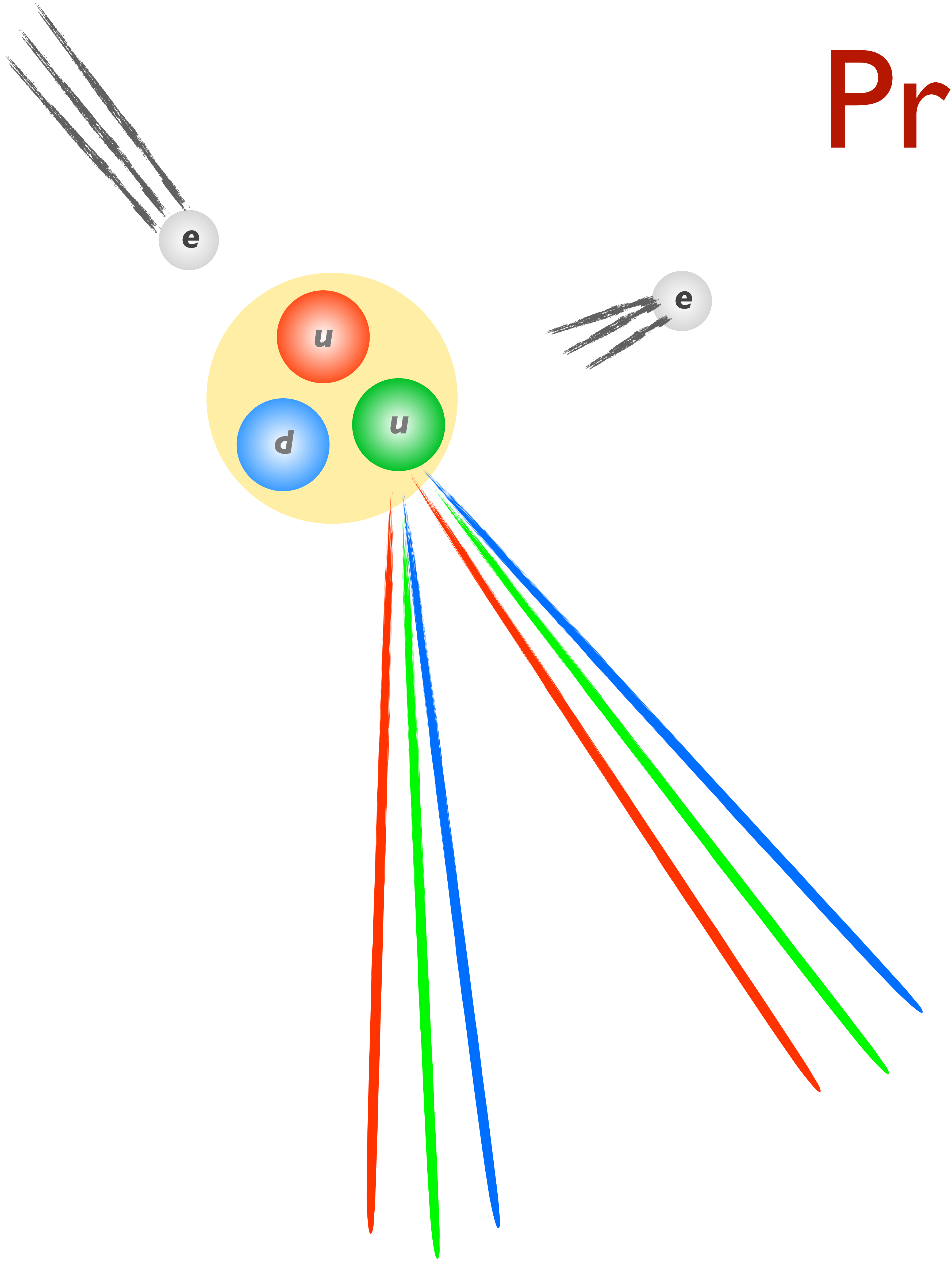


Probing the strong interaction with DIS event shapes

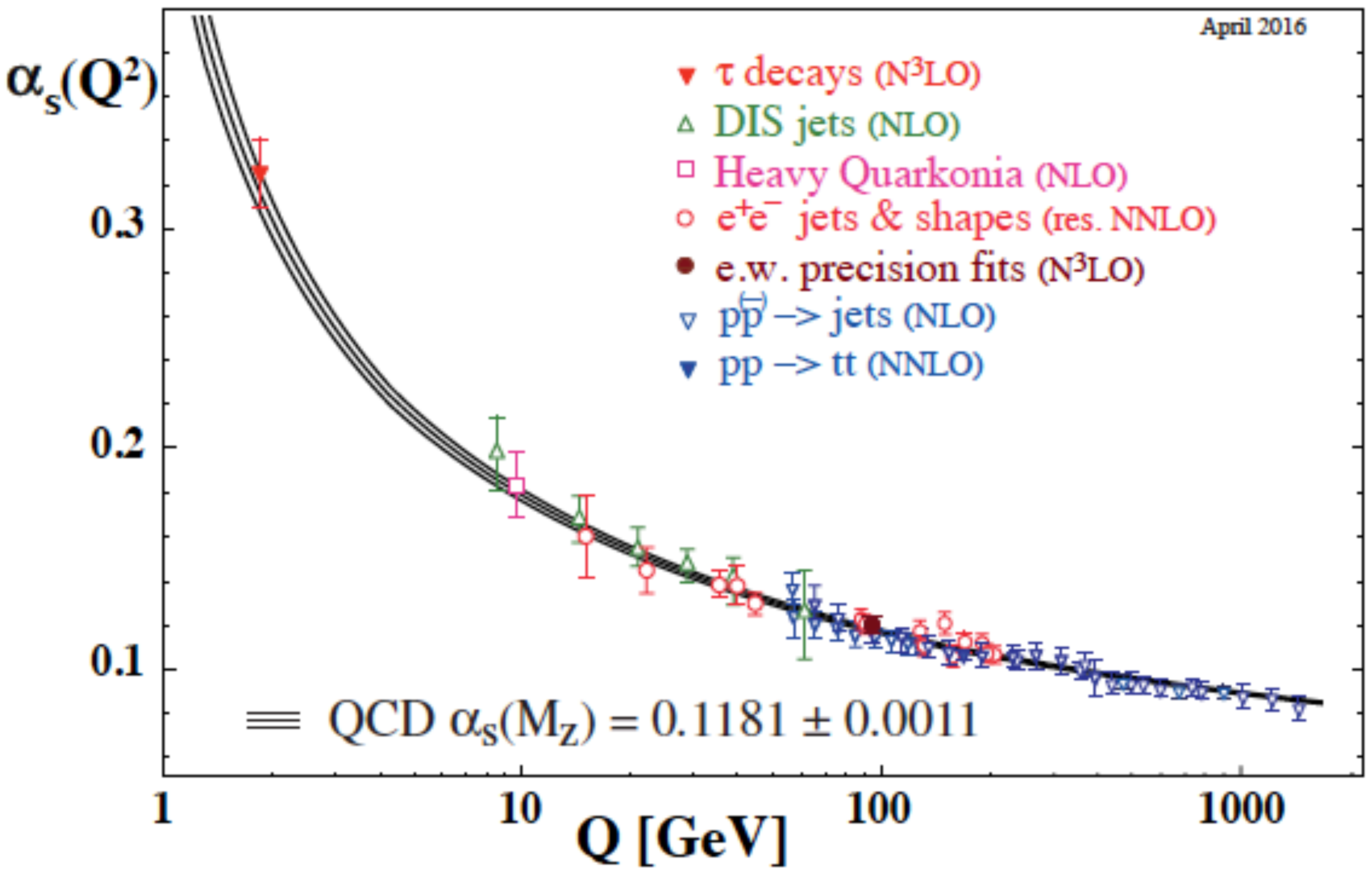
Christopher Lee, LANL

In collaboration with
Daekyoung Kang & Iain Stewart

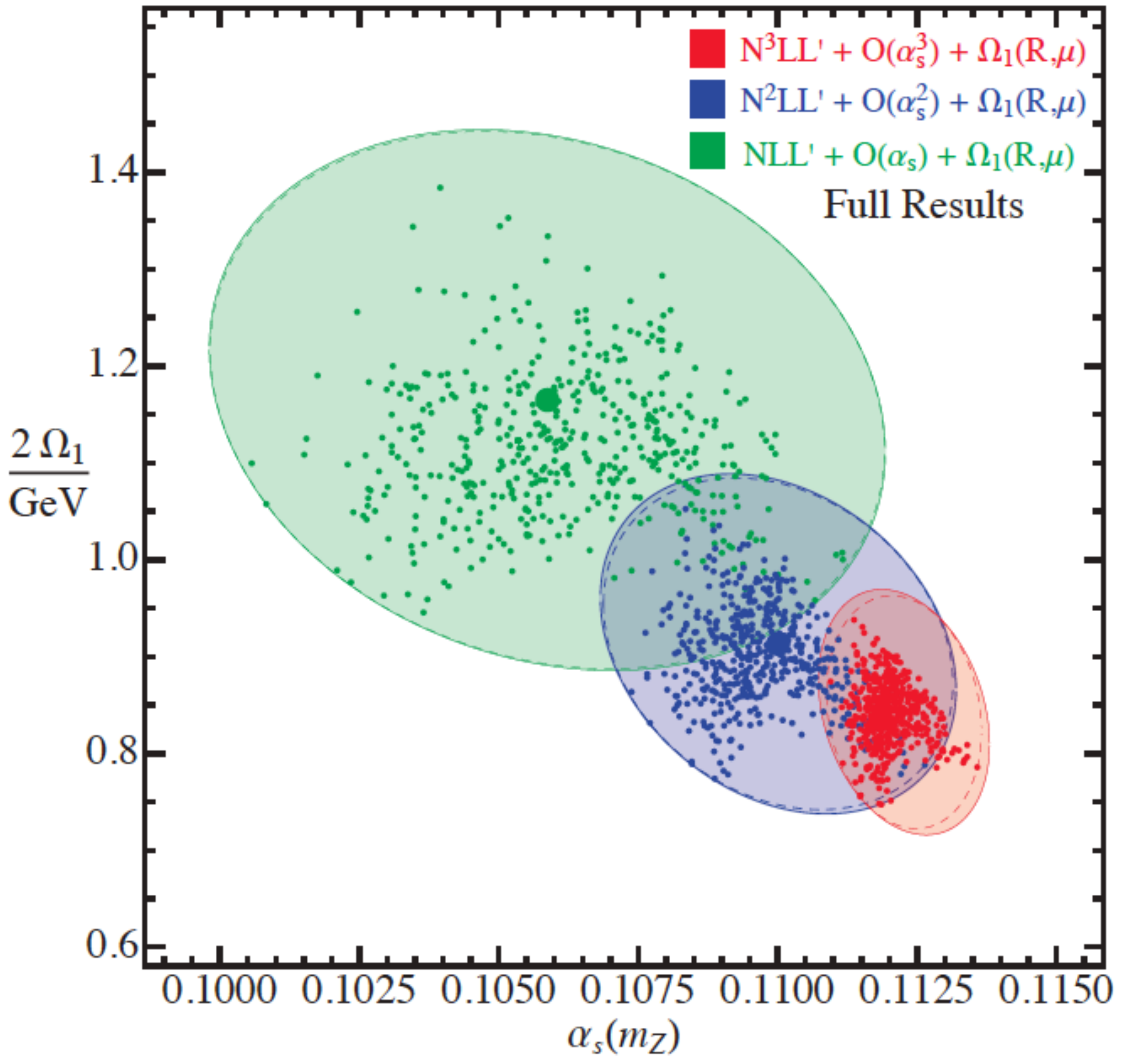
EIC Jets & HQ Working Group Meeting
April 6, 2020

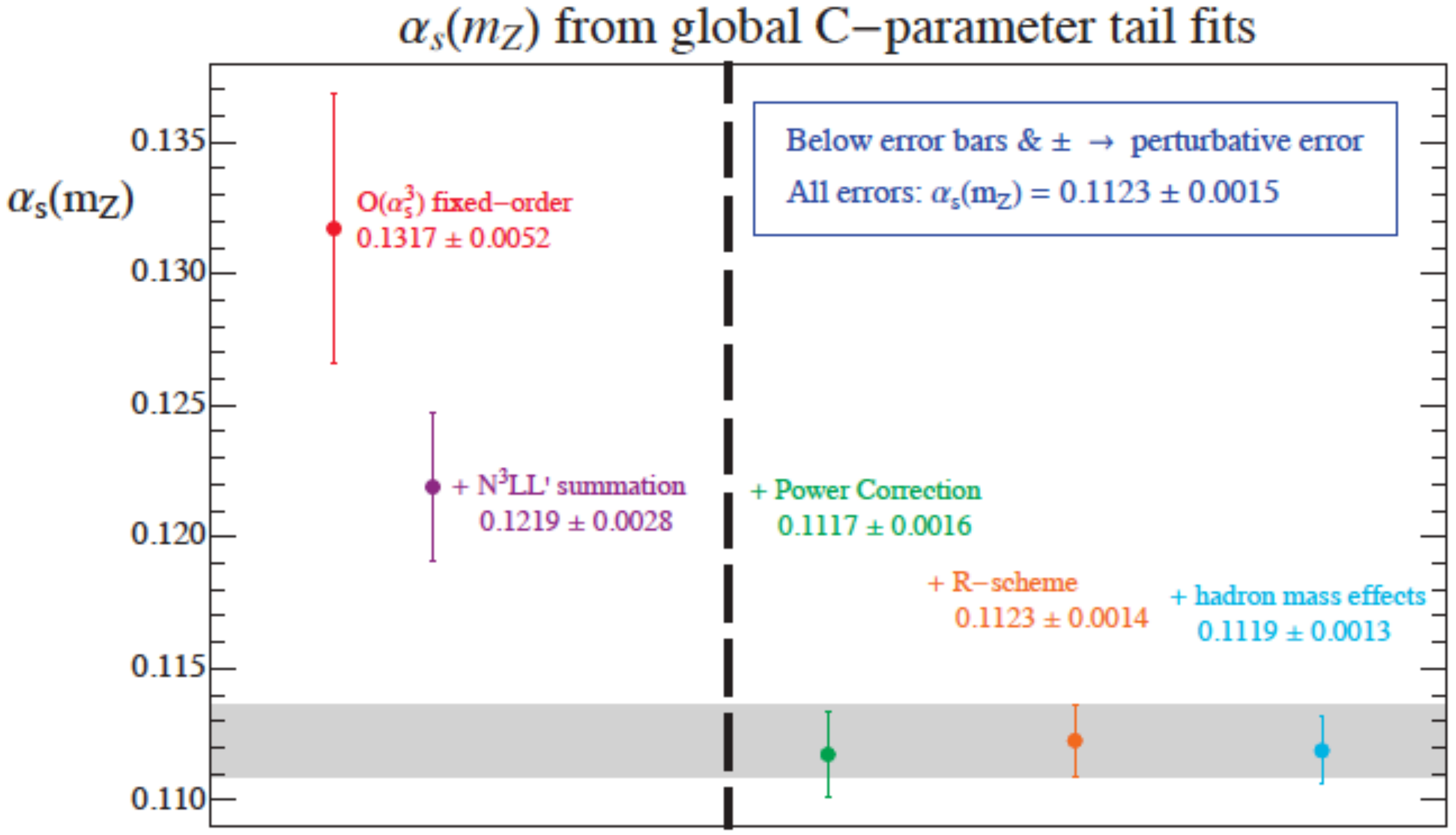
Event shapes and the strong coupling

PDG 2016:

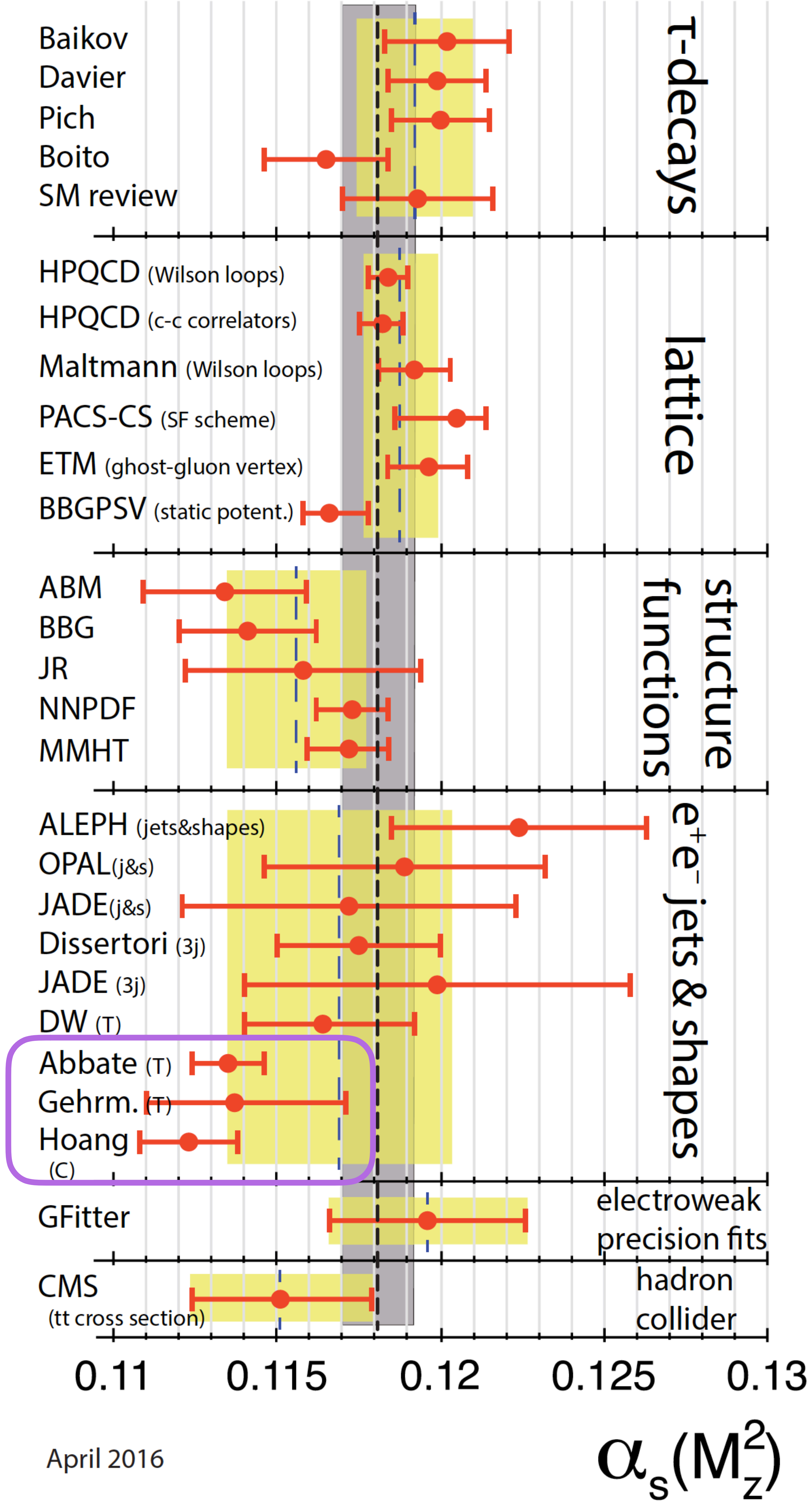


Hoang et al., PRD 91 (2015) 094018:

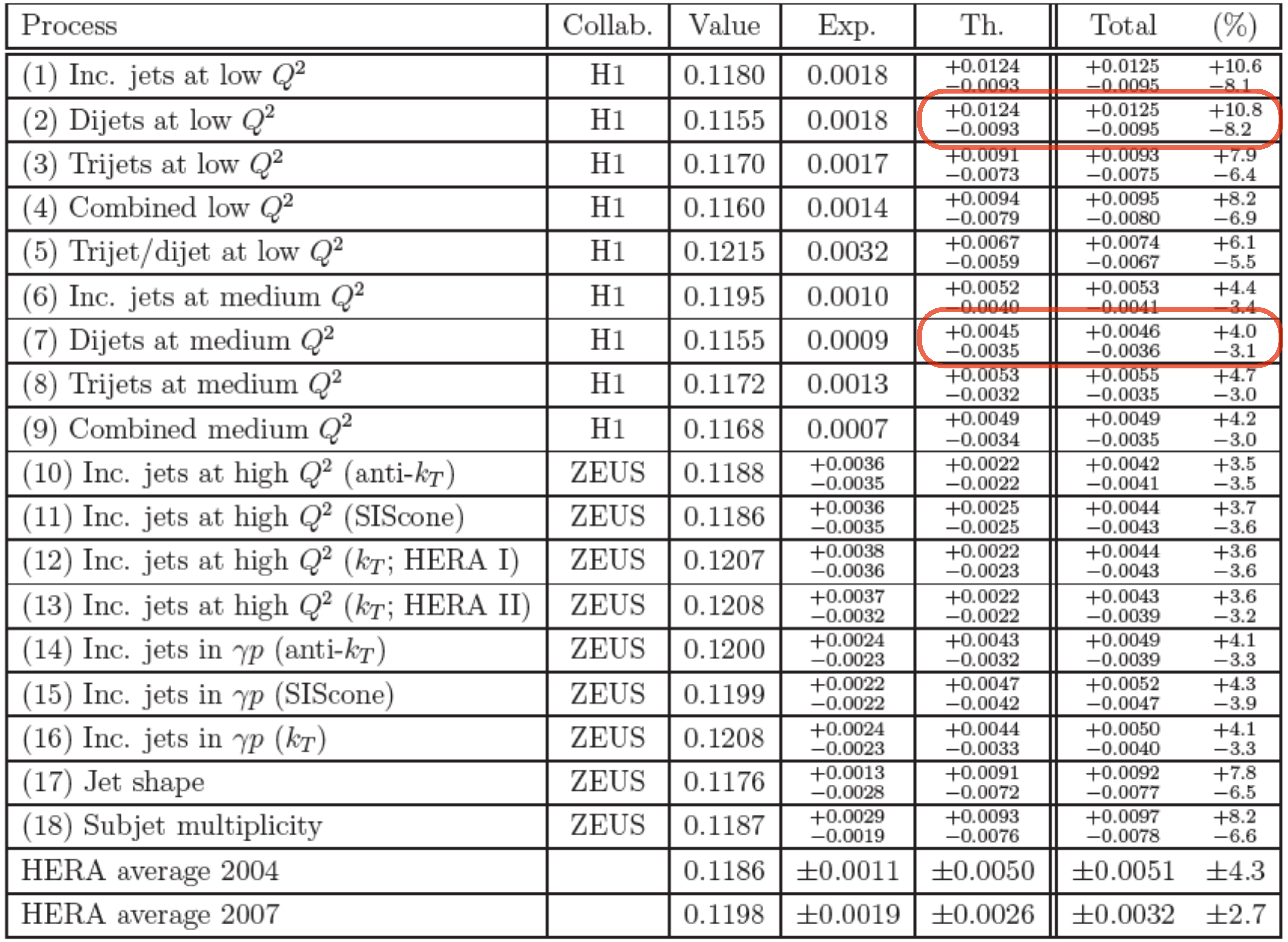




Event shapes



Jets in DIS and the strong coupling



Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	±0.0011	±0.0050	±0.0051	±4.3
HERA average 2007		0.1198	±0.0019	±0.0026	±0.0032	±2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

Extractions from exclusive jet cross sections have order 10% uncertainty, dominated by theory

Improve to level of e^+e^-?

C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

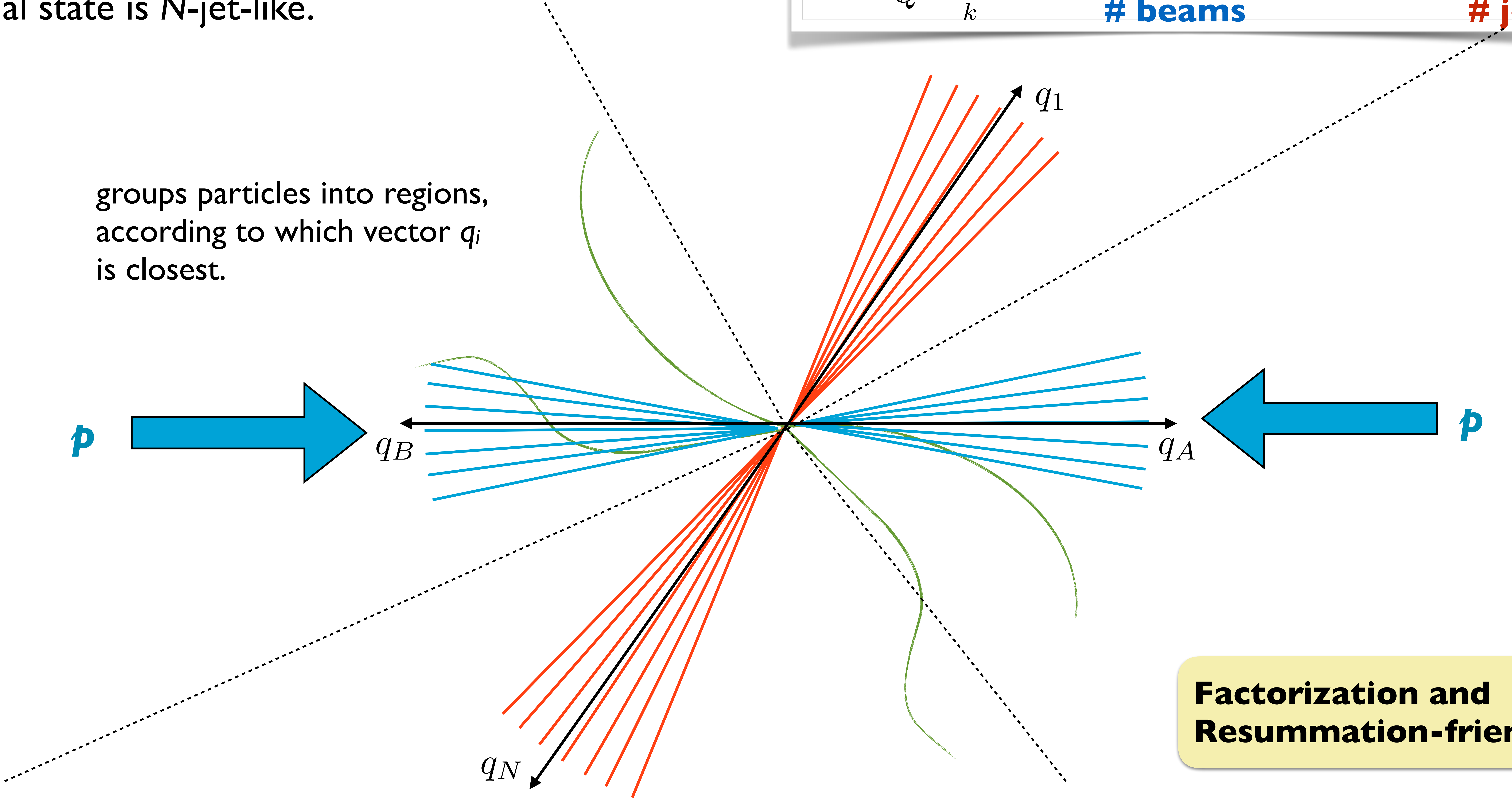
N-jettiness

Stewart, Tackmann, Waalewijn (2010)

- A global event shape measuring degree to which final state is N-jet-like.

$$\tau_N = \frac{2}{Q^2}\sum_k \min\{\underbrace{q_A\cdot p_k, q_B\cdot p_k}_{\#\text{ beams}}, \underbrace{q_1\cdot p_k,\ldots,q_N\cdot p_k}_{\#\text{ jets}}\}$$

groups particles into regions, according to which vector q_i is closest.

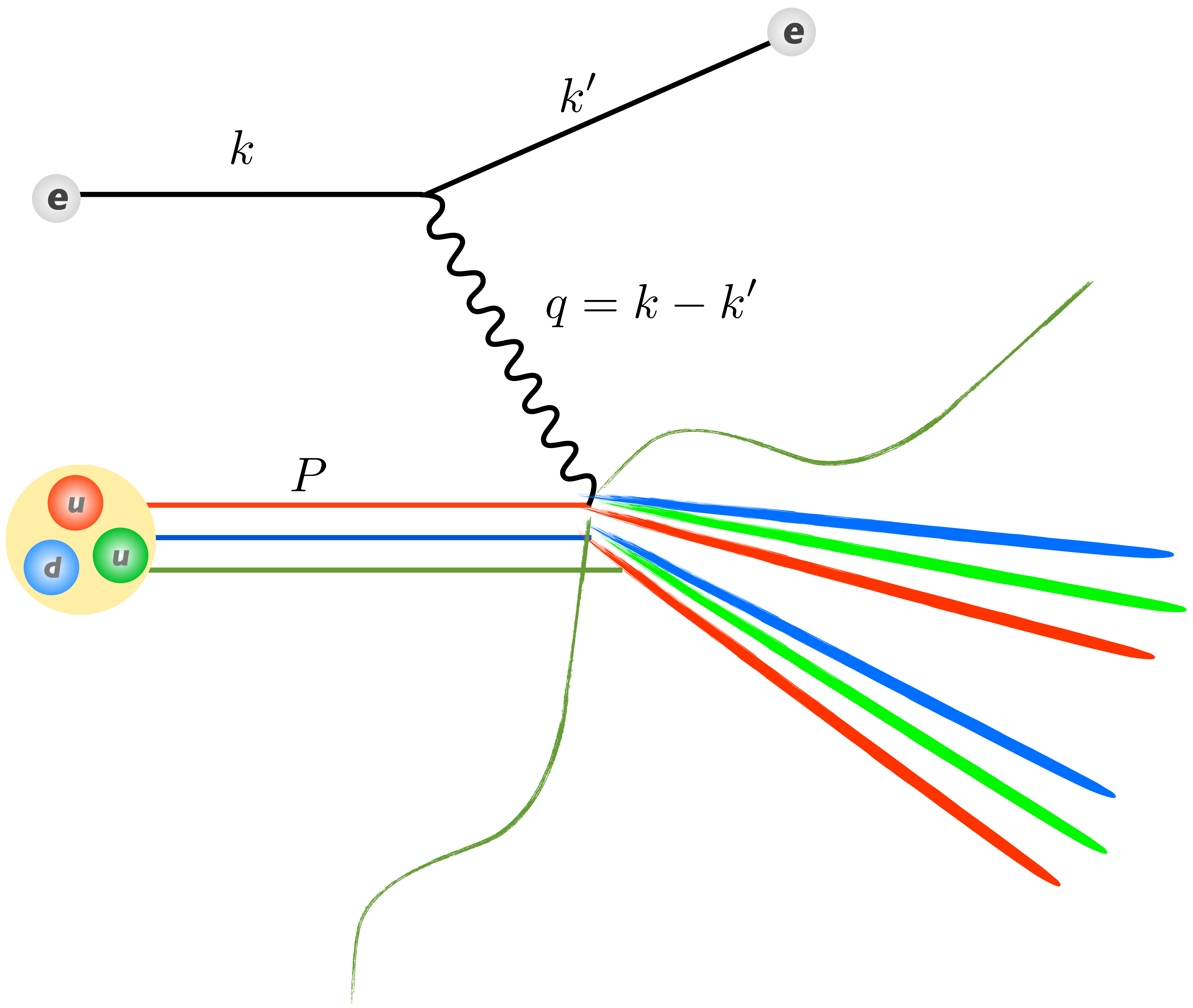


Factorization and Resummation-friendly

1-Jettiness in DIS

D. Kang, CL, I. Stewart (2013)

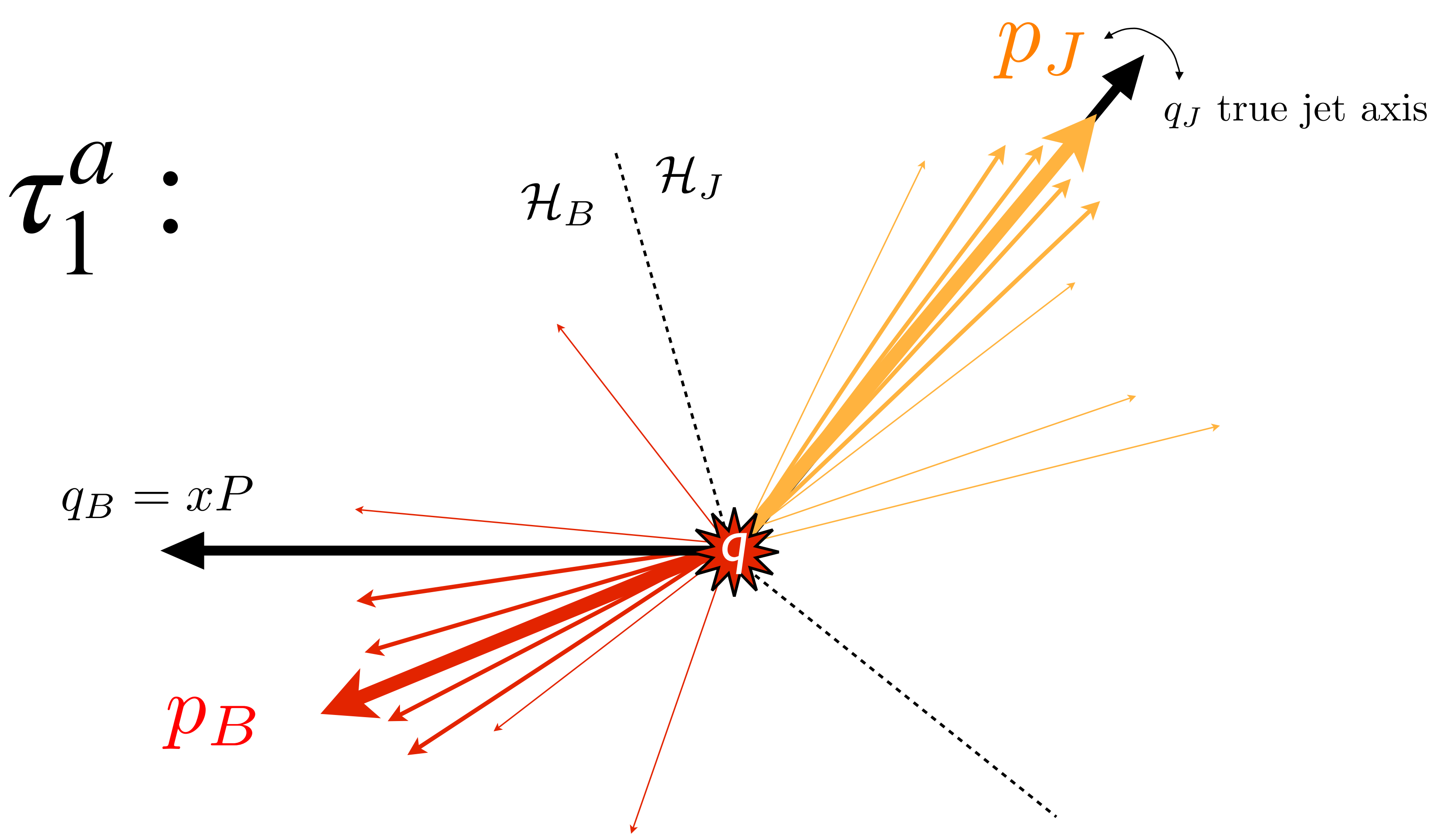
also Z. Kang, Liu, Mantry, Qiu (2012, 2013)



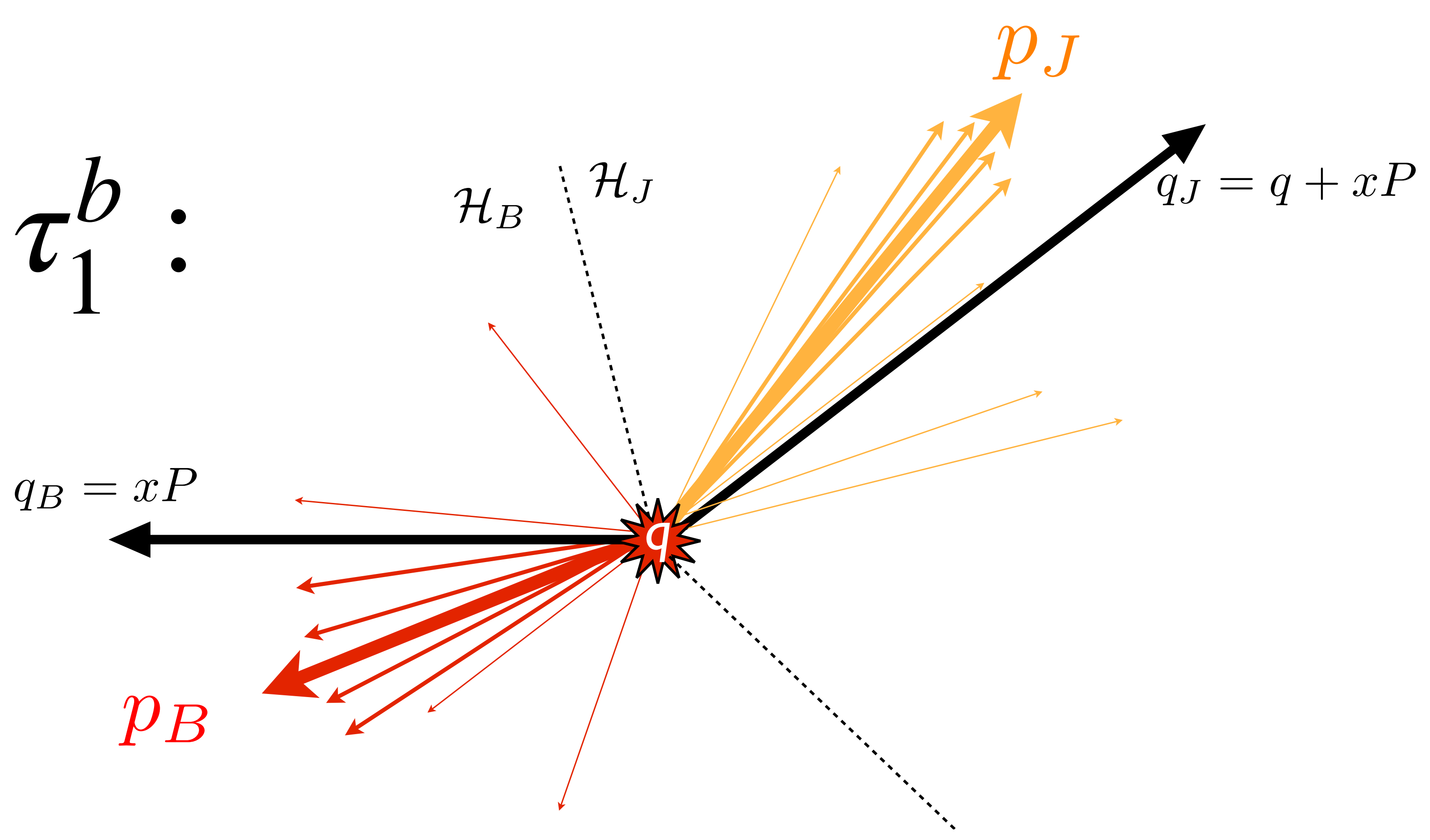
- "1-jettiness" in DIS measures final states with beam radiation + one additional jet

$$\tau_1 = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

τ_1^a :



τ_1^b :



- Different choices of axes are possible: different sensitivity to ISR transverse momentum

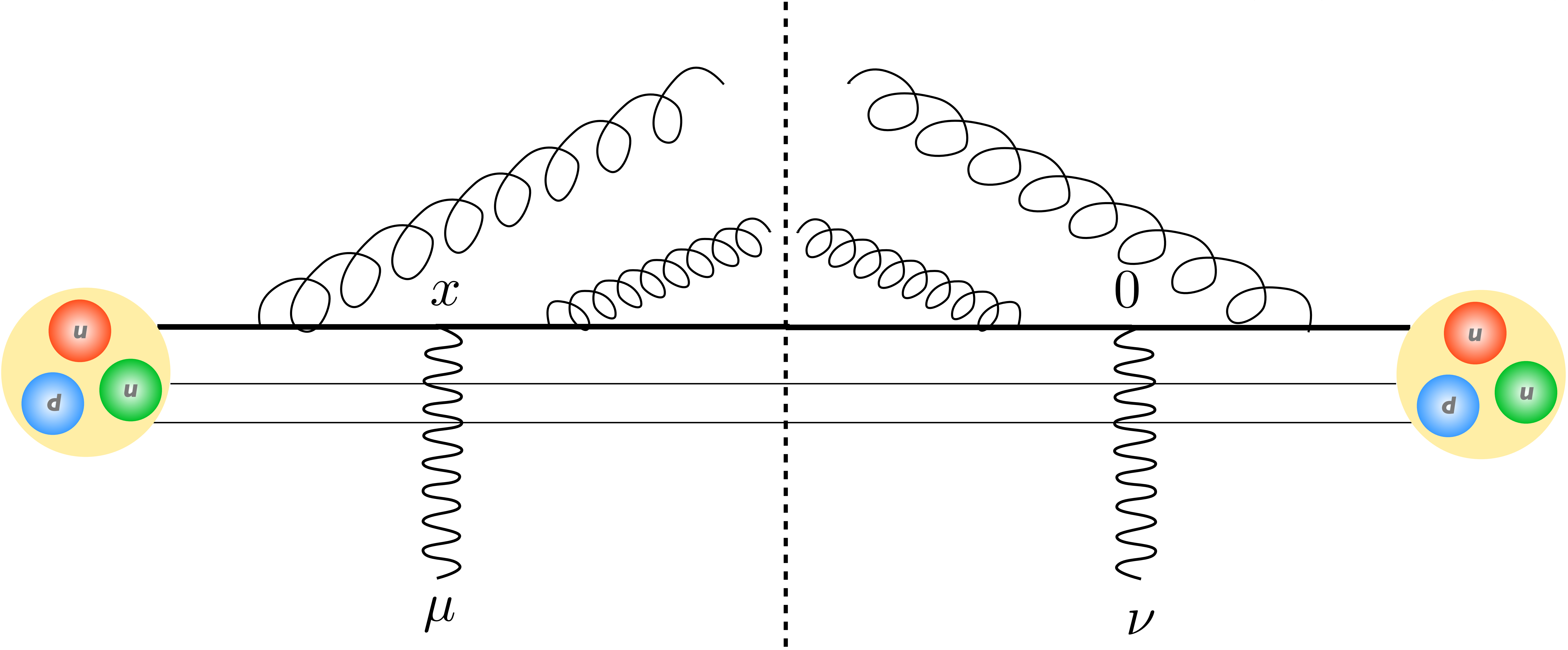
Factorization Theorem for DIS thrust

Start in QCD:

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = \underbrace{L_{\mu\nu}(x,Q^2)}_{\text{leptonic tensor}}\underbrace{W^{\mu\nu}(x,Q^2,\tau_1)}_{\text{hadronic tensor}}$$

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x}\langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1-\hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for DIS thrust

Factor collinear and soft matrix elements:

hard function

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \boxed{C^*(Q^2,\mu)C(Q^2,\mu)}\, \delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

soft function

$$\times \boxed{\langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B} Y_{n'_J}](0)|0\rangle}$$

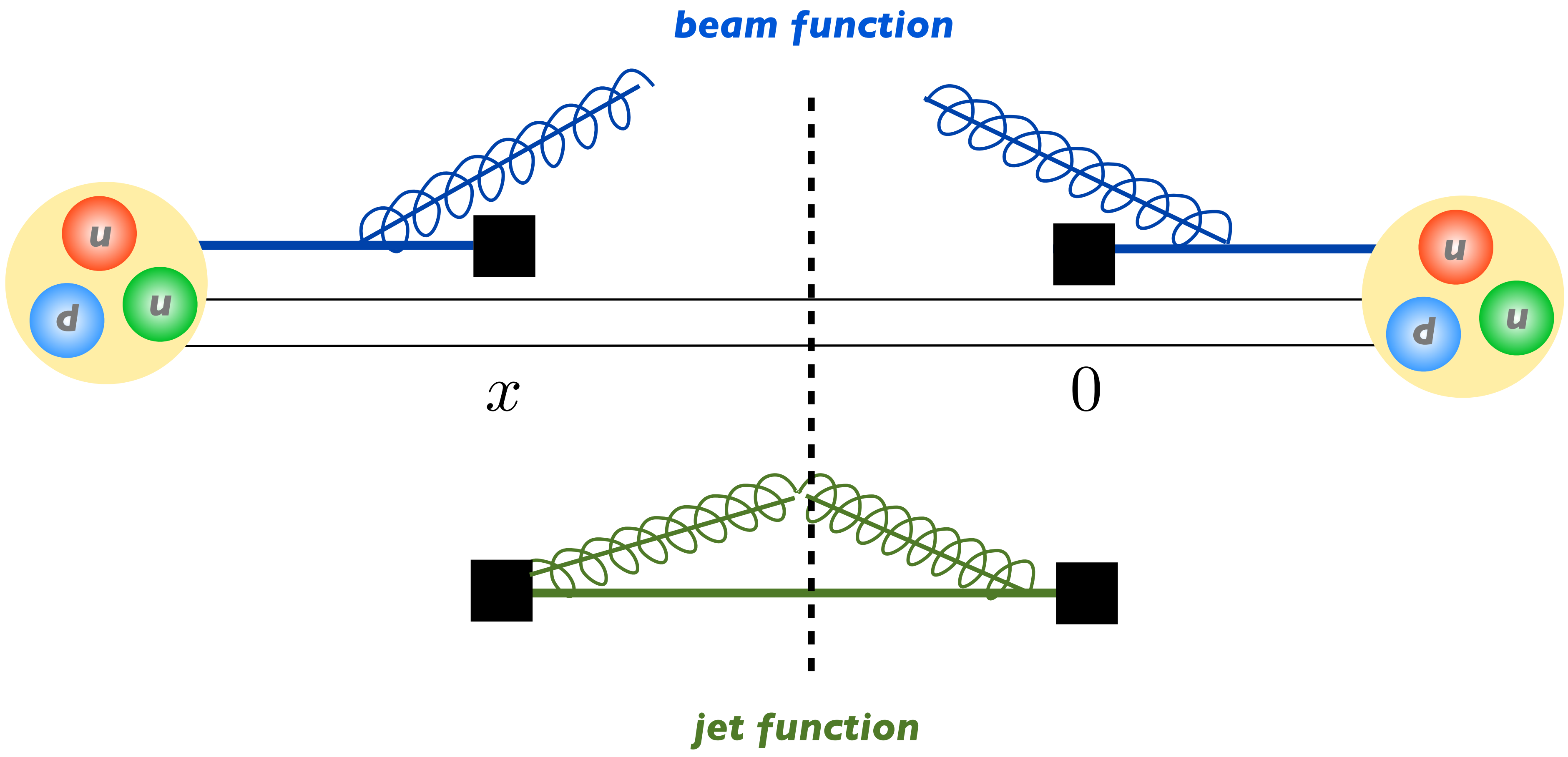
beam function

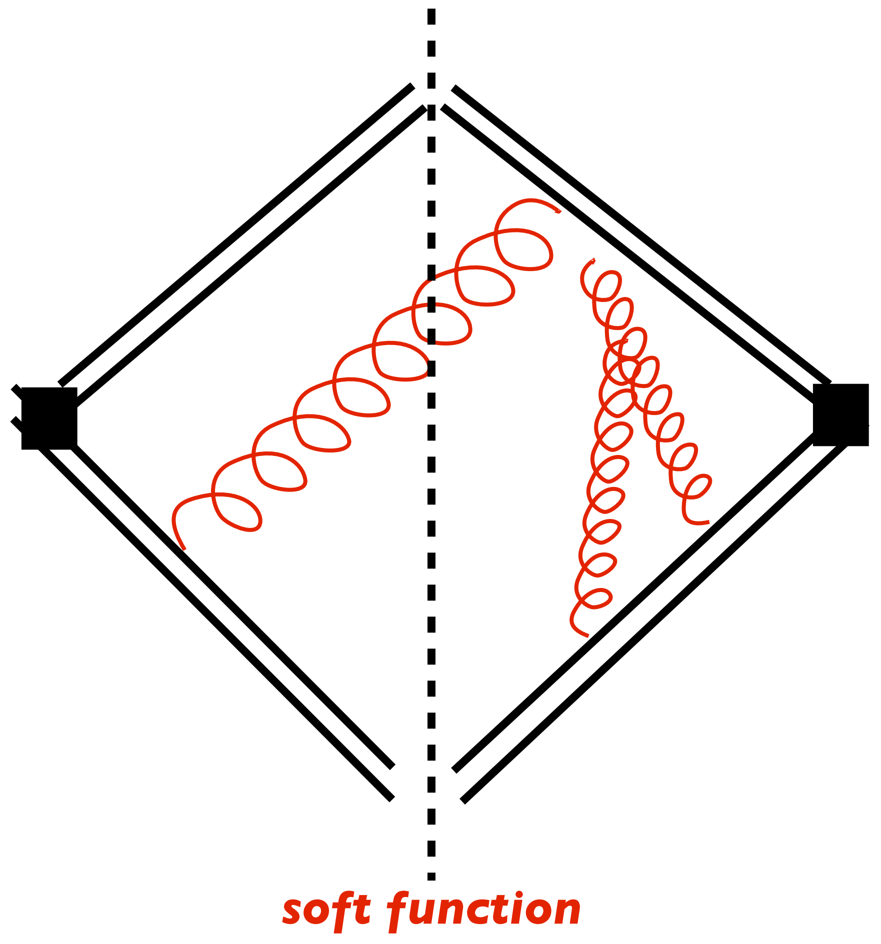
$$\times \boxed{\langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle}$$

jet function

$$\times \boxed{\langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle}$$

(+ permutations)



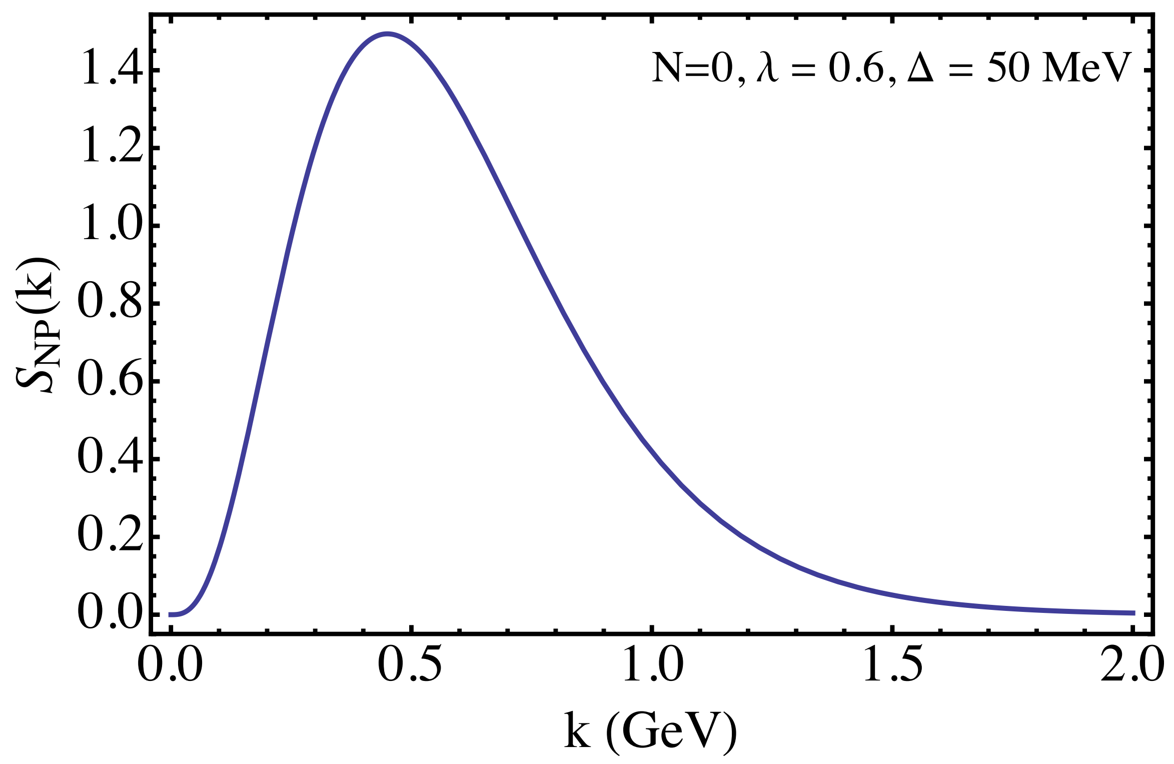


$$\Rightarrow \quad \frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S\, \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$

$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu)S(k_S,\mu)$$

Nonperturbative corrections

- In general, soft function expressed as convolution of perturbative part and nonperturbative shape function:



$$S(k_S,\mu) = \int dl\, S_{\rm PT}(k_S - l,\mu) S_{\rm NP}(l)$$

- For large enough $\tau\ (k_S)$, leading effect is a shift:

$$\langle e\rangle = \langle e\rangle_{\rm PT} + c_e\frac{\Omega_1}{Q}$$

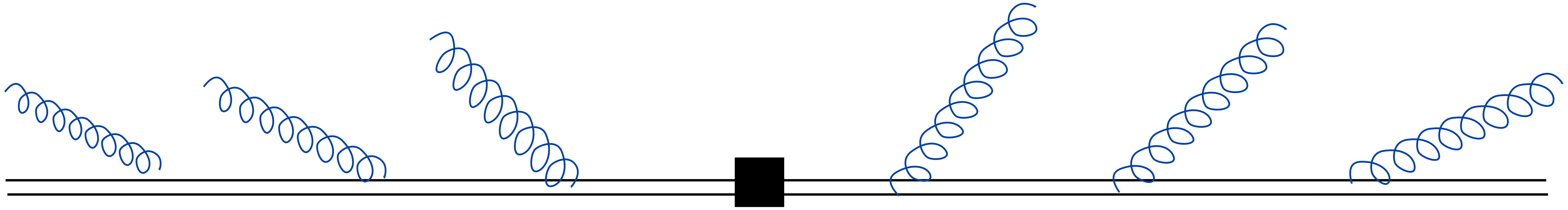
c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter

- Rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C}\mathrm{Tr}\langle 0|\overline{Y}^\dagger_{\bar n} Y^\dagger_n \underset{\downarrow}{\mathcal{E}_T}(\eta) Y_n \overline{Y}_{\bar n}|0\rangle$$

"energy flow" operator



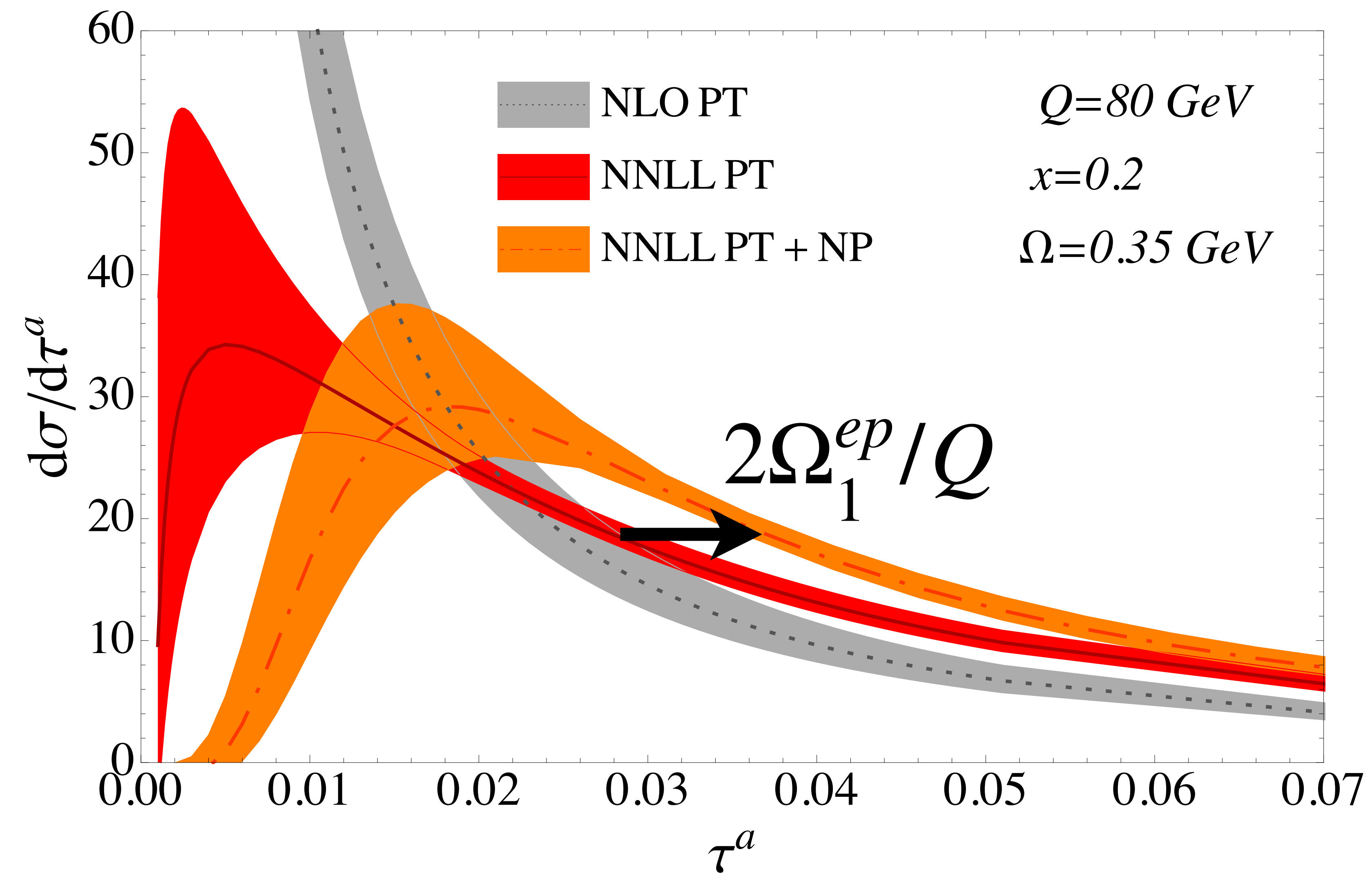
soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

DIS thrust cross sections

Resummed to N³LL accuracy:

D. Kang, CL, Stewart
(2015 and in progress)

include a nonperturbative shape function:



can be related to other
DIS event shapes

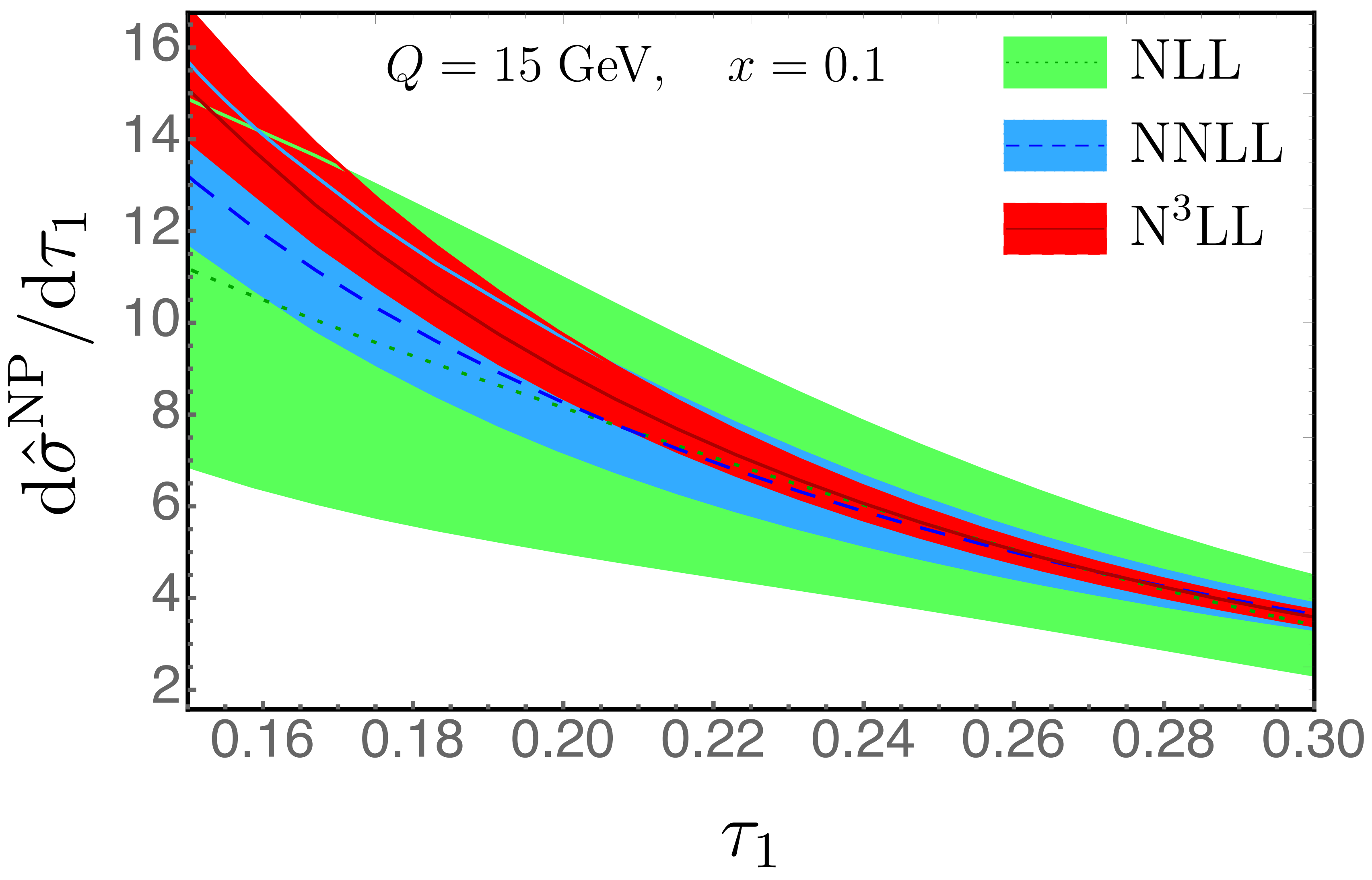
D. Kang, CL,
Stewart (2013)

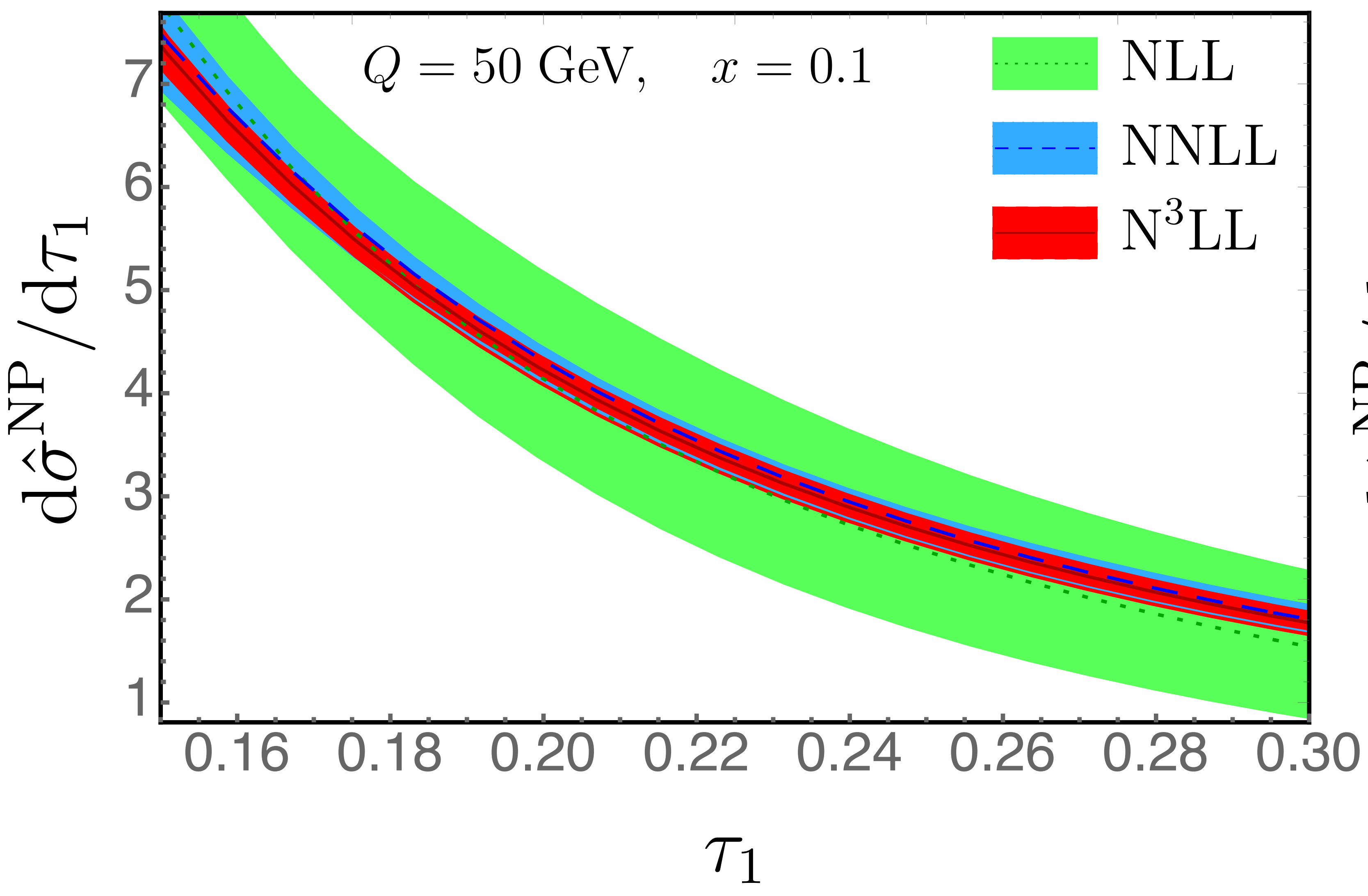
or even *pp* to 1 jet
observables

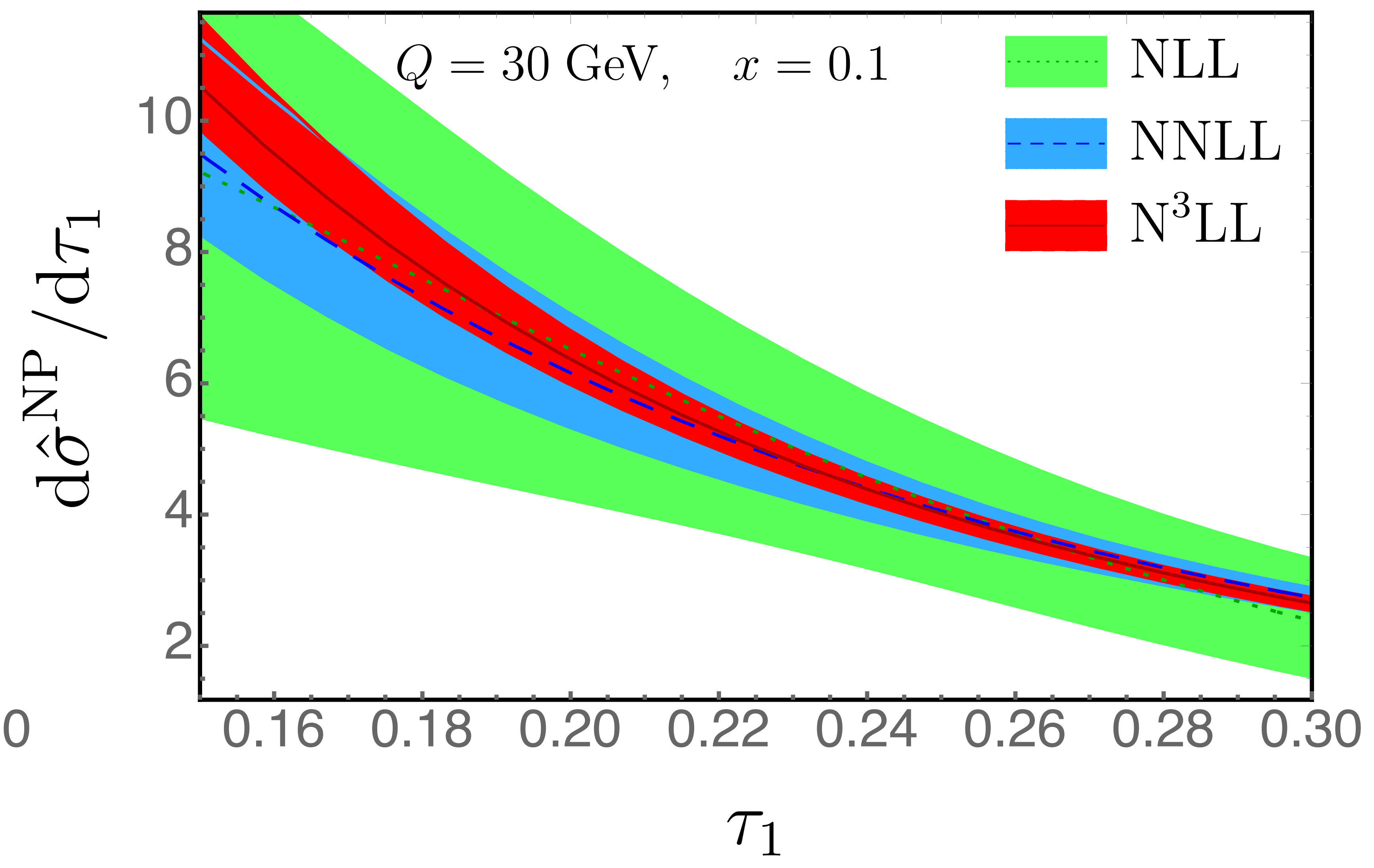
Stewart, Tackmann,
Waalewijn (2014)

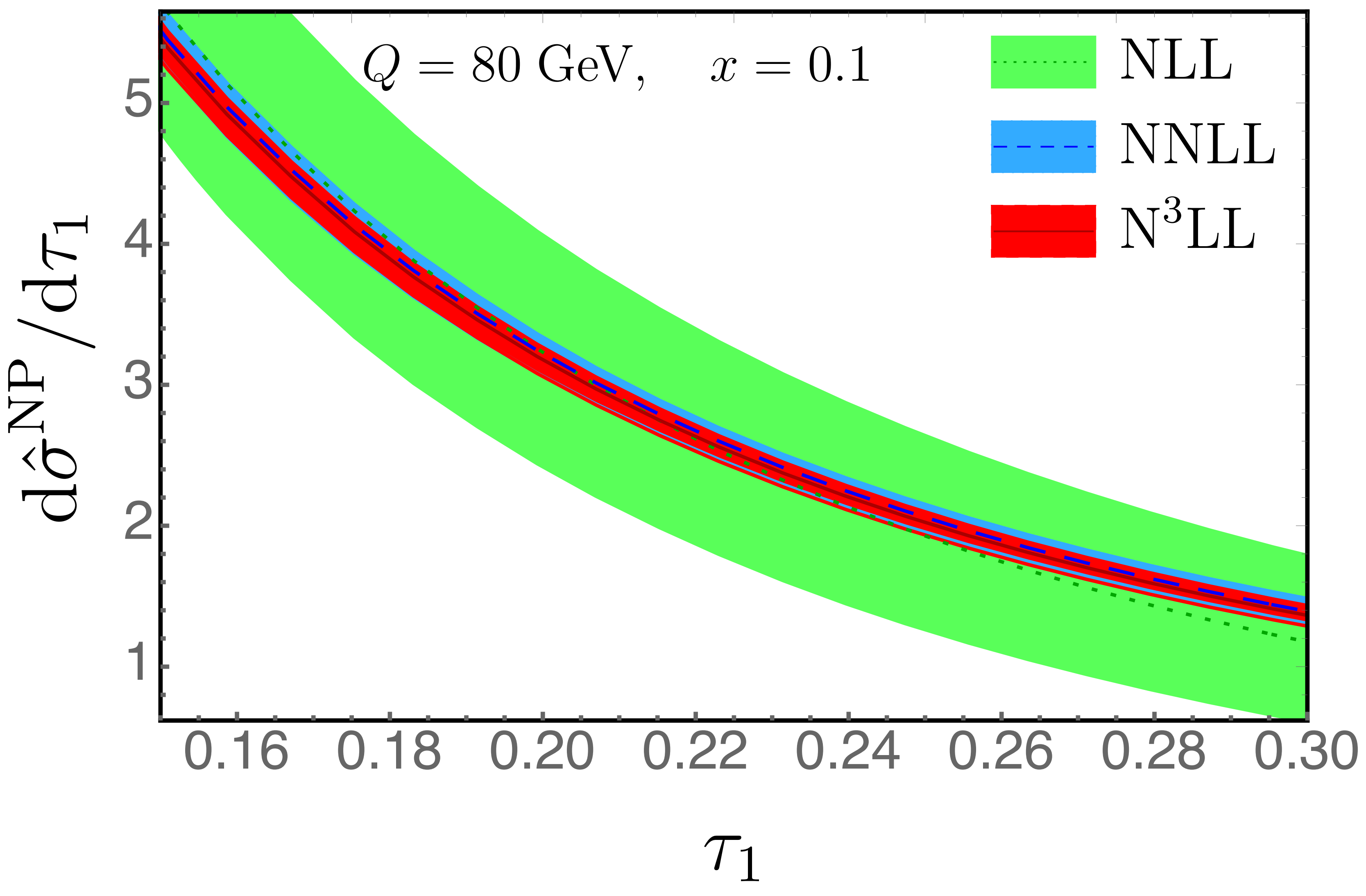
DIS thrust cross sections

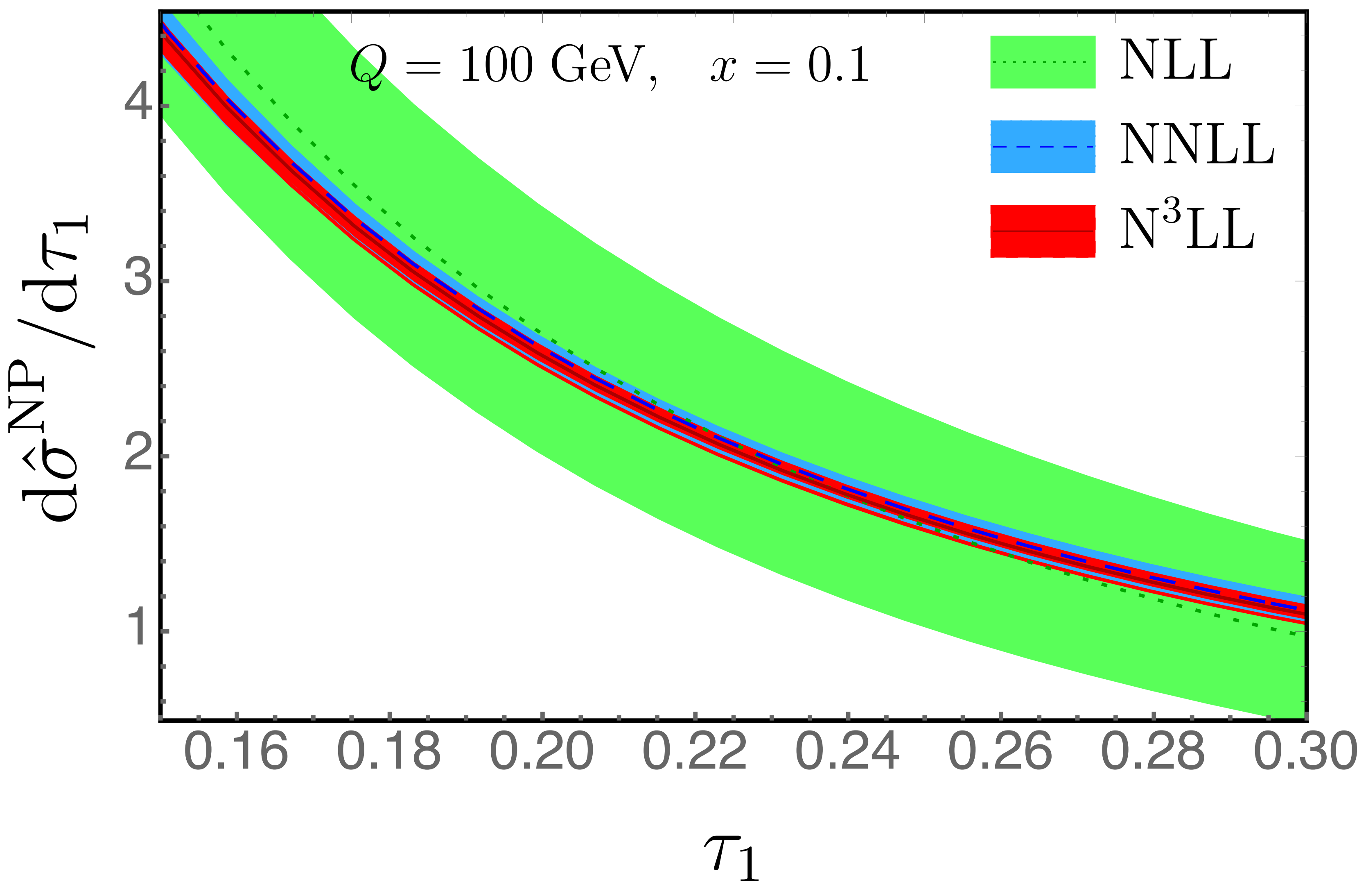
Tail region, fixed *x*, low to high *Q*:





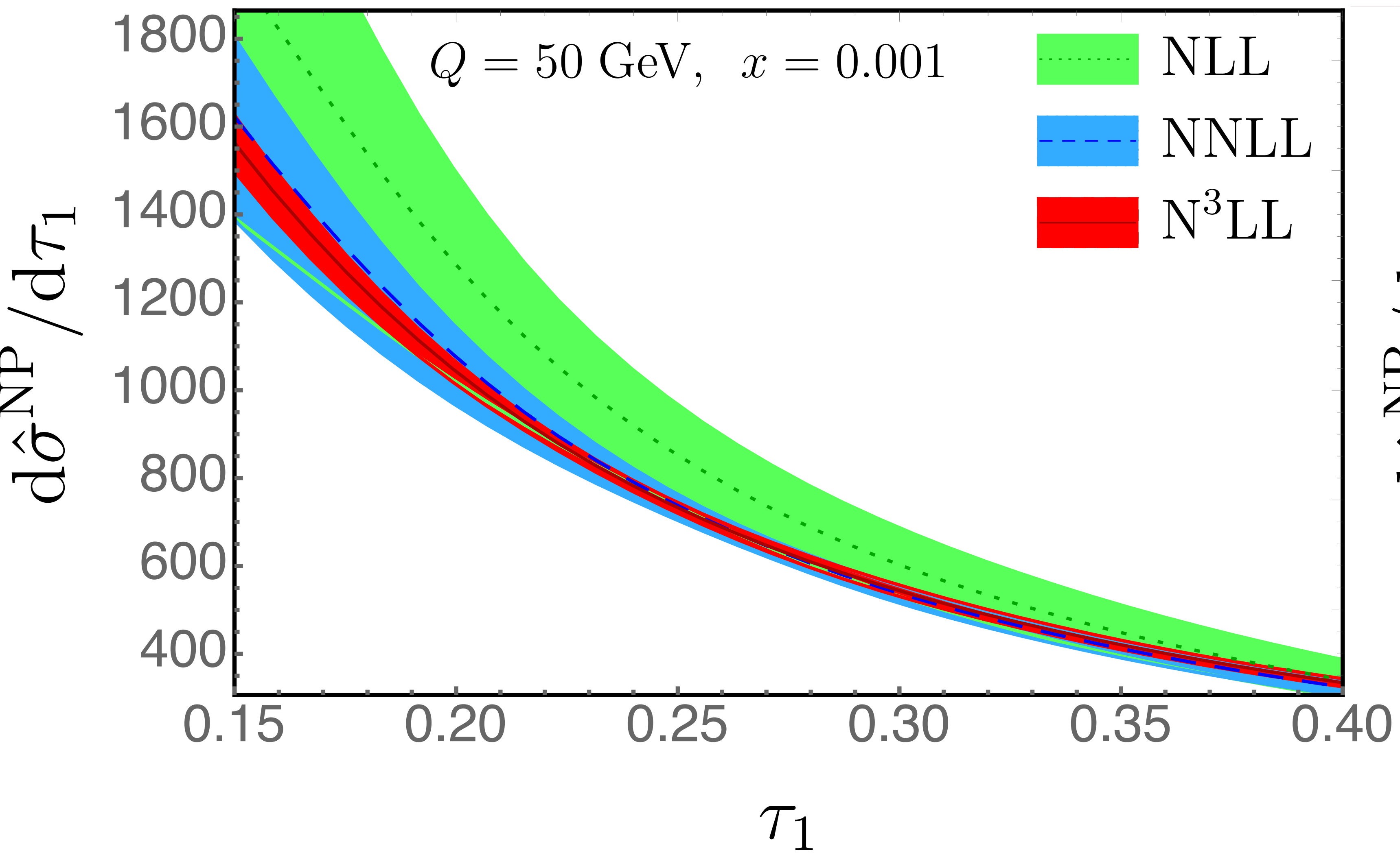


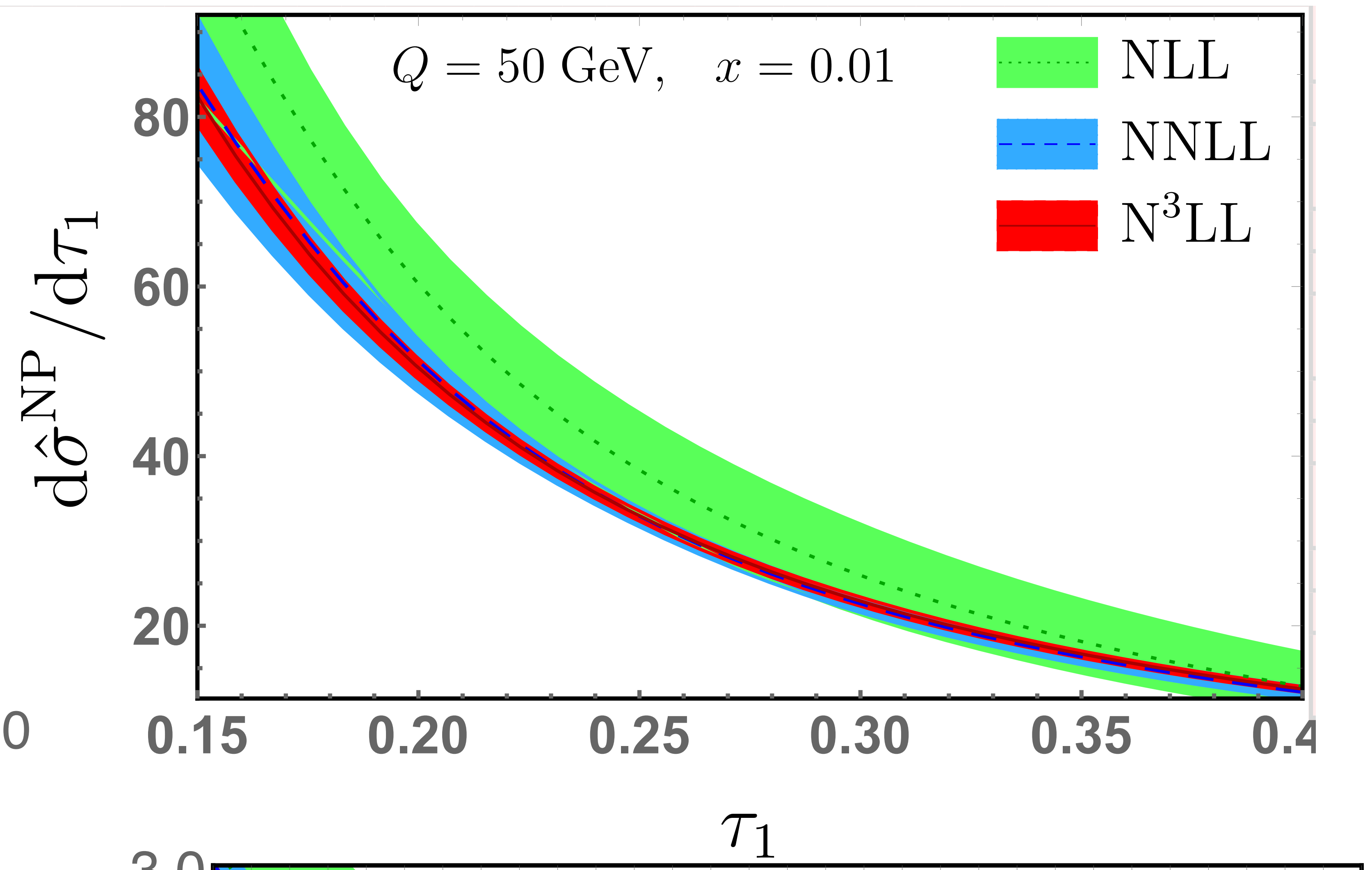


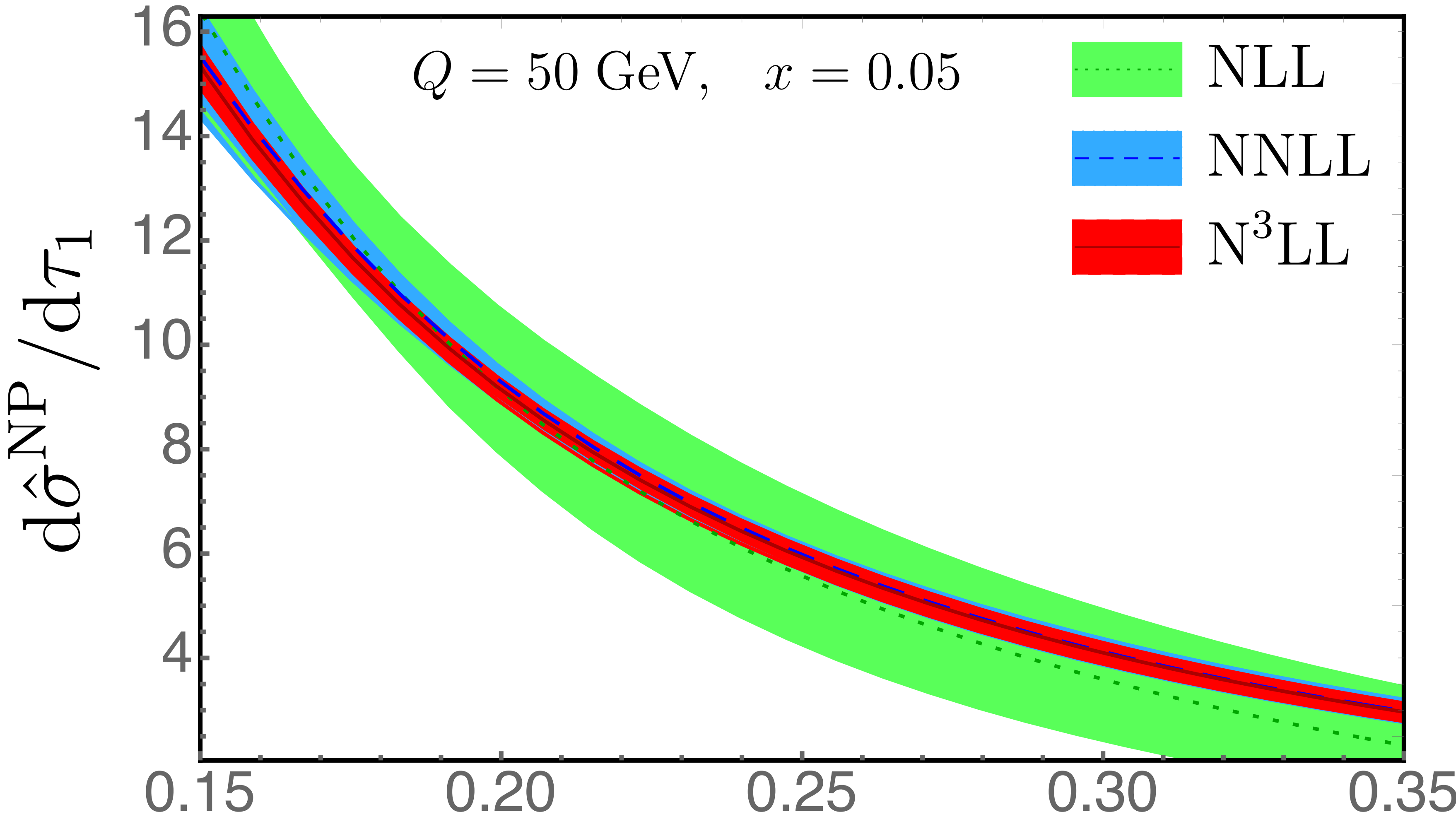


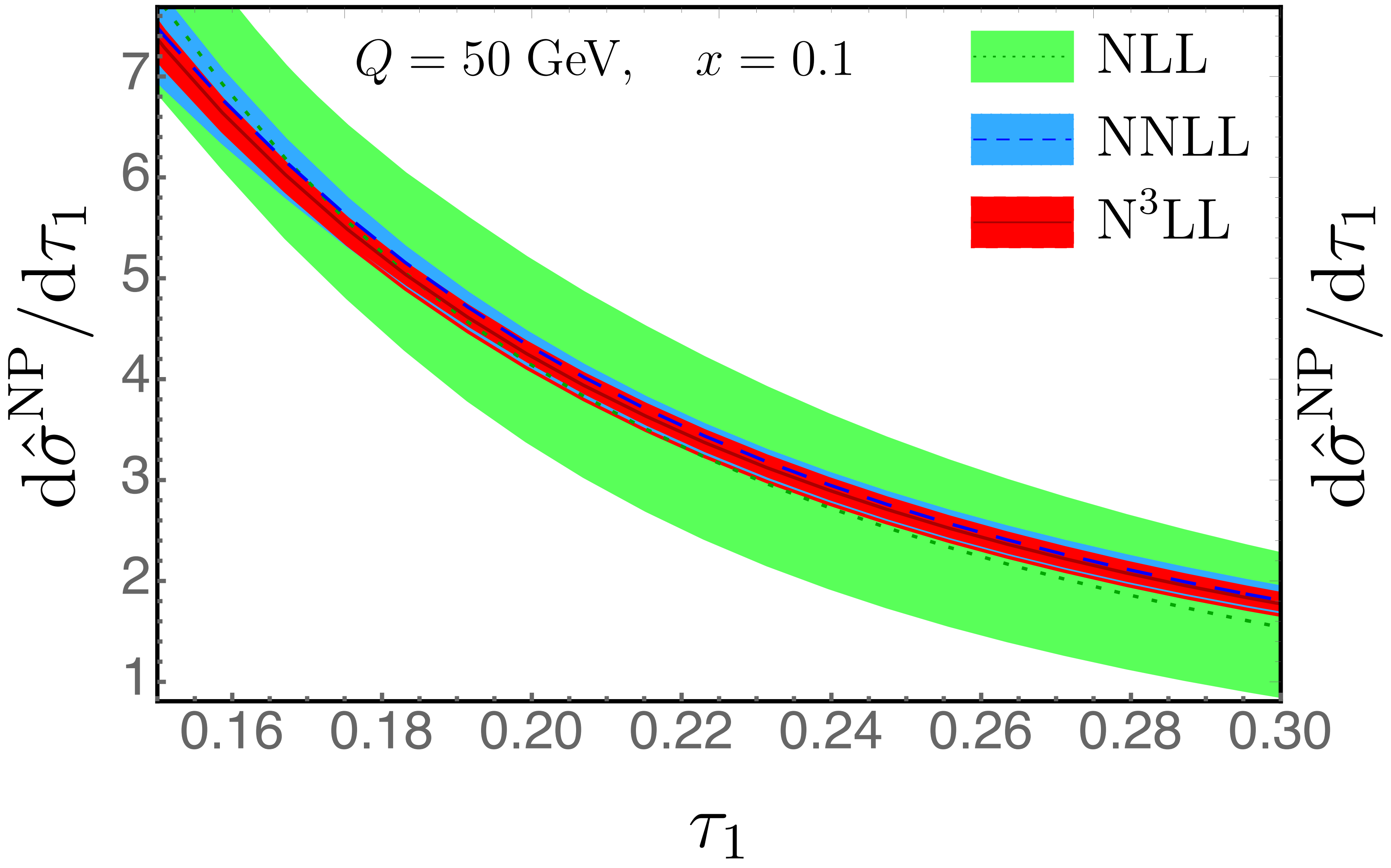
DIS thrust cross sections

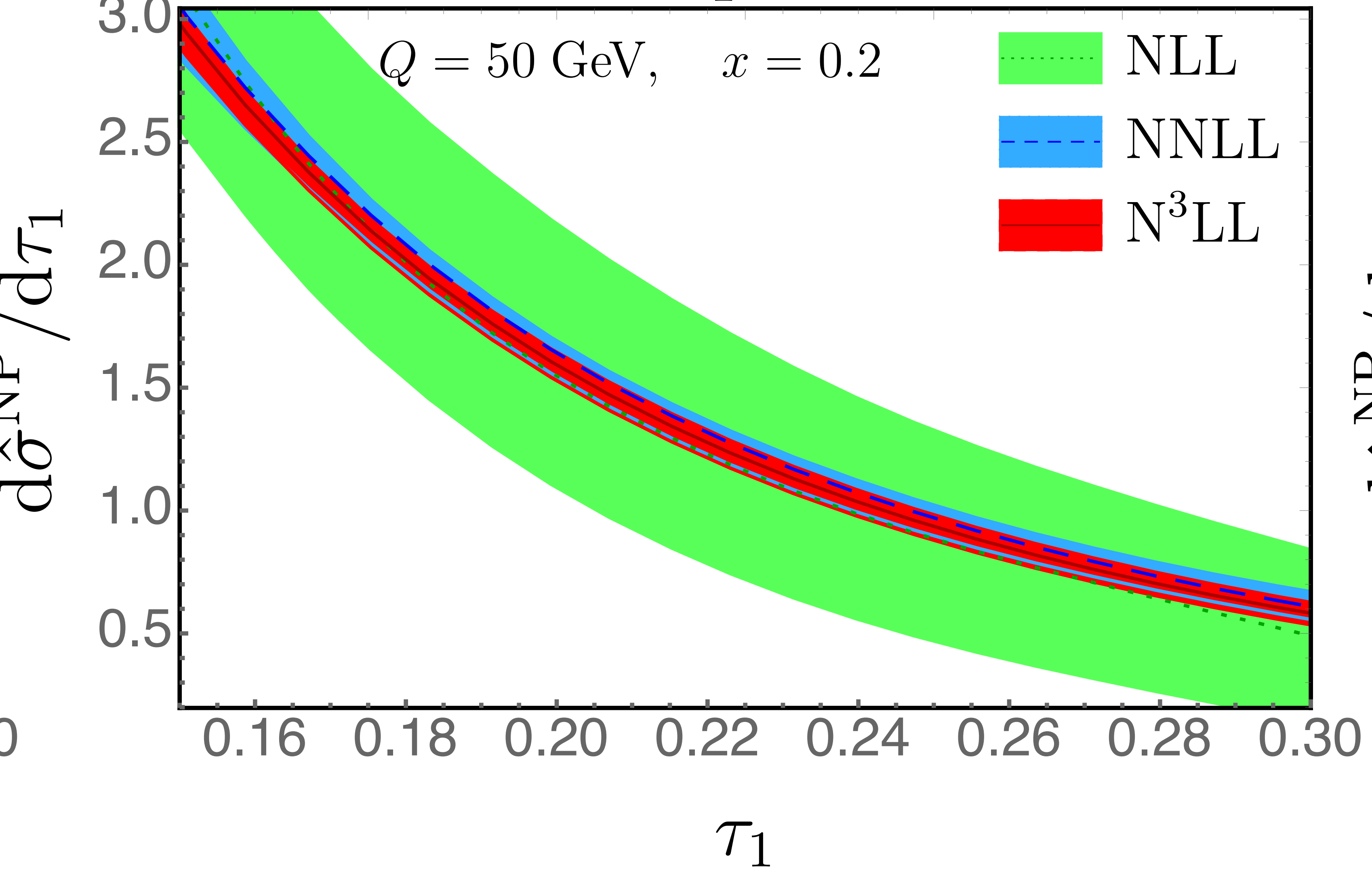
Tail region, fixed *Q*, low to high *x:*

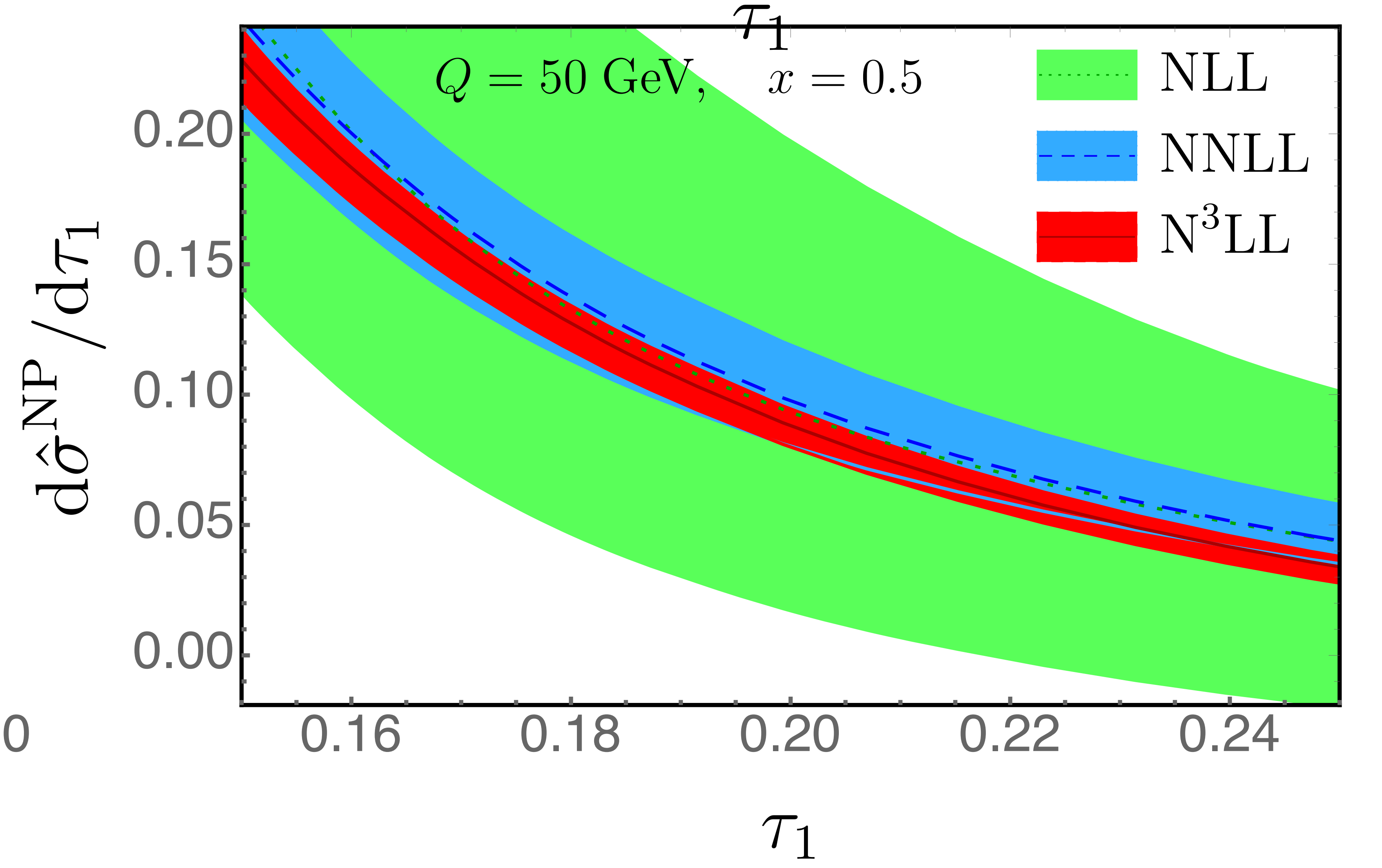






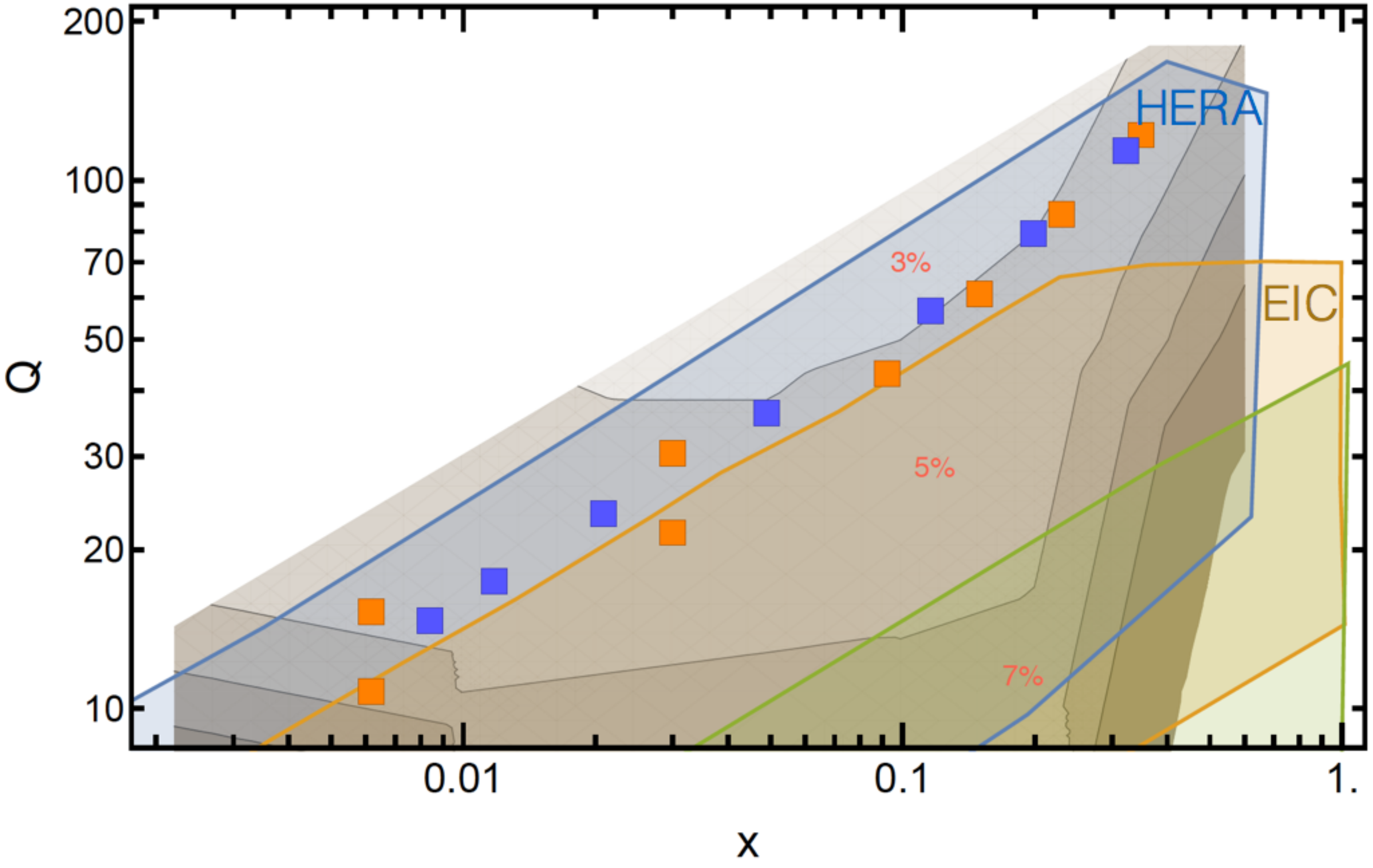




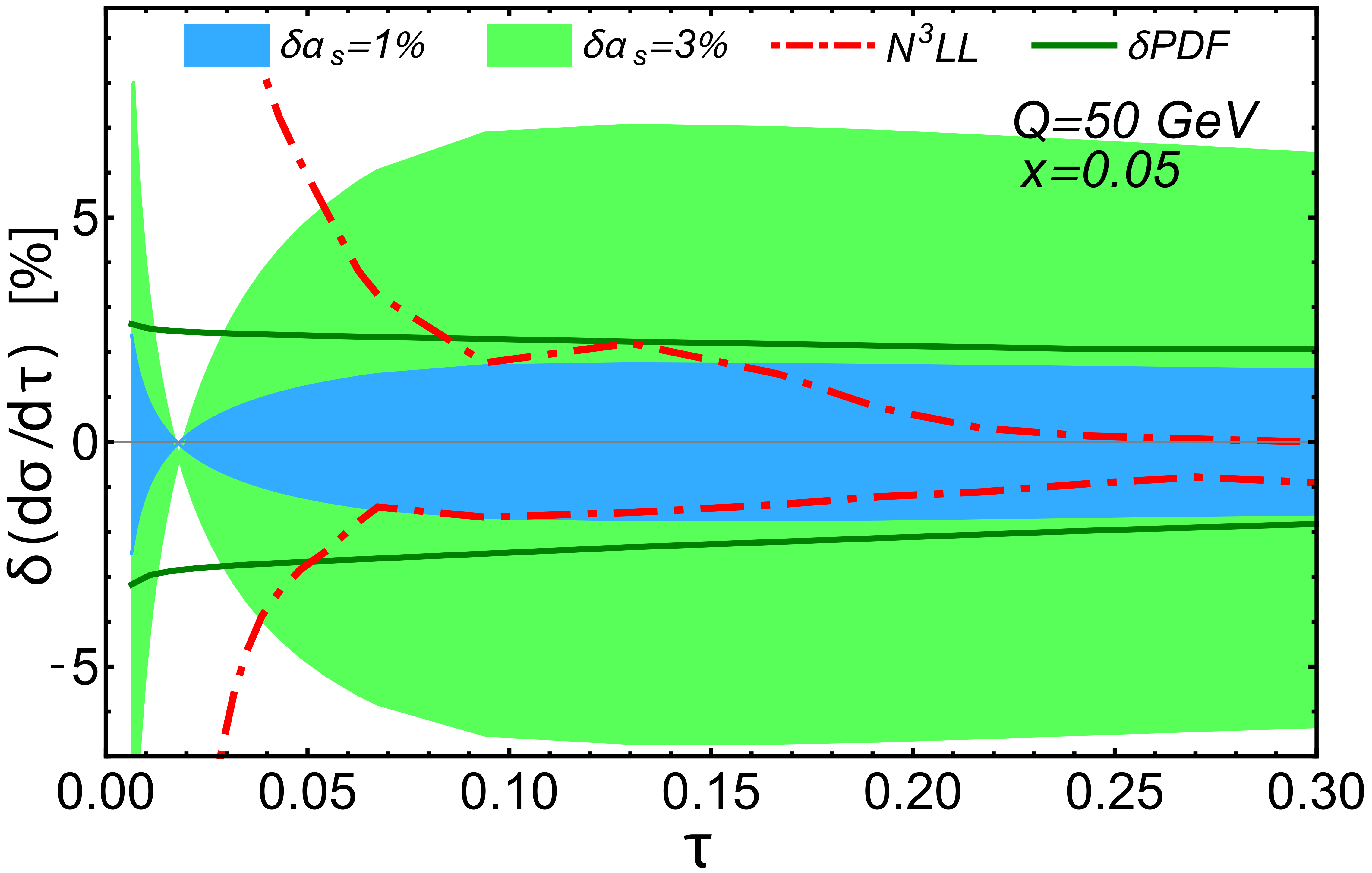


Experimental sensitivity and strong coupling

Current theoretical uncertainty
vs. HERA or EIC coverage:



Current theoretical uncertainty on the order of 1% sensitivity to α_s and PDF uncertainties:



D. Kang, CL, Stewart
(2015 and in progress)

Outlook

- N^{3}LL resummed predictions for DIS thrust to be published soon.
- Event shapes in DIS promising candidates for precision determination of strong coupling, PDFs, and hadronization corrections
- Results from HERA or an EIC may shed light on “low” value of LEP event shape determinations of strong coupling