Pion and Kaon in Basis Light-front Quantization

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Workshop on Pion and Kaon Structure Functions at the EIC 06/04/2020

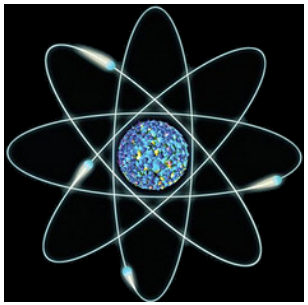
Outline

- Basis Light-front Quantization approach
- Application to π and K
  - Leading Fock sector (based on NJL interacton)
  - With one dynamical gluon
- Summary and Future Plan

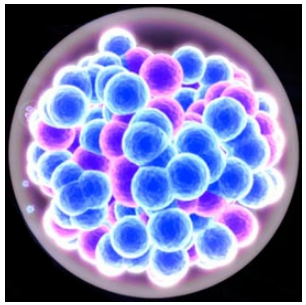
Hamiltonian Formalism

- Schrödinger equation universally describes different physics：

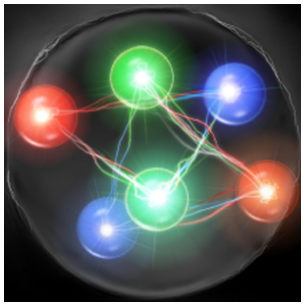
$$H|\psi\rangle = E|\psi\rangle$$



atom



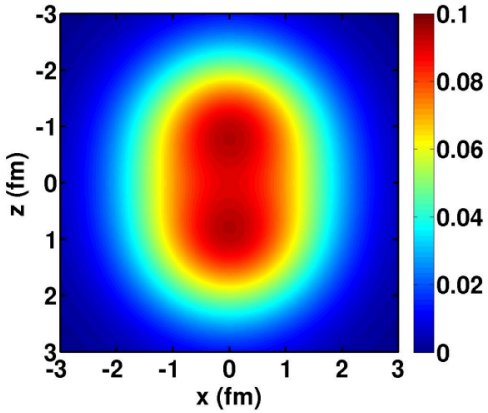
nucleus



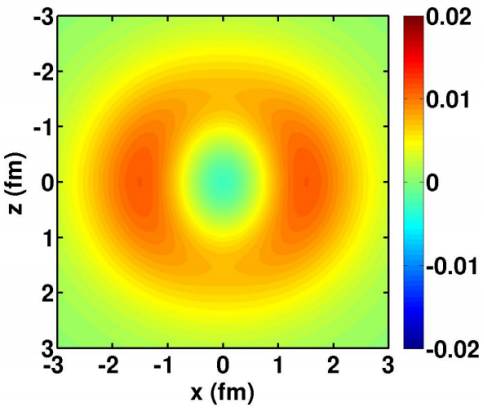
hadron

Nonrelativistic, few-body **Nonrelativistic, many-body** **Relativistic, many-body**

- Wave functions encode full information of the system



Proton density



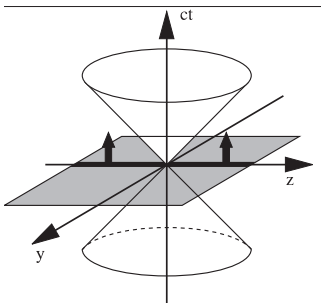
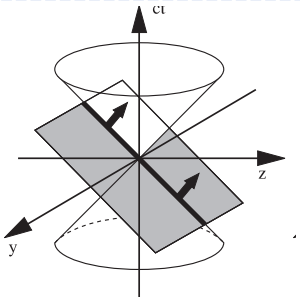
Neutron – proton density

^{9}Be

Adapted from Yang Li's talk

Light-front Quantization

[Dirac, 1949]

Equal time quantization	Light-front quantization
$t \equiv x^0$	$t \equiv x^+ = x^0 + x^3$
	
x^1, x^2, x^3	$x^- = x^0 - x^3,$ $x^\perp = x^{1,2}$
$P^0, \vec{P}$	$P^- = P^0 - P^3,$ $P^+ = P^0 + P^3, P^\perp = P^{1,2}$
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$	$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + P_\perp^2}{P^+}$

- Not just a coordinate transformation.
- New theory !!!

Advantages:

- Frame-independent wave functions
- Simple vacuum structure
- No square root in Hamiltonian P^-

Basis Light-front Quantization

[Vary et al, 2008]

- Nonperturbative eigenvalue problem

$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$

  - P^-: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - P^-_β: eigenvalue for $|\beta\rangle$

- Evaluate observables for eigenstate

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

- Fock sector expansion
  - Eg. $|\boldsymbol{\pi}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}gg\rangle + d|q\bar{q}q\bar{q}\rangle + \ldots$

- Discretized basis
  - Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$.
  - Longitudinal: plane-wave basis, labeled by k.
  - Basis truncation:

$$\sum_i(2n_i + |m_i| + 1) \leq N_{max},$$
$$\sum_i k_i = K.$$

N_{max}, K are basis truncation parameters.

Large N_{max} and K : High UV cutoff & low IR cutoff

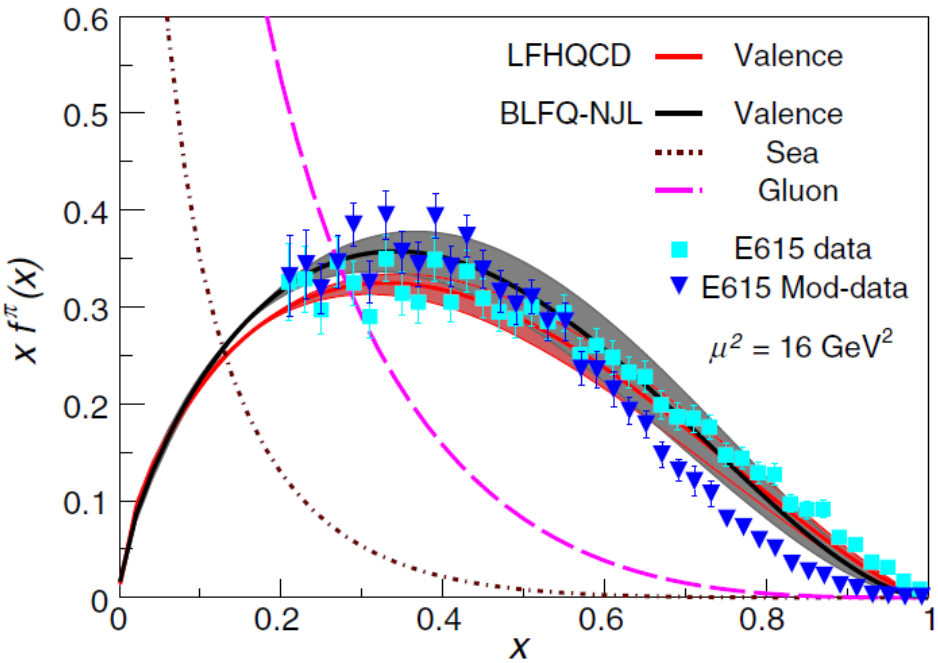
Application to π and K

PDF from BLFQ and QCD Evolution for Light Mesons

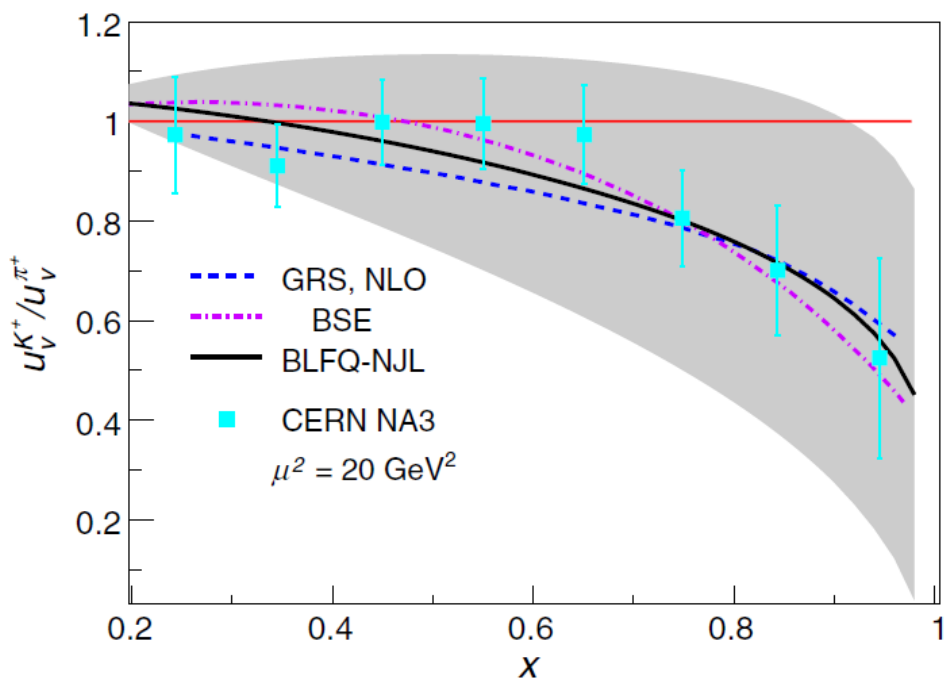
$$H_{\text{eff}} = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

PDF for the valence quark result from the light-front wave functions obtain by diagonalizing the effective Hamiltonian.

Pion PDF



Valence *u* PDF Kaon/Pion

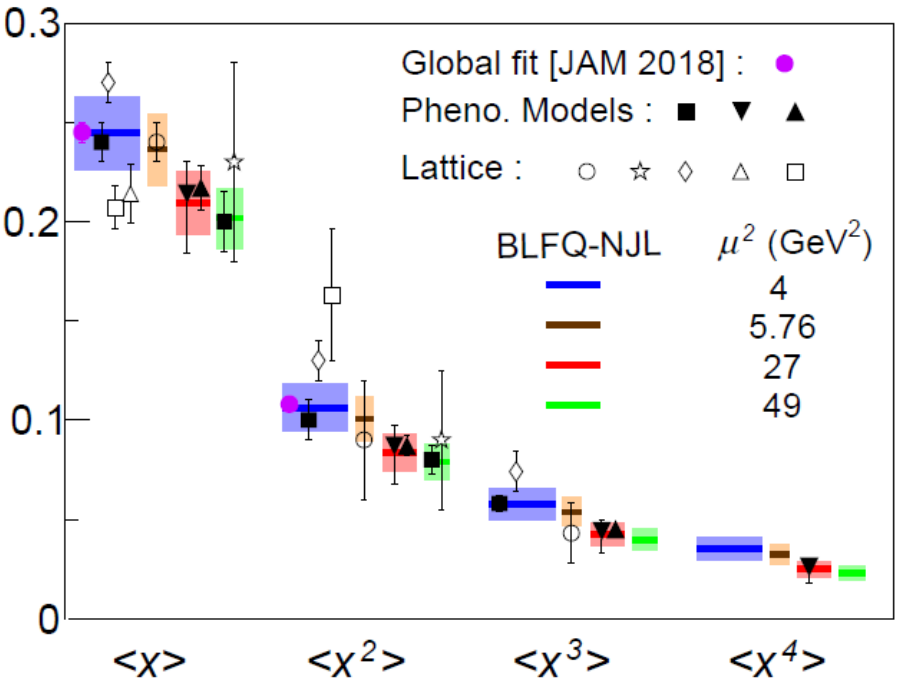


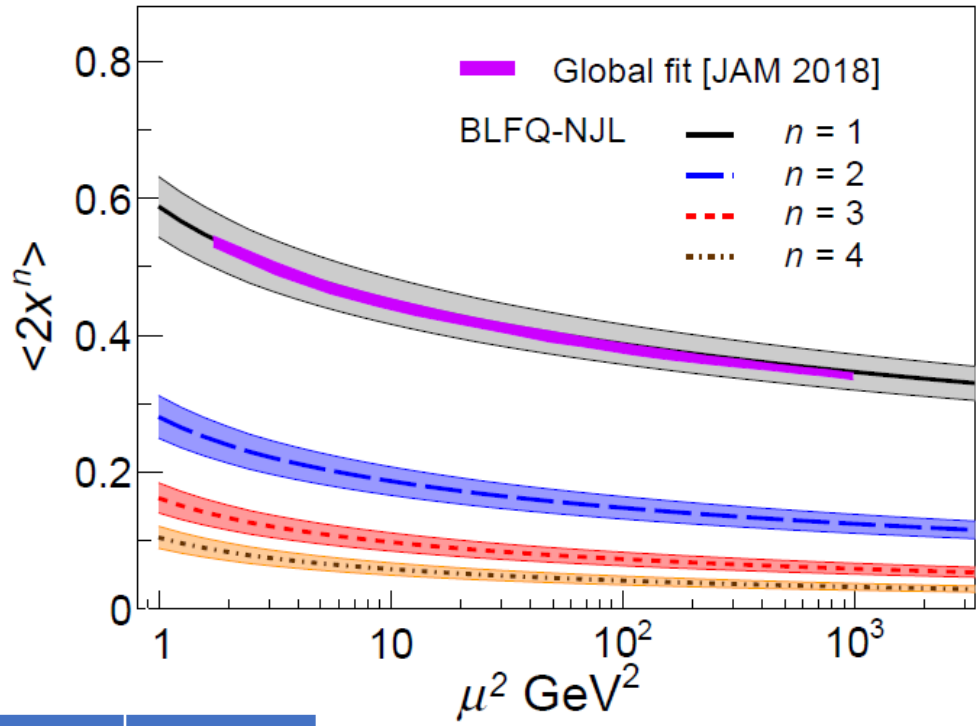
[Lan, Mondal, Jia, Zhao, Vary, PRL122, 172001(2019)]

Agree with experimental results

The moments of pion valence quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x, \mu^2),\ \ n = 1, 2, 3, 4.$$





$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ-NJL	**0.489**	**0.398**	**0.113**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

Agree with other results

[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

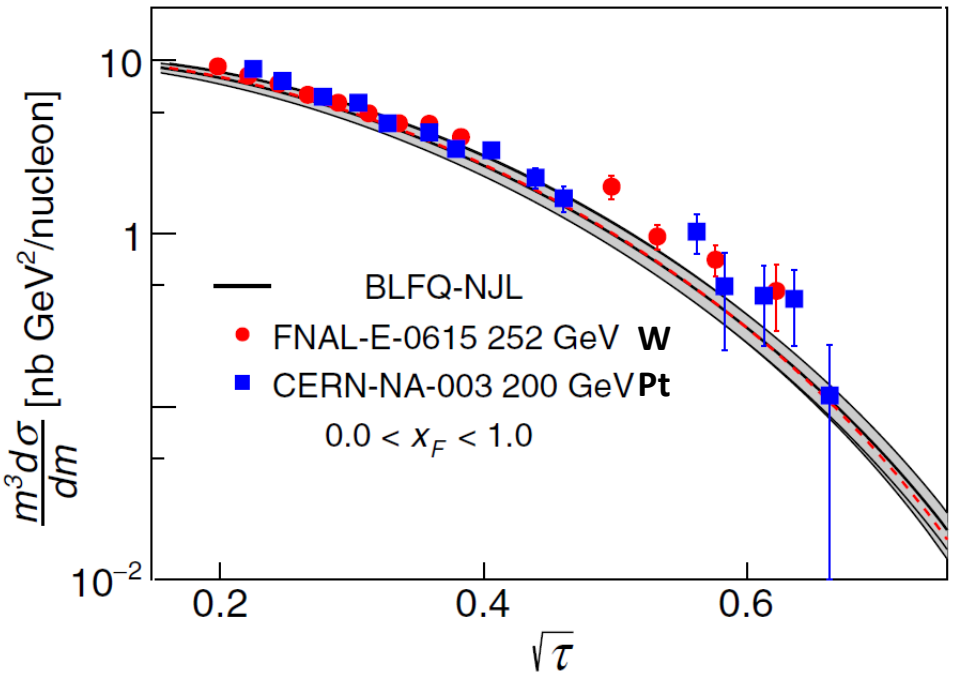
Drell-Yan cross section

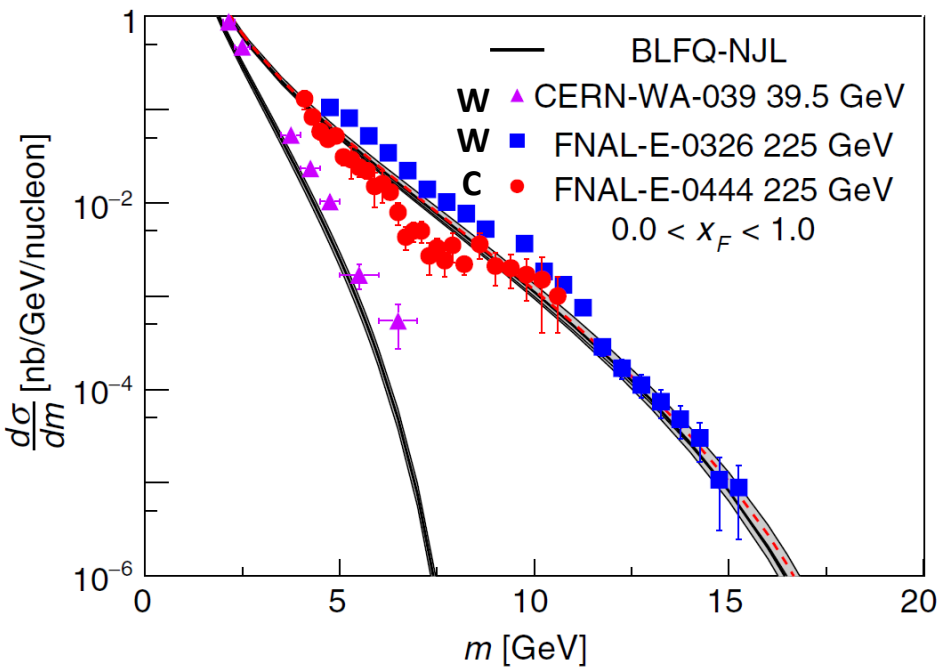
$$\pi^- \text{ Nucleus} \to \mu^+ \mu^- \text{ X}$$

[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

[nCTEQ 2015]

$$\frac{m^3 d^2\sigma}{dm dY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$





Agree with experimental data (**FNAL E615, 326, 444, & CERN NA3, WA-039**).

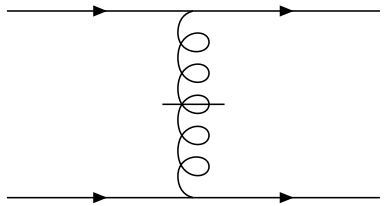
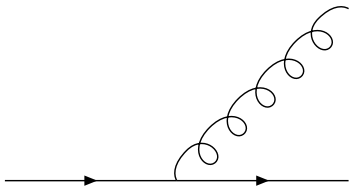
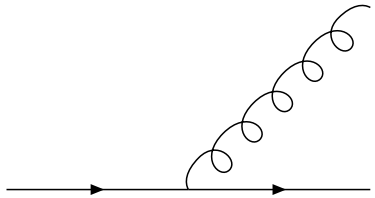
[Lan, Mondal, Jia, Zhao, Vary, PRD101,034024(2020)]

$$|\pi\rangle = |q\bar{q}\rangle + \cdots$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Interaction Part of Hamiltonian

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$\boldsymbol{H}_{\mathbf{int}}$	$\lvert q\bar{q}\rangle$	$\lvert q\bar{q}g\rangle$
$\langle q\bar{q}\rvert$		
$\langle q\bar{q}g\rvert$		0

$$P^- = \frac{\overrightarrow{k_\perp^2} + m_q^2}{x} + \frac{\overrightarrow{k_\perp^2} + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^{\,2} - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + \boldsymbol{H}_{\mathbf{int}}$$

Pion *Spectrum, DC, Radii*

$\pi^+ \longrightarrow \mu^+\nu$ $\pi^- \longrightarrow \mu^-\bar{\nu}$

$$\langle r_c^2 \rangle = 6\frac{\partial}{\partial Q^2}F(Q^2)|_{Q^2\to 0}$$

$$F(Q^2) = \sum_i \int dx_i H(x_i, 0, Q^2)$$

$$\langle 0|\bar{\Psi}(0)\gamma^+\gamma_5\Psi(0)|\Psi_\pi(P,s)\rangle = iP^+ f_P,$$

$$\frac{f_P}{2\sqrt{2N_C}} = \int [d^3k]\Psi_2,$$

	m_{π^+} [MeV]	m_{ρ^+} [MeV]	f_{π^+} [MeV]	f_{ρ^+} [MeV]	$\sqrt{\langle r_c^2\rangle}\|_{\pi^+}$[fm]	norm1
SetI	139.50	775.26	230.1	233.8	0.747	0.639
Set II	139.5	775.2	161.3	154.7	0.489~1.31	0.522
PDG	***139.57***	***775.26±0.25***	***130.2±1.7***	***221±2***	***0.672±0.008***	
BLFQ-NJL	*139.57*	*775.23±0.04*	*202.10*	*100.12*	*0.68±0.05*	

[Tanabashi, et al, PRD(2018)]

[Jia, Vary, PRC(2018)]

Preliminary

Set I

$N_{\max} = 8, K_{\max} = 9, M_J = 0$
$m_q = 0.33$ GeV, $m_g = 0.60$ GeV,
$\kappa = 0.77$ GeV, $b = 0.49$ GeV,
$\alpha = 0.284$, $m_f = 3.38$ GeV

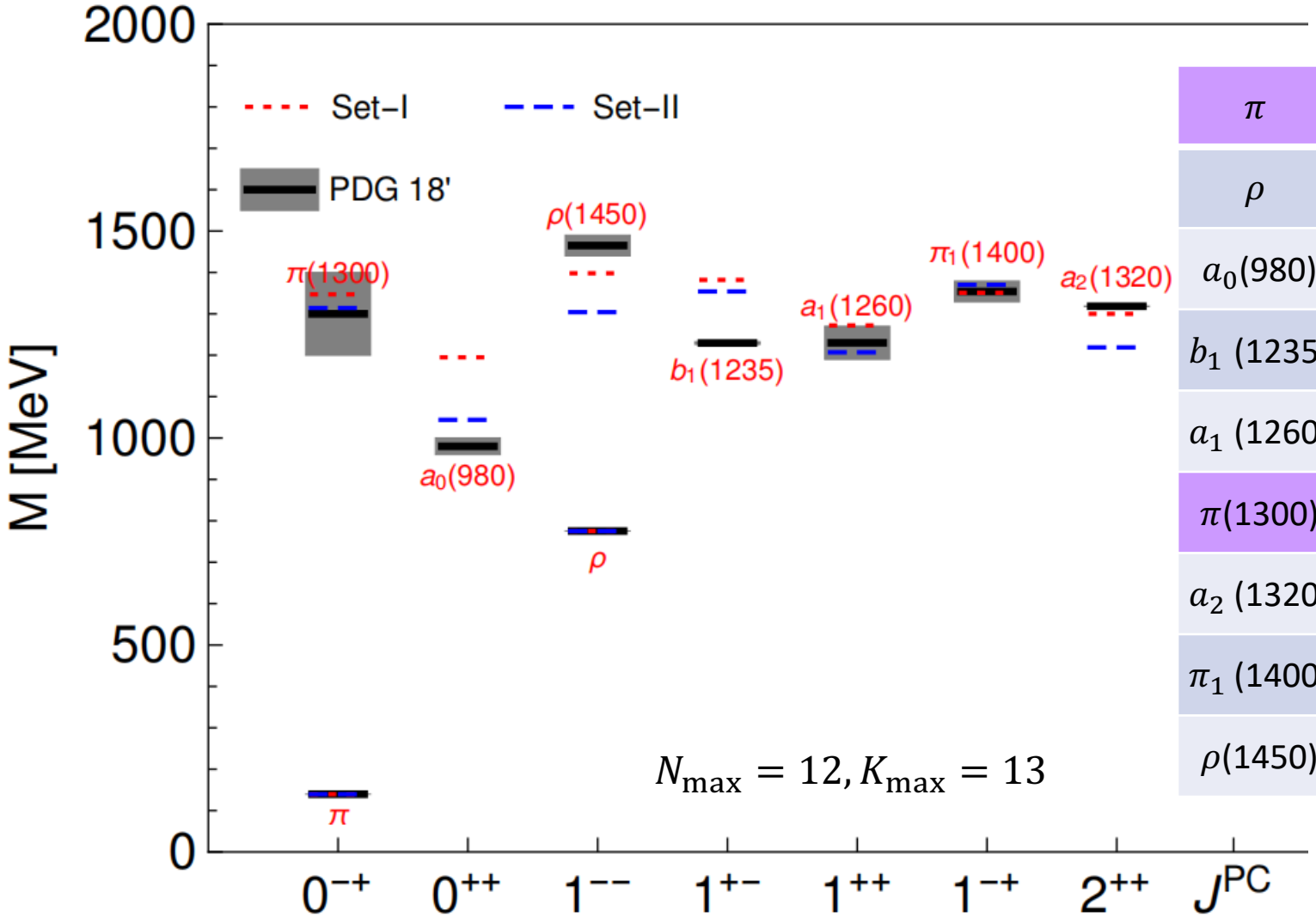
Set II

$N_{\max} = 12, K_{\max} = 13, M_J = 0$
$m_q = 0.39$ GeV, $m_g = 0.56$ GeV,
$\kappa = 0.84$ GeV, $b = 0.33$ GeV,
$\alpha = 0.318$, $m_f = 4.80$ GeV

[Lan, et al, in preparation]

Spectrum

$\pi^+ \longrightarrow \mu^+\nu$ $\pi^- \longrightarrow \mu^-\bar{\nu}$



	norm1	DC[MeV]
π	0.522	161
ρ	0.523	154
$a_0(980)$	0.376	0
b_1 (1235)	0.299	0
a_1 (1260)	0.333	16
$\pi(1300)$	0.259	61
a_2 (1320)	0.325	0
π_1 (1400)	0.001	0
$\rho(1450)$	0.311	55

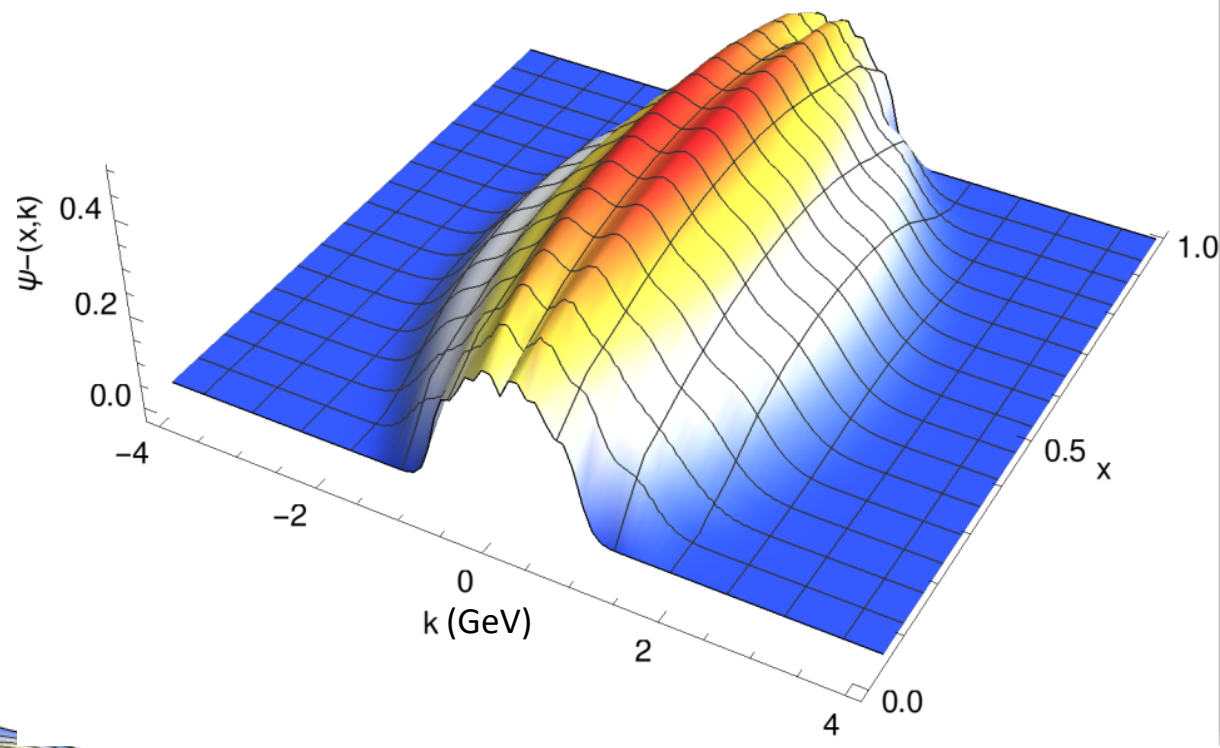
$m_J = \sum_i (m_i + s_i)$ $m_P = (-i)^{2J} P$

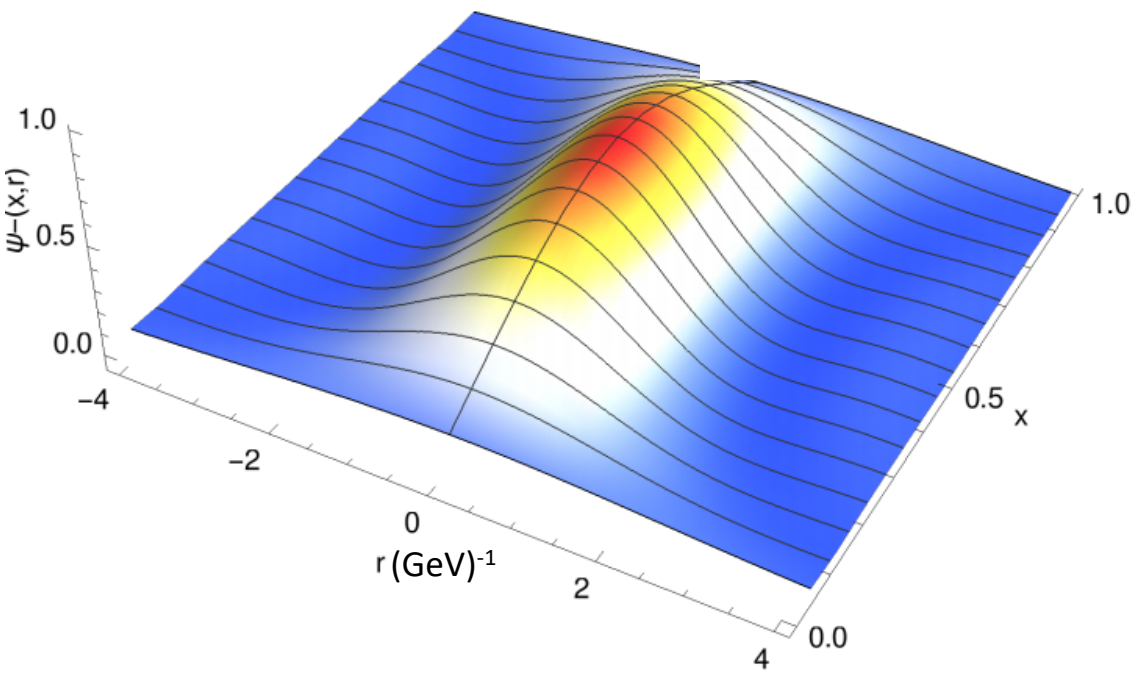
[Lan, et al, in preparation]

Preliminary

Pion Wavefunction
(↑↓- ↓↑ component)

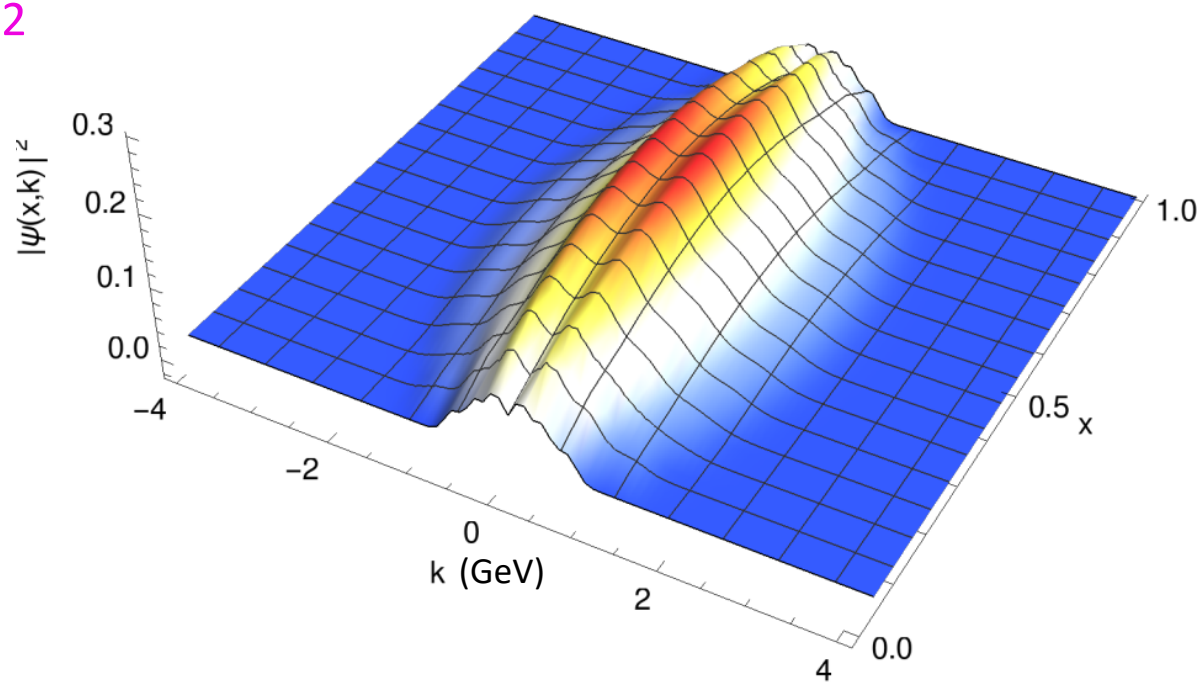
Momentum space

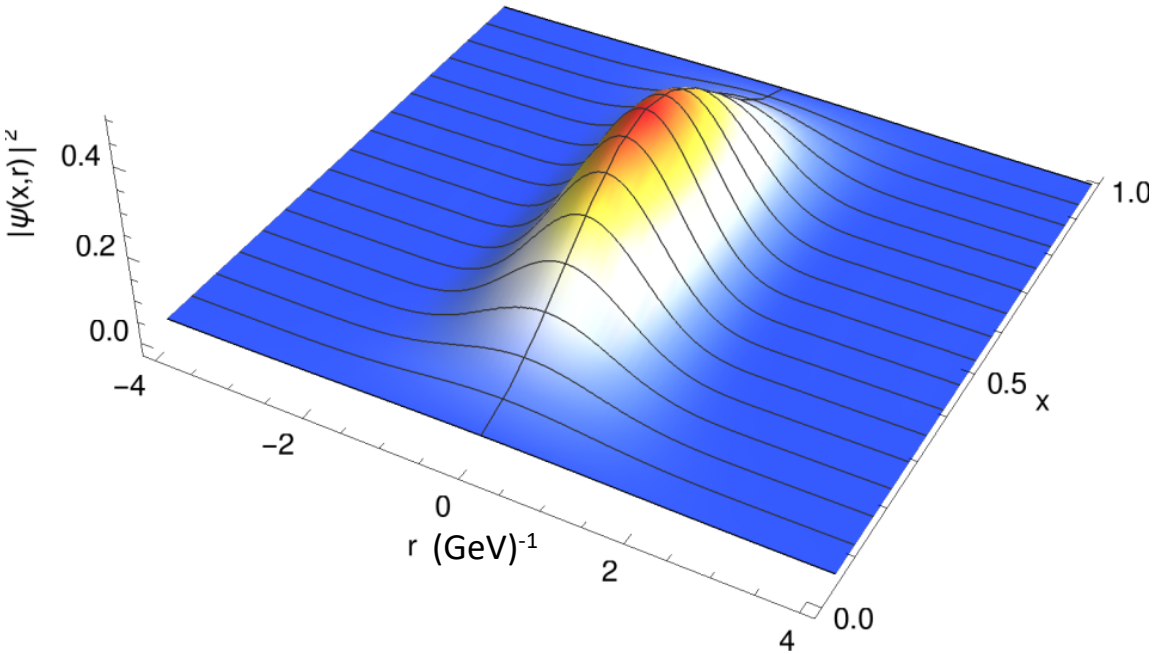




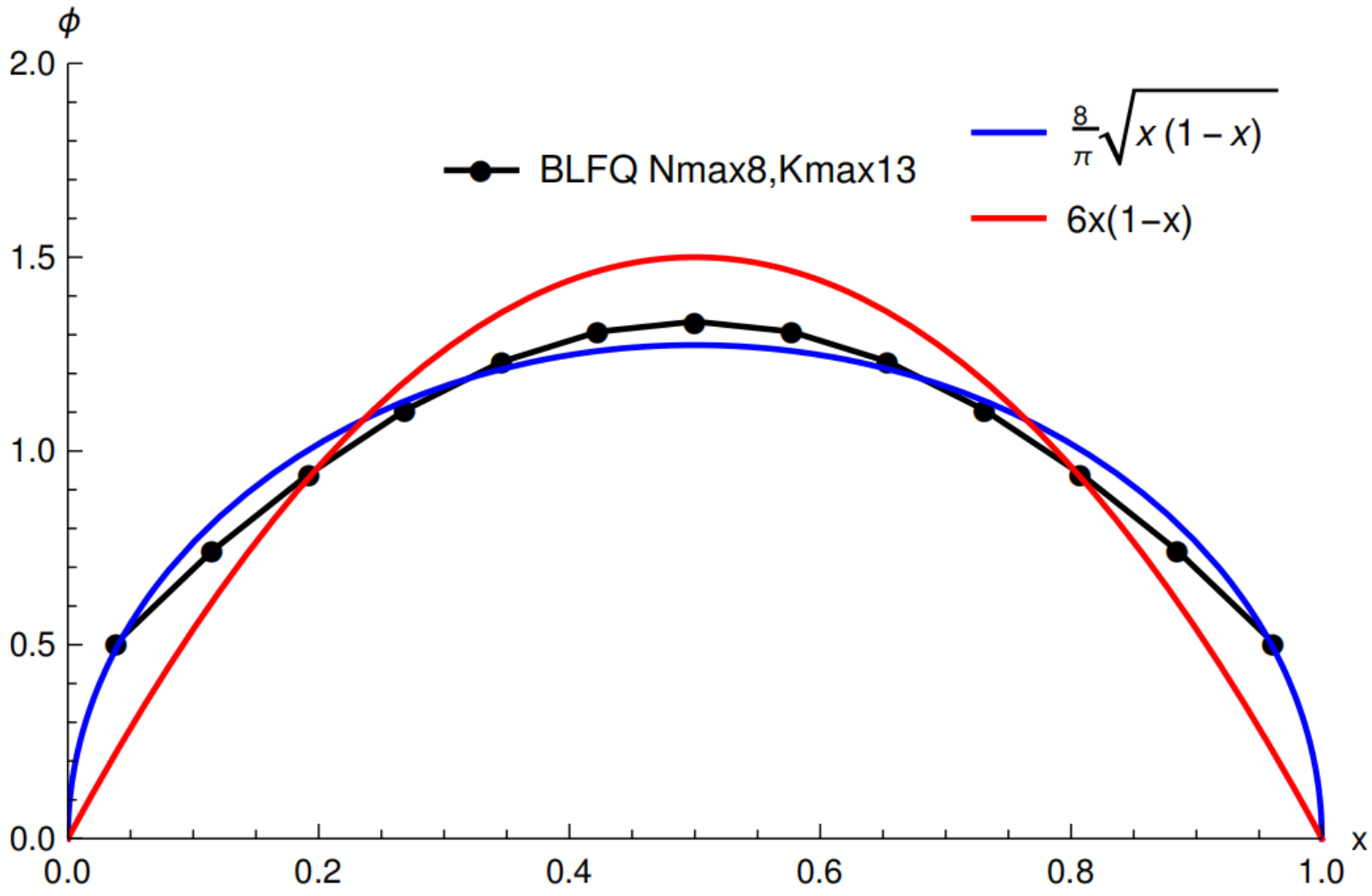
Coordinate space

|Pion Wavefunction|^2
(↑↓- ↓↑ component)

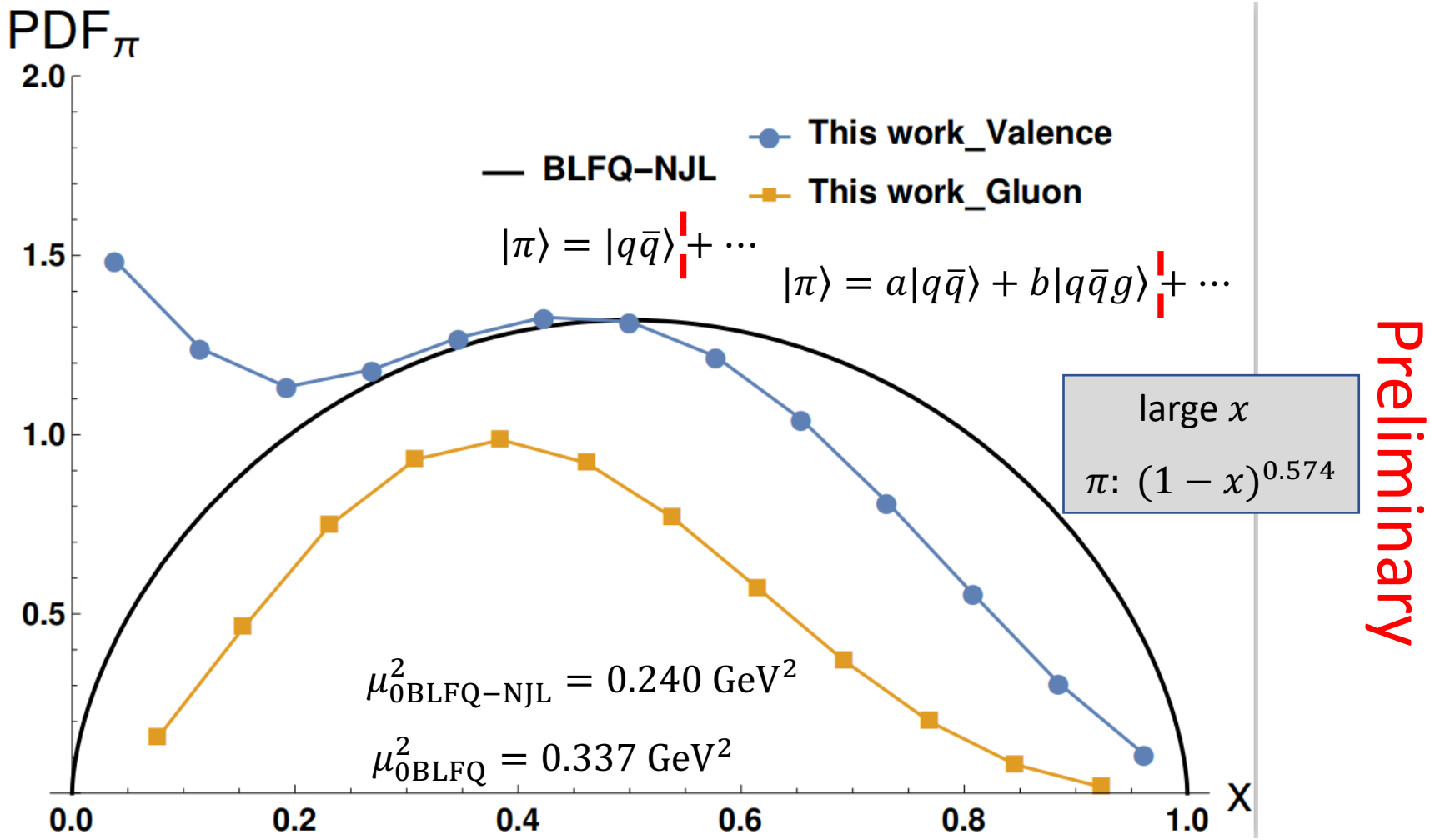




PDA


ϕ
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
BLFQ Nmax8,Kmax13
$\frac{8}{\pi}\sqrt{x(1-x)}$
6x(1−x)

Pion initial PDF



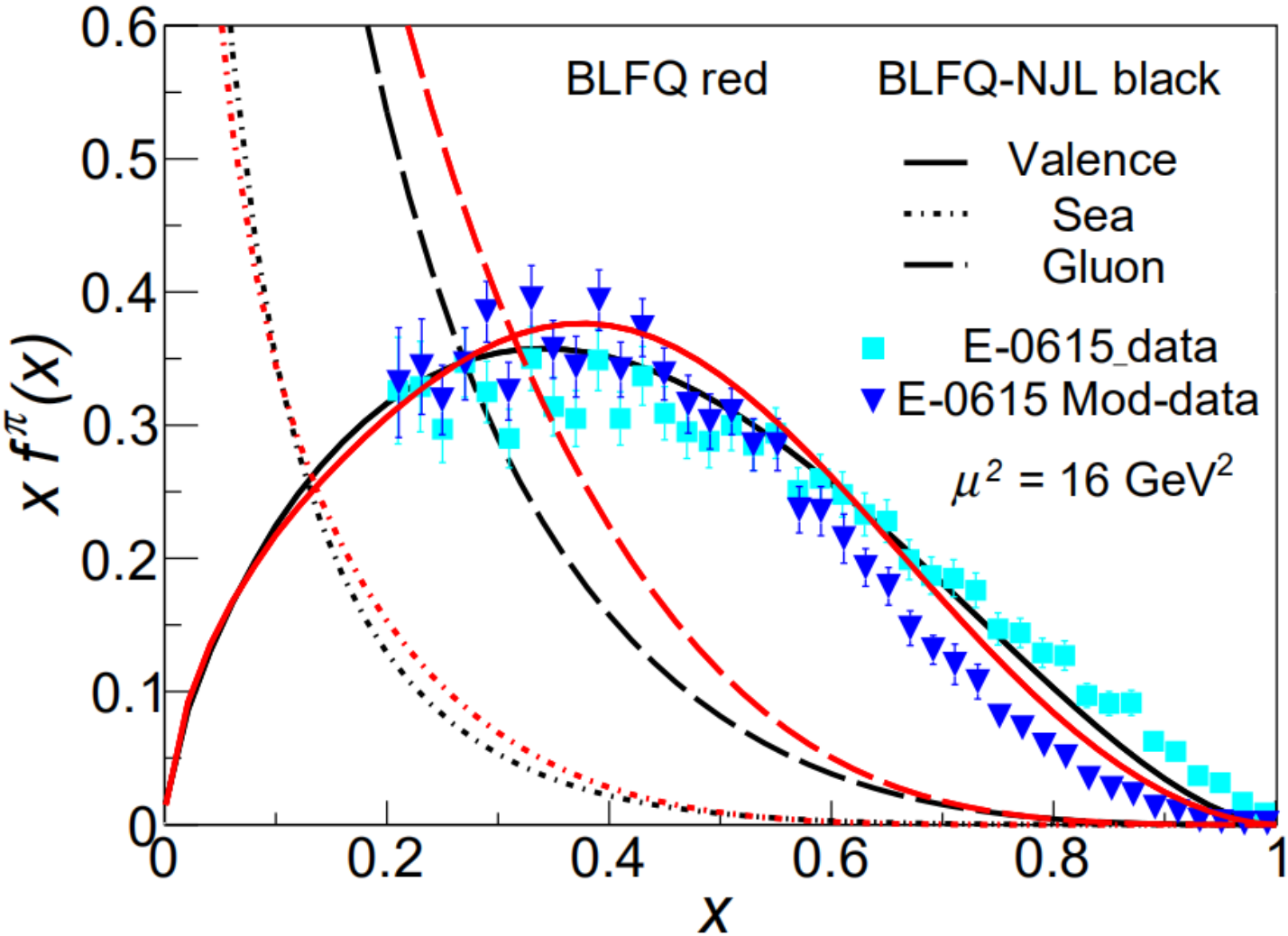
Valence close to BLFQ-NJL result at large x, more than BLFQ-NJL result at small x; we have gluon in initial PDF.

[Lan, et al, in preparation]

Pion PDF

[Lan, et al, in preparation]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

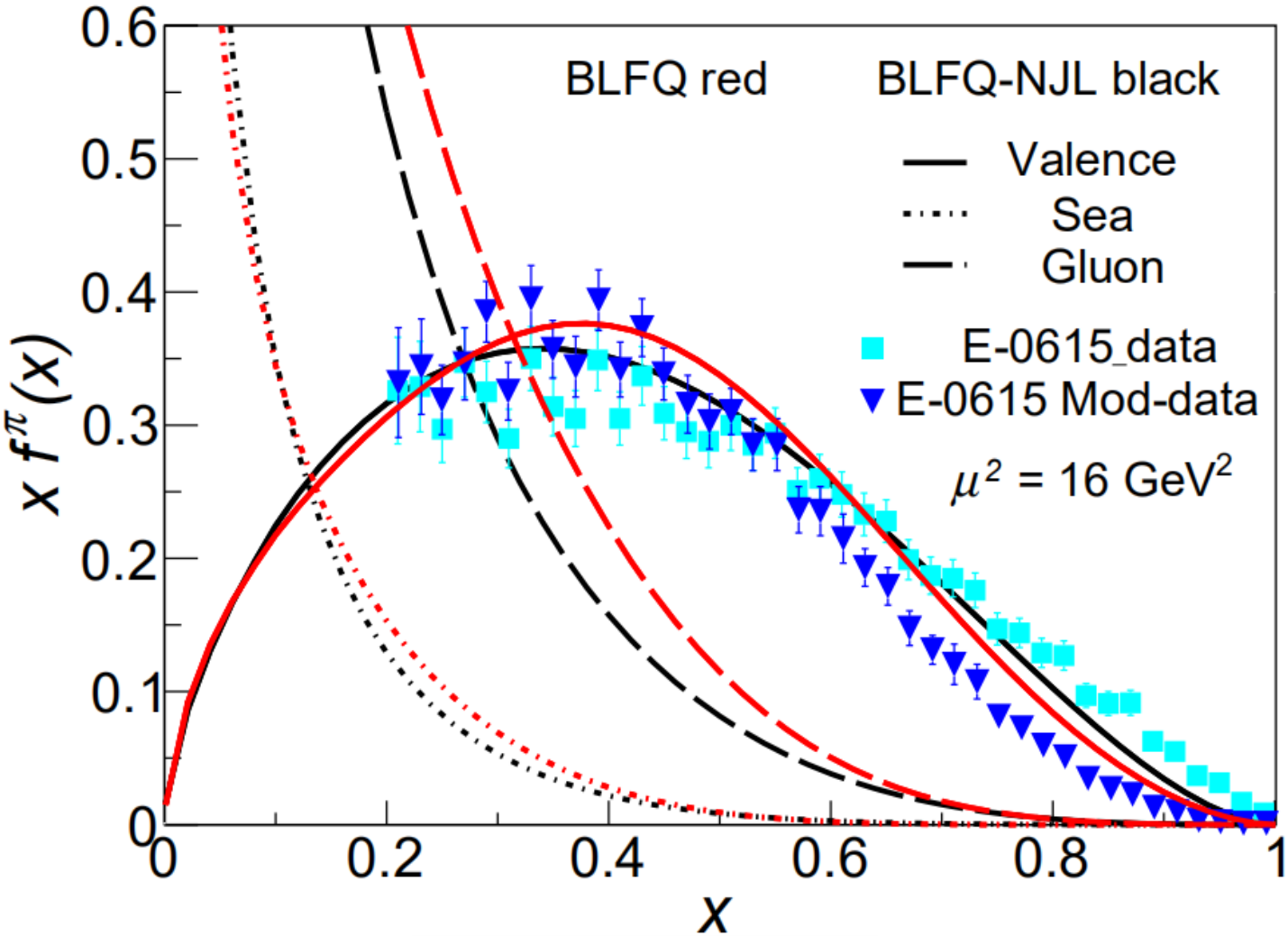


Preliminary

- Valence agrees with E615 results
- More gluon than BLFQ-NJL result
- Sea close to BLFQ-NJL result

Pion PDF

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



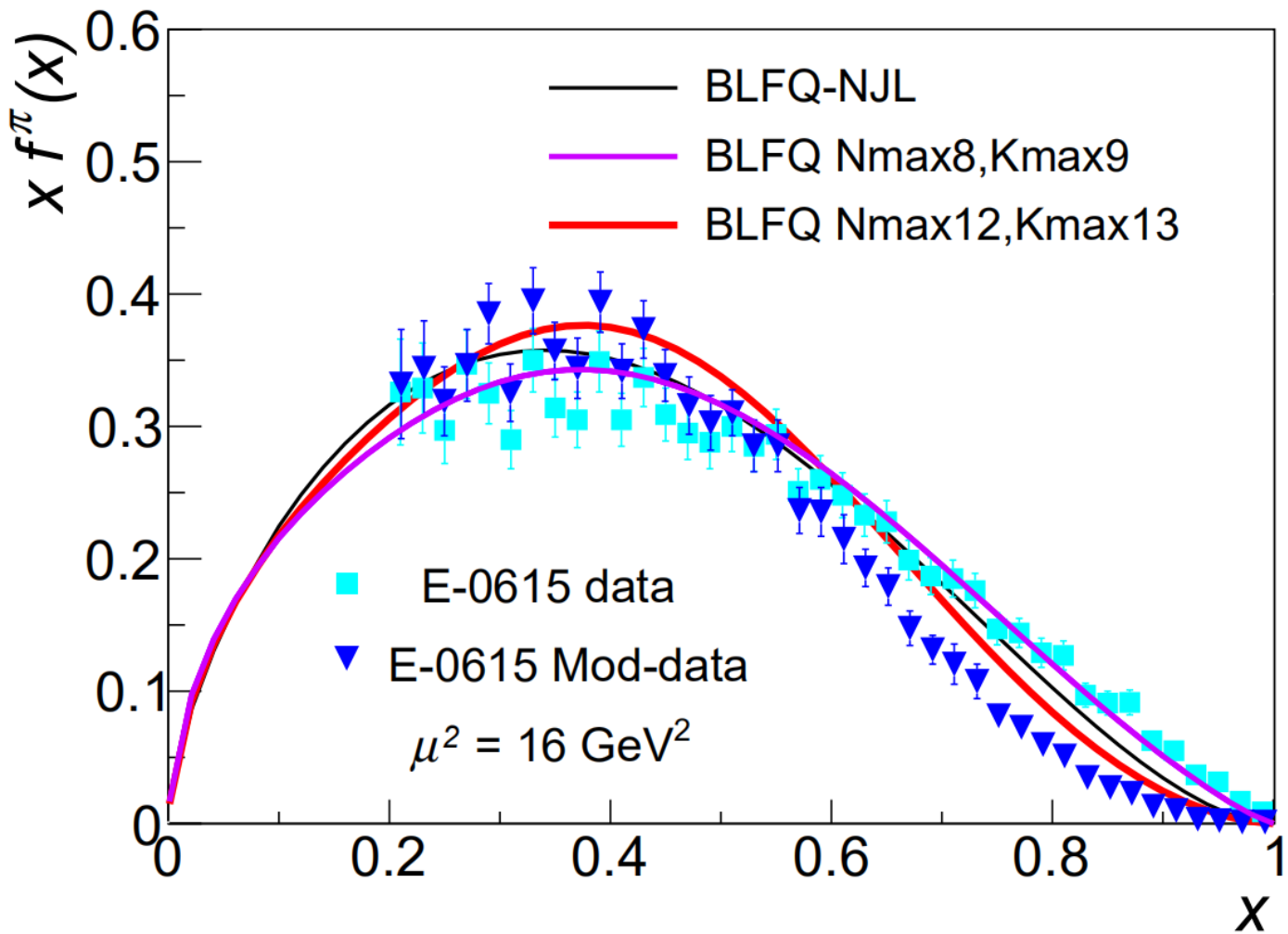
Preliminary

$\langle x \rangle$ @ 4 GeV^2	Valence	Gluon	Sea
BLFQ	**0.492**	**0.414**	**0.093**
[Ding *et. al.*, BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

[Lan, et al, in preparation]

Pion PDF

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



Preliminary

- Larger basis size ⟹ more realistic pion decay constant
  ⟹ narrower in longitudinal momentum ⟹ prefer Mod-data

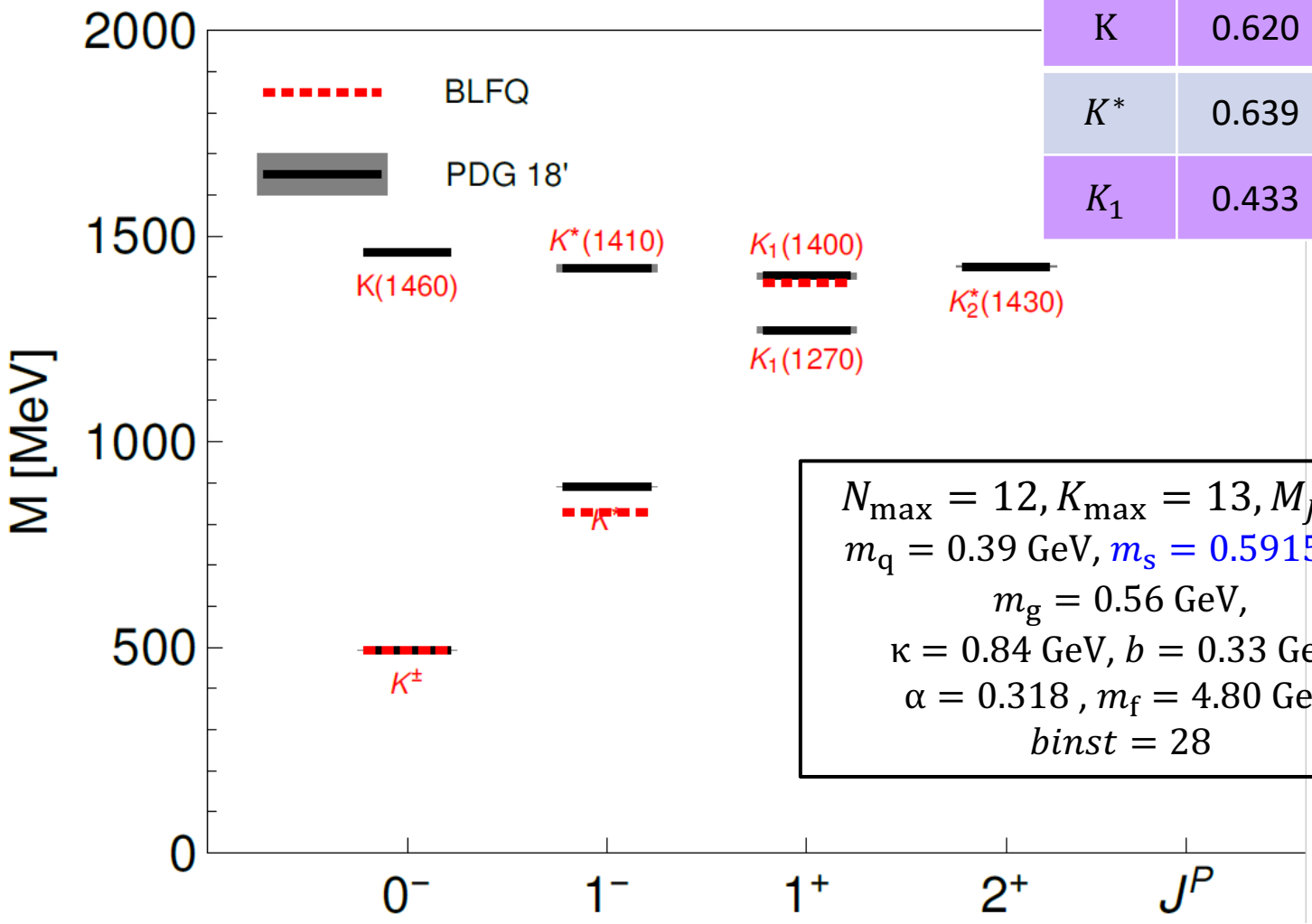
[Lan, et al, in preparation]

Light meson *in progress*

Kaon *Spectrum*

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$

	Norm1	DC[MeV]
K	0.620	176
K^*	0.639	170
K_1	0.433	18

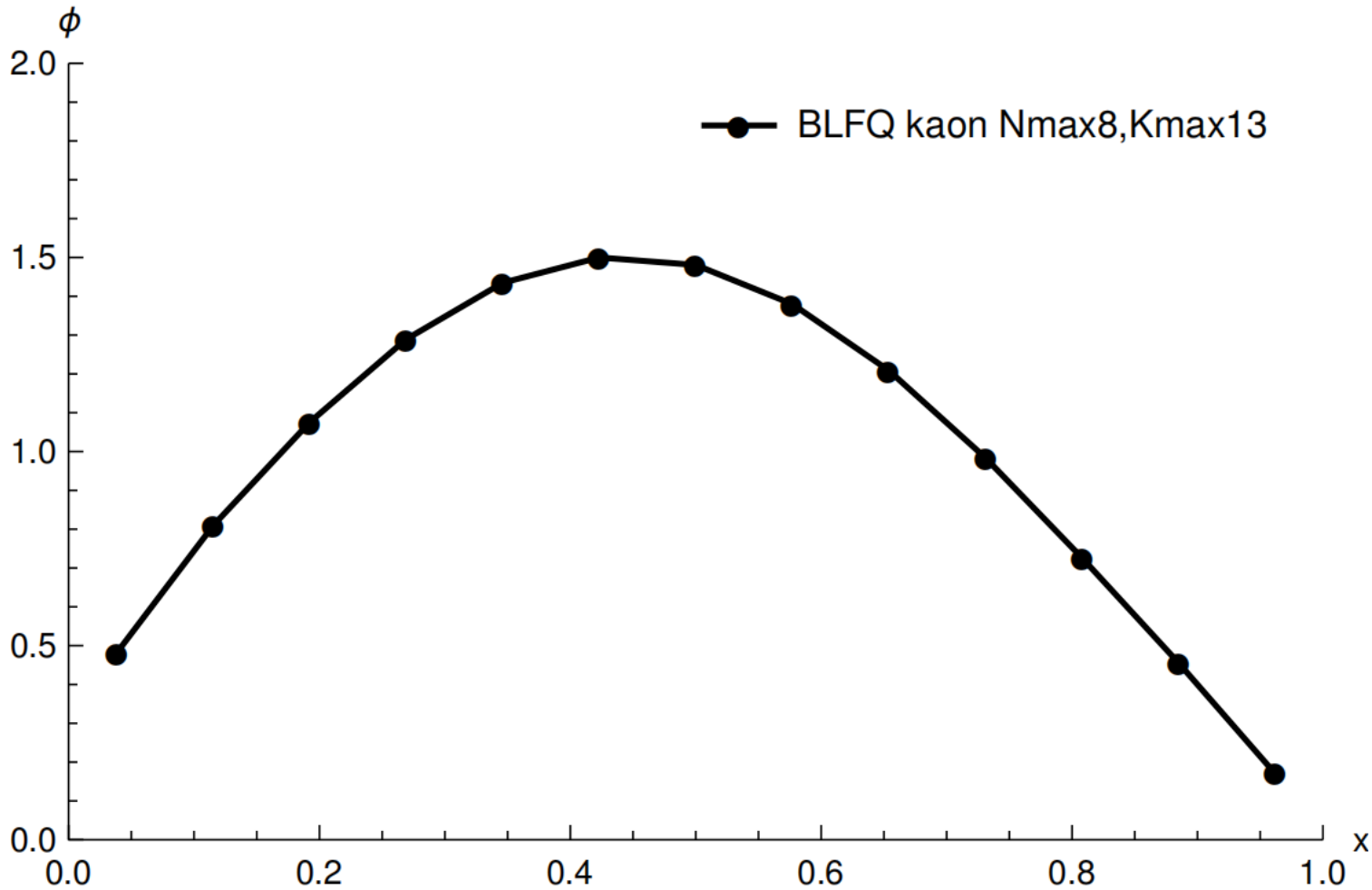


$N_{\max} = 12, K_{\max} = 13, M_J = 0$
$m_q = 0.39$ GeV, $m_s = 0.5915$ GeV
$m_g = 0.56$ GeV,
$\kappa = 0.84$ GeV, $b = 0.33$ GeV,
$\alpha = 0.318$, $m_f = 4.80$ GeV,
$binst = 28$

Preliminary

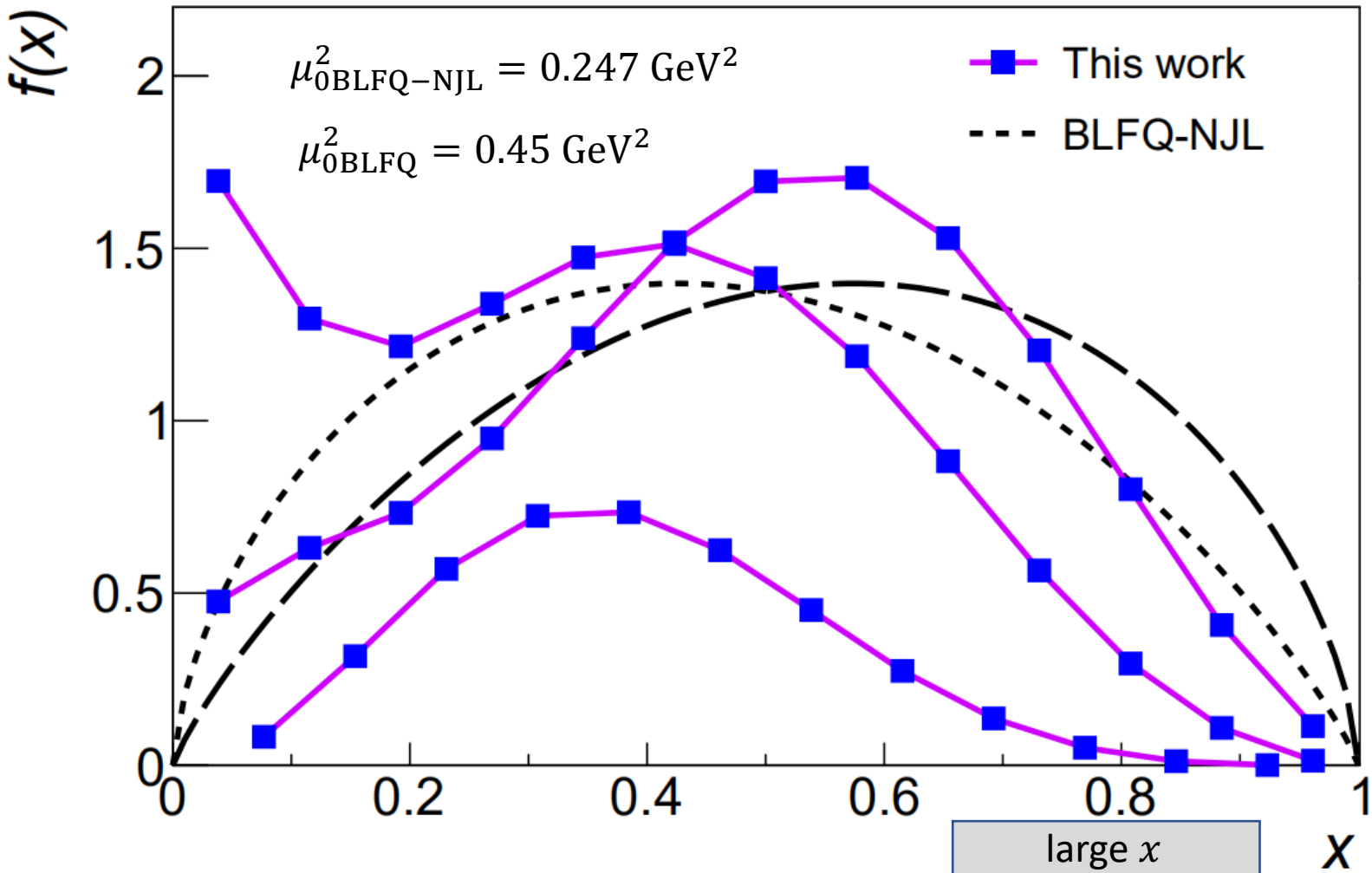
[Lan, et al, in preparation]

PDA



Kaon initial PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



large x

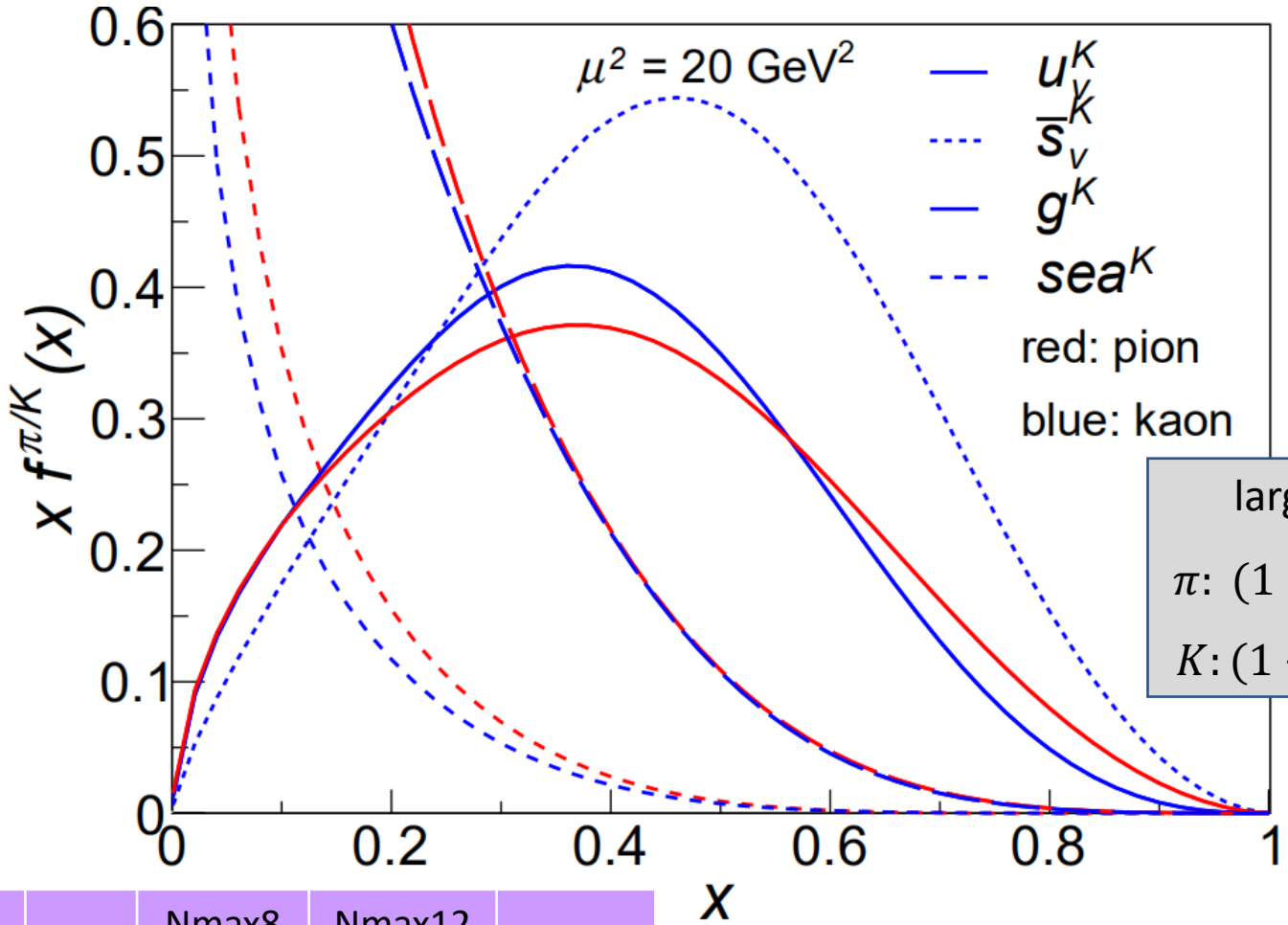
$K: (1-x)^{1.05}$

Preliminary

[Lan, et al, in preparation]

Kaon PDF

$$|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$$



large x

π: $(1-x)^{1.588}$

K: $(1-x)^{1.911}$

Preliminary

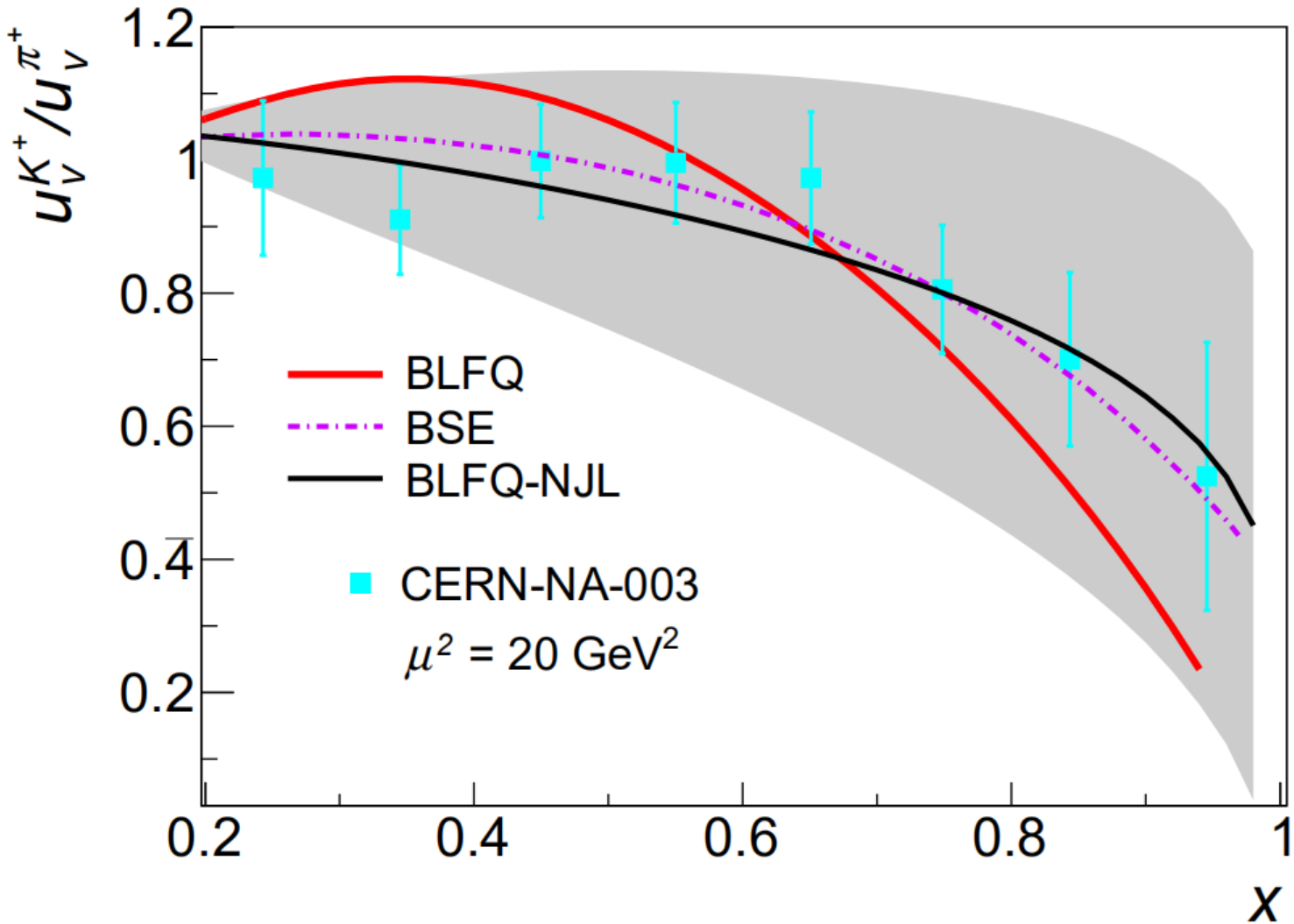
DC [MeV]	**exp.**	Nmax8 Kmax9	Nmax12 Kmax13	?
pion	**131**	233	161	?
kaon	**160**	256	176	?

[Lan, et al, in preparation]

It seems that larger Nmax/Kmax need to be considered in next step

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$

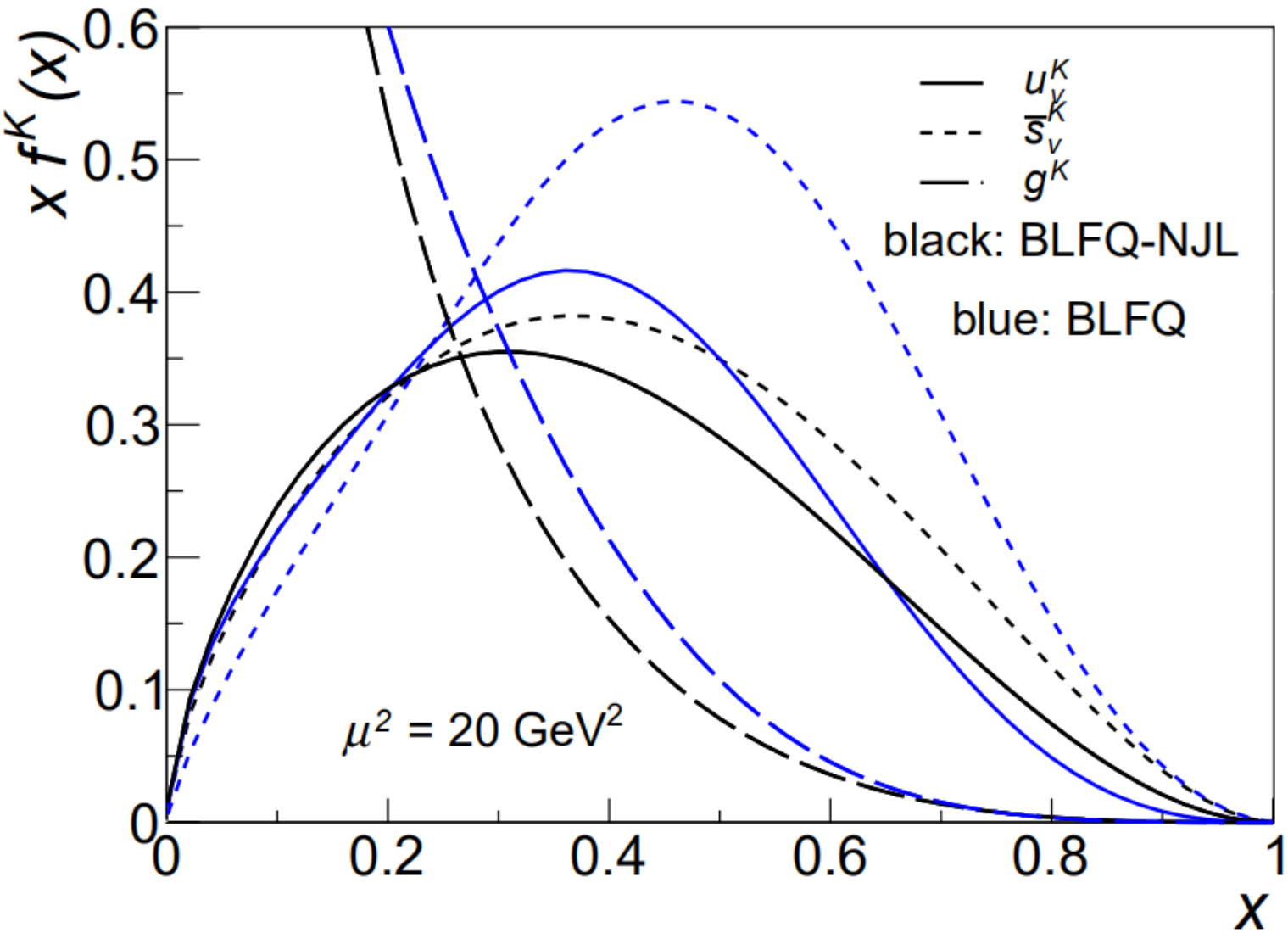


Preliminary

[Lan, et al, in preparation]

Kaon PDF

$|K\rangle = |u\bar{s}\rangle + \cdots$ **VS** $|K\rangle = a|u\bar{s}\rangle + b|u\bar{s}g\rangle + \cdots$



Preliminary

[Lan, et al, in preparation]

Conclusions

- Basis Light-front Quantization:
  - Nonperturbative approach to relativistic many-body bound states
- Light-front Hamiltonian ⟶ Wavefunction ⟶ Observables
  - Mass spectrum ⟷ structure
- Systematically expandable by including higher Fock sectors

-$|\text{Baryon}\rangle = |qqq\rangle + |qqqg\rangle + |qqqq\bar{q}\rangle + \cdots$
-$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + \cdots$
-$|\text{Exotic hadrons}\rangle = |q\bar{q}q\bar{q}\rangle + \cdots$

Future Plans

- Meson: heavy quarkonium ➡ heavy-light meson ➡ strange meson
- Baryon: nucleon ➡ excited nucleon ➡ baryons with s, b, c quarks…
- Expansion in Fock sectors: $|q\bar{q}q\bar{q}\rangle$ in meson and $|qqqq\bar{q}\rangle$ in baryon
- Evaluation of observables: FF, PDF, DY cross section, PDA, GPD, TMD, GTMD…
- Tremendous amount of possibilities…

Thank you !

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