Charge symmetry violation

Phiala Shanahan, MIT

Image Credit: 2018 EIC User's Group Meeting
Charge symmetry violation

Charge symmetry

$180^\circ$ rotation about the ‘2’ axis in isospin space

$P_{CS} = e^{i\pi T_2}$

Partonic charge symmetry relations

\begin{align*}
    u^p(x, Q^2) &= d^n(x, Q^2) \\
    d^p(x, Q^2) &= u^n(x, Q^2) \\
    s^p(x, Q^2) &= s^n(x, Q^2) \\
    c^p(x, Q^2) &= c^n(x, Q^2)
\end{align*}

Analogous for antiquark PDFs

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Charge symmetry violation

Charge symmetry is not a symmetry of nature

\[ m_u \neq m_d \]
\[ Q_u \neq Q_d \]

Strong \( \rightarrow \) quark masses
QED \( \rightarrow \) photon radiation

Define "CSV” PDF combinations

\[ \delta u(x) \equiv u^p(x) - d^n(x) \]
\[ \delta d(x) \equiv d^p(x) - u^n(x) \]

Consider CSV in

- Valence quark PDFs
  \[ u_v(x) \equiv u(x) - \bar{u}(x) \]
  \[ d_v(x) \equiv d(x) - \bar{d}(x) \]

- Sea quark PDFs \( \leftrightarrow \) gluon PDF
  \[ \delta g(x) = g^p(x) - g^n(x) \]

In Eq. (86), \( g(x) = g^p(x) = g^n(x) \). Piller and Thomas showed that CSV contributions in the...
Implications of CSV at the EIC

(SI)DIS cross-sections at the EIC

Constraints on nucleon and nuclear PDFs
- CSV
- Heavy flavour
- Sea quarks
- Gluons
- Nuclear effects

Tests of the SM via precision measurements of electroweak parameters

Phiala Shanahan, MIT
In Eq. (7), we have

d\sigma^{L,R}_{NC} = \frac{4\pi\alpha^2 s}{Q^4} \left[ xy^2 F_1^\gamma(x, Q^2) + f_1(x, y) F_2^\gamma(x, Q^2) \right]
+ \frac{Q^2}{(Q^2 + M_Z^2)} v_\ell \pm a_\ell \frac{2\sin \theta_W \cos \theta_W}{2 \sin \theta_W \cos \theta_W}
\times \left[ xy^2 F_1^Z(x, Q^2) + f_1(x, y) F_2^Z(x, Q^2) \right]
+ \frac{Q^2}{(Q^2 + M_Z^2)^2} \left[ xy^2 F_1^Z(x, Q^2) + f_2(y) x F_3^Z(x, Q^2) \right]
+ f_1(x, y) F_2^Z(x, Q^2) \pm f_2(y) x F_3^Z(x, Q^2) \right).

**Careful!** Nuclear vs nucleon PDFs (even for deuteron)

**Neutral current interactions**

\[ l^- \gamma \rightarrow l^- \]
\[ N \rightarrow X \]

*Drop assumption of CSV*

*chase through CSV terms in all structure functions*
CSV in DIS cross-sections

- Drop assumption of CSV
- Chase through CSV terms in all structure functions

### Charged current interactions

\[
\begin{align*}
\frac{d^2\sigma_{cc}^{l^+(l^-)}}{dx\,dy} &= \frac{\pi s}{2} \left( \frac{\alpha}{2\sin^2\theta_W (M_W^2 + Q^2)} \right)^2 \\
&\times \left[ x y^2 F_1^W(x, Q^2) + f_1(y) F_2^W(x, Q^2) \right] \\
&\quad + f_2(y) x F_3^W(x, Q^2) \right].
\end{align*}
\]

### Example for isoscalar target \(N_0\)

\[
2 F_1^{W^+ N_0}(x, Q^2) \rightarrow u^+(x) + d^+(x) + 2s(x) + 2\bar{c}(x) - \delta u(x) - \delta d(x)
\]

**Note:** CSV contributions aren't purely valence

---

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Charge symmetry violation

How large is CSV in parton distribution functions?

General quark model arguments:

Fractional partonic CSV

\[
\frac{\delta q}{q} \sim \frac{\delta m}{\langle M \rangle}
\]

Quark mass differences ~1-5MeV

Effective mass of system w/ quark removed
- Valence quark: ~500MeV
- Sea quark ~1.3GeV

Expectation:

CSV in valence quark PDFs > CSV in sea quark PDFs
Charge symmetry violation

How large is CSV in the parton distribution functions?

- Partonic CSV not directly resolved in experiment: bounds at few\%-10\%
  [Indirect evidence: global fits accommodate CSV]

- Theory and lattice QCD calculations suggest ~1\% level in valence PDFs
  - Lattice QCD $\rightarrow$ lowest moments
  - Models: for moderate $x$ ($x > \sim 0.1$)
    \[
    |\delta u_v(x) + \delta d_v(x)| \ll |\delta u_v(x) - \delta d_v(x)|
    \]

- Small, BUT could explain significant fraction of NuTEV anomaly

Young, PES, Thomas [arXiv:1312.4990]
Indirect measure of Paschos-Wolfenstein ratio:

\[ R_{PW} = \frac{\sigma_{\nu A}^{NC} - \sigma_{\bar{\nu} A}^{NC}}{\sigma_{\nu A}^{CC} - \sigma_{\bar{\nu} A}^{CC}} \rightarrow \frac{1}{2} - \sin^2 \theta_W \]

This simplification relies on assumptions:

- Exact charge symmetry
- Vanishing partonic strangeness \( s(x) - \bar{s}(x) \)
- Isoscalar nucleus with no nuclear effects
- No higher-twist effects

Discrepancy from SM
Correction to the Paschos-Wolfenstein ratio from CSV

$$\Delta R_{PW}^{CSV} = \frac{1}{2} \left( 1 - \frac{7}{3} \sin^2 \theta_W \right) \frac{\langle x \rangle \delta u^- - \langle x \rangle \delta d^-}{\langle x \rangle u^- + \langle x \rangle d^-}$$

Extensive literature discussing further corrections incl. Non-isoscalar nucleus, strangeness

- Bentz, Cloet, Londergan & Thomas PLB(2010)
- Davidson, Forte, Gambino, Rius, Strumia JHEP 02 (2002) 037
- ...

Moments of PDFs

$$\langle x \rangle_q = \int x q(x) dx$$

Calculable in lattice QCD

Phiala Shanahan, MIT
CSV moments from lattice QCD

Indirect lattice QCD determination of first moment of CSV PDFs
[Shanahan, Thomas & Young, PRD(2013)094515]

- For small breaking in the u-d quark masses $m_\delta \equiv (m_d - m_u)$

$$
\langle x \rangle_{\delta u} \simeq \frac{m_\delta}{2} \left[ \left( -\frac{\partial \langle x \rangle^p_u}{\partial m_u} + \frac{\partial \langle x \rangle^p_u}{\partial m_d} \right) - \left( -\frac{\partial \langle x \rangle^n_d}{\partial m_u} + \frac{\partial \langle x \rangle^n_d}{\partial m_d} \right) \right]
$$

Charge symmetry

$$
\langle x \rangle_{\delta u} \simeq m_\delta \left[ -\frac{\partial \langle x \rangle^p_u}{\partial m_u} + \frac{\partial \langle x \rangle^p_u}{\partial m_d} \right]
$$

- SU(3) symmetry: fit isospin symmetric lattice results for hyperons (exploit use of non-physical quark masses in lattice QCD)

Determines CSV parameters in EFT
CSV moments from lattice QCD

Indirect lattice QCD determination of first moment of CSV PDFs

[Shanahan, Thomas & Young, PRD(2013)094515]

Our result

\[ \langle x \rangle_{\delta u} = -0.0023(7) \]
\[ \langle x \rangle_{\delta d} = 0.0017(4) \]

This result + CSV from QED parton evolution

NuTeV anomaly reduced by \(~1\sigma\)
Sin$^2\theta_W$ at the EIC

Constrain $\sin^2 \theta_W$ using parity-violating e-D scattering

$$A_{PV}^{eD}(x, y) = \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ a_1^d + f(y)a_3^d \right]$$

$$= -\frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{9}{10} \left[ \left( 1 - \frac{20}{9} \sin^2 \theta_W \right) + (1 - 4\sin^2 \theta_W) \frac{1 - (1-y)^2}{1 + (1-y)^2} \right]$$

Assumptions:
- CSV negligible
- Impulse approximation in scattering
- No higher-twist contributions
- Sea quarks negligible

**Sin^2θ_W at the EIC**

**Constrain sin^2 θ_W using parity-violating e-D scattering**

\[ A_{PV}^{eD}(x, y) = \frac{-G_F Q^2}{4\sqrt{2}\pi} \left[ a_1^d + f(y) a_3^d \right] \]

CSV terms contribute to both couplings

\[ a_1^d \rightarrow a_1^{(0)} + \delta_{CSV} a_1^d \]
\[ a_3^d \rightarrow a_3^{(0)} + \delta_{CSV} a_3^d \]

\[ \frac{\delta_{CSV} a_1^d}{a_1^{d(0)}} = \left[ -\frac{3}{10} + \frac{2g^u_v + g^d_v}{2(2g^u_v - g^d_v)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)} \]
\[ \frac{\delta_{CSV} a_3^d}{a_3^{d(0)}} = \left[ -\frac{3}{10} + \frac{2g^u_A + g^d_A}{2(2g^u_A - g^d_A)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)} \]

**x-dependent CSV PDFs, not just moments**
- Lattice QCD calculation was first moment only
- Model x-dependence
Only first moment from lattice QCD calculation

Constrain simple parameterisation of x-dependence

\[ \delta_q(x) = \kappa q x^{-1/2} (1 - x)^4 (x - 1/11) \]

Young, PES, Thomas [arXiv:1312.4990]
Only first moment from lattice QCD calculation

Constrain simple parameterisation of x-dependence

\[ \delta_q(x) = \kappa_q x^{-1/2} (1 - x)^4 (x - 1/11) \]

Including photon radiation

Wang, Thomas, Young, PLB(2016)
**Sin$^2\theta_W$ at the EIC**

CSV contribution to parity-violating asymmetry is at the sub-percent level

![Graph showing CSV contribution to parity-violating asymmetry with lattice result and caveats.](image)

**Caveats:**
- Model form assumed, fit from one moment
- Lattice systematics ignored (no continuum, inf vol limit, no QED)
- Chiral extrapolation used
CSV in WNC couplings from e-D DIS

Note that CSV affects couplings $a_1^d$, $a_3^d$ in the same way (x-dep)

\[
\frac{\delta^{(CSV)} a_1^d}{a_1^{d(0)}} = \left[ -\frac{3}{10} + \frac{2g_v^u + g_v^d}{2(2g_v^u - g_v^d)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)}
\]

\[
\frac{\delta^{(CSV)} a_3^d}{a_3^{d(0)}} = \left[ -\frac{3}{10} + \frac{2g_A^u + g_A^d}{2(2g_A^u - g_A^d)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)}
\]

Use this to distinguish CSV effects on WNC couplings from possible new physics signatures in couplings

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} \left[ \bar{e} \gamma^\mu \gamma_5 e \left( C_{1u} \bar{u} \gamma_\mu u + C_{1d} \bar{d} \gamma_\mu d \right) + \bar{e} \gamma^\mu e \left( C_{2u} \bar{u} \gamma_\mu \gamma_5 u + C_{2d} \bar{d} \gamma_\mu \gamma_5 d \right) \right]
\]

\[
a_1^d = \frac{6}{5} \left( 2C_{1u} - C_{1d} \right) \left[ 1 + \text{(CSV)} + \text{(new)} \right]
\]

\[
a_3^d = \frac{6}{5} \left( 2C_{2u} - C_{2d} \right) \left[ 1 + \ldots \right]
\]

\[\text{e.g., Leptophobic } Z' \text{ affects only } a_3^d\]
CSV in WNC couplings from e-D DIS

Note that CSV affects couplings $a^d_1$, $a^d_3$ in the same way (x-dep)

$$
\frac{\delta^{(CSV)} a^d_1}{a^d_1} = \left[ -\frac{3}{10} + \frac{2g^u + g^d}{2(2g^u - g^d)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)}
$$

$$
\frac{\delta^{(CSV)} a^d_3}{a^d_3} = \left[ -\frac{3}{10} + \frac{2g^u + g^d}{2(2g^u - g^d)} \right] \frac{\delta u(x) - \delta d(x)}{u(x) + d(x)}
$$

$$
\begin{align*}
a^d_1 &= \frac{6}{5}(2C_{1u} - C_{1d})[1 + (\text{CSV}) + (\text{new}) + (\text{sea}) + (\text{TMC}) + (\text{HT})] \\
a^d_3 &= \frac{6}{5}(2C_{2u} - C_{2d})[1 + \ldots]
\end{align*}
$$

Theory/ lattice QCD suggests CSV contributions to the couplings is at the sub-percent level

[Y.X.Zhao (SoLID Collaboration)]

Phiala Shanahan, MIT
How can we constrain CSV?

• Strongest upper limit by comparing $F_2$ in charged current reactions induced by neutrinos with $F_2$ for charged lepton DIS, on isoscalar targets

• Coupled knowledge of heavy quark PDFs and CSV
  Sea quark contributions suppressed at larger $x$, more sensitive to CSV

$$R_c(x) \equiv \frac{F_2^\gamma N_0(x) + x[s^+(x) + c^+(x)]}{5F_2^W N_0(x) / 18} / 6$$

$$R_c(x) \approx 1 + \frac{3(\delta u^+(x) - \delta d^+(x))}{10 \sum_j q_j^+(x)} .$$

• Neutral current structure function at EIC
• Charged current structure function at EIC with sufficiently high beam intensity?

Also: neutrino-nucleon DIS on heavy targets:

$$\Delta x F_3(x) = x F_3^W(x) - x F_3^W(x) ;$$

$$\Delta x F_3^{N_0}(x) \rightarrow x \left[ 2(s^+(x) - c^+(x)) + \delta d^+(x) - \delta u^+(x) \right].$$

• Role for EIC: precise nuclear correction factors

Phiala Shanahan, MIT
Semi-inclusive pion production

Lepton DIS on isoscalar nuclear targets

\[
\frac{1}{\sigma_N(x)} \frac{d\sigma^h_N(x,z)}{dz} = \frac{N^Nh(x,z)}{\sum_i e_i^2 q_i^N(x)}
\]

Yield of hadron h per scattering from nucleon N

\[R^\Delta(x,z) = 8 \left( \frac{N^{D\pi^-}(x,z)}{1 + 4\Delta(z)} - \frac{N^{D\pi^+}(x,z)}{4 + \Delta(z)} \right) \]

\[= C^\Delta(z) \left[ R_{CS}(x) + R_{SV}(x,z) \right] \]

CSV

Sea-valence interference term, less important at large x

\[R_{CS}(x) = \frac{4(\delta d_v(x) - \delta u_v(x))}{3(u_v(x) + d_v(x))} \]

- CSV terms substantial for \(x > 0.4\)
- Determine CSV via measurement of x-dep of R for fixed z

Requires that factorisation be valid to a few percent
Test of weak current relation

Compare charge-changing interactions from electron/positron scattering on isospin-0 nucleus

\[ R_W(x) \equiv \frac{2 \left( F_{2W^+}^D(x) - F_{2W^-}^D(x) \right)}{F_{2W^+}^D(x) + F_{2W^-}^D(x)} - \frac{\delta d_v(x) - \delta u_v(x) + 2(s_v(x) - c_v(x))}{\sum_j q_j^x(x)} \equiv R_{CSV}(x) + R_S(x). \]

Theory calculations suggest

- Opposite sign \( \delta d_v(x), \delta u_v(x) \) effects add in magnitude
- \( x \)-dep of CSV and strange is different (note strange must have node)

### Sea quark CSV: sum rules

<table>
<thead>
<tr>
<th>Expectation: CSV in valence quark PDFs</th>
<th>CSV in sea quark PDFs</th>
</tr>
</thead>
<tbody>
<tr>
<td>First moments of valence quark CSV vanish by quark normalisation conditions</td>
<td></td>
</tr>
<tr>
<td>Sum rules in moments isolate sea quark CSV</td>
<td></td>
</tr>
<tr>
<td>Note: sea quark CSV ↔ gluon CSV via evolution</td>
<td></td>
</tr>
</tbody>
</table>

#### CSV vs sea quark flavour asymmetry

- **e.g., Gottfried sum rule**

\[
S_G \equiv \int_0^1 dx \left( \frac{F_2^{\ell p}(x) - F_2^{\ell n}(x)}{x} \right) = \frac{1}{3} - \frac{2}{3} \int_0^1 dx \left[ \bar{d}^p(x) - \bar{u}^p(x) \right] + \frac{2}{9} \int_0^1 dx \left[ 4\delta\bar{d}(x) + \delta\bar{u}(x) \right].
\]
"pseudo-CSV" EMC effect

- **Isovector EMC effect** for nuclei with $N \neq Z$
  \[ R_{N,Z}(x) = \frac{F_{2}^{(N,Z)}(x)}{F_{2}^{(d)}(x)} \]

- **Isovector EMC has similar signature to CSV → "pseudo-CSV"**

- **Full flavour dependence of EMC effects**
  \[ R_{N,Z}^{(3)}(x) = \frac{u_{N,Z}(x) - d_{N,Z}(x)}{u_{d}(x) + d_{d}(x)} \]
  
  - Challenging in experiment (e.g., MINERvA)
  - Moments accessible in LQCD

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**Graphs**


- Preliminary results

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**Preliminary**

Phiala Shanahan, MIT
Parton physics from Lattice QCD

**Precision Era**
- Fully-controlled w/ few-percent errors within ~5y
- Static properties of nucleon incl. spin, flavour decomp.
- Mellin moments of PDFs, GPDs

**Early Era**
- Fully-controlled w/ ~15-percent errors within ~5y
- Nuclear structure A<5
- Spin, flavour decomp. of EMC-type effects

**Exploratory Era**
- First calculations, timeline for controlled calculations unclear
- x-dependence of PDFs
- TMDs

Phiala Shanahan, MIT
Prospects for CSV from Lattice QCD

- State of the art lattice QCD calculations include QED and isospin breaking \([M_n-M_p: \text{Borsanyi et al. Science 347 (2015) 1452}]

- On EIC timescale:

  
  
  \begin{itemize}
  
  \item Low moments of CSV PDFs
  
  \item EMC effect in moments (isovector)
  
  \item CSV PDFs including x-dependence
  
  \end{itemize}

  
  
  \[\text{SPECULATIVE}\]
EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD: Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- **BUT** EMC effects in moments are very small

Classic EMC effect is defined in $F_2$:

$$F_2(x, Q^2) = \sum_{q=u,d,s...} x e_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$$

Number density of partons of flavour $q$

$x$-integrals of numerator and denominator

$$\int_0^1 dx x^n q(x, Q^2)$$
EMC effects in Mellin moments

First investigation of EMC-type effects from LQCD: Nuclear effects in Mellin moments of PDFs

- Calculable from local operators
- **BUT** EMC effects in moments are very small

Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

- First QCD determination of momentum fraction of nuclei
- Few-percent determination of quark momentum fraction
  - ~10% determination of strange quark contributions

Isoscalar (BARE)

\[ \langle \pi \rangle_{\text{bare}} \sim 800 \text{ MeV} \]
Matrix elements of the Energy-Momentum Tensor in light nuclei

first QCD determination of momentum fraction of nuclei

- Few-percent determination of quark momentum fraction
- ~10% determination of strange quark contributions

Strange (BARE) \( \langle x \rangle_H \)

\[ m_\pi \sim 800 \text{ MeV} \]
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

- First QCD determination of momentum fraction of nuclei
- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

Ratio of quark momentum fraction in nucleus to nucleon

- Small mixing with gluon EMT operators (neglected)
- Sum rule constraint

<table>
<thead>
<tr>
<th>$R_{pp}$</th>
<th>$R_{^{2}H}$</th>
<th>$R_{^{3}H}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.02</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Normalized to proton result

$\pi^{0}$ mass $\sim 800$ MeV

PRELIMINARY
Momentum fraction of nuclei

Matrix elements of the Energy-Momentum Tensor in light nuclei

- First QCD determination of momentum fraction of nuclei
- Bounds on EMC effect in moments at ~few percent level, consistent with phenomenology

Ratio of quark momentum fraction in nucleus to nucleon

- No mixing
- No sum rule constraint

\[ \frac{u}{d} \approx 800 \text{ MeV} \]

Normalised to proton result
Momentum fractions of nuclei

- First determination of all components of momentum decomposition of light nuclei
- Small mixing between quark and gluon EMT operators neglected
- Constraint on either quark or gluon EMC in this quantity implies constraint on the other from sum rules:

\[ m_\pi \sim 800 \text{ MeV} \]

\[ \langle x \rangle_N = \langle x \rangle_H \]

\[ R_{pp} = 1.0 \]

\[ A = 2, 3 \]

\[ m_\pi \sim 800 \text{ MeV} \]

\[ \langle x \rangle_N = \langle x \rangle_H \]

\[ A = 1, 2, 3 \]
Momentum fractions of nuclei

- Work in progress at close-to-physical values of the quark masses

First evidence of signals in physical-point data
Implications of parton CSV at the EIC

(SI)DIS cross-sections at the EIC

Constraints on nucleon and nuclear PDFs
Disentangle contributions from
- CSV
- Heavy flavour
- Sea quarks
- Gluons
- Nuclear effects

Tests of the SM via precision measurements of electroweak parameters
Implications of parton CSV at the EIC

CSV relevant in cross-sections/asymmetry measurements at the percent-level

- Cross-sections
- CSV
- Heavy flavour
- Sea quarks
- Gluons
- Nuclear effects

Disentangle contributions from

Constraints on CKM, ZFITTER and nuclear PDFs

Phiala Shanahan, MIT