# Implementation of the Goloskokov-Kroll model pseudoscalar meson production in PARTONS

Kemal Tezgin

University of Connecticut

*kemal.tezgin@uconn.edu*

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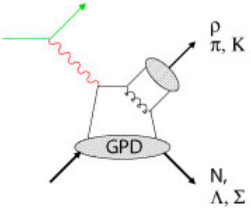
# Chiral-odd GPDs

- Four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\; |p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p,\lambda).$$

  where $i = 1, 2$ is the transversity index [Diehl '03]

- Accessible through exclusive meson production processes



# Chiral-odd GPDs

- Factorization for electroproduction of mesons, only for longitidunally polarized photons, has been proven [Collins-Frankfurt-Strikman '97]
- For transversely polarized photons, cross section is power suppressed by $1/Q$ [Collins-Frankfurt-Strikman '97]
- But in some kinematics, it is apparent that transversely polarized photons contributes substantially [HERMES Collaboration, arXiv:0907.2596[hep-ex]]

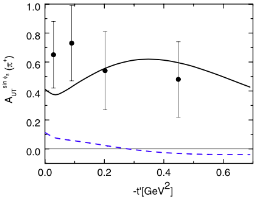



**Fig. 2** (Color online) The $\sin\phi_s$ moment for a transversely polarized target at $Q^2 \simeq 2.45$ GeV$^2$ and $W = 3.99$ GeV. The prediction from our handbag approach is shown as a *solid line*. The *dashed line* is obtained disregarding the twist-3 contribution. Data are taken from [10]

Figure: [Goloskokov-Kroll '10]

# Goloskokov-Kroll Model

- Goloskokov-Kroll(GK) model for pseudoscalar meson production considers the region of small $\xi$ and small $-t$, but large $Q^2$ and $W$.
  [Goloskokov-Kroll '10]
- Contributions from transversely polarized photons can be computed as a twist-3 effect in the handbag mechanism
- In pseudoscalar meson production, the following amplitudes are relevant

$$\mathcal{M}_{0+,0+} = \sqrt{1-\xi^2}\frac{e}{Q}[\langle\tilde{H}\rangle - \frac{\xi^2}{1-\xi^2}\langle\tilde{E}\rangle]$$

$$\mathcal{M}_{0-,0+} = \frac{e}{Q}\frac{-t'}{2m}[\xi\langle\tilde{E}\rangle]]$$

$$\mathcal{M}_{0-,++} = \sqrt{1-\xi^2}e\,\langle H_T\rangle$$

$$\mathcal{M}_{0+,\mu+} = -\frac{e}{4m}\sqrt{-t'}\langle\bar{E}_T\rangle$$

# Goloskokov-Kroll Model

- Generically, $\langle F \rangle$ represents a convolution of a GPD $F$ with an appropriate subprocess amplitude

$$\langle F \rangle = \sum_{\lambda} \int_{-1}^{1} dx \, \mathcal{H}_{\mu'\lambda,\mu\lambda}(x, \xi, Q^2) \, F(x, \xi, t)$$

  where $\lambda$ denotes unobserved helicities of the partons.
- Subprocesses are calculated in the so-called modified perturbative approach: Transverse momenta of the quark and the anti-quark in the meson are kept and gluon radiations are taken into account through Sudakov factor
- GPDs appear in the following combination

$$F^0(x, \xi, t) = \frac{1}{\sqrt{2}}\Big(e_u \, F^u(x, \xi, t) - e_d \, F^d(x, \xi, t)\Big)$$

$$F^+(x, \xi, t) = F^u(x, \xi, t) - F^d(x, \xi, t)$$

# Goloskokov-Kroll Model

- In impact space

$$\mathcal{H}_\pi = \int d\tau d^2\vec{b}\, \hat{\Psi}_\pi(\tau, -\vec{b})\hat{\mathcal{F}}^i_\pi(\bar{x}, \xi, \tau, Q^2, \vec{b})\alpha_s(\mu_R)\; exp\big( - S(\tau, \vec{b}, Q^2)\big)$$

- Hard scattering kernels has the following forms in momentum space

$$\mathcal{F}^q_{\pi^0} = \frac{N_c^2 - 1}{2N_c}\sqrt{\frac{2}{N_c}}\frac{Q}{\xi}\Big[\frac{1}{k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi) - i\epsilon} - \frac{1}{k_\perp^2 - \bar{\tau}(\bar{x}-\xi)Q^2/(2\xi) - i\epsilon}\Big]$$

$$\mathcal{F}^q_{\pi^+} = \frac{N_c^2 - 1}{2N_c}\sqrt{\frac{2}{N_c}}\frac{Q}{\xi}\Big[\frac{e_d}{k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi) - i\epsilon} - \frac{e_u}{k_\perp^2 - \bar{\tau}(\bar{x}-\xi)Q^2/(2\xi) - i\epsilon}\Big]$$

- A Gaussian meson wave function is used at twist-2

$$\Psi_\pi(\tau, \vec{b}) \sim \tau(1-\tau)exp\Big[\frac{\tau(\tau-1)}{4}\frac{\vec{b}^2}{a_\pi^2}\Big]$$

# Goloskokov-Kroll Model

- Sudakov factor has the form

$$S(\tau, b, Q) = s(\tau, b, Q) + s(\bar{\tau}, b, Q) - \frac{4}{\beta_0}\, ln\, \frac{ln(\mu_R/\Lambda_{QCD})}{\hat{b}}$$

where

$$s(\tau, b, Q) = \frac{8}{3\beta_0}\Big(\hat{q}\, ln\Big(\frac{\hat{q}}{\hat{b}}\Big) - \hat{q} + \hat{b}\Big) + NLL$$

$$\hat{b} = -ln(b\, \Lambda_{QCD})$$

$$\hat{q} = ln\big(\tau Q/(\sqrt{2}\Lambda_{QCD})\big)$$

- Twist-3 meson wave function

$$\Psi_\pi(\tau, \vec{b}) \sim exp\Big[-\frac{\vec{b}^2}{8a_\pi^2}\Big]\, I_0(\frac{\vec{b}^2}{8a_\pi^2})$$

# Goloskokov-Kroll Model GPDs

- GPDs are constructed from double distribution ansatz

$$F_i^a(\bar{x},\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta \, \delta(\rho + \xi\eta - \bar{x}) f_i^a(\rho,\eta,t)$$

where for valence-quark GPDs;

$$f_i(\rho,\eta,t) = exp[(b_i - \alpha_i' \, ln\, \rho)t] \, F_i^a(\rho,\xi=t=0) \, \frac{3}{4} \frac{(1-\rho)^2 - \eta^2}{(1-\rho)^3} \Theta(\rho)$$

- Parameters of the forward limits

$\tilde{H}$ : DSSV, Phys. Rev. D **80**, 034030 (2009)

$H_T$: ABM, Phys. Rev. D **86**, 054009 (2012) and DSSV, Phys. Rev. D **80**, 034030 (2009)

$\tilde{E}$ : LHPC Collaboration, Phys. Rev. D **77**, 094502 (2008)

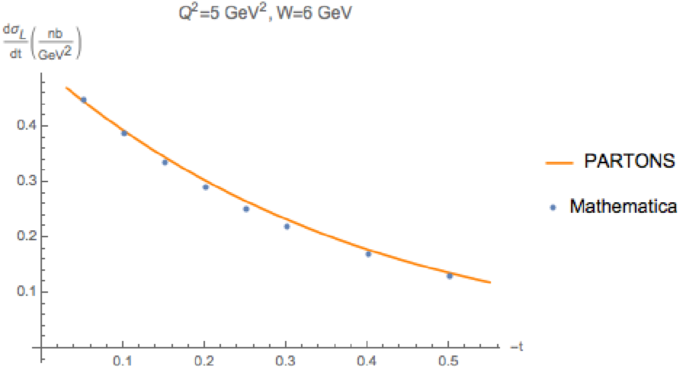
$\bar{E}_T$ : QCDSF and UKQCD Collaborations, Phys. Rev. Lett. **98**, 222001 (2007)

# Goloskokov-Kroll Model

- So altogether, to compute the longitudinal cross section, we need: Twist-2 meson wave function, kernel, running coupling, Sudakov factor and GPDs $\tilde{H}$ and $\tilde{E}$.
- To compute the transverse cross section, we need: Twist-3 meson wave function, kernel, running coupling, Sudakov factor and GPDs $H_T$ and $\bar{E}_T$.
- 3 dimensional integrals, over $\bar{x}, \tau$ and $b$, are performed in impact space
- In many different processes, meson wavefunction has the same structure.
- Sudakov factor has also the same structure
- $\pi^+$ electroproduction also receives a pion pole contribution, besides the handbag contribution

# PARTONS vs. Mathmematica

- Comparision between the PARTONS and Mathmematica codes (slightly different GPDs parametrization for $\tilde{H}$)



# PARTONS vs. Mathmematica

- Comparision between the PARTONS and Mathmematica codes at amplitude level (the same GPDs parametrization).

$Q^2 = 5GeV^2,\ W = 6GeV$

$$M_{0p0p}|_{t=-0.2} = 0.173902 - 0.150831i \ \rightarrow\ 0.1737324 - 0.1509104i$$
$$M_{0m0p}|_{t=-0.2} = 0.00422273 - 0.0502173i \ \rightarrow\ 0.0041224 - 0.0502987i$$
$$M_{0p0p}|_{t=-0.4} = 0.136403 - 0.108212i \ \rightarrow\ 0.1362792 - 0.1082896i$$
$$M_{0m0p}|_{t=-0.4} = 0.00742887 - 0.0477006i \ \rightarrow\ 0.0073482 - 0.0477759i$$