Parton distribution functions and beyond

Khépani Raya-Montaño

In collaboration with:

L. Chang, C.D. Roberts, J. Rodriguez-Quintero + ...

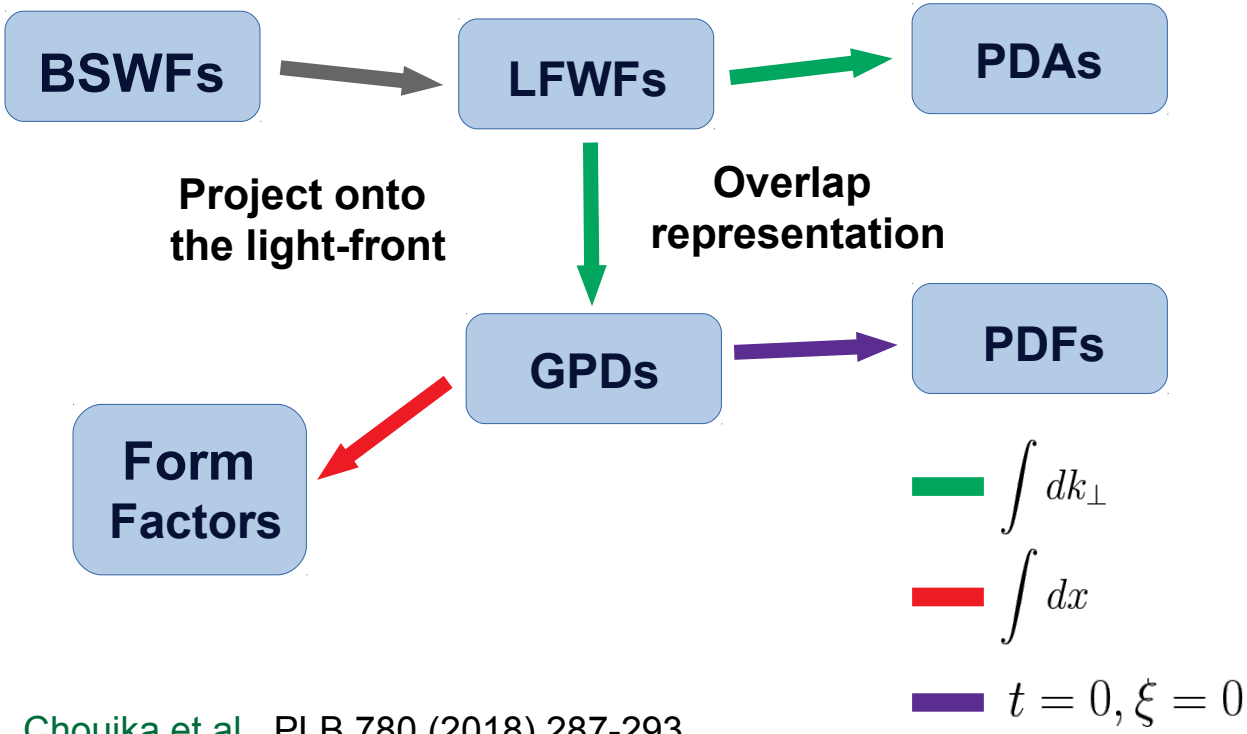
Pion and Kaon Structure Functions at the EIC

June 2 – 5, 2020. CFNS Virtual Meeting

The Light-front wave function (LFWF) approach

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion/kaon structure.



The idea:
Compute ***everything*** from the **LFWF.**

The inputs:
Solutions from quark **DSE** and meson **BSE**.

The alternative inputs:
Model **BSWF** from realistic DS-BS ***predictions***.

N. Chouika et al., PLB 780 (2018) 287-293.
C. Mezrag et al., FBS 57 (2016) no.9, 729-772

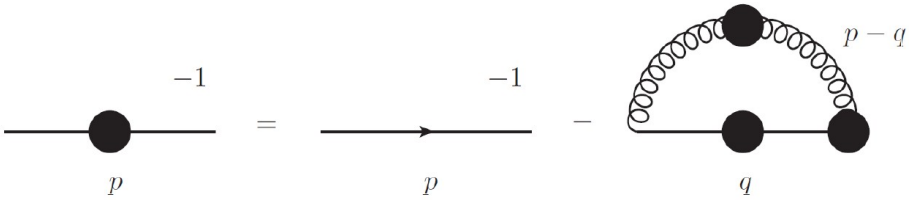
Bethe-Salpeter wavefunction

- BSWF: sandwich of the Bethe-Salpeter amplitude and quark propagators:

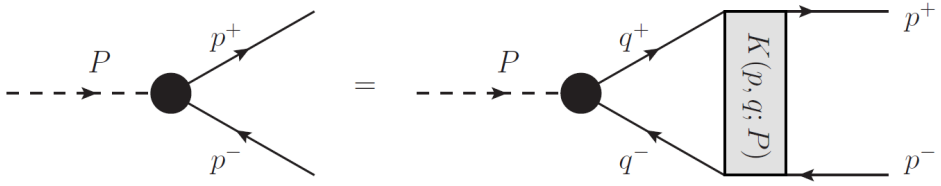
$$\chi_H(k_-^H; P_H) = S_q(k)\Gamma_H(k_-^H; P_H)S_{\bar{q}}(k - P_H)\ ,\ k_-^H = k - P_H/2\ .$$

$P^2 = -m_H^2$: **meson's mass**; Γ_H **BS amplitude**; $S_{q(\bar{q})}$ **quark (antiquark) propagator**

- Quark propagator and BSA should come from solutions of:



Quark DSE



Meson BSE

- **Alternative** first step: **construct** a *theory driven* **guess**.

LFWF: model

- Starting with the Kaon as a example, we employ a Nakanishi-like representation:

$$n_K \chi_K^{(2)}(k_-^K; P_K) = \underbrace{\mathcal{M}(k; P_K)}_{1} \int_{-1}^{1} d\omega \, \underbrace{\rho_K(\omega)}_{2} \underbrace{\mathcal{D}(k; P_K)}_{3} \, ,$$

1: Matrix structure (leading BSA):

$$\mathcal{M}(k; P_K) = -\gamma_5 [\gamma \cdot P_K M_u + \gamma \cdot k (M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \, ,$$

2: Sprectral weight: To be described later.

3: Denominators: $\mathcal{D}(k; P_K) = \Delta(k^2, M_u^2) \Delta((k - P_K)^2, M_s^2) \hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \, ,$

where: $\Delta(s,t) = [s+t]^{-1}, \; \hat{\Delta}(s,t) = t \Delta(s,t) \, .$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: model

- **Algebraic** manipulation yields:

$$\chi_K^{(2)}(k_-^K; P_K) = \mathcal{M}(k; P_K) \int_0^1 d\alpha \, 2\chi_K(\alpha; \sigma^3(\alpha)) \ , \ \sigma = (k - \alpha P_K)^2 + \Omega_K^2 \ ,$$

Depends on Feynman and model **parameters**.

Scalar function, **reads** as:

$$\chi_K(\alpha; \sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv \right] \frac{\rho_K(\omega)}{n_K} \frac{\Lambda_K^2}{\sigma^3} \ .$$

- **$\rho_K(\omega)$** will play a **crucial role** in determining the meson's observables.
- Realisitc **DSE predictions** will help us shape it.

Pion PDF as benchmark!

M. Ding et al.
Chin.Phys. 44 (2020) no.3, 031002
PRD 101 (2020) no.5, 054014

(M. Dings's talk)

Light-front wavefunction

- The **pseudoscalar LFWF** can be written:

$$f_K \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \mathrm{tr}_{CD} \int_{dk_\parallel} \delta(n \cdot k - x n \cdot P_K) \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K) \ .$$

- The **moments** of the distribution:

$$< x^m >_{\psi_K^{\uparrow\downarrow}} = \int_0^1 dx x^m \psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{1}{f_K n \cdot P} \int_{dk_\parallel} \left[\frac{n \cdot k}{n \cdot P}\right]^m \gamma_5 \gamma \cdot n \chi_K^{(2)}(k_-^K; P_K)$$

$$\int_0^1 d\alpha \alpha^m \left[\frac{12}{f_K} \mathcal{Y}_K(\alpha; \sigma^2)\right] \ , \quad \mathcal{Y}_K(\alpha; \sigma^2) = [M_u(1-\alpha) + M_s \alpha] \mathcal{X}(\alpha; \sigma_\perp^2) \ .$$

Uniqueness of Mellin moments →

$$\boxed{\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)}$$

✓ Compactness of this result is a merit of the algebraic model.

S-S Xu et al., PRD 97 (2018) no.9, 094014.

Light-front wavefunction

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K}\mathcal{Y}_K(x; \sigma_\perp^2)$$

- Thus, the **LFWF** is heavily influenced by **$\rho_K(\omega)$.**

$$\mathcal{Y}_K(\alpha;\sigma^2) = [M_u(1-\alpha) + M_s\alpha]\mathcal{X}_K(\alpha;\sigma_\perp^2)\ , \quad \chi_K(\alpha;\sigma^3) = \left[\int_{-1}^{1-2\alpha} d\omega \int_{1+\frac{2\alpha}{\omega-1}}^{1} dv + \int_{1-2\alpha}^{1} d\omega \int_{\frac{\omega-1+2\alpha}{\omega+1}}^{1} dv\right] \frac{\rho_K(\omega)}{n_K}\frac{\Lambda_K^2}{\sigma^3}\ .$$

- The explicit form of **$\rho_K(\omega)$** **controls** the shape of **PDAs, GPDs, PDFs**, etc.

$$\Rightarrow \psi_K^{\uparrow\downarrow}(x, k_\perp^2) \sim \int d\omega \ \cdots \rho_K(\omega) \cdots$$

- For example:

$$\rho_\pi(\omega) \sim (1-\omega^2)$$

Asymptotic model
(chiral limit)

$$\phi(x) \sim x(1-x)$$

Asymptotic PDA

$$q(x) \sim [x(1-x)]^2$$

Parton-like profile

C. Mezrag et al., PLB 741 (2015) 190-196.
C. Mezrag et al., FBS 57 (2016) no.9, 729-772

Light-front wavefunction

$$\psi_K^{\uparrow\downarrow}(x, k_\perp^2) = \frac{12}{f_K} \mathcal{Y}_K(x; \sigma_\perp^2)$$

- Our choice is given by **experience** and careful **analyses**.

$$u_G \rho_G(\omega) = \frac{1}{2b_0^G} \left[\mathrm{sech}^2 \left(\frac{\omega - \omega_0^G}{2b_0^G} \right) + \mathrm{sech}^2 \left(\frac{\omega + \omega_0^G}{2b_0^G} \right) \right] \left[1 + \omega \, v_G \right] ,$$

Parameters

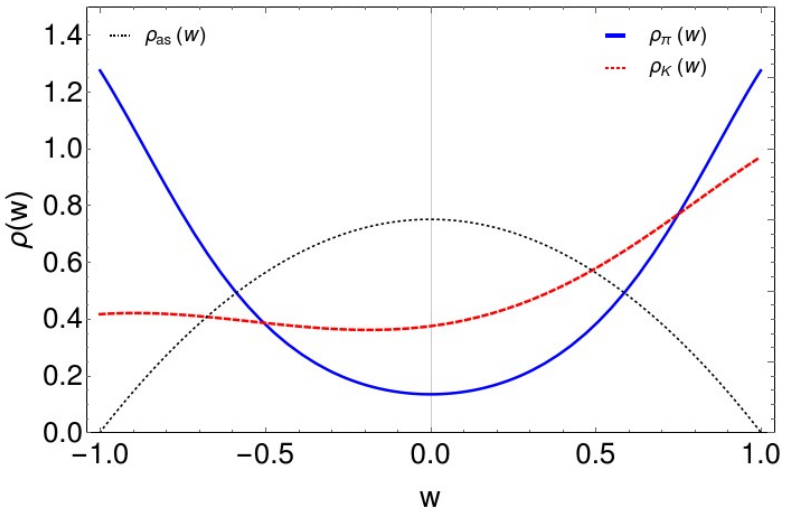
Λ_π	b_0^π	ω_0^π	ν_π	Λ_K	b_0^K	ω_0^K	ν_K
M_u	0.275	1.23	0	$2M_s$	0.5	1.3	0.4

$$m_\pi = 0.140 \text{ GeV}, m_K = 0.49 \text{ GeV}$$

$$M_u = 0.31 \text{ GeV}, M_s = 1.2 \; M_u$$

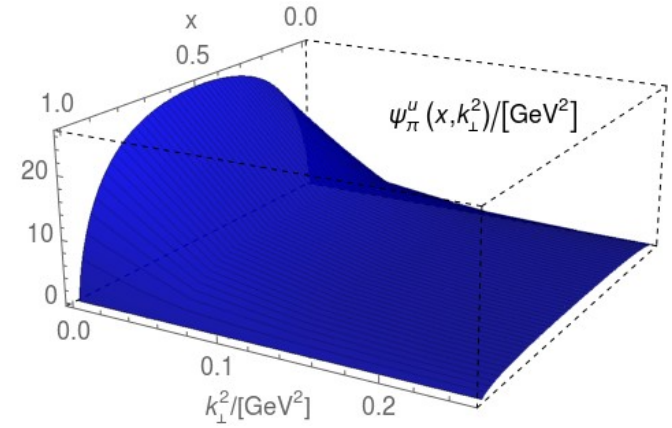
Typical values of **constituent** quark masses, from **realistic** DSEs **solutions**.

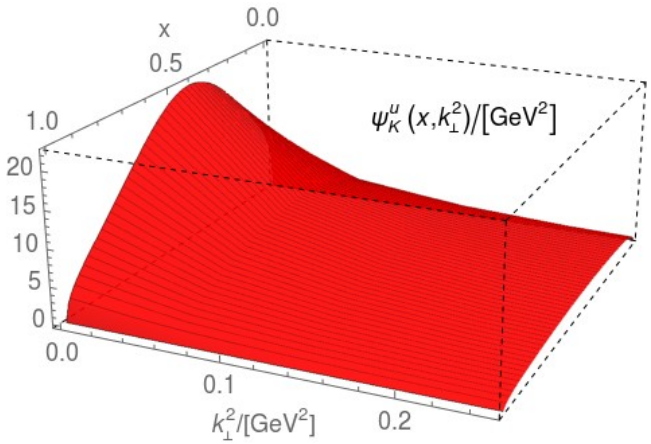
Profiles

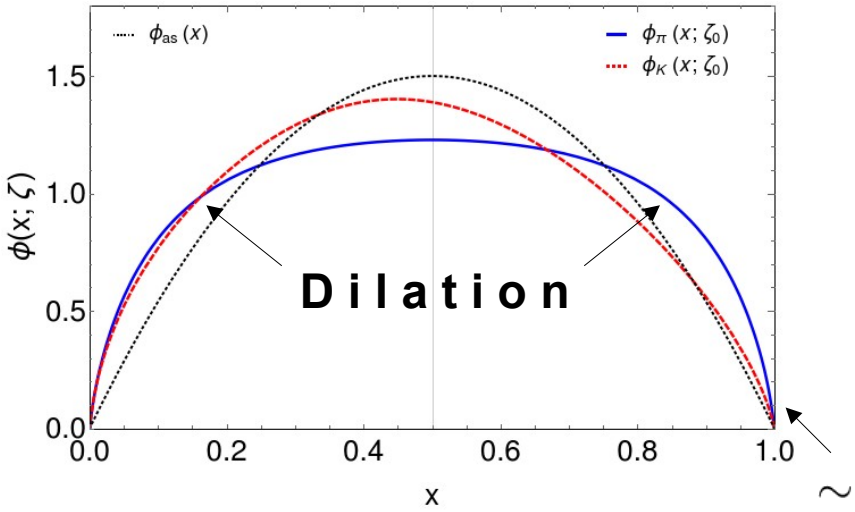


LFWFs and PDAs

$$\phi_M(x) = \frac{1}{16\pi^3}\int d^2\vec{k}_\perp \psi_M^{\uparrow\downarrow}(x, k_\perp^2)$$







$\sim (1-x)^{1+\gamma}$

- ✓ **Smooth fall** at the endpoints
- ✓ **Broad** and concave functions of *x*
- ✓ *Sufficient* **broadening,** consistent with **DCSB**

  [+] L. Chang et al., PRL 110 (2013) no.13, 132001

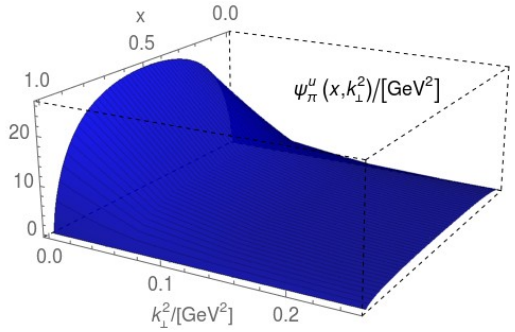
- ◆ Kaon asymmetry lead by the difference M_s - M_u

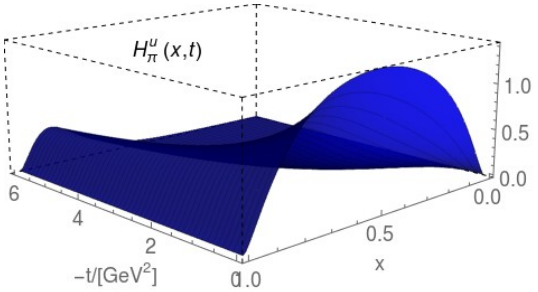
LFWFs and GPDs

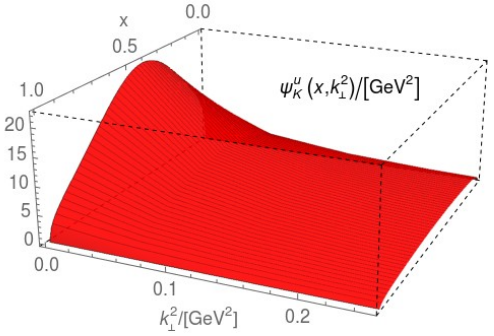
LFWFs → GPDs

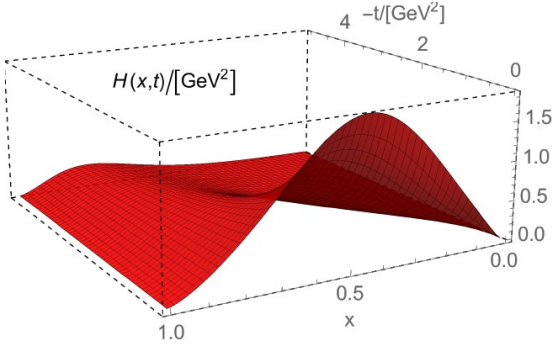
- In the **overlap representation**, the **GPD** reads as:

$$H_M^q(x,\xi,t) = \int \frac{d^2\mathbf{k}_\perp}{16\,\pi^3}\Psi^*_{u\bar{f}}\left(\frac{x-\xi}{1-\xi},\mathbf{k}_\perp+\frac{1-x}{1-\xi}\frac{\Delta_\perp}{2}\right)\Psi_{u\bar{f}}\left(\frac{x+\xi}{1+\xi},\mathbf{k}_\perp-\frac{1-x}{1+\xi}\frac{\Delta_\perp}{2}\right)$$









- ✓ Valence content only
- ✓ Valid in the **DGLAP** region
- ✓ ***Compatible*** with diagram approach

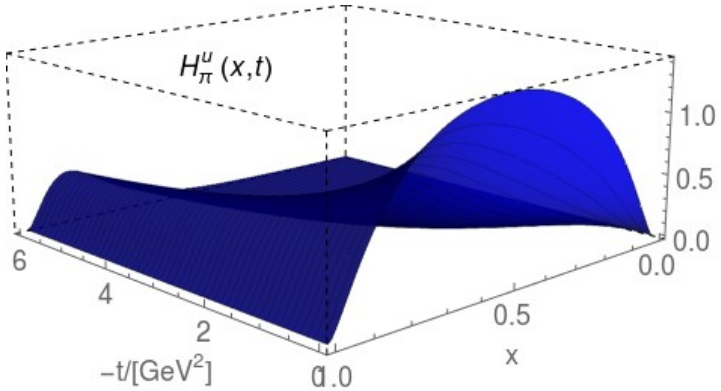
(J R-Q's, C. Mezrag's talks)

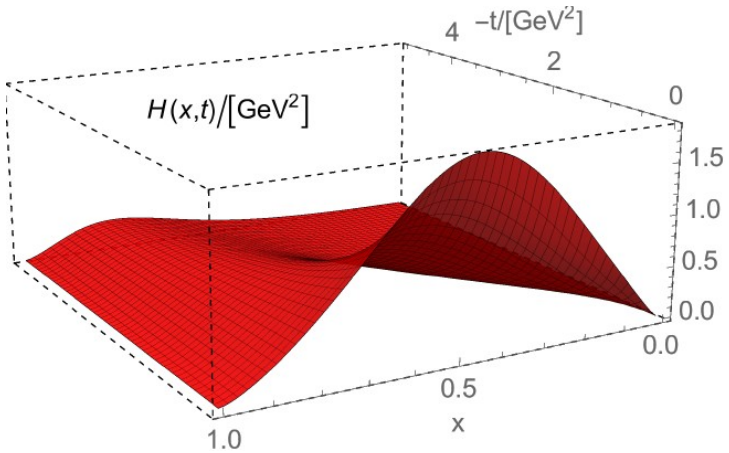
LFWFs and PDFs

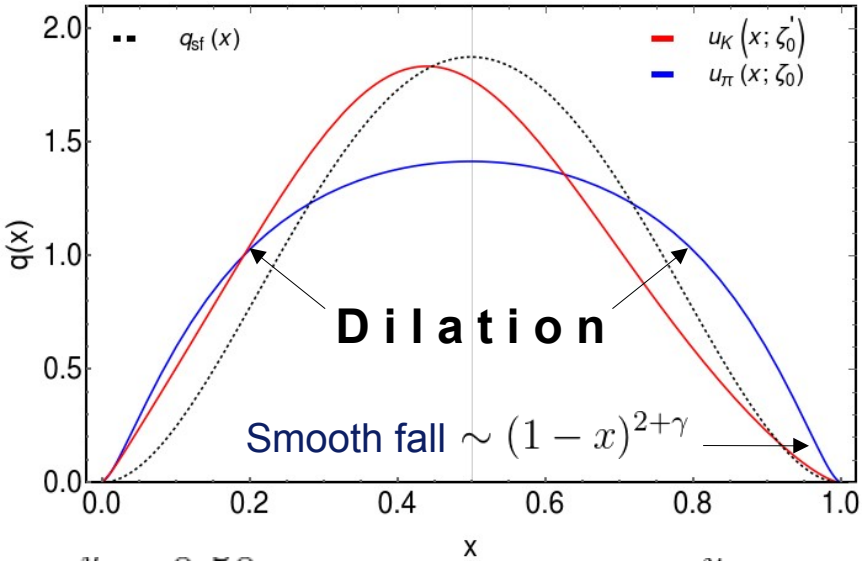
GPD → PDF

- The **PDF** is obtained from the **forward limit** of the **GPD**.

$$q(x) = H(x,0,0)$$







$$<x>^u_\pi = 0.50 \qquad <x>^u_K = 0.45$$

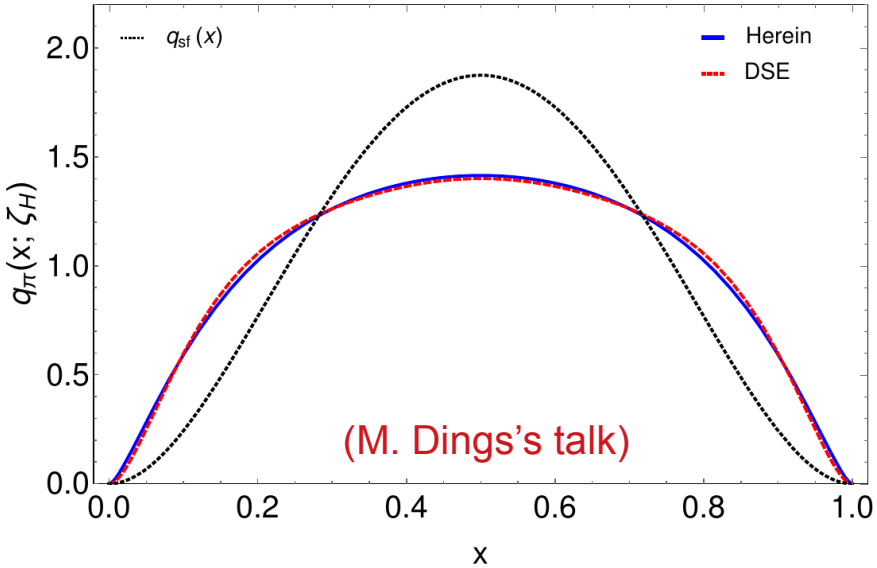
- ζ_H: **all** the momentum is carried by the valence-quarks.
- **Defined** from the **PI** effective charge.

(J. Rodriquez-Quintero's talk)

Pion PDFs

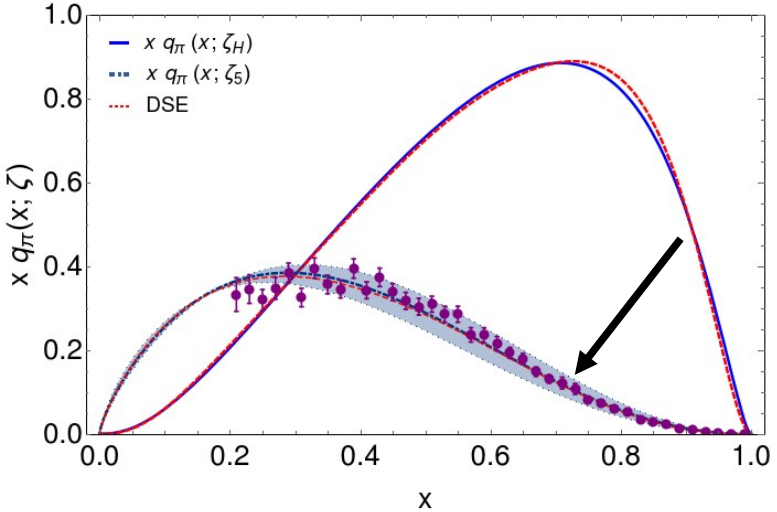
GPD → PDF

- We use the DSE prediction of the PDF as **benchmark** to get a **realistic LFWF**.



ζ_H	$<x^2>_\pi$	$<x^4>_\pi$	$<x^6>_\pi$
DSE	0.3020	0.1460	0.0857
Herein	0.3020	0.1461	0.0861

- At ζ_H, we see a **small deviation** from the *realistic* PDF.
- Practically ***irrelevant*** after DGLAP **evolution**.



DSE: M. Ding et al. Chin.Phys. 44 (2020) no.3, 031002. PRD 101 (2020) no.5, 054014

Form Factors

GPD → FFs

- **Electromagnetic** form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\; H_M^q(x,\xi,t)$$

Can safely take **ξ = 0**

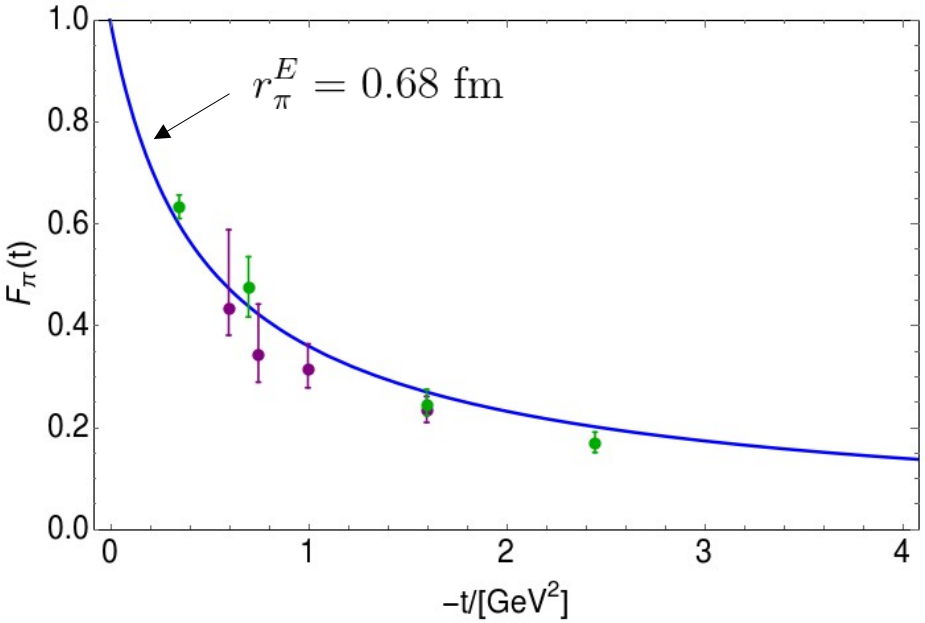
"Polinomiality"

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_{\bar{f}} F_M^{\bar{f}}(\Delta^2)$$

Weighed by electric charges

- **Isospin symmetry**

$$F_{\pi^+}(-t) = F_{\pi^+}^u(-t)$$



Data: G.M. Huber et al. PRC 78 (2008) 045202

Form Factors

GPD → FFs

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2 \Theta_1^M(t)$$

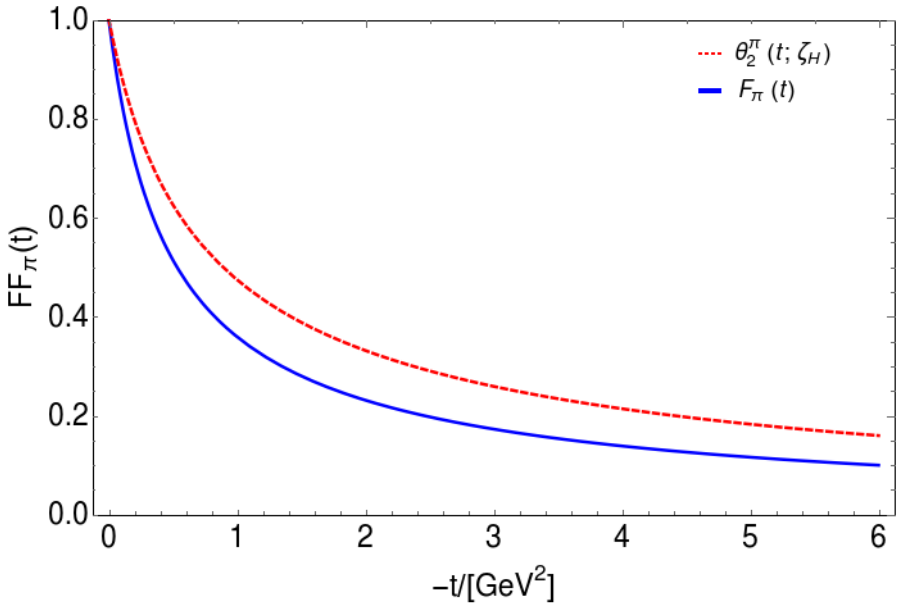
  - ✓ Directly obtained if **ξ = 0**
    - × **ERBL** region needed. **"D-term"**
  - ➔ **Isospin symmetry**

$$\Theta_2^\pi(t) = \int_0^1 dx\, 2x H_\pi^u(x,0,t)$$

- **ERBL extension** is possible:

N. Chouika et al., EPJC 77 (2017) 12, 906

(C. Mezrag's talk)



$$r_\pi^E = 0.68 \text{ fm}\ ,\ r_\pi^{\theta_2} = 0.56 \text{ fm}$$

Evolved Distributions

DGLAP + PI charge

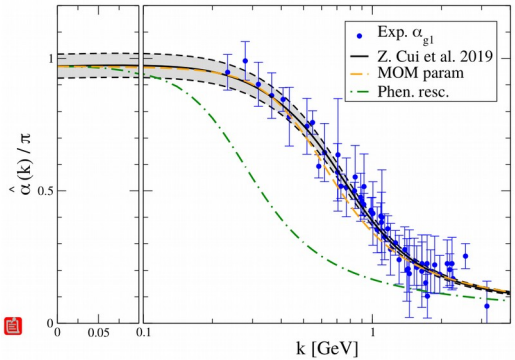
Idea: Define an effective coupling such that the equations below are exact.

$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y} q(y;t) P\left(\frac{x}{y}\right)$$

- or

$$\frac{d}{dt}M_n(t) = -\frac{\alpha(t)}{4\pi}\gamma_0^n M_n(t)$$

i.e. no LO, NLO, etc:
all orders are there



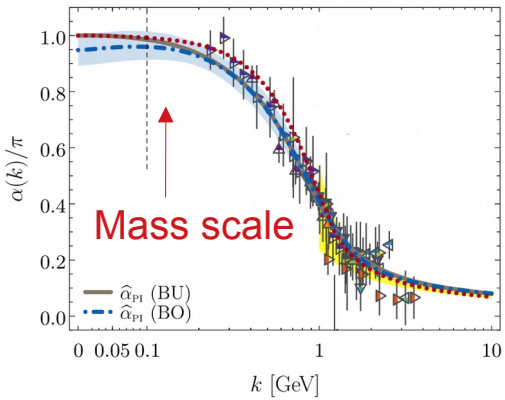
Z-F Cui et al., arXiv:1912.08232

… and identify, not tune, the (initial) **hadron scale ζ_H**.

(fully dressed quasiparticles are the correct degrees of freedom)

- Features of the **PI effective** charge lead to the **answer**.

J. R-Q et al., arXiv:1909.13802.



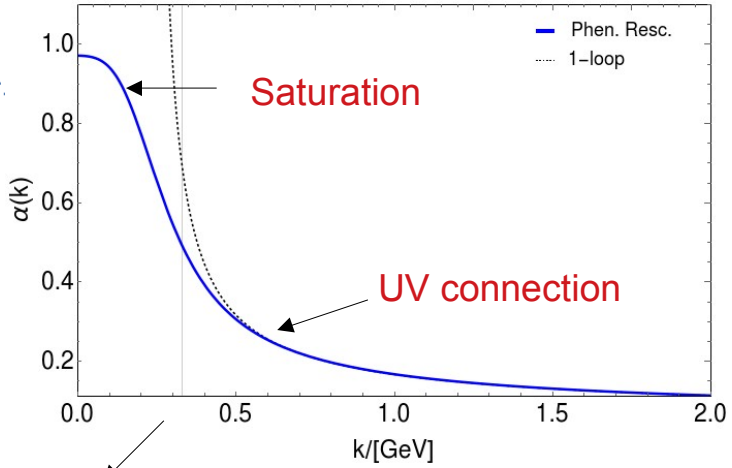
D. B. et al., PRD 96 (2017) no.5, 054026.
J. R-Q. et al., FBS 59 (2018) no.6, 121.

Evolved distributions

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.

Hadron scale: $\zeta_H \sim 0.33\ \mathrm{GeV}$

"All orders effective coupling"

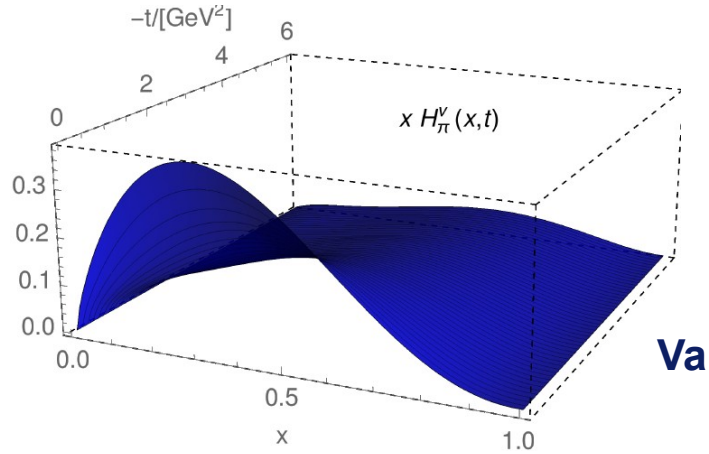


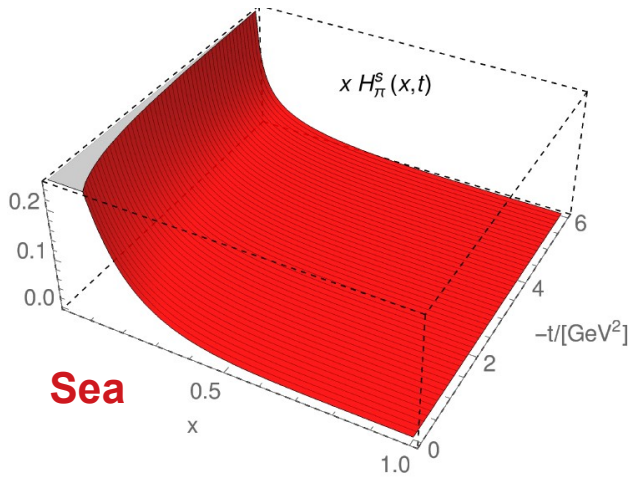
$\zeta_H = m_G$

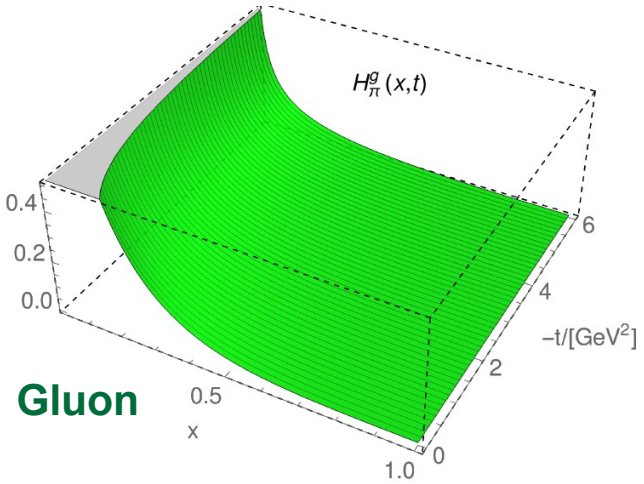
Evolved distributions: GPDs

$\zeta = 5.2$ GeV

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** GPDs are obtained.



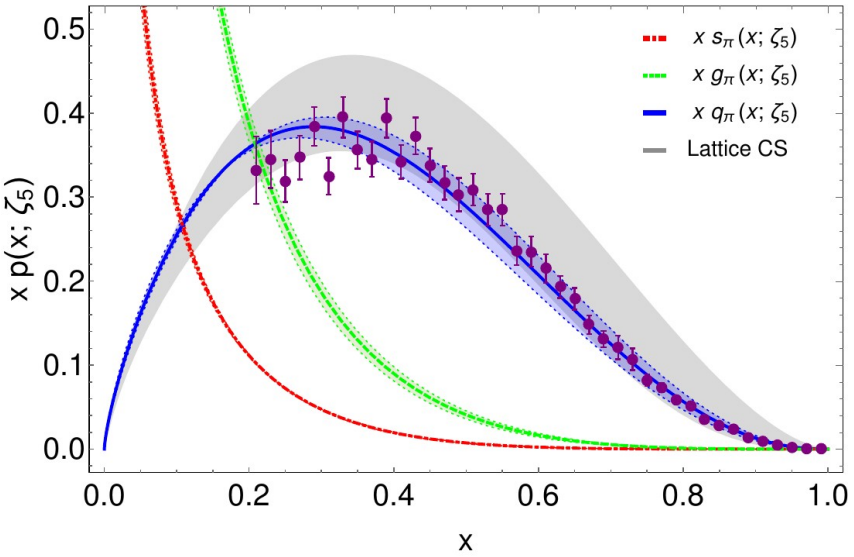




Evolved distributions: PDFs

$\zeta = 5.2$ GeV

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The forward limit corresponds to the **PDFs**.

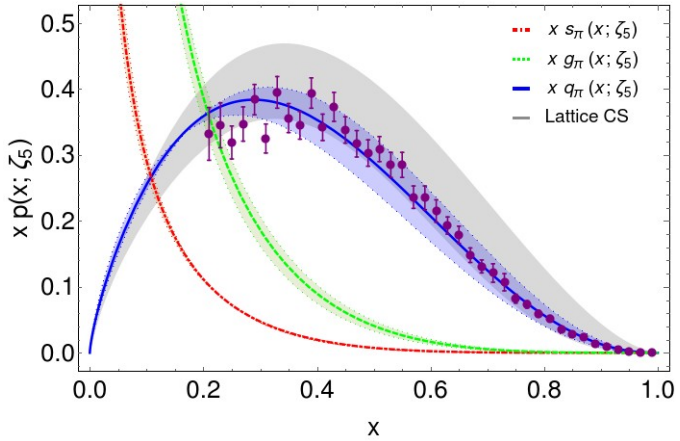


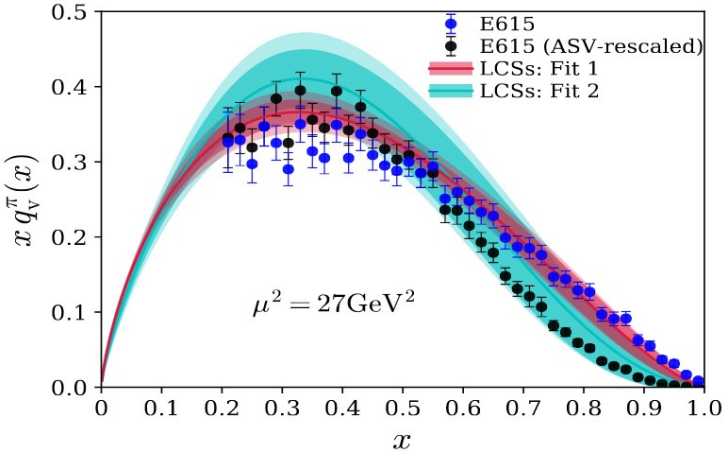
ζ	$<x>_V$	$<x>_G$	$<x>_S$
2 GeV	0.483(42)	0.411(24)	0.106(18)
5.2 GeV	0.412(36)	0.449(19)	0.138(17)

Data: M. Aicher et al., PRL 105 (2010) 252003

Lattice: R.S. Sufian et al., arXiv:2001.04960

Evolved distributions: PDFs





- Satisfactory agreement with novel **Lattice** **"Cross-Section"** result:

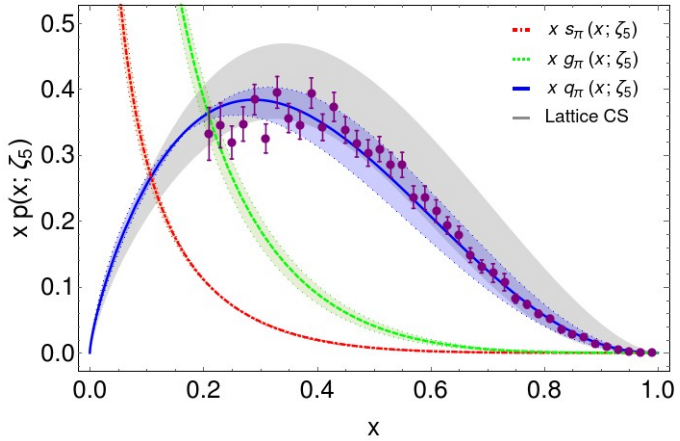
  R.S. Sufian et al., arXiv:2001.04960

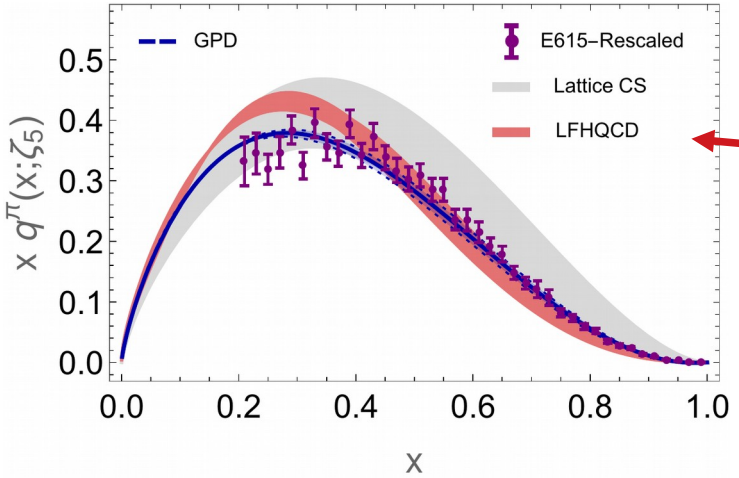
  … which **favors** a large-x exponent **'2':**

> The resultant PDFs obtained are in agreement with the $q_v^\pi(x)$ extracted from the experimental data. Our analysis indicates that a $(1-x)^2$-behavior of the $q_v^\pi(x)$ at large x is preferred. Future calculations with finer

(D. Richards' talk)

Evolved distributions: PDFs





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- Light-front hamiltonian QCD can also **accomodate** such behavior:

  L. Chang et al., arXiv:2001.07352

"Rule – 2"

(L. Chang's talk)

➔ Different treatments arrive at, essentially, the ***same*** prediction. Such **confluence** cannot go unnoticed.

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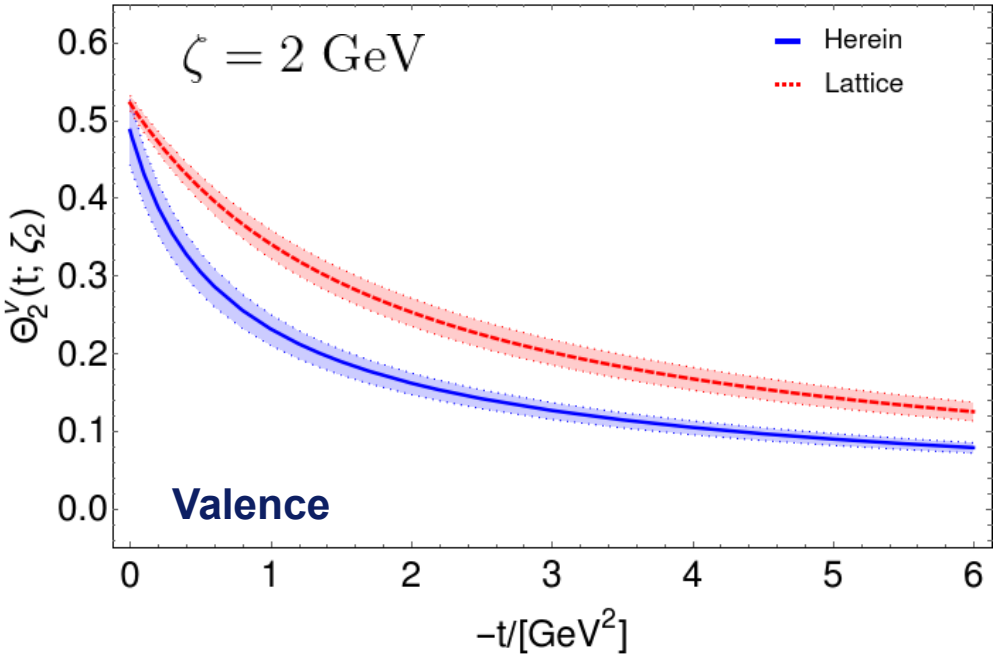
- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
- **GFF** θ_2 comes from the *off-forward* <x>.



Herein: softer GFF, $\Theta_2(0)/2 = 0.24(2)$

Lattice: D. Brommel, PhD. Thesis 2007

$\Theta_2(0)/2 = 0.26(4) \qquad (m_\pi^2 > 0.3\ \mathrm{GeV}^2)$

Evolved distributions: GFFs

$\zeta = 2\ \mathrm{GeV}$

- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
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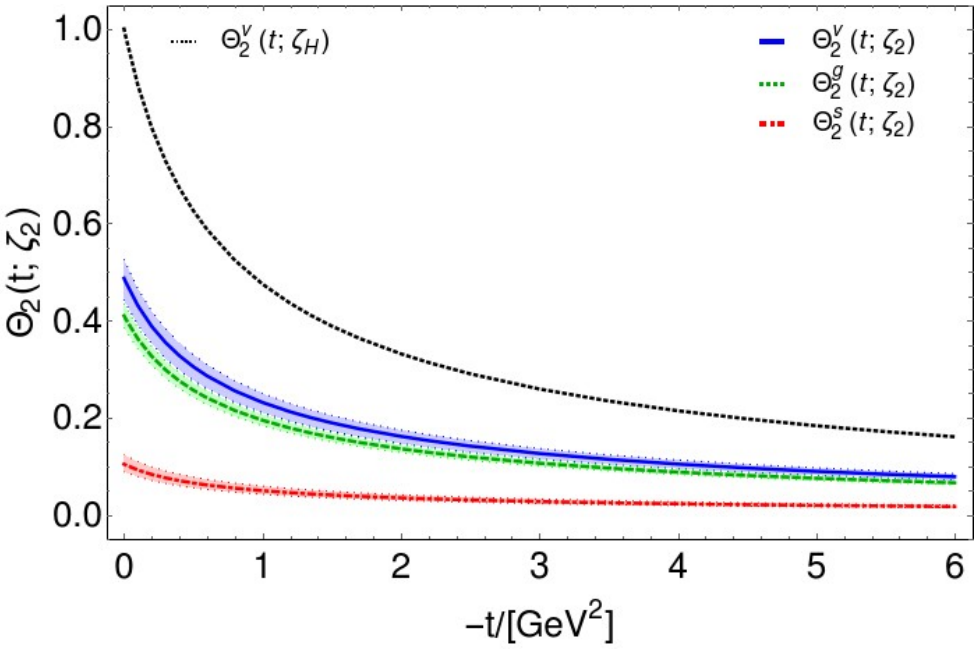
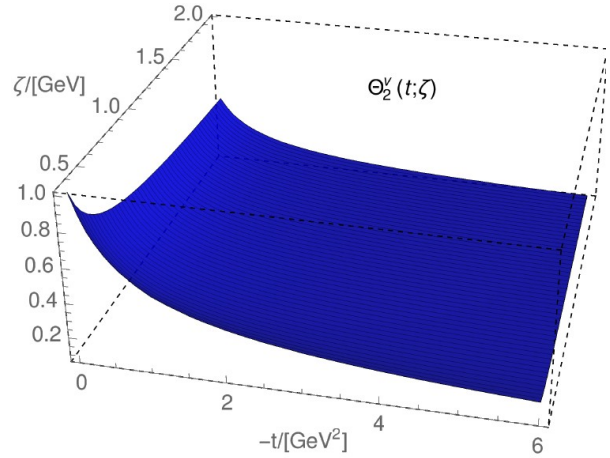


- Naturally, one has the **sum rule:**

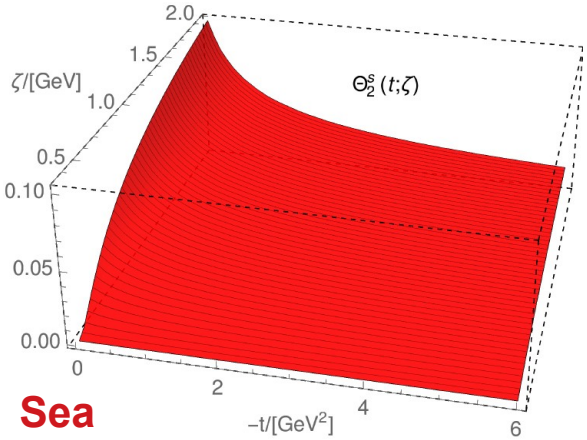
$$\Theta_2^v(t;\zeta) + \Theta_2^s(t;\zeta) + \Theta_2^g(t;\zeta) = \Theta_2^v(t;\zeta_H),\ \forall\zeta$$

Evolved distributions: GFFs

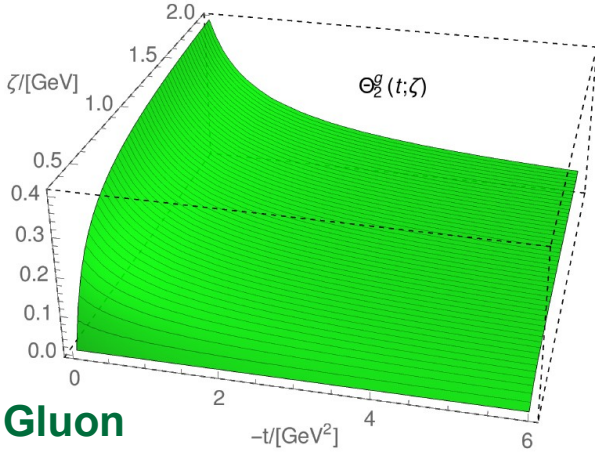
- Starting with **valence** distributions, at *hadron scale*, and generate **gluon** and **sea** distributions via evolution equations.
- Thus **gluon** and **sea** **GPDs** are obtained.
- The *forward* limit corresponds to the **PDFs**.
- **GFF** θ_2 comes from the *off-forward* <x>.
- Straightforward steps yield a "**3D**" **picture** of θ_2. (**ζ** and ***t*** dependence)



Valence



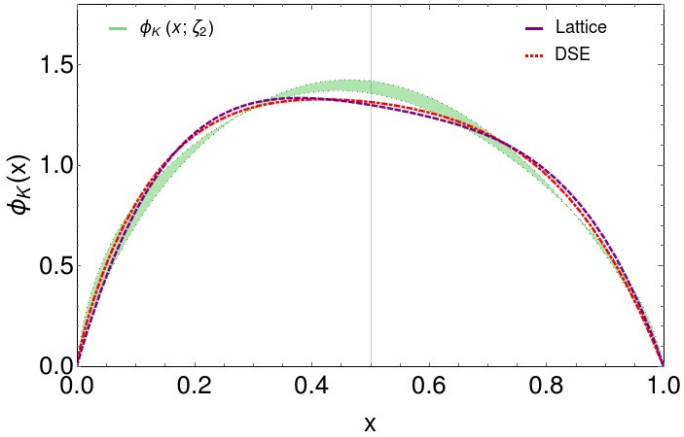
Sea



Gluon

Kaon and Pion PDF

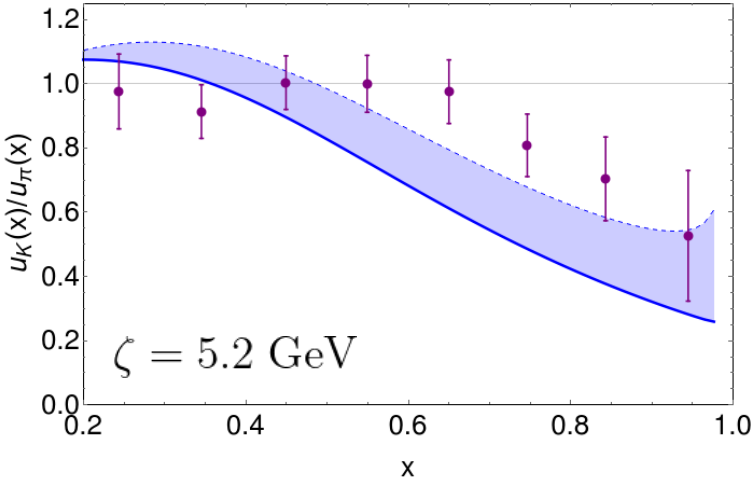
- Producing an ***acceptable*** **PDA** makes us confident on our **PDF/GPD** predictions.



$\zeta = 2$ GeV	Herein	Lattice	DSE
$<(2x-1)^1>_K$	-0.033(8)	-0.032(12)	-0.040
$<(2x-1)^2>_K$	0.225(8)	0.231(4)	0.231

Lattice: G.S. Bali et al., JHEP 1908 (2019) 065

DSE: C. Shi et al., PLB 738 (2014) 512-518



- Momentum fractions in **Kaon:**

ζ	$<x>_u$	$<x>_s$	$<x>_G$	$<x>_S$
2 GeV	0.218(19)	0.276(23)	0.410(25)	0.098(17)
5.2 GeV	0.186(16)	0.239(20)	0.445(19)	0.128(17)

- ✓ Compatible with *ongoing* **DSE** calculations.

(C.D. Roberts' talk)

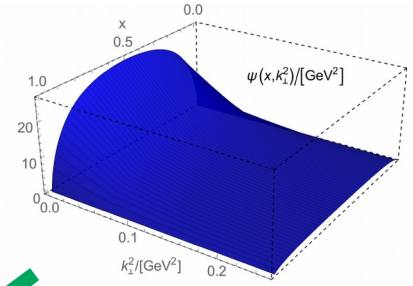
Summary

Summary: Pion

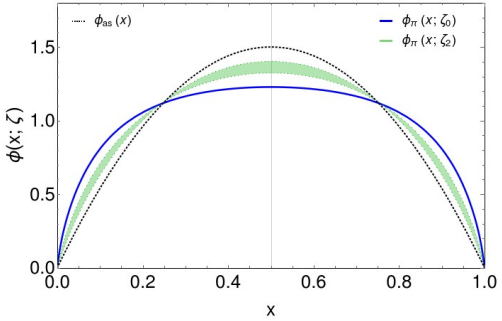
- Using **DSE prediction** of pion PDF as **benchmark**, we modeled the pion **BSWF**.

BSWFs

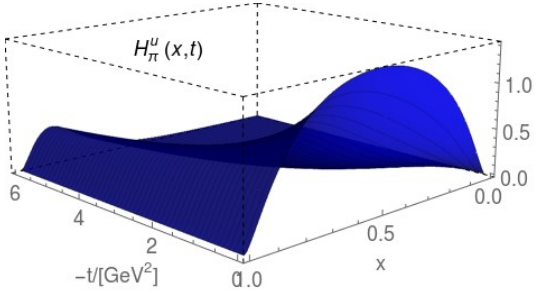
$\int dk_\perp$



LFWF



PDA



GPD

DSE: Chin.Phys. 44 (2020) no.3, 031002
PRD 101 (2020) no.5, 054014

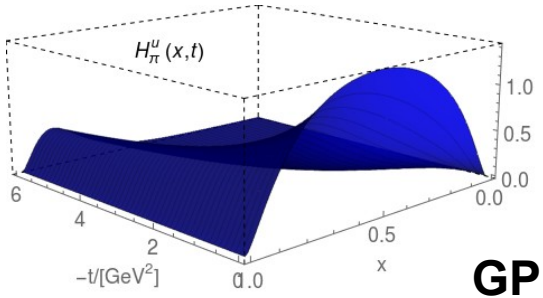
- **Consistent** features of the **PDA**:
  - ✓ Broad and concave at real world scales.
  - ✓ Agreement with Lattice and DSE results.

- The **GPD** obtained from the **overlap** representation.
  - Limited to the **DGLAP** region.
  - Compatible with the diagram approach.
  - Extension to ERBL region is possible.

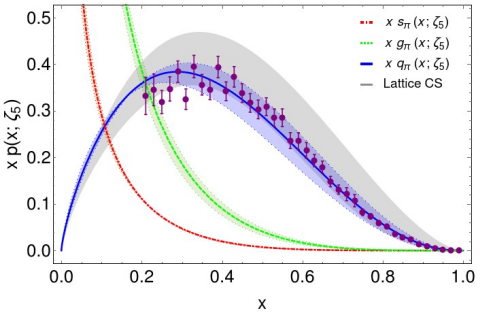
N. Chouika et al., EPJC 77 (2017) 12, 906

(C. Mezrag's talk)

Summary: Pion



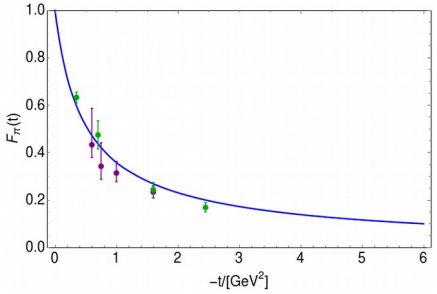
GPD



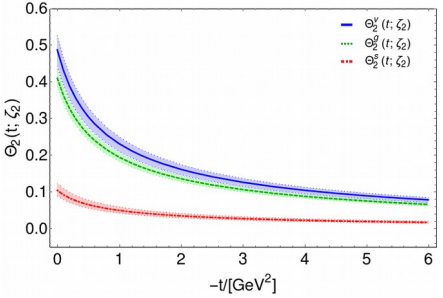
PDF

$\int dx$

$t = 0, \xi = 0$



EFF

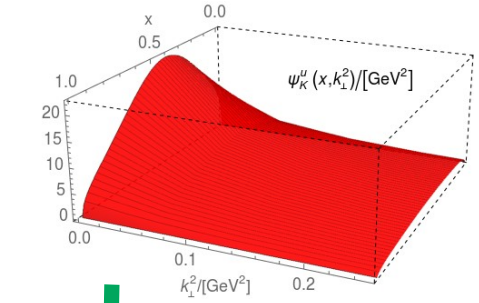


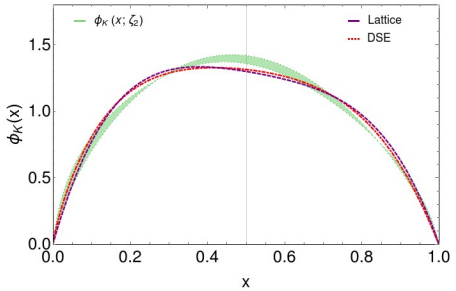
GFF

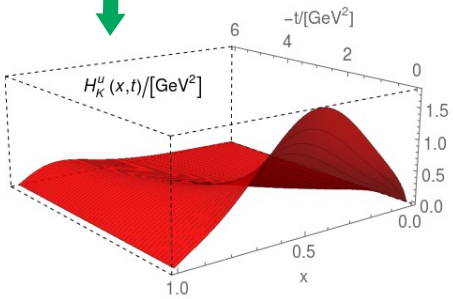
- Connection of the **PDF** with **DSE predictions** implies:
  - ✓ Keen agreement with reanalyzed data.
  - ✓ Large-x behavior as predicted by pQCD.
  - ✓ Compatible with novel Lattice results.
- **EFF *consistent*** with empirical data.
  - ➔ One can trust the off-forward quantities.
  - ✗ ERBL region <u>still needed.</u>
- Intimate ***connection*** with the **running coupling**:
  - ✓ PI effective charge → effective charge for PDF evolution.
  - ✓ ***Definition*** of the <u>hadron scale</u>.

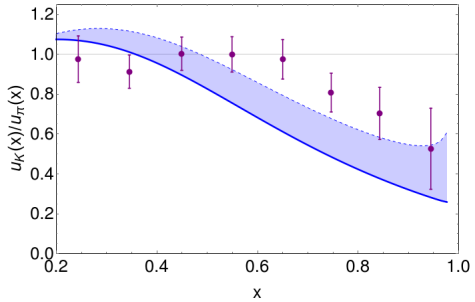
➔ Both **LFWF** and **GPD** are **promising candidates** to be the true objects.

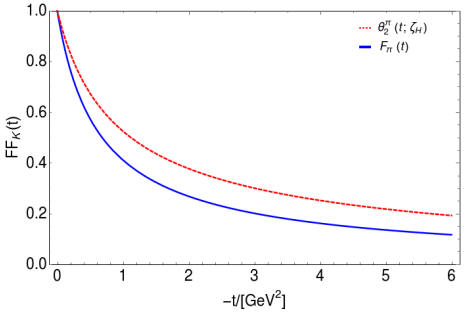
Summary: Kaon

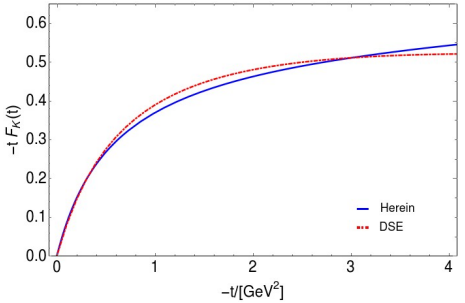












- **Qualitative** features of the distributions are *properly* **captured**.
- **Specific** numbers <u>can be improved</u>:
  - ✓ **DSE** predictions of the kaon PDF will further **constrain** our **LFWF**.
    - ➔ Then we can achieve the same degree of confidence as with the pion.
- **Gluon** and **sea** kaon GPDs are **within reach**.
  - ➔ A comprehensive picture could be obtained.
- Kaon demands **more work**.