# **Update – ongoing PDF studies for EIC IRG**

Tim Hobbs, JLab EIC Center & CTEQ@SMU May 21<sup>st</sup> 2020



 $|S_{f}|$  for  $g(x,\mu)$  ct14 hera2 NNLO  $10^{3}$  $10 \,\mathrm{GeV} \, e^{\pm} \, imes \, 250 \,\mathrm{GeV} \, p$  $-N_{pt} = 295-$ highlighted range:  $|S_f|$  $|S_f| > 0.25$  $10^{2}$ -1.2 µ [GeV] 1.0 0.8 0.6 0.4 0.2  $10^{1}$ 0 • NC  $e^-p$  $\blacktriangle$  CC  $e^+p$  $\blacksquare$  CC  $e^-p$ • NC  $e^+p$ 10<sup>-5</sup> 10<sup>-</sup> 1 10<sup>-6</sup>  $10^{-4}$  $10^{-2}$  $10^{-1}$ 











#### 2<sup>nd</sup> EIC Yellow Report Workshop; Pavia Univ. 20-22 May 2020

# CT18 parton distributions

CTEQ

focus remains on PDF impacts of inclusive and heavy-flavor tagged EIC data



advocate fast methods in addition to event-level studies and eventual fitting
 [Hessian profiling with <u>ePump</u> and <u>PDFSense</u>]

developments: (i) ongoing fits; (ii) event-level studies; (iii) theory predictions

### we have a dedicated effort to explore PDF impacts of the EIC

Collaborators and consultants						
Pavel Nadolsky Fred Olness Bo-Ting Wang Steve Sekula	Southern Methodist					
Sayipjamal Dulat	Xinjiang Univ.					
CP. Yuan	Michigan State					
Migual Arratia	UC Riverside					
Alberto Accardi Yulia Furletova	Jefferson Lab					

 the goal: use recentlydeveloped tools for PDF global analyses to examine the PDF pulls of EIC pseudodata

> 1803.02777 1806.07950 1904.00022 1907.00988 2001.07862



• needed for the Yellow Report Initiative: <u>quick, unambiguous</u> PDF impact metrics

 $\rightarrow$  in turn, these can be incorporated into the YR workflow:

iteratively, machine design  $\rightarrow$  simulation  $\rightarrow$  physics

 $\rightarrow$  speed, simplicity, clarity can help ensure robust and timely convergence  $_3$ 

## high-energy EIC pseudodata

- reach in center-of-mass energy,  $20 \leq \sqrt{s} \leq 140\,{
m GeV}$ 

 $\rightarrow$  luminosities 2-3 decades greater than at HERA

→ á la HERA, the combination of precision & kinematic coverage provide constraining 'lever arm' on QCD evolution

 $\rightarrow$  QCD evolution: (high x, low Q)  $\leftrightarrow$  (low x, high Q)

- - generated based on CT14 $_{\rm HERA2}$  NNLO PDF fit

### Hessian profiling [ePump] for EIC impacts on PDF errors

ePump: Schmidt, Pumplin, and Yuan; PRD98 (2018) no.9, 094005



EIC pseudodata supersede fixed-target DIS information in CT fits

• reweighting strongly depends on tolerance; CT-like tolerance used here

 $\rightarrow$  complementary approaches welcome! 5



# | $S_{\rm f}$ | for $d(x,\mu)$ CT14 Hera2 NNLO

 note: with PDFSense, already employing kinematical maps for

#### in development: these maps can eventually be interfaced with bin-by-bin fast stat. uncertainty estimates

 $\rightarrow$  for 1 fb<sup>-1</sup>; can recover stat. uncertainties for desired lumi.

	102	$ S_{f}  > 0.25$											$ \sim f $					I		
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9		
5	3 697 277	2 004 394	1335307	984 580	771 579	629 786	529 176	454 253	396 474	2 346 683	1063724	526723	250 610	108 060	39648	11096	1838	80		
10	563 217	331 880	228 507	171 010	134 964	110 518	92 967	79792	69 577	407 584	179 488	86233	39818	16648	5906	1588	250	10		
15	70225	94 233	67 300	51929	41850	34770	29 552	25 555	22 409	132 685	58477	27828	12692	5232	1826	481	74	3		
20	Θ	19037	25757	20153	16537	13972	12040	10528	9314	56377	25241	12009	5451	2231	771	201	30	1		
25	0	Θ	3446	8360	7677	6504	5655	4994	4459	27743	12740	6097	2766	1128	388	100	15	1		
30	0	Θ	Θ	308	2630	3441	2978	2626	2353	15001	7098	3432	1562	637	218	56	8	Θ		
35	0	Θ	Θ	Θ	Θ	364	1260	1524	1360	8704	4232	2074	949	387	133	34	5	Θ		
40	0	Θ	Θ	Θ	Θ	Θ	Θ	187	555	5324	2653	1320	608	249	85	22	3	Θ		
45	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	2818	1731	873	406	167	57	15	2	Θ		
50	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	1207	1172	596	280	116	40	10	1	Θ		
55	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	421	822	418	198	82	28	7	1	Θ		
60	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	44	594	301	143	60	21	5	1	Θ		
65	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	280	222	105	44	15	4	1	Θ		
70	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	91	169	79	33	12	3	0	Θ		
75	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	5	116	61	25	9	2	Θ	Θ		
80	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	Θ	45	48	20	7	2	0	Θ		
85	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	Θ	7	38	16	5	1	Θ	Θ		
90	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	Θ	Θ	18	13	4	1	Θ	Θ		
95	0	Θ	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	Θ	Θ	3	10	4	1	Θ	Θ		
100	Θ	Θ	Θ	Θ	Θ	Θ	Θ	0	Θ	0	0	Θ	0	5	3	1	0	0		

GeV

for NC *e*-*p* at  $\sqrt{s} = 140 \text{ GeV}_{r,00} + \frac{Y_{-}}{2}xW_{3}^{+} - \frac{y^{2}}{2}W_{L}^{+}$ 

Х

С

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- in addition to fast impact methods, direct fitting is available for end-stage analyses
- results align closely with fast methods; *i.e.*, **PDFSense** and **ePump**



#### EIC pseudo-data fitted against legacy CT data sets

- beyond fitting EIC over present data, comparisons with legacy data are instructive
- in CT, vFe dimuon production (NuTeV, CCFR) are important constraints on s(x,Q)
- especially without  $e^+$ , EIC CC inclusive DIS data struggle to compete



Unraveling PDFs' flavor dependence is challenging; multiple EIC channels/processes needed



#### note PDFs' different orders-of-mag.!

NC DIS: sensitivity to d-type quarks  $\frac{1}{4}$  that of u-type

$$\sigma \propto \frac{4}{9}(u_+ + c_+) + \frac{1}{9}(d_+ + s_+ + b_+)$$

CC DIS: lower accuracy (1/10 lumi.)

high x (>0.1) inclusive DIS

- ightarrow u-quark dominates
- ightarrow d-quark  $\frac{1}{2}$  of u, but harder to access in NC DIS (above)

 $ightarrow ar{d} + ar{u} \; \sim \;$  few percent of u

...1% error on  $u \to$  50-100% error on  $\bar{d} + \bar{u}$ 

ightarrow for x~0.1,  $s \approx \overline{s} \approx \overline{d} - \overline{u} < 0.1(\overline{d} + \overline{u})$  ightarrow at x>0.5, no separation for  $\overline{u}, \overline{d}, \overline{s}$  9 (ii) <u>case study:</u> accessing nucleon strangeness in (tagged) CC processes?

 $\rightarrow$  in CT, main inputs are from vDIS on heavy nuclei (nuclear corrs. relevant)

 $\rightarrow$  of interest: the strange suppression ratio,  $R_s = \frac{s+\bar{s}}{\bar{u}+\bar{d}}$ 

 $\rightarrow$  typical QCD fits find R<sub>s</sub> ~ 0.5; ATLAS W/Z production favors R<sub>s</sub> ~ 1

# $\rightarrow$ question: can CC charm jet production off <u>proton</u> distinguish small from large R<sub>s</sub>?





sensitivity of CC charm jet production to strange suppression

see talks: S. Sekula (Inclusive/SIDIS/Jets-HQ Joint) & M. Arratia

- 100 fb<sup>-1</sup> CC DIS (10M simulated events), at 10x275 GeV ( $e^{-}$  on p);  $Q^{2}$  > 100 GeV<sup>2</sup>
- even assuming conservative charm-tagging efficiency, event-level discrimination potential is substantial, relative to statistical uncertainties



## (iii) input theory grids from modern CT global fits

CT-based theory grids are currently in development

for preliminary testing and benchmarking, we have initially made available: 1.234100E-04 1.507330E-0 112530E-04 7.465860E-04 9. 737950E-03 8.229750E-03 1.00  $F_{2}^{\gamma}(900); F_{L}^{\gamma}(901)$ 427360E-02 9.071800E-02 2.635970E-01 2. 488120E-01 5.611440E-01 5.73 5.998290E-01 6. 165310E-01 7.326320E-01 7.49 390E-01 8.007370E-(930);  $F_L^{W^-}$  (931);  $F_3^{W^-}$  (932) 559400E-01 8.623030E-01 8.687  $F_2^W$ 355070E-01 9.424620E-01 9.494700E-01 6800F-01 8 882 396640E+00 4.001450E+00 4.75000 468640E+01 7.507240E+01 1.047120E+02 0720E+00 8.758190E+0 2.143800E+02 3.152120E+0 901 930 931 932 7.02993E-01 2.68314E-02 3.34988E+00 9.58268E-01 2.67969E-02 3.37103E+00 9.63083E-01 3.41008E-02 4.34177E+00 1.23546E+00 -5.49899E  $\rightarrow$  based on CT18 NNLO 1.22029E+00 5.09282E-02 5.60916E+00 1.50347E+00 -6.28726E 696E+00 8.47902E-02 7.10935E+00 1.83204E+00 1.37909E-01 8.86982E+00 2.22522E+00 -7.64617 2.36476E+00 2.12566E-01 1.09098E+01 2.68973E+00 -8.11648E-01  $\rightarrow$  other targets (e.g., deuteron), boson/SF 3.10932E-01 1.32566E+01 3.23596E+00 -8.40145 4.35331E-01 1.59289E+01 3.87542E+00 -8.45608 combinations can be generated as needed 5.88655E-01 1.89499E+01 4.61887E+00 -8.26580E 7.71996E-01 2.23491E+01 5.47507E+00 -7.83729E 1.01494E+00 2.65775E+01 6.61457E+00 27277E+00 3.13436E+01 7.84127E+00  $\rightarrow$  different theory settings also quickly done 9.06879E+00 1.56691E+00 3.67284E+01 9.26204E+00 -5.13664E 1.89701E+00 4.28131E+01 1.09051E+01 26086E+00 4.96774E+01 • pQCD choices [  $O(\alpha_{e}), m_{e}, \dots$  ] 2.65660E+00 5.74157E+01 1.49432E+01 .07804E+00 6.61715E+01 1.74206F+01 -8.68745E 2.03048F+01 51927E+00 7.61371E+01 3.97423E+00 8.74733E+01 2.35942E+01 alternate fits: 4.43726E+00 1.00317E+02 2.71613E+01 -6.30205E 4.90898E+00 1.15002E+02 3.08598E+01 5.38836E+00 1.31991E+02 3.43119E+01 5.88410E+00 1.51426E+02 3.66519E+01  $\rightarrow$  CT18X (modified DIS scale) 6.42754E+00 1.73169E+02 3.74007E+01 7.10023E+00 1.97291E+02 3.74439E+01 5.05488E+01 2.73262E-02 3.30746E+00 9.29410E-01 → CT18Z (ATLAS W/Z: s-PDF) 2.73087E-02 3.32808E+00 9.34205E-01 6.99809E-01 9.35754E-01 3.54460E-02 4.27011E+00 1.20284E+00 -5.42529E-01

## meanwhile: the Snowmass kickoff overlaps with this meeting...

 the PDF/pQCD focuses inside the YR IRG (as well as HQ/SIDIS) are central to Snowmass objectives [which has comparable timescale]

a community is coalescing around this topic



LPC Workshop on

## PHYSICS CONNECTIONS BETWEEN THE LHC AND EIC

Fermilab LHC Physics Center (LPC) November 13-15, 2019

Exploring physics intersections between LHC phenomenology and a future Electron-Ion Collider (EIC) program via:

- Precision QCD
- Monte Carlo Event Generators
- Lattice QCD

Organizing Committee Tim Hobbs (Chair, SMU) Abhay Deshpande (BNL) Jianwei Qiu (JLab) Rik Yoshida (ANL)

Local Organizing Committee Radja Boughezal (ANL/Northwestern) John Campbell (FNAL) Olga Evdokimov (UIC) Stefan Hoeche (FNAL) Frark Petriello (ANL/Northwestern) LPC Events Committee Gabriele Benelli (Brown) Kevin Pedro (FNAL) LPC Coordinators Cecilia Gerber (UIC) Sergo Jindariani (FNAL)

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https://indico.cern.ch/e/LHCEICPhysics

Electroweak/neutrino phenomenology

Machine learning & computation

BSM physics searches





 as a machine for precision QCD, this will have serious implications for Energy Frontier efforts

e.g., PDF improvements  $\rightarrow$  SM precision

- the EIC will intersect multiple Snowmass frontiers/topics
  - ightarrow coordination within the EICUG needed

full program available:

https://indico.cern.ch/e/LHCEICPhysics

# conclusions, future steps

#### • post-Temple updates

 $\rightarrow$  validation of fast methods with direct pseudo-data fits

- $\rightarrow$  reproduce fast-method conclusions
- $\rightarrow e^+$  data play a vital role in extending EIC impact

also relevant for CSV,  $\,s 
eq \bar{s}\,$ 

 $\rightarrow$  initial examination of event-level sensitivity of CC DIS charm jet prod.

 $\rightarrow$  provides another avenue to unravel light-quark sea:  $s(x), R_s(x)$ see talks: S. Sekula (Inclusive/SIDIS/Jets-HQ Joint) & M. Arratia

- $\rightarrow$  input theory grids according to CT are in production; initial grids released
  - $\rightarrow$  extension/rapid updates possible based on need
- going forward: build on these studies by examining dependence on systematics



supplementary material -

#### sensitivities can be aggregated for direct comparisons of exps



### sensitivities can be aggregated for direct comparisons of exps







#### $|S_f|$ for g(x, $\mu$ ), CT14 HERA2 NNLO



II) visualizing impacts with PDFSense

Phys.Rev. D98 (2018) 094030



х

### $|S_f|$ for $\sigma_H 0$ 14 TeV, CT14<sub>HERA2</sub> NNLO



B.-T. Wang, TJH, S. Doyle, J. Gao, T.-J. Hou, P. M. Nadolsky, F. I. Olness

Phys.Rev. D98 (2018) 094030

(magnitude of PDF pull of each datum)

 $|S_f|$ 

1.2

1.0

0.8

0.6

0.4

0.2

0

 after the aggregated HERA data, inclusive jet production – greatest total sensitivity!

► large correlations for E866, BCDMS, CCFR, CMS WASY, Z p<sub>T</sub> and ttbar production, but smaller numbers of highly-sensitive points

#### LHC Run-1 gluon PDF impact in CT14 $\rightarrow$ CT18(Z)



• while LHC Run-1 data drive important PDF improvements, including for the gluon at high-, low-x, the effect is relatively incremental

the EIC tomography program will deliver high-precision DIS

 by measuring the nucleon's multi-dimensional wave function with high precision, the EIC will hugely constrain proton collinear structure



 DIS cross sections from EIC will supercede the bulk of fixed-target information in contemporary QCD fits; provide an 'anchor-point' to resolve systematic PDF tensions



- the impact of an EIC upon the theoretical predictions for inclusive Higgs production arises from a very broad region of the kinematical space it can access
- impact rather closely tied to that of the integrated gluon PDF:





# An EIC would drive lattice phenomenology

arXiv:1904.00022 [hep-ph]

- A high-luminosity lepton-hadron collider will impose very tight constraints on many lattice observables; below, the isovector first moment and qPDF; this is crucial for benchmarking!





• PDFSense identifies the most sensitive experiments with high confidence and in accord with other methods such as the LM scans. It works the best when the uncertainties are nearly Gaussian, and experimental constraints agree among themselves [arXiv:1803.02777]

#### PDFSense predictions can be validated against actual fits



- PDFSense successfully predicts the highest impact data sets before fitting, as shown in this illustration for the large x PDF ratio  $\,d/u\,$
- Lagrange Multiplier scans provide an independent test of which datasets most drive the global fit in connection with specific PDFs

#### HERA and fixed-target (BCDMS, NMC) data are dominant!

 $\sin^2 \theta_W$  (and, eventually,  $M_W$ )

...as a follow-on to Alesandro's EW-focused overview:

important PDF correlations for the ATLAS extraction of  $\sin^2 heta_W$ 

# Example: $\sin^2 \theta_{weak} \equiv s2w$ measured by ATLAS 8 TeV

Correlation, sinθ<sub>w</sub> (ATLAS 8 TeV CB) and f(x,Q) at Q=81.45 GeV 2018/11/11, PRELIMINARY, CT14 NNLO



Strongest correlations of s2w with  $u_{val}$ ,  $d_{val}$  at  $0.005 \lesssim x \lesssim 0.2$ 

weak correlations with  $\bar{u}$ ,  $\bar{d}$  ,  $\bar{s}$ , g

 $u_{val}$ ,  $d_{val}$  changed between CT10 and CT14 [1506.07433, Sec. 2B]

It is instructive to explore the data pulls on  $u_{val}, \, d_{val}$ 

rather than the costly LM scans, we can examine a "cheaper" measure which yields comparable information

the L2 sensitivity



 $L_2$  sensitivity. Take  $X = f_a(x_i, Q_i)$  or  $\sigma(f)$ ;  $Y = \chi_E^2$  for experiment E. Find  $\Delta Y(\vec{z}_{m,X})$  for the displacement  $|\vec{z}_{m,X}| = 1$  along the direction  $\vec{\nabla}X/|\vec{\nabla}X|$  (corresponding to  $\Delta \chi_{tot}^2 = T^2$  and  $X(\vec{z}) = X(0) + \Delta X$ ):

$$S_{f,L_2} \equiv \Delta Y(\vec{z}_{m,X}) = \vec{\nabla}Y \cdot \vec{z}_{m,X} = \vec{\nabla}Y \cdot \frac{\vec{\nabla}X}{|\vec{\nabla}X|}$$
  
or,  $\sim \operatorname{Corr}[f_a, \chi_E^2]$   $= \Delta Y \cos \varphi$ 

...extent to which total  $\chi^2_{_{\rm F}}$  of specific expts. correlates with x-dep. of PDFs



CT18 NNLO,  $d_V(x,Q)(x, 100 \text{ GeV})$ 



CT18Z NNLO, s(x, 2 GeV)



L<sub>2</sub> sensitivity, strangeness: CT18Z

#### Most sensitive experiments

246 LHCb8Zeer	12
<del>248</del> ATL7ZW.xF	12
250 LHCb8WZ	12
<del>251</del> ATL8DY	12
<del>542</del> CMS7jtR7y6T	26
<del>545</del> CMS8jtR7T	26
HERAIPII	26
102 BcdF2dCor	

- --124--- NuT∨NuChXN
- ---<del>125</del>--- NuT∨NbChXN
- ----126--- CcfrNuChXN
- ----127---- CcfrNbChXN
- ---<mark>201</mark>--- e605
- ----<del>203</del>--- e866f
- ----204---- e866ppxf

A tension trend between DIS (HERA I+II, CCFR, NuTeV) and Drell-Yan (ATLAS 7 Z/W, LHCb W/Z, E866 pp, ...) experiments

pronounced effect of ATLAS 7 TeV Z/W data!

 the CTEQ-TEA global analysis relies on the Hessian formalism for its error treatment

 $\chi_{E}^{2}(\vec{a}) = \sum_{i=1}^{N_{pt}} r_{i}^{2}(\vec{a}) + \sum_{\alpha=1}^{N_{\lambda}} \overline{\lambda}_{\alpha}^{2}(\vec{a}) \qquad \text{nuisance parameters to handle correlated errors}$   $r_{i}(\vec{a}) = \frac{1}{s_{i}} \left( T_{i}(\vec{a}) - D_{i,sh}(\vec{a}) \right)$ these result in systematic  $D_{i} \rightarrow D_{i-1}(\vec{a}) = D_{i} - \sum_{j=1}^{N_{\lambda}} \beta_{i-j} \overline{\lambda}_{j-j}(\vec{a})$ 

these result in systematic shifts to data central values:

$$D_i \to D_{i,sh}(\vec{a}) = D_i - \sum_{\alpha=1}^{N_{\lambda}} \beta_{i\alpha} \overline{\lambda}_{\alpha}(\vec{a})$$

• a 56-dimensional parametric basis  $\vec{a}$  is obtained by diagonalizing the Hessian matrix H determined from  $\chi^2$  (following a 28-parameter fit)

use this basis to compute 56component "normalized" residuals :

$$\delta_{i,l}^{\pm} \equiv \left( r_i(\vec{a}_l^{\pm}) - r_i(\vec{a}_0) \right) / \langle r_0 \rangle_E$$

where 
$$\langle r_0 
angle_E \equiv \sqrt{rac{1}{N_{pt}}\sum_{i=1}^{N_{pt}}r_i^2(ec{a}_0)}$$



... but how does the behavior of these residuals relate to the fitted PDFs and their uncertainties?

for example, how does the PDF uncertainty (at specific x,  $\mu$ ) correlate with the residual associated with a theoretical prediction at the same x,  $\mu$ ?

examine the Pearson correlation over the 56-member PDF error set between a PDF of given flavor and the residual



[X,Y] are exactly (anti-)correlated at the far (right) left above.

 we may then evaluate correlations between arbitrary PDF-derived quantities over the ensemble of error sets ([X,Y] may be PDFs, cross sections, residuals,...):

$$\operatorname{Corr}[X,Y] = \frac{1}{4\Delta X \Delta Y} \sum_{j=1}^{N} (X_j^+ - X_j^-)(Y_j^+ - Y_j^-) \qquad \Delta X = \frac{1}{2} \sqrt{\sum_{j=1}^{N} (X_j^+ - X_j^-)^2}$$

# Correlation $C_f$ and sensitivity $S_f$

The relation of data point i on the PDF dependence of f can be estimated by:

•  $C_f \equiv \operatorname{Corr}[\rho_i(\vec{a})), f(\vec{a})] = cos\phi$  $\vec{\rho}_i \equiv \vec{\nabla} r_i / \langle r_0 \rangle_E$  -- gradient of  $r_i$  normalized to the r.m.s. average residual in expt E;

$$\left(\vec{\nabla}r_i\right)_k = \left(r_i(\vec{a}_k^+) - r_i(\vec{a}_k^-)\right)/2$$



$$\operatorname{Corr}[X,Y] = \frac{1}{4\Delta X\Delta Y} \sum_{j=1}^{N} (X_j^+ - X_j^-)(Y_j^+ - Y_j^-)$$

 $\mathcal{N}$ 

 $C_f$  is **independent** of the experimental and PDF uncertainties. In the figures, take  $|C_f| \ge 0.7$  to indicate a large correlation.

• 
$$S_f \equiv |\vec{\rho}_i| \cos\varphi = C_f \frac{\Delta r_i}{\langle r_0 \rangle_E}$$
 -- projection of  $\vec{\rho}_i(\vec{a})$  on  $\vec{\nabla} f$ 

 $S_f$  is proportional to  $\cos\varphi$  and the ratio of the PDF uncertainty to the experimental uncertainty. We can sum  $|S_f|$ . In the figures, take  $|S_f| > 0.25$  to be significant.

## 2<sup>nd</sup> aside: kinematical matchings

 $\cap$ 

 residual-PDF correlations and sensitivities are evaluated at parton-level kinematics determined according to leading-order matchings with physical scales in measurements

deeply-inelastic 
$$\mu_i \approx Q|_i, \ x_i \approx x_B|_i$$
  
scattering:

$$x_i$$
: parton mom. fraction

$$\mu_i$$
 : factorization scale

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#### hadron-hadron collisions:

scattering:

$$AB \to CX$$
  $\mu_i \approx Q|_i, \ x_i^{\pm} \approx \frac{Q}{\sqrt{s}} \exp(\pm y_C)\Big|_i$ 

 $Q = 2p_{Tj}, y_C = y_j$ single-inclusive jet production:

$$t\bar{t}$$
 pair production:  $Q = m_{t\bar{t}}, \ y_C = y_{t\bar{t}}$  etc...

 $d\sigma/dp_T^Z$  measurements:  $Q = \sqrt{(p_T^Z)^2 + (M_Z)^2}, \ y_C = y_Z$