Pion, Kaon and Proton Mass Understanding

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June 2, 2020
On January 9, 2020:

*The U.S. DOE announced the selection of BNL as the site for the Electron-Ion Collider*

A new era to explore the emergent phenomena of QCD!

A long journey – a joint effort of the full community:

“... answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics.”

... three profound questions:

How does the mass of the nucleon arise?
How does the spin of the nucleon arise?
What are the emergent properties of dense systems of gluons?
Emergent Hadron Properties from QCD

- **Mass – intrinsic to a particle:**
  
  = Energy of the particle when it is at the rest

  ✧ QCD energy-momentum tensor in terms of quarks and gluons
  
  \[ T^{\mu\nu} = \frac{1}{2} \psi \left[ iD^{\mu}(\gamma^{\nu}) \right] \psi + \frac{1}{4} g^{\mu\nu} F^{2} - F^{\mu\alpha} F_{\alpha}^{\nu} \]

  ✧ Hadron Mass:
  
  \[ M_{h}(\mu) = \left. \frac{\langle h(p) | \int d^{3}x T^{00}(\mu) | h(p) \rangle}{\langle h(p) | h(p) \rangle} \right|_{\text{Rest frame}} \]

- **Spin – intrinsic to a particle:**
  
  = Angular momentum of the particle when it is at the rest

  ✧ QCD angular momentum density in terms of energy-momentum tensor
  
  \[ M^{\alpha\mu} = T^{\alpha\nu} x^{\mu} - T^{\alpha\mu} x^{\nu} \]

  \[ J^{i} = \frac{1}{2} \varepsilon^{ijk} \int d^{3}x M^{0jk} \]

  ✧ Hadron Spin:
  
  \[ S_{h}(\mu) = \left. \frac{\langle h(p), s | J^{z}(\mu) | h(p), s \rangle}{\langle h(p), s | h(p), s \rangle} \right|_{\text{Rest frame}} \]

---

If we do not understand hadron mass & spin, we do not know QCD!
The Proton Mass

- Nucleon mass – dominates the mass of visible world:

  - Higgs mechanism is not enough!!!
  - How does QCD generate the nucleon mass?

  "... The vast majority of the nucleon’s mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. ..."

Higgs mechanism is not enough!!!

“Mass without mass!”

REACHING FOR THE HORIZON

The 2015 Long Range Plan for Nuclear Science

How to quantify and verify this, theoretically and experimentally?
Nucleon Mass vs Pion Mass

- Nucleon mass from Lattice QCD calculation:

\[ M_N = 800 \text{ MeV} + m_\pi \]

*Unexpected behavior!!*
Hadron Mass from Lattice QCD

- Hadron mass from Lattice QCD calculation:

  ![Graph showing hadron masses from Lattice QCD]

  How does QCD generate this? The role of quarks vs. that of gluons?
The Proton Mass: from Models to QCD

- **Dynamical scale:**
  - Asymptotic freedom \iff\ confinement:
  - A dynamical scale, \( \Lambda_{QCD} \), consistent with \( \frac{1}{R} \sim 200 \text{ MeV} \)

- **Bag model:**
  - Kinetic energy of three quarks: \( K_q \sim \frac{3}{R} \)
  - Bag energy (bag constant B): \( T_b = \frac{4}{3} \pi R^3 B \)
  - Minimize \( K_q + T_b \): \( M_p \sim \frac{4}{R(\text{fm})} 197.3(\text{MeVfm}) \)
  - Proton radius:
    \( R(\text{PDG}) = 0.8414(19) \text{ fm} \)
  - Proton mass:
    \( M_p \sim 936 - 940 \text{ MeV} \)
    \( M_p(\text{PDG}) = 938.272 \text{ MeV} \)

- **Meson is different, especially the pion!**

- **Constituent quark model:**
  - Spontaneous chiral symmetry breaking:
    *Massless quarks gain \( \sim 300 \text{ MeV mass when traveling in vacuum}*
    \( M_p \sim 3 m_q^{\text{eff}} \sim 900 \text{ MeV} \)
Pion, Kaon and Proton Mass in QCD

- **QCD:**
  \[ \mathcal{L}_{\text{QCD}}(m) = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \bar{\psi}_i [i(\gamma^\mu D_\mu)_{ij} - m \delta_{ij}] \psi_j \]

- **Hadron mass (in any frame):**
  \[ \langle h(p)|T^{\mu\nu}|h(p)\rangle \propto p^\mu p^\nu \quad \longrightarrow \quad \langle h(p)|T^\mu h(p)\rangle \propto p^2 = M_h^2 \]
  Same \( T^{\mu\nu} \) for:
  \[ \langle P(p)|T^\mu |P(p)\rangle = M_P^2 \sim (938 \text{ MeV})^2 \]
  \[ \langle \pi(p)|T^\mu |\pi(p)\rangle = M_\pi^2 \sim (139 \text{ MeV})^2 \ll M_P^2 \]
  \[ \langle K(p)|T^\mu |K(p)\rangle = M_K^2 \sim (497 \text{ MeV})^2 \]

- **Poincare Invariance:**
  \[ \partial_\mu T^{\mu\nu} = 0 \]

- **Global scale invariance for \( \mathcal{L}_{\text{QCD}}(m = 0): \)**
  \[ x \rightarrow e^{-\alpha x} \]
  Classically,
  \[ T^{\mu\nu} g_{\mu\nu} = T^\mu = 0 \]
  \[ \mathcal{L}_{\text{QCD}}(m = 0): \]
  Dilatonic current:
  \[ D_\mu \equiv T^\mu x^\mu \]
  \[ \partial^\mu D_\mu = 0 = [\partial^\mu T^\mu] x^\mu + T_{\mu\nu} g^{\mu\nu} \]

  No massive bound state!
Sources of hadron mass:

Quantum mechanically,

\[
T^\alpha = \frac{\beta(g)}{2g} F^{\mu\nu,a} F^a_{\mu\nu} + \sum_{q=u,d,s} m_q (1 + \gamma_5) \bar{\psi}_q \psi_q
\]

- QCD trace anomaly
- Chiral symmetry breaking

\[
\beta(g) = -(11 - 2n_f/3) \frac{g^3}{(4\pi)^2} + \ldots
\]

Need to know: \[\langle h(p) | F^{\mu\nu,a} F^a_{\mu\nu} | h(p) \rangle\] and \[\langle h(p) | \bar{\psi}_q \psi_q | h(p) \rangle\] independently!

Isolate the traceless term:

\[
T^{\mu\nu} = \overline{T}^{\mu\nu} + \hat{T}^{\mu\nu}
\]

- Traceless term:
  \[
  \overline{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^\alpha_\alpha
  \]

- Trace term:
  \[
  \hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^\alpha_\alpha
  \]
Decomposition and Sum Rules

- mass in hadron’s rest frame:
  - Hadron state:
    \[ |P\rangle \quad \text{with the normalization:} \quad \langle P|P\rangle = (E/M)(2\pi)^3 \delta^3(0) \]
  - Hamiltonian:
    \[ H_{\text{QCD}} = \int d^3\vec{x} T^{00}(0, \vec{x}) \quad \langle P|H_{\text{QCD}}|P\rangle = (E^2/M_p)(2\pi)^3 \delta^3(0) \]
  - QCD energy-momentum tensor:
    \[ T^{\mu\nu} = \bar{T}^{\mu\nu} + \hat{T}^{\mu\nu} \quad \langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu / M_p \quad \text{No } g^{\mu\nu} \text{ term!} \]
    \[ \langle P|\bar{T}^{\mu\nu}|P\rangle = (P^\mu P^\nu - \frac{1}{4} M_p^2 g^{\mu\nu}) / M_p \quad \langle P|\hat{T}^{\mu\nu}|P\rangle = \frac{1}{4} M_p g^{\mu\nu} \]
    \[ \frac{\langle P|\int d^3x \bar{T}^{00}|P\rangle}{\langle P|P\rangle} \quad \text{at rest} \quad \frac{3}{4} M_p \quad \text{“Traceless” term} \]
    \[ \frac{\langle P|\int d^3x \hat{T}^{00}|P\rangle}{\langle P|P\rangle} \quad \text{at rest} \quad \frac{1}{4} M_p \quad \text{“Trace” term} \]

- sum rules for proton mass:
  \[ M_p = \frac{\langle P|\int d^3x T^{00}|P\rangle}{\langle P|P\rangle} \quad \text{at rest} \quad M_q + M_g + M_m + M_a \]

  - relativistic motion
  - \(\chi\) symmetry breaking
  - quantum fluctuation
  - quark energy
  - gluon energy
  - quark mass
  - trace anomaly
Decomposition and Sum Rules

- **Identities:**
  \[
  \frac{\langle P | \int d^3 x \, T^\alpha_\alpha | P \rangle}{\langle P | P \rangle} = 4 \frac{\langle P | \int d^3 x \, \tilde{T}^{00} | P \rangle}{\langle P | P \rangle} = \frac{4}{3} \frac{\langle P | \int d^3 x \, \tilde{T}^{00} | P \rangle}{\langle P | P \rangle} \at \text{rest}
  \]

- **Traceless terms:**
  \[
  \tilde{T}^{\mu\nu} = \tilde{T}_q^{\mu\nu} + \tilde{T}_g^{\mu\nu}
  \]
  \[
  \tilde{T}_q^{\mu\nu} = \frac{1}{2} \gamma_\nu \left( D^{(\mu)} - g^{\mu\nu} \right) \gamma_\nu - \frac{1}{4} g^{\mu\nu} \gamma_m \gamma_m
  \]
  \[
  \tilde{T}_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu_\alpha
  \]

  \[
  \frac{\langle P | \tilde{T}_q^{\mu\nu} | P \rangle}{\langle P | P \rangle} = a(\mu^2) \left( P^\mu P^\nu - \frac{1}{4} M_p^2 g^{\mu\nu} \right) / M_p
  \]
  \[
  \langle P | \tilde{T}_g^{\mu\nu} | P \rangle = [1 - a(\mu^2)] \left( P^\mu P^\nu - \frac{1}{4} M_p^2 g^{\mu\nu} \right) / M_p
  \]

  \[
  \frac{\langle P | \int d^3 x \, \tilde{T}^{00}_q | P \rangle}{\langle P | P \rangle} = a(\mu^2) \frac{3}{4} M_p
  \]
  \[
  \frac{\langle P | \int d^3 x \, \tilde{T}^{00}_g | P \rangle}{\langle P | P \rangle} = [1 - a(\mu^2)] \frac{3}{4} M_p
  \at \text{rest}
  \]

- **Trace terms:**
  \[
  \hat{T}^{\mu\nu} = \hat{T}_m^{\mu\nu} + \hat{T}_a^{\mu\nu}
  \]

  \[
  \langle P | \hat{T}_m^{\mu\nu} | P \rangle = b \frac{1}{4} M_p g^{\mu\nu}
  \]
  \[
  \langle P | \hat{T}_a^{\mu\nu} | P \rangle = [1 - b] \frac{1}{4} M_p g^{\mu\nu}
  \]

  \[
  \frac{\langle P | \int d^3 x \, \hat{T}^{00}_m | P \rangle}{\langle P | P \rangle} = b \frac{1}{4} M_p
  \]
  \[
  \frac{\langle P | \int d^3 x \, \hat{T}^{00}_a | P \rangle}{\langle P | P \rangle} = [1 - b] \frac{1}{4} M_p
  \at \text{rest}
  \]
Roles of quarks and gluons?

✧ Quark energy contribution:
\[ H_q = \int d^3 \vec{x} \bar{\psi} (-i \vec{D} \cdot \vec{\alpha}) \psi, \]
\[ M_q = \left| \frac{\langle P | H_q | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = (a - b) \frac{3}{4} M_p \]

✧ Gluon energy contribution:
\[ H_g = \int d^3 \vec{x} \frac{1}{2} (E^2 + B^2) \]
\[ M_g = \left| \frac{\langle P | H_g | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = (1 - a) \frac{3}{4} M_p \]

✧ Quark mass contribution:
\[ H_m = \int d^3 \vec{x} \bar{\psi} m \psi \]
\[ M_m = \left| \frac{\langle P | H_m | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = b M_p \]

✧ Trace anomaly contribution:
\[ H_a = \int d^3 \vec{x} \frac{9 \alpha_s}{16\pi} (E^2 - B^2) \]
\[ M_a = \left| \frac{\langle P | H_a | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = (1 - b) \frac{1}{4} M_p \]

\[ M_p = \left| \frac{\langle P | \int d^3 x T^{00} | P \rangle}{\langle P | P \rangle} \right|_{\text{at rest}} = M_q + M_g + M_m + M_a \]

Need to connect “a” and “b” to physical observables independently!
Decomposition and Sum Rules

Physical meaning of “a”:

\[ \langle P | \bar{T}_q^{\mu\nu} | P \rangle \equiv a(\mu^2)(P^\mu P^\nu - \frac{1}{4} M_p^2 g^{\mu\nu})/M_p \]

\[ \bar{T}_q^{\mu\nu} = \frac{1}{2} \bar{\psi} i \gamma^\mu D^{(\mu} \gamma^\nu) \psi - \frac{1}{4} g^{\mu\nu} \bar{\psi} m \psi ; \]

\[ \bar{T}_g^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^{\nu}_\alpha . \]

Let \( \mu \to + \) and \( \nu \to + : \)

\[ a_q(\mu^2) = \int dx \, x \left[ q(x, \mu^2) + \bar{q}(x, \mu^2) \right] \quad a_g(\mu^2) = \int dx \, x \, g(x, \mu^2) \]

Total momentum fraction carried by the quarks & gluons are measurable

Physical meaning of “b”:

\[ \langle P | \bar{T}_m^{\mu\nu} | P \rangle \equiv b \frac{1}{4} M_p g^{\mu\nu} \]

\[ b = \frac{1}{M_p} \frac{\langle P | \int d^3 \bar{x} \bar{\psi} m \psi | P \rangle}{\langle P | P \rangle} \]

Need independent measurement of the trace anomaly!

\[ T_\alpha^\alpha = \frac{\beta(g)}{2g} F^{\mu\nu,a} F^{\alpha}_{\mu\nu} + \sum_{q=u,d,s} m_q (1 + \gamma_m) \bar{\psi}_q \psi_q \]

QCD trace anomaly \quad Chiral symmetry breaking
Decomposition and Sum Rules

- **Quark mass contribution – the “b”-term:**
  \[ b M_p = \langle P | m_u \bar{u}u + m_d \bar{d}d | P \rangle + \langle P | m_s \bar{s}s | P \rangle + \ldots \text{(heavy flavors)} \]

- **The first term – πN σ-term:**
  \[ \sigma_{\pi N} = \hat{m} \langle N | \bar{u}u + \bar{d}d | N \rangle \]
  with \( \hat{m} = (m_u + m_d)/2 \)
  Both lattice QCD and phenomenological analyses give:
  \[ \sigma_{\pi N} \sim 45 - 50 \text{ MeV} \]

- **The second term – strange scalar charge:**
  **Light strange quark:**
  \[ \sigma_{Kn} = (\hat{m} + m_s) \langle N | \bar{u}u + \bar{d}d + 2\bar{s}s | N \rangle / 4 \]
  Both lattice QCD and phenomenological analyses give:
  \[ \sigma_{Kn} \sim 360 - 400 \text{ MeV} \]
  Need measurement and calculation of trace anomaly!
The derivation of above decomposition is valid for all hadrons except “massless” one:

At the chiral limit, \( m_\pi = 0 \)

\[
\langle \pi(p) | T^\mu_\mu | \pi(p) \rangle = m_\pi^2 \to 0
\]

DCSB – see Roberts’ talk

Do not have to choose the rest frame:

\[
T^\alpha_\alpha = \frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu} + \sum_{q=u,d,s} m_q (1 + \gamma_m) \bar{\psi}_q \psi_q \quad \Rightarrow \quad \langle h(p) | T^\alpha_\alpha | h(p) \rangle = M_h^2
\]

Need to know: \( \langle h(p) | F^{\mu\nu,a} F^{a}_{\mu\nu} | h(p) \rangle \) and \( \langle h(p) | \bar{\psi}_q \psi_q | h(p) \rangle \) independently!

Contribution from the traceless term:

\[
\langle P | \bar{T}^{\mu\nu} | P \rangle = (P^\mu P^\nu - \frac{1}{4} M_p^2 g^{\mu\nu}) / M_p
\]

We do not have to choose “00” components and the rest frame

We could choose \( \mu \to + \) and \( \nu \to - \) (or \( \perp \perp \)):

\[
M_p = 2 \langle h(p) | \bar{T}^{+-}_0 | h(p) \rangle
\]

Need to find new observables sensitive to these high-twist operators!

Ji, 2003.04478

Aslan, Qiu

In preparation
Pion and Kaon Structure

- Pion decays, and there is no stable pion target

- Pion beam:
  
  Talking advantage of time-dilation, \( \pi + p \rightarrow \ell^+ \ell^- + X \) Drell-Yan process

  Precision of pion structure depends on our knowledge of proton structure

- Lattice QCD:
  
  - using a vector-axial-vector correlation as an example

  \[
  \frac{1}{2} [T_{v5}^{\mu\nu}(\xi, p) + T_{5v}^{\mu\nu}(\xi, p)] = \frac{\xi^4}{2} \langle h(p) | (J_\mu^v(\xi/2) J_5^\nu(-\xi/2) + J_5^\mu(\xi/2) J_\nu^v(-\xi/2)) | h(p) \rangle \\
  \equiv \epsilon^{\mu\nu\alpha\beta} p_\alpha \xi_\beta \overline{T}_1(\omega, \xi^2) + (p^\mu \xi^\nu - \xi^\mu p^\nu) \overline{T}_2(\omega, \xi^2)
  \]

  - Collinear factorization:

  \[
  \overline{T}_i(\omega, \xi^2) = \sum_{f=q, \bar{q}, g} \int_0^1 \frac{dx}{x} f(x, \mu^2) C_i^f(\omega, \xi^2; x, \mu^2) + O[||\xi||/\text{fm}]
  \]

  - Lowest order coefficient functions:

  \[
  C_1^{q(0)}(\omega, \xi^2; x) = \frac{1}{\pi} x (e^{ix\omega} + e^{-ix\omega})
  \]

  \[
  T_1(\omega, \xi^2) \equiv \int \frac{d\omega}{2\pi} e^{-ix\omega} \overline{T}_1(\omega, \xi^2) \\
  = \frac{1}{\pi^2} (q(\omega, \mu^2) - \bar{q}(\omega, \mu^2)) \equiv \frac{1}{\pi^2} q_v(\omega, \mu^2)
  \]

Vanishes under \( T \)

Sufian et al.

PRD99 (2019) 074507
Current-Current Correlators

- “Lattice cross section” of V-A current correlator:

- Extracted pion valence quark distribution:

Sufian et al. JLab PRD99 (2019) 074507
Sufian et al. @ DNP19
Comparison of pion PDF from different approaches

Good LCSs:

Conway et al
WRH
ASV
JAM
LFHQCD
DSE
This calculation

\( \mu^2 = 27 \text{GeV}^2 \)

\( m_\pi = 416 \text{MeV} \), \( P = 0.6 - 1.5 \text{GeV} \)

pseudo-PDFs:
M. Constantinou @ DNP19

[B. Joo et al. (JLab-W&M), arXiv:1909.08517]

\( \mu^2 = 27 \text{GeV}^2 \)

\( m_\pi = 416 \text{MeV} \), \( \nu \leq 4.7 \)

quasi-PDFs:

\( m_\pi = 310 \text{MeV} \), \( P = 1.74 \text{GeV} \)


\( \mu = 3.2 \text{ GeV} \)

\( \gamma_t; P_z = 1.29 \text{ GeV} \)

\( m_\pi = 300 \text{MeV} \), \( P = 1.29 \text{GeV} \)
Hadron mass is an emergent phenomenon of QCD dynamics

Decomposition of hadron mass (or sum rule) is not unique:

- Individual terms are local matrix elements of quark/gluon operators
- None of the individual terms are physical, owing to the confinement
- A decomposition is valuable iff individual terms can be measured or calculated independently with controllable approximations

Good decomposition should work for both proton, pion, and ...

\[
\langle P(p)|T^\mu_\mu|P(p)\rangle = M^2_P \sim (938 \text{ MeV})^2
\]
\[
\langle \pi(p)|T^\mu_\mu|\pi(p)\rangle = M^2_\pi \sim (139 \text{ MeV})^2
\]
\[
\langle K(p)|T^\mu_\mu|K(p)\rangle = M^2_K \sim (497 \text{ MeV})^2
\]

We can also get mass decomposition from traceless term, and calculate new sets of operators in LQCD or match these HT matrix elements to experimental observables, ...

Thanks!