JAM Pion PDF Analysis Including Resummation

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Workshop on Pion and Kaon Structure Functions at the EIC

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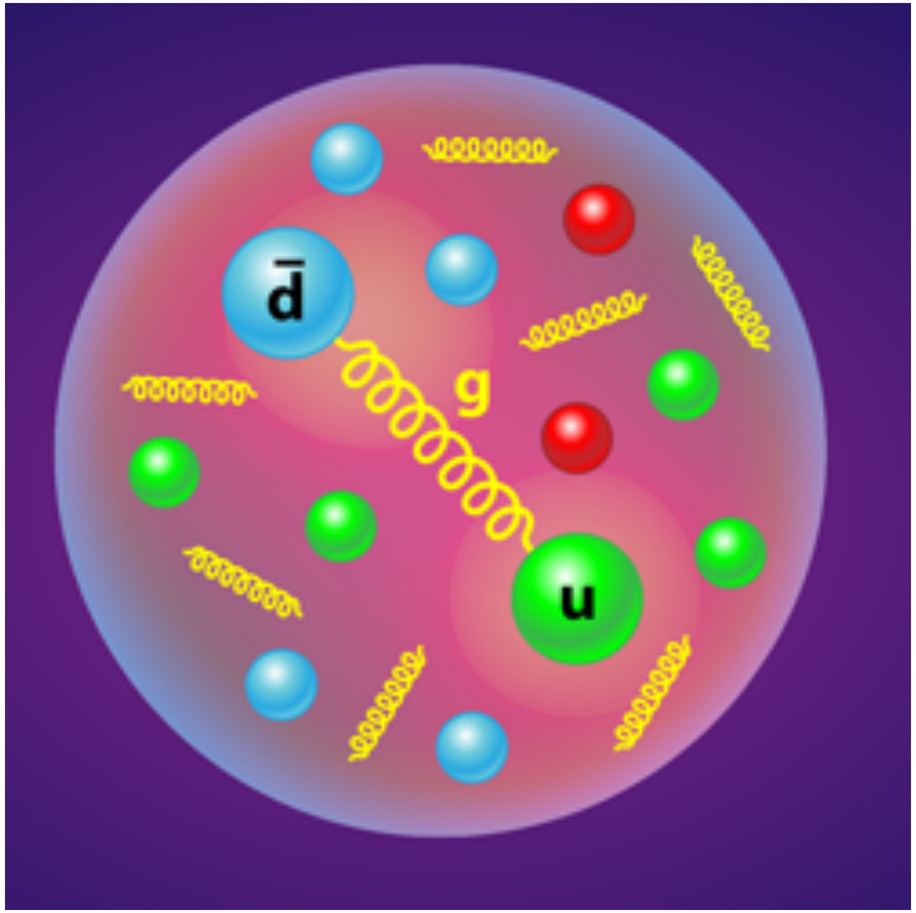
Introduction/Motivation

Pions

- Pion is the Goldstone boson associated with chiral symmetry breaking
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a $q\bar{q}$ bound state



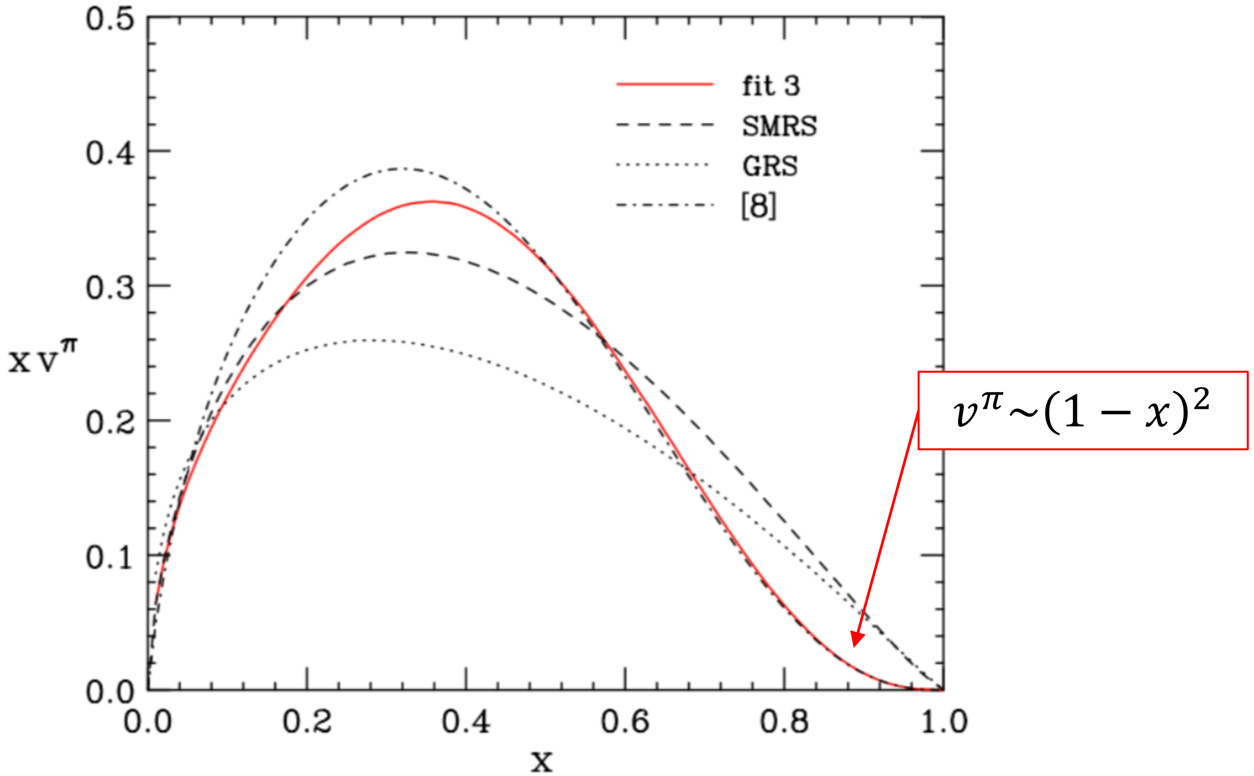
Theoretical Interest

- Behavior of PDF as $x_\pi \to 1$ ($\mathrm{v}_\pi \sim (1 - x_\pi)^{2\beta}$) has theoretical interest
- Active debate as to whether $\beta = 1$ or $1/2$

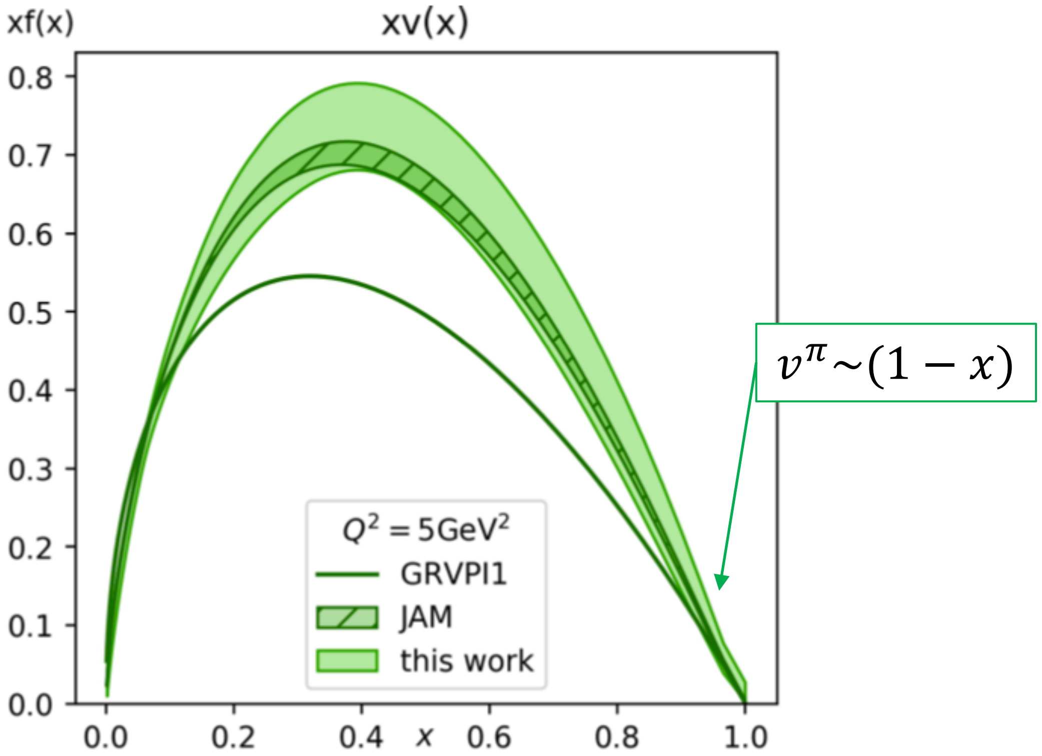
Theoretical Interest

- Recent lattice calculations as well as phenomenologically determined valence quark PDFs using threshold resummation indicate $\beta = 1$ as opposed to fixed order ($\beta = 1/2$)
- Our analysis with threshold resummation will have impact on this question

Recent Pion Phenomenology



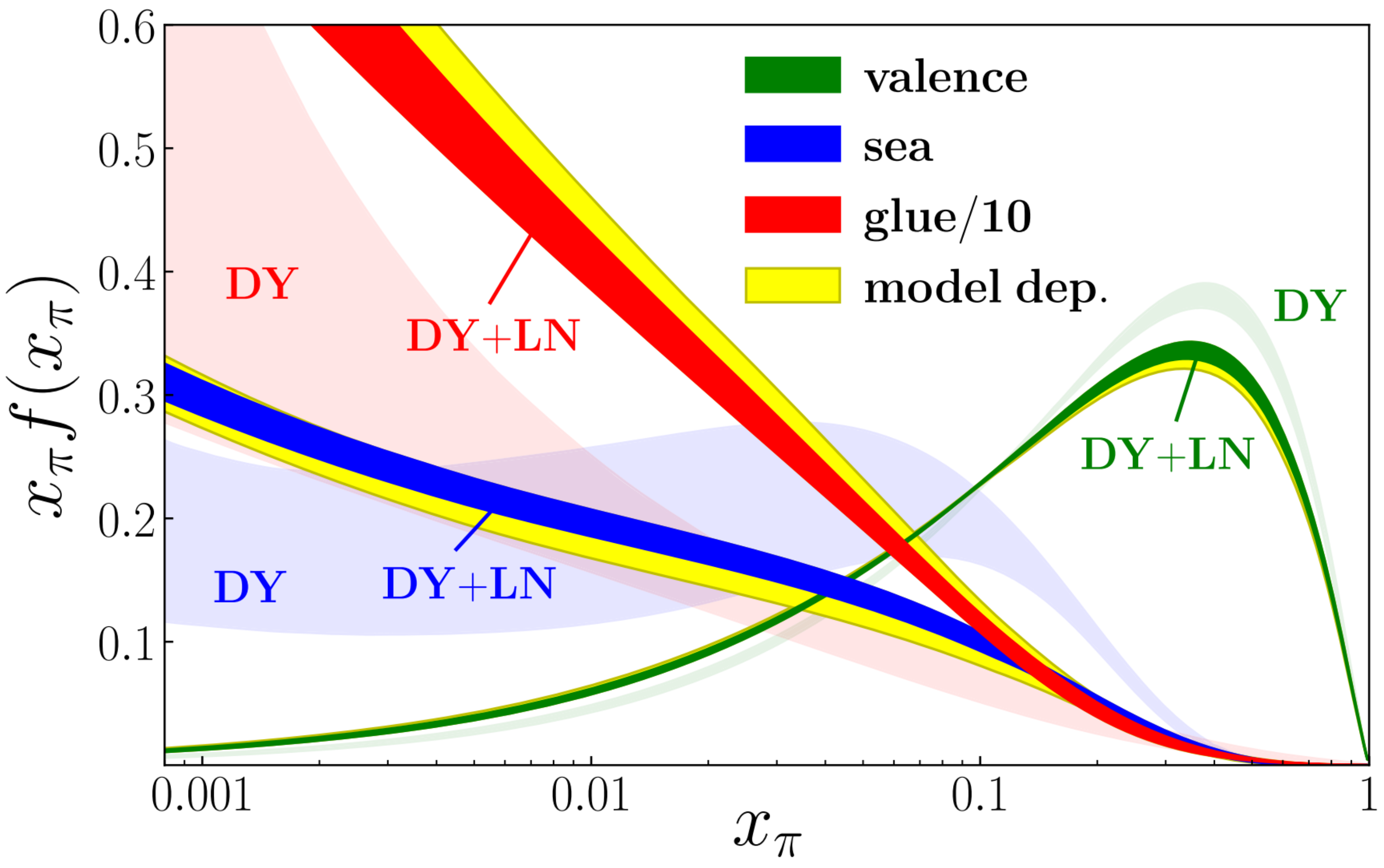
- Recent (M. Aicher, et al, 2010) pion fit to DY data
- Fit uses soft gluon resummation



- Recent (I. Novikov, et al, 2020, xFitter) pion fit to DY and prompt photon data
- Fit uses NLO in α_S

JAM 18 Pion PDFs

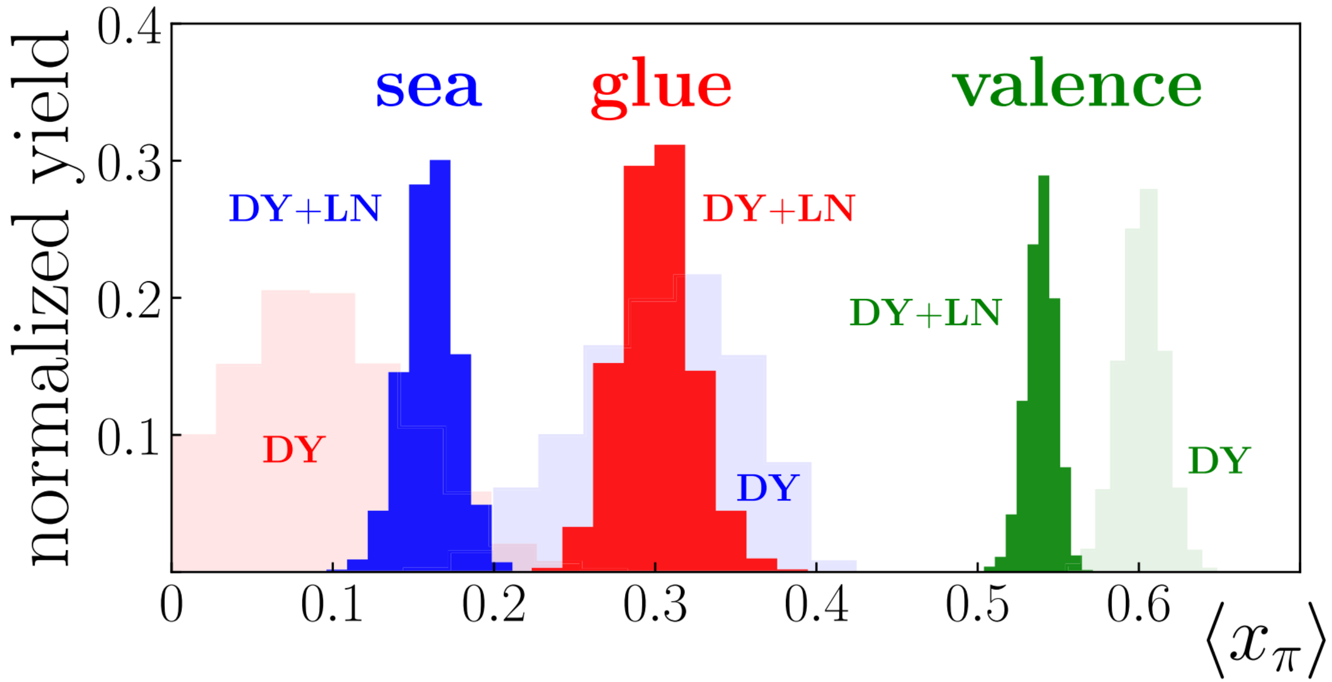
- Valence, sea, and gluon distributions were extracted in an NLO analysis
- Drell-Yan (DY) only fit then include the Leading Neutron (LN)



PB, N. Sato, W. Melnitchouk, C. –R. Ji, Phys. Rev. Lett. **121**, 152001 (2018)

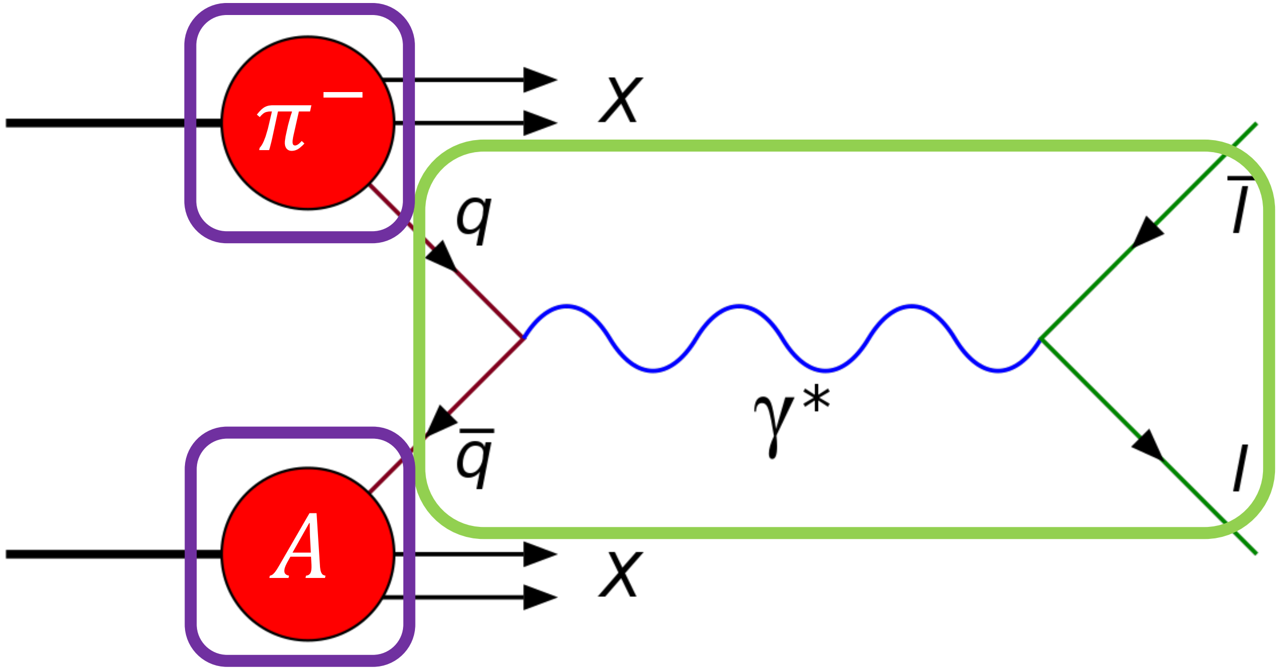
JAM 18 Momentum Fractions

- We also compute the momentum fractions for each flavor
- Large difference in in the gluon and sea $\langle x_\pi \rangle$ from a DY to a DY+LN analysis
- Gluon carries ~30% of the momentum fraction at the initial scale



Observables

Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^A(x_A, \mu) \otimes \hat{\sigma}_{i,j}(x_\pi, x_A, Q/\mu)$$

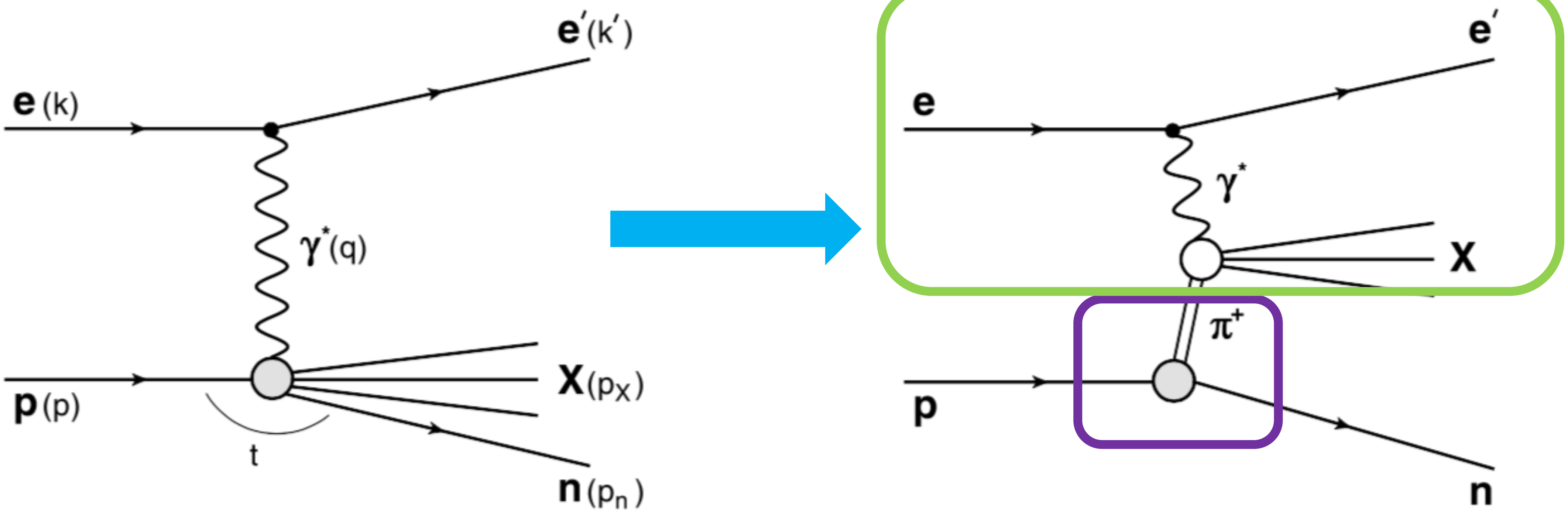
DY Observable and x_F

- Observables in $\pi^- W$ DY experiments such as E615 and NA10 are

$$\frac{d\sigma}{dx_F dQ^2}$$

- It's important to note that while x_F is measured, the parton momentum fraction is *NOT*
- The relation to parton momentum fraction $x_F = x_1^0 - x_2^0$ only holds at leading order, where $x_{1,2}^0 = \sqrt{\tau} e^{\pm Y}$, where $\tau = \frac{Q^2}{S}$ and Y is the rapidity
- In an NLO analysis, interpretation cannot remain

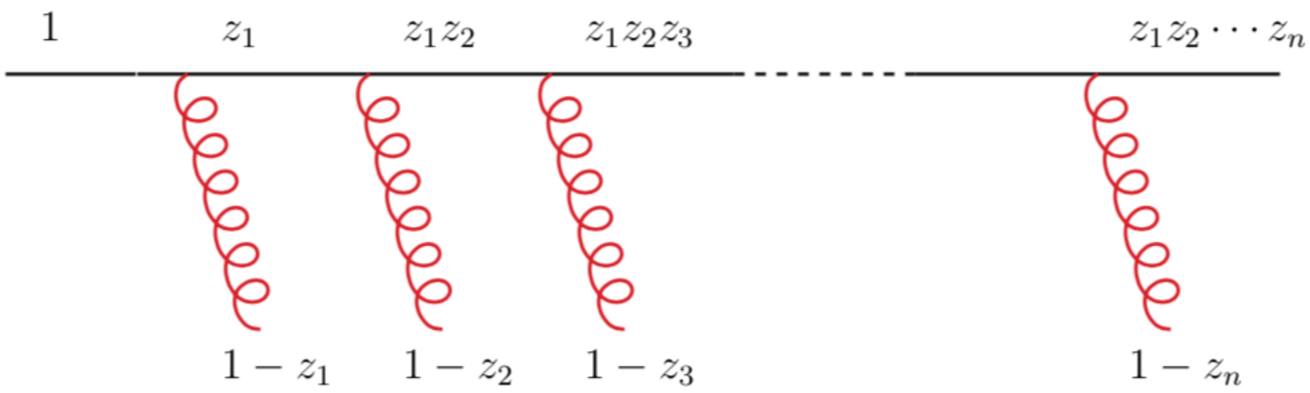
Leading Neutron (LN)



$$\frac{d\sigma}{dx\,dQ^2\,dy} \sim f_{p\to\pi^+n}(y) \times \sum_q \int_{x/y}^{1} \frac{d\xi}{\xi} C(\xi)\, q\left(\frac{x/y}{\xi}, \mu^2\right)$$

Threshold Resummation in Drell-Yan

Soft Gluon Resummation



- The goal is to sum the contributions of the soft gluon emissions from the quark line to all orders of α_S
- Can perturbatively calculate these emissions to all orders of α_S
- Here, z_i near 1

Exponentiation in Mellin space

- The matrix elements of emitted soft gluons that carry large logarithms are factorized in the Eikonal approximation
- Phase space only factorizes in Mellin space
- Summing over all orders of α_S leads to exponentiation of the Mellin space coefficients

$$\sum_{n=1}^{\infty} C^{(n)}(N) = \sum_{n} \frac{1}{n!} [C^{(1)}_{\mathrm{soft}}(N)]^n$$
$$= \exp\left(C^{(1)}_{\mathrm{soft}}(N)\right)$$

Full Hard Kernel to Calculate

$$C^{\mathrm{N}^p\mathrm{LO}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) = \sum_{j=0}^{p} \alpha_s^j C^{(j)}(N) + C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s) - \sum_{j=0}^{p} \frac{\alpha_s^j}{j!}\left[\frac{d^j C^{\mathrm{res}}_{\mathrm{N}^k\mathrm{LL}}(N,\alpha_s)}{d\alpha_s^j}\right]_{\alpha_s=0}$$

Fixed order Kernel
Already have calculated this at NLO!

Resummation Kernel.
Calculate such as Leading Log,
or Next-to-Leading Log

Matching coefficients
Need to subtract in
order to avoid double
counting

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

An NLO calculation gathers the $\mathcal{O}(\alpha_S)$ terms

	LL	NLL	…	N^{p}LL
LO	1	--	…	--
NLO	$\alpha_S \log(N)^2$	$\alpha_S \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^{k}LO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Add the columns to the rows

	LL	NLL	…	N^pLL
LO	1	--	…	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	…	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2 (\log(N)^2, \log(N)^3)$	…	--
…	…	…	…	…
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	…	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^pLL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2(\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^kLO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k\left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Drell-Yan Rapidity Distribution

- Formulate resummation in Mellin space for Q^2(or τ) distribution
- For rapidity distribution, introduce a Fourier transform

$$\sigma(N,M) = \int_0^1 d\tau \tau^{N-1} \int_{-\ln 1/\sqrt{\tau}}^{\ln 1/\sqrt{\tau}} dY\, e^{iMY} \frac{d\sigma}{dQ^2 dY}$$

- To compare with data, must invert back to momentum fraction and rapidity space

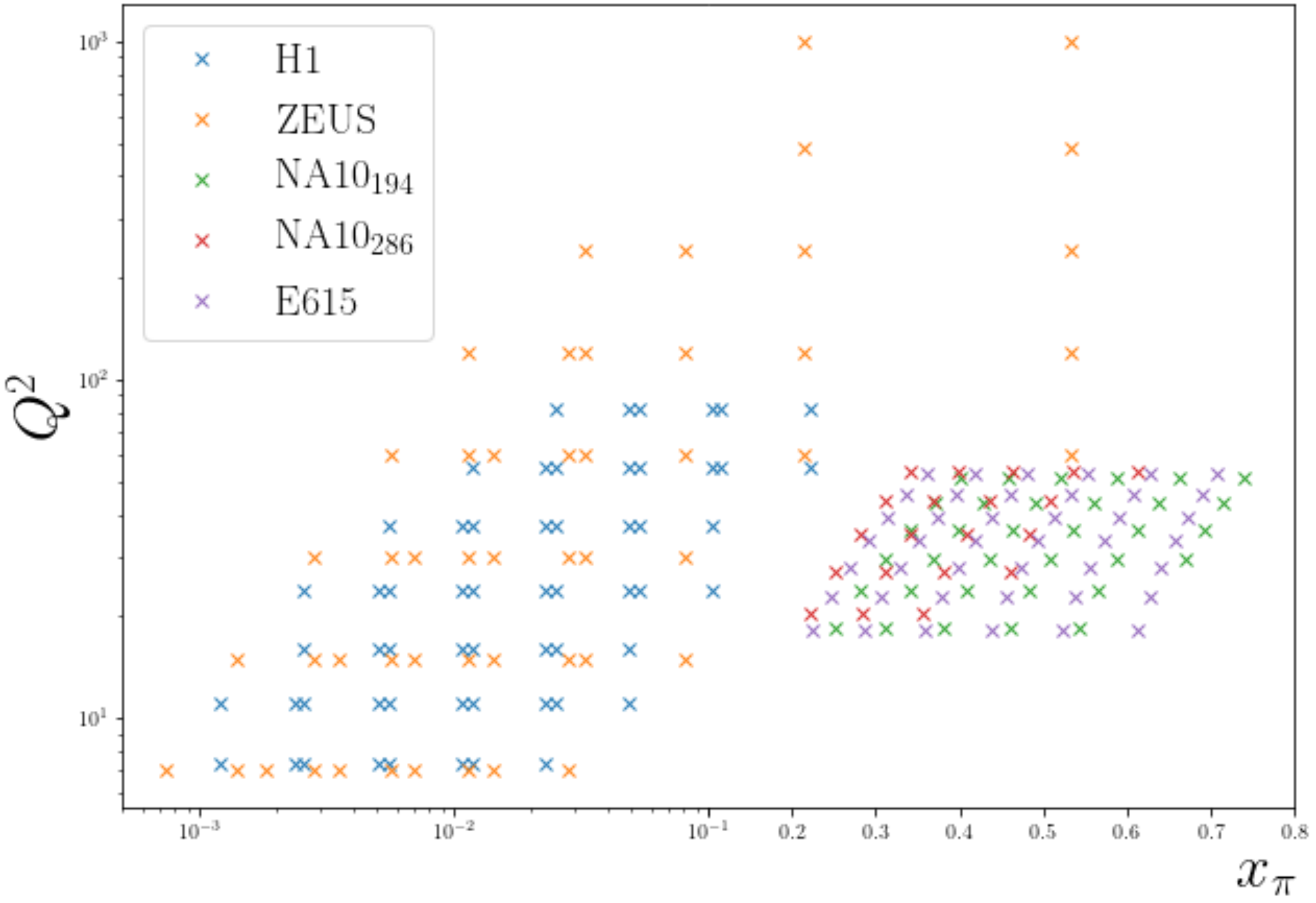
$$\frac{d\sigma}{dQ^2 dY} = \int_{-\infty}^{\infty} \frac{dM}{2\pi} e^{-iMY} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \sigma(N,M)$$

Non-trivial! But beyond the scope of this talk

Extraction Procedure

Kinematic Coverage

- We want to be able to fit simultaneously the Drell-Yan and Leading Neutron data
- We can shape the pion PDFs at both high- and low-x_π with both datasets
- E615, NA10 – DY
- H1, ZEUS – LN



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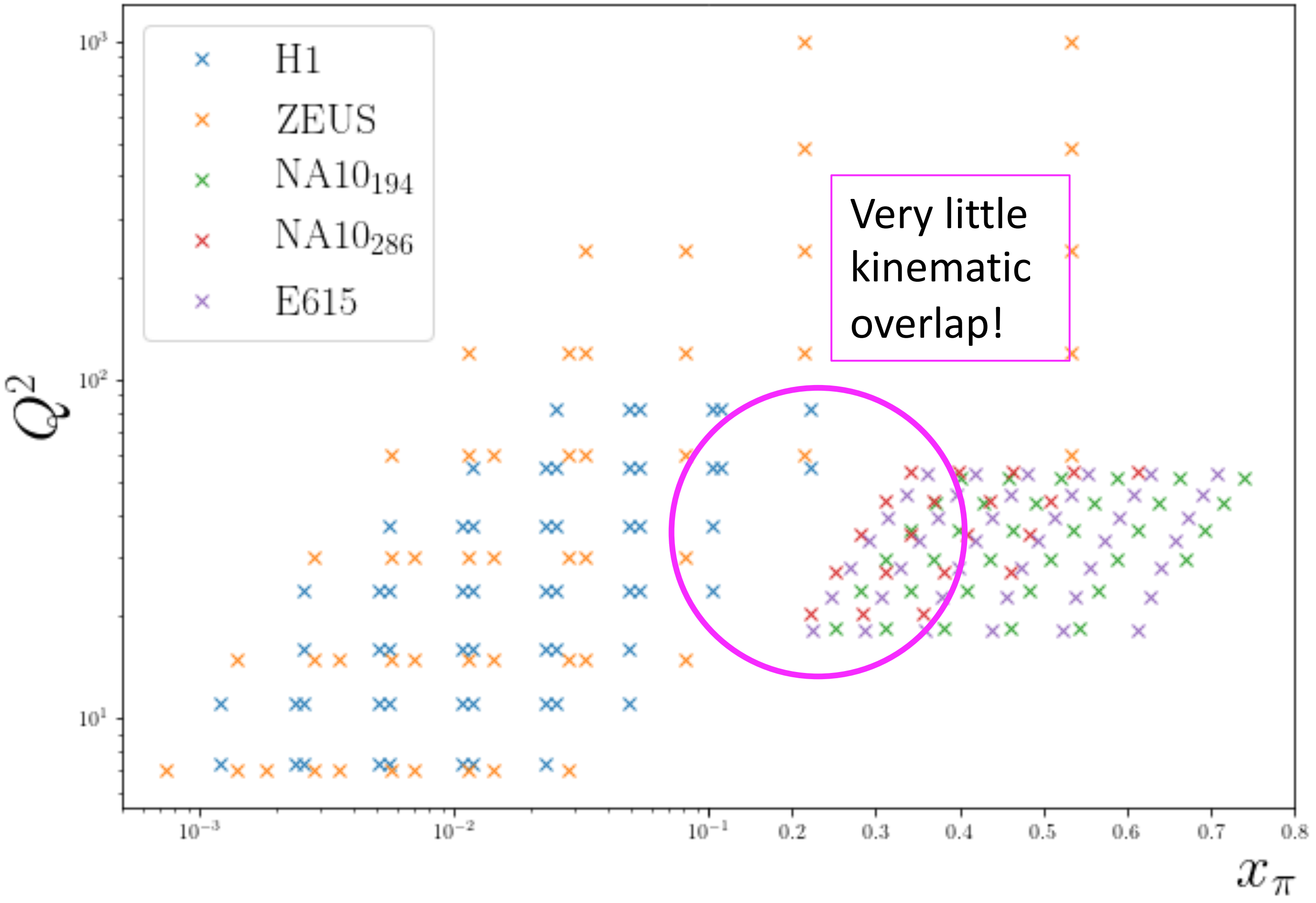


Parameterization of the PDF (in terms of π^-)

Vector of parameters

- Each PDF is parameterized as $$f(x_\pi, Q_0^2; \mathbf{a}) = \frac{N}{B(2+\alpha, \beta)} x_\pi^\alpha (1 - x_\pi)^\beta$$ $\boxed{Q_0^2 = m_c^2}$
- We equate the valence distributions: $\bar{u}_v^{\pi^-} = d_v^{\pi^-}$
- We equate the light sea distributions: $u^{\pi^-} = \bar{d}^{\pi^-} = u_s^{\pi^-} = d_s^{\pi^-} = s = \bar{s}$
- Parameters are reduced by the quark sum rule and momentum sum rule

Quark sum rule $$\int_0^1 dx_\pi q_v^\pi = 1$$

Momentum Sum Rule $$\int_0^1 dx_\pi x_\pi (2q_v^\pi + 6q_s^\pi + g^\pi) = 1$$

Monte Carlo

- Using Bayesian statistics, we describe the probability

$$\mathcal{P}(\mathbf{a}|\text{data}) \propto \mathcal{L}(\text{data}|\mathbf{a})\pi(\mathbf{a})$$

- We quantify the expectation value and variance of our observable $\mathcal{O}$ as a function of the parameter set $\boldsymbol{a}_i$

$$E[\mathcal{O}] = \frac{1}{N}\sum_i \mathcal{O}(\mathbf{a}_i)$$

$$V[\mathcal{O}] = \frac{1}{N}\sum_i \left[\mathcal{O}(\mathbf{a}_i) - E[\mathcal{O}]\right]^2$$

Multi-Step Strategy

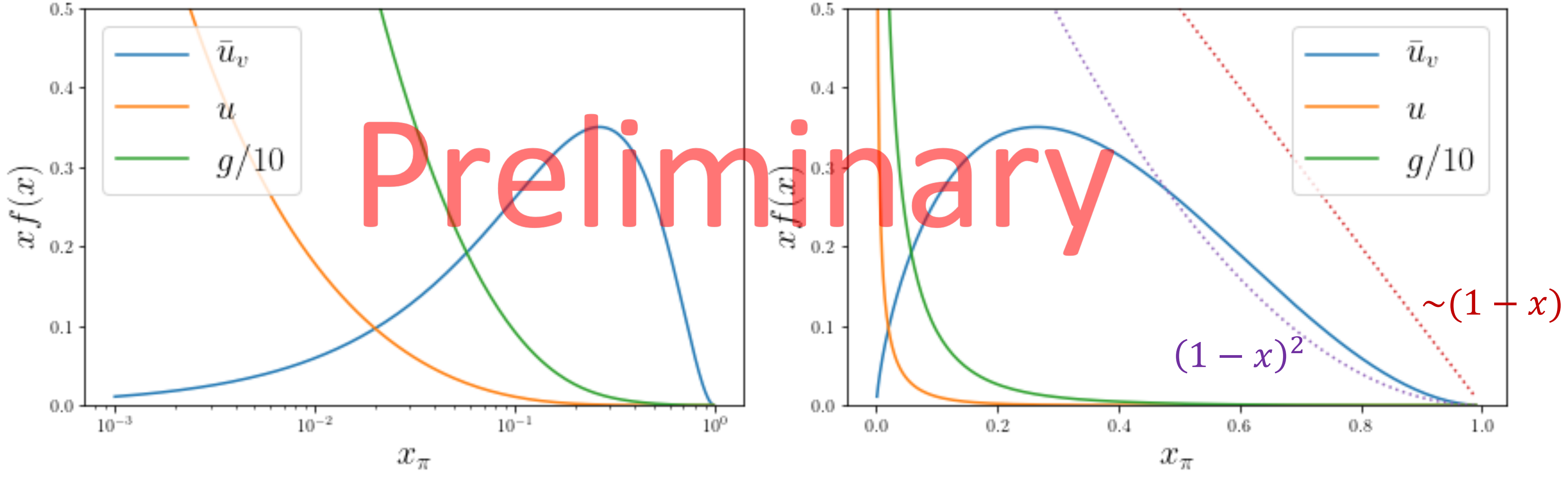
- Fitting PDFs to many types of observables all at once is time consuming and slows the fit
- We start with many replicas with flat priors to fit to one observable, the $\pi^{-}W$ DY data
- The posteriors from that fit are used as the priors for the next fit, which includes the LN data

Results

Single Fit to DY

- Apply kinematic cuts of $0 < x_F < 0.8$ and $4.16 < Q < 7.5$ GeV to avoid mesonic resonances
- Fit to only E615 data and achieve a χ^2/npts of 93.14/55

Here, $Q^2 = 10$ GeV²



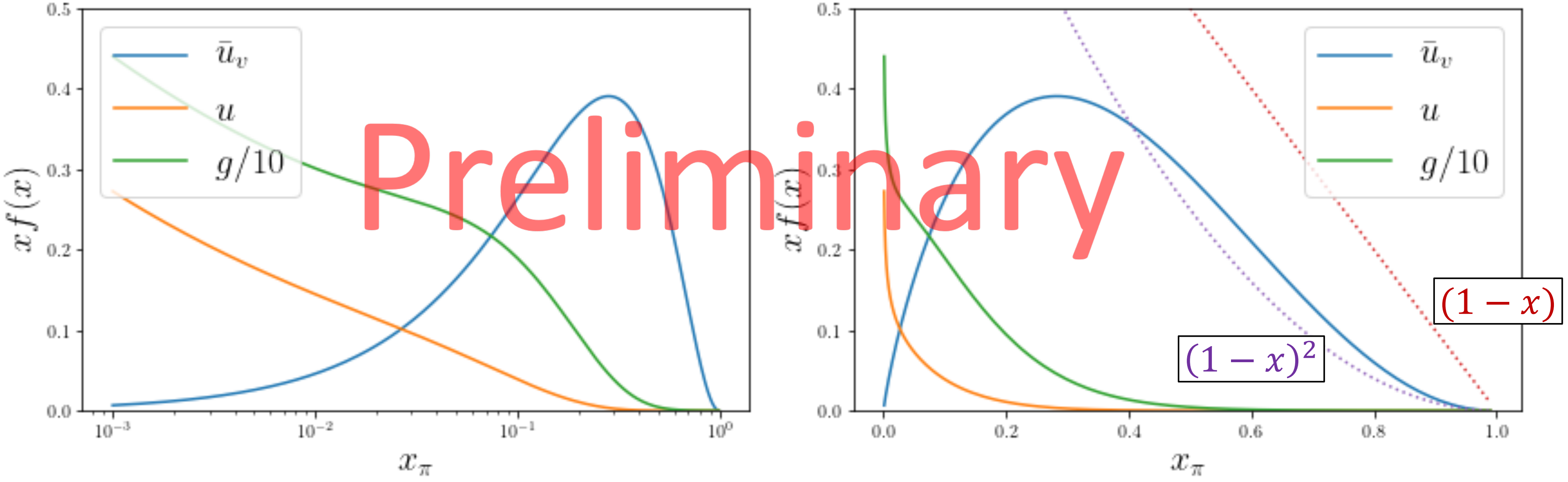
Log Scale

Linear Scale

Single Fit to DY and LN

- Include the H1 dataset from the LN experiments
- The H1 dataset has χ^2/npts=18.95/58, and the E615 data has χ^2/npts=84.12/55

Here, $Q^2 = 10$ GeV²

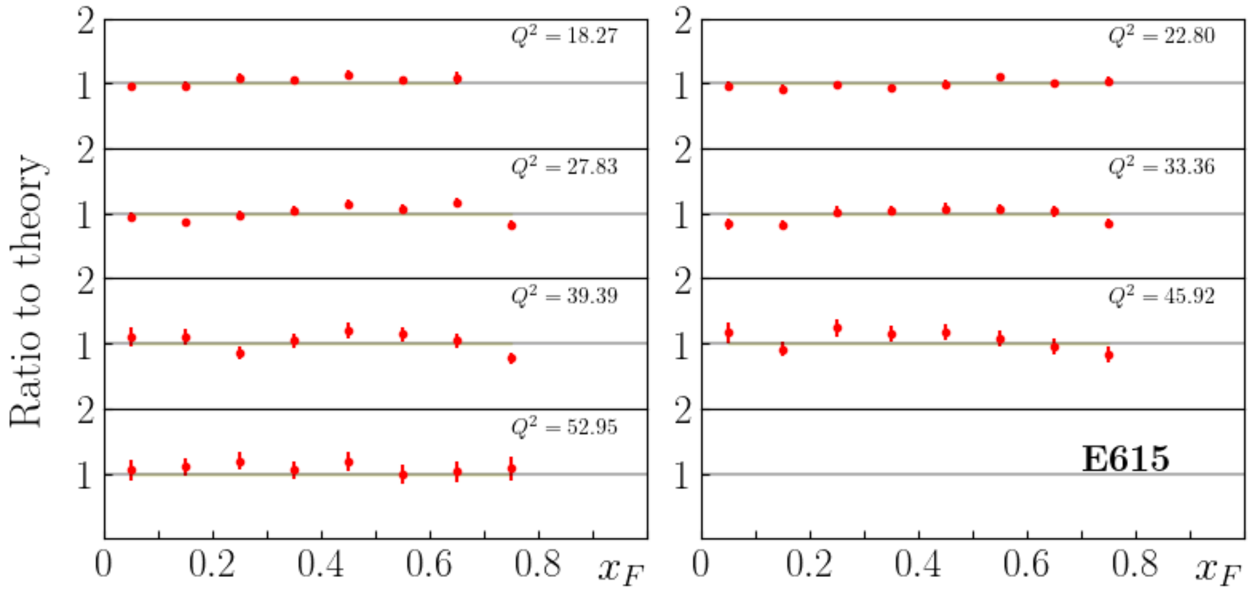


Log Scale

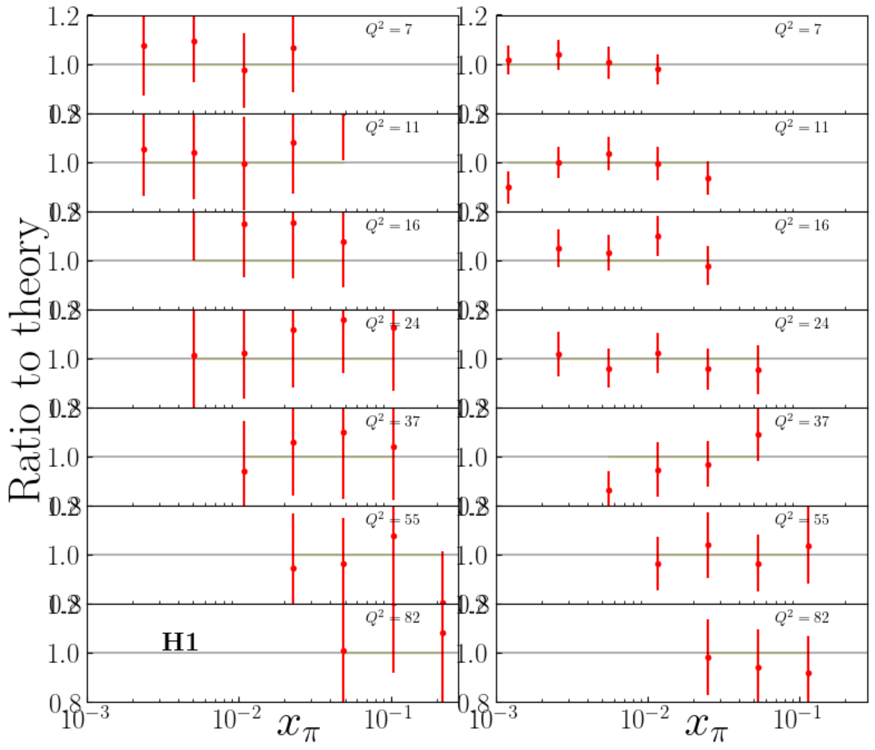
Linear Scale

Data and Theory Agreement

- Show the data divided by the theory and see good agreement



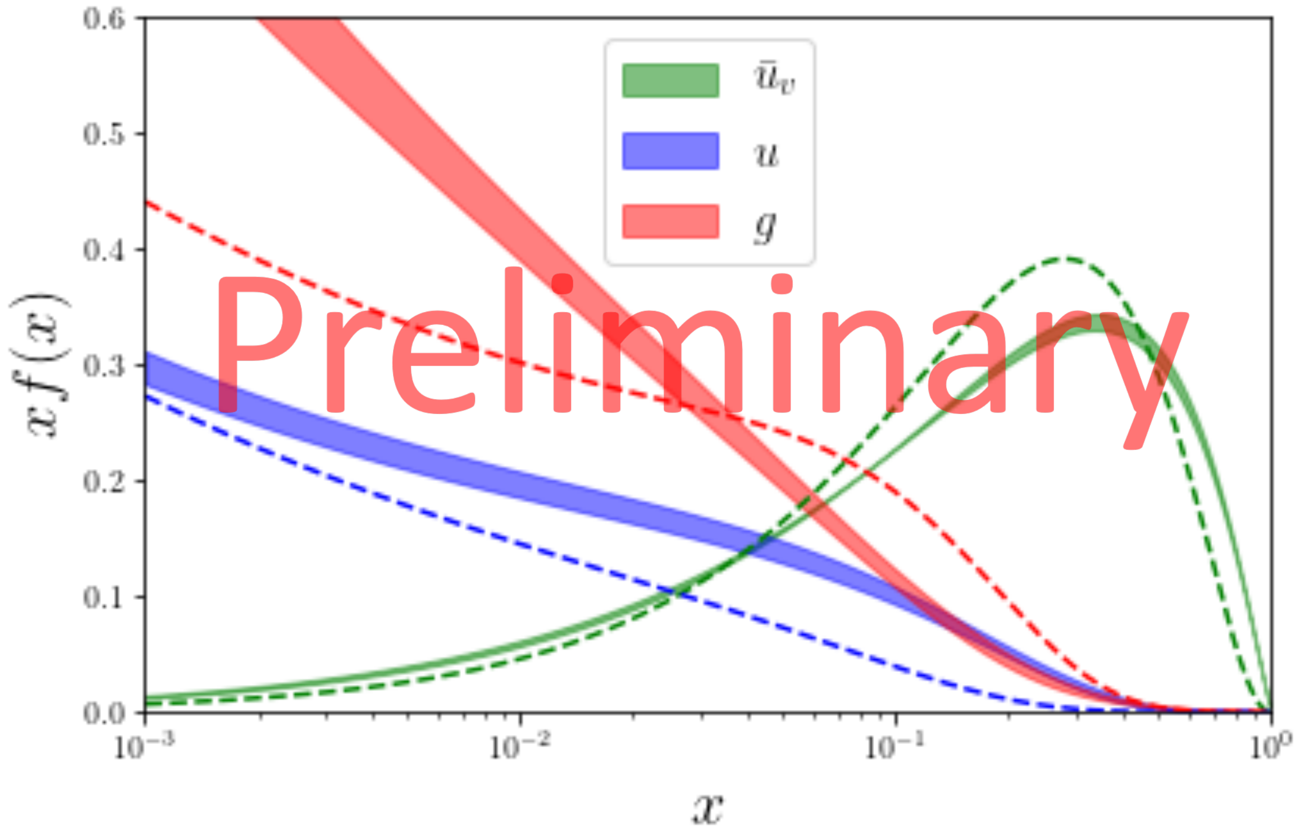
Drell-Yan E615 data



Leading Neutron H1 data

Comparison to JAM18 Pion PDFs

- Available from https://github.com/JeffersonLab/jam18pion
- Bands are without resummation, and dashed lines are latest fit with resummation
- A softer fall off at high x in resummation fit
- Sea is more suppressed than fixed order



Momentum Fraction Comparisons

- We can calculate the momentum fractions in comparison with the JAM18 PDF analysis

	JAM18 Pions	Resummation Single Fit
$\langle x_\pi \rangle_{val}$	0.54 ± 0.01	0.52
$\langle x_\pi \rangle_{sea}$	0.16 ± 0.02	0.022
$\langle x_\pi \rangle_{glu}$	0.30 ± 0.02	0.46

- The sea is *much* lower, the gluon is considerably higher

Challenges

- A χ^2 penalty had to be placed in order to avoid a fit with negative sea
- Fits show that the sea is effectively 0 in the DY region
- Because of the lack of data, there is a void in the constraints on the sea and the gluon at large x_π
- Because there are three unknown functions and only DY, LN, and Q^2 evolution as observables, we cannot validate universality of PDFs
- More precision data is needed

Conclusions

Next Steps

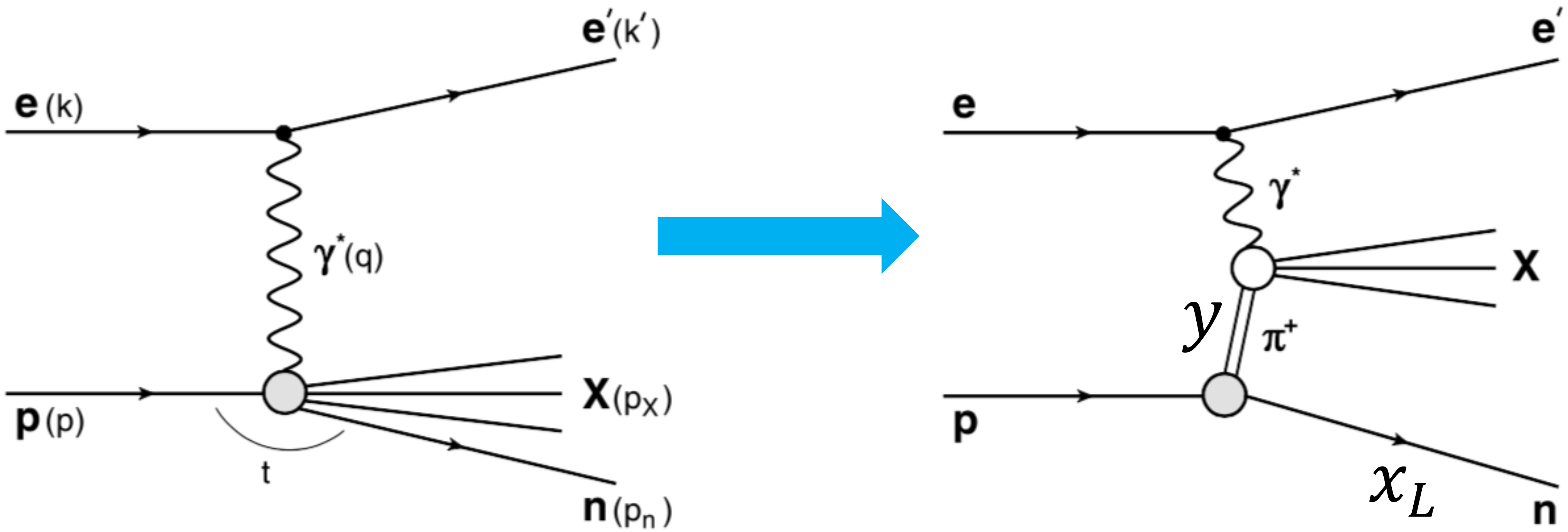
- Will include the NA10 DY data and the ZEUS data
- Do Monte Carlo analysis to quantify uncertainties
- Investigate different resummation prescriptions and approximations such as Borel prescription and Double Mellin transformations

Summary and Conclusions

- Soft gluon resummation allows us to sum the large logarithms that could potentially spoil perturbation at larger orders of α_S
- Fits done with resummation indicate a softer fall off at high-x_π
- Resummation is still a work in progress
- More data with large p_T will be sensitive to large-x_π PDFs

Backup Slides

Leading Neutron (LN)



- One pion exchange occurs when x_L is near 1
- When t is very small, exchanged pion is almost on-shell

Leading Neutron (LN)

Splitting Function:

$$f_{\pi N}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y[k_\perp^2 + y^2 M^2]}{x_L^2 D_{\pi N}^2} |\mathcal{F}|^2$$

Where $y = k^+/p^+ = x/x_\pi$,
$g_A = 1.267$, $f_\pi = 93\text{MeV}$

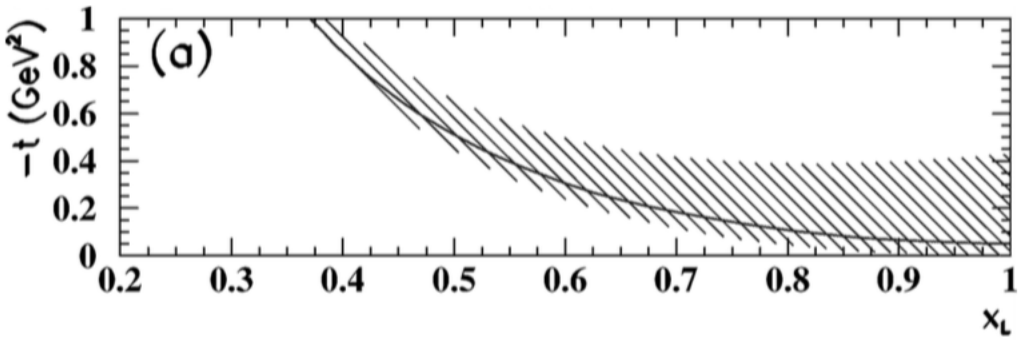
$$D_{\pi N} \equiv t - m_\pi^2 = -\frac{1}{1-y}[k_\perp^2 + y^2 M^2 + (1-y)m_\pi^2]$$

$$\mathcal{F} = \begin{cases} \text{(i)} \quad \exp\left((M^2 - s)/\Lambda^2\right) & s\text{-dep. exponential} \\ \text{(ii)} \quad \exp\left(D_{\pi N}/\Lambda^2\right) & t\text{-dep. exponential} \\ \text{(iii)} \quad (\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & t\text{-dep. monopole} \\ \text{(iv)} \quad \bar{x}_L^{-\alpha_\pi(t)} \exp\left(D_{\pi N}/\Lambda^2\right) & \text{Regge} \\ \text{(v)} \quad \left[1 - D_{\pi N}^2/(\Lambda^2 - t)^2\right]^{1/2} & \text{Pauli-Villars} \end{cases}$$

UV regulators used in the literature

High x_L

- At low t, the neutron carries the majority of the longitudinal momentum of the proton, x_L
- In this region, we can guarantee that a proton has split into a pion and a neutron (as opposed to another particle)



Setting resummation up

- Because of the Eikonal approximation, in the soft limit, matrix elements of large numbers of emitted gluons can be factorized as such:

$$\mathcal{M}_n(z_1, \ldots, z_n) \overset{\text{soft}}{\simeq} \frac{1}{n!} \prod_{i=1}^{n} \mathcal{M}_1(z_i)$$

- Even though the amplitudes factorize in z-space in that way, the phase space does not because of the presence of a delta function for conservation of momentum

$$\delta(z - z_1 z_2 \ldots z_n).$$

Setting resummation up

- In Mellin space, however, we do have factorization of the phase space,

$$\int_0^1 dz z^{N-1} \delta(z - z_1 z_2 ... z_n) = z_1^{N-1} z_2^{N-1} ... z_n^{N-1}$$

- So for hard kernels, the nth emission is written as:

$$C^{(n)}(N) \overset{\text{soft}}{\simeq} \frac{1}{n!} \left[C^{(1)}_{\text{soft}}(N) \right]^n$$

- Where $C^1_{\text{soft}}(N)$ is the hard kernel for one soft gluon emitted from the quark line

Resummation Calculation

- We need to calculate the following

$$\alpha_S C^{(1)}_{\text{soft}}(N) = 2\frac{C_F}{\pi}\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2}^{(1-z)^2 Q^2}\frac{dk_\perp^2}{k_\perp^2}\alpha_S(k_\perp^2)$$

- If α_S is a constant, the dz integrand returns the Mellin transform of $\left(\frac{\log(1-z)^2}{1-z}\right)_+$

Resummation Calculation

- Looking at the terms of the LL

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$$

$$\lambda \sim b_0 \alpha_S \ln N$$

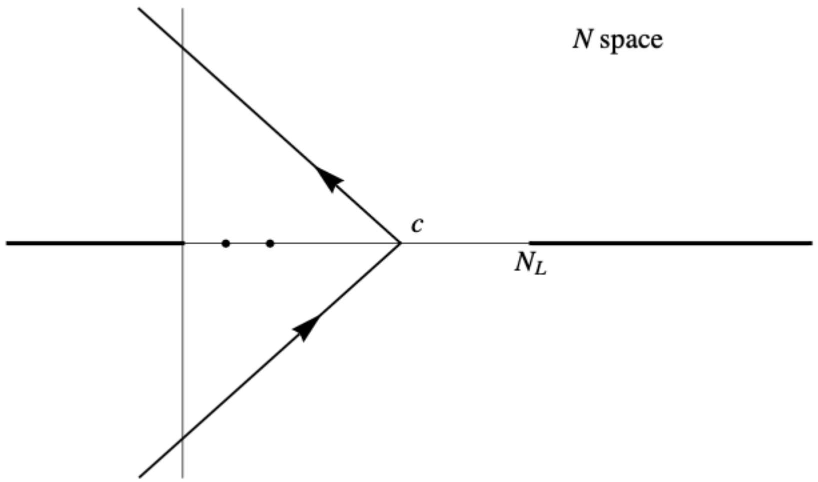
- In blue, we see a potential problem, *i.e.* when the argument of the log is 0:

$$N_L = \exp(1/2\alpha_S b_0)$$

- This describes the Landau pole, which must be handled (ambiguity on how to do this – why there are multiple prescriptions!)

Minimal Prescription

- Avoid the Landau pole by enclosing the Mellin contour to the left of the Landau pole but to the right of the normal right-most pole



Bayesian Statistics

- The probability of the parameter set $\vec{a}$ given the data is

$$\mathcal{P}(\vec{a}|\mathrm{data}) = \frac{1}{Z}\mathcal{L}(\mathrm{data}|\vec{a})\pi(\vec{a})$$

- Where Z is the Bayesian evidence, π is the Bayesian priors, and the likelihood function is

$$\mathcal{L}(\mathrm{data}|\vec{a}) = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$

Bayesian Statistics

- The χ^2 function is

$$\chi^2_{\text{expt}}(\vec{a}) = \sum_i \frac{(D_i + S_i - T_i(\vec{a})/N)^2}{\sum_j (\alpha_{i,j})^2}$$

- D is each data point, S is the systematic shift associated with correlated uncertainties, $T(\vec{a})$ is the theory calculation, based on the parameter set, N is the overall normalization for the experiment, and α are the uncorrelated statistical uncertainties

Benefit of high-p_T data on large x_π

- In the p_T-dependent cross section, must integrate over momentum fraction

$$\frac{d\sigma}{dQ^2 dY dp_T^2} = \int_{x_a^{\min}}^{1} dx_a f_a(x_a) f_b(x_b) \hat{\sigma}$$

$$x_a^{\min} = \frac{x_1 - \tau}{1 - x_2}$$

$$x_{1,2} = \sqrt{\frac{p_T^2 + Q^2}{s}} e^{\pm Y}$$

- Where $x_a^{\min}$ grows as p_T grows
- Integrating over a smaller region of high x will allow PDFs to be more sensitive to the data in that region