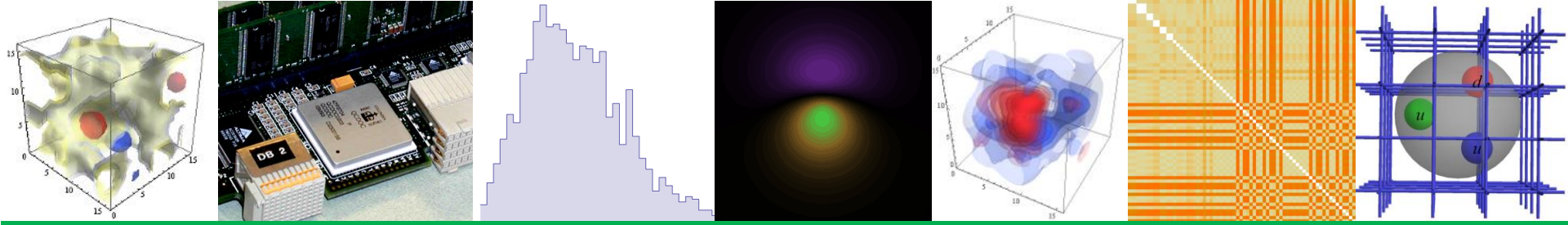


Pion and Kaon Distribution from Lattice QCD

HUEY-WEN LIN

This work is supported by the NSF under grant PHY 1653405
"CAREER: Constraining Parton Distribution Functions for New-Physics Searches"

Outline

§ Brief LaMET Introduction with PDF Examples

§ Pion and Kaon PDFs and Pion GPD

H. Lin et al., 2003.14128; J. Chen, HL, J. Zhang, 1904.12376

§ Pion and Kaon Distribution Amplitudes

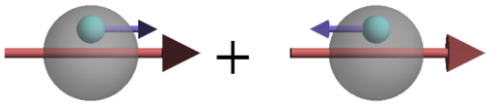
R. Zhang et al.(MSULat), 2005.13955

Thanks to MILC collaboration for sharing 2+1+1 HISQ lattices

PDFs on the Lattice

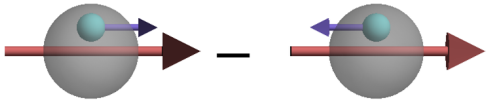
§ Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

spin-dependent transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

§ True distribution can only be recovered with all moments

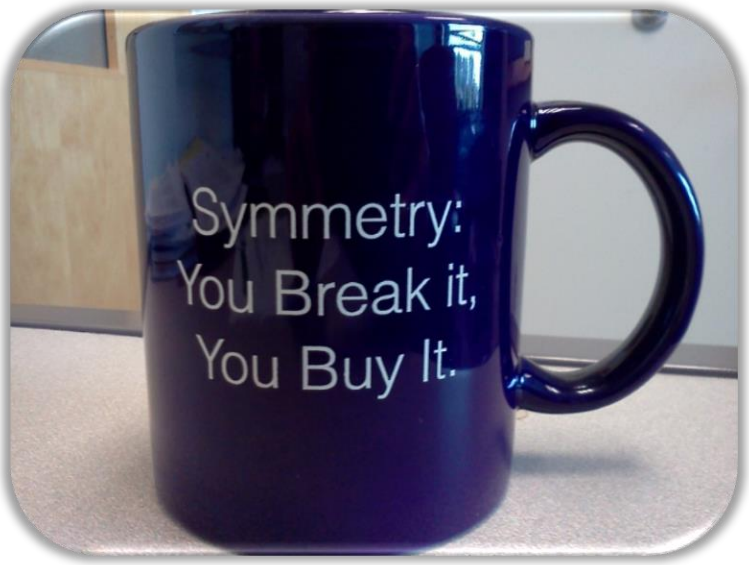
PDFs on the Lattice

§ Limited to the lowest few moments

- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem

§ Relative error grows in higher moments

- Calculation would be costly
- Cannot separate valence contrib. from sea



PDFs on the Lattice

§ Limited to the lowest few moments
- For higher moments, all ops mix with lower-dimension ops
- No practical proposal yet to overcome this problem

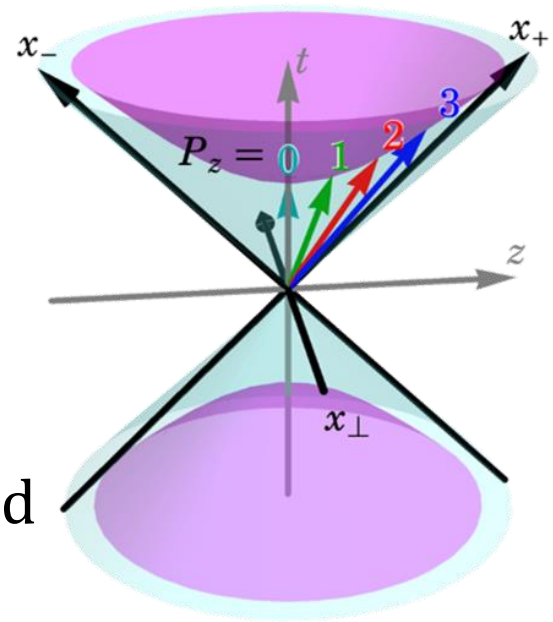
§ Relative error grows in higher moments
- Calculation would be costly
- Cannot separate valence contrib. from sea

§ New Strategy: Xiangdong Ji, PRL 111, 039103 (2013);

§ Adopt lightcone description for PDFs

§ Calculate finite-boost quark distribution
- In $P_z \to \infty$ limit, parton distribution recovered
- For finite P_z, corrections are applied through effective theory

§ Feasible with today's resources!

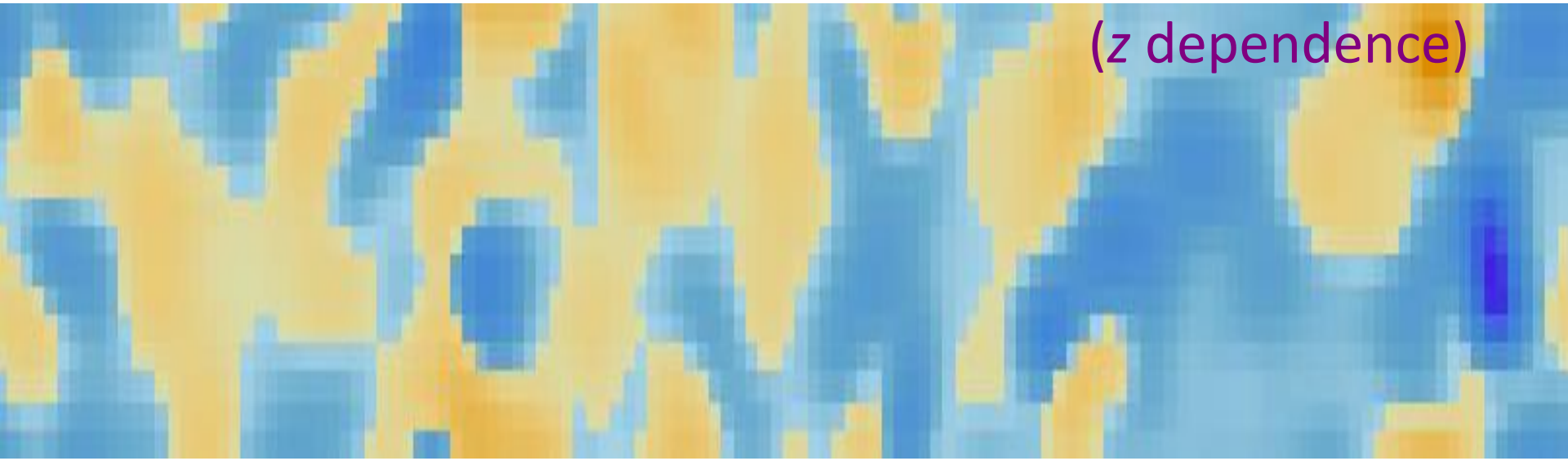


LaMET Recipe

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

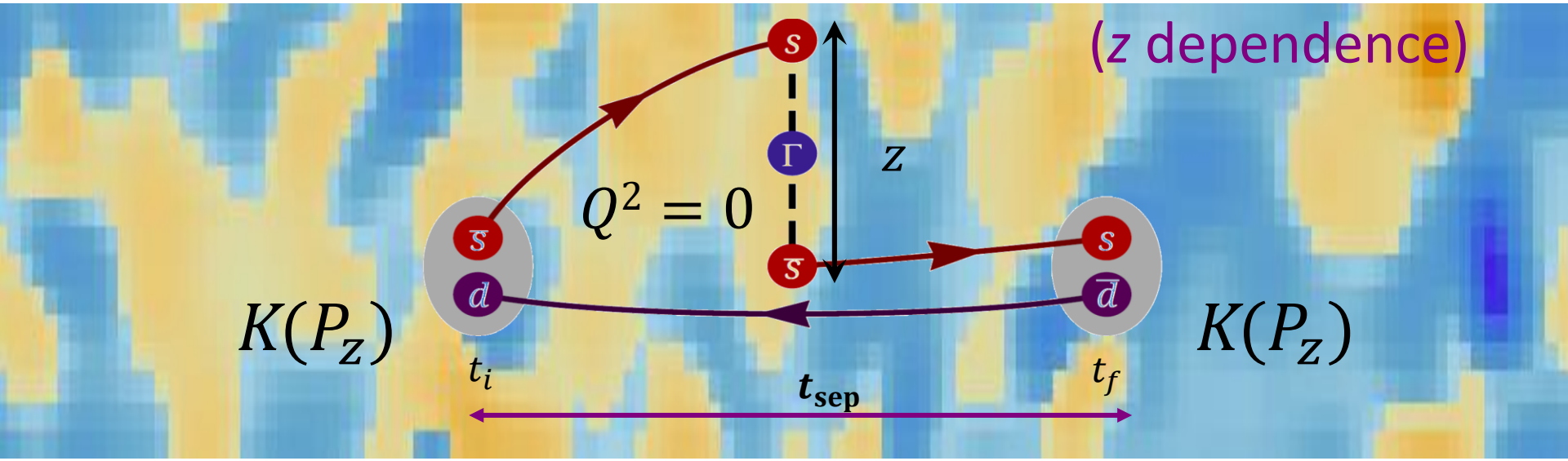
See the talk by David Richard (Wednesday) for a nice overview of other lattice x-dependent methods

LaMET Recipe

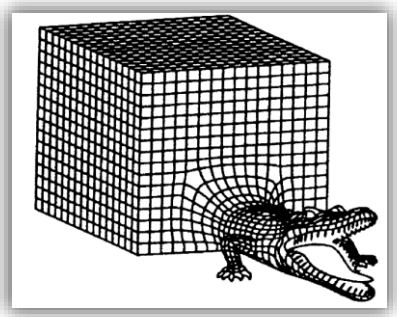
Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices



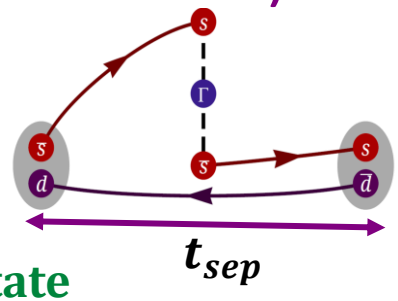
LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

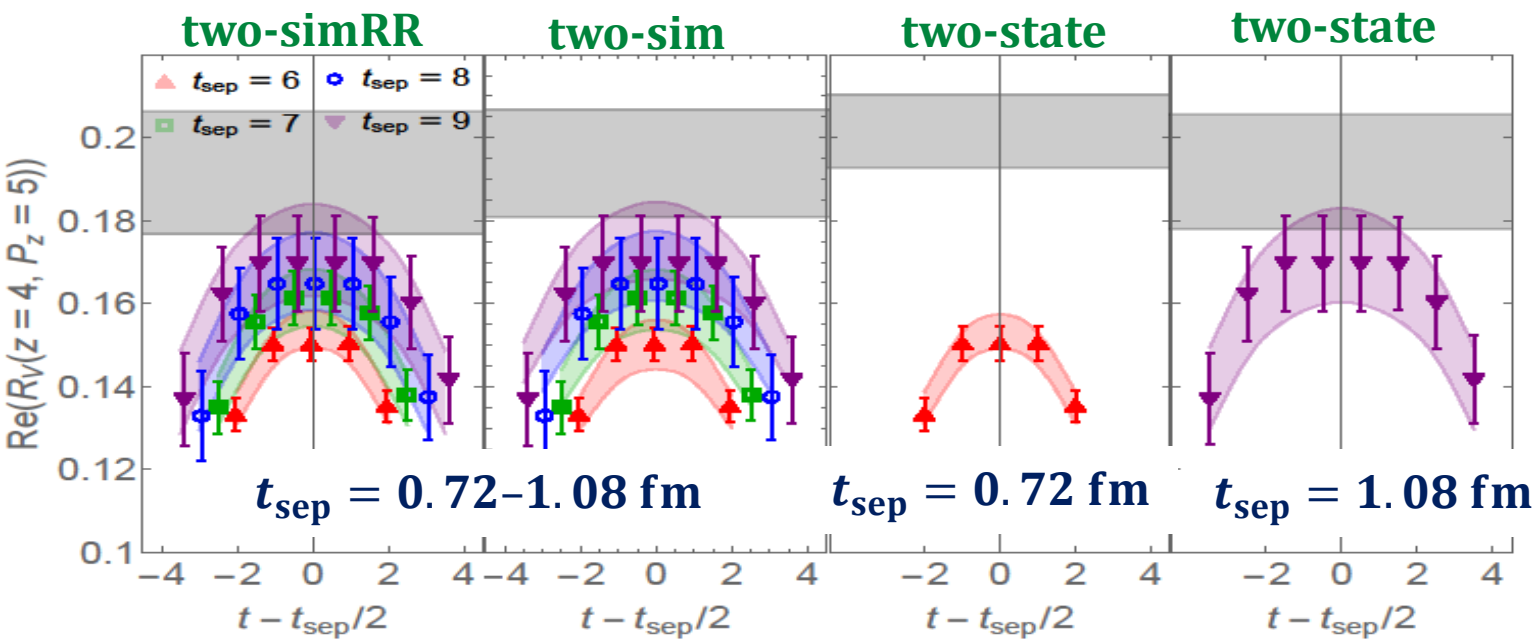
(z dependence)



Systematics: excited-state contamination

Kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

$P_z \approx 1.3$ GeV, $z = 4$, real (plot by Zhouyou Fan)



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

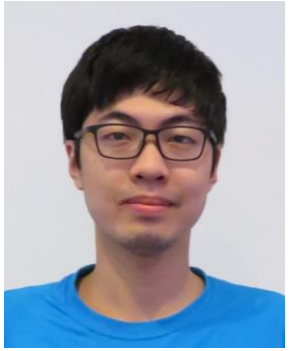
1) Calculate meson matrix elements on the lattice

(z dependence)

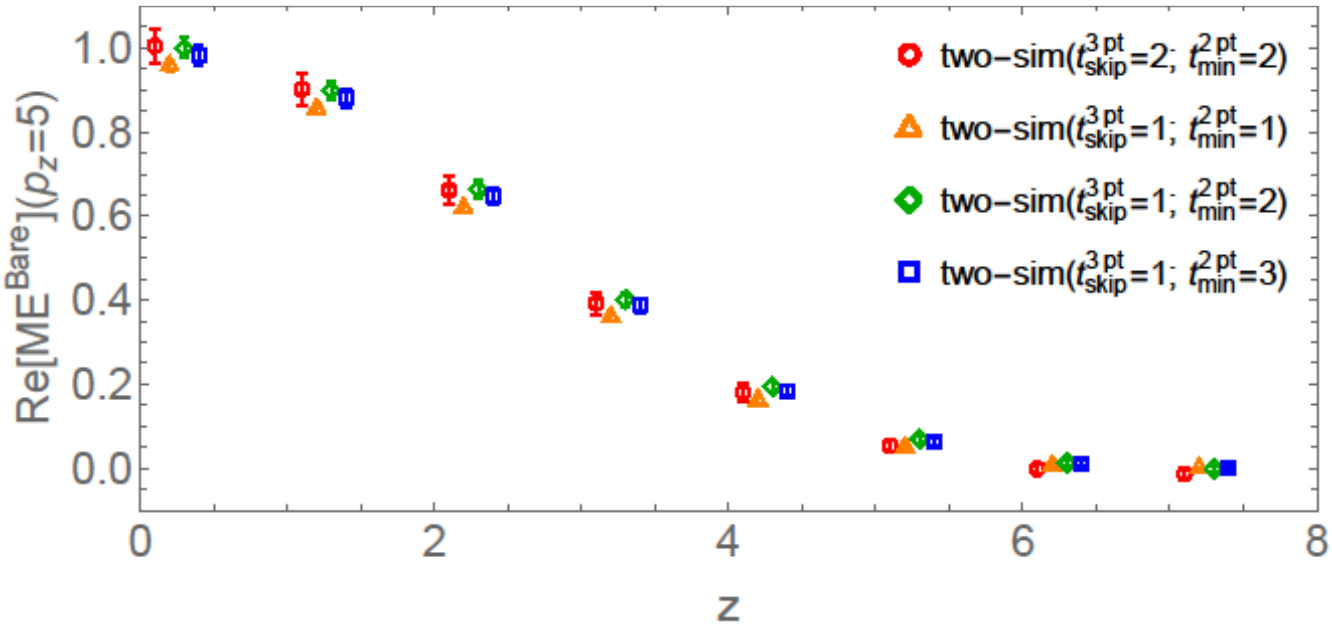
Systematics: stability in extracting matrix elements

Kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

$P_z \approx 1.3$ GeV



Zhouyou Fan

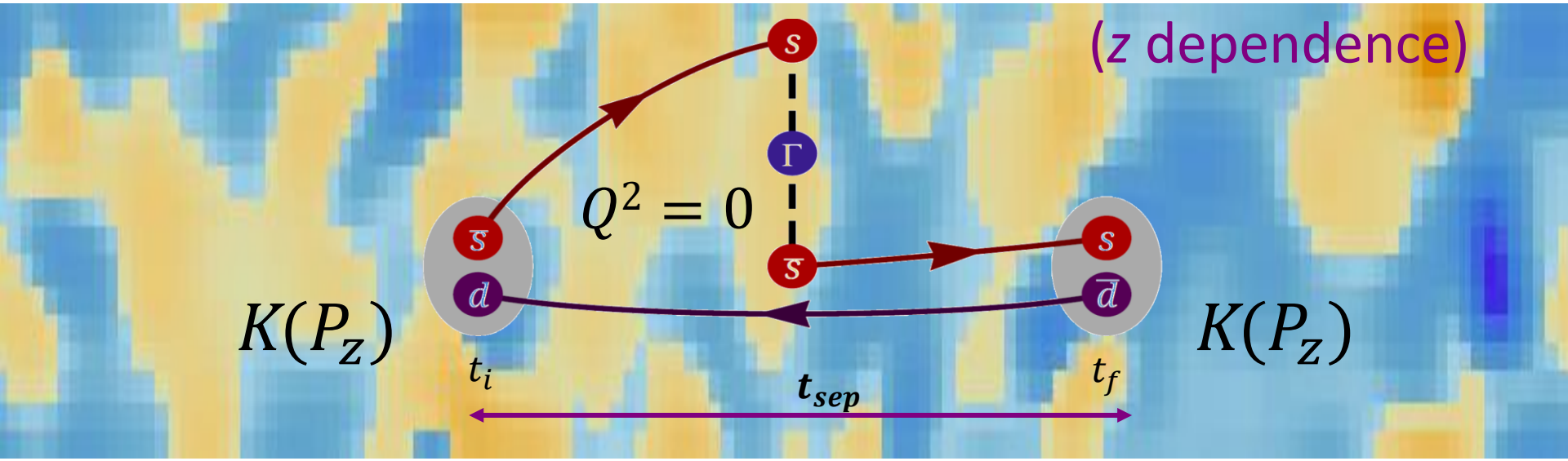


LaMET Recipe

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

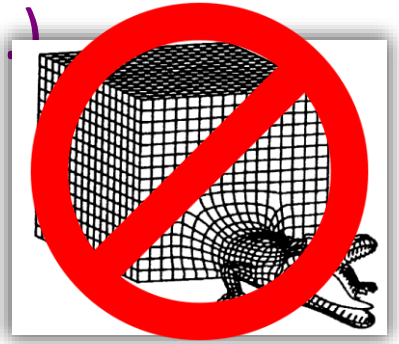
1) Calculate meson matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Excited-state removal; nonperturbative renorm.
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(z dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\, \Gamma \exp\left[-ig \int_0^z dz'\, A_z(z')\right] \psi(0) \right| P \right\rangle$$

$x = k_z/P_z$

lattice z coordinate

product of lattice gauge links

hadron momentum $P_\mu = \{P_t, 0, 0, P_z\}$

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(*z* dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\, \Gamma \exp\left[-ig \int_0^z dz'\, A_z(z')\right] \psi(0) \right| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate meson matrix elements on the lattice

(*z* dependence)

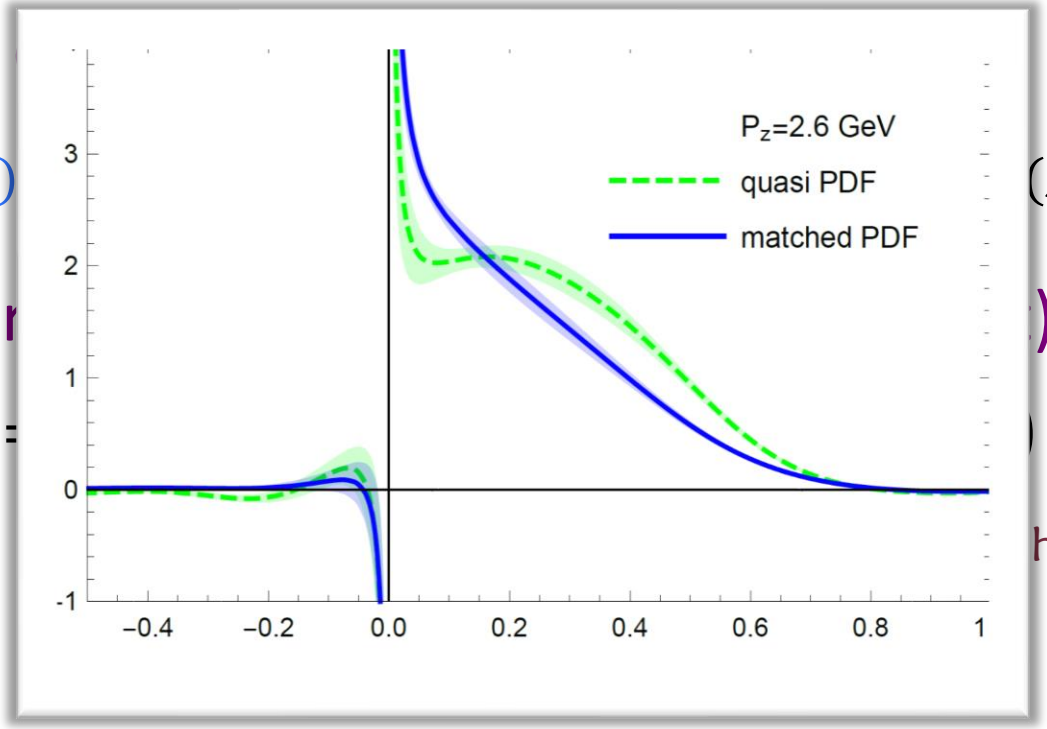
2) Compute

$\tilde{q}(x,\mu,P_z)$ $(z')\Big]\psi(0)\Big|P\Big\rangle$

3) Recover tr)

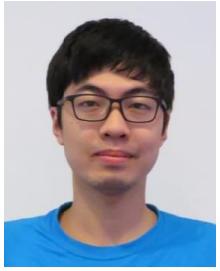
$\tilde{q}(x,\mu,P_z)$ = $+\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$

hen et al, 1603.06664



§ Matching is a crucial step in recovering the true lightcone distribution

New Results on Meson Structure

+ HWL

Rui Zhang (MSU) Zhouyou Fan (MSU) Jiunn-Wei Chen (NTU) Jian-Hui Zhang (BNU)

H. Lin et al., 2003.14128

2003.14128

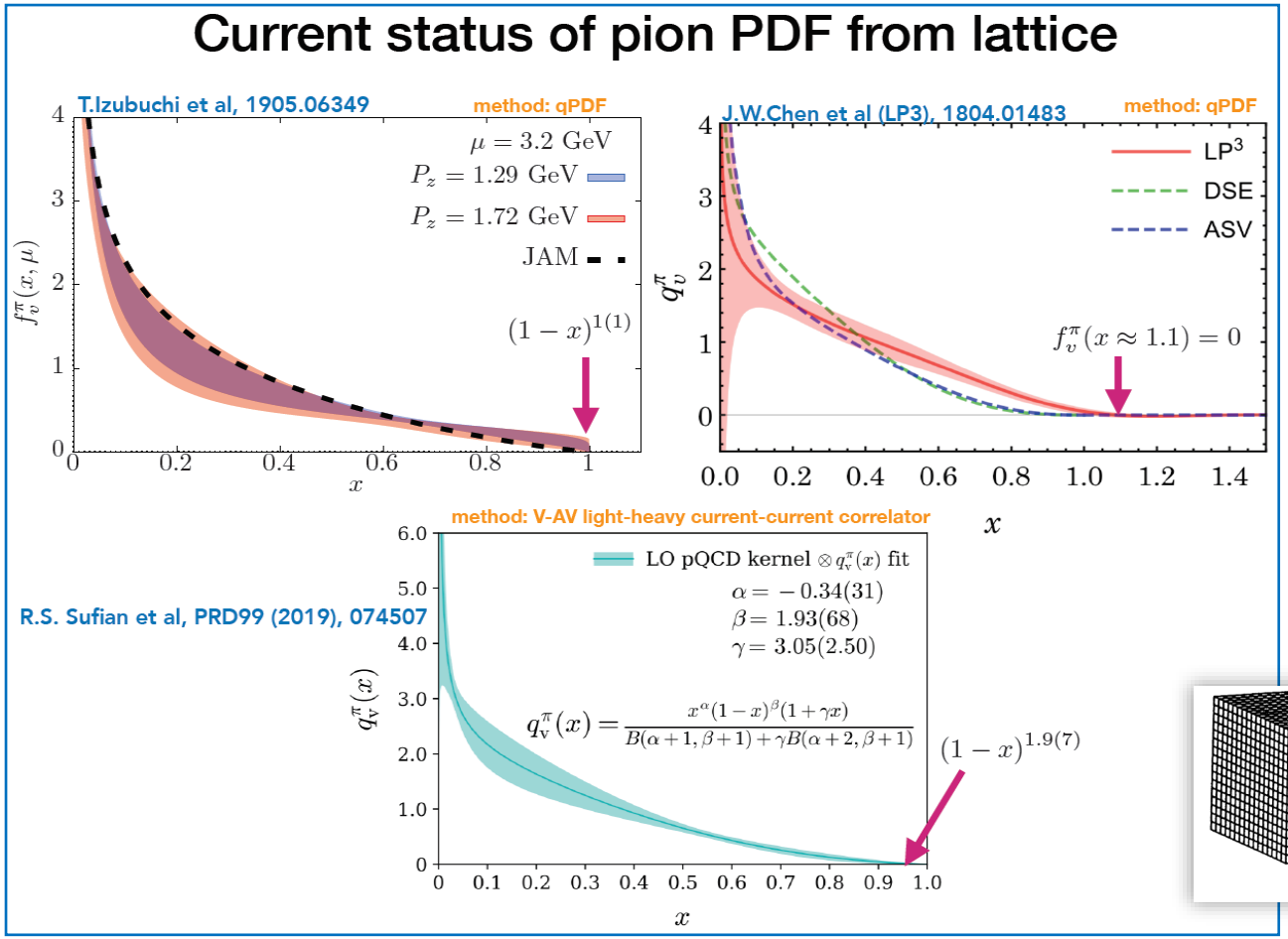
Pion Valence-Quark PDF

§ Status as of Summer 2019

Slide by Nikhil Karthik @ Lattice 2019

$M_\pi \approx 310$ MeV

$M_\pi \approx 426$ MeV



§ Single-ensemble calculation

- Non-physical pion mass, single lattice spacing, single volume

2003.14128

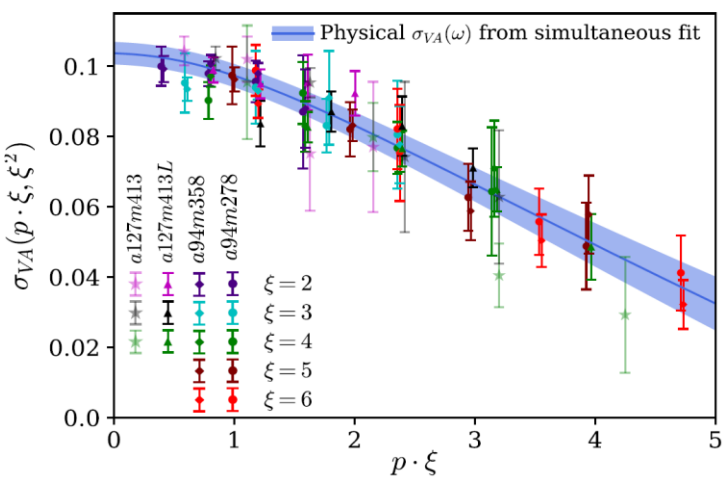
Pion Valence-Quark PDF

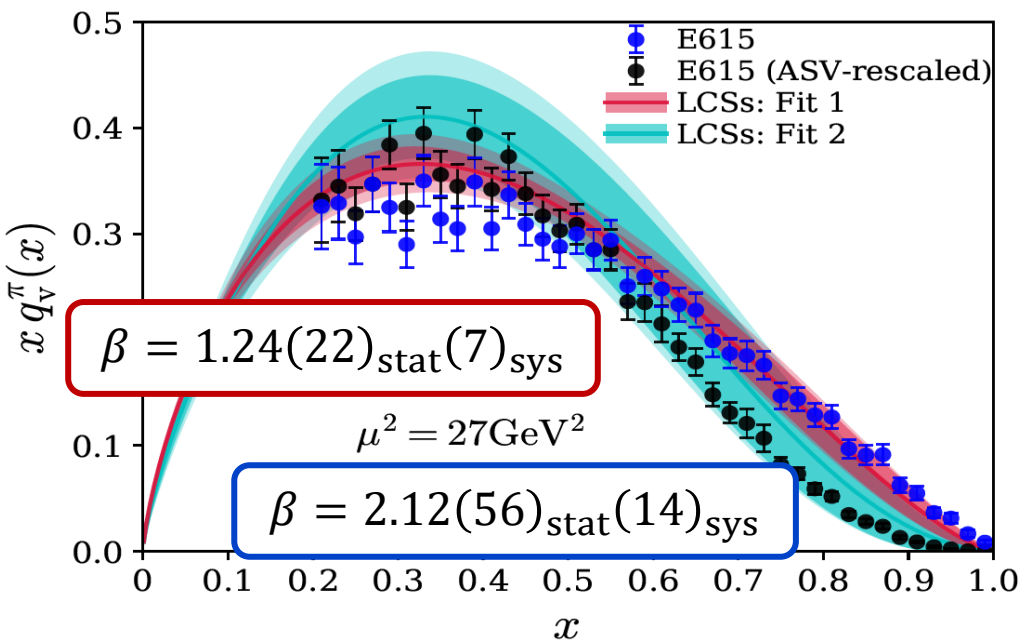
§ Results from JLab-W&M/LCS method

- $M_\pi = \mathbf{278}, 358, 413$ MeV with $a = 0.094, 0.127$ fm
- Extrapolated to physical limit (shown as blue band)
- Renormalized $Z_{V,A}$ in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

R. S. Sufian, et al, 2001.04960

$$q_v^\pi(x) = \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{B(\alpha+1,\beta+1) + \gamma B(\alpha+2,\beta+1)}$$



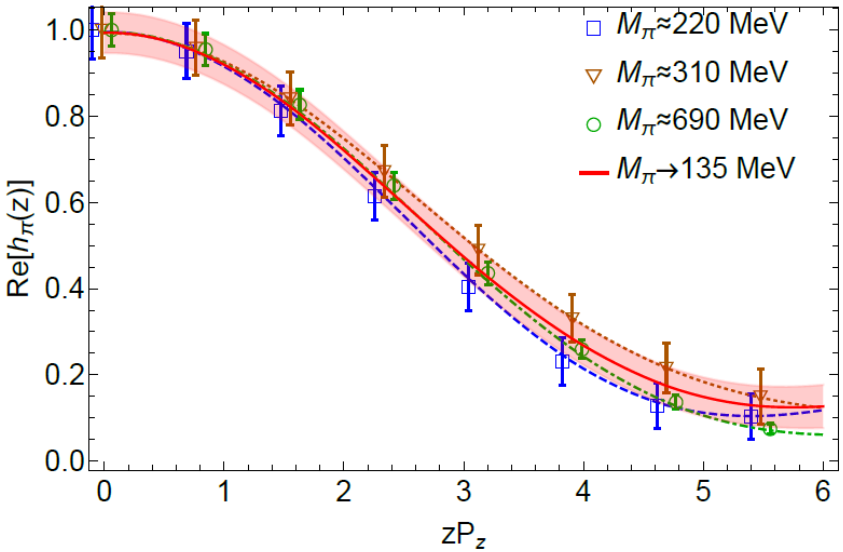


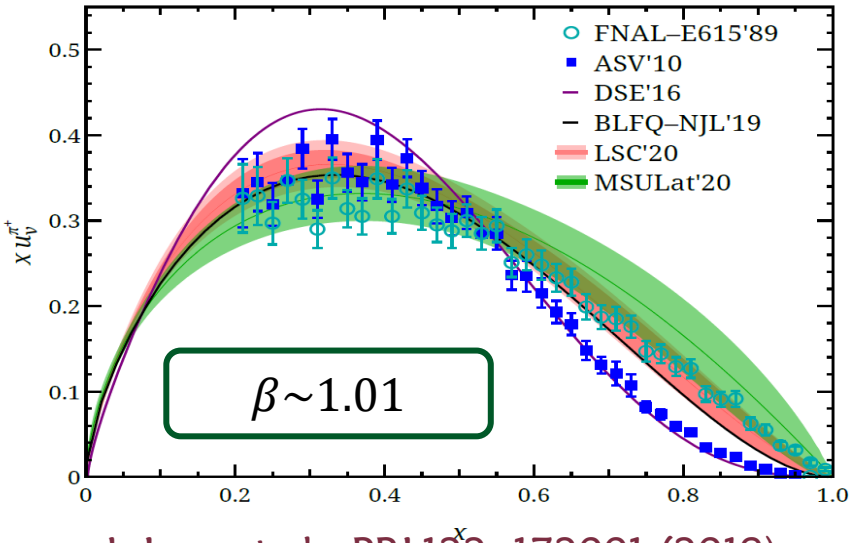
Pion Valence-Quark PDF

§ Results from MSULat/quasi-PDF method

- $M_\pi = \mathbf{220}, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- Extrapolated to physical limit (shown as pink/green band)
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

H. Lin et al. (MSULat), 2003.14128



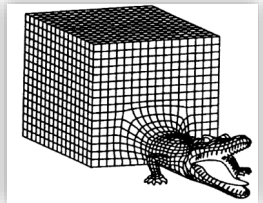


J. S. Conway et al., PRD39, 92 (1989).
M. Aicher et al, PRL105, 252003 (2010), 1009.2481.
C. Chen et al, PRD93, 074021 (2016), 1602.01502.

J. Lan, et al, PRL122, 172001 (2019), 1901.11430;
PRD101, 034024 (2020), 1907.01509.
R. S. Sufian, et al, 2001.04960

Pion Gluon PDF

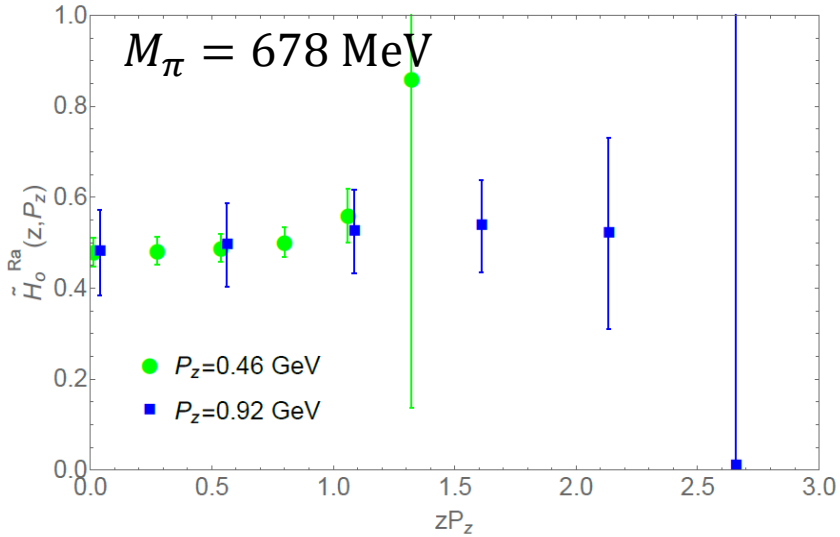
§ Pioneering first glimpse into pion gluon PDF using LaMET



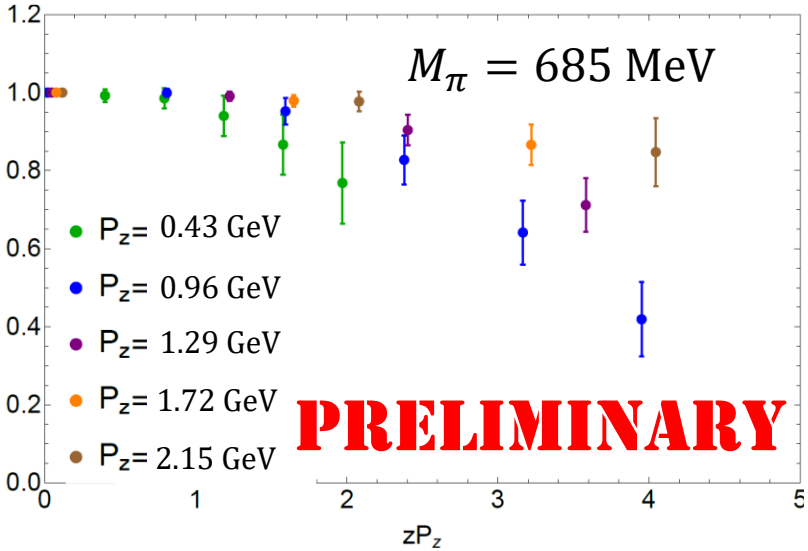
- Promising results using coordinate-space comparison, but signal does not go far in z

Fan. et al, Phys. Rev. Lett. 121, 242001 (2018)

- Lattice calculation #1: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass



- Lattice calculation #2: clover/2+1+1 HISQ, 0.15fm, 310-MeV sea pion mass with increased momenta (normalized by $\langle x \rangle_g$)



(plot by Zhouyou Fan)

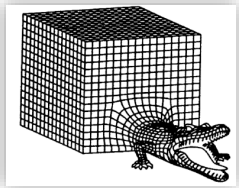
First Lattice GPDs

§ Pioneering first glimpse into pion GPD using LaMET

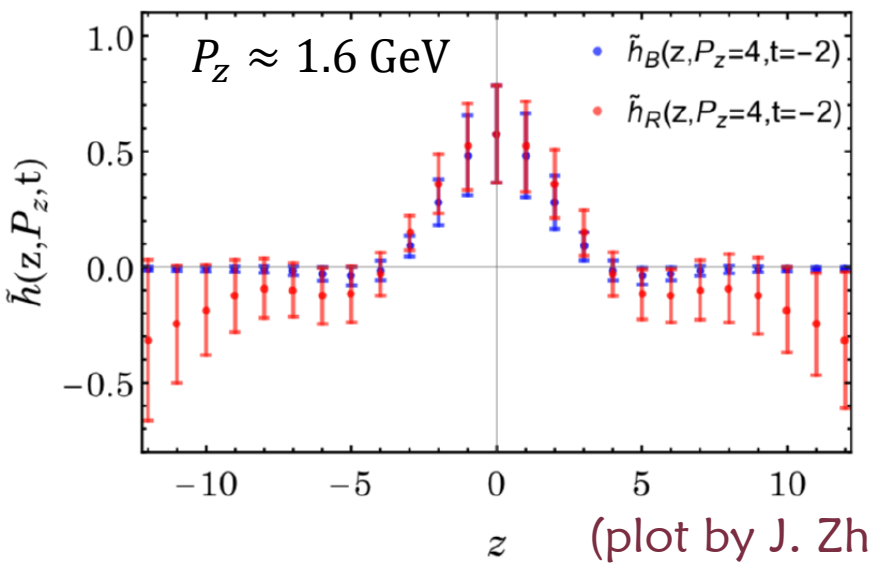
- Lattice details: clover/HISQ, 0.12fm, **310-MeV** pion mass

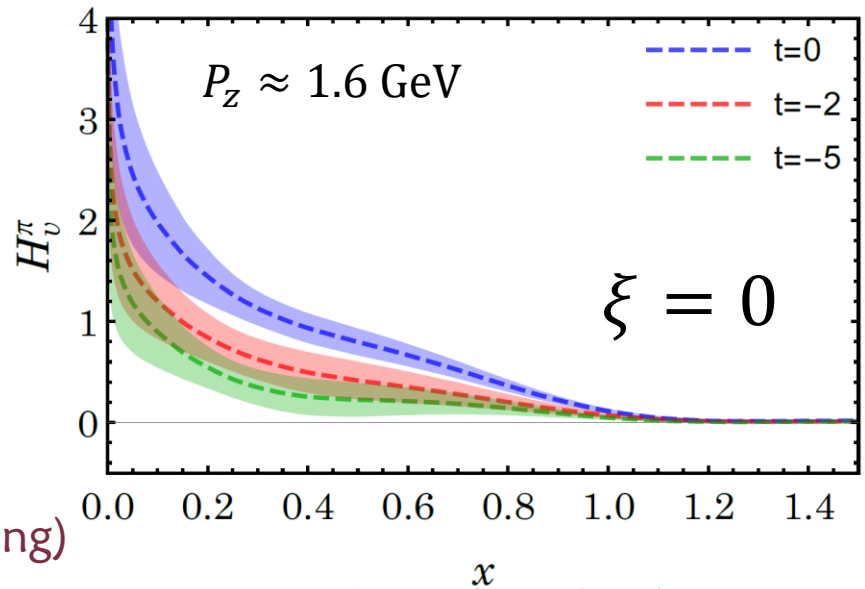
$P_z \approx 1.3, 1.6$ GeV

J. Chen, HL, J. Zhang, 1904.12376



$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right)\gamma^+\Gamma\left(\frac{\eta^-}{2},-\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$





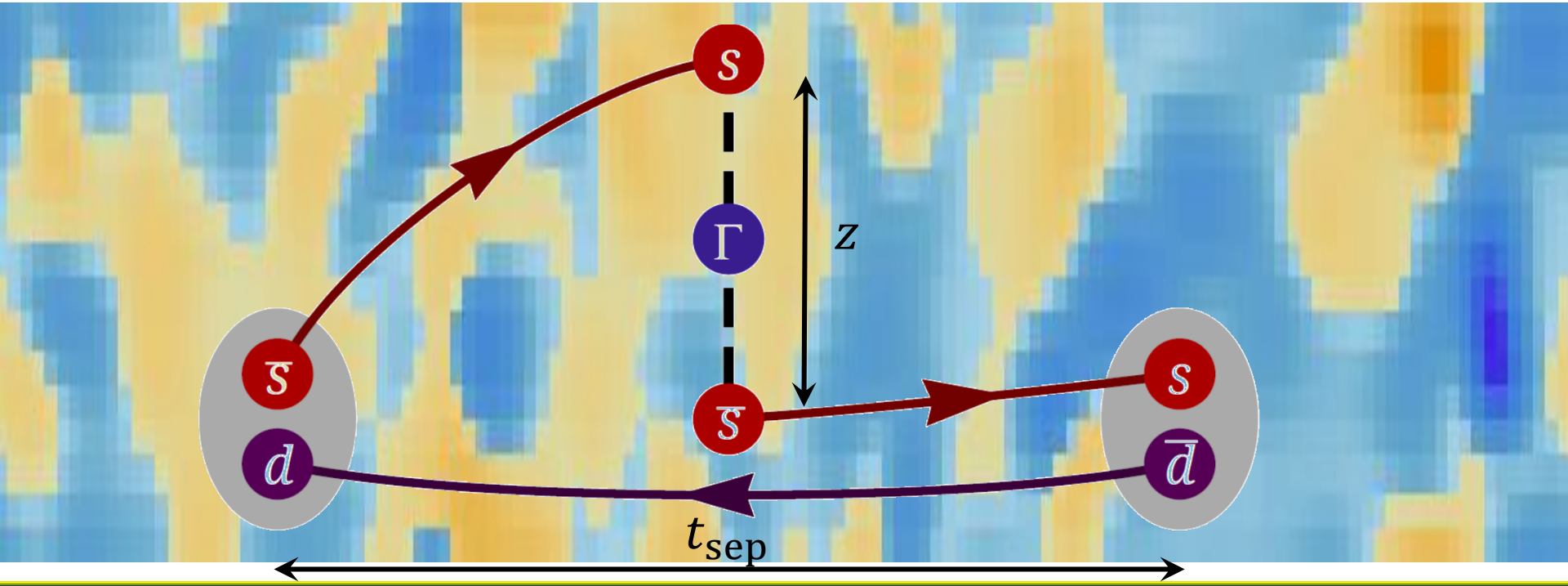
(plot by J. Zhang)

See talk by Colin Egerer, Raza Sufian (Friday)

First Kaon PDFs

§ The first study of *x*-dependent kaon PDF on the lattice

- $M_\pi \in \{220, 310, 690\ (\eta_s)\}$ MeV
- $a = 0.06, 0.12$ fm
- $M_\pi^{\min} L = 5.5$
- $t_{\text{sep}} = 0.72\text{–}1.08$ fm

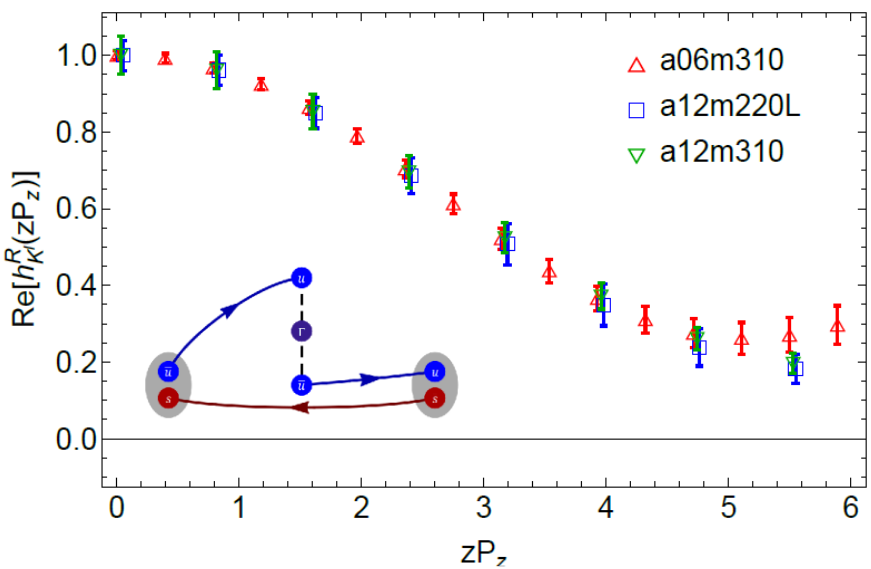


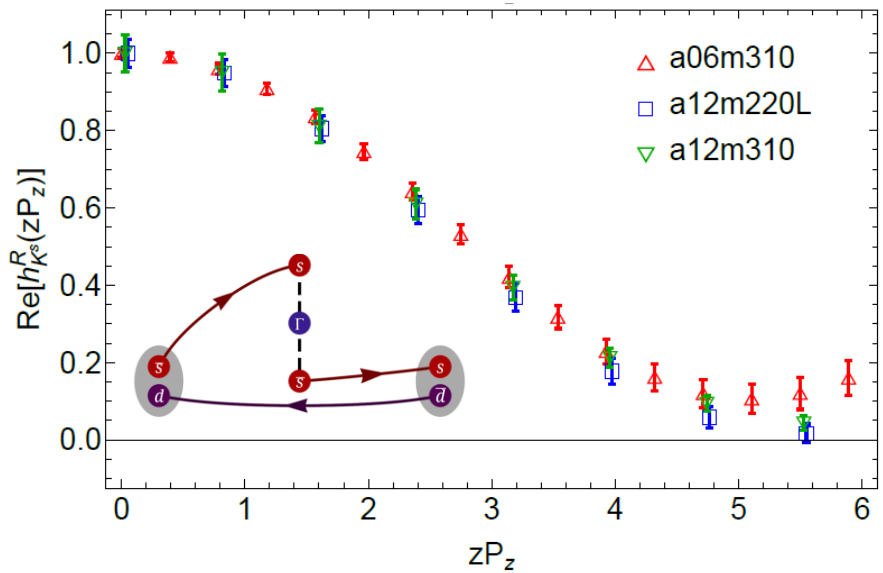
First Kaon PDFs

§ We renormalized in RI/MOM scheme

$\rightsquigarrow \mu_R = 2.4$ GeV

Systematics: Continuum extrapolation $a \to 0$



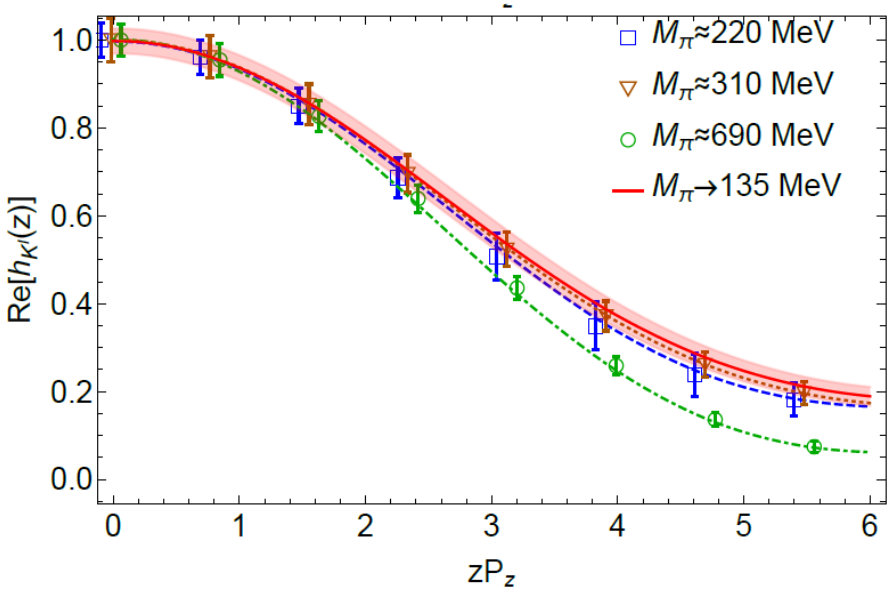


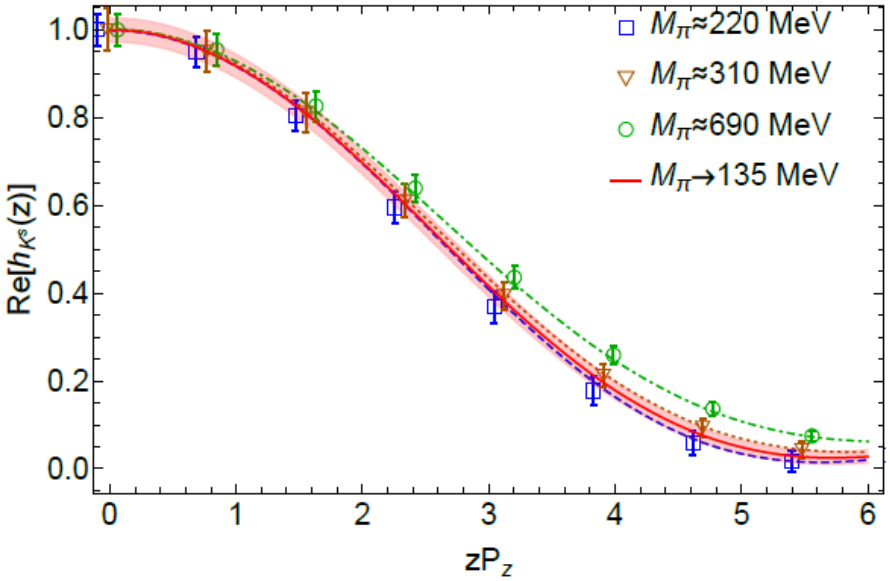
§ Mild dependence on the lattice spacing and pion masses

First Kaon PDFs

§ We renormalized in RI/MOM scheme

- $\mu_R = 2.4$ GeV
- Extrapolate to physical limit (shown as pink band)





§ Combine with matching to yield $\overline{\text{MS}}$ results

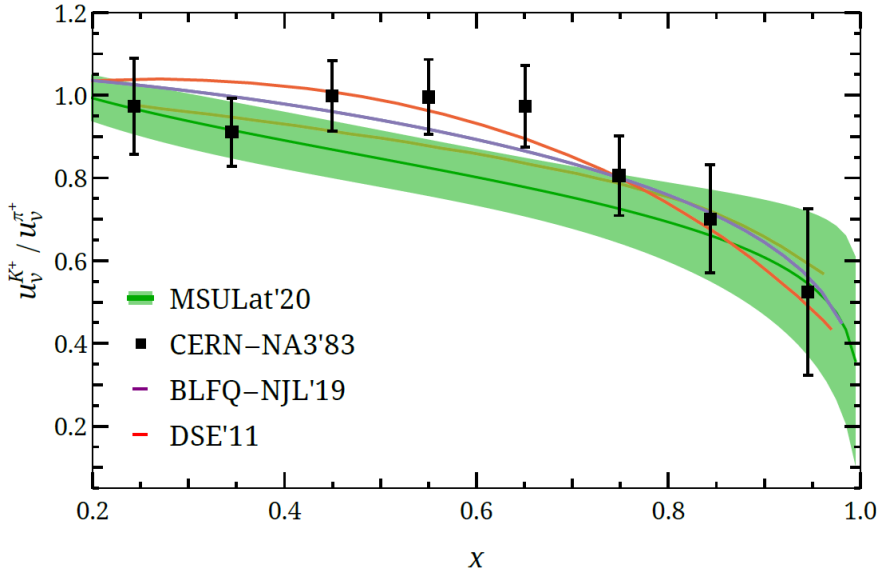
(plot by Rui Zhang)

First Kaon PDFs

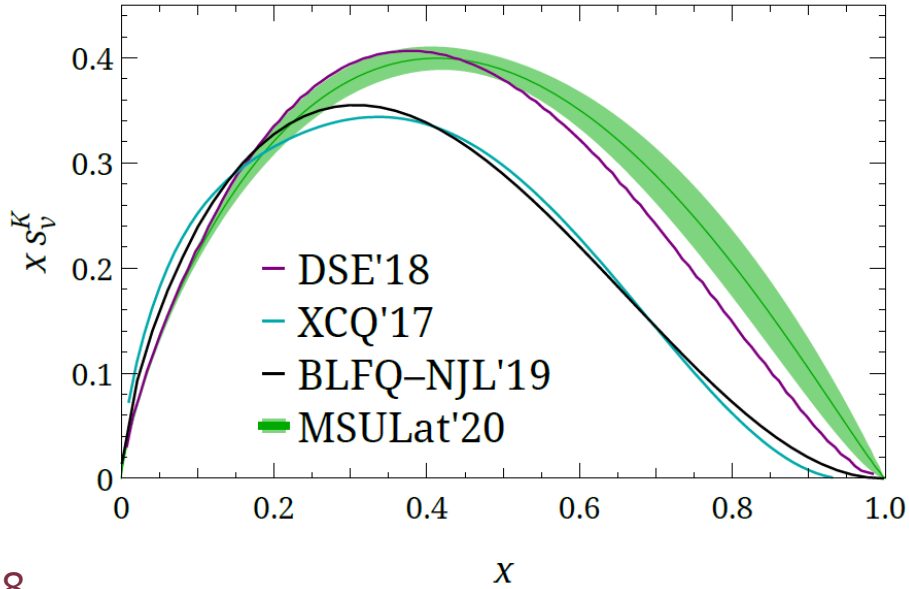
§ LaMET steps 2 & 3: Extracting the lightcone distribution from the physical-limit lattice matrix elements

H. Lin et al. (MSULat), 2003.14128

Kaon-to-pion ratio for up-quark PDF



Kaon strange-quark PDF



CERN-NA3'83: J. Badier et al. (NA3), Z. Phys. C18, 281 (1983).

BLFQ-NJL'19: J. Lan, et al, PRL122, 172001 (2019); Phys. Rev. D101, 034024 (2020)

DSE'11: T. Nguyen et al., PRC83, 062201 (2011),

DSE'18: K. D. Bednar,et al, PRL124, 042002 (2020)

XCD'17: A. Watanabe et al, PRD97, 074015 (2018)

New Results on Meson DA

Rui Zhang (MSU)

Carson Honkala (MSU)

Jiunn-Wei Chen (NTU)

+ HWL

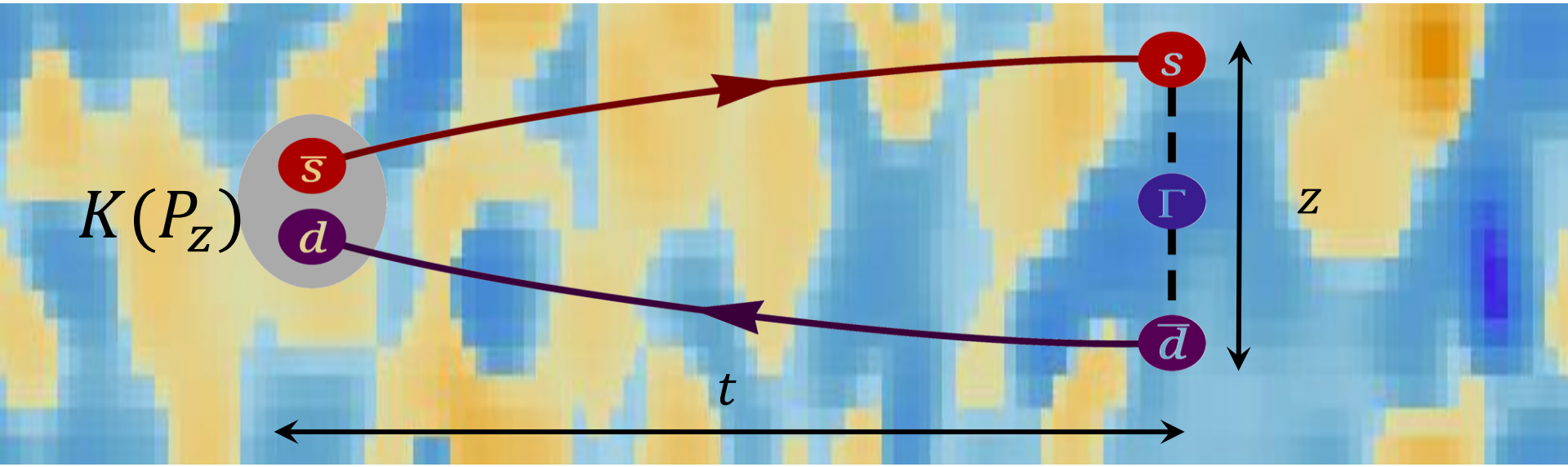
R. Zhang et al.(MSULat), 2005.13955

Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

- $M_\pi \in \{\mathbf{310}, 690\ (\eta_s)\}$ MeV
- $a \in \{0.06, 0.09, 0.12\}$ fm
- $M_\pi^{\min} L = 4.5$

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\,\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$

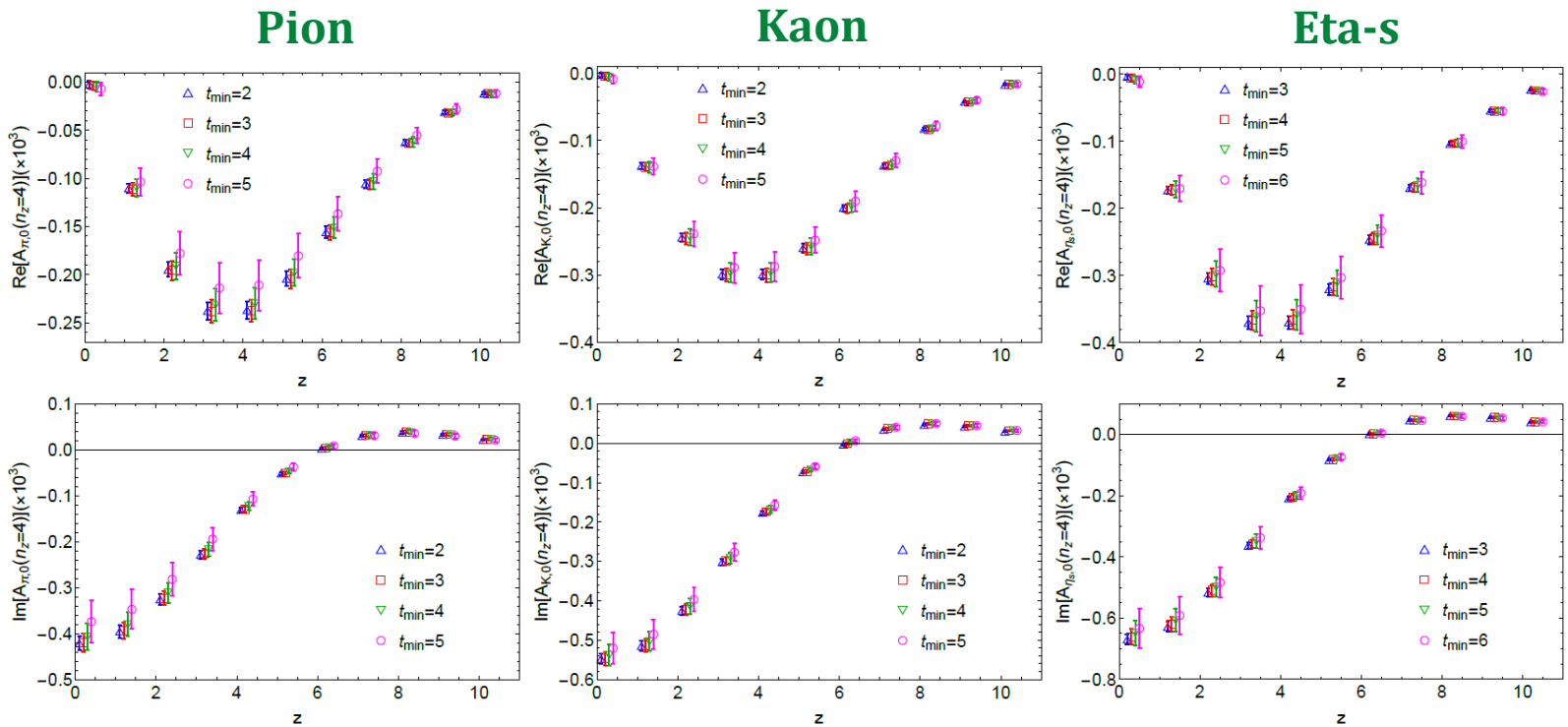


Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

Systematics: stability in extracting matrix elements

- $a = 0.06$ fm, 310-MeV ensemble example

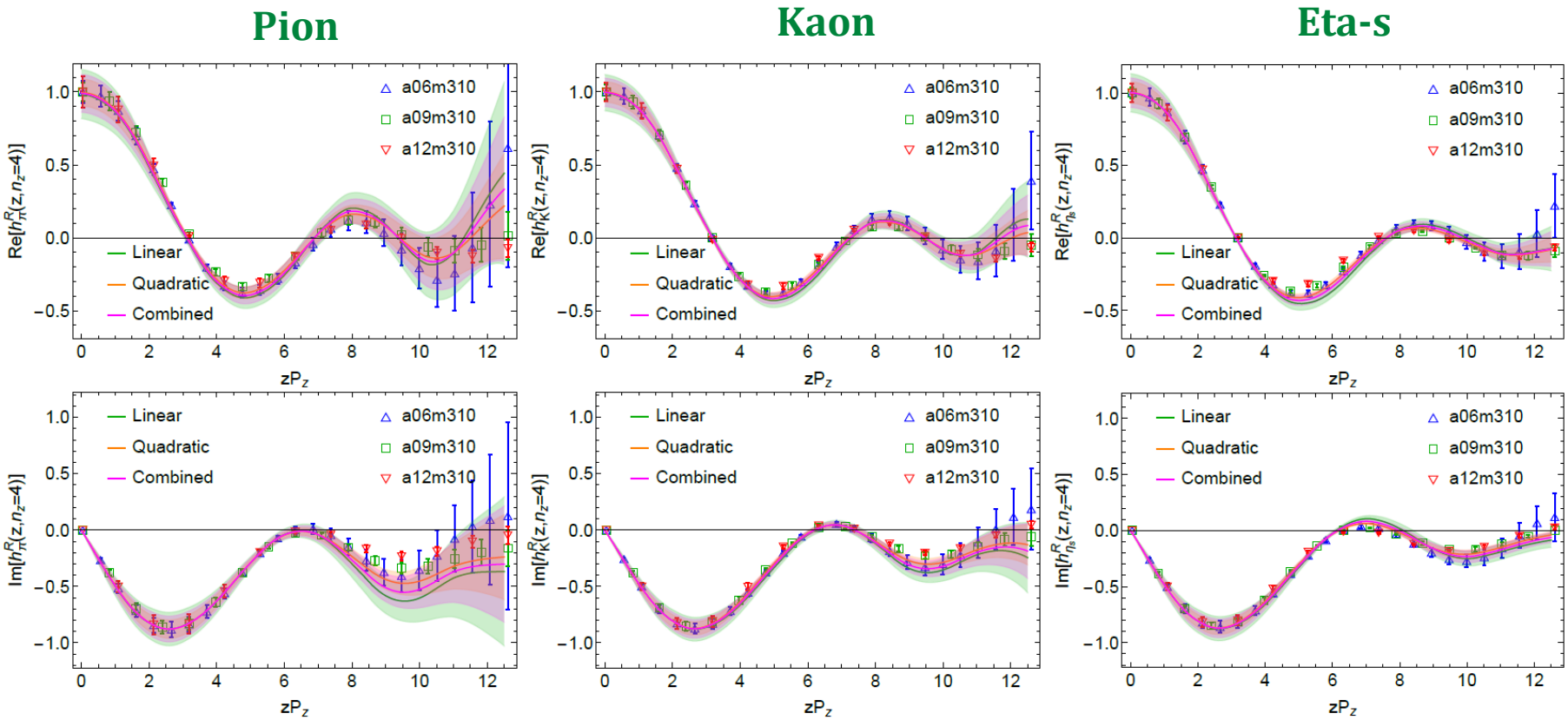


R. Zhang et al., 2005.13955

Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

Systematics: Continuum extrapolation $a \to 0$



R. Zhang et al., 2005.13955

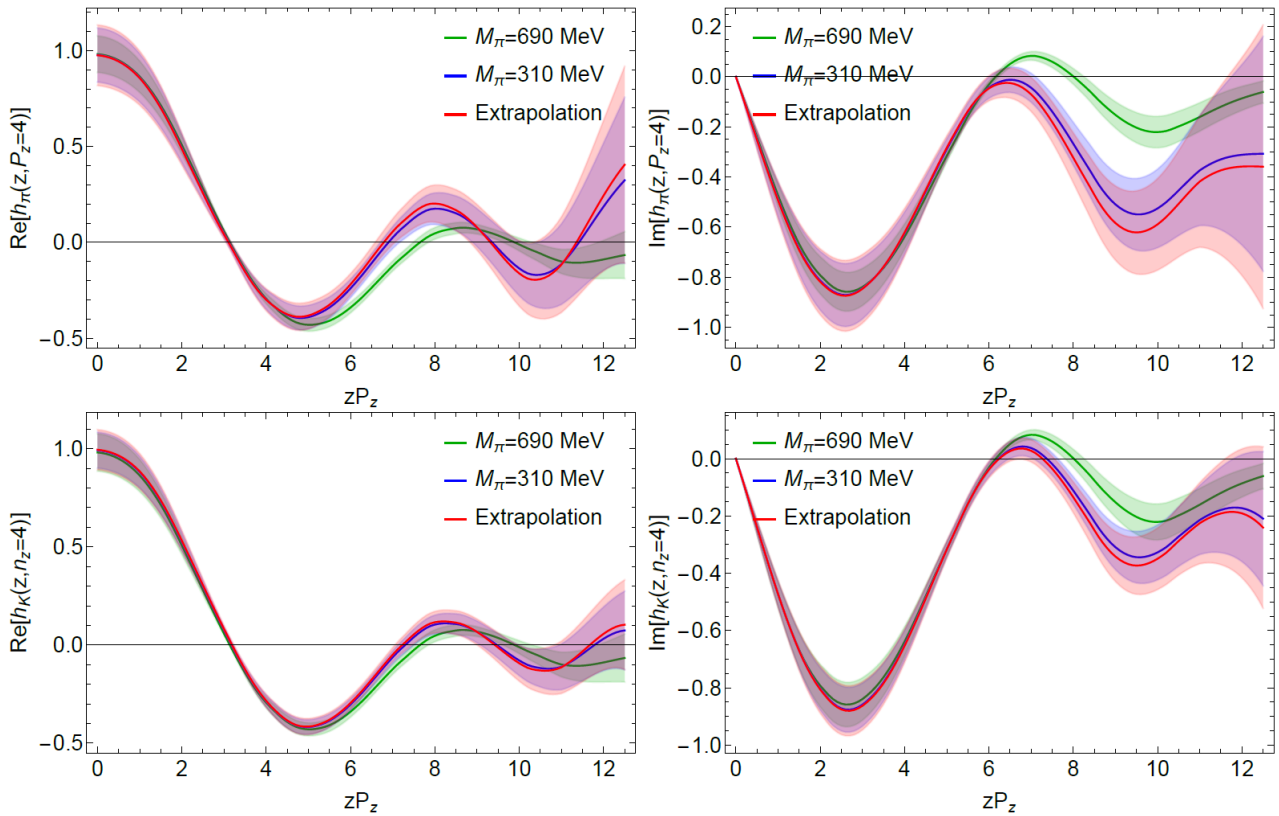
Pion and Kaon DA

§ The first continuum-limit study of x-dependent meson DA on the lattice

Systematics: Continuum extrapolation $m_\pi \to m_\pi^{\text{phys}}$

Pion

Kaon



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Pion and Kaon DA

§ Extract the DA distribution from the physical-continuum matrix elements

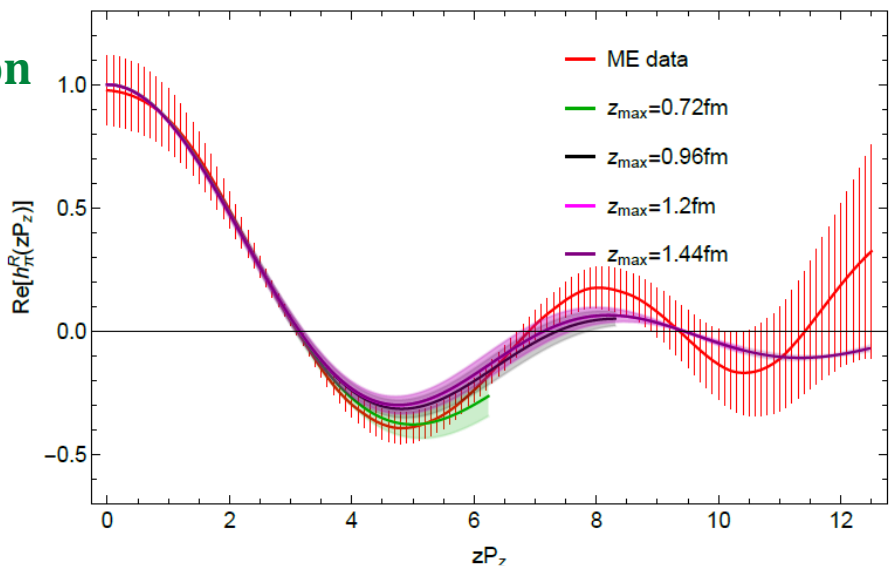
R. Zhang et al. (MSULat), 2005.13955

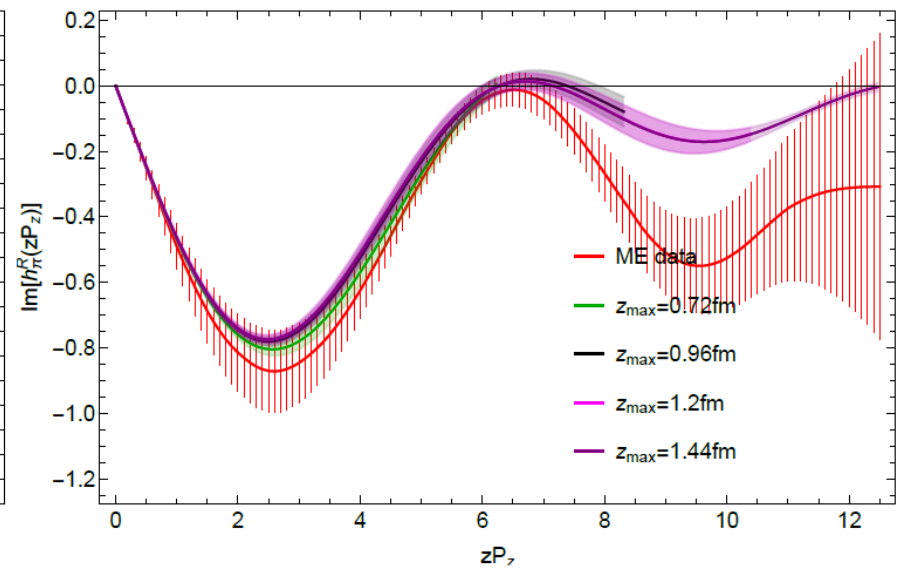
$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx\, e^{i(1-x)zP_z} \int_0^1 dy\, C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y)$$

$$f_{m,n}(x) = \frac{1}{B(m+1,n+1)} x^m (1-x)^n \qquad B(m+1,n+1) = \int_0^1 dx\, x^m (1-x)^n$$

➰ 1st method: fit to the functional form

Pion

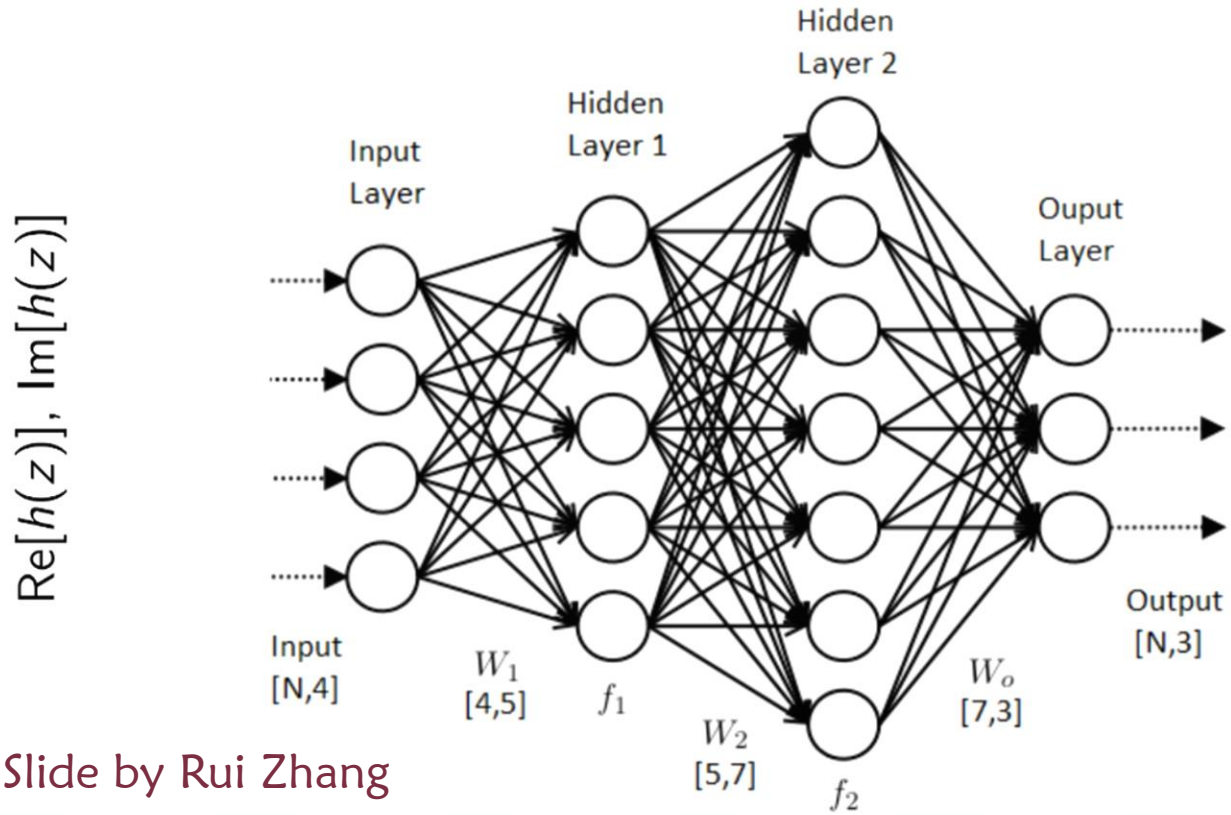




Pion and Kaon DA

Machine Learning - A Promising Solution?

Machine learning models are effective in extracting complicated dependence of the output data on input data.



Slide by Rui Zhang

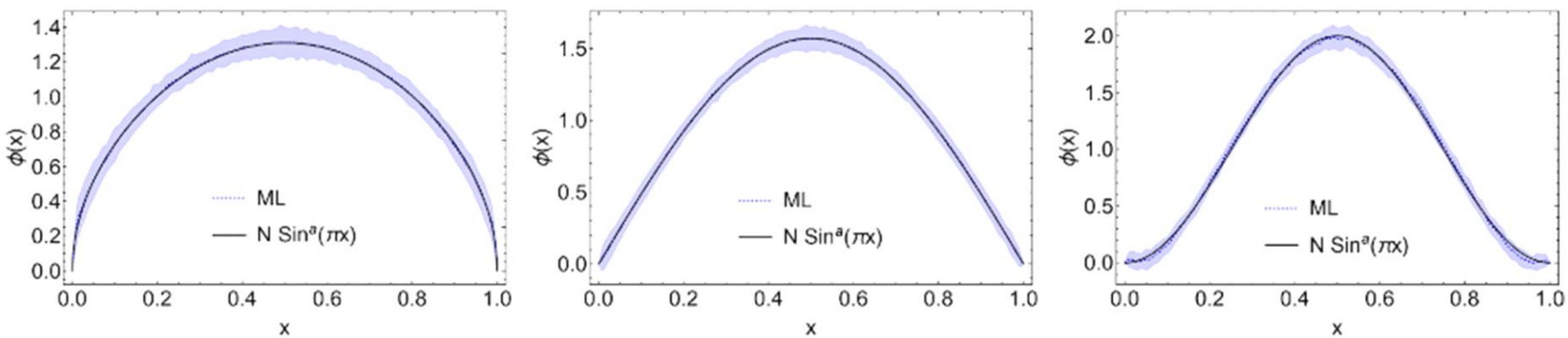
Pion and Kaon DA

Machine Learning - preliminary attempts

Slide by Rui Zhang

Training: 10,000 Pseudo-data from functional form $x^a(1-x)^b/B[a+1,b+1]$ with $a,b>0$. With random relative noise added.

Extrapolation test: Generate $h(z)$ from $f(x) = Nsin^a(\pi x)$ with $a = 0.5, 1, 2$. Add 1000 random noise $\sigma(z) = 0.1e^{0.1z}h(z)$ to $h(z)$. Estimate mean and error of the prediction on 1000 samples.



Pion and Kaon DA

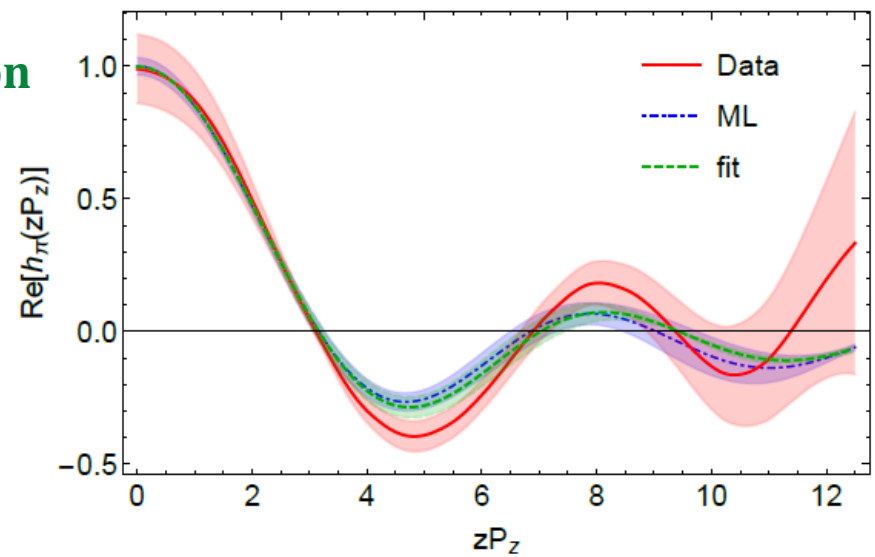
§ Extract the DA distribution from the physical-continuum matrix elements

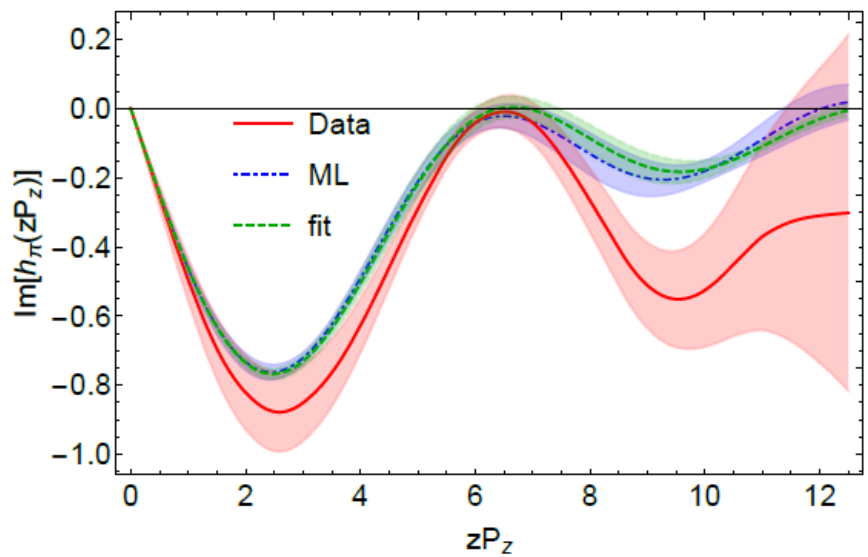
R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x, y, \left(\frac{\mu^R}{p_z^R}\right)^2, \frac{P_z}{\mu^R}, \frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

➰ 2nd method: use machine learning to determine f

Pion





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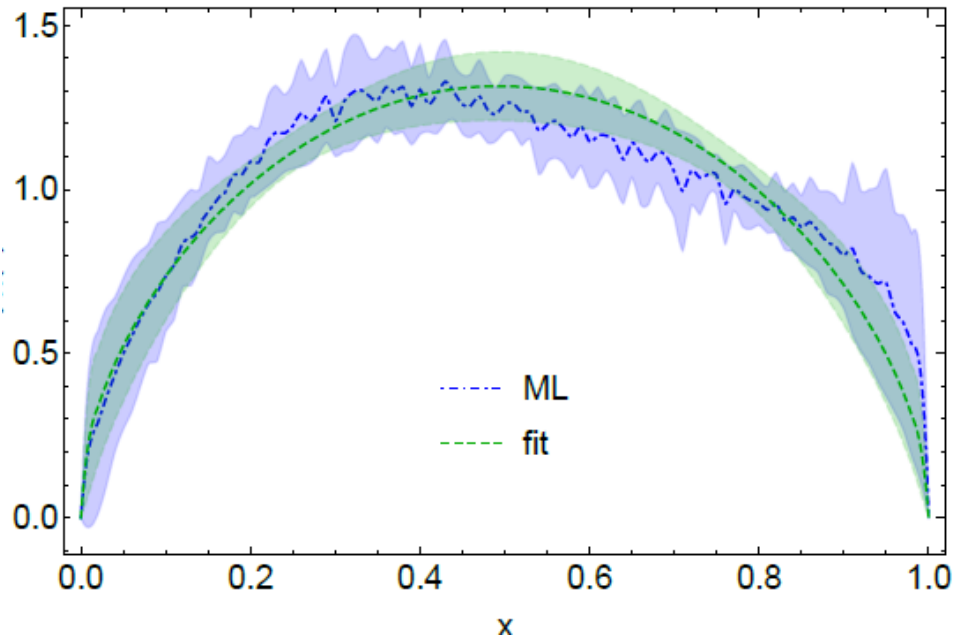
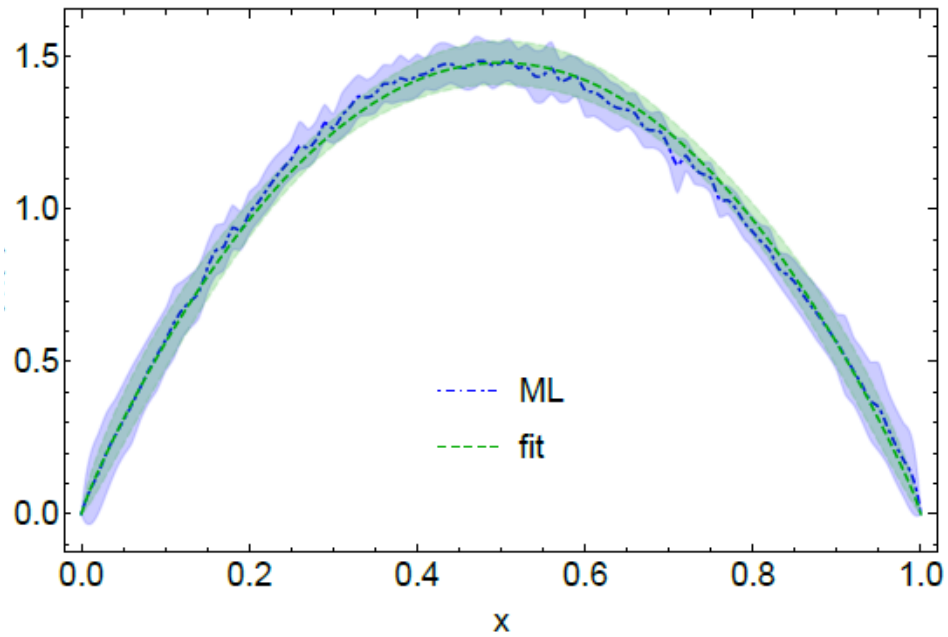
Pion



Kaon



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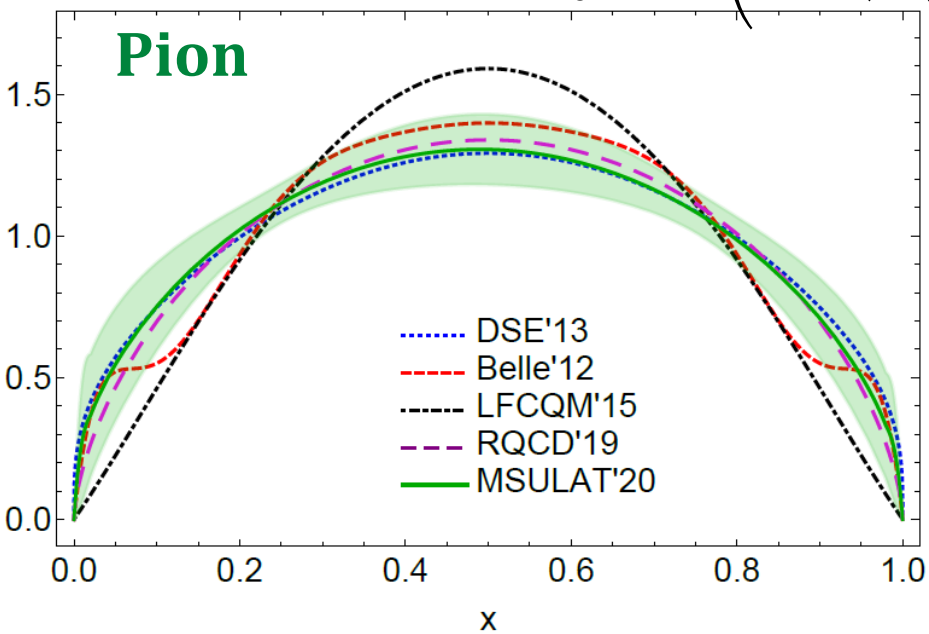
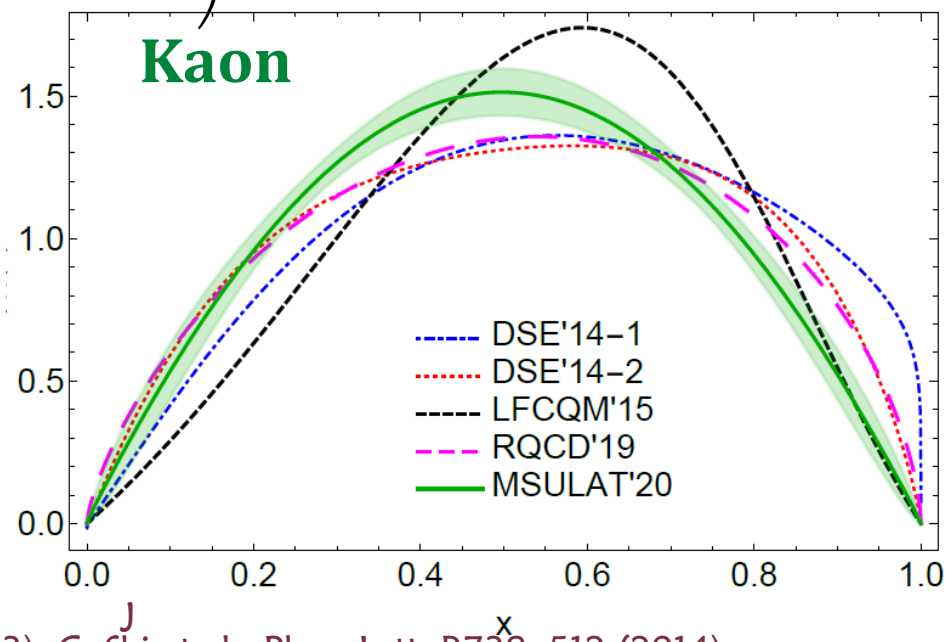




DES'13: L. Chang et al., Phys. Rev. Lett. 110, 132001 (2013); C. Shi et al., Phys. Lett. B738, 512 (2014)
Belle'12: S. Agaev et al., Phys. Rev. D86, 077504 (2012);
LFCQM'15:J. P. B. C. de Melo et al., AIP Conf. Proc. 1735, 080012 (2016);
RQCD'19: G. S. Bali et al.,J HEP 08, 065 (2019); DSE'14:

Summary & Outlook

Exciting time for studying meson structure on the lattice

§ Overcoming longstanding obstacle to full *x*-distribution

- Most importantly, this can be done with today's computer

§ Progress made in pion and kaon structure

- First look at kaon PDF, and pion GPD
- Continuum-limit pion and kaon DAs

§ Future improvement

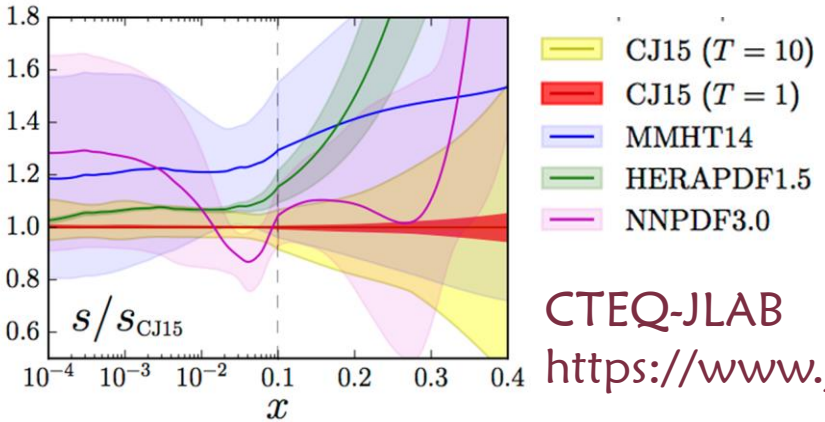
- Lighter (or physical) pion-mass and larger momentum
- Explore more ways to reduce the parameter-dependence of distribution functions

Thanks to MILC collaboration for sharing lattices and NSF CAREER Award under grant PHY 1653405

Backup Slides

First Lattice Strange PDF

§ Large uncertainties in global PDFs



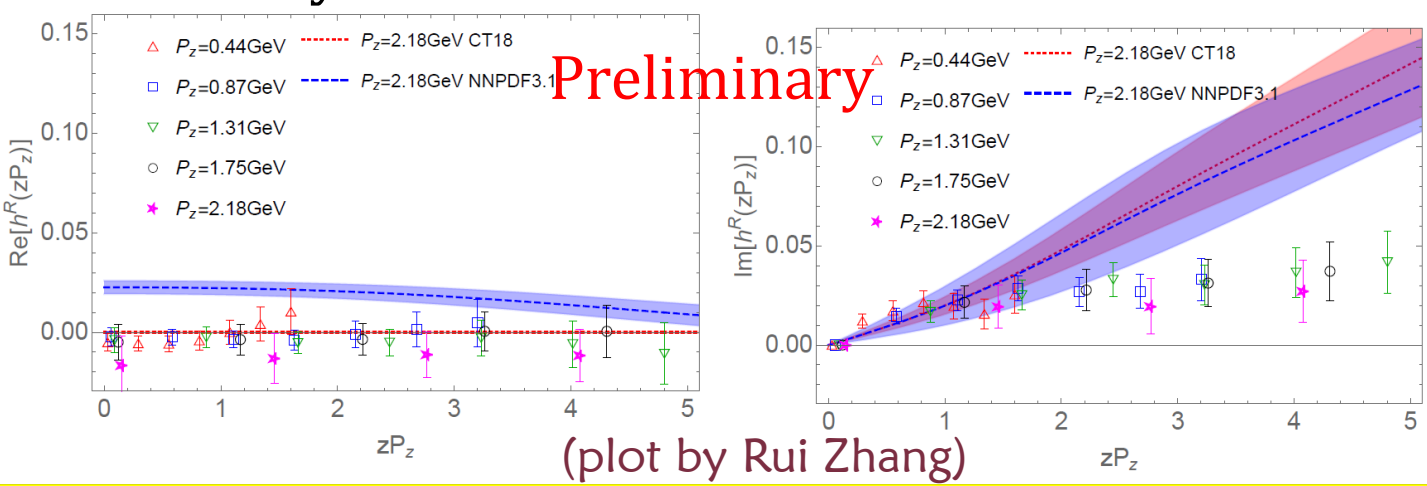
CTEQ-JLAB
https://www.jlab.org/theory/cj/

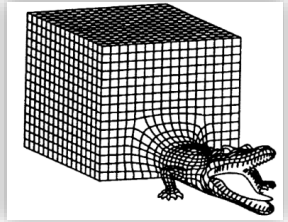
- Assumptions imposed due to lack of precision data

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

§ Results by MSULat/quasi-PDF method

- Preliminary calculation: clover on 2+1+1 HISQ, $M_\pi \approx 310$ MeV





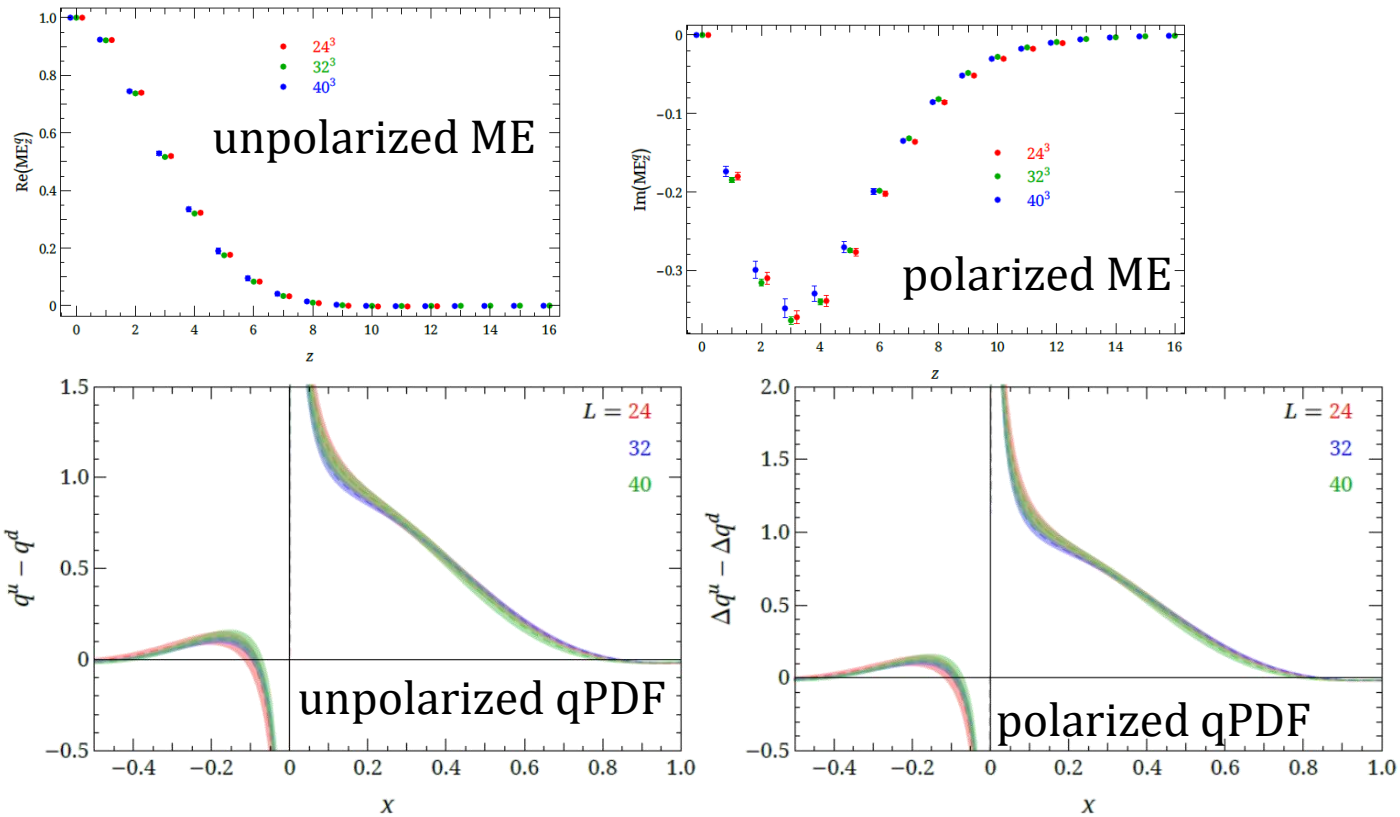
(plot by Rui Zhang)

Systematics Study

§ First finite-volume study in quasi-PDFs

- Clover on 2+1+1 HISQ, $M_\pi \approx \mathbf{220}$ MeV, $a \approx 0.12$ fm
- $M_\pi L \approx 3.3, 4.4, 5.5$, $P_z \approx 1.3$ GeV

HL, R, Zhang, Phys.Rev.D 100 (2019) 7, 074502



Superfine Lattice Spacing

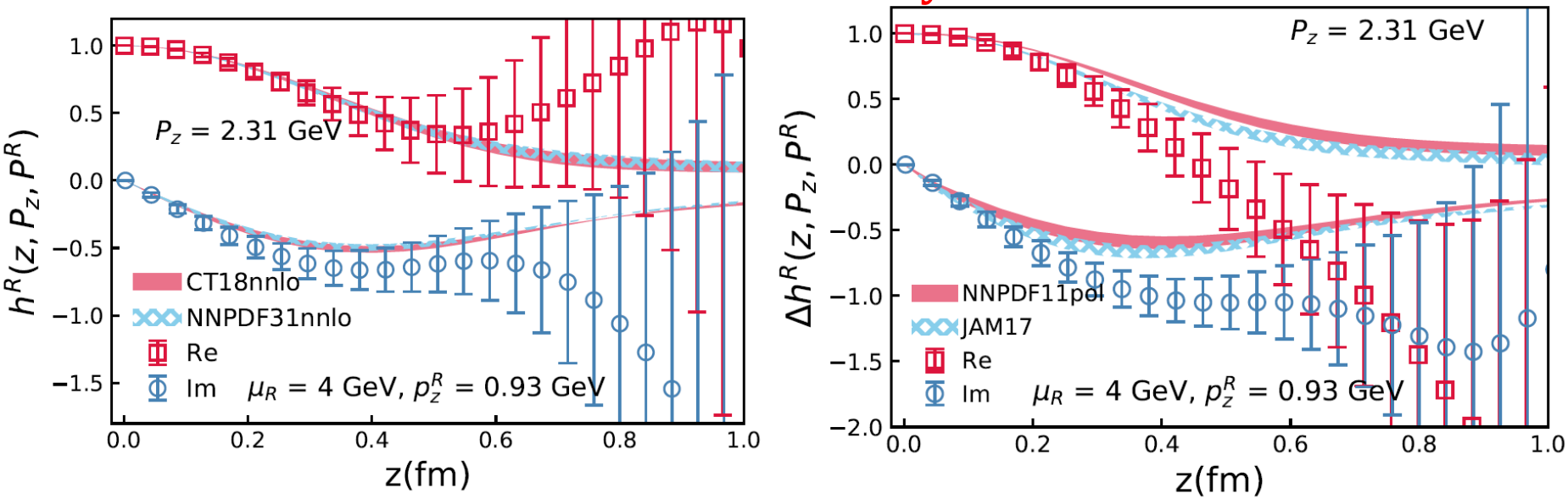
§ Approaching continuum limit

- Important for all x-dependent methods
  Large momentum required to reach $x < 0.1$ reliably
  $(aP_z)^n$ systematics should be small
- First work done with superfine lattice spacing, $a \approx 0.042$ fm

Unpolarized ME

Preliminary

Polarized ME



(plot by Xiang Goa)

Machine-Learning Prediction

Input
$X_i = (O_i^1, O_i^2, \dots)$

ML Model →

Output
$\hat{O}_i$

R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

All Data
- Labeled (lb) Data
  - Training (tr) Data
  - Bias-Correction (BC) Data
- Unlabeled (ul) Data

Prediction with bias correction Yoon et al., PRD 2018:

$$\langle C_{\text{pred,BC}} \rangle = \langle C_{\text{pred}} \rangle_{\text{ul}} + \langle C_{\text{BC}} - C_{\text{pred}} \rangle_{\text{BC}}$$

Machine-Learning Prediction

Input
$X_i = (O_i^1, O_i^2, \dots)$

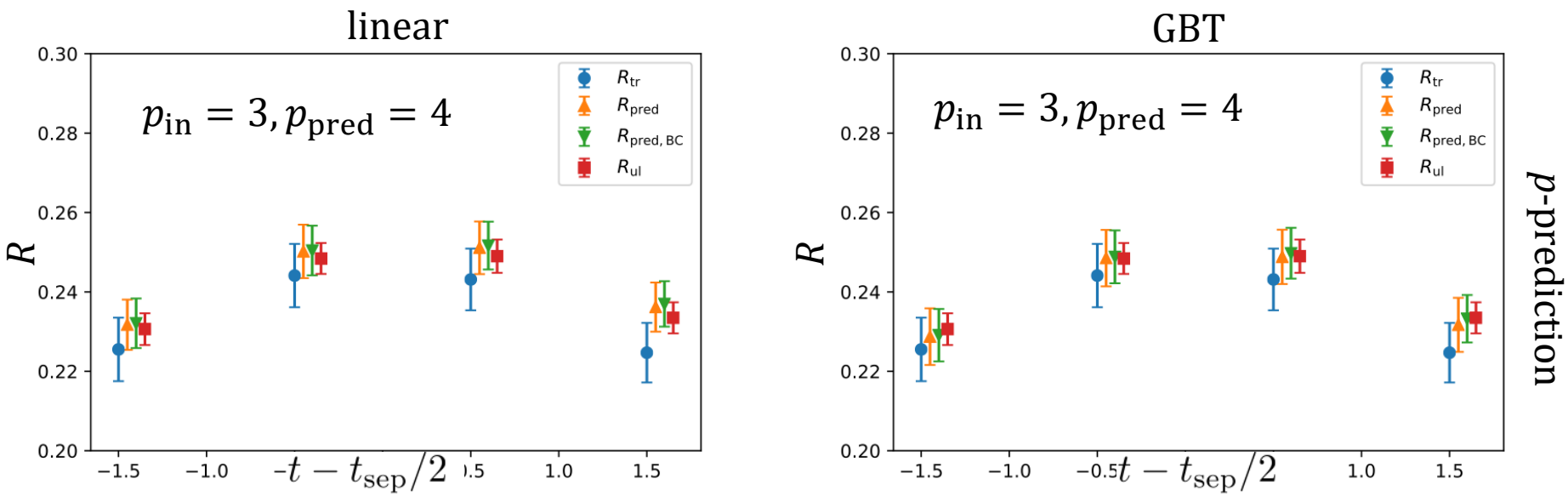
ML Model →

Output
$\hat{O}_i$

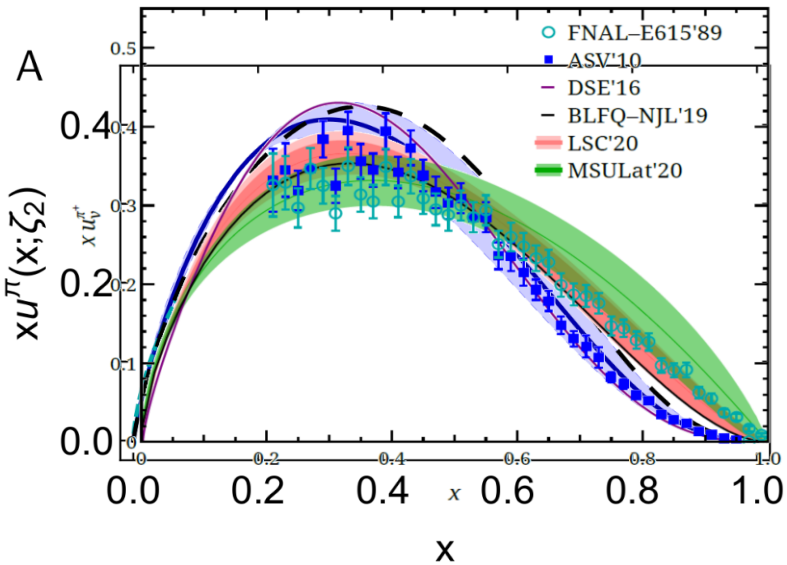
R. Zhang et al (MSULat), Phys. Rev. D 101, 034516 (2020)

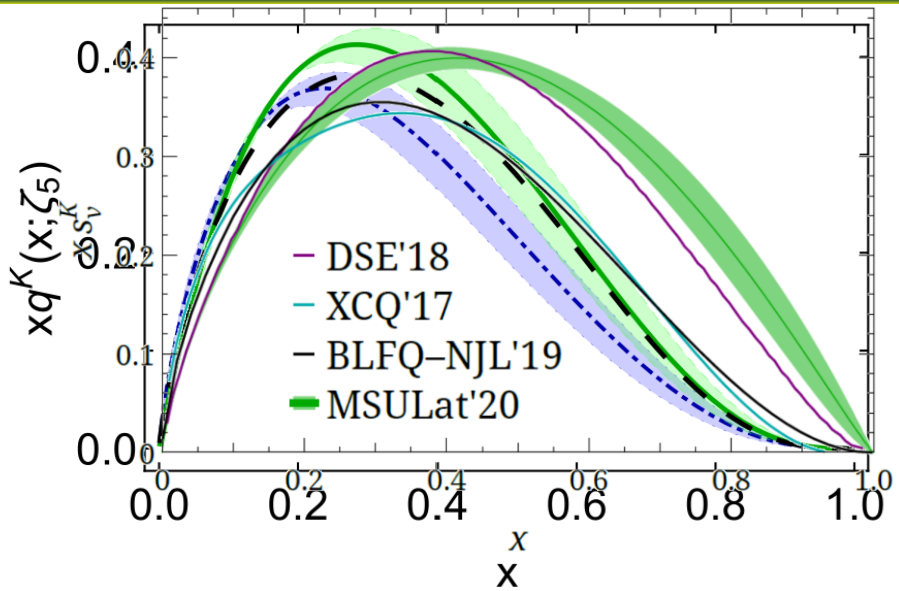
§ Multiple quasi-PDF data sets studied (meson DA, gluon/kaon PDFs)

- Example kaon PDF at 220-MeV ensemble



(plot by Rui Zhang)





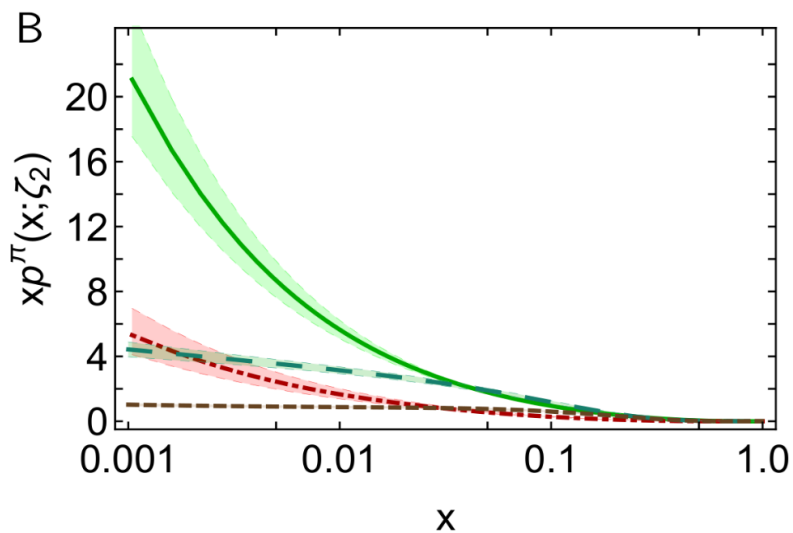


FIG. 13. Solid green curve – kaon's valence $\bar{s}$-quark distribution defined at ζ_H by Eq. (45) and Table III–middle, evolved $\zeta_H \to \zeta_5$ using the procedure explained in Sec. IV A, including the splitting function modification in Eq. (58a) with $\sigma_{ss} = 1$. Dot-dashed blue curve – kaon's valence u-quark distribution, unchanged from Fig. 11. Dashed black curve – central $\bar{s}$-quark distribution from Fig. 11, *i.e.* obtained with mass-independent evolution. (The bands bracketing the central DFs reflect the uncertainty in the $k^2 = 0$ value of the PI charge, Fig. 1.)

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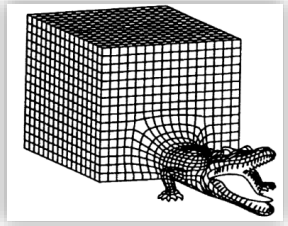
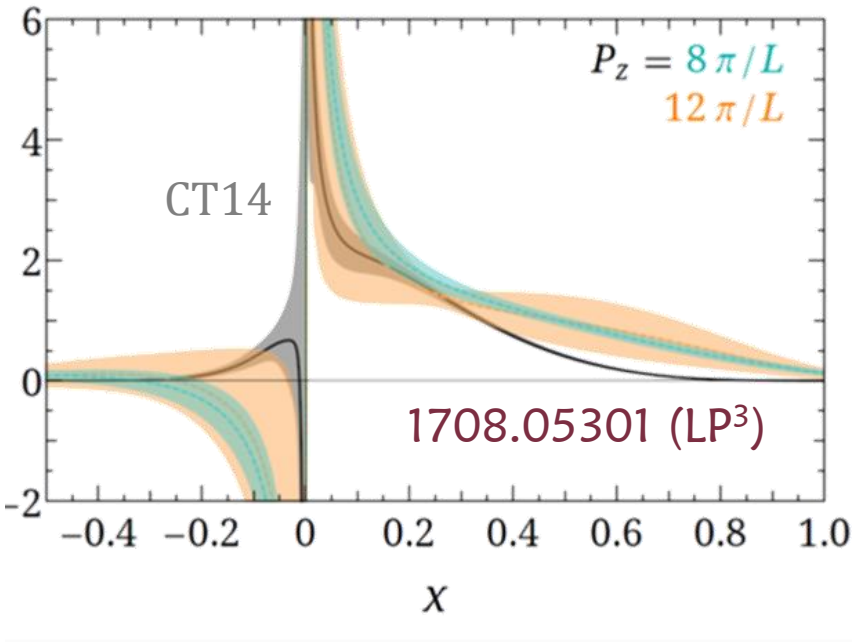
References	Sea quarks	Valence quarks	$\langle\xi^2\rangle_\pi$	$\langle\xi^2\rangle_K$	Renormalization	a (fm)	M_π (MeV)	$M_\pi L$
MSULat'20 (this work)	2+1+1f HISQ	clover	0.244(30)	0.198(16)	RI-MOM	0.06–0.012	310–690	4.4–10
RQCD'19 [106]	2+1f clover	clover	0.234(6)(6)	0.231(4)(6)	RI'-SMOM	0.039–0.086	130–420	3.6–6.4
RQCD'17 [105]	2+1f clover	clover	0.2077(43)	N/A	RI'-SMOM	0.086	222–420	3.9–5.8
RQCD'15 [104]	2f clover	clover	0.236(4)(4)	N/A	RI'-SMOM	0.06–0.08	150–260	3.4–4.8
RBC/UKQCD'10 [103]	2+1f DWF	DWF	0.28(1)(2)	0.26(1)(2)	RI'/MOM	0.11	330–670	4.5–9.2
QCDSF'07 [102]	2f clover	clover	0.260(39)	0.260(6)	RI/MOM	0.06–0.085	580–1170	4.6–9.6

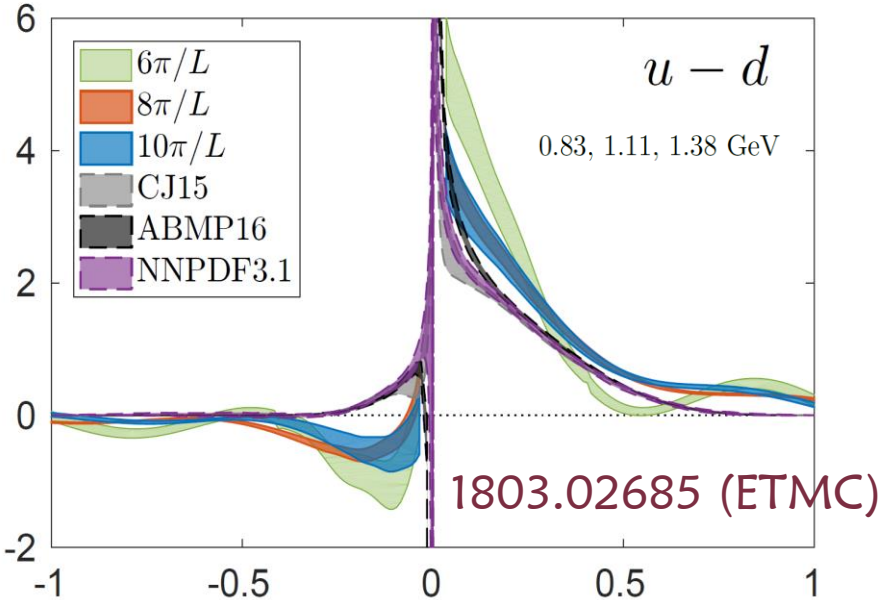
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed



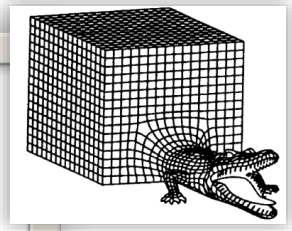


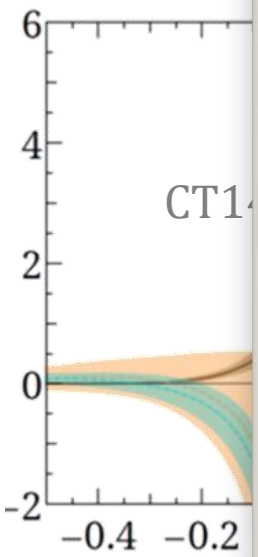


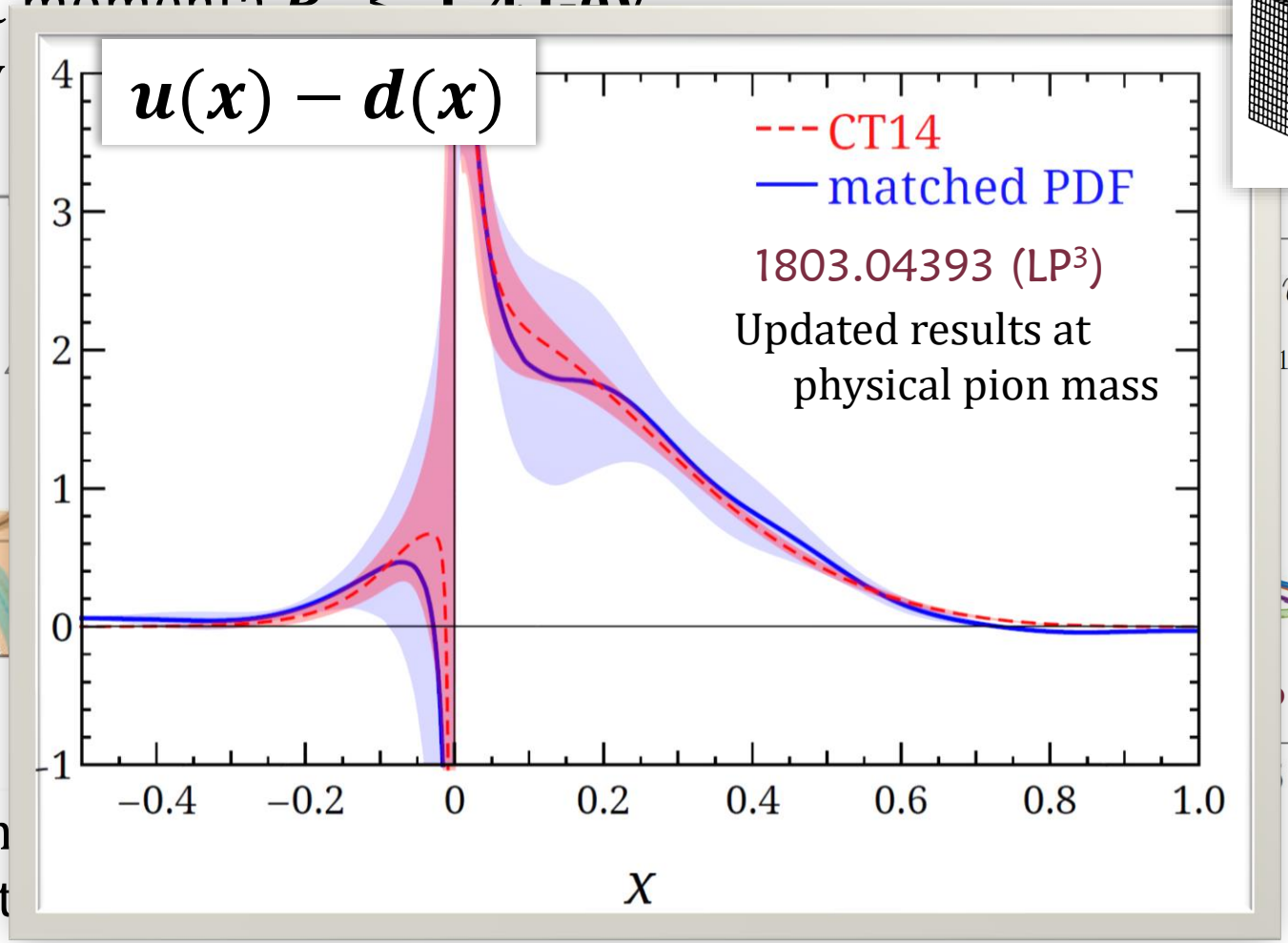
Physical Pion Mass Results

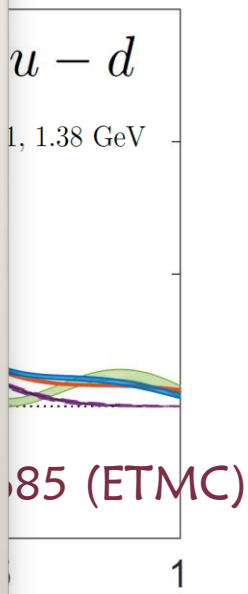
§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z < 1.4$ GeV
- Study









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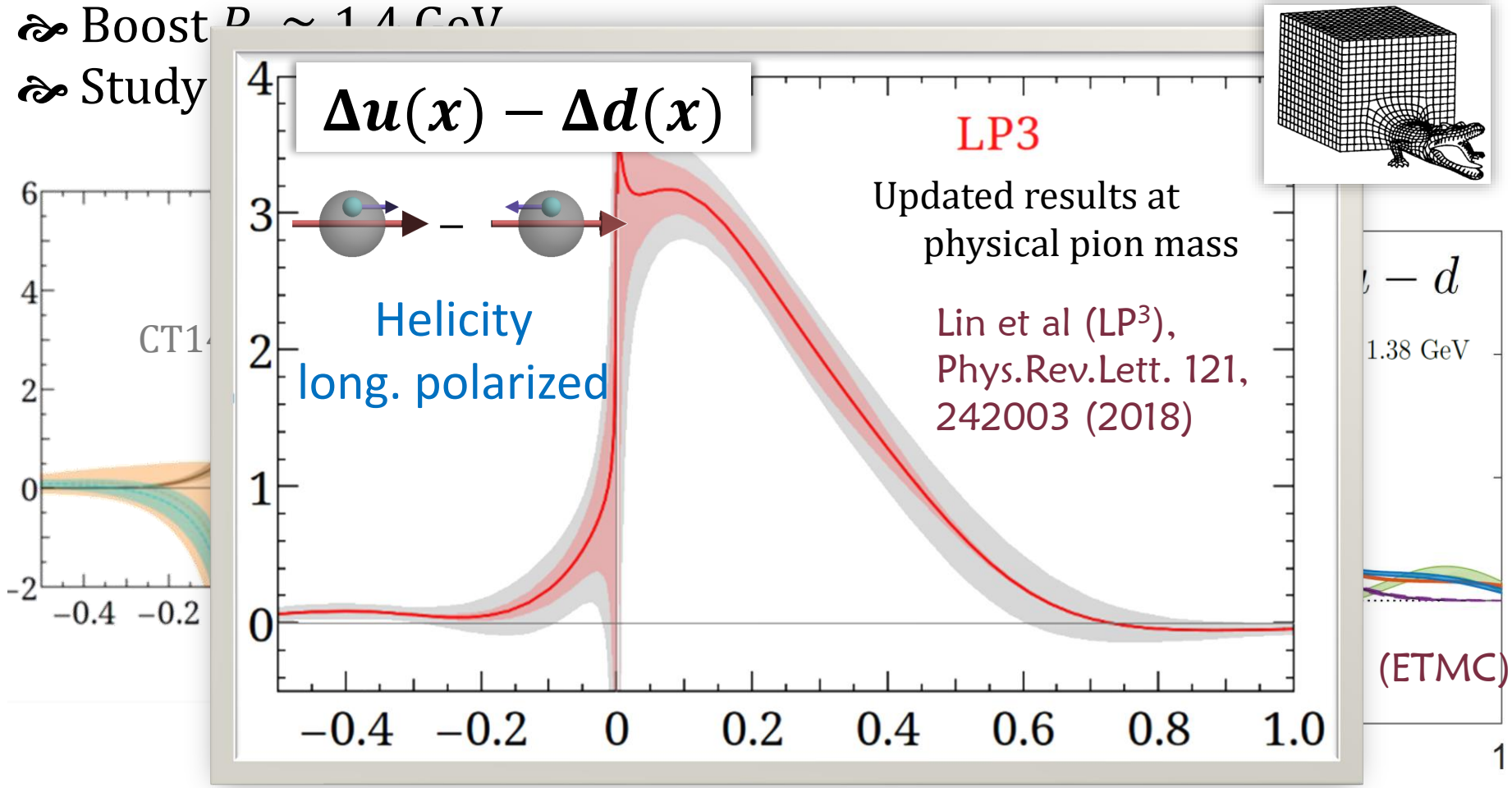
less likely to exactly coincide with global fits.

Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study



Physical Pion Mass Results

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