Center for Frontiers in Nuclear Science

Workshop on
Pion and Kaon Structure Functions
at the EIC

2-5 June 2020
Online

Pion, Kaon and Proton Mass Understanding

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June 2, 2020

Jefferson Lab

U.S. DEPARTMENT OF ENERGY | Office of Science

JSA

U.S. - based Electron-Ion Collider

- ❑ **On January 9, 2020:**

  The U.S. DOE announced the selection of BNL as the site for the Electron-Ion Collider

  → *A new era to explore the emergent phenomena of QCD!*

- ❑ **A long journey – a joint effort of the full community:**

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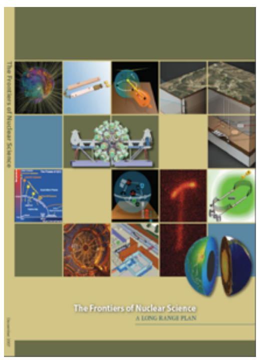
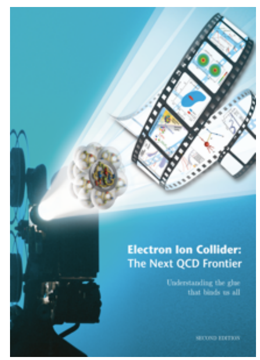
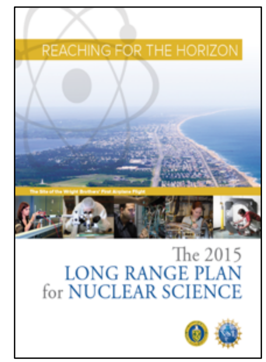
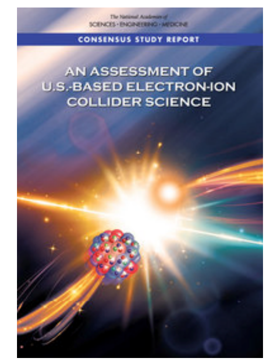








…

"… answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics."

… three profound questions:

How does the mass of the nucleon arise?

How does the spin of the nucleon arise?

What are the emergent properties of dense systems of gluons?

Emergent Hadron Properties from QCD

❑ Mass – intrinsic to a particle:

= Energy of the particle when it is at the rest

✧ QCD energy-momentum tensor in terms of quarks and gluons

$$T^{\mu\nu} = \frac{1}{2}\overline{\psi}\left[iD^{(\mu}\gamma^{\nu)}\right]\psi + \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F_{\alpha}{}^{\nu}$$

Operator - Same for all hadrons

✧ Hadron Mass:

$$M_h(\mu) = \left.\frac{\langle h(p)|\int d^3x\, T^{00}(\mu)\,|h(p)\rangle}{\langle h(p)|h(p)\rangle}\right|_{\text{Rest frame}}$$

$h = \pi, K, p, n, \ldots$

Depending on the state

❑ Spin – intrinsic to a particle:

= Angular momentum of the particle when it is at the rest

✧ QCD angular momentum density in terms of energy-momentum tensor

$$M^{\alpha\mu\nu} = T^{\alpha\nu}x^{\mu} - T^{\alpha\mu}x^{\nu} \qquad J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{0jk}$$

Operator - Same for all hadrons

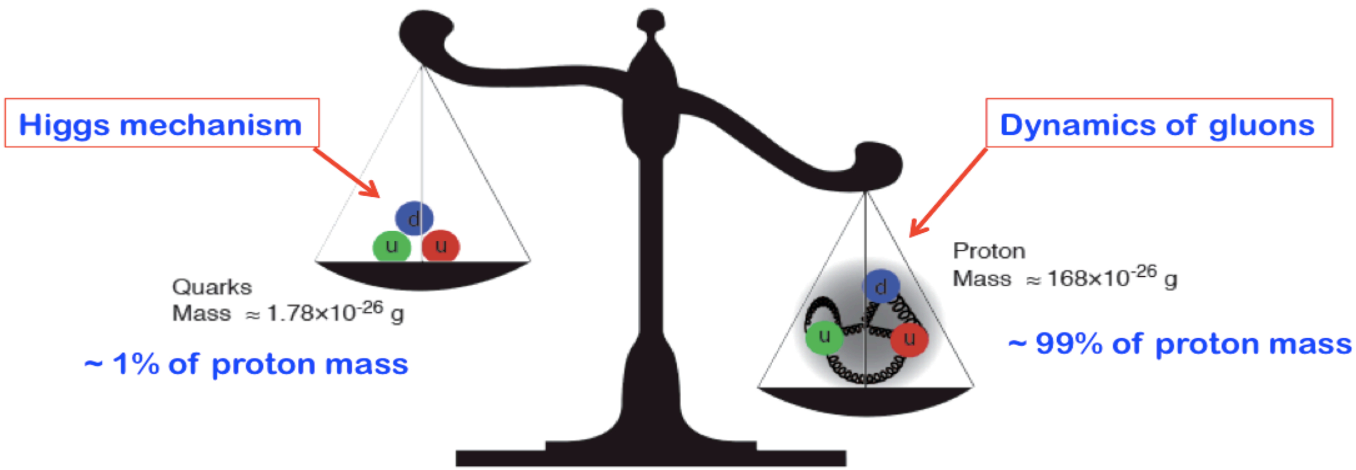
✧ Hadron Spin:

$$S_h(\mu) = \left.\frac{\langle h(p),s|\,J^z(\mu)\,|h(p),s\rangle}{\langle h(p),s|h(p),s\rangle}\right|_{\text{Rest frame}}$$

If we do not understand hadron mass & spin, we do not know QCD!

The Proton Mass

- ❑ **Nucleon mass – dominates the mass of visible world:**



Higgs mechanism is not enough!!! *“Mass without mass!”*

- ❑ **How does QCD generate the nucleon mass?**

  “… The vast majority of the nucleon’s mass is due to quantum fluctuations of quark-antiquark pairs, the gluons, and the energy associated with quarks moving around at close to the speed of light. …”

  REACHING FOR THE HORIZON

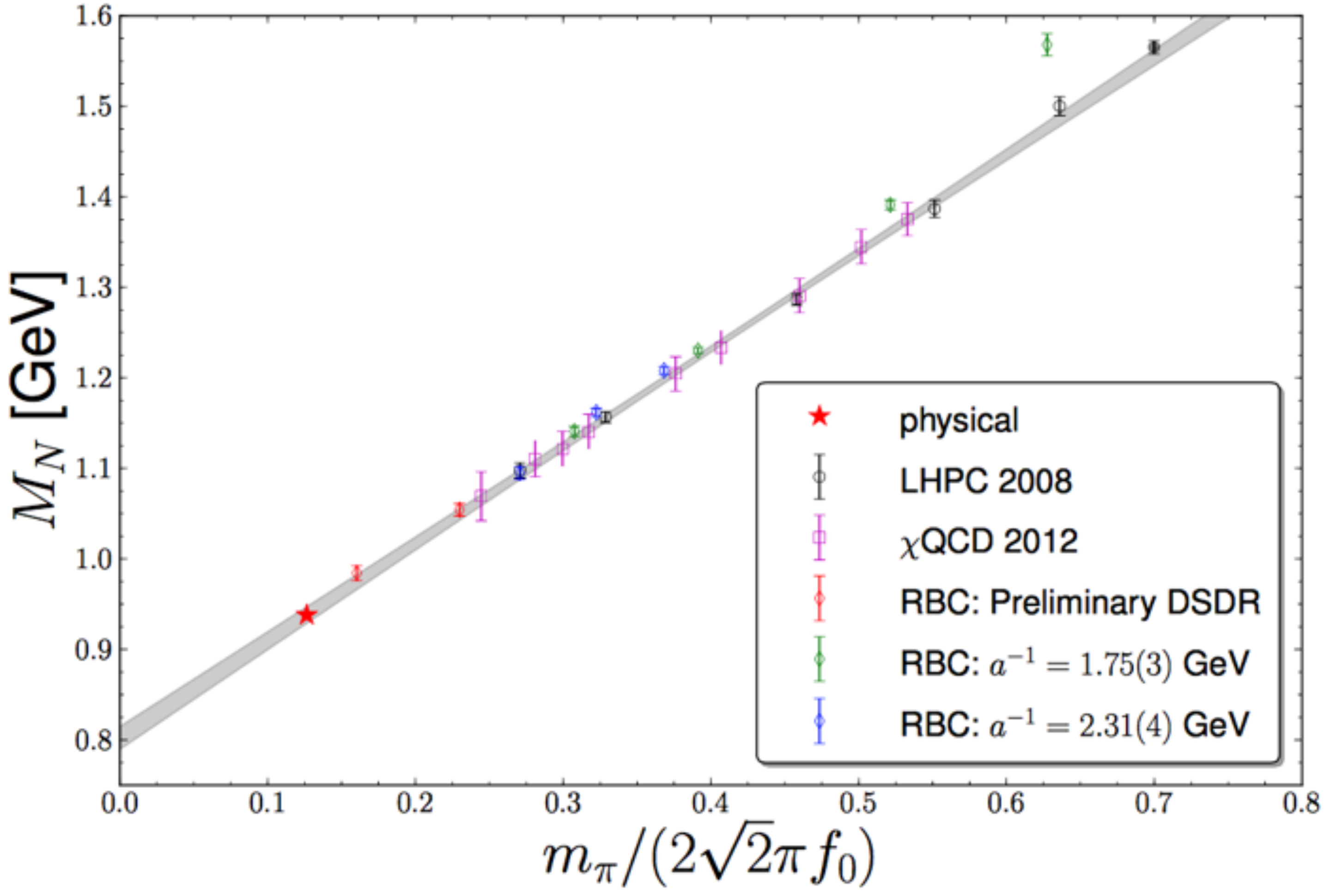
  The 2015 Long Range Plan for Nuclear Science

How to quantify and verify this, theoretically and experimentally?

Nucleon Mass vs Pion Mass

Martin Savage @ Temple meeting

- **Nucleon mass from Lattice QCD calculation:**

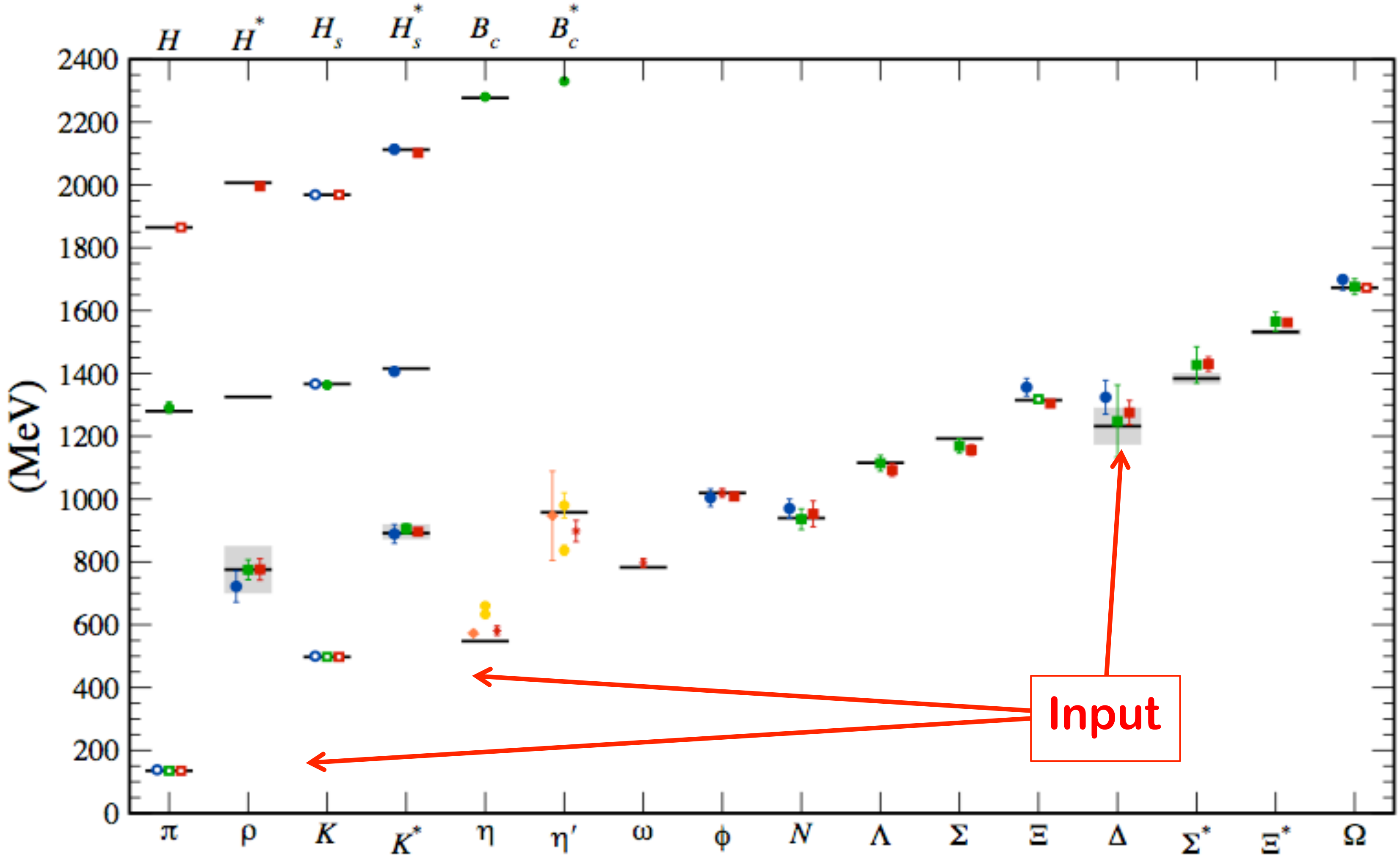


$$M_N = 800 \text{ MeV} + m_\pi$$

Unexpected behavior !!

Hadron Mass from Lattice QCD

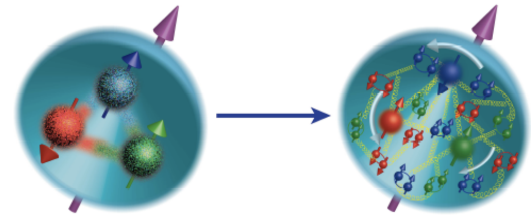
- ❑ **Hadron mass from Lattice QCD calculation:**



How does QCD generate this? The role of quarks vs. that of gluons?

The Proton Mass: from Models to QCD

❑ Dynamical scale:



✧ **Asymptotic freedom** $\longleftrightarrow$ **confinement:**

➡ **A dynamical scale,** Λ_{QCD} **, consistent with** $\dfrac{1}{R} \sim 200\ \mathrm{MeV}$

❑ Bag model:

✧ **Kinetic energy of three quarks:** $K_q \sim 3/R$

✧ **Bag energy (bag constant B):** $T_b = \dfrac{4}{3}\pi R^3\, B$

✧ **Minimize** $K_q + T_b$**:** $M_p \sim \dfrac{4}{R(\mathrm{fm})} 197.3(\mathrm{MeV fm})$

✧ **Proton radius:**

$$R(\mathrm{PDG}) = 0.8414(19)\ \mathrm{fm}$$

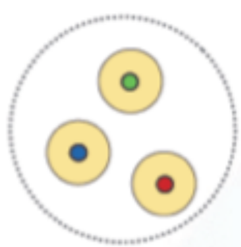
✧ **Proton mass:**

➡ $M_p \sim 936 - 940\ \mathrm{MeV}$

$$M_p(\mathrm{PDG}) = 938.272\ \mathrm{MeV}$$

Meson is different, especially the pion!

❑ Constituent quark model:



✧ **Spontaneous chiral symmetry breaking:**

Massless quarks gain ~300 MeV mass when traveling in vacuum

➡ $M_p \sim 3\, m_q^{\mathrm{eff}} \sim 900\ \mathrm{MeV}$

Pion, Kaon and Proton Mass in QCD

❑ QCD:

$$\mathcal{L}_{\rm QCD}(m) = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \overline{\psi}_i \left[i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij} \right] \psi_j$$

❑ Hadron mass (in any frame):

$$\langle h(p) | T^{\mu\nu} | h(p) \rangle \propto p^\mu p^\nu \quad \Rightarrow \quad \langle h(p) | T^\mu_\mu | h(p) \rangle \propto p^2 = M_h^2$$

Same $T^{\mu\nu}$ for:

$$\langle P(p) | T^\mu_\mu | P(p) \rangle = M_p^2 \sim (938\,\mathrm{MeV})^2$$

$$\langle \pi(p) | T^\mu_\mu | \pi(p) \rangle = M_\pi^2 \sim (139\,\mathrm{MeV})^2 \quad \ll M_p^2$$

$$\langle K(p) | T^\mu_\mu | K(p) \rangle = M_K^2 \sim (497\,\mathrm{MeV})^2$$

❑ Poincare Invariance:

$$\partial_\mu T^{\mu\nu} = 0$$

❑ Global scale invariance for $\mathcal{L}_{\rm QCD}(m=0)$: $\quad x \to e^{-\alpha} x$

Classically,

$$T^{\mu\nu} g_{\mu\nu} = T^\mu_\mu = 0$$

Dilaton current: $\mathcal{D}_\mu \equiv T_{\mu\nu}\, x^\mu$

$$\partial^\mu \mathcal{D}_\mu = 0 = \left[\partial^\mu T_{\mu\nu} \right] x^\mu + T_{\mu\nu}\, g^{\mu\nu}$$

No massive bound state!

Pion, Kaon and Proton Mass in QCD

❑ Sources of hadron mass:

Quantum mechanically,

Vacuum expectation breaks chiral symmetry

$$T^{\alpha}_{\alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} \underbrace{m_q (1+\gamma_m) \overline{\psi}_q \psi_q}_{\text{Chiral symmetry breaking}}$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \dots$$

Need to know: $\langle h(p)|F^{\mu\nu,a} F^{a}_{\mu\nu}|h(p)\rangle$ and $\langle h(p)|\overline{\psi}_q \psi_q|h(p)\rangle$ independently!

❑ Isolate the traceless term:

$$T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$$

Traceless term: $\overline{T^{\mu\nu}} \equiv T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\alpha}$

Trace term: $\widehat{T^{\mu\nu}} \equiv \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\alpha}$

Decomposition and Sum Rules

X. Ji, PRL (1995)

❑ Mass in hadron's rest frame:

✧ **Hadron state:**

$|P\rangle$ **With the normalization:** $\langle P|P\rangle = (E/M)(2\pi)^3\delta^3(\mathbf{0})$

✧ **Hamiltonian:**

$$H_{\rm QCD} = \int d^3\vec{x}\, T^{00}(0,\vec{x}) \qquad \langle P|H_{\rm QCD}|P\rangle = (E^2/M_p)(2\pi)^3\delta^3(0)$$

✧ **QCD energy-momentum tensor:**

$$T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}} \qquad \langle P|T^{\mu\nu}|P\rangle = P^\mu P^\nu/M_p$$

No $g^{\mu\nu}$ term!

$$\langle P|\overline{T}^{\mu\nu}|P\rangle = \left(P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu}\right)/M_p \qquad \langle P|\widehat{T}^{\mu\nu}|P\rangle = \frac{1}{4}M_p\, g^{\mu\nu}$$

$$\left.\frac{\langle P|\int d^3x\, \overline{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \frac{3}{4}M_p \qquad \left.\frac{\langle P|\int d^3x\, \widehat{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \frac{1}{4}M_p$$

"Traceless" term *"Trace" term*

❑ Sum rules for Proton Mass:

Relativistic motion

χ Symmetry Breaking

Quantum fluctuation

$$M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \overbrace{M_q + M_g}^{\text{Relativistic motion}} + M_m + M_a$$

Quark Energy

Gluon Energy

Quark Mass

Trace Anomaly

Decomposition and Sum Rules

❑ Identities:

$$\frac{\langle P|\int d^3x\, T^{\alpha}_{\ \alpha}|P\rangle}{\langle P|P\rangle} = 4\left.\frac{\langle P|\int d^3x\, \widehat{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \frac{4}{3}\left.\frac{\langle P|\int d^3x\, \overline{T}^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}}$$

❑ Traceless terms:

$$\overline{T}^{\mu\nu} = \overline{T}^{\mu\nu}_{q} + \overline{T}^{\mu\nu}_{g} \qquad \bar{T}^{\mu\nu}_{q} = \frac{1}{2}\bar{\psi}i\overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi - \frac{1}{4}g^{\mu\nu}\bar{\psi}m\psi, \qquad \bar{T}^{\mu\nu}_{g} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}_{\ \alpha}$$

$$\Rightarrow \quad \langle P|\overline{T}^{\mu\nu}_{q}|P\rangle \equiv a(\mu^2)(P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_p$$

$$\langle P|\overline{T}^{\mu\nu}_{g}|P\rangle \equiv [1-a(\mu^2)](P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_P$$

$$\left.\frac{\langle P|\int d^3x\, \overline{T}^{00}_{q}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = a(\mu^2)\,\frac{3}{4}\,M_p \qquad \left.\frac{\langle P|\int d^3x\, \overline{T}^{00}_{g}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = [1-a(\mu^2)]\,\frac{3}{4}\,M_p$$

❑ Trace terms:

$$\widehat{T}^{\mu\nu} = \widehat{T}^{\mu\nu}_{m} + \widehat{T}^{\mu\nu}_{a} \quad \Rightarrow \quad \langle P|\widehat{T}^{\mu\nu}_{m}|P\rangle \equiv b\,\frac{1}{4}M_p\, g^{\mu\nu} \qquad \langle P|\widehat{T}^{\mu\nu}_{a}|P\rangle \equiv [1-b]\,\frac{1}{4}M_p\, g^{\mu\nu}$$

$$\left.\frac{\langle P|\int d^3x\, \widehat{T}^{00}_{m}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = b\,\frac{1}{4}\,M_p \qquad \left.\frac{\langle P|\int d^3x\, \widehat{T}^{00}_{a}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = [1-b]\,\frac{1}{4}\,M_p$$

Decomposition and Sum Rules

X. Ji, PRL (1995)

❑ Roles of quarks and gluons?

✧ **Quark energy contribution:**

$$H_q = \int d^3\vec{x}\; \bar{\psi}(-i\mathbf{D}\cdot\alpha)\psi, \qquad M_q = \left.\frac{\langle P|H_q|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (a-b)\,\frac{3}{4}\,M_p$$

✧ **Gluon energy contribution:**

$$H_g = \int d^3\vec{x}\; \frac{1}{2}(\mathbf{E}^2+\mathbf{B}^2) \qquad M_g = \left.\frac{\langle P|H_g|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-a)\,\frac{3}{4}\,M_p$$

✧ **Quark mass contribution:**

$$H_m = \int d^3\vec{x}\; \bar{\psi}m\psi \qquad M_m = \left.\frac{\langle P|H_m|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = b\,M_p$$

✧ **Trace anomaly contribution:**

$$H_a = \int d^3\vec{x}\; \frac{9\alpha_s}{16\pi}\,(\mathbf{E}^2-\mathbf{B}^2) \qquad M_a = \left.\frac{\langle P|H_a|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = (1-b)\,\frac{1}{4}\,M_p$$

➡ Relativistic motion | χ Symmetry Breaking | Quantum fluctuation

$$M_p = \left.\frac{\langle P|\int d^3x\, T^{00}|P\rangle}{\langle P|P\rangle}\right|_{\text{at rest}} = \overbrace{M_q + M_g}^{\text{Relativistic motion}} + M_m + M_a$$

Quark Energy | Gluon Energy | Quark Mass | Trace Anomaly

➡ **Need to connect "a" and "b" to physical observables independently!**

Decomposition and Sum Rules

❑ Physical meaning of "a":

$$\langle P|\overline{T}_q^{\mu\nu}|P\rangle \equiv a(\mu^2)(P^\mu P^\nu - \frac{1}{4}M_p^2\, g^{\mu\nu})/M_p$$

$$\bar{T}_q^{\mu\nu} = \frac{1}{2}\bar{\psi} i \overleftrightarrow{D}^{(\mu}\gamma^{\nu)}\psi - \frac{1}{4}g^{\mu\nu}\bar{\psi}m\psi, \qquad \bar{T}_g^{\mu\nu} = \frac{1}{4}g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}.$$

Let $\mu \to +$ **and** $\nu \to +$:

$$\Rightarrow \quad a_q(\mu^2) = \int dx\, x\left[q(x,\mu^2) + \bar{q}(x,\mu^2)\right] \qquad a_g(\mu^2) = \int dx\; x\; g(x,\mu^2)$$

Total momentum fraction carried by the quarks & gluons are measurable

❑ Physical meaning of "b":

$$\langle P|\widehat{T}_m^{\mu\nu}|P\rangle \equiv b\,\frac{1}{4}M_p\, g^{\mu\nu} \qquad b = \frac{1}{M_p}\frac{\langle P|\int d^3\vec{x}\;\overline{\psi}\, m\, \psi|P\rangle}{\langle P|P\rangle}$$

Need independent measurement of the trace anomaly!

$$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g}F^{\mu\nu,a}F^a_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s}\underbrace{m_q(1+\gamma_m)\overline{\psi}_q\psi_q}_{\text{Chiral symmetry breaking}}$$

Decomposition and Sum Rules

- Quark mass contribution – the "b"-term:

$$b\,M_p = \langle P|m_u\overline{u}u + m_d\overline{d}d|P\rangle + \langle P|m_s\overline{s}s|P\rangle + ...(\text{heavy flavors})$$

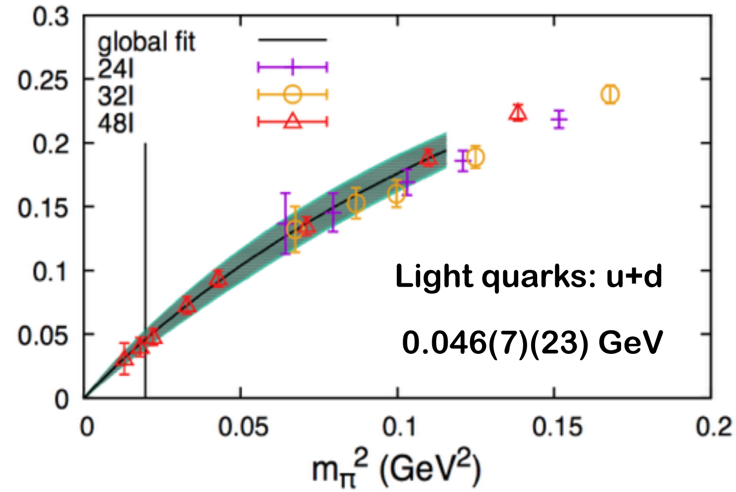
- The first term – πN σ-term:

$$\sigma_{\pi N} = \hat{m}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

with $\hat{m} = (m_u + m_d)/2$

Both lattice QCD and phenomenological analyses give:

$$\sigma_{\pi N} \sim 45 - 50\ \text{MeV}$$



YBY, PRD94, 054503 (2016)

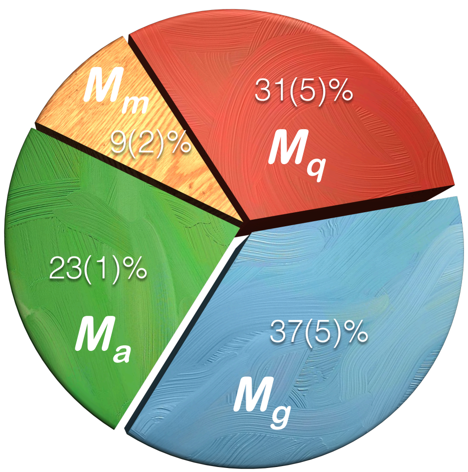
- The second term – strange scalar charge:

Light strange quark:

$$\sigma_{KN} = (\hat{m} + m_s)\langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N\rangle/4$$

Both lattice QCD and phenomenological analyses give:

$$\sigma_{KN} \sim 360 - 400\ \text{MeV}$$



Need measurement and calculation of trace anomaly!

Pion and Kaon Mass

- The derivation of above decomposition is valid for all hadrons except "massless" one:

At the chiral limit, $m_\pi = 0$

$$\langle \pi(p)|T^\mu_\mu|\pi(p)\rangle = m_\pi^2 \to 0$$

DCSB – see Roberts' talk

- Do not have to choose the rest frame:

$$T^\alpha_{\ \alpha} = \frac{\beta(g)}{2g} F^{\mu\nu,a} F^a_{\mu\nu} + \sum_{q=u,d,s} m_q(1+\gamma_m)\overline{\psi}_q \psi_q \quad \Rightarrow \quad \langle h(p)|T^\alpha_{\ \alpha}|h(p)\rangle = M_h^2$$

Need to know: $\langle h(p)|F^{\mu\nu,a} F^a_{\mu\nu}|h(p)\rangle$ and $\langle h(p)|\overline{\psi}_q \psi_q|h(p)\rangle$ independently!

- Contribution from the traceless term:

$$\langle P|\overline{T}^{\mu\nu}|P\rangle = (P^\mu P^\nu - \frac{1}{4} M_p^2\, g^{\mu\nu})/M_p$$

We do not have to choose "00" components and the rest frame

We could choose $\mu \to +$ and $\nu \to -$ (or $\perp\perp$):

$$M_p = 2\,\langle h(p)\,|\overline{T}^{+-}(0)\,|h(p)\rangle$$

Ji, 2003.04478

Aslan, Qiu
In preparation

Need to find new observables sensitive to these high-twist operators!

Pion and Kaon Structure

- ❑ **Pion decays, and there is no stable pion target**
- ❑ **Pion beam:**

  Talking advantage of time-dilation, $\pi + p \to \ell^+\ell^- + X$ Drell-Yan process

  Precision of pion structure depends on our knowledge of proton structure

- ❑ **Lattice QCD:**

  – using a vector-axial-vector correlation as an example

$$\frac{1}{2}\left[T^{\mu\nu}_{v5}(\xi,p) + T^{\mu\nu}_{5v}(\xi,p)\right] = \frac{\xi^4}{2}\langle h(p)|\left(\mathcal{J}^\mu_v(\xi/2)\,\mathcal{J}^\nu_5(-\xi/2) + \mathcal{J}^\mu_5(\xi/2)\,\mathcal{J}^\nu_v(-\xi/2)\right)|h(p)\rangle$$
$$\equiv \epsilon^{\mu\nu\alpha\beta}\,p_\alpha\xi_\beta\,\widetilde{T}_1(\omega,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)\,\widetilde{T}_2(\omega,\xi^2)$$

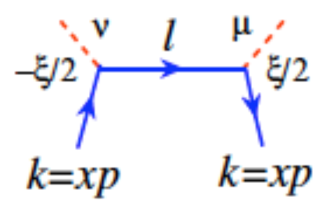
Vanishes under T

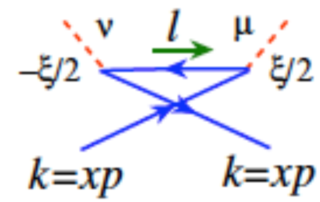
✧ Collinear factorization:

$$\widetilde{T}_i(\omega,\xi^2) = \sum_{f=q,\bar{q},g}\int_0^1 \frac{dx}{x}\, f(x,\mu^2)\, C^f_i(\omega,\xi^2;x,\mu^2) + \mathcal{O}\left[|\xi|/\mathrm{fm}\right]$$

✧ Lowest order coefficient functions:

$$C^{q(0)}_1(\omega,\xi^2;x) = \frac{1}{\pi^2}\,x\left(e^{ix\omega} + e^{-ix\omega}\right)$$





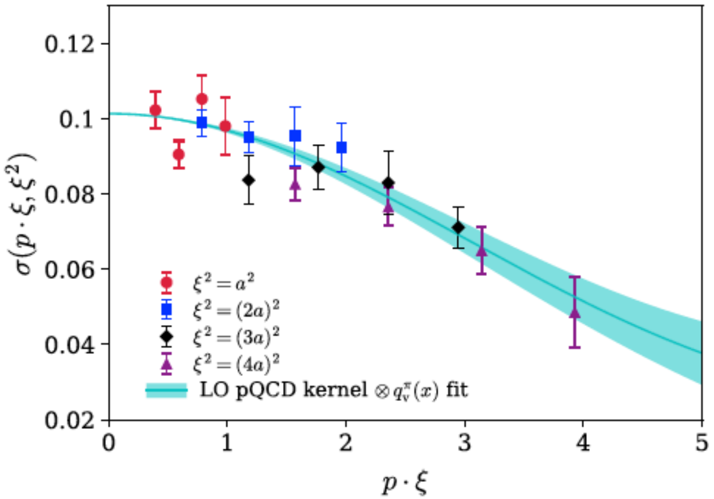
$$T_1(\widetilde{x},\xi^2) \equiv \int \frac{d\omega}{2\pi} e^{-i\widetilde{x}\omega}\,\widetilde{T}_1(\omega,\xi^2)$$
$$= \frac{1}{\pi^2}\left(q(\widetilde{x},\mu^2) - \overline{q}(\widetilde{x},\mu^2)\right) \equiv \frac{1}{\pi^2} q_v(\widetilde{x},\mu^2)$$

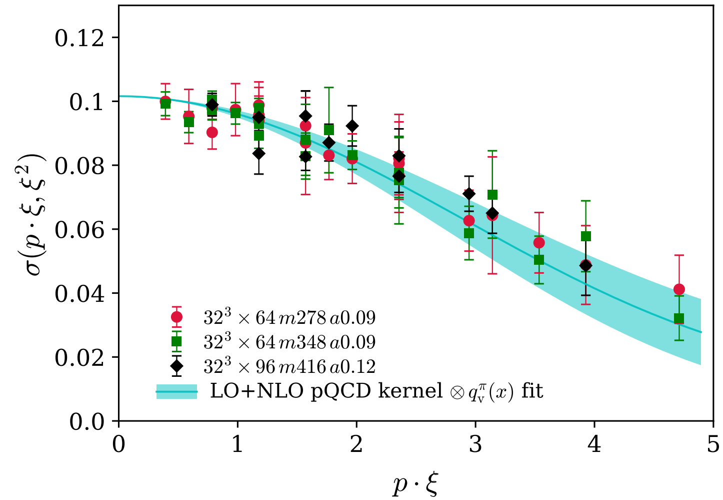
Sufian et al.
PRD99 (2019) 074507

Current-Current Correlators

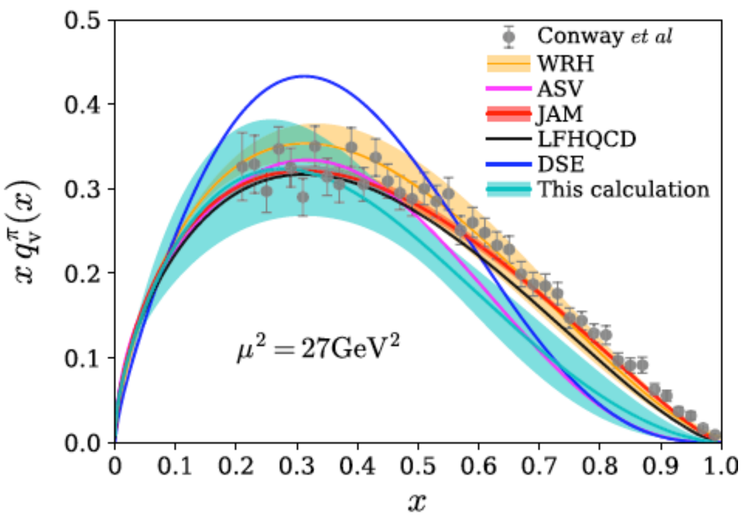
Sufian et al. JLab
PRD99 (2019) 074507

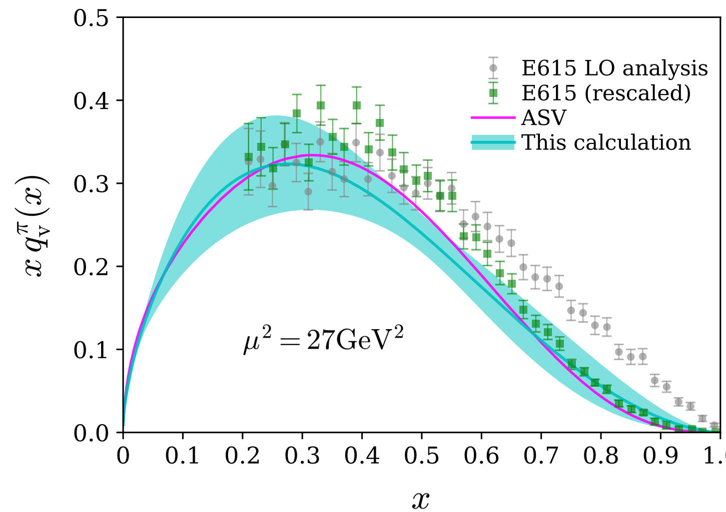
- "Lattice cross section" of V-A current correlator:





- Extracted pion valence quark distribution:





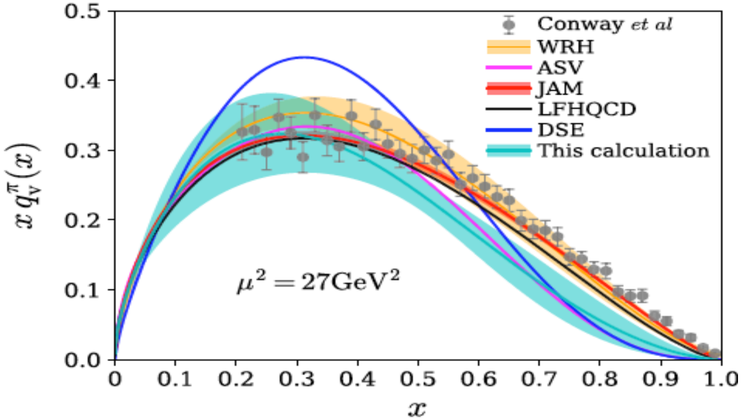
Sufian et al. @ DNP19

Comparison of pion PDF from different approaches

M. Constantinou @ DNP19

★ Good LCSs:

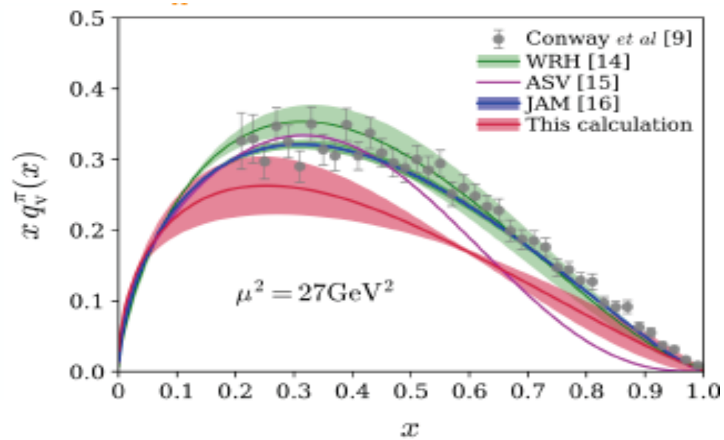
[R. Sufian et al. (JLab - W&M), Phys. Rev. D 99 (2019) 074507, arXiv:1901.03921]



$m_\pi = 416\,\mathrm{MeV}$, $P = 0.6 - 1.5\,\mathrm{GeV}$

★ pseudo-PDFs:

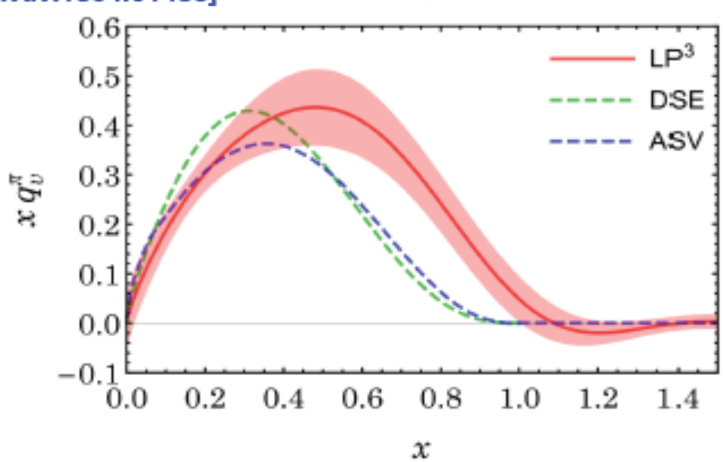
[B. Joo et al. (JLab-W&M), arXiv:1909.08517]



$m_\pi = 416\,\mathrm{MeV}$, $\nu \leq 4.7$

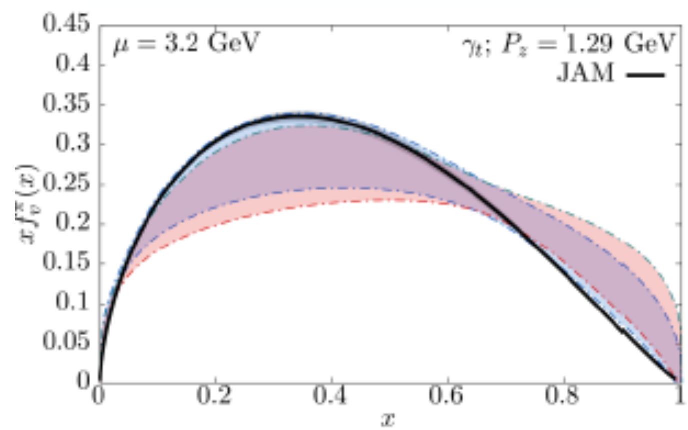
★ quasi-PDFs:

[J.-H. Zhang et al. (LP3), Phys. Rev. D 100, 034505 (2019), arXiv:1804.01483]



$m_\pi = 310\,\mathrm{MeV}$, $P = 1.74\,\mathrm{GeV}$

[T. Izubuchi et al. (BNL-SBU-UConn), Phys. Rev. D 100, 034516 (2019) arXiv:1905.06349]



$m_\pi = 300\,\mathrm{MeV}$, $P = 1.29\,\mathrm{GeV}$

Summary and outlook

- Hadron mass is an emergent phenomenon of QCD dynamics
- Decomposition of hadron mass (or sum rule) is not unique:

  Individual terms are local matrix elements of quark/gluon operators

  None of the individual terms are physical, owing to the confinement

  A decomposition is valuable iff individual terms can be measured or calculated independently with controllable approximations
- Good decomposition should work for both proton, pion, and …

$$\langle P(p)|T^{\mu}_{\mu}|P(p)\rangle = M_p^2 \sim (938\,\mathrm{MeV})^2$$
$$\langle \pi(p)|T^{\mu}_{\mu}|\pi(p)\rangle = M_\pi^2 \sim (139\,\mathrm{MeV})^2$$
$$\langle K(p)|T^{\mu}_{\mu}|K(p)\rangle = M_K^2 \sim (497\,\mathrm{MeV})^2$$

- We can also get mass decomposition from traceless term, and calculate new sets of operators in LQCD or match these HT matrix elements to experimental observables, …

Thanks!