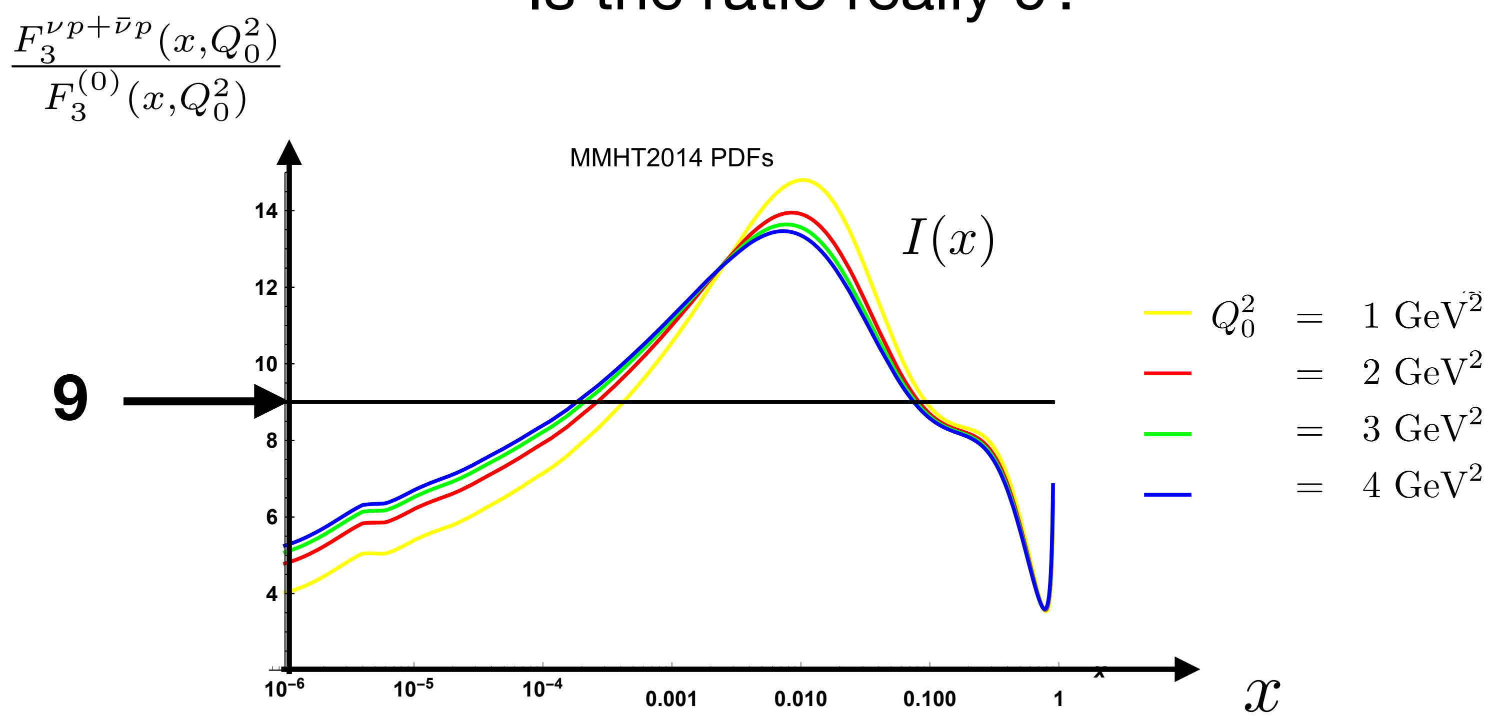


also include high-weight data points from DIS region at $Q^2 = 2\ \mathrm{GeV}^2$

Fit parameters:

$$A_{p+n} = 2.16(3)$$
$$c = 0.61(1)$$
$$\Lambda_R = 0.49(7)$$

Is the ratio really 9?



No! But the observable correction is proportional to $\int_0^1 dx F_3(x,Q^2)$ so taking a ratio of 9 is still meaningful.

Two choices:

$$A_{p-n} = \frac{A_{p+n}}{I(x)} \Rightarrow \Box^{\gamma W}_{A,\mathrm{Reg}} = 0.34 \times 10^{-3}$$ (PDF-dependent)

$$A_{p-n} = \frac{A_{p+n}}{9} \Rightarrow \boxed{\Box^{\gamma W}_{A,\mathrm{Reg}} = 0.38(4)_{\mathrm{sys}}(3)_{\mathrm{stat}} \times 10^{-3}}$$

The Regge contribution is poorly constrained, and multiple models lead to a similar central value, *e.g.* from gZ axial contribution to Qweak:

$$F^{(0)}_{3,\mathrm{Reg}} = \frac{1+\Lambda^2/Q_0^2}{1+\Lambda^2/Q^2}\, F^{(0)}_{3,\mathrm{DIS}}(x,Q_0^2), \qquad \Lambda \approx 0.8\ \mathrm{GeV}$$

P.G. Blunden *et al.*, Phys. Rev. Lett. **107**, 081801 (2011)

Background Contribution:

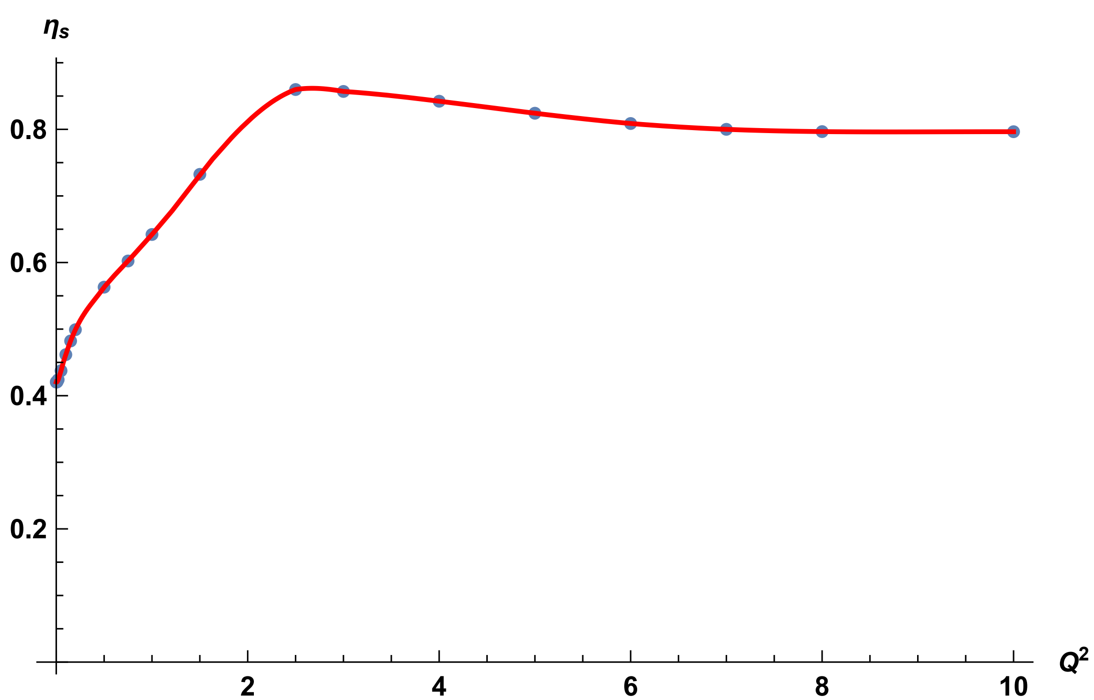
The background is a smoothly decreasing curve which goes to 0 at the pion threshold and matches the DIS and Regge regions at $W^2 = 4\ \mathrm{GeV}^2$

By using PDF info and valence quark arguments one can show the proportionality statement:

$$F_{3,\mathrm{bgd}}^{(0)} \sim F_{1,\mathrm{bgd}}^{\gamma\gamma} \qquad \text{at fixed } Q^2$$

Rescaled Bosted-Christy parametrization:

$$F_{3,\mathrm{bgd}}^{(0)} = \eta_S(Q^2)\frac{W^2 - M^2}{8\pi^2\alpha}\left[1+\frac{W^2-(M+m_\pi)^2}{Q^2+Q_0^2}\right]^{-1}\Sigma_{i=1}^{2}\frac{\sigma_T^{NR,i}(0)[W-(M+m_\pi)]^{(i+1/2)}}{(Q^2+a_i^T)^{(b_i^T+c_i^TQ^2+d_i^TQ^4)}}$$

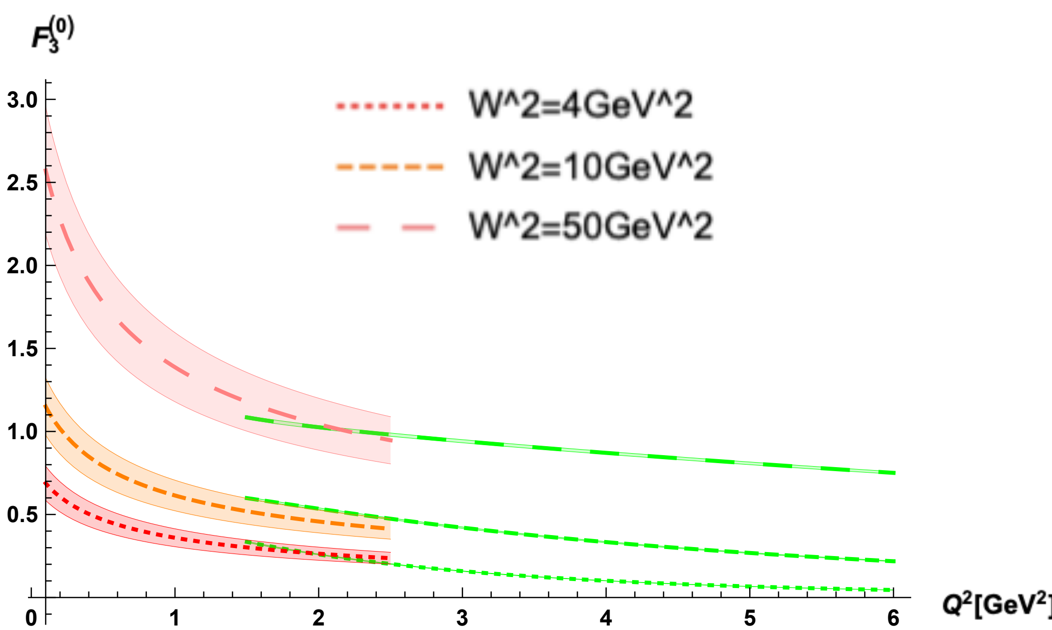


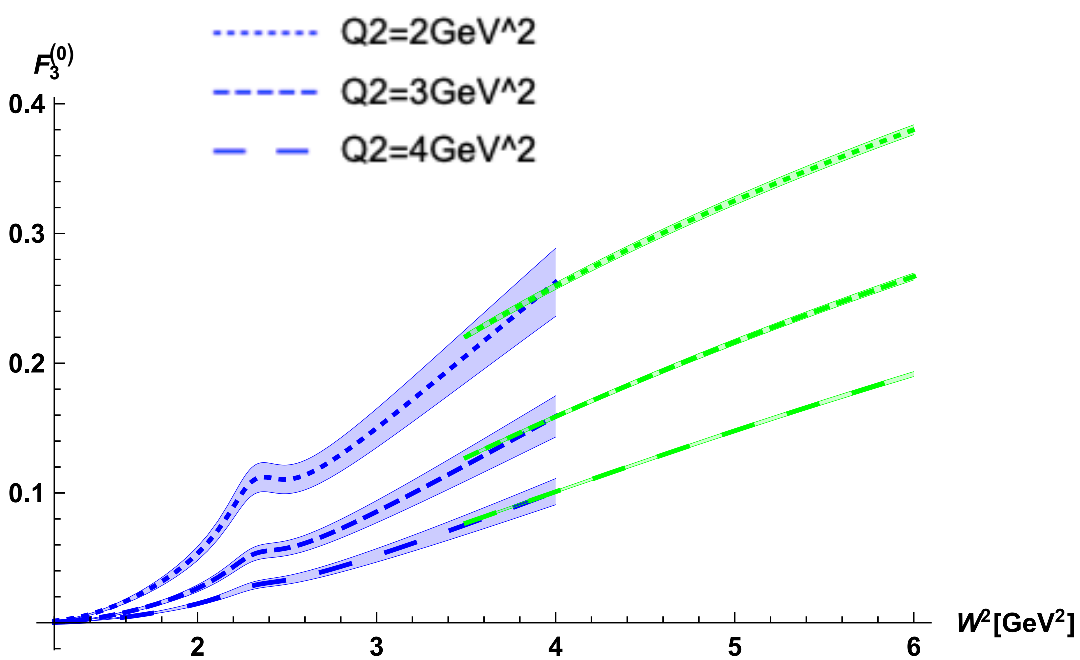
$$\boxed{\Box_{A,\mathrm{bgd}}^{\gamma W} = 0.15(1)\times 10^{-3}}$$

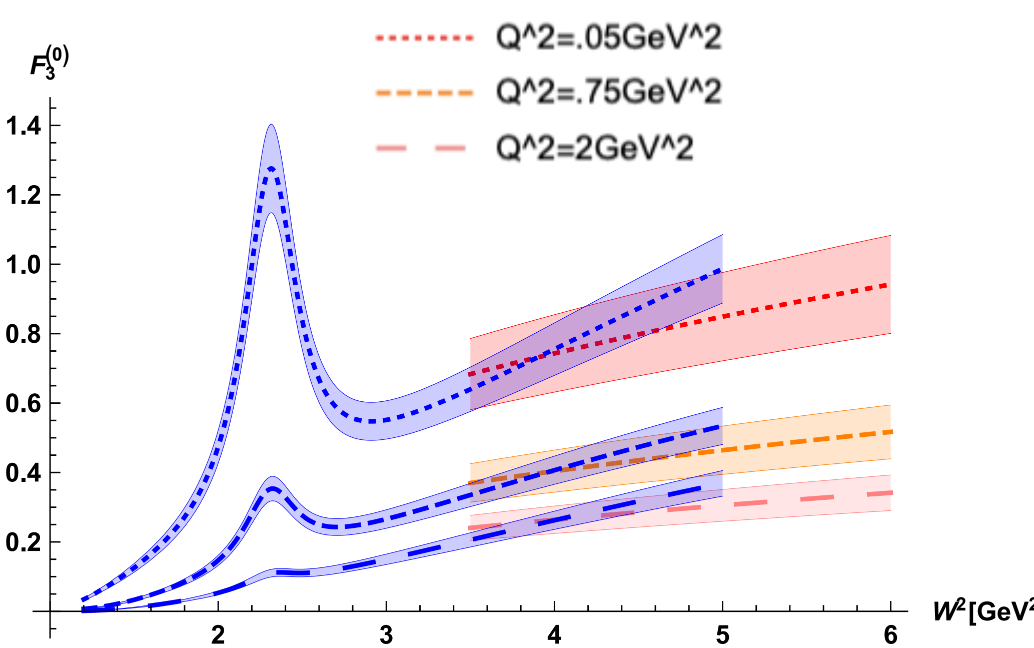
this is a similar box contribution as compared to extending the DIS & Regge models up to x=1

Boundary Matching:

- The SF should be continuous over all region boundaries
- All models agree within uncertainties
- The boundaries shown $Q_0^2 = 2\ \mathrm{GeV}^2, W_{\min}^2 = 4\ \mathrm{GeV}^2$ are not unique, and we find the total Box correction is insensitive to their choice





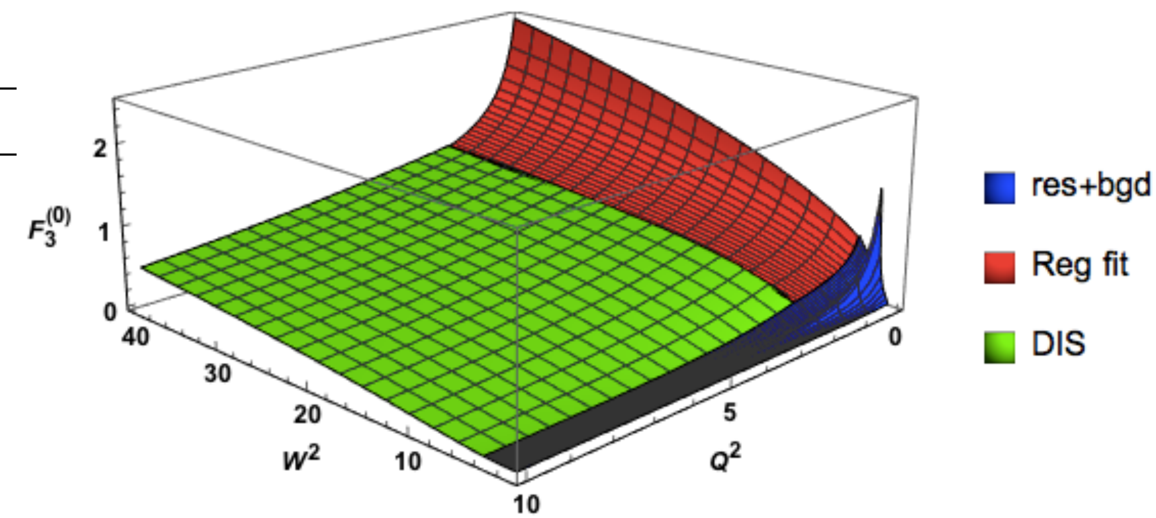


Revising Vud: Simple Approach

Total Box correction:

$\square_A^{\gamma W}(\times 10^{-3})$	**SBM**	SGRM	CMS
elastic	1.04(6)	1.06(6)	0.99(10)
resonance	0.04(2)	–	–
DIS + high-Q^2 bgd	2.29(2)	2.17(0)*	2.16(2)*
Regge + low-Q^2 bgd	0.52(7)	0.56(8)	0.36(7)
total	**3.89(10)**	3.79(10)	3.51(12)

*computed at $\alpha(0) = 1/137.036$



Extract V_{ud} from super allowed beta decays:

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

$$\Delta_R^V = 0.017007 + 2\square_A^{\gamma W}$$

includes re-summed log

$$\Delta_R^V = 0.02479(20)$$

$$\boxed{\Sigma_{CKM}^{3\times 3} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = .9983(4)} \neq 1$$

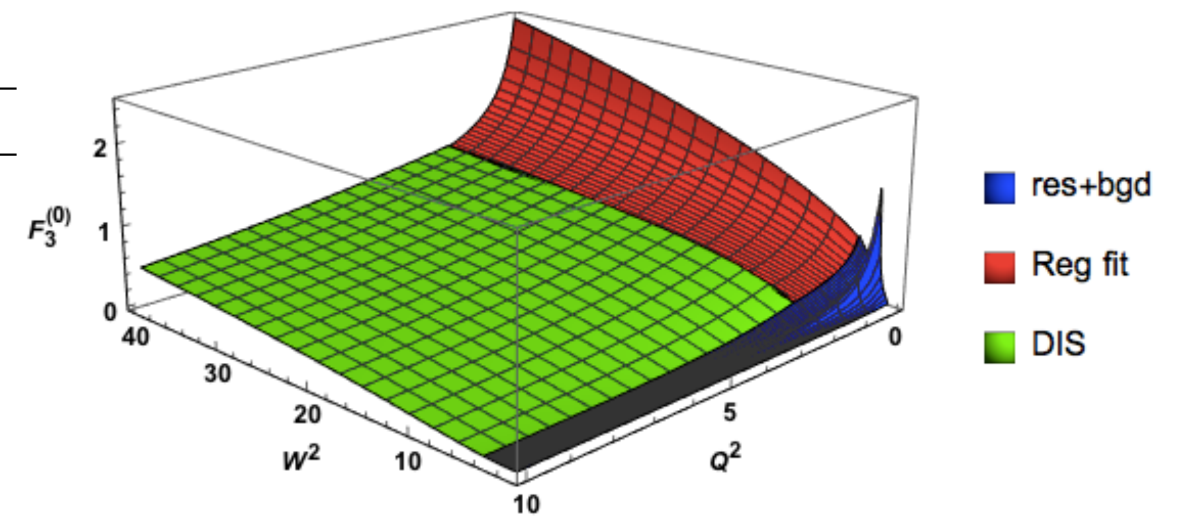
The effect of computing $\square_A^{\gamma W}$ via a dispersion relation has led to the top row of the CKM matrix to fall short of unitarity by 4σ

Revising Vud: CMS Approach

Total Box correction:

$\Box_A^{\gamma W}(\times 10^{-3})$	**SBM**	SGRM	CMS
elastic	1.04(6)	1.06(6)	0.99(10)
resonance	0.04(2)	–	–
DIS + high-Q^2 bgd	2.29(2)	2.17(0)*	2.16(2)*
Regge + low-Q^2 bgd	0.52(7)	0.56(8)	0.36(7)
total	**3.89(10)**	3.79(10)	3.51(12)

*computed at $\alpha(0) = 1/137.036$



Extract V_{ud} from super allowed beta decays:

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)} \qquad \Delta_R^V = .01671 + 1.022\left[2\Box_A^{\gamma W}(Q^2 \geq Q_0^2) + .00014\right]$$

$$+1.065\left[2\Box_A^{\gamma W}(Q^2 < Q_0^2) + 2\Box_{A,el}^{\gamma W}\right]$$ (used by CMS)

$$\Delta_R^V = .02486(26)$$

$$\boxed{\Sigma_{CKM}^{3\times 3} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = .9982(5)} \quad \neq 1$$

The effect of computing $\Box_A^{\gamma W}$ via a dispersion relation has led to the top row of the CKM matrix to fall short of unitarity by 3.8σ

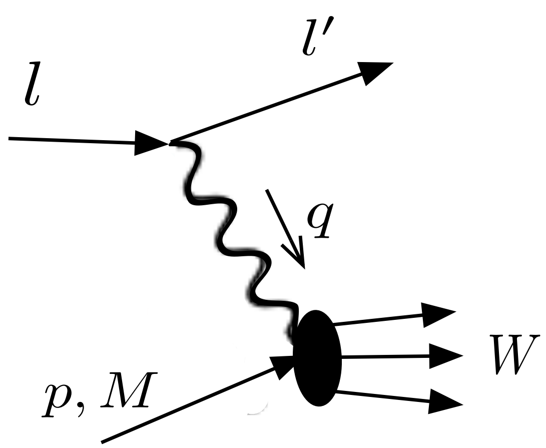
EIC Contribution?

- One could improve the Vud extraction by better constraining the neutral current axial structure functions:

$$F_3^{\nu p+\bar{\nu}p} = F_{3,p}^{\gamma Z} + F_{3,n}^{\gamma Z}$$

$$F_3^{(0)} = F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z}$$

Need more data at low Q, low x



$$x = \frac{Q^2}{Q^2+W^2-M^2}$$

$$eN \to eX$$

$e^{\pm}$ *DIS* cross section:

$$\frac{d^2\sigma^{NC}}{dxdy} = \frac{4\pi\alpha^2}{xyQ^2}\eta^{NC}\left\{\left(1-y-\frac{x^2y^2M^2}{Q^2}\right)F_2^{NC} + y^2 x F_1^{NC} \mp \left(y-\frac{y^2}{2}\right)xF_3^{NC}\right\}$$

$$F_3^{NC} \sim -(g_A^e \pm \lambda g_V^e)\eta_{\gamma Z} x F_3^{\gamma Z} + \ldots$$